Department of Physics and Astronomy

UCL

Driven-dissipative superfluids:

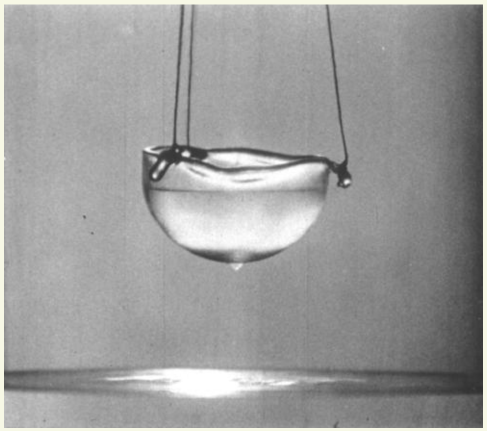
from a compact Kardar-Parisi-Zhang dynamics to a rigid state

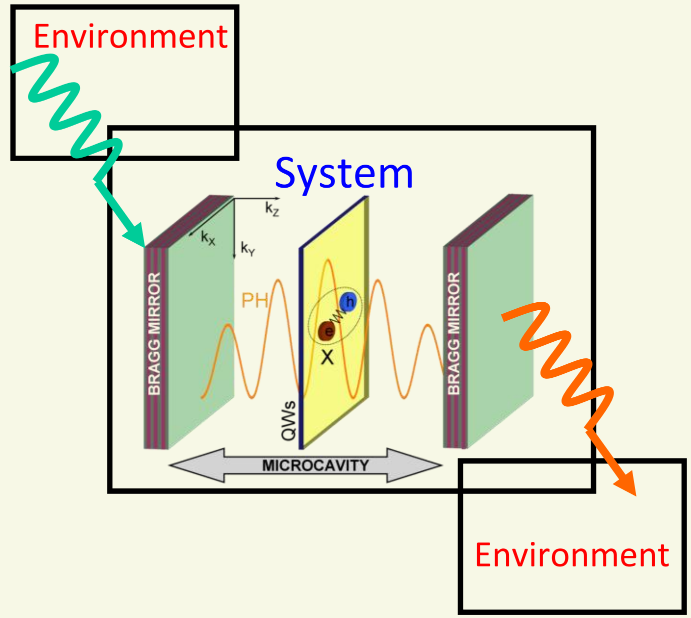
Marzena Szymańska

KITP, Aug 2018

Photonic Platforms: driven-dissipative quantum fluids

