STABLE ISOTOPES + RARE GASES ①

Oxygen Isotopes

- Oxygen has 3 isotopes, 16, 17 + 18.
  - 17 is very minor, so most terrestrial work utilizes $^{18}O/^{16}O$ ratios ($16/18 \sim 500$)

- For mantle studies, the ~~util~~ utility of oxygen derives from the fact that the equilibrium fractionation of oxygen is strongly temperature dependent, largest at low T, almost vanishing at high T.

- For example, at 1200°C $\left[\begin{array}{l} \Delta \text{melt–olivine} \sim +0.36\%_{\circ} \\ \Delta \text{plag–olivine} \sim +0.7\%_{\circ} \end{array} \right]$

- whereas at earth surface temperatures ($\sim 20°C$)
  $$\left[\begin{array}{l} \Delta\ CaCO_3\text{–water} \sim +30\%_{\circ} \\ \Delta\ \text{Silica–water} \sim +35\%_{\circ} \\ \Delta\ \text{clay minerals–water} \sim +25\%_{\circ} \end{array} \right.$$

- Therefore, oxygen isotopes are a good marker for materials that have been "processed" through subaerial conditions.

Mantle oxygen

- olivines from a large variety of mantle peridotites (>100 samples, spinel + garnet facies, hydrous) and anhydrous

  average @ $\boxed{5.18\%_{\circ}}$ $\pm 0.1\%_{\circ}$

- average altered upper oceanic crust = $+9\%_{\circ}$ (518A)

- average oceanic pelagic sediment $\sim +15\%_{\circ}$

- average deep oceanic crust (gabbros) $\sim 4.4\%_{\circ}$ (735B)
  - note that high temperature exchange with seawater ($\delta^{18} \equiv 0$ by definition) can lead to low $\delta^{18}O$ values.

- This shows how minerals being equilibrated with seawater start at high $\Delta^{18}O$, and approach $\delta^{18}=0$ at temperatures of 300–800°C.

- They "cross" the $\delta^{18}O$ of "mantle" at 150–350°C, so ~~that~~ could have highly exchanged rocks that still look like pristine mantle!

- Note many Icelandic basalt melts have $\delta^{18}O$ down to +2‰ — way below "primary" mantle values

- Current thinking for Iceland low $\delta^{18}O$ basalts:
    - interaction of seawater and older deep high T basaltic substrate (lower crust?), to give low $\delta^{18}O$
    - assimilation or digestion of these materials by magma chamber processes, generating low $\delta^{18}O$ melts.
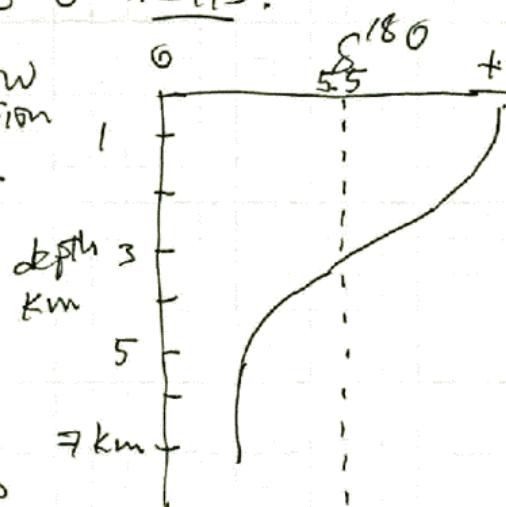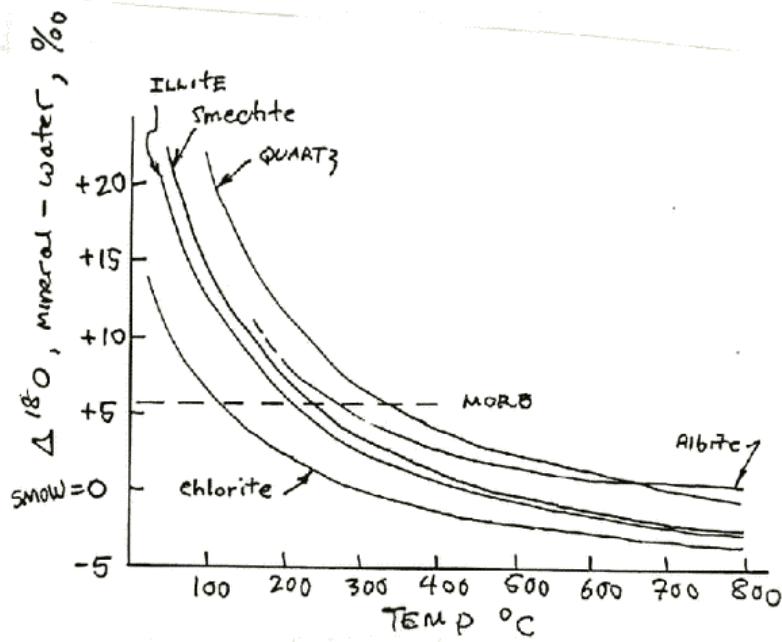
    - ophiolites show both low and high-T interaction with seawater ⟶

- oceanic basalts (olivine "reference")

$$\left[\begin{array}{lll} 127 & MORB & +5.3‰ \\ 156 & OIB's & +5.1‰ \end{array} \right]$$

- Most enriched EM2 basalt (highest 87/86 Sr) = +5.49‰
    - so this is slightly higher than "primary mantle" value of +5.2‰ — would "allow" ≤ 2% sediment)

(right margin, red)

— plus in the future most of the periodic Table as a result of ICPMMS technology!

⊛ A ton of other elements that show isotopic fractionation that is useful in mantle studies (Li, B, C, S, Fe) —

STABLE ISOTOPES and Rare GASES

(1) { not important { for deep earth }

HELIUM   has 2 isotopes { $^3He \rightarrow$ primordial + cosmogenic }
                           { $^4He \rightarrow$ primordial + radiogenic }
                           (U+Th decay))

- Measurements normally referenced to Helium in atmosphere

$$\{ \; ^4/_3 He = 719,400 \; \}$$
$$\{ \; ^3/_4 He = 1.39 \times 10^{-6} = \} R_a$$

So a sample with $^3/_4 He = 10 R_a = 1.39 \times 10^{-5}$, etc.

- As noted earlier, change of $^4/_3 He$ with time depends on $\mu = \left(^{238}U/_{3He} \right) + \left(^{235}U/_{3He} \right) + \left(\frac{^{232}Th}{_{3He}} \right)$

(where Th contribution for normal)
(Th/U ratios is ~ 50%)

- TERRESTRIAL RESERVOIRS
  - Atmosphere = 1 R_a
  - Continental Crust << 1 R_a (due to high abundance of)
                                 (U+Th relative to ^3He)
  - DMM ~ 8 R_a
  - OIB reservoirs ~ 3 - 43 R_a
  - BULK Earth - not known, but assumed to be higher than highest measured mantle sample (i.e. > 43 R_a)
  - CORE - unknown, but assumed by some to be a repository of lots of primordial ^3He (+ little U+Th)

COSMIC RESERVOIRS

| Solar Photosphere (= solar nebula) | = 318 R_A |
| Jupiter | = 120 R_A |
| Meteorites, trapped component | = 90 R_A |

- Usual assumption, earth started with Solar He.

