Compactifying the Cubic Code

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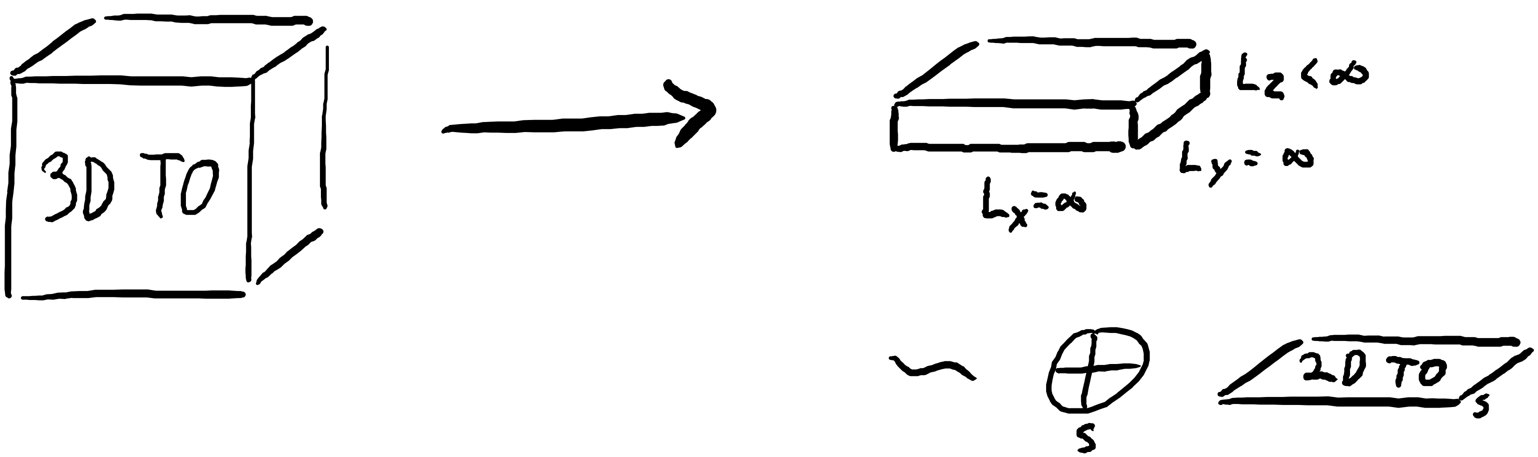
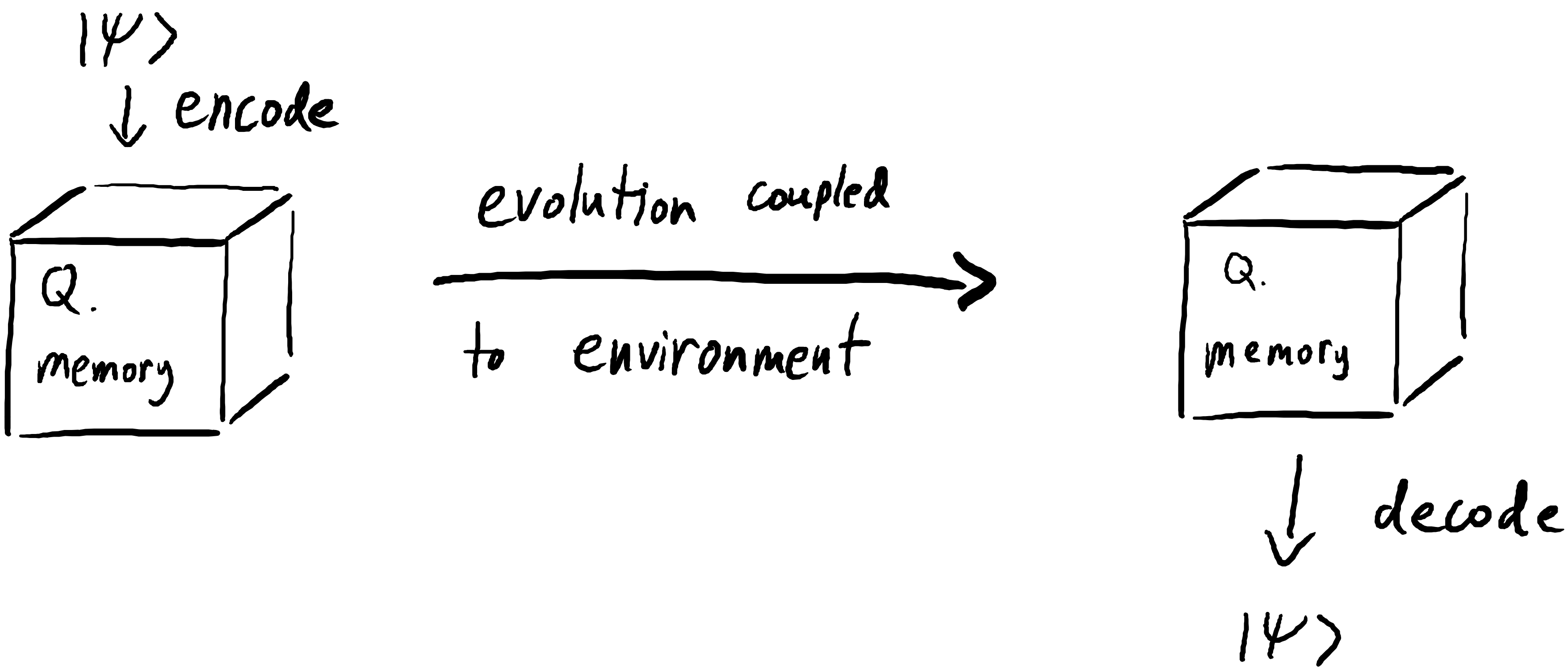
Postdoc in Cheng group at Yale