2D Light-matter quantum fluids with drive and decay



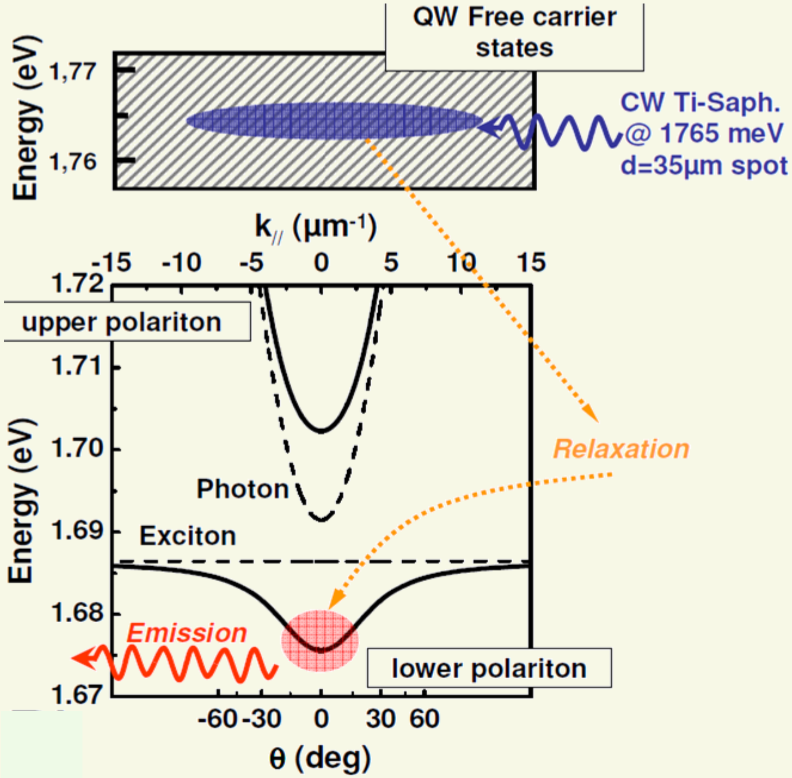


Polaritons
Photon BEC
Circuit QED systems
Atoms in cavities
....

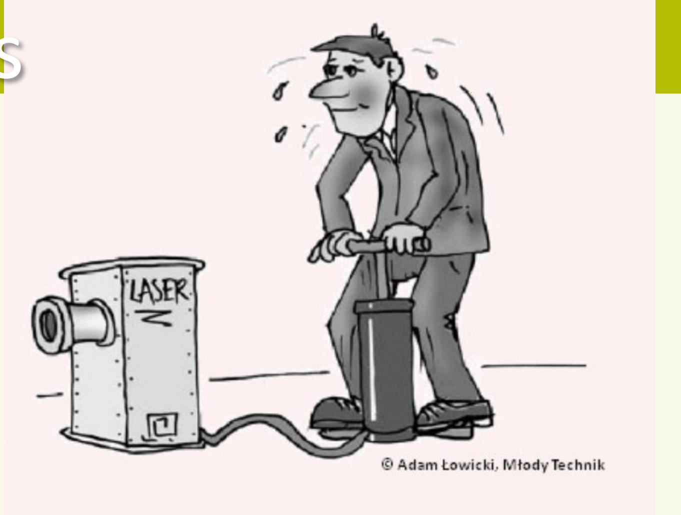
Different Drives

Incoherent

- ✓ Preserves U(1) symmetry
- ✓ Gapless Goldstone mode
- ✓ Violates particle number conservation (f-sum rule may not hold)

