Geophysical Inverse Problem: an Introduction

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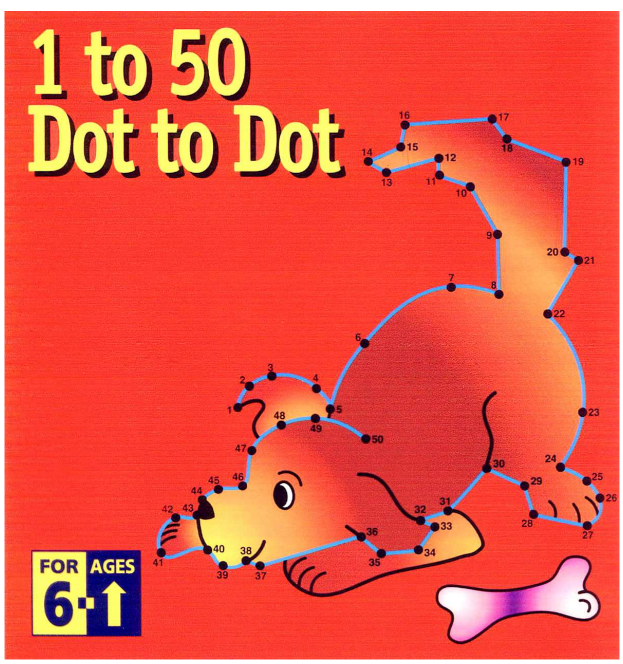
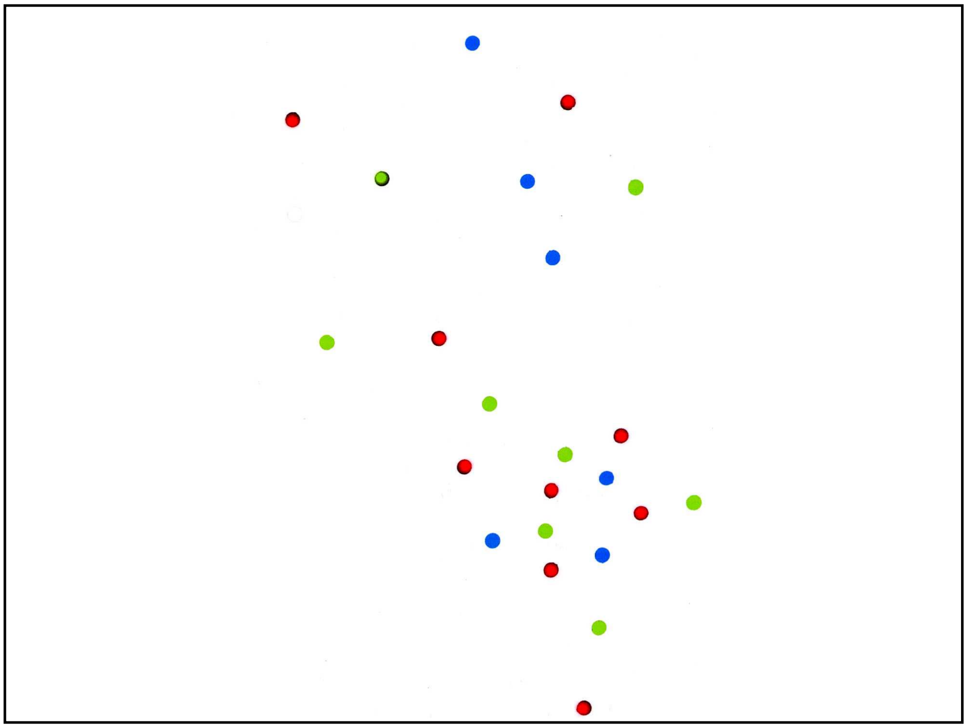
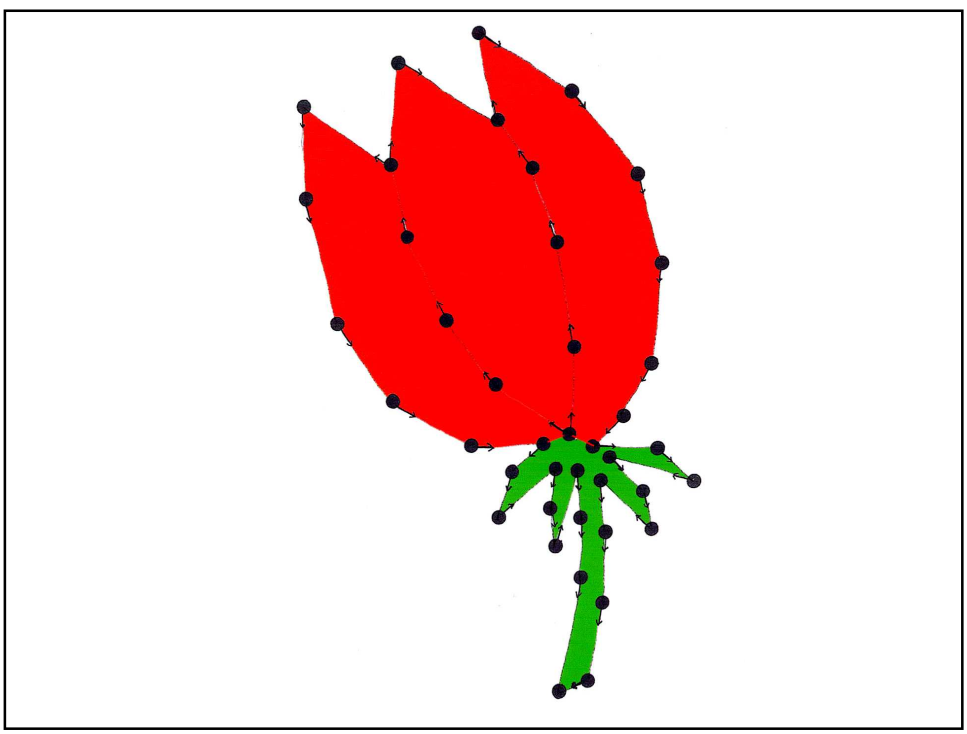
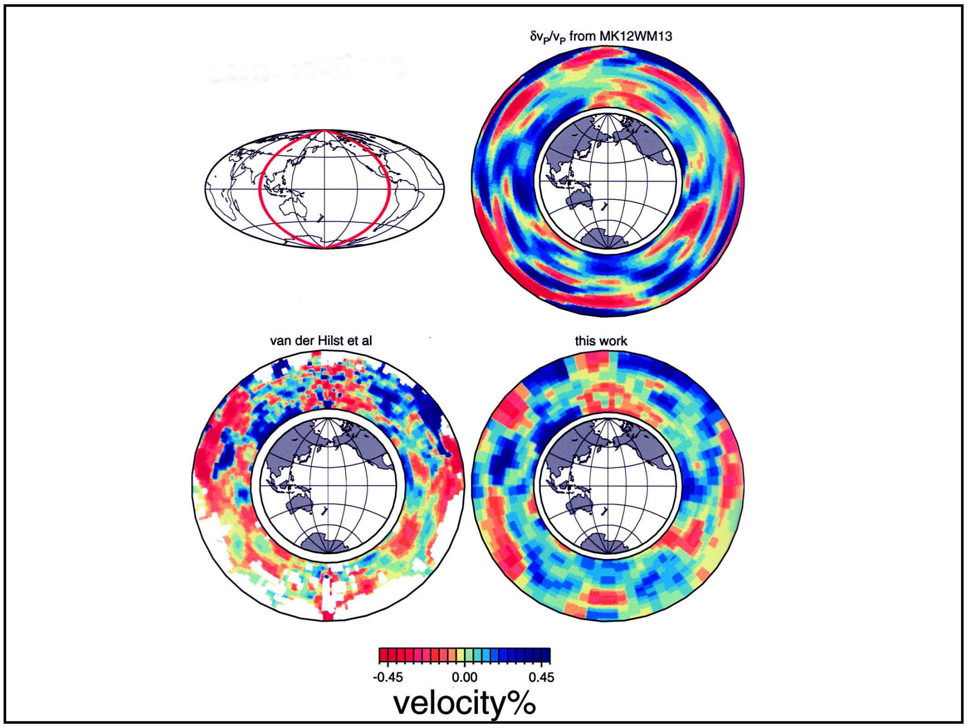
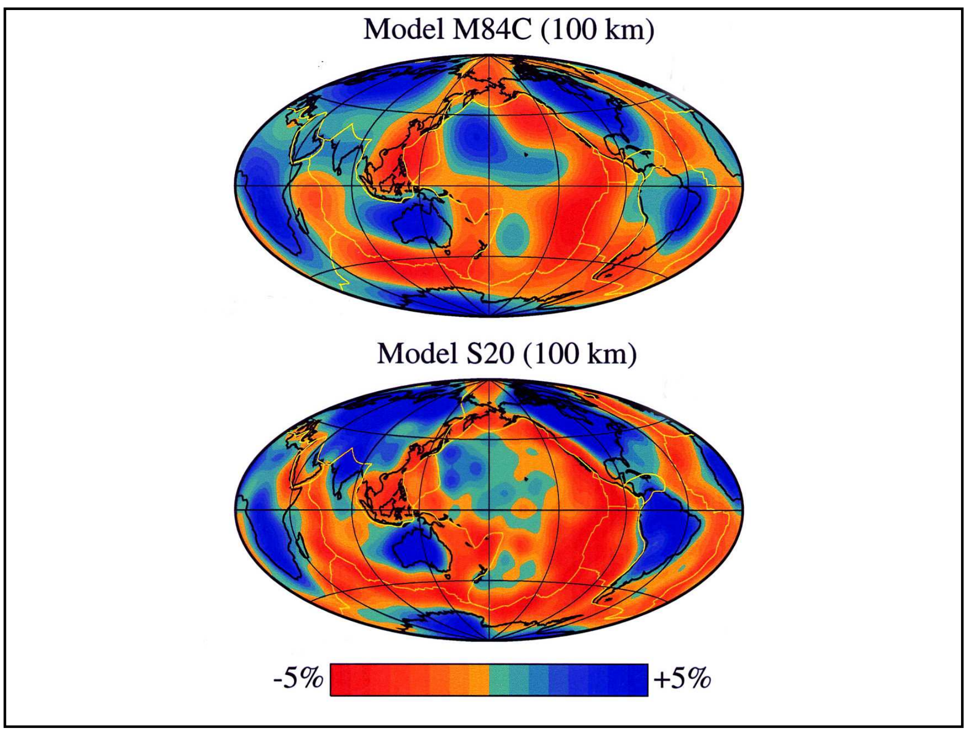
July 12, 2004



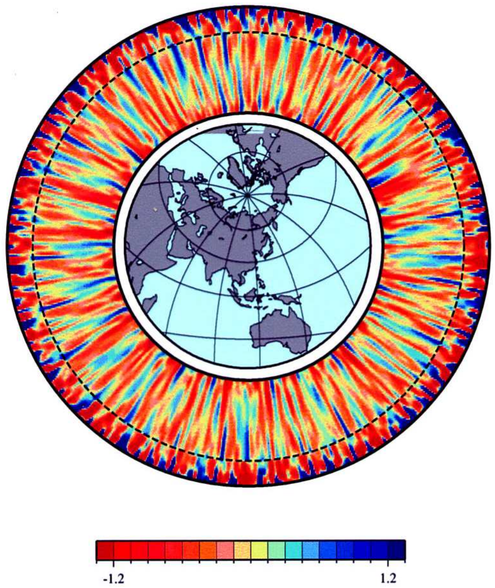





δvP/vP from MK12WM13
van der Hilst et al
this work
-0.45
0.00
0.45
velocity%


Model M84C (100 km)
Model S20 (100 km)
-5%
+5%

High Resolution



Linear Inverse Problem

Solve

$$\mathbf{A} \cdot \mathbf{x} = \mathbf{d};$$

subject to the least squares condition:

$$||\mathbf{A} \cdot \mathbf{x} - \mathbf{d}||_2 = \min$$

and one or more regularization constraints:

$$||\mathbf{x}||_2 = \min$$
$$||\nabla \mathbf{x}||_2 = \min$$
$$||\nabla^2 \mathbf{x}||_2 = \min$$

These can be combined:

$$||\mathbf{A} \cdot \mathbf{x} - \mathbf{d}||_2 + \eta_0^2 ||\mathbf{x}||_2 + \eta_1^2 ||\nabla \mathbf{x}||_2 + \eta_2^2 ||\nabla^2 \mathbf{x}||_2 = \min$$

By selecting the size of the coefficients η_0, η_1 or η_2 the influence of the regularization with respectet to the rms of the model, its integrated gradient or the laplacian can be controlled.