Joint work with Arpit Dua & Meng Cheng

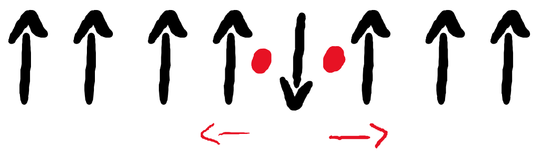
Overview

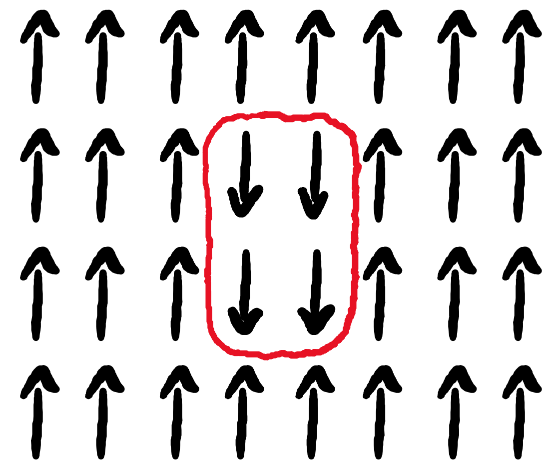


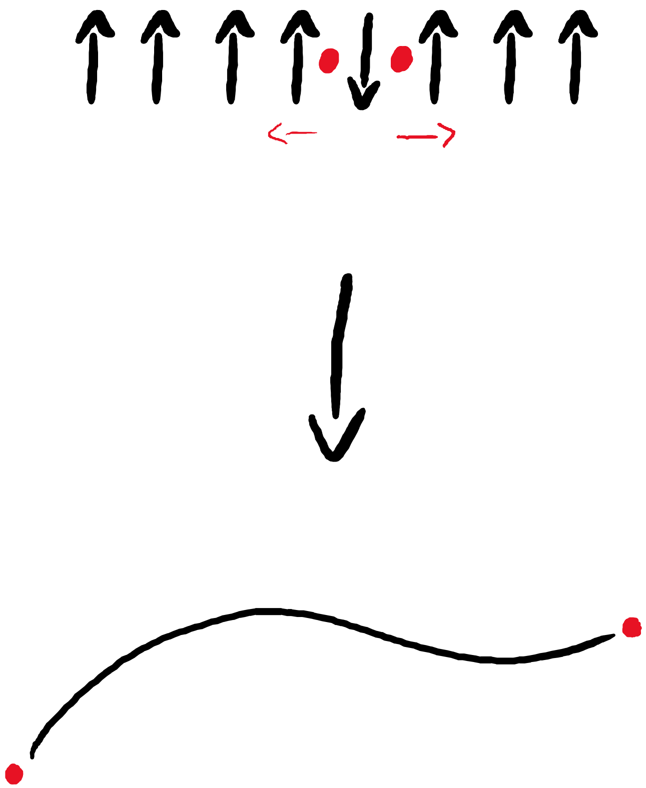
Background



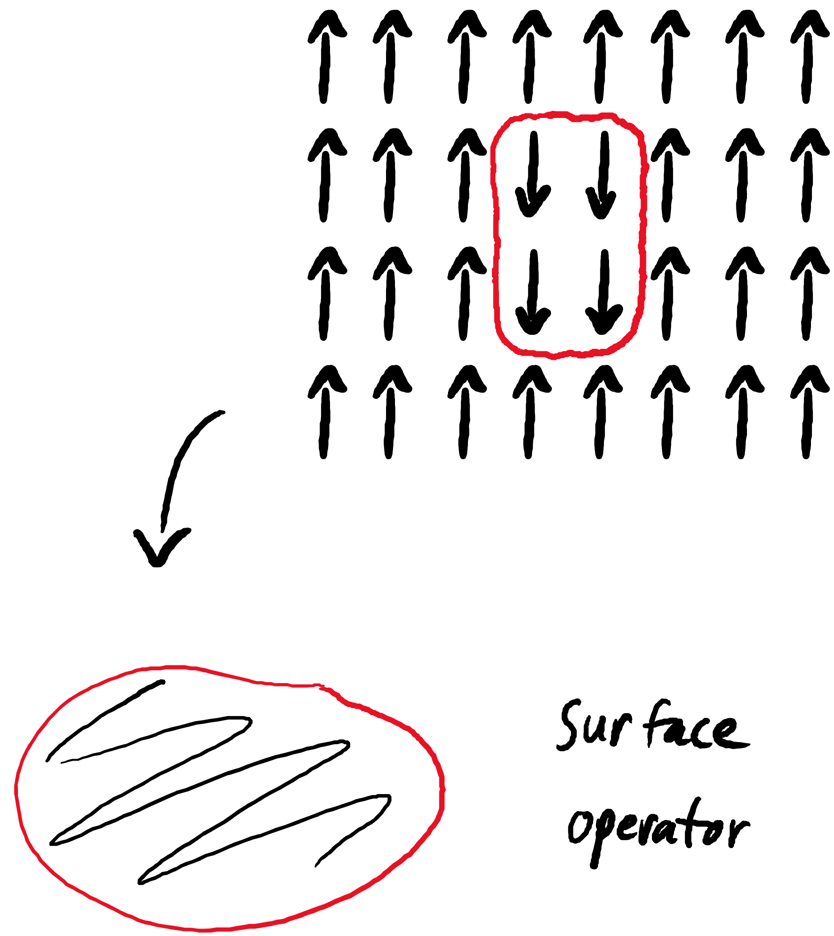
$|\psi\rangle$
encode
Q. memory
evolution coupled
to environment
Q. memory
decode
$|\psi\rangle$