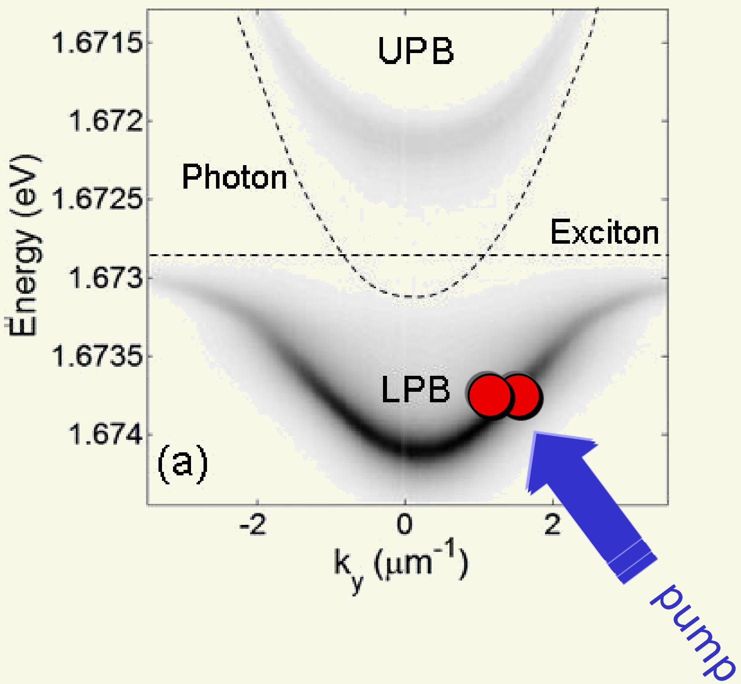


Different Drives



Coherent

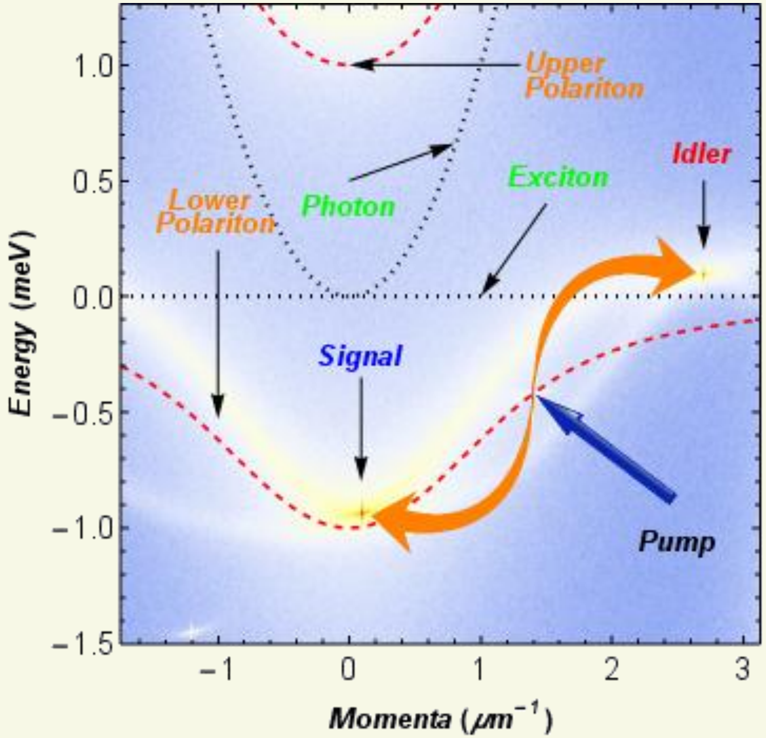
- ✓ Breaks U(1) symmetry
- ✓ No gapless Goldstone mode
- ✓ Violates particle number conservation (f-sum rule may not hold)
- ✓ Violates Galilean invariance



Different Drives

Parametric

- ✓ Preserves U(1) symmetry
- ✓ Gapless Goldstone mode
- ✓ Violates particle number conservation (f-sum rule may not hold)
- ✓ Spatial inhomogeneity – additional knob



Incoherently Driven Bosons in 2D

Generic model for incoherently driven-dissipative bosons

$$id\psi(\mathbf{r},t) = \left[-\frac{\nabla^2}{2m} + g|\psi(\mathbf{r},t)|^2 + i(\gamma - \kappa - \Gamma|\psi(\mathbf{r},t)|^2)\right]\psi(\mathbf{r},t)dt + dW$$

$$\frac{\gamma}{1+|\psi(\mathbf{r},t)|^2/n_S}$$ more non-linear drive

Wiener noise $$\langle dW^*(\mathbf{r}',t)dW(\mathbf{r},t)\rangle = \frac{\gamma+\kappa+\Gamma|\psi(\mathbf{r},t)|^2}{dV}\delta_{\mathbf{r},\mathbf{r}'}dt$$

$$\frac{\frac{\gamma}{1+|\psi(\mathbf{r},t)|^2/n_S}+\kappa}{dV}$$

Another model used in the context of semiconductor microcavities

$$id\psi(\mathbf{r},t) = \left[-\frac{\nabla^2}{2m} + g|\psi(\mathbf{r},t)|^2 + g_R n_R(\mathbf{r},t) + i(R\left[n_R(\mathbf{r},t)\right] - \kappa)\right]\psi(\mathbf{r},t)dt + dW$$

$$n_{\mathrm{R}}(\mathbf{r},t)t = P(\mathbf{r}) - \gamma_R n_R(\mathbf{r},t) - R[n_R(\mathbf{r},t)]|\psi(\mathbf{r},t)|^2$$

$$\langle dW^*(\mathbf{r}',t)dW(\mathbf{r},t)\rangle = \frac{R\left[n_R(\mathbf{r},t)\right]+\kappa}{dV}\delta_{\mathbf{r},\mathbf{r}'}dt$$