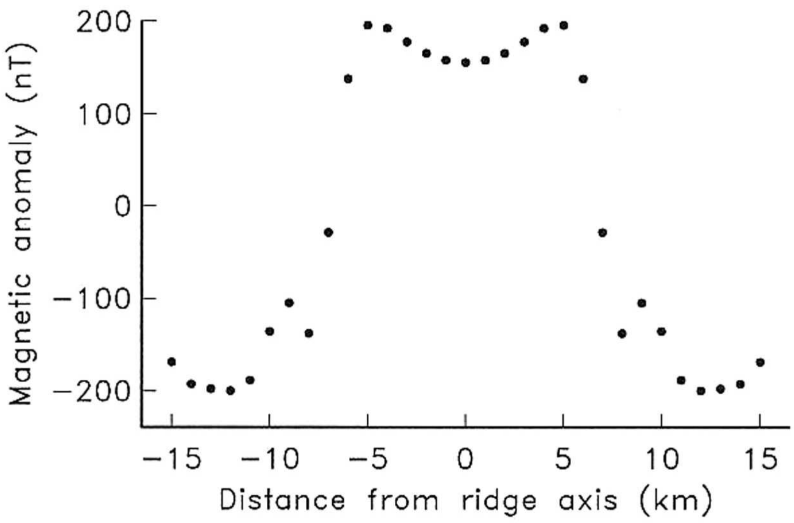


Figure 2.01a: An idealized marine magnetic anomaly at a mid-ocean ridge. The solid dots are the values of the vertical component of the magnetic field.

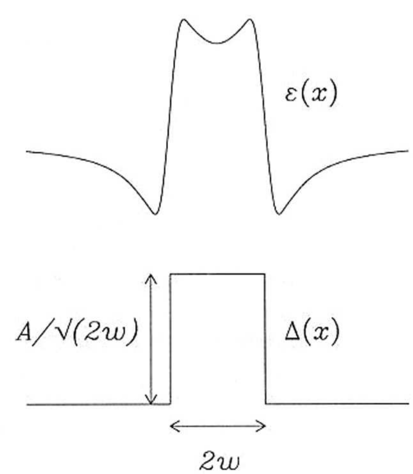


Figure 2.04a: The crustal magnetization $\Delta(x)$, a boxcar function, generates a magnetic anomaly $\varepsilon(x)$ at the ocean surface in the idealized system under consideration.

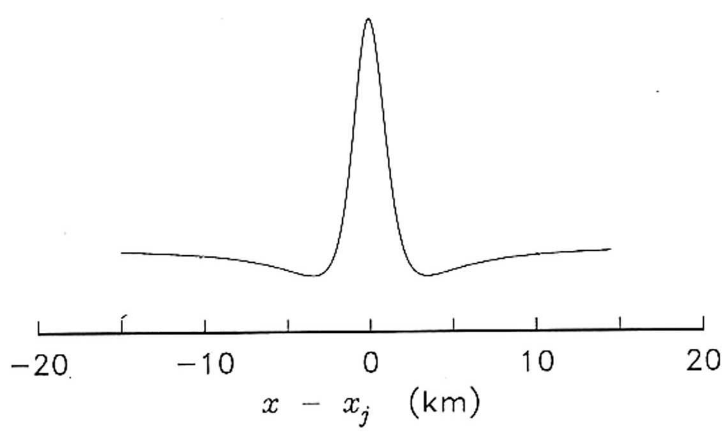


Figure 2.06a: Shown as a solid line, the L_2 representer g_j in equation (1) for the magnetic anomaly data.

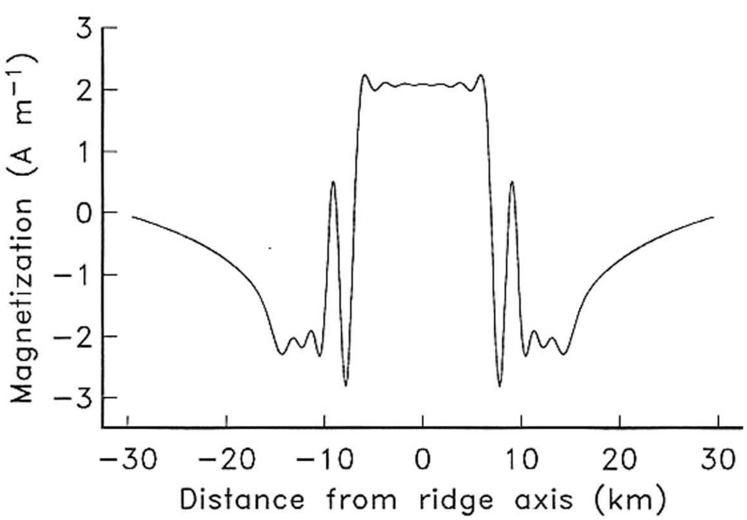


Figure 2.06c: The magnetization model with the smallest norm in the space W_2^1 generating the given magnetic anomaly values. Notice the somewhat smaller oscillations of the magnetization compared with those of figure 2.06b.

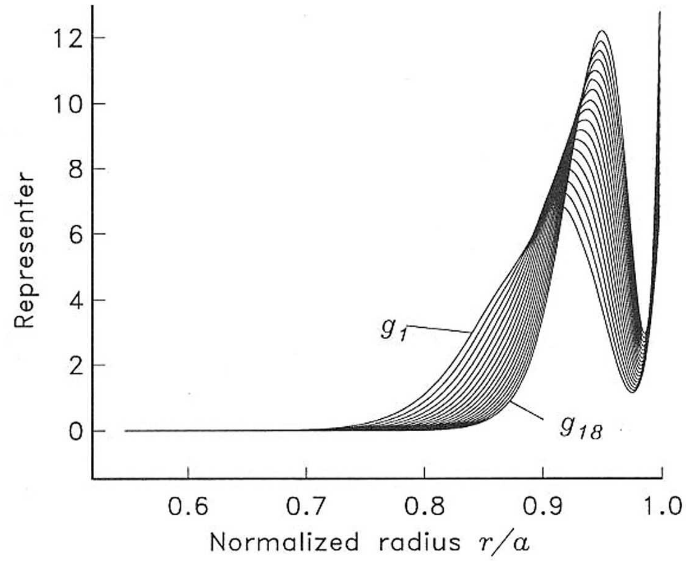
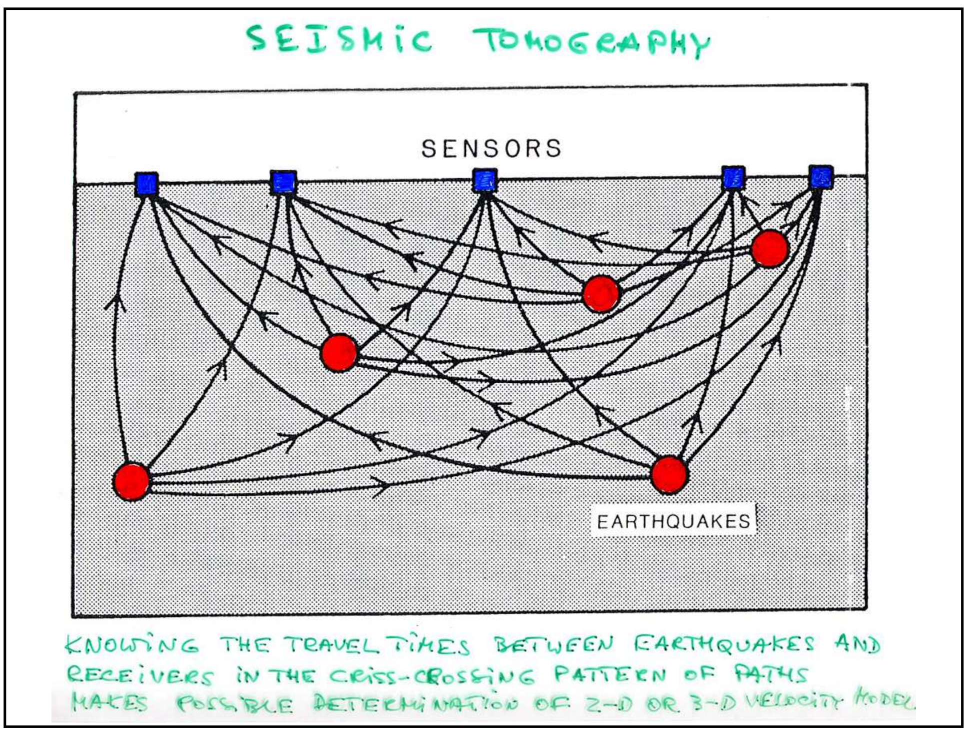


Figure 2.08b: The 18 representers in L_2 for dissipation associated with the fundamental modes $l = 21, 22, \cdots 38$ whose decay parameters appear in figure 2.08a.



Parameterization

We represent a function on a surface of a sphere by superposition of basis functions:

$$f(\vartheta, \varphi) = \sum_i c_i \cdot g_i(\vartheta, \varphi).$$

The basis function may be **local** (splines), such that they vanish at some distance from point i. A frequently used form of a local basis function is the block representation:

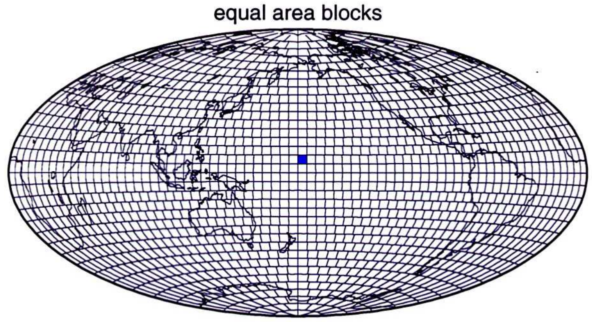
$$g_i(\vartheta, \varphi) = \begin{cases} 1 & \text{if } \vartheta_i^- \leq \vartheta \leq \vartheta_i^+ \text{ and } \varphi_i^- \leq \varphi \leq \varphi_i^+ \\ 0 & \text{otherwise} \end{cases}$$

A common form of representation of **global** basis functions are spherical harmonics, which are orthogonal on the surface of the sphere:

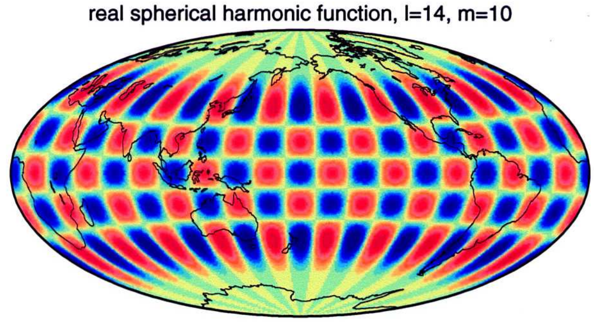
$$g_i(\vartheta, \varphi) = \begin{cases} p_\ell^m(\vartheta) \cos m\varphi & \text{or} \\ p_\ell^m(\vartheta) \sin m\varphi. \end{cases}$$

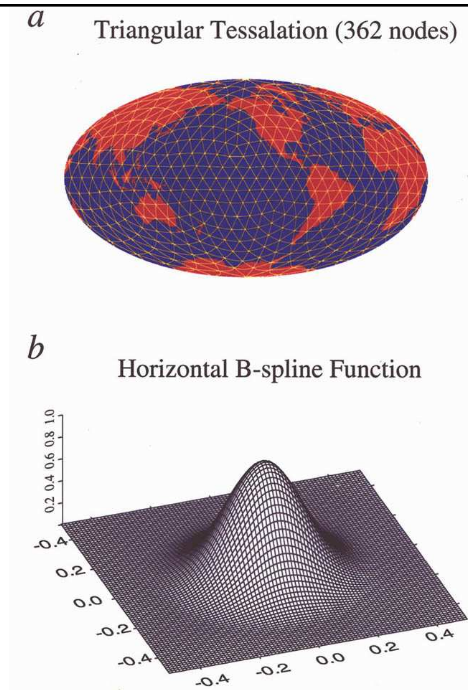
In the first example, the representation is simple in the **space domain**, in the other, it is equally simple in the **wavenumber domain**. The choice depends on the kind of observations that are interpreted and on the kind of information that we want to obtain.