②

Upper mantle (DMM)

MOST N-MORB = 8 ± 1 Ra
(and higher + lower)
(adjacent to plumes)

MORB

$^3He/^4He$ (R_A)
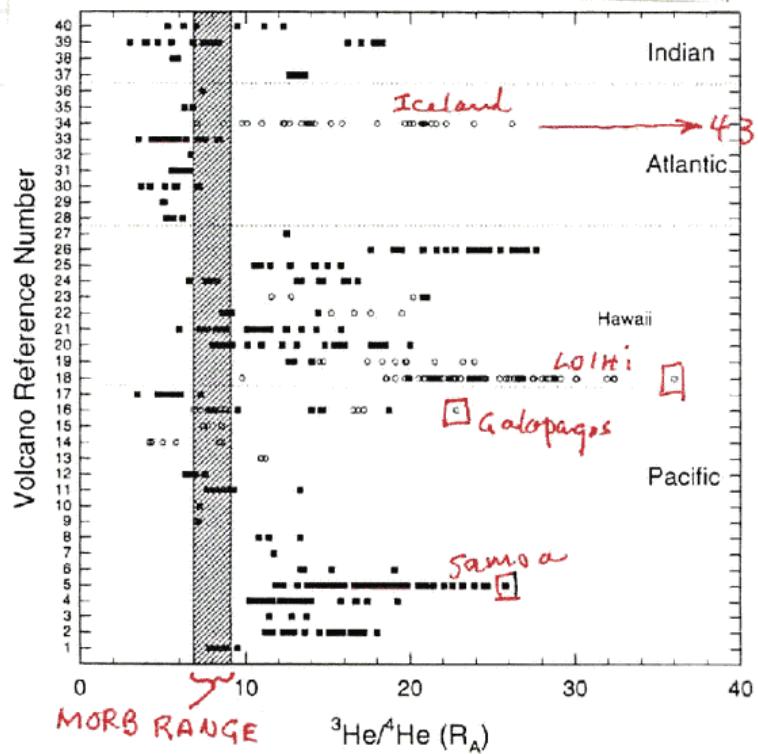
← OIBS
Highest 3/4He

| LOIHi | — | 36 Ra |
Iceland	—	43 Ra
Galapagos	—	26 Ra
Samoa	—	24 Ra

$^3He/^4He$ (R_A)

Log [^3He] cc STP/g   Number

OIBS →

LOWEST 3/4He
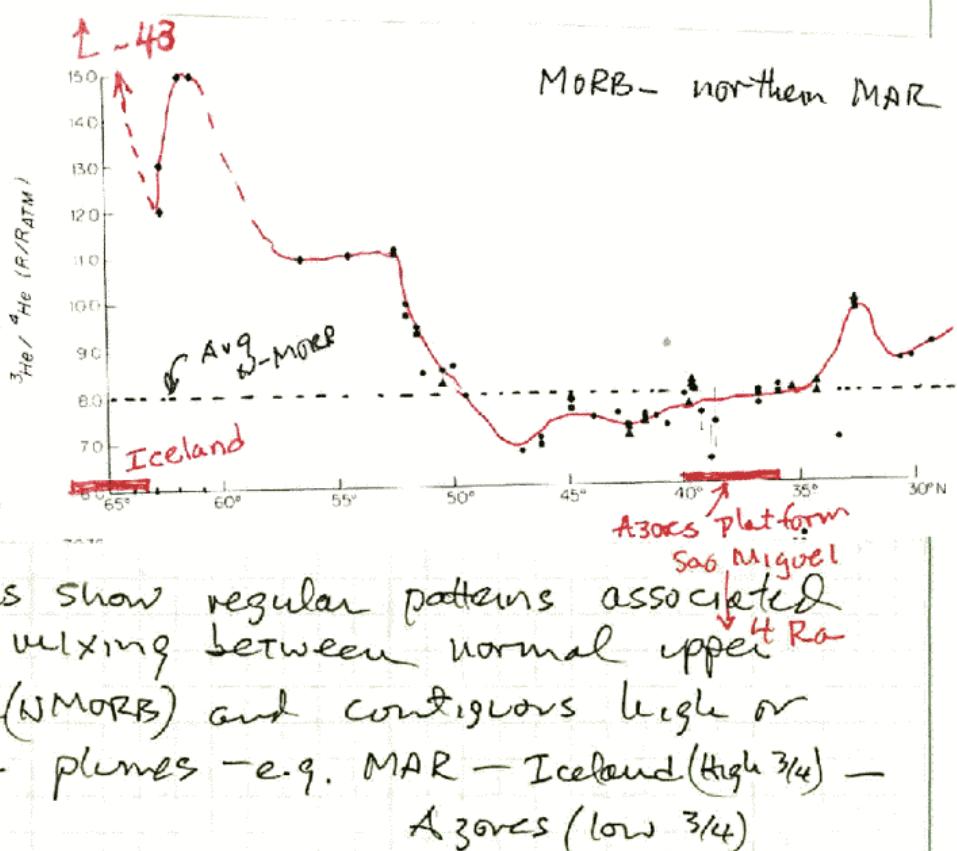
Sao Miguel, Azores — 4 Ra
Heard Island — 3 Ra
St. Helena — 4 Ra
Guadalupe — 4 Ra

(but some of these
may represent
atmospheric He
contamination
during magma
chamber AFC
processes)

Volcano Reference Number

MORB RANGE

$^3He/^4He$ (R_A)

③

↑ ~43

MORB— northern MAR

$^3He/\,^4He$ (R/R$_{ATM}$)

Avg N-MORB

Iceland

A30cs Platform
Sao Miguel
↓ 4 Ra

- Many places show regular patterns associated with mixing between normal upper mantle(NMORB) and contiguous high or low He plumes —e.g. MAR — Iceland (High 3/4) — Azores (low 3/4)
- Iceland-MAR transect is remarkable, with high 3/4 He showing up some 1700km south of Iceland

He Evolution Models
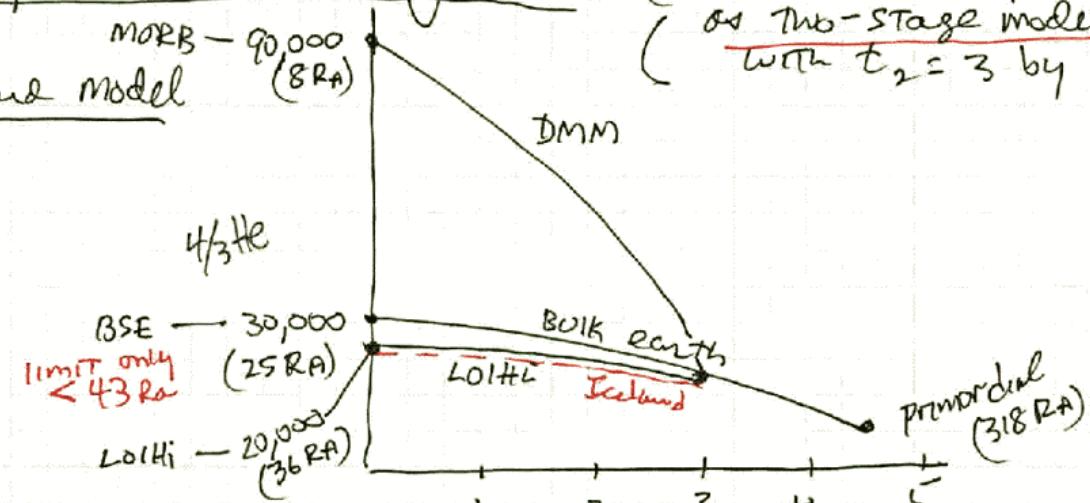
- high 3/4 He generally regarded as evidence for the "least-degassed", Therefore "most primordial" mantle.
- The thinking here is that mantle that melts under a ridge (or anywhere) will partition He into the melt preferentially to U+Th, & He will subsequently be lost to atmosphere during basalt eruption (degassing).
- No question that ridge melting does "inject" lots of 3He into oceans and then → atmosphere (3/4 He "signal" from EPR can be seen in water column as far west as Samoa— 7000km
- IS NOT OBVIOUS that He preferentially partitions into the melt (leaving high $\frac{U+Th}{He}$ residue → t → low 3/4He