Free phase: gapless phase mode, all other excitations gapped

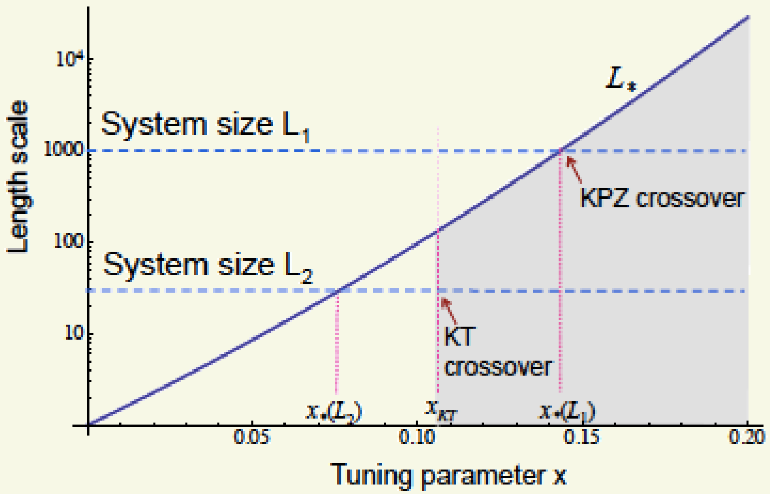
Kardar–Parisi–Zhang (KPZ) Theory

[Altman et al, *PRX* 2015]

Treating phase fluctuations exactly

$$\partial_t\theta = D_x\partial_x^2\theta + D_y\partial_y^2\theta + \boxed{\frac{\lambda_x}{2}(\partial_x\theta)^2 + \frac{\lambda_y}{2}(\partial_y\theta)^2} + \bar{\zeta}(\mathbf{x},t)$$

KPZ non-linearity



At large distances

$$\langle\phi^*(r)\phi(0)\rangle \sim e^{-r^{2\chi}}, \quad \chi \approx 0.37$$

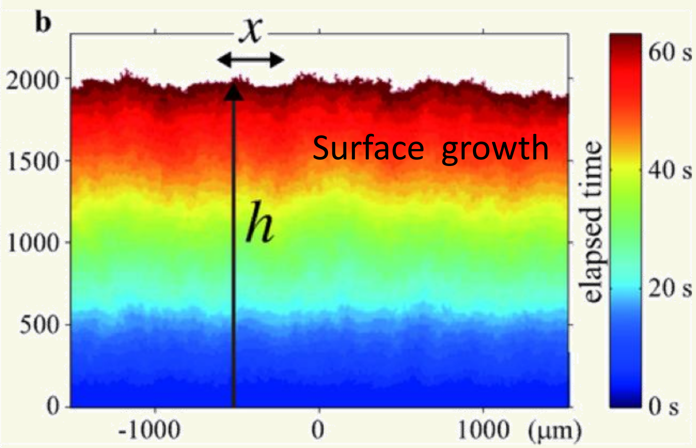
Stretched exponential (faster then algebraic) decay of coherence but superfluidity survives

KPZ – why interesting?

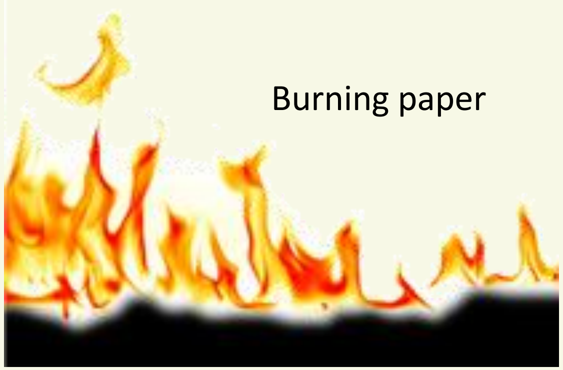
Solving KPZ equation   Martin Hairer, Warwick