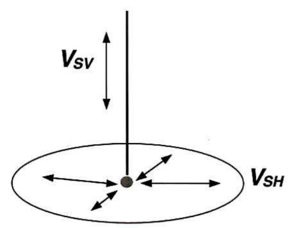
equal area blocks



real spherical harmonic function, l=14, m=10







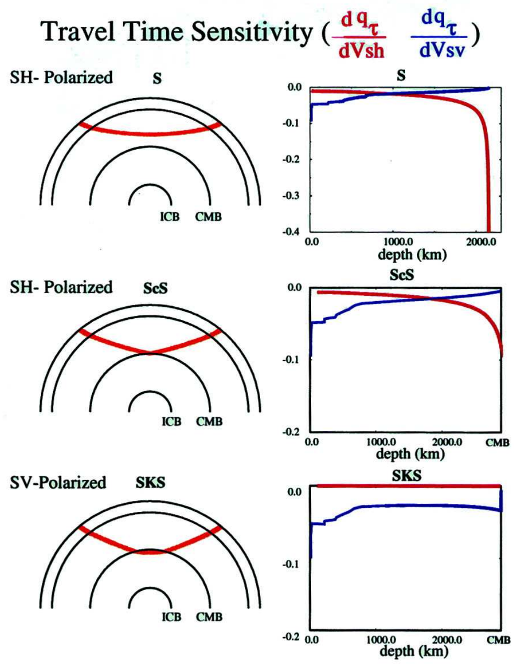
$$A = \rho V_{PH}^2$$

$$C = \rho V_{PV}^2$$

$$N = \rho V_{SH}^2$$

$$L = \rho V_{SV}^2$$

$$F = \eta (A - 2L)$$



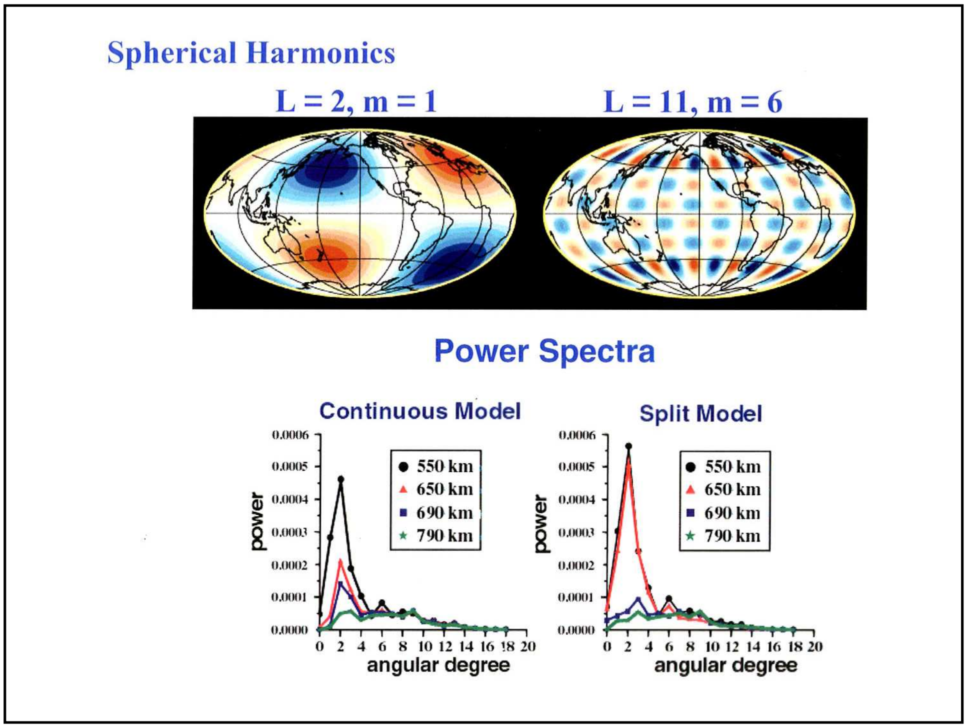
Power Spectrum

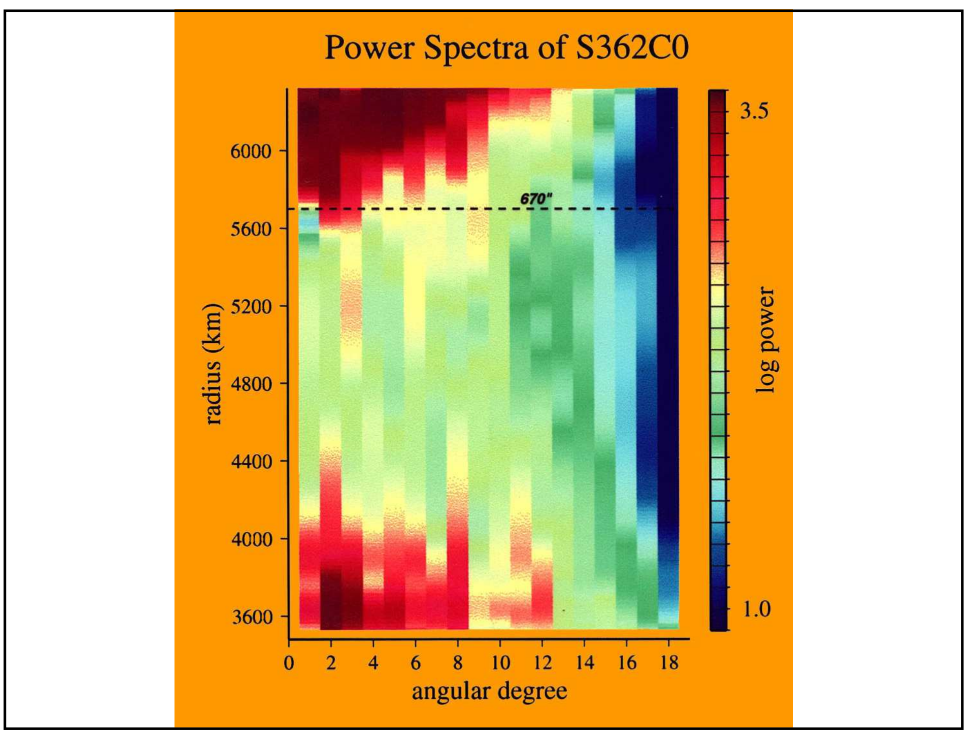
With a function $f(\vartheta, \varphi)$ described on a spherical surface by:

$$f(\vartheta, \varphi) = \sum_{\ell=0}^{\infty} \sum_{m=0}^{\ell} (A_\ell^m \cos m\varphi + B_\ell^m \sin m\varphi)\, p_{\ell m}(\cos\vartheta);$$

the *power spectrum* is defined for a harmonic degree ℓ as:

$$G_\ell = (A_\ell^0)^2 + \frac{1}{2} \sum_{m=1}^{\ell} (A_\ell^m)^2 + (B_\ell^m)^2.$$


Spherical Harmonics
L = 2, m = 1
L = 11, m = 6
Power Spectra
Continuous Model
Split Model
power
550 km
650 km
690 km
790 km
angular degree


Power Spectra of S362C0
radius (km)
6000
5600
5200
4800
4400
4000
3600
670"
3.5
1.0
log power
0 2 4 6 8 10 12 14 16 18
angular degree

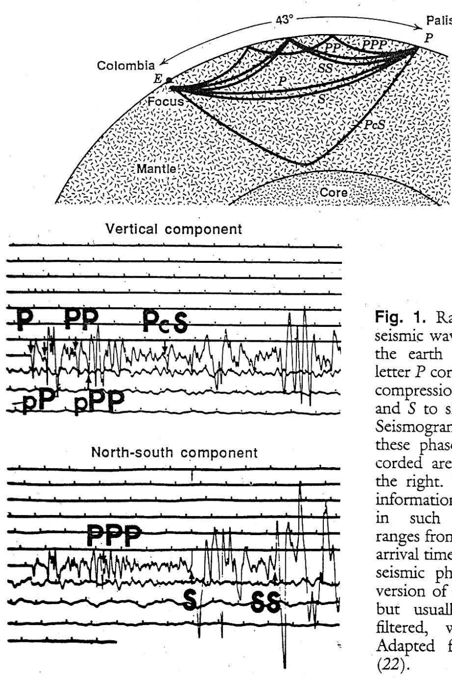
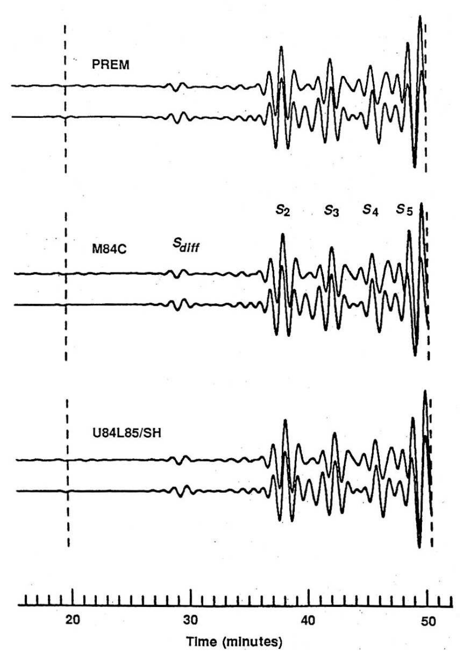
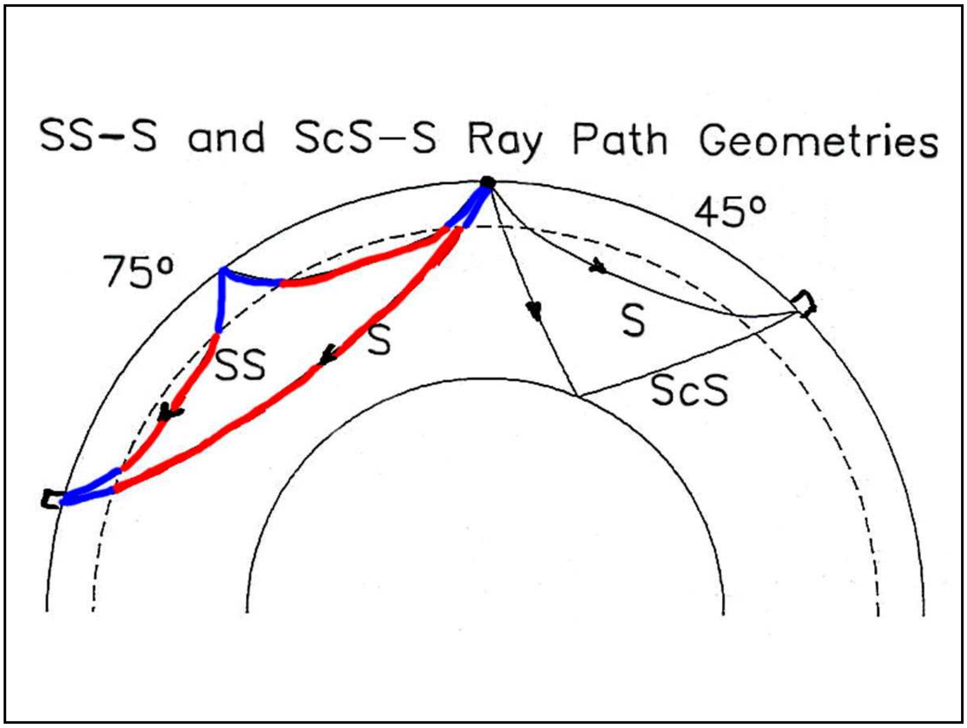
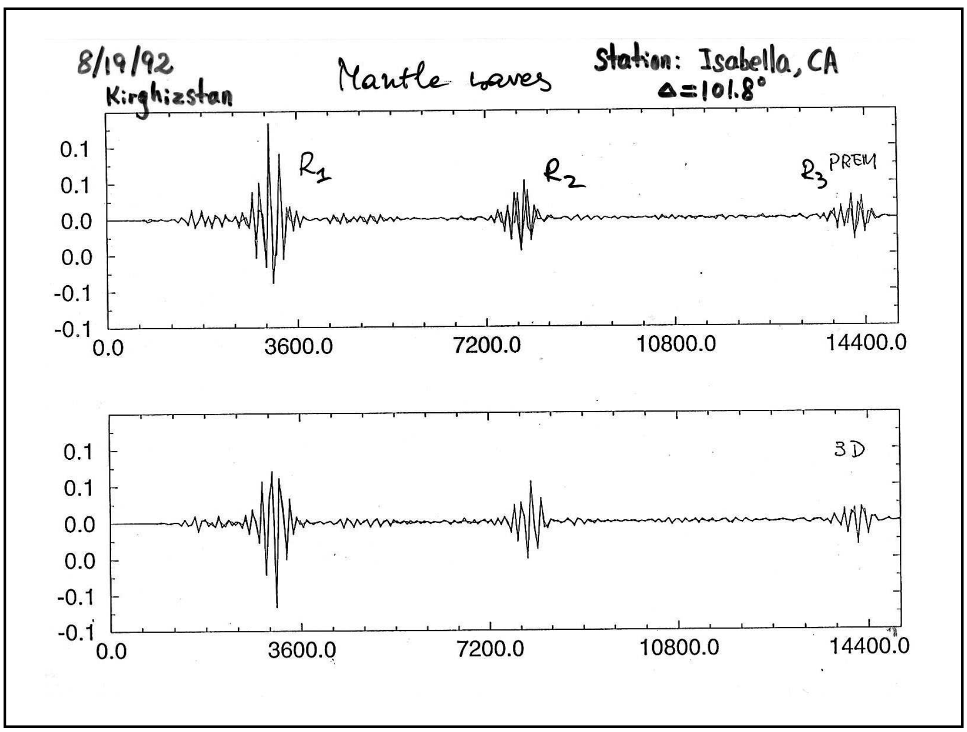
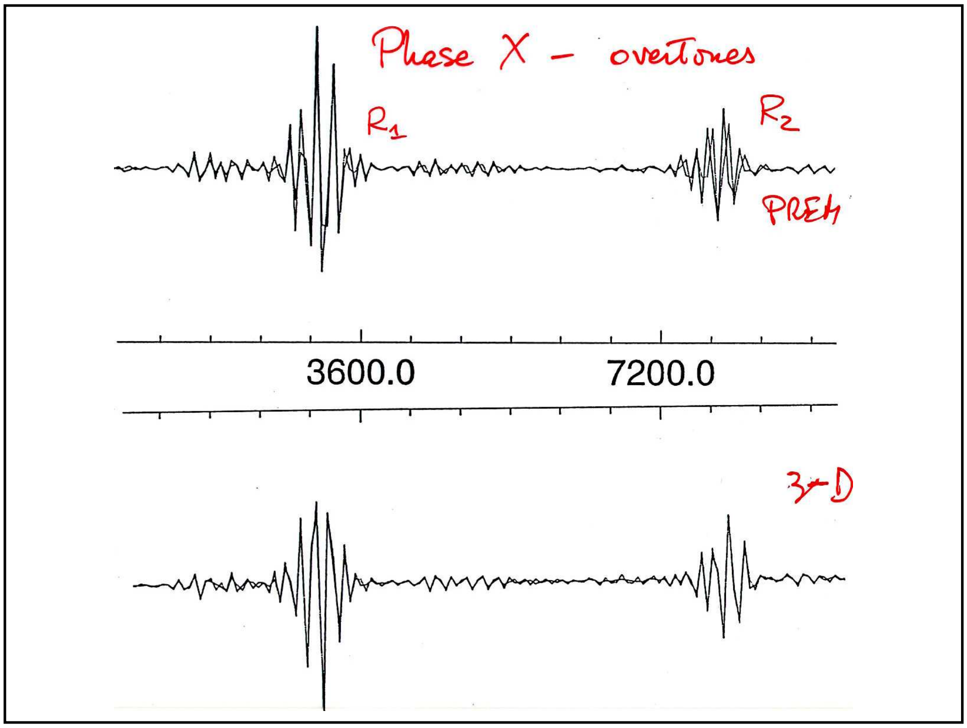