(4)
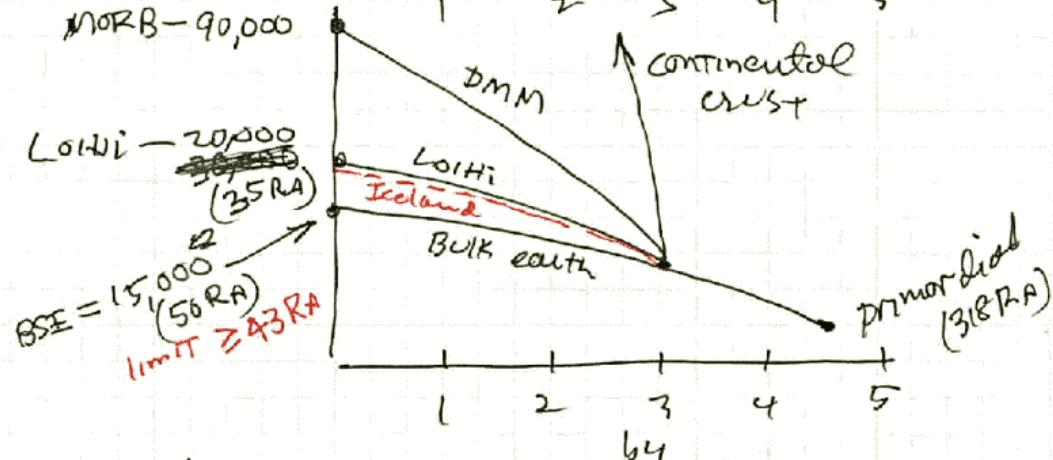
Two possible evolutionary models (shown for simplicity
as two-stage models,
with $t_2 = 3$ by)

NON-Standard Model


MORB — 90,000
(8 RA)

DMM

$4/3$He

BSE — 30,000
(25 RA)
limit only
< 43 Ra

BULK earth

LOIHI   Iceland

LoiHi — 20,000
(36 RA)

primordial
(318 RA)

1   2   3   4   5

STANDARD
MODEL

MORB — 90,000

DMM

continental
crust

LOIWI — 20,000
(35 RA)

LoiHi
Iceland

BULK earth

BSE = 15,000
(50 RA)
limit ≥ 43 RA

primordial
(318 RA)

1   2   3   4   5
by

• Standard model — He more incompatible than U+Th,
melting leaves residue with higher $\frac{U+Th}{3\,He}$
than Bulk earth, grows to $3/4$ He < Bulk earth
By this model, BULK earth ≥ highest $3/4$ He measured

• Non-Standard model — He is ~~less~~ more compatible
than U+Th, melting residue will
have lower $\frac{U+Th}{3\,He}$ than bulk earth,
therefore Bulk earth $3/4$ He < highest measured
or B

Who cares? — The non-standard model challenges
the basic assumption that the highest
$3/4$ He mantle is the least processed, or
most primordial
— Bulk earth $3/4$ He in the non-standard model
then is virtually unconstrained (< 43 Ra only)
so could be 5 RA, 8 RA (=MORB), 36 Ra (LoiHi), whatever!

(5)

He diffusion in the mantle

- Many people assume He diffuses so rapidly in the mantle that it will be "decoupled" from other isotopes, and dispersed from its parent reservoir.

- ocean crust will be degassed in ^{3}He, but enriched in U+Th (relative to DMM source)

- over time, this high $\frac{U+Th}{^3He}$ will lead to very low 3/4 He

- re-injection of crust into mantle, plus aging, will provide a source of low 3/4 He —

- will this "infect" a large volume of mantle surrounding re-cycled ocean crust?

relevant diffusion coefficients

	1756°C	2000°C
1 atm { He in Cpx	3.5×10^{-6}	2.4×10^{-5} cm²/sec
He in olivine	2.4×10^{-6}	3.2×10^{-5}
Ca in olivine	1.8×10^{-9}	2.8×10^{-8}
Fe⇄Mg perovskite	3.7×10^{-16}	$\sim 10^{-15}$
(24 GPa)		

- For diffusion of a species out of a slab (of small thickness) ~~of thickness t~~, for $\frac{C_x}{C_{center}} = 0.38$,

  Rule-of-thumb is: $L = 4\sqrt{Dt}$

  i.e. L is "peak width" at $C_x/C_c = 0.38$

⑥
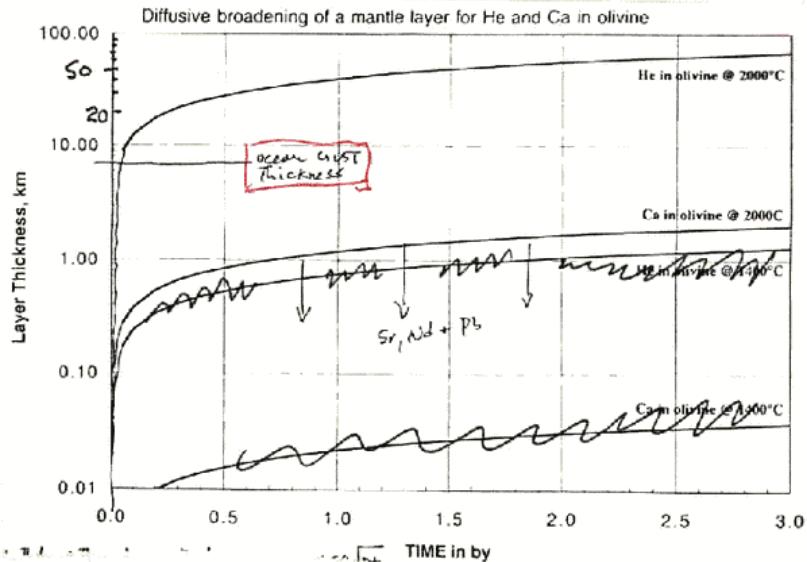
— dispersal of He + Ca in olivine layer at 2000°C

Diffusive broadening of a mantle layer for He and Ca in olivine

— He produces a 70km thick layer after 2 b.y.

— Ca only disperses < 2 km

— So, for He, a 7 km ocean crust will "infect" a volume 10× its size with He.