2014 Fields Medal

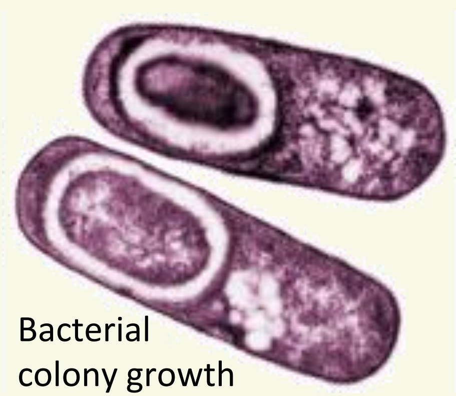
Universality class for a wide range of non-equilibrium phenomena in 1D



Takeuchi et al, 2011



J Maunuksela et al, PRL,1997



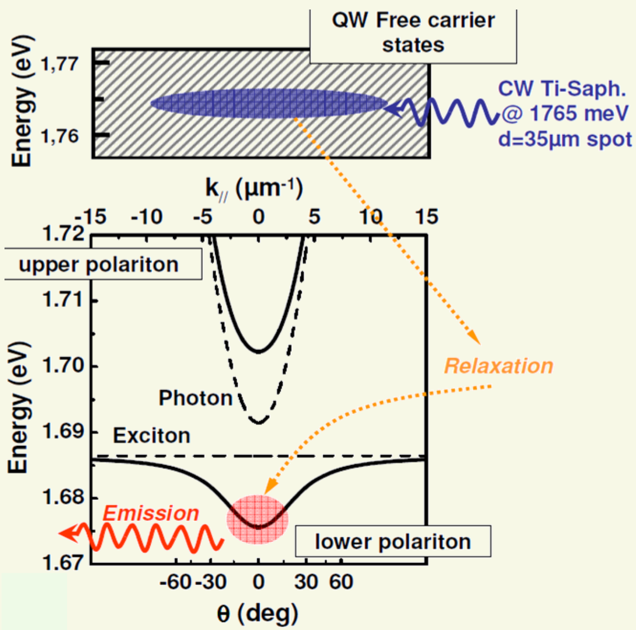
J Wakita et al, 1997

To date no experimental realisation of KPZ in 2D

KPZ Order with Polaritons

$$\partial_t\theta = D_x\partial_x^2\theta + D_y\partial_y^2\theta + \frac{\lambda_x}{2}(\partial_x\theta)^2 + \frac{\lambda_y}{2}(\partial_y\theta)^2 + \bar{\zeta}(\mathbf{x},t)$$

KPZ non-linearity



Incoherently pumped microcavity:

L_1 unrealistically large away from BKT

KPZ order

killed by vortices → exponential decay of correlations

or shows up on too large length scales → power-law decay of correlations within system size

Compact KPZ Equation and Vortices

Interactions between V and AV in KPZ equation

$$V(R) = \underbrace{\frac{1}{\epsilon}\log\left(\frac{R}{D_c}\right)}_{\text{Attractive as in XY model}} - \underbrace{\frac{a}{3\epsilon^3}\log^3\left(\frac{R}{D_c}\right)}_{\text{Repulsive or attractive depending on anisotropy}}$$