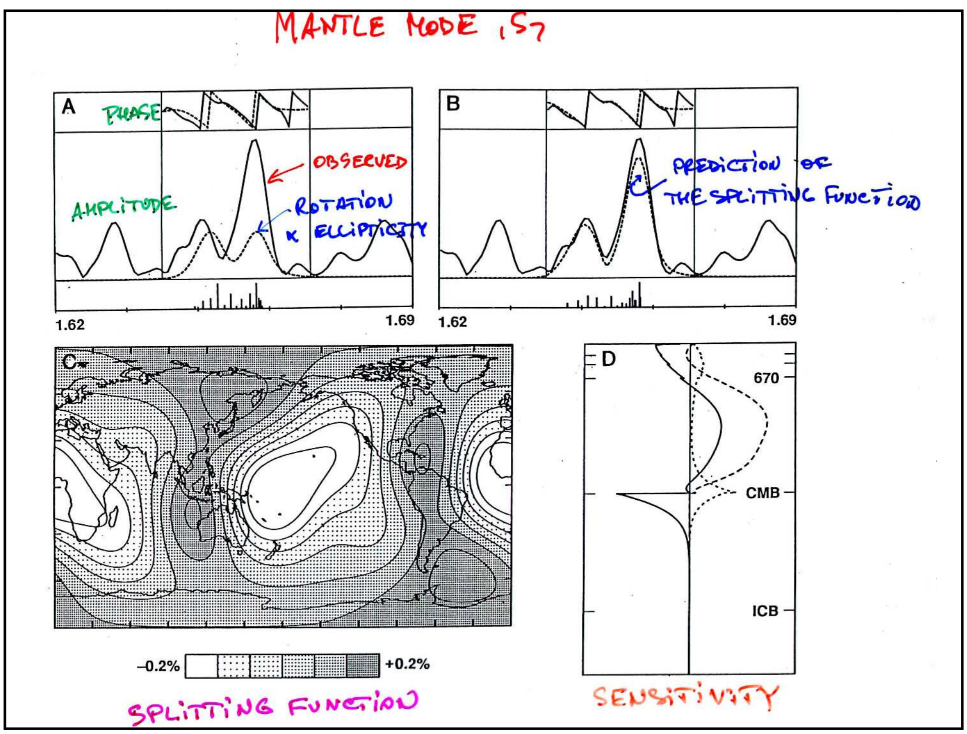
Fig. 1. Ray paths of seismic waves through the earth (left). The letter *P* corresponds to compressional waves and *S* to shear waves. Seismograms in which these phases were recorded are shown on the right. The use of information contained in such recordings ranges from reading of arrival times of various seismic phases to inversion of a complete, but usually low-pass filtered, wave train. Adapted from Donn (*22*).

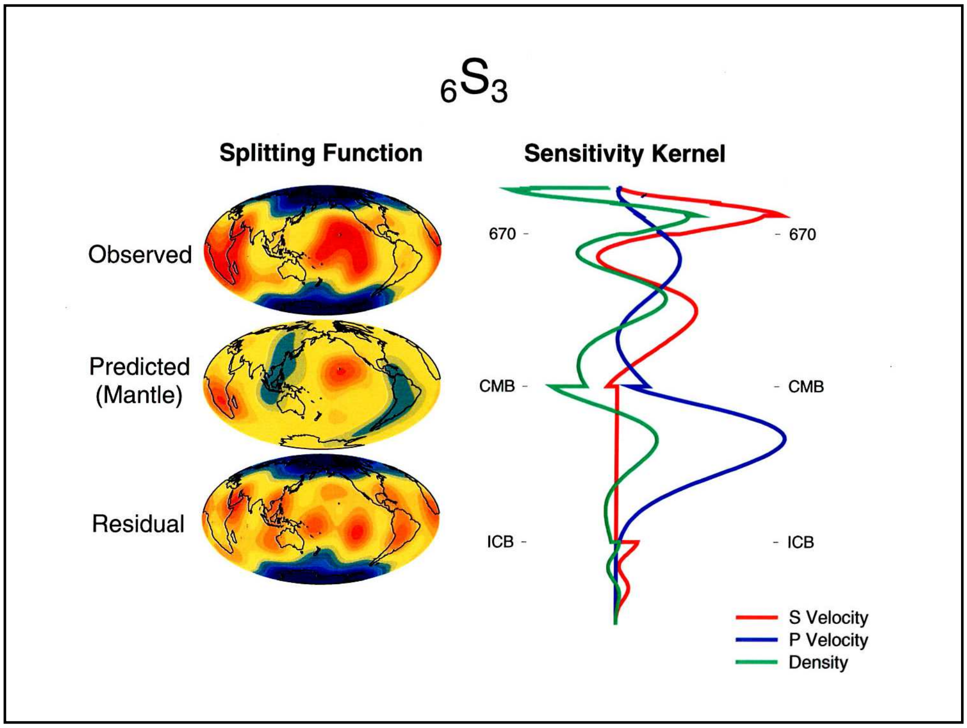



SS−S and ScS−S Ray Path Geometries
45°
75°
S
SS
S
ScS

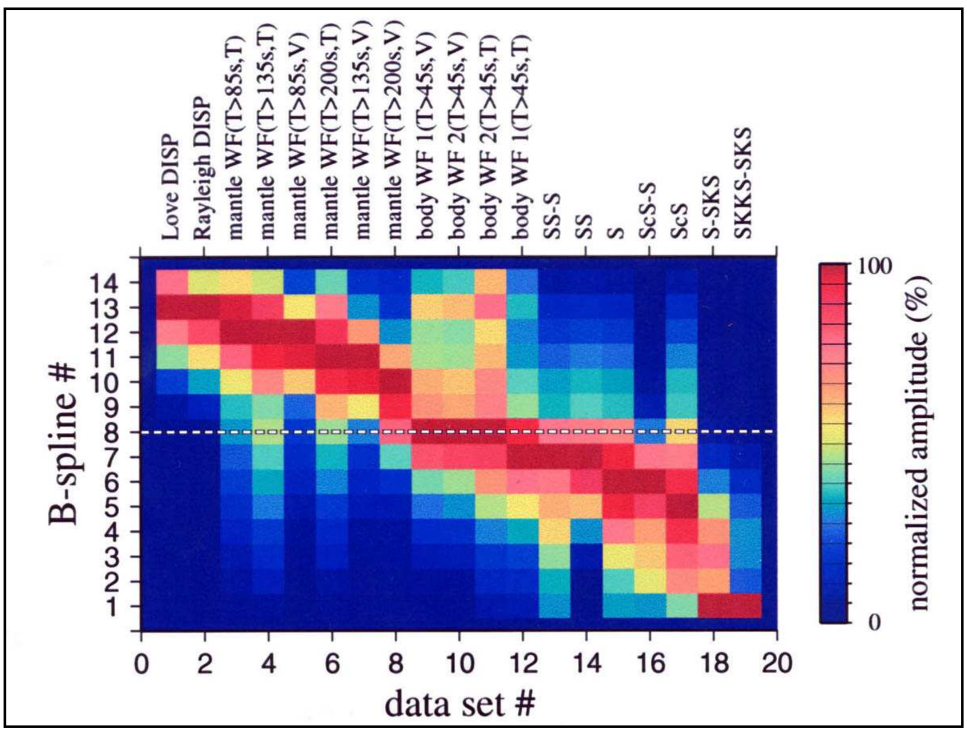

8/19/92
Kirghizstan
Mantle waves
Station: Isabella, CA
Δ=101.8°
R1
R2
R3
PREM
3D
0.1
0.0
-0.1
0.0
3600.0
7200.0
10800.0
14400.0


Phase X – overtones
R1
R2
PREM
3600.0
7200.0
3-D

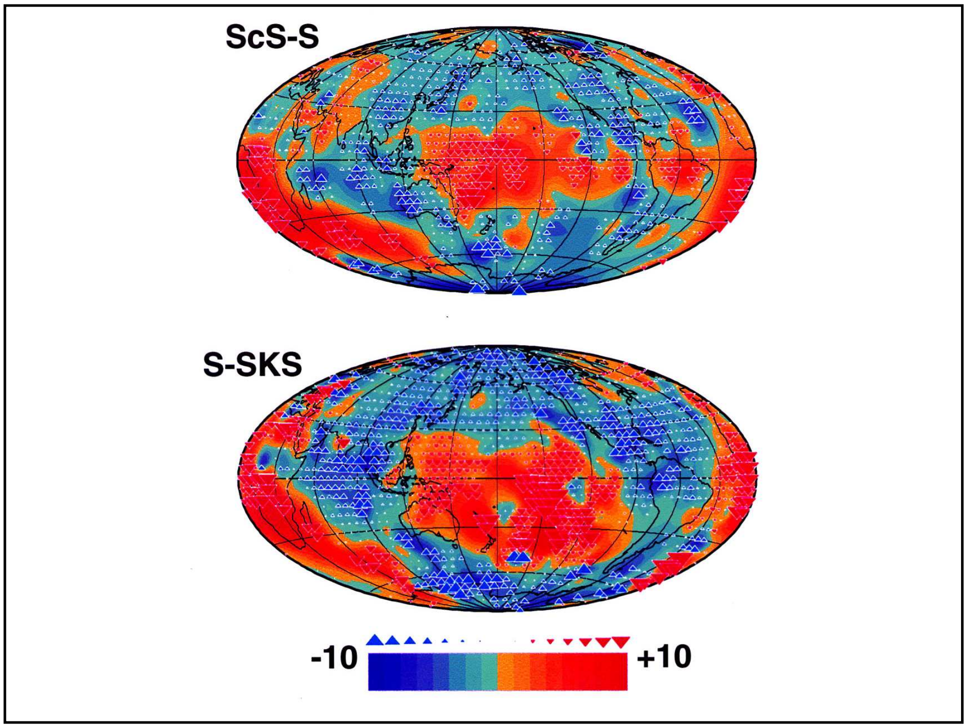

MANTLE MODE 1S7
A
PHASE
AMPLITUDE
OBSERVED
ROTATION & ELLIPTICITY
1.62
1.69
B
PREDICTION OF THE SPLITTING FUNCTION
1.62
1.69
C
–0.2%
+0.2%
SPLITTING FUNCTION
D
670
CMB
ICB
SENSITIVITY