NEON
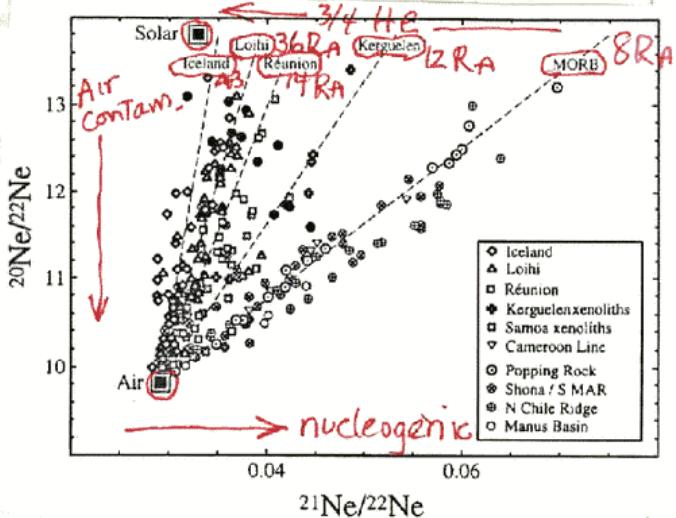
3 ISOTOPES
[
20 — primordial
21 — primordial plus nucleogenic
22 — primordial
]

• ^{21}Ne formation by α reactions from U, Th, on ^{18}O : $^{18}O(\alpha,n)^{21}Ne$

• Atmospheric Ne is different from Solar (nebula) Ne either due to mass fractionation (escape of light Ne), or due to addition of "late-veneer" meteoritic Ne

• High $^{21}/^{22}Ne$ reflect nucleogenic ^{21}Ne additions due to reservoir of high $\frac{U+Th}{^{22}Ne}$


• all data converge of atmospheric Ne, due to atmospheric contamination in magma chambers or during recycling of surficial "stuff" into mantle.

• Each OIB domain generates its own mixing array, with the highest 3/4 samples lowest $(\frac{U+Th}{^3He})$ having the lowest $^{21}/^{22}Ne$ (low $\frac{U+Th}{^{22}Ne}$). Consistent with degassing of $\underline{He}$ and $\underline{Ne}$ from DMM

⑦

<u>XENON</u> 9 isotopes, 6 major ones, all are basically primordial
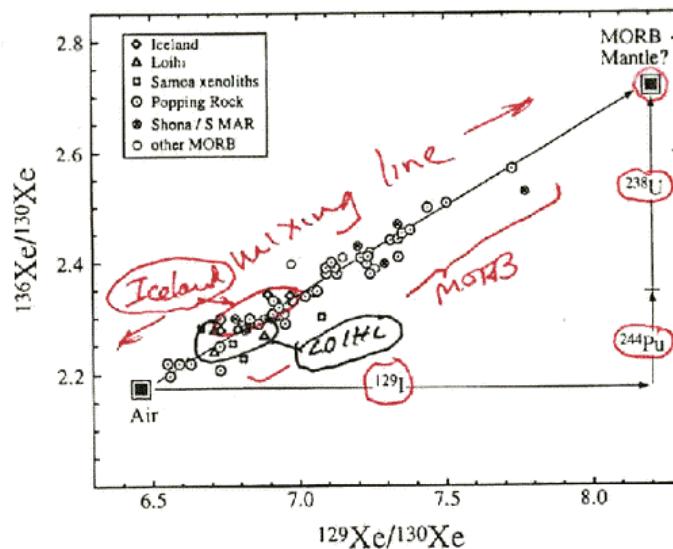
with •^{129}Xe having a contribution from
^{129}I extinct radioactivity ($t_{1/2} = 17$ my)

• ^{136}Xe has contributions from both
extinct ^{244}plutonium ($t_{1/2} = 80$ my)

<u>and</u> fission of ^{238}U

• as with Ne, most
mantle seems to be
a mixture of MORB
(+OIB?) mantle and
atmospheric Xe.

• The high ^{129}Xe
signature reflects
very early retention
of the decay of ^{129}I.

• Lack of mixing
between MORB
mantle and atmosphere
for ~4.4 by.

①

RADIOGENIC ISOTOPE SYSTEMS

General equation:

$${}^{87}Sr_t = {}^{87}Sr_0 + {}^{87}Rb \left[e^{\lambda T} - e^{\lambda t} \right)$$
— divide by reference ${}^{86}Sr$

$$R_t = R_0^T + \mu \left[e^{\lambda T} - e^{\lambda t} \right] \hookleftarrow$$

$$\begin{cases} R_0 = \text{initial isotope ratio at time } T \\ R_t = \text{isotope ratio at time } t \\ \mu = \text{parent/daughter ratio today} \end{cases}$$

For $t = 0$ (today)

$$R_{now} = R_0 + \mu \left[e^{\lambda T} - 1 \right]$$

P	D	Reference Isotope	λ	R	$R_{chondrite}$	μ
238 U → 206 Pb		204 Pb	1.5512E-10	206/204 Pb	9.307	238U/204Pb
235 U → 207 Pb		204 Pb	9.844E-10	207/204 Pb	10.294	235U/204Pb
232 Th → 208 Pb		204 Pb	4.948E-11	208/204 Pb	29.476	232Th/204Pb
87 Rb → 87 Sr		86 Sr	1.42E-11	87/86 Sr	0.69880	87Rb/86Sr
147 Sm → 143 Nd		144 Nd	6.54E-12	143/144 Nd	0.506675	147Sm/144Nd
176 Lu → 176 Hf		177 Hf	1.865E-11	176/177 Hf	0.27978	176Lu/177Hf
187 Re → 187 Os		188 Os	1.67E-11	187/188 Os	0.09531	187Re/188Os

②

U-Th-Pb-He system

Note the powerful coupling in this system.

- $^{238}U/^{235}U$ is constant in the solar system, today $\equiv 137.88$

- ^{238}U, ^{235}U + ^{232}Th all have a Pb isotope daughter

- along with the Pb daughter, all three produce ^4He daughters as well (from the α decay)

$$
\begin{cases}
^{238}U & \text{produces } 8 \text{ } \alpha\text{'s per decay} \quad \left(\frac{238-206}{4}=8\right) \\
^{235}U & \text{produces } 7 \text{ } \alpha\text{'s } \text{``} \quad \text{``} \\
^{232}Th & \text{``} \qquad 6 \text{ } \alpha\text{'s } \text{``} \quad \text{``}
\end{cases}
$$

- So we also have a $(U+Th/He)$ "system"

$$
\left(\frac{4}{3}He\right)_t = \left(\frac{4}{3}He\right)_0 + \frac{^{238}U}{^3He}\left[8\left(e^{\lambda_{238}T}-e^{\lambda_{238}t}\right) + \frac{7}{137.88}\left(e^{\lambda_{235}T}-e^{\lambda_{235}t}\right) \right.
$$
$$
\left. + 6\frac{^{232}Th}{^{238}U}\left(e^{\lambda_{232}T}-e^{\lambda_{232}t}\right)\right]
$$

- This $\left(\frac{U+Th}{He}\right)$ system will become important later in the lectures when we talk about the ^3He "tracer" system. Note that if we thought about He as we do with all the other radiogenic systems, we would always talk about $^4He/^3He$ (i.e. radiogenic daughter over primordial "reference" isotope)

- To all of your infinite confusion, however, we always "talk" about $^3He/^4He$!! (Because high $^3/4He$ is equated to "more primordial". This is like saying high $\frac{^{86}Sr}{^{87}Sr}$ is the "most primordial" or "least processed".

- we'll see how why both statements (for both He + Sr) are wrong.