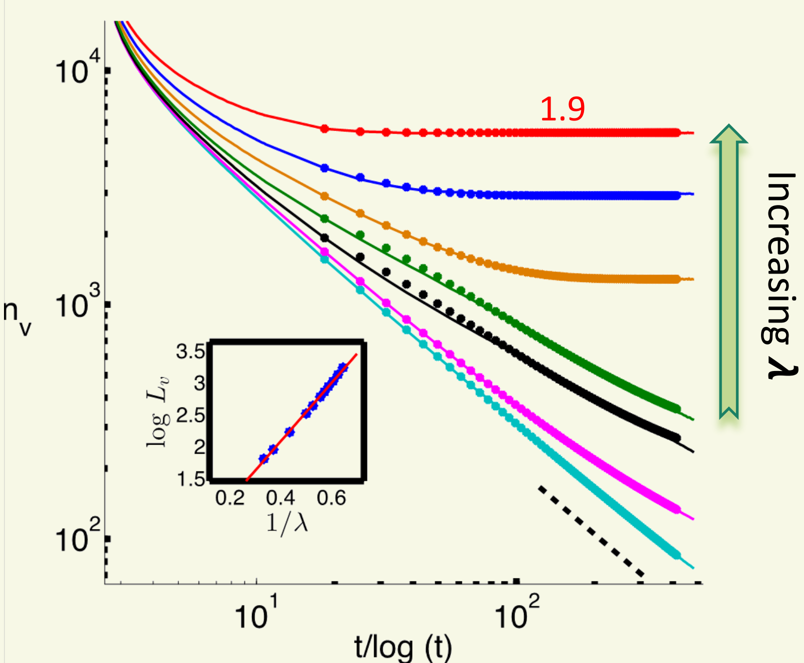
[Sieberer et al, 2018]

For isotropic system interactions repulsive at $L_v = D_c \exp\left(2\sqrt{\epsilon}D/\lambda\right)$

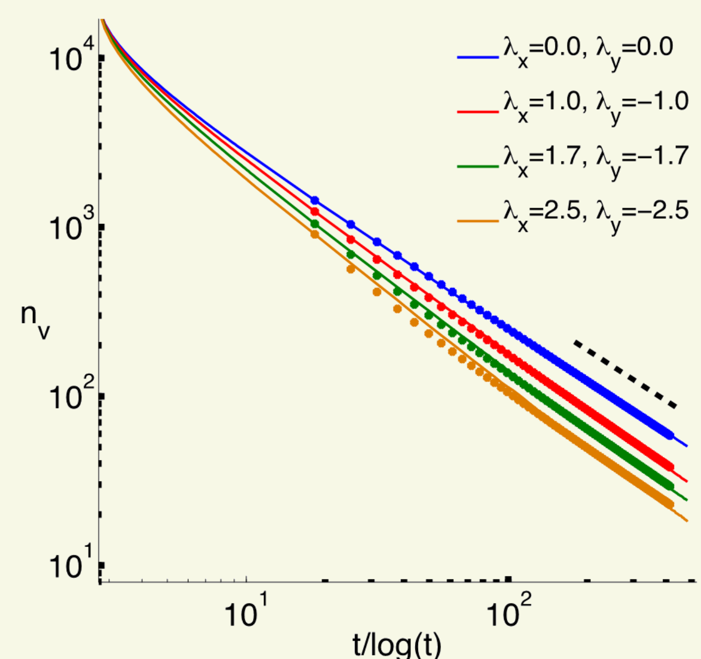
For strongly anisotropic interactions always attractive and enhanced with respect to non-driven system

Phase ordering kinetics after quench

Isotropic



Strongly Anisotropic



Compact KPZ Equation and Vortices

Interactions between V and AV in KPZ equation

$$V(R) = \underbrace{\frac{1}{\epsilon}\log\left(\frac{R}{D_c}\right)}_{\text{Attractive as in XY model}} - \underbrace{\frac{a}{3\epsilon^3}\log^3\left(\frac{R}{D_c}\right)}_{\text{Repulsive or attractive depending on anisotropy}}$$