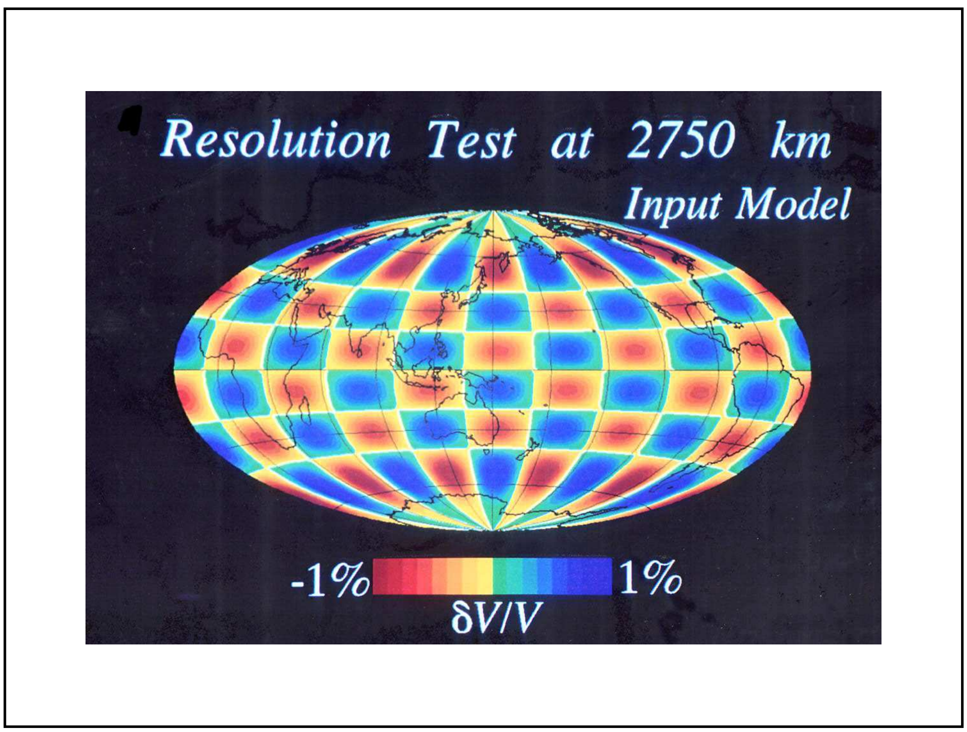


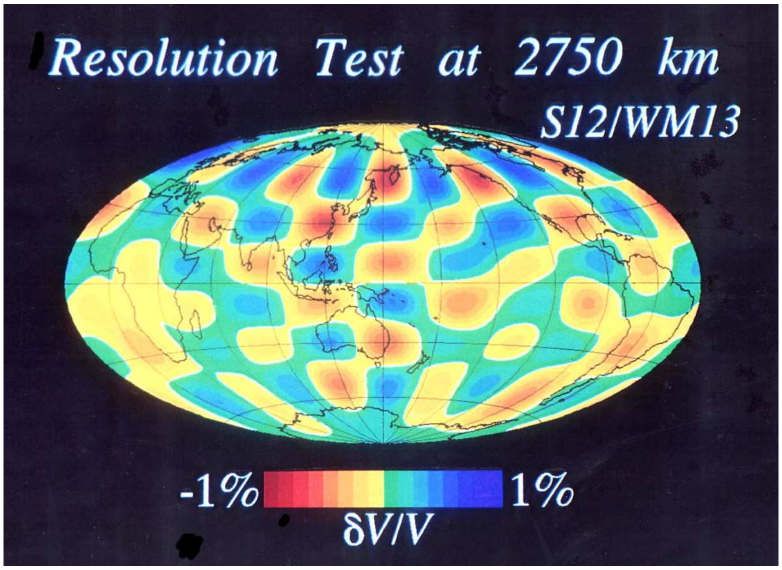
$_6S_3$
Splitting Function
Sensitivity Kernel
Observed
Predicted (Mantle)
Residual
670
CMB
ICB
S Velocity
P Velocity
Density

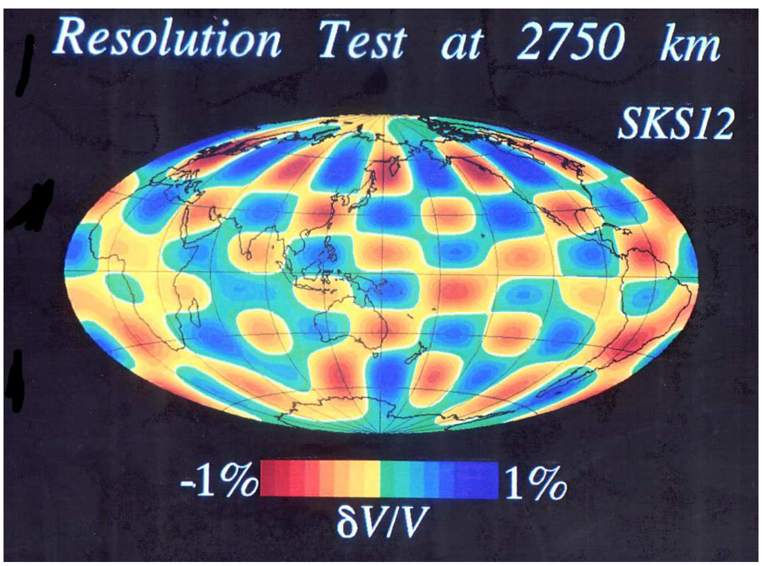


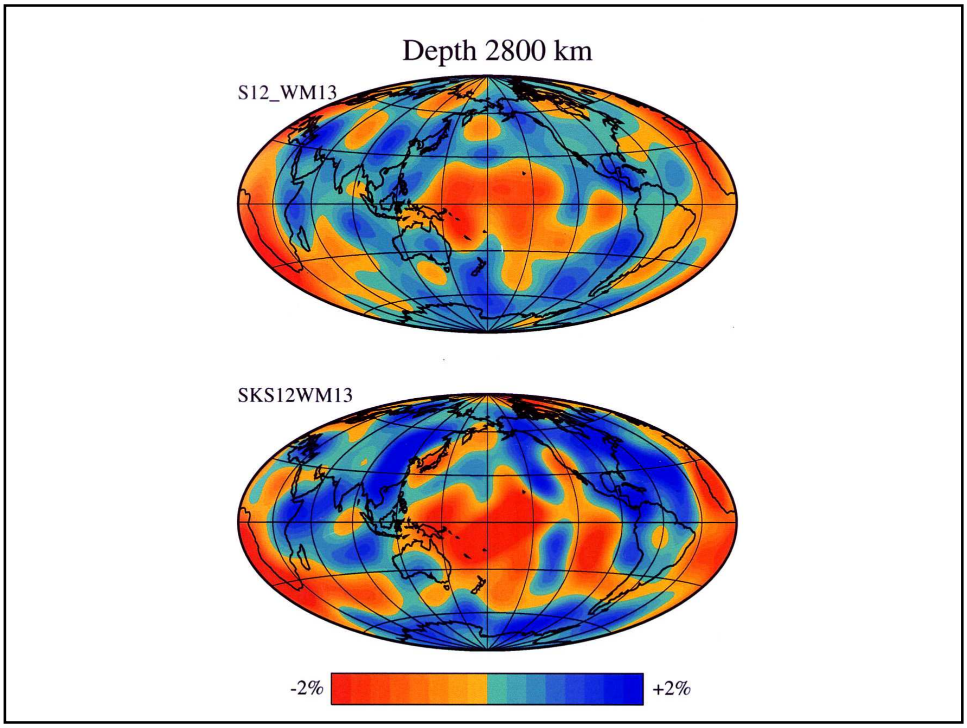
Love DISP
Rayleigh DISP
mantle WF(T>85s,T)
mantle WF(T>135s,T)
mantle WF(T>85s,V)
mantle WF(T>200s,T)
mantle WF(T>135s,V)
mantle WF(T>200s,V)
body WF 1(T>45s,V)
body WF 2(T>45s,V)
body WF 2(T>45s,T)
body WF 1(T>45s,T)
SS-S
SS
S
ScS-S
ScS
S-SKS
SKKS-SKS
B-spline #
data set #
normalized amplitude (%)
100
0