MANTLE ISOTOPE EVOLUTION MODELS

③
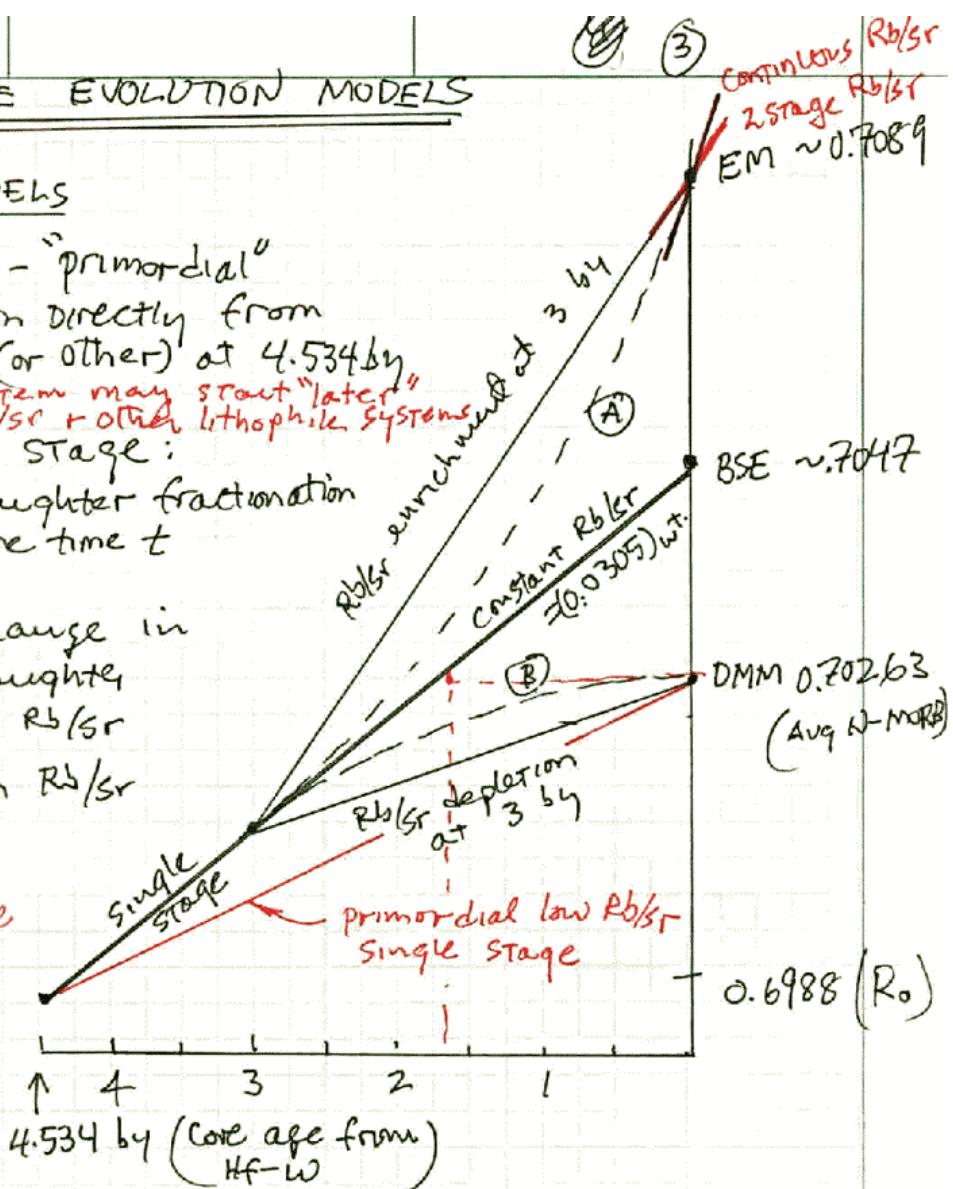
CLASSES OF MODELS

1) Single Stage - "primordial"
   Evolution directly from
   BSE (or other) at 4.534 by,
   (note - U/Pb system may start "later"
   than Rb/Sr + other lithophile systems)

2) TWO (multi) stage:
   • parent/daughter fractionation
     at some time t

3) Continuous change in
   parent/daughter
   Ⓐ increase in Rb/Sr
   Ⓑ decrease in Rb/Sr

* evolution lines are
  nearly linear for
  Rb/Sr ($t_{1/2}$ = 50 by)

Left margin (red):
* DIFFERENT "STARTS" for different "Clocks"
  U/Pb clock
• Pb into core starts before earth
• Rb clock may start before earth condensation of Rb starts
  is finished accreting.
• Pb into core starts U/Pb clock

AMPAD
22-141 50 SHEETS
22-142 100 SHEETS
22-144 200 SHEETS

Diagram labels:
- CONTINUOUS Rb/Sr
- 2 stage Rb/Sr
- EM ~0.7089
- Rb/Sr enrichment at 3 by
- Ⓐ
- constant Rb/Sr
- (0.0305) wt.
- BSE ~.7047
- Ⓑ
- DMM 0.70263 (Avg N-MORB)
- single stage
- Rb/Sr depletion at 3 by
- primordial low Rb/Sr single stage
- 0.6988 (R₀)

X-axis: 4 3 2 1
4.534 by (Core age from Hf-W)

• Note that all these models can "get to" present
  day 87/86 Sr as desired. What is different
  is the Rb/Sr ratio today for the mantle source
  — slope of evolution line today is the present
    Rb/Sr of each source
  — for DMM, Rb/Sr: Single stage > Two stage > continuous

• Note that once BSE 87/86 is >0.7026 (DMM today),
  no model will get to DMM from BSE
  — This age limit is 1.64 by (i.e. point at which
    BSE 87/86 = 0.70263)

• a "continuous" model seems reasonable for
  upper (DMM) mantle, which we think has
  been continuously depleted over time by
  removal of continental crust (i.e. crust has
  been formed at all ages since at least 4.40 by)

④

- we have given single stage evolution above
- Two Stage Evolution

$$R_2 = R_0 + \mu_1 [e^{\lambda T} - e^{\lambda t}] + \mu_2 [1 - e^{\lambda t}]$$

$$\begin{bmatrix} \mu_1 = \text{stage 1 parent/daughter} \\ \mu_2 = \text{stage 2 } \quad " \quad " \\ R_0 = \text{initial ratio} \\ R_2 = \text{ratio today after stage 2} \end{bmatrix}$$

- Continuous Evolution (for Rb/Sr for example)

    - First order transport process where parent + daughter are removed (added) at a rate proportional to their instantaneous concentration

    i.e. $$\frac{d(^{86}Sr)}{dt} = -\alpha \, Sr$$

    $$\frac{d(^{87}Rb)}{dt} = -\lambda \,^{87}Rb - \gamma \,^{87}Rb$$

    - Combined transport coefficient $K = (\gamma - \alpha)$

 $$R_t = R_0 + \frac{\mu_0 \lambda \, [1 - \exp[-(\lambda + K)t]]}{\lambda + K}$$
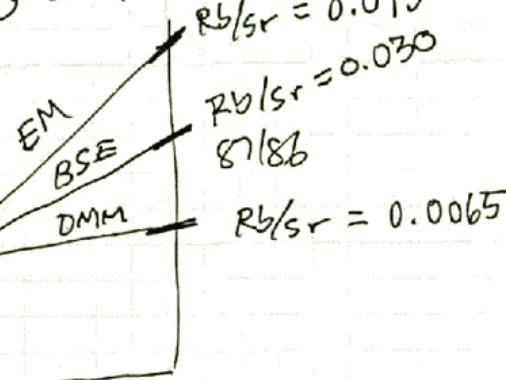
- To link back to our earlier He discussion (3/4 He versus 4/3 He), saying high 3/4 He is "primordial" is like saying high $^{86}/^{87}$ Sr is "primordial" — i.e. MORB has the highest $^{86}/^{87}$ Sr of present day mantle — but is clearly not the most "primordial" (or "unprocessed")