String operator



Surface operator

3D Topological Phases

TQFT

- point + loop excitations
- String + membrane operators

e.g. Toric code

3D Topological Phases

TQFT

- point + loop excitations
- String + membrane operators

e.g. Toric code

Fracton: - point excitations

I

- Some mobility restrictions
- String operators

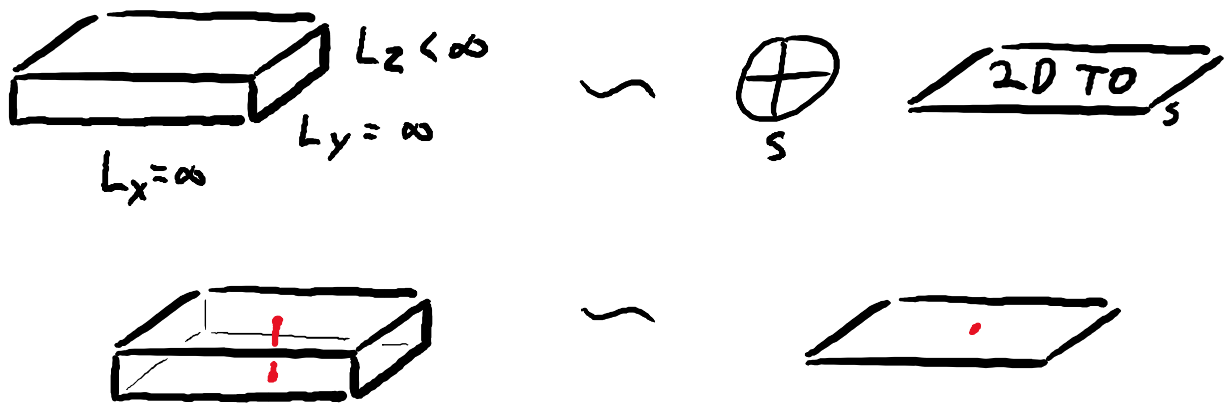
e.g. X-cube

II

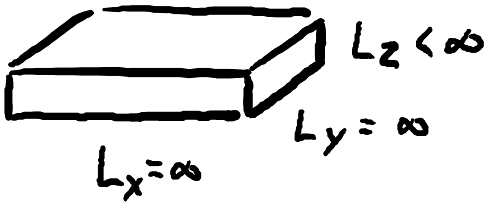
- Immobile excitations
- no string operators

e.g. Cubic code

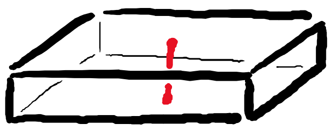
Compactification



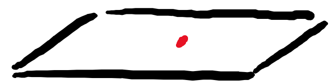
Compactification



$\sim \bigoplus_S$



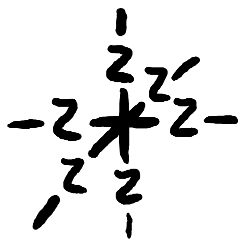
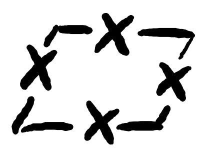
$\sim$



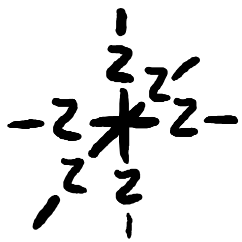
Thm. 2D CSS code TO $\sim$ (Toric code)$^{\otimes n}$

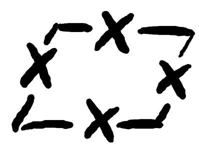
$n_{\underline{v}}$ – an "order parameter" for fractons?

3D Toric Code

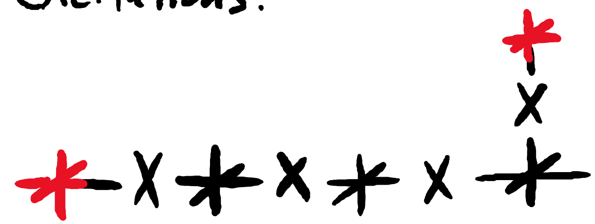
$$H = -\sum_{v}$$  $$-\sum_{p}$$ 

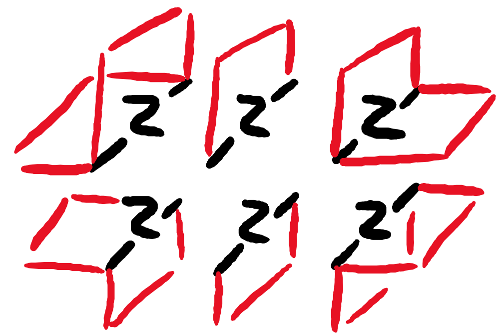
3D Toric Code

$H = -\sum_v$ 

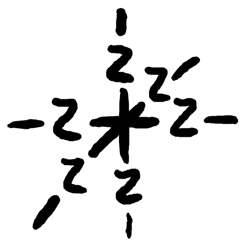
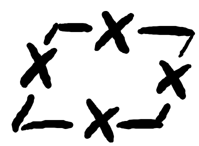
$- \sum_p$ 

excitations:





3D Toric Code

$H = -\sum_v$  $-\sum_p$ 

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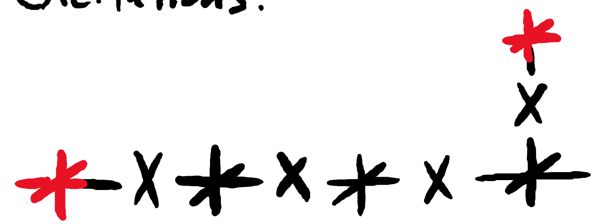
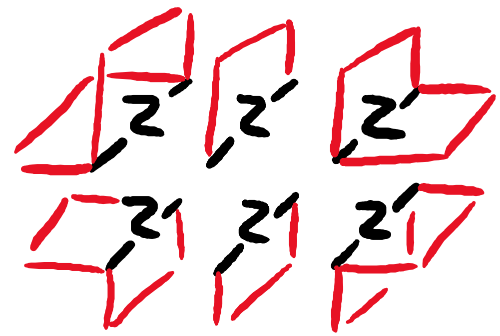
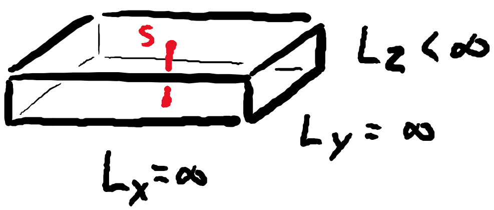


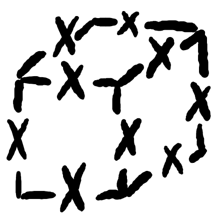
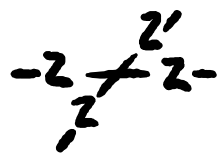


Compactification:

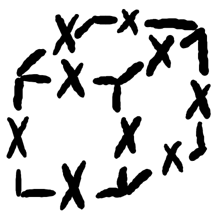


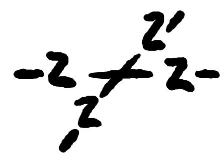
$\sim \bigoplus_{s=\pm}$ Toric code

X-cube

$H = -\sum_c$  $-\sum_v$  $+ \dots$

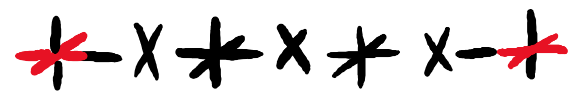
X-cube

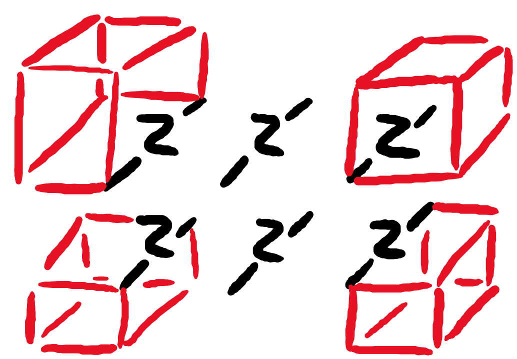
$H = -\sum_c$ 

$-\sum_v$ 

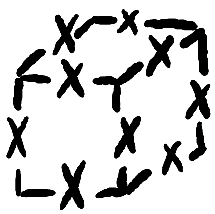
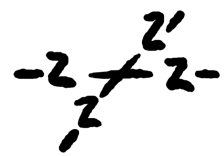
$+ \dots$

excitations:

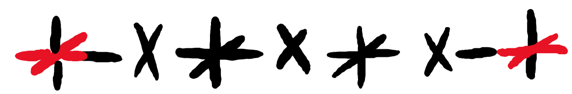


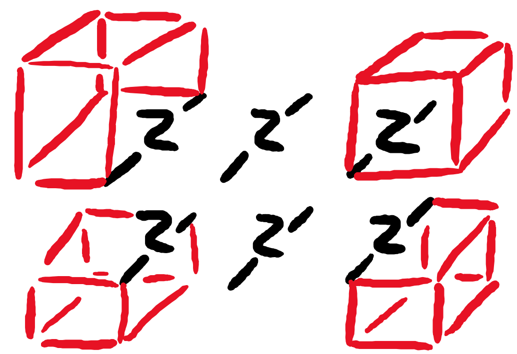


X-cube

$$H = -\sum_c$$  $$-\sum_v$$  $$+\ldots$$

Excitations:





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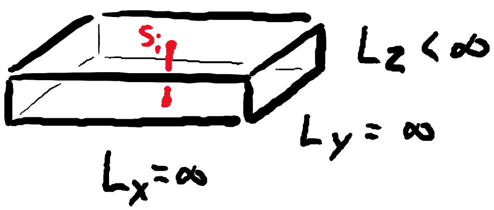
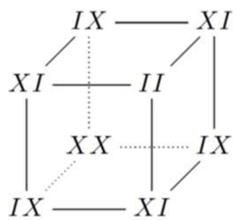
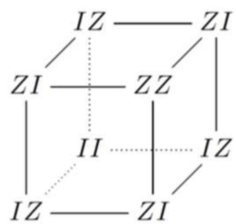


$$\sim \bigoplus_{s_i = \pm} (\text{Toric code})^{\otimes (L_z - 1)}$$

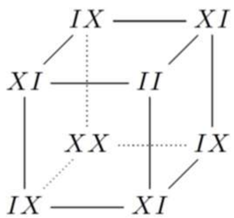
Cubic Code

$H = -\sum_{c}$ 

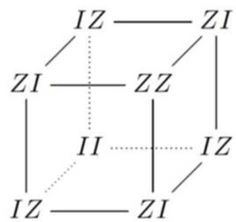
$+$ 

Cubic Code

$H = -\sum_c$



$+$



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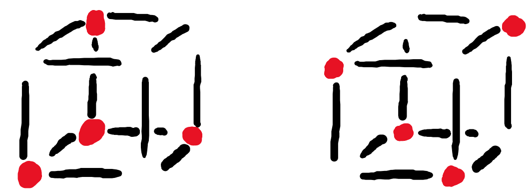

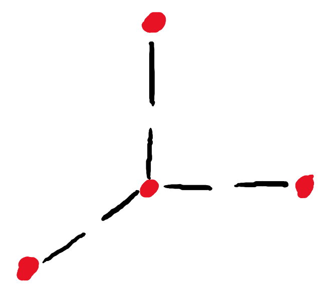

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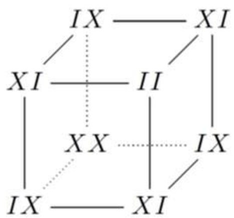
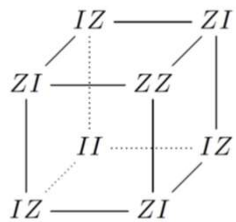
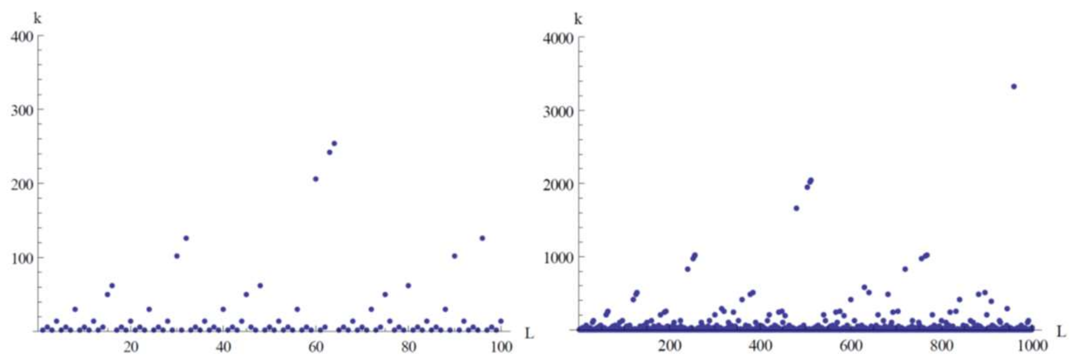
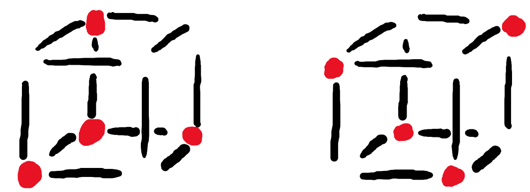
$H = -\sum_c$ 

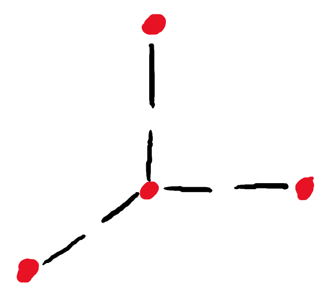
$+$ 



$k = \log$ GSD on $L \times L \times L$ torus, from Haah's thesis.

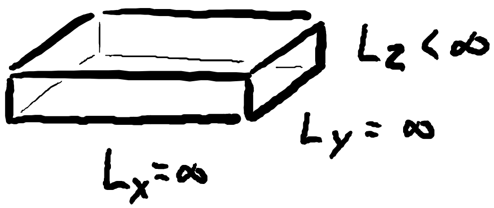
excitations:





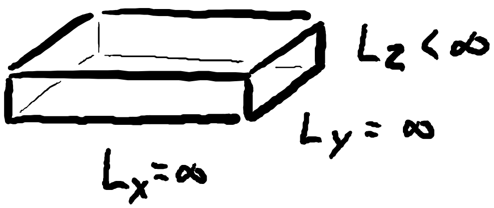
$$k = \begin{cases} 2, & L = 2^p + 1 \\ 4L - 2, & L = 2^p \end{cases}$$