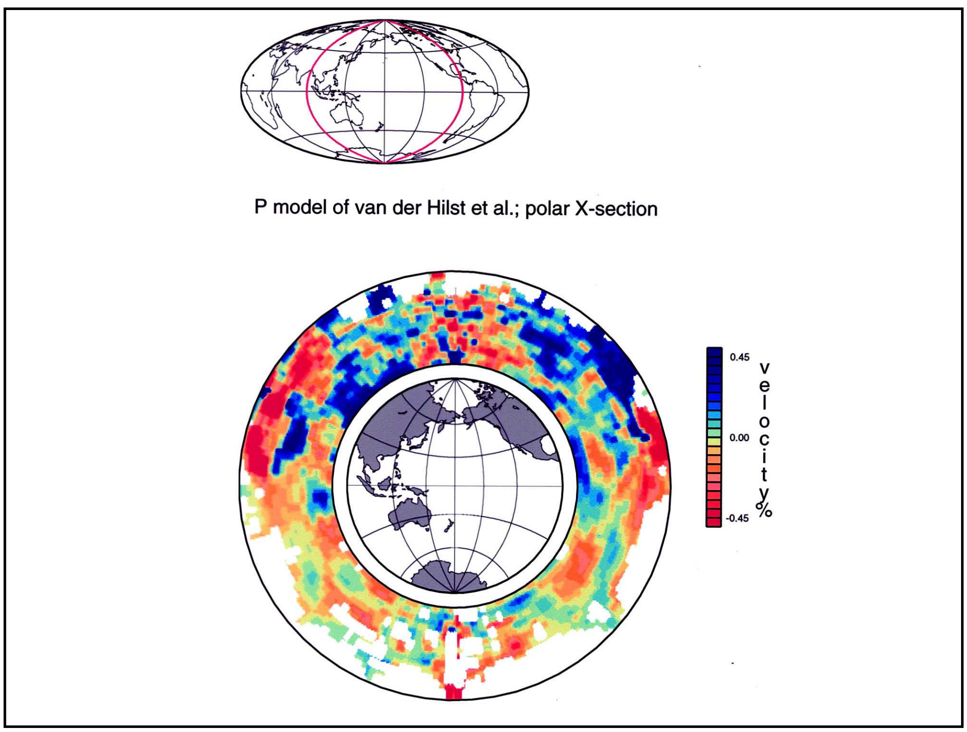

ScS-S
S-SKS
-10
+10

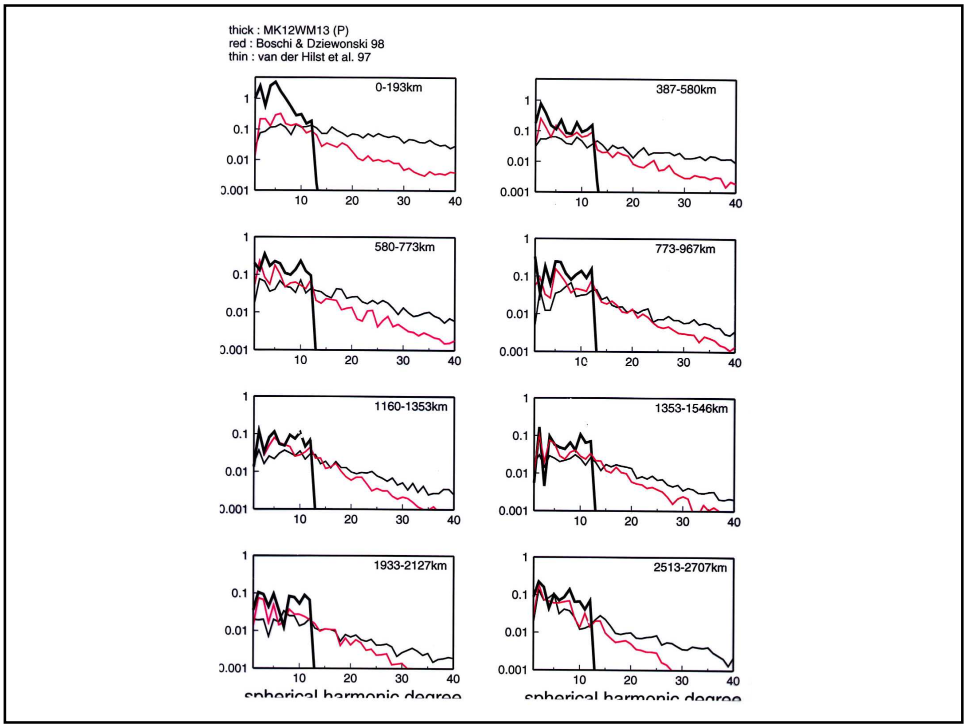

Resolution Test at 2750 km
Input Model
-1%
1%
δV/V

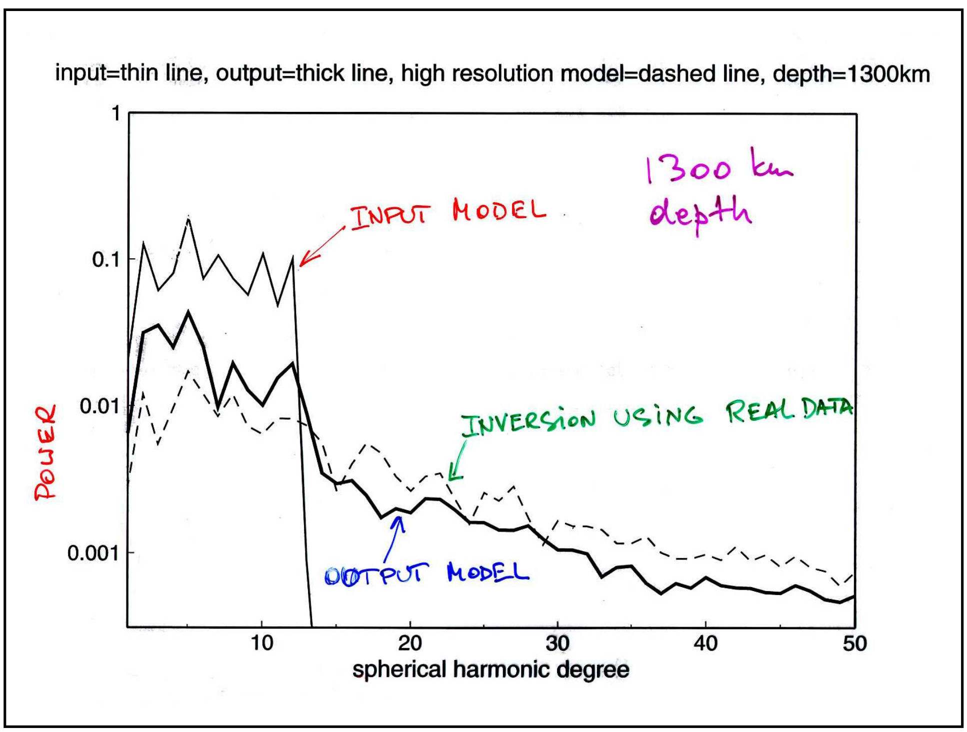

Resolution Test at 2750 km
S12/WM13
-1%
1%
δV/V

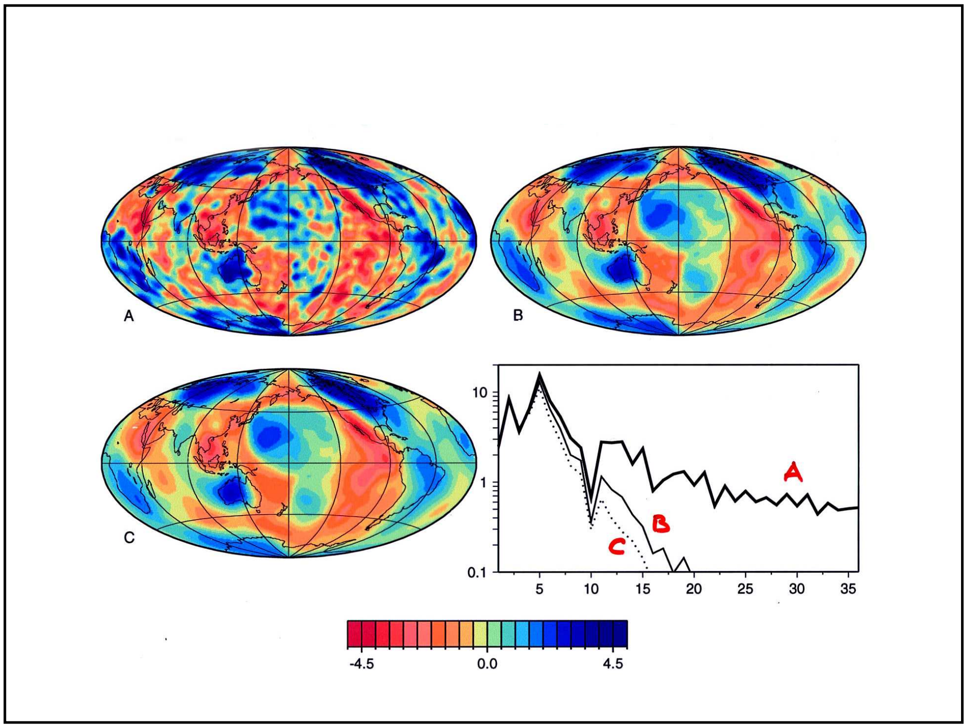

Resolution Test at 2750 km
SKS12
-1%
1%
δV/V

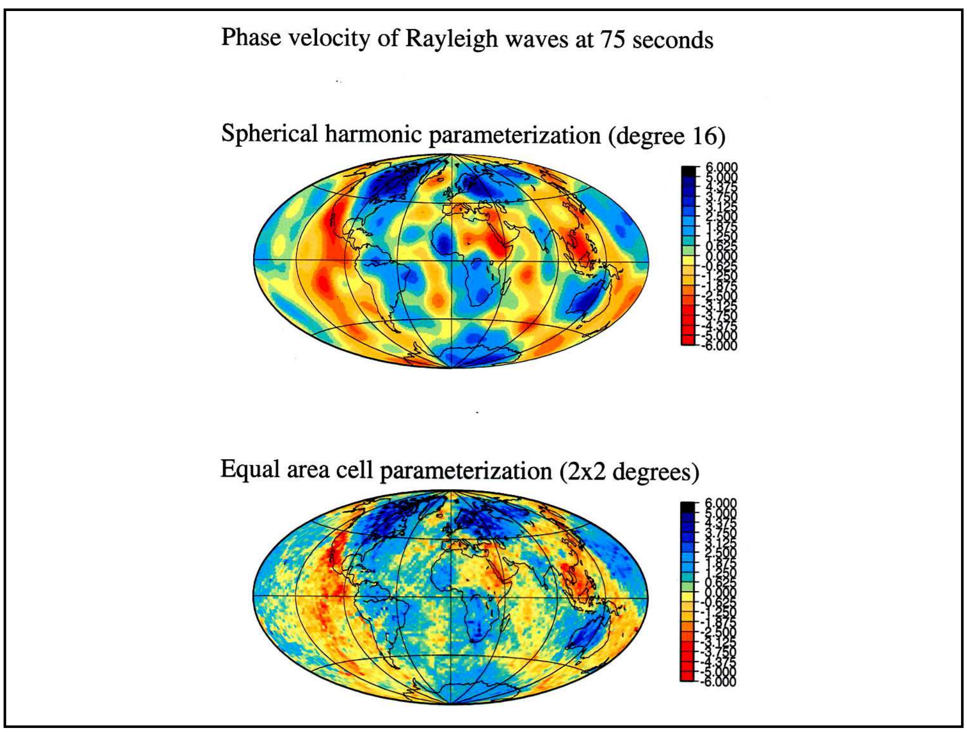

Depth 2800 km
S12_WM13
SKS12WM13
-2%
+2%

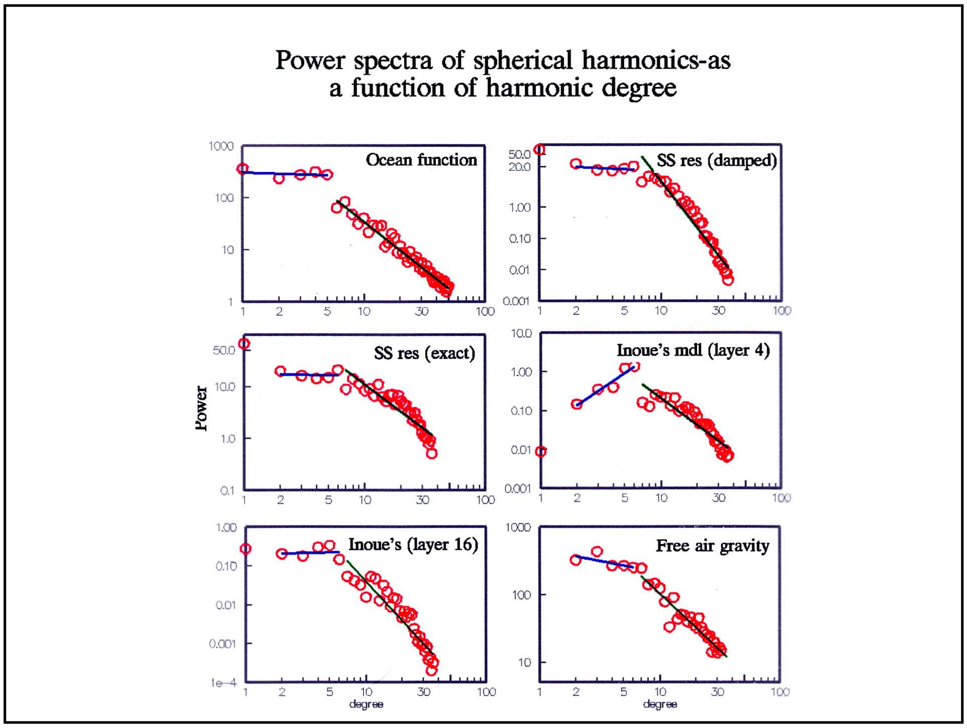

P model of van der Hilst et al.; polar X-section
0.45
0.00
-0.45
velocity%

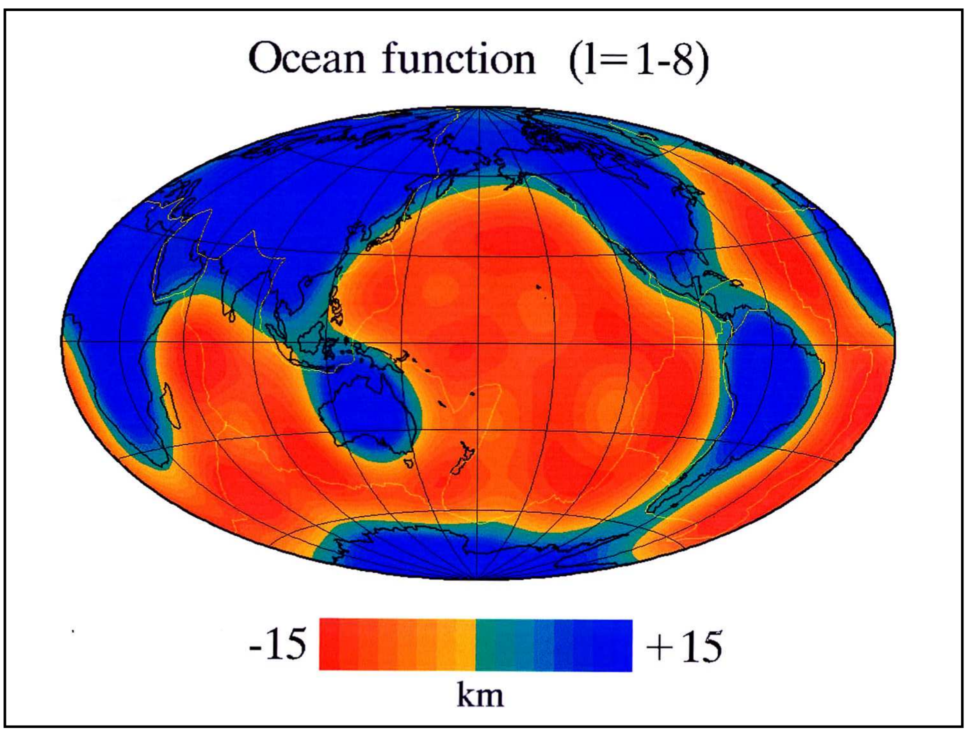

thick : MK12WM13 (P)
red : Boschi & Dziewonski 98
thin : van der Hilst et al. 97
0-193km
387-580km
580-773km
773-967km
1160-1353km
1353-1546km
1933-2127km
2513-2707km
spherical harmonic degree