Isotope constraints on mantle source
Spidergrams
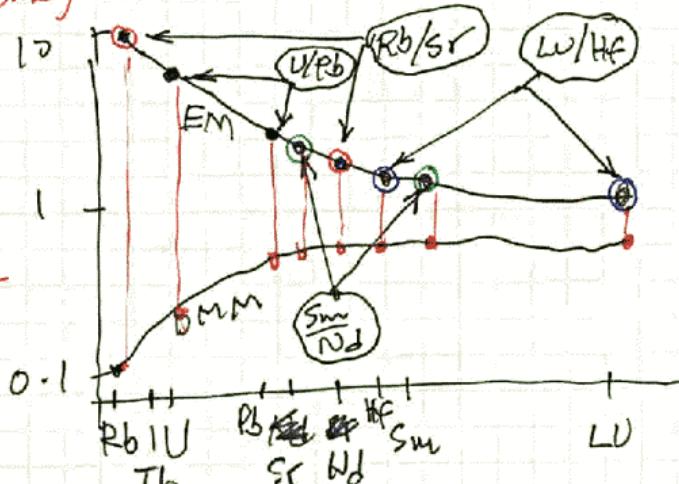
• Note from the evolution figure (p.3) that present day isotope ratios in various mantle domains provides "model" constraints on source parent/daughter ratios"

— These models involve assumption re "age" of fractionation and 1, 2 or continuous stages, but are STILL very strong constraints (especially in a relative sense, since same model "age" can be used for all systems)

Rb/sr = 0.073

Rb/sr = 0.030

$^{87}/^{86}$

Rb/sr = 0.0065

EM

BSE

DMM

• Do this → C | JE

✓ JUST for DMM source (EM source as shown is too enriched)

U/Pb   Rb/sr   Lu/Hf

EM

DMM

Sm/Nd

1.0

1

0.1

Rb|U   Rb   Hf
Th   Sr Nd   Sm   Lu

Δ Pb to explain full oceanic mantle range

Nd 0.51233 → 0.51325 ΔSm/Nd = 24%

$^{206}/^{204}$ Pb 17.5 → 21.9 ΔU/Pb = 90%

• INFERRED SOURCE versus DERIVED Melts

— as discussed already, the source composition and derived melt compositions should be directly "relatable" through partition coefficients and melting models.

remember, for batch modal melting:

$$C_{melt} = \frac{C_0^{solid}}{D + F(1-D)}$$

unfortunately, approaching the problem from both ends doesn't always "meet in the middle"

(6)

For example —

many shield basalts in Samoa have:

$\begin{cases} ^{87/86}Sr = 0.7046 \quad (\approx \text{bulk earth}) (= 0.7047) \\ ^{143/144}Nd = 0.51280 \quad (> \text{bulk earth}) (= 0.51264) \\ ^{176/177}Hf = 0.28300 \quad (> \text{bulk earth}) (= 0.28284) \end{cases}$

infers source Lu/Hf > BULK earth

whereas measured "melt" parent-daughter
ratios in these basalts are:

$\begin{cases} Rb/Sr = 0.041 \quad (> BSE = 0.030) & 1.37 \\ Sm/Nd = 0.247 \quad (< BSE = 0.325) & 1.32 \\ Lu/Hf = 0.050 \quad (\ll BSE = 0.239) & 4.8 \end{cases}$

factor

Summary (more incompatible / less incompatible)

	from isotopes	observed melts			factor
Rb/Sr	~ 0.030	0.041	>80 ?		~ 1.37
Nd/Sm	< 3.08	4.05	0.64	0.26	> 1.31
Hf/Lu	< 4.18	20	0.59	.06	> 4.8

Cpx Garnet

Kd's

• while partitioning involving
Cpx and garnet are in the
right "direction" to create the
necessary fractionations between an isotope-derived
source composition and the observed melts,

the actual quantitative modelling necessary to
really fit the data only work if the
melt fractions are very small, and in the
case of Hf/Lu are difficult to accomplish even
with very small F, and lots of garnet.

• This procedure works fine for DMM ⇌ NMORB but
fails for many OIB. Points to the need for a
auto-enrichment process (autometasomatism) in most OIB melting regimes.

* USing mantle Melts To infer Source Composition

MELTING + FRACTIONATION

Albarede, Intro. to Geochemical Modelling, Cambridge Press 1995

* F = fraction of melt

* D = Bulk partition coefficient = $x^\alpha D^\alpha + x^\beta D^\beta + x^\gamma D^\gamma + \ldots$

(x^α etc = mass fraction of phase α in source)

P = Bulk non-modal partition coefficient
$$= Y^\alpha D^\alpha + Y^\beta D^\beta + Y^\gamma D^\gamma + \cdots$$

(Y^α etc = mass fraction of phase α entering melt)

$[$ as $F \to O$, $P \to D$ $]$

BATCH MODAL MELTING

A   * $$C_\ell = \frac{C_s^o}{D + F(1-D)}$$

• Batch melt

as $F \to 0$, $C_\ell / C_s^o \to \frac{1}{D}$

B   * $$C_s = \frac{D C_s^o}{D + F(1-D)}$$

• residual solid

BATCH NON-MODAL MELTING

$$C_\ell = \frac{C_s^o}{D + F(1-P)}$$

• Batch melt

$$C_s = \frac{C_s^o}{(1-F)}\left[\frac{D - FP}{D + F(1-P)}\right]$$

• residual solid

FRACTIONAL MODAL MELTING

$$C_\ell = \frac{C_s^o}{D}(1-F)^{(\frac{1}{D}-1)}$$

• instantaneous melt

C.   * $$C_\ell = \frac{C_s^o}{F}\left[1 - (1-F)^{1/D}\right]$$

• aggregated melt

as $F \to 0$, $C_\ell / C_s^o \to 1/D$

D.   * $$C_s = C_s^o(1-F)^{(\frac{1}{D}-1)}$$

• RESIDUAL SOLID

* Even if we know D's exactly, and measure C_ℓ's precisely, always a trade-off between $F + C_s^o$

FRACTIONAL NON-MODAL MELTING

$$C_\ell = \frac{C_s^o}{D}\left(1 - \frac{PF}{D}\right)^{(1/P - 1)} \qquad \bullet \text{ instantaneous melt}$$

$$C_\ell = \frac{C_s^o}{F}\left[1 - \left(1 - \frac{PF}{D}\right)^{1/P}\right] \qquad \bullet \text{ aggregated melt}$$

$$\dot{C}_s = \frac{C_s^o}{(1-F)}\left(1 - \frac{PF}{D}\right)^{1/P} \qquad \bullet \text{ residual solid}$$

CRITICAL MELTING

ϕ = porosity, mass fraction.

$$C_{residue} = C_o\left(1 - F\right)^{\left[\frac{(1-\phi)(1-D)}{(1-\phi)D + \phi}\right]}$$