Compactifying Cubic Code



$$\sim (\text{Toric code})^{\otimes n(L_z)}$$

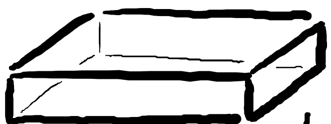
Compactifying Cubic Code



$$\sim \left(\text{Toric code} \right)^{\otimes n(L_z)}$$

Topological entanglement entropy =

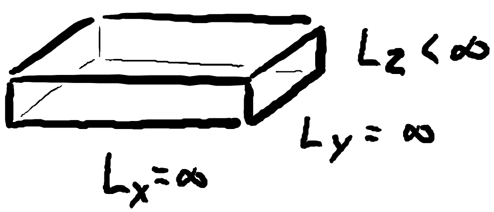
Compactifying Cubic Code



$\sim (\text{Toric code})^{\otimes n(L_z)}$

Topological entanglement entropy $= -2L_z$

Compactifying Cubic Code



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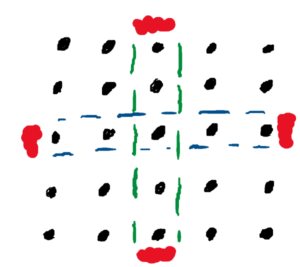
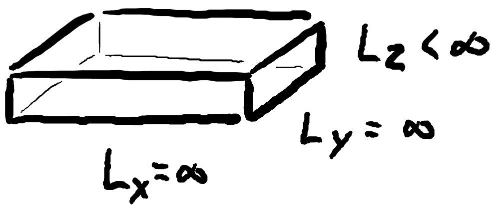
String operator pairs =



Compactifying Cubic Code

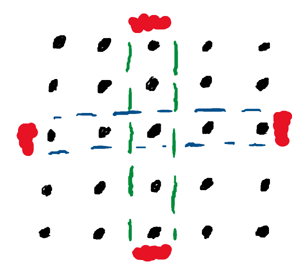


$$\sim \left(\text{Toric code}\right)^{\otimes n(L_z)}$$

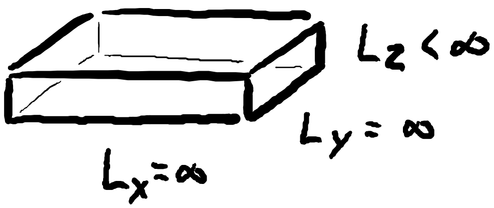
Topological entanglement entropy $= -2L_z$

String operator pairs $= \begin{cases} 2L_z, & j=0 \\ 2(L_z - 2^{i+1}), & j \neq 0 \end{cases}$

for $L_z = 2^i 3^j p$



Compactifying Cubic Code



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Topological entanglement entropy $= -2L_z$

$$\# \text{ String operator pairs} = \begin{cases} 2L_z & , j=0 \\ 2(L_z - 2^{i+1}) & , j \neq 0 \end{cases}$$

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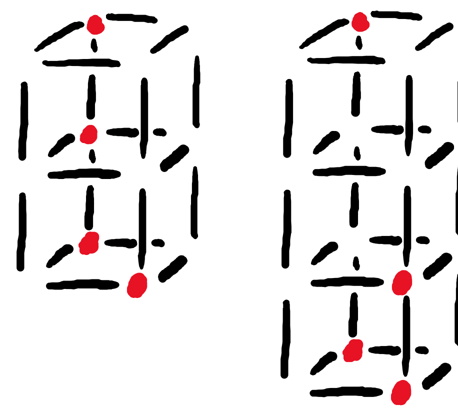


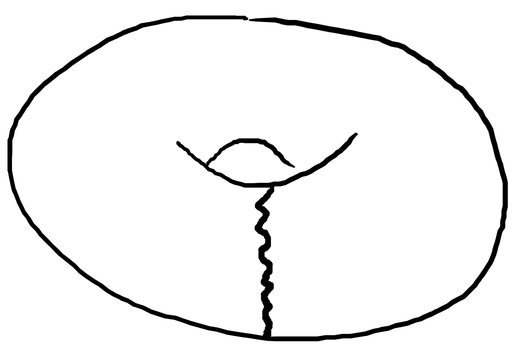
Compactifying Cubic Code

How to match GSD on 3D torus ?

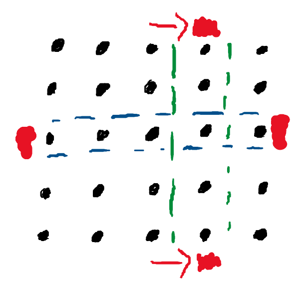
Compactifying Cubic Code

How to match GSD on 3D torus?

Twisted b.c. from nontrivial action of translation.



$L_x \neq k \cdot O_{T_x}$



Compactifying Cubic Code

O_{T_x} :   2   4   2   8   30

Compactifying Cubic Code

O_{T_x}:	2	4	2	8	30
	4	126	16	126	60

Compactifying Cubic Code

O_{T_x}:

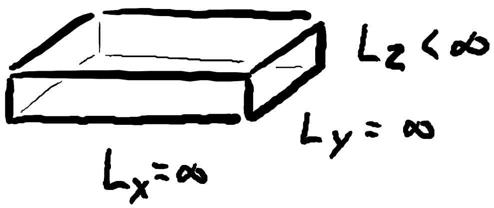
2	4	2	8	30
4	126	16	126	60
682	8	2730	252	30

Compactifying Cubic Code

$\mathcal{O}_{T_X}$:

$\mathcal{O}_{T_X}$:	2	4	2	8	30
	4	126	16	126	60
	682	8	2730	252	30
	32	510	252	19418	120
	126	1364			

Compactifying Cubic Code



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Topological entanglement entropy $= -2L_z$

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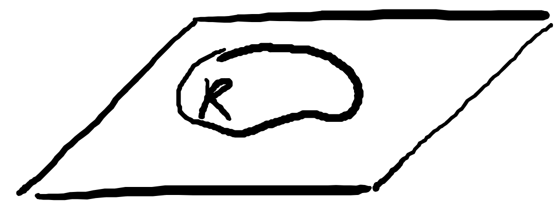
for $L_z = 2^i 3^j p$

what!?