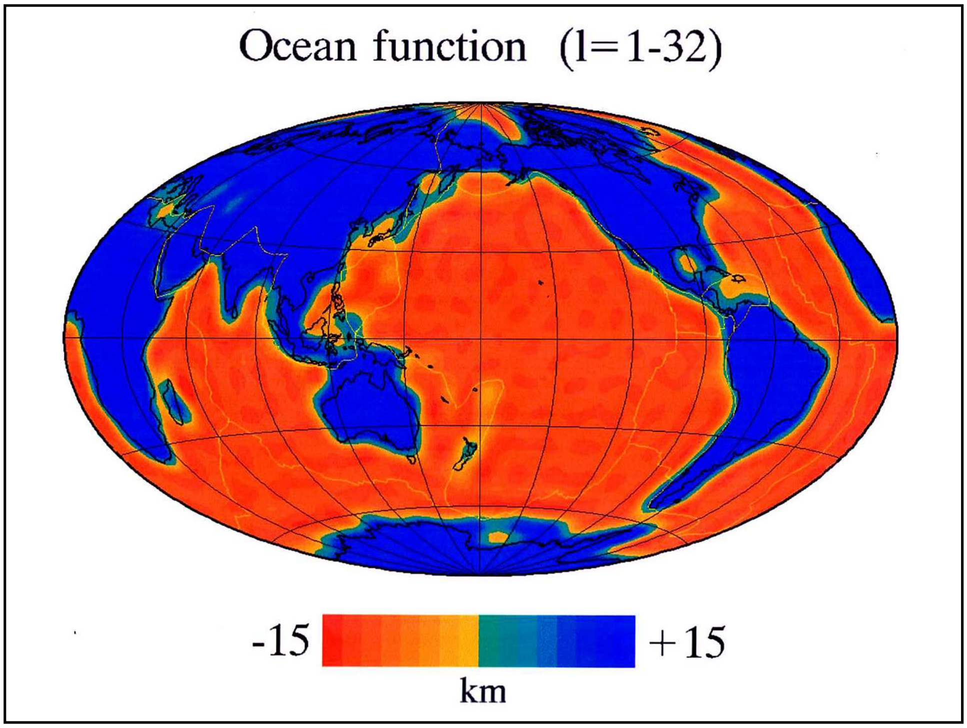

input=thin line, output=thick line, high resolution model=dashed line, depth=1300km
INPUT MODEL
1300 km depth
INVERSION USING REAL DATA
OUTPUT MODEL
POWER
spherical harmonic degree

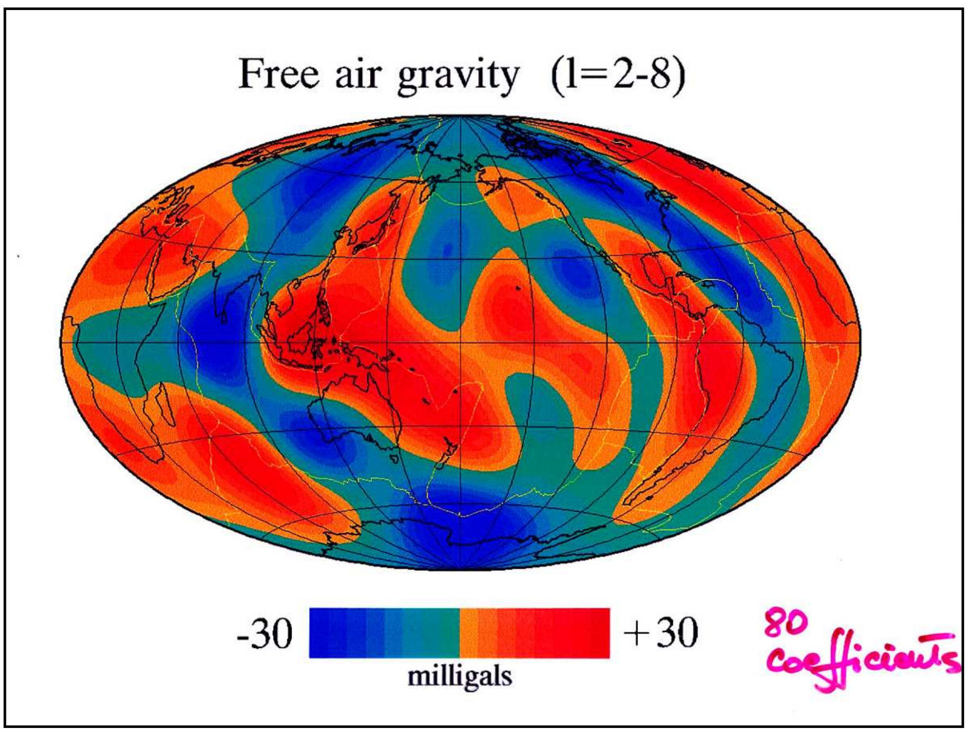

A
B
C
10
1
0.1
5
10
15
20
25
30
35
-4.5
0.0
4.5

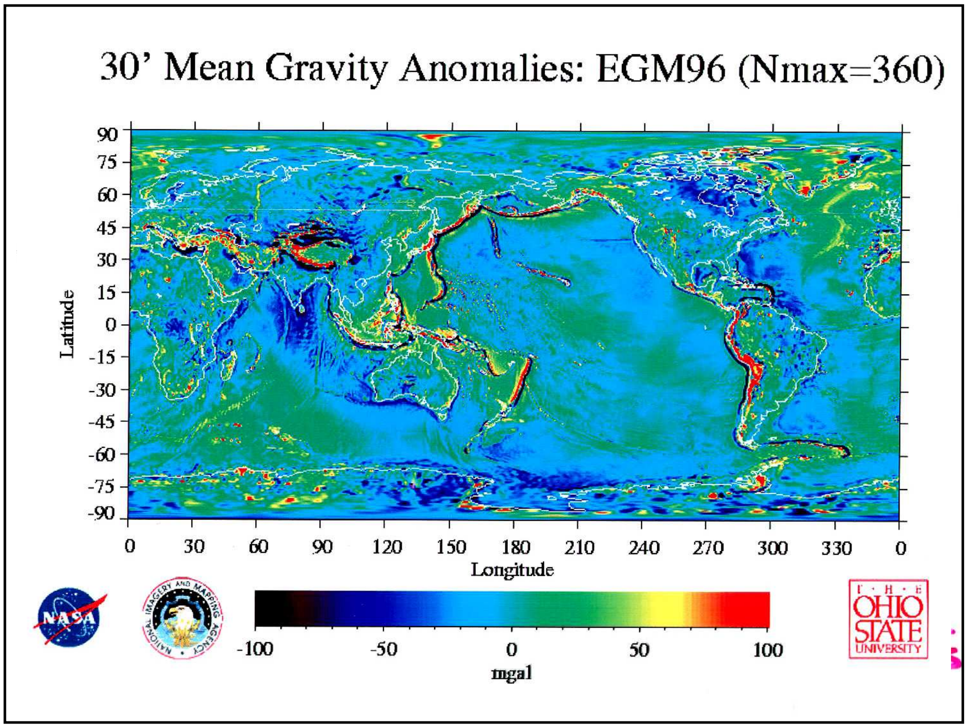

Phase velocity of Rayleigh waves at 75 seconds
Spherical harmonic parameterization (degree 16)
Equal area cell parameterization (2x2 degrees)

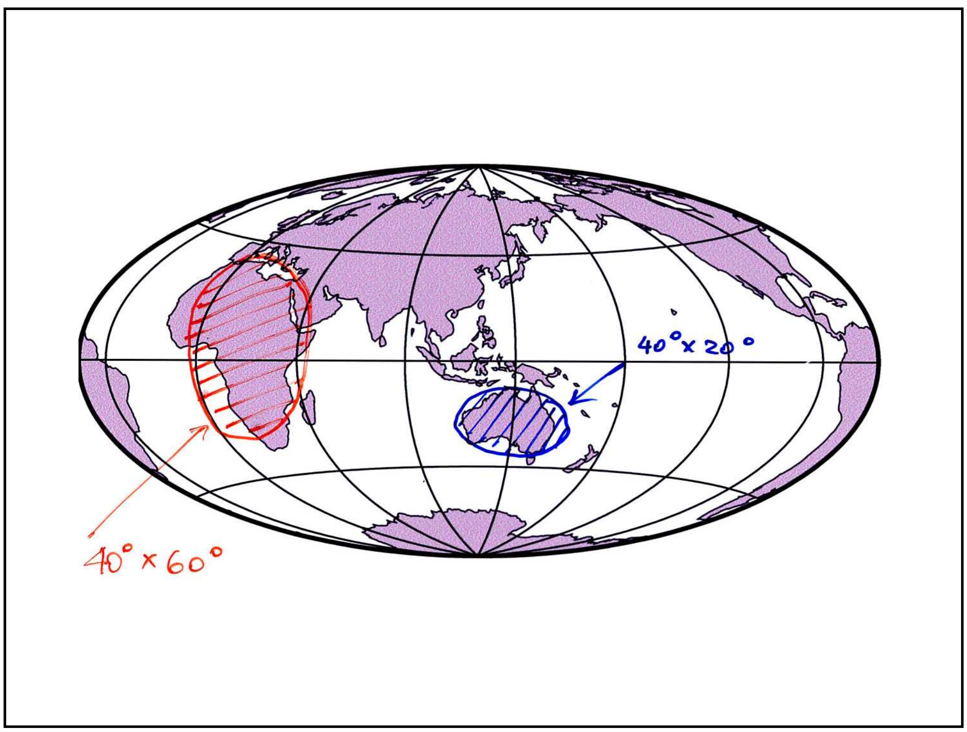

Power spectra of spherical harmonics-as
a function of harmonic degree
Ocean function
SS res (damped)
SS res (exact)
Inoue's mdl (layer 4)
Inoue's (layer 16)
Free air gravity
Power
degree
degree