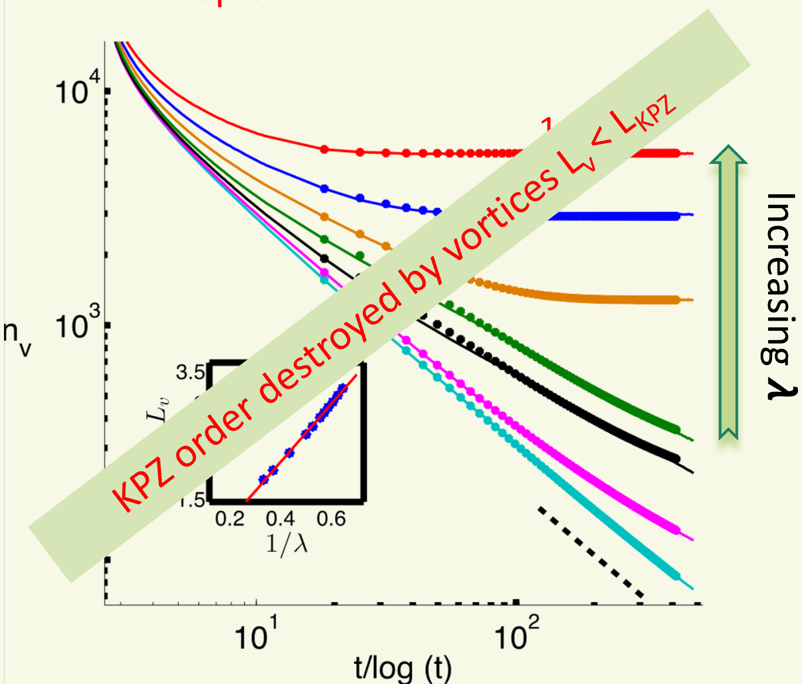
[Sieberer et al, 2018]

For isotropic system interactions repulsive at $L_v = D_c \exp\left(2\sqrt{\epsilon}D/\lambda\right)$

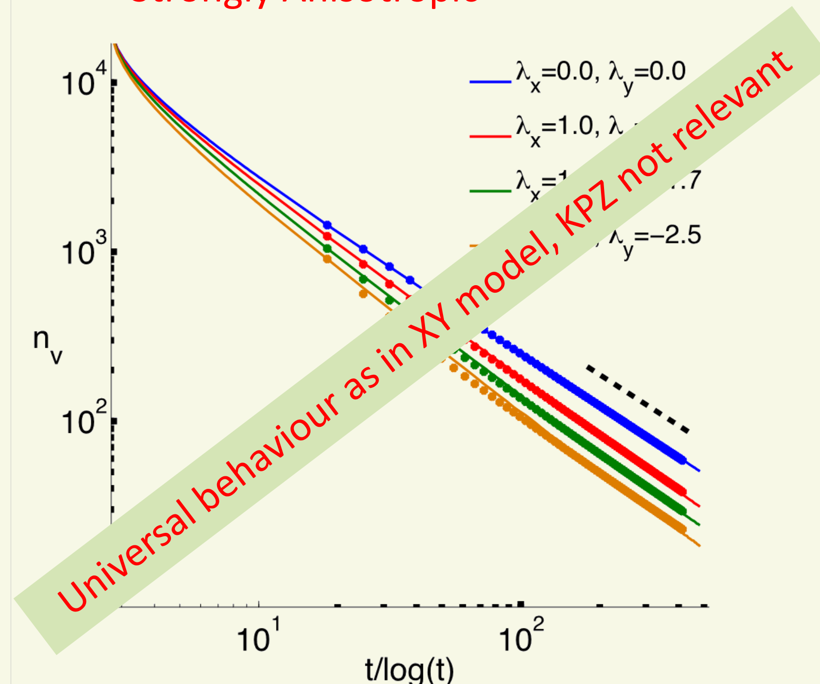
For strongly anisotropic interactions always attractive and enhanced with respect to non-driven system