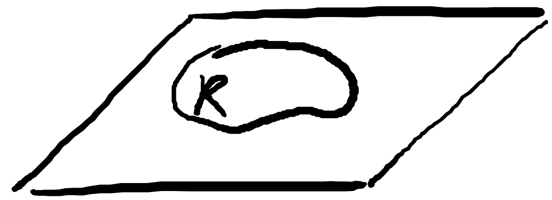
Spurious Topological Entanglement Entropy

$$S_R = a|\partial R| - \gamma + \ldots$$



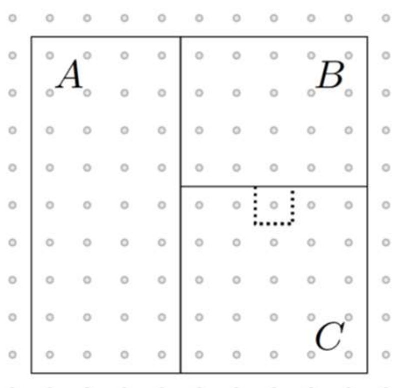
Spurious Topological Entanglement Entropy

$$S_R = a|\partial R| - \gamma + \dots$$

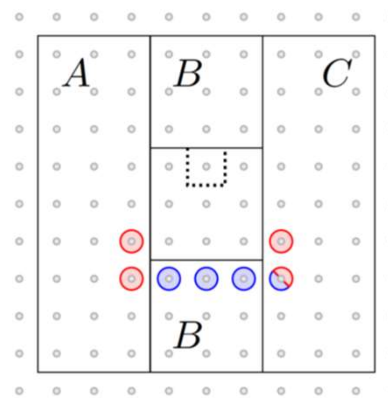


$$S_{topo}^{(1\text{ or }2)} = S_A + S_B + S_C - S_{AB} - S_{BC} - S_{AC} + S_{ABC} = -(1\text{ or }2)\,\gamma, \quad \gamma = \log D$$

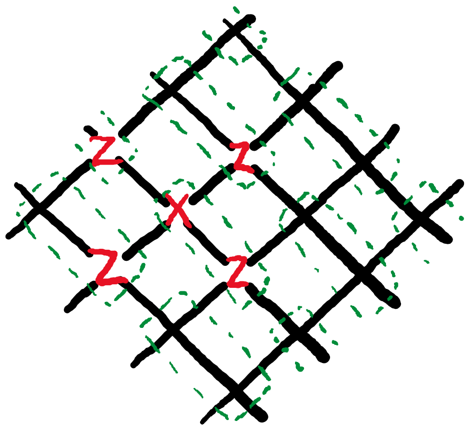
1.



2.



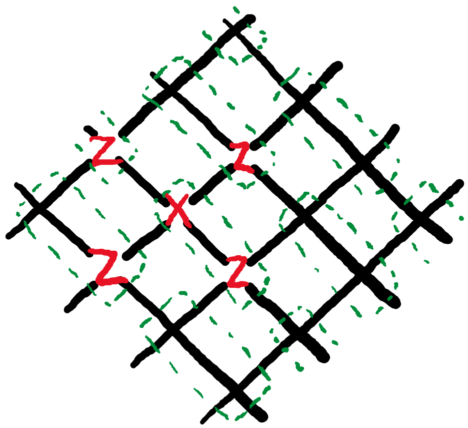
Cluster State



$$|\text{Cluster}\rangle = \prod_{\langle ij \rangle} CZ_{i,j} \, |+\rangle^{\otimes N}$$

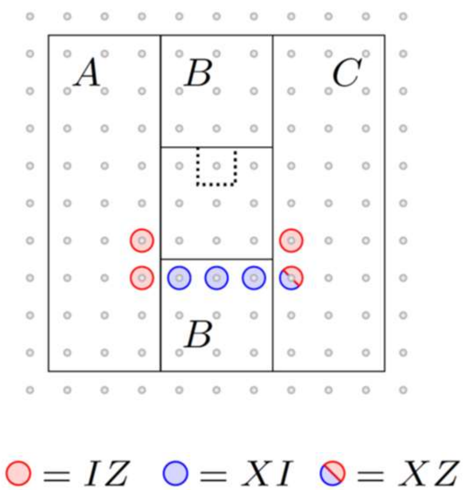
$$\Rightarrow \gamma = 0$$

Cluster State

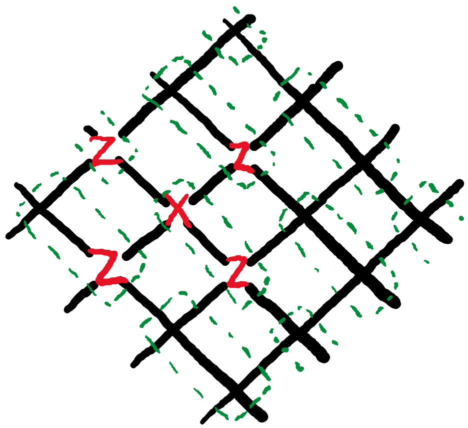


$$|\text{cluster}\rangle = \prod_{\langle ij\rangle} CZ_{i,j} \, |+\rangle^{\otimes N}$$