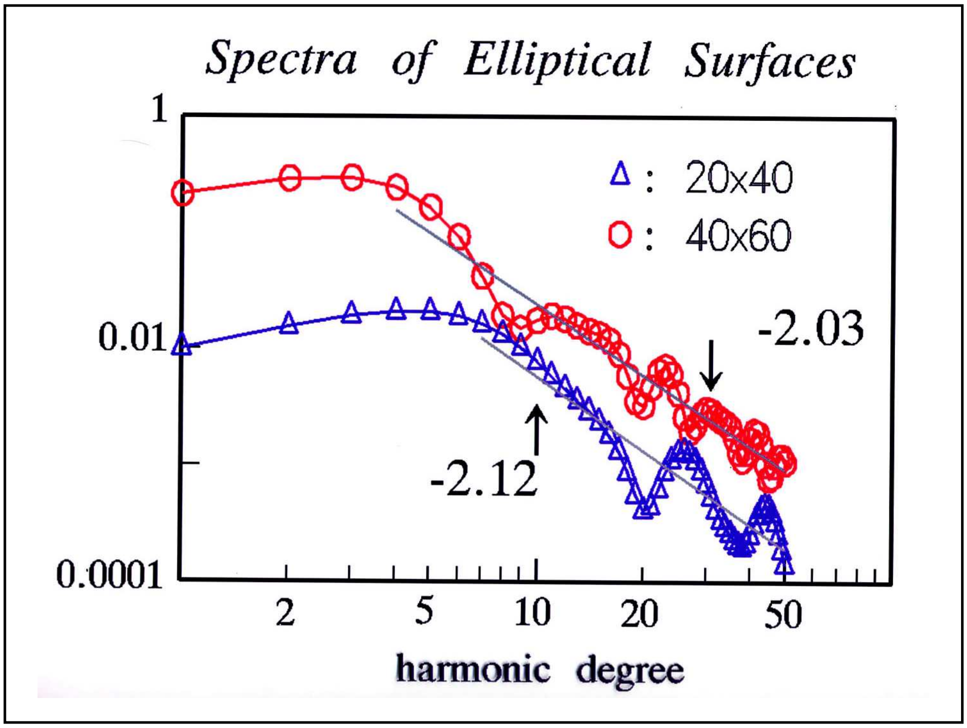

Ocean function (l=1-8)
-15
+15
km

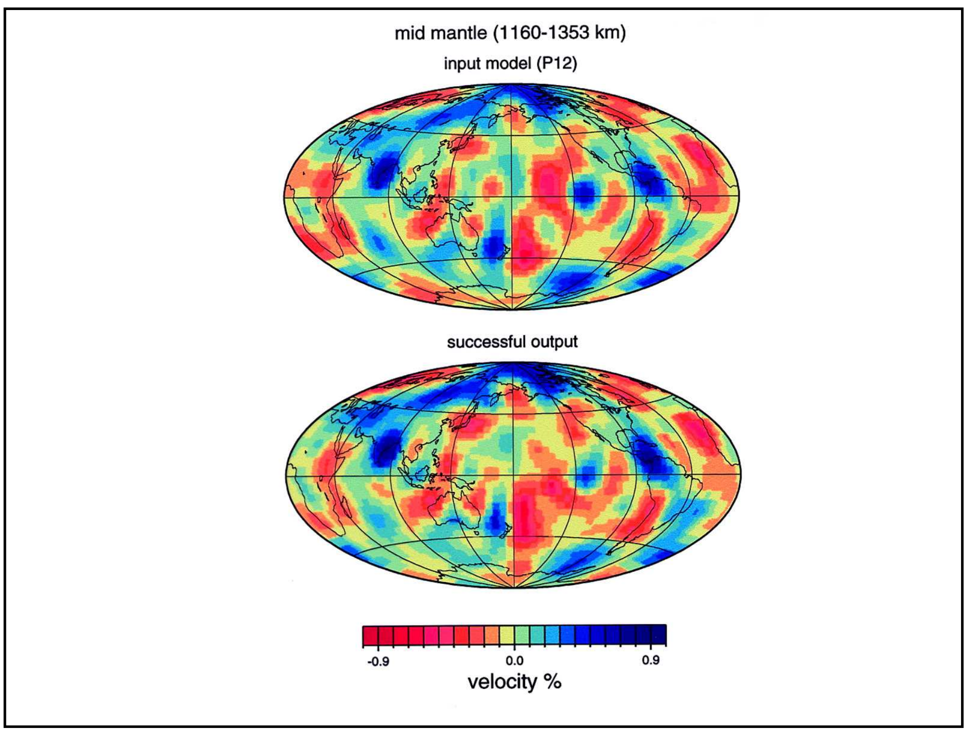

Ocean function (l=1-32)
-15
+15
km

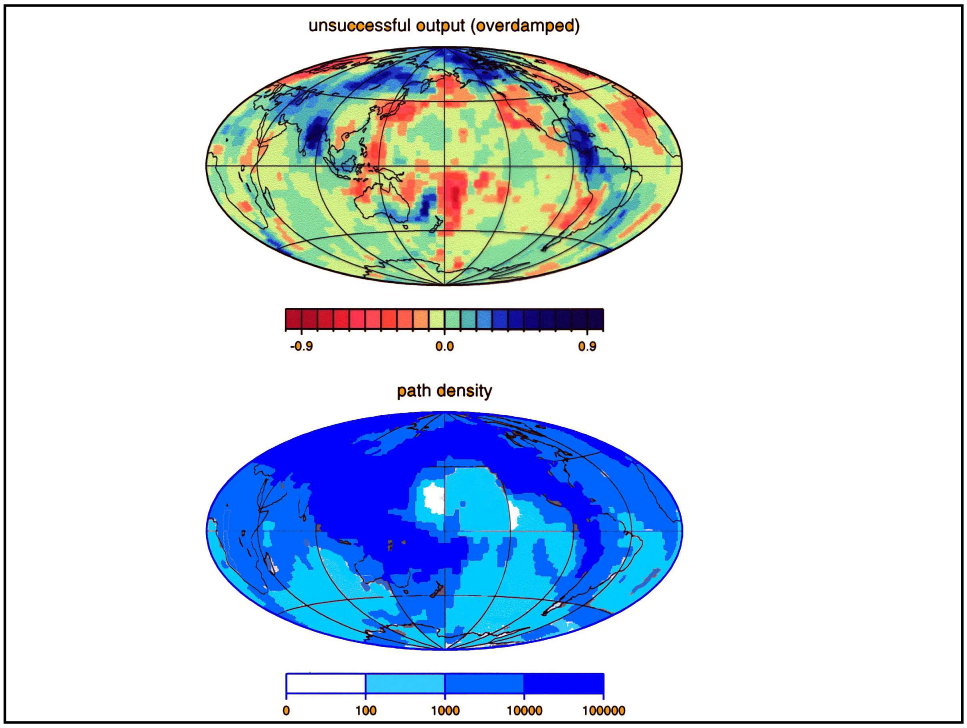

Free air gravity (l=2-8)
-30
+30
milligals
80 coefficients

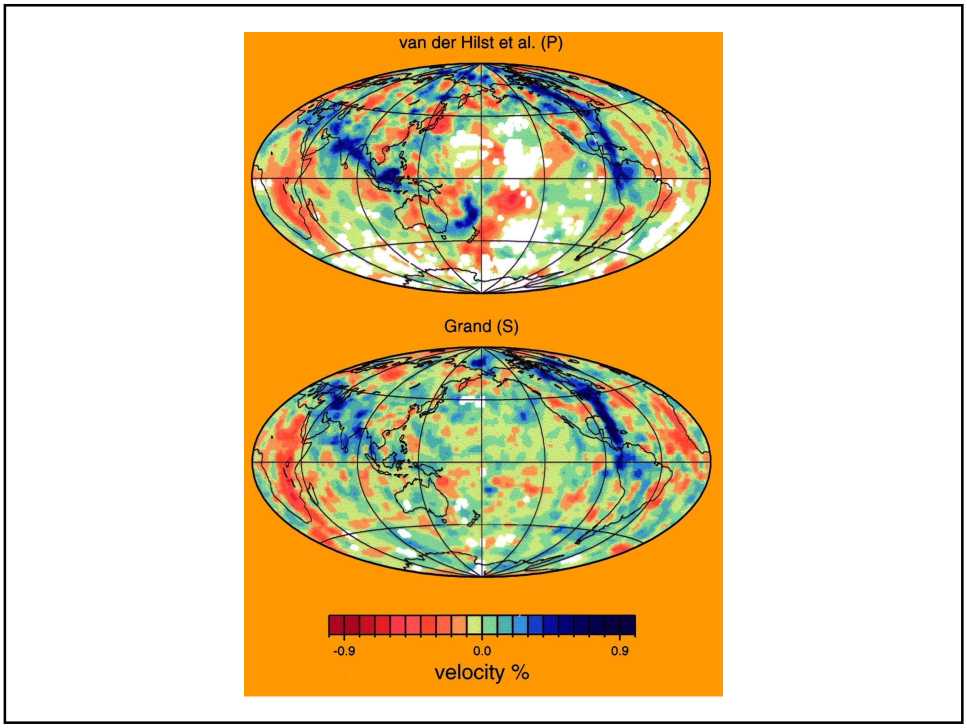

30' Mean Gravity Anomalies: EGM96 (Nmax=360)
Latitude
90
75
60
45
30
15
0
-15
-30
-45
-60
-75
-90
0
30
60
90
120
150
180
210
240
270
300
330
0
Longitude
-100
-50
0
50
100
mgal

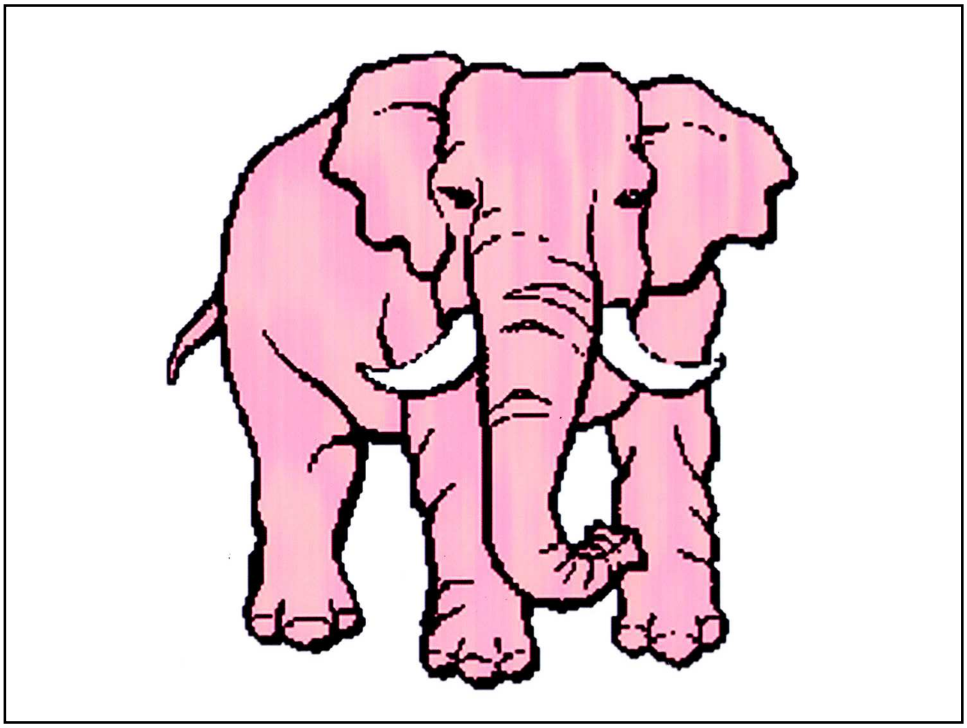

40°x 20°
40° x 60°

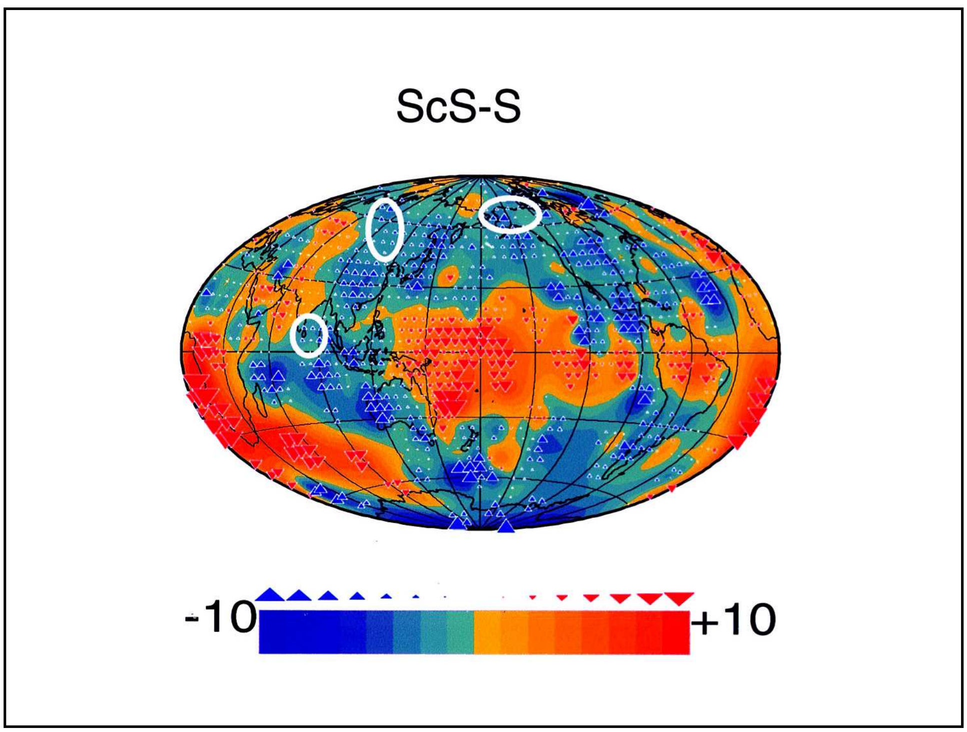

Spectra of Elliptical Surfaces
1
0.01
0.0001
: 20x40
: 40x60
-2.03
-2.12
2
5
10
20
50
harmonic degree


mid mantle (1160-1353 km)
input model (P12)
successful output
-0.9
0.0
0.9
velocity %


unsuccessful output (overdamped)
-0.9
0.0
0.9
path density
0
100
1000
10000
100000


van der Hilst et al. (P)
Grand (S)
-0.9
0.0
0.9
velocity %




ScS-S
-10
+10