- where $C_{residue}$ = residual solid
 plus trapped melt

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C_L versus F
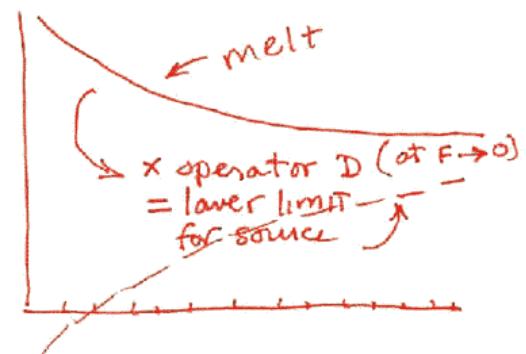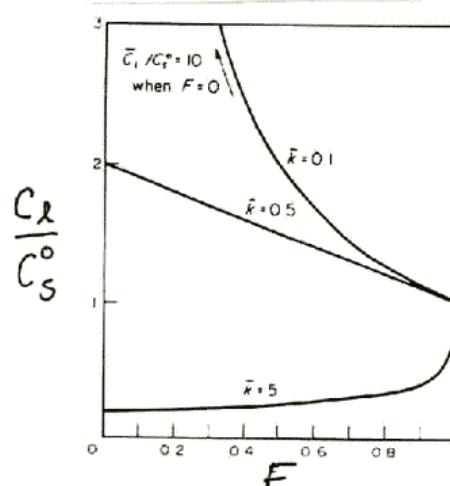
- C_ℓ, aggregated liquid, for modal fractional melting (relative to C_s^o):

as $F \to 0$, $\left(\begin{smallmatrix}\text{but blows up}\\ \text{at } F \equiv 0\end{smallmatrix}\right)$

$$\frac{C_\ell}{C_s^o} \to \frac{1}{D}$$

a, $F \to 1$

$$\frac{C_\ell}{C_s^o} = 1 \quad \left(\begin{smallmatrix}\text{i.e.}\\ C_\ell \to C_s^o\end{smallmatrix}\right)$$

$\bar{C}_\ell / C_s^o = 10$
when $F = 0$

$\bar{k} = 0.1$

$\bar{k} = 0.5$

$\frac{C_\ell}{C_s^o}$

$\bar{k} = 5$

F

← melt

x operator D (at $F \to 0$)
= lower limit
for source

Figure annotations (handwritten):

approx Bulk D for middle of a peridotite spidergram

For D = 0.01

D = 0.002 produces higher Ce's at low F, but approaches curve for D = 0.01 above F = 2%
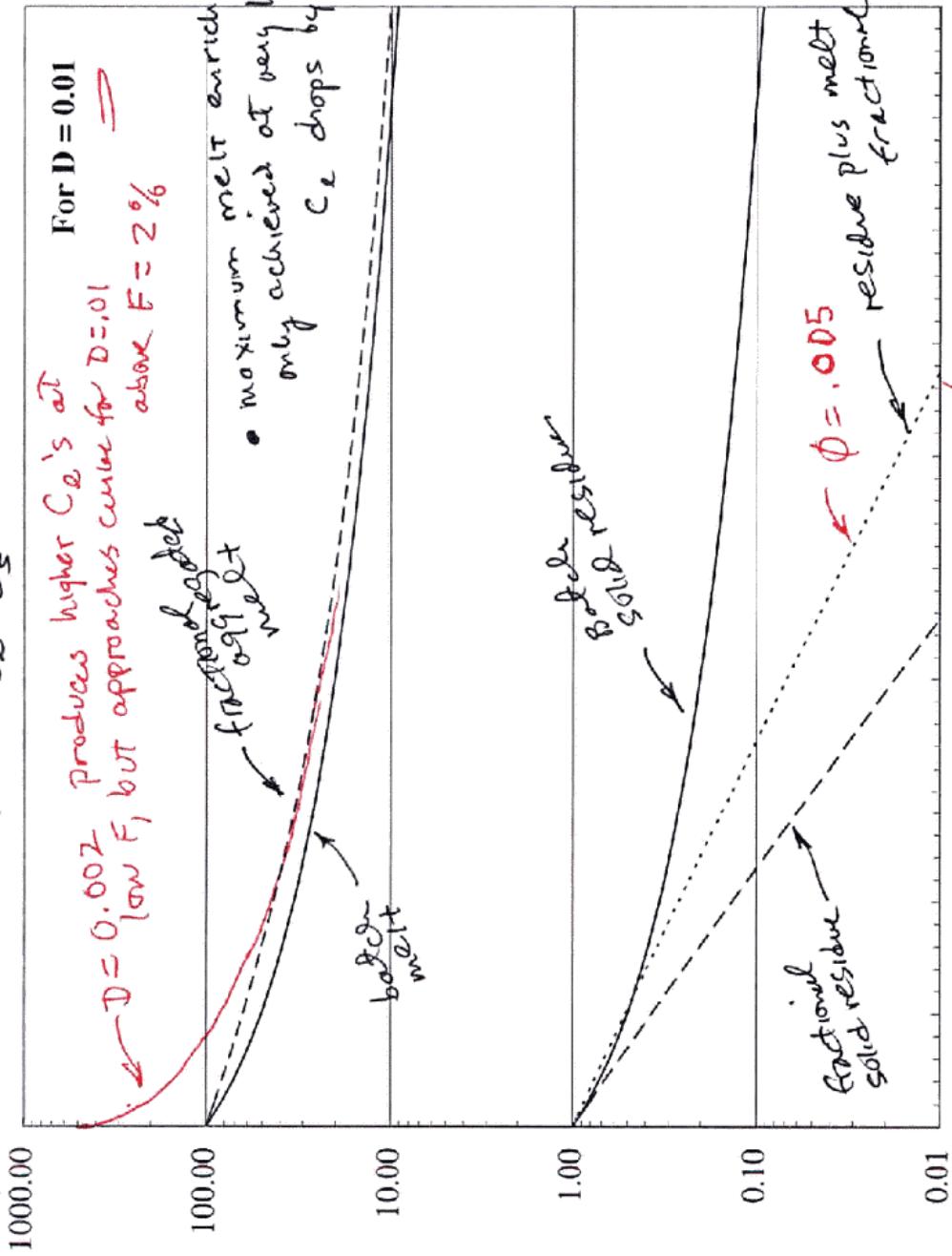
* • Batch + aggregated melts are very similar
ƒ • Both start (F=0) at $Ce/C_0 = \frac{1}{D}$
* • Both end (F=1) at $Ce = C_0 = C_s$

• maximum melt enrichment is only achieved at very low F (for low D's)

Ce drops by 5x (for D=.002) with F ~ 0.8%

fractionated aggregated melt

batch melt

Batch solid residue

φ = .005

residue plus melt in porosity!

residue plus melt fractions

Cresidue = Csolid plus residual melt

Fractional solid residue

• fractional (modal) melting depletes residue very rapidly!
○ critical melting is between batch + fractional

Legend:
— batch Cl
---- fract'l Cl
— batch Cs
— · — fract'l Cs
········· critical Cres

Y-axis: Concentration at F/Initial Concentration (1000.00, 100.00, 10.00, 1.00, 0.10, 0.01)

X-axis: Melt Fraction (0, 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.1)