$$S^{(1)}_{topo} = -2 \, , \quad S^{(2)}_{topo} = -2$$



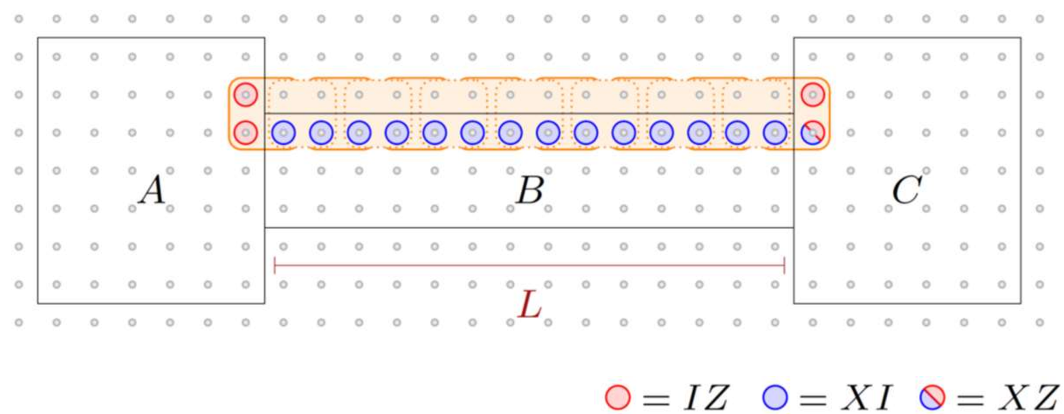
Cluster State



$$|\text{Cluster}\rangle = \prod_{\langle ij\rangle} CZ_{i,j}\, |+\rangle^{\otimes N}$$

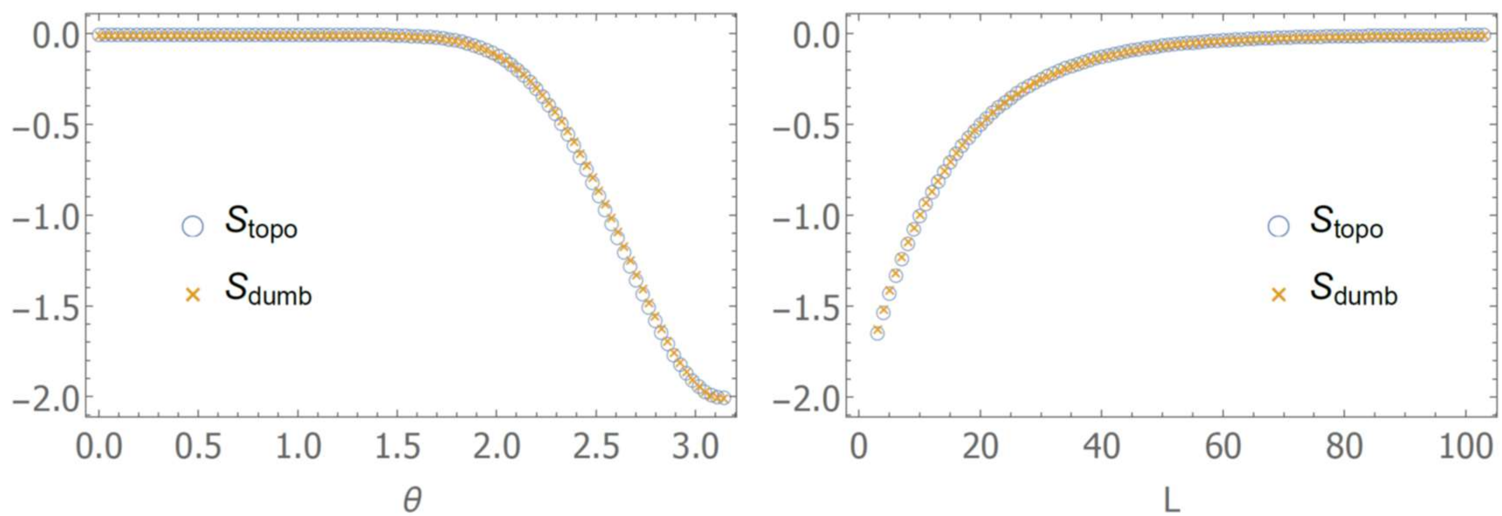
$$S^{(1)}_{\text{topo}} = -2 \quad , \quad S^{(2)}_{\text{topo}} = -2$$

$$S^{\text{dumb}}_{\text{topo}} = -2$$



Deformed Cluster State

$$|\theta\rangle = \prod_{\langle ij\rangle} Ce^{i\theta}_{i,j}\,|+\rangle^{\otimes N}\ , \quad |0\rangle = |+\rangle^{\otimes N}\ , \quad |\pi\rangle = |\text{cluster}\rangle\ .$$



In Summary

☆ Compactification provides useful info. on 3D TOs, particularly fracton models.

☆ Compactifying cubic code yields 2D toric code layers,

+ surprises: enriched by translation sym.

Subsystem sym. & spurious TEE.