③

USE of MELTS to DERIVE SOURCE

- For batch modal melting, can derive source characteristics and partition coefficients by "comparing" different elements
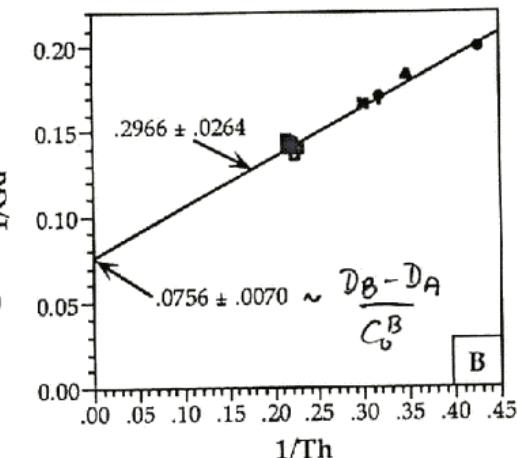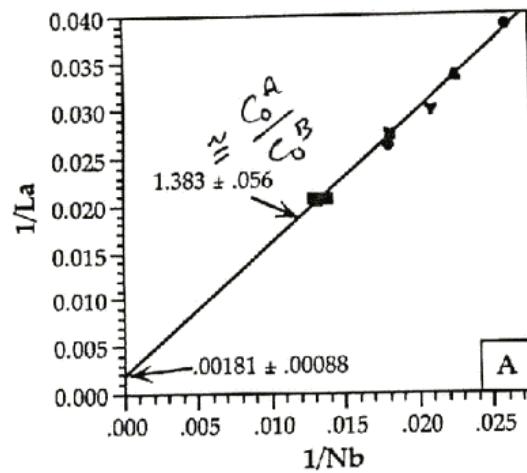
  ※ $C_\ell/C_0 = \dfrac{1}{D + F(1-D)}$

    – do for two elements A + B
    – plot reciprocals $\left(x = \dfrac{1}{A}, \; y = \dfrac{1}{B}\right)$

  ∞ Slope = $\dfrac{C_0^A}{C_0^B} \dfrac{(1 - D_B)}{(1 - D_A)} \sim \dfrac{C_0^A}{C_0^B} \left(\begin{array}{c}\text{for}\\\text{small}\\ D\end{array}\right)$

- Intercept = $\dfrac{D_B - D_A}{C_0^B(1 - D_A)} \sim \dfrac{D_B - D_A}{C_0^B}(\text{"})$

Result for the whole spidergram,
   Hobbs Lineament
   ⟶
   $\left(D_{Dy} \text{ set to } 0.5 \, PUM\right)$
   ⟶ gives $F = 1.6 \sim 3.2\%$

Note typical source feature,
   that: $^{87}/_{86} = 0.7028$, but
 $Rb/Sr \sim PUM$
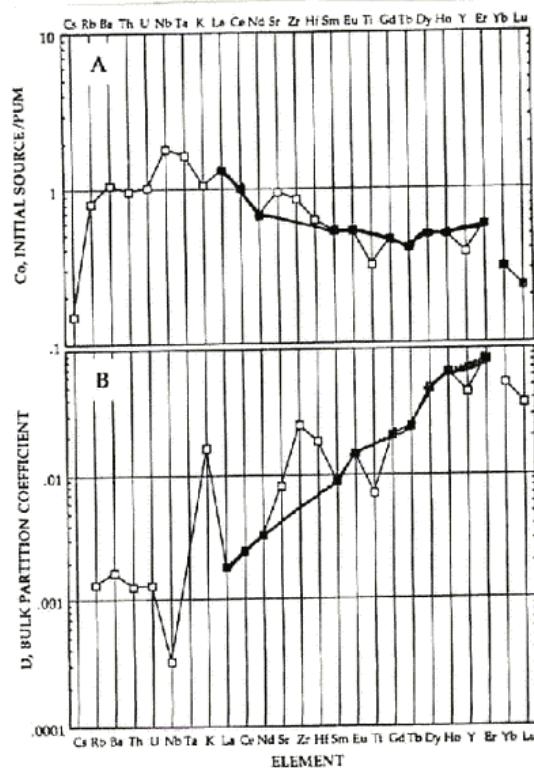
   $^{143}/_{144} = 0.51289$, but
   $Nd/Sm > PUM$
   $(Sm/Nd < PUM)$

   – i.e. the "enriched" source pattern must be __Recent__

Top graph: 1/La (y-axis, 0.000–0.040) vs 1/Nb (x-axis, .000–.025). Slope $\approx \dfrac{C_0^A}{C_0^B}$, $1.383 \pm .056$. Intercept $.00181 \pm .00088$. Label A.

Middle graph: 1/Gd (y-axis, 0.00–0.20) vs 1/Th (x-axis, .00–.45). $.2966 \pm .0264$. $.0756 \pm .0070 \sim \dfrac{D_B - D_A}{C_0^B}$. Label B.

Bottom graphs: A — C_0, INITIAL SOURCE/PUM; B — D, BULK PARTITION COEFFICIENT vs ELEMENT (Cs Rb Ba Th U Nb Ta K La Ce Nd Sr Zr Hf Sm Eu Ti Gd Tb Dy Ho Y Er Yb Lu).

④

USE OF RESIDUES TO DERIVE C_0's
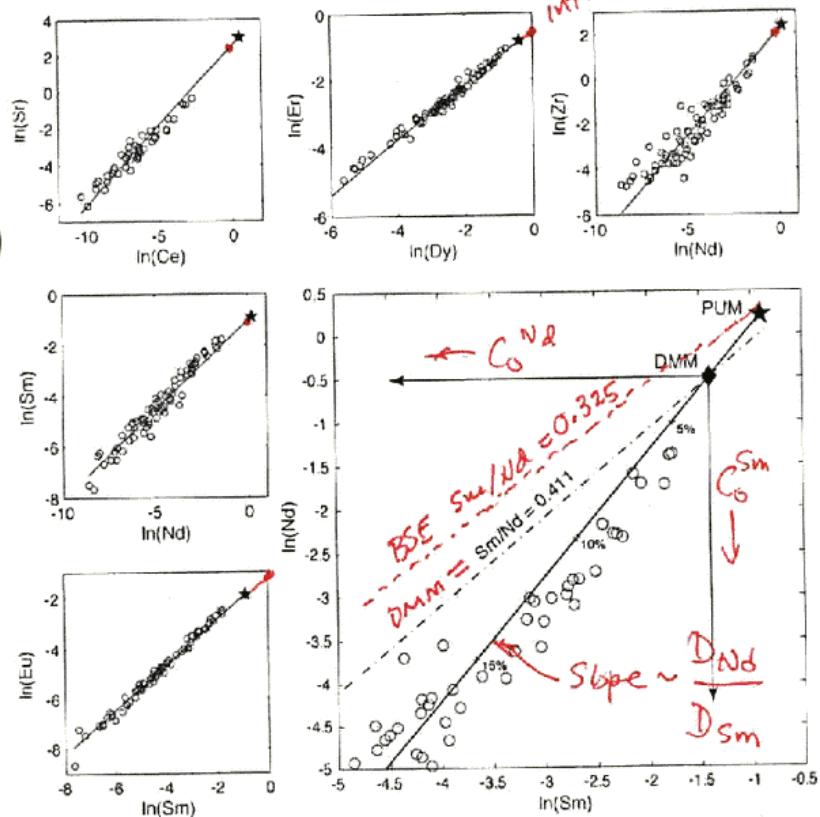
- For fractional modal melting, plot of $\log C_X$ vs $\log C_Y$ will be linear:

by equating eqn D for 2 species (eliminating F), can get ratios of D's + C_s^0's

$\mathrm{intercept} = \dfrac{C_0^{Sm}}{(C_0^{Nd})^R}$

✷ $R = \text{slope} = \dfrac{D_B(1-D_A)}{D_A(1-D_B)}$

$\sim \dfrac{D_B}{D_A}\left(\begin{array}{c}\text{for small}\\ D\end{array}\right)$

✷ $\mathrm{Intercept} = \dfrac{C_0^A}{(C_0^B)^R}$

✷   ~~the~~ WILL give the "shape" of a source spidergram, but NOT absolute C_s^0's .

✷ FIX one C_s^0, can "pin" the spidergram

— Do this for BULK earth, by using Sm/Nd of chondrites (= constant) = 0.325