Phase ordering kinetics after quench

Isotropic



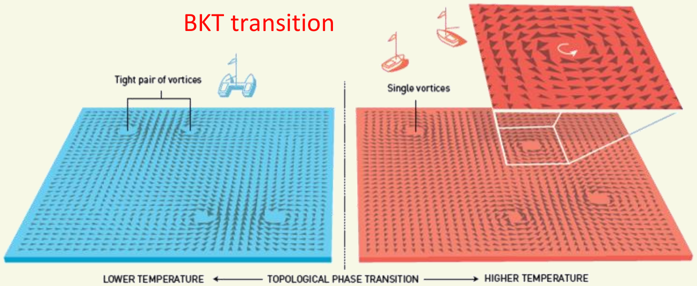
Strongly Anisotropic



Topological Phase Transition

$$id\psi(\mathbf{r},t) = \left[-\frac{\nabla^2}{2m} + g|\psi(\mathbf{r},t)|^2_- + i(\gamma - \kappa - \Gamma|\psi(\mathbf{r},t)|^2_-)\right]\psi(\mathbf{r},t)dt + dW$$

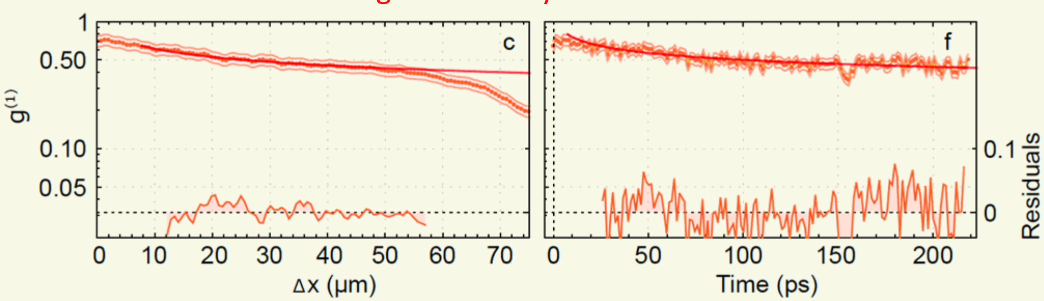
BKT transition



2016 Nobel Prize

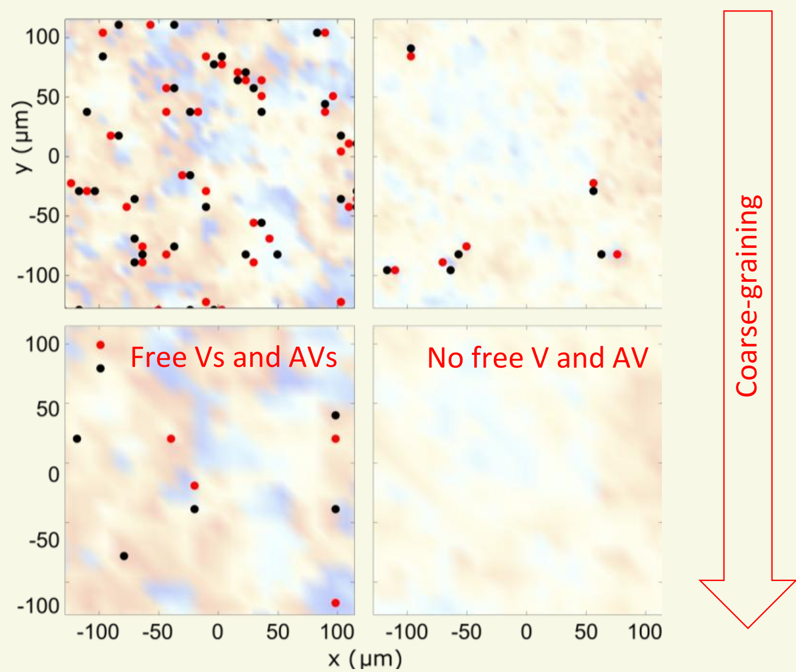
Equilibrium BKT in polariton fluid

Algebraic decay of coherence



$$|g^{(1)}(x,-x)| = A|2x|^{-\alpha}$$

$$\boxed{\alpha_s = \alpha_t < 1/4}$$



BKT crossover

[D. Caputo, Nature Materials (2017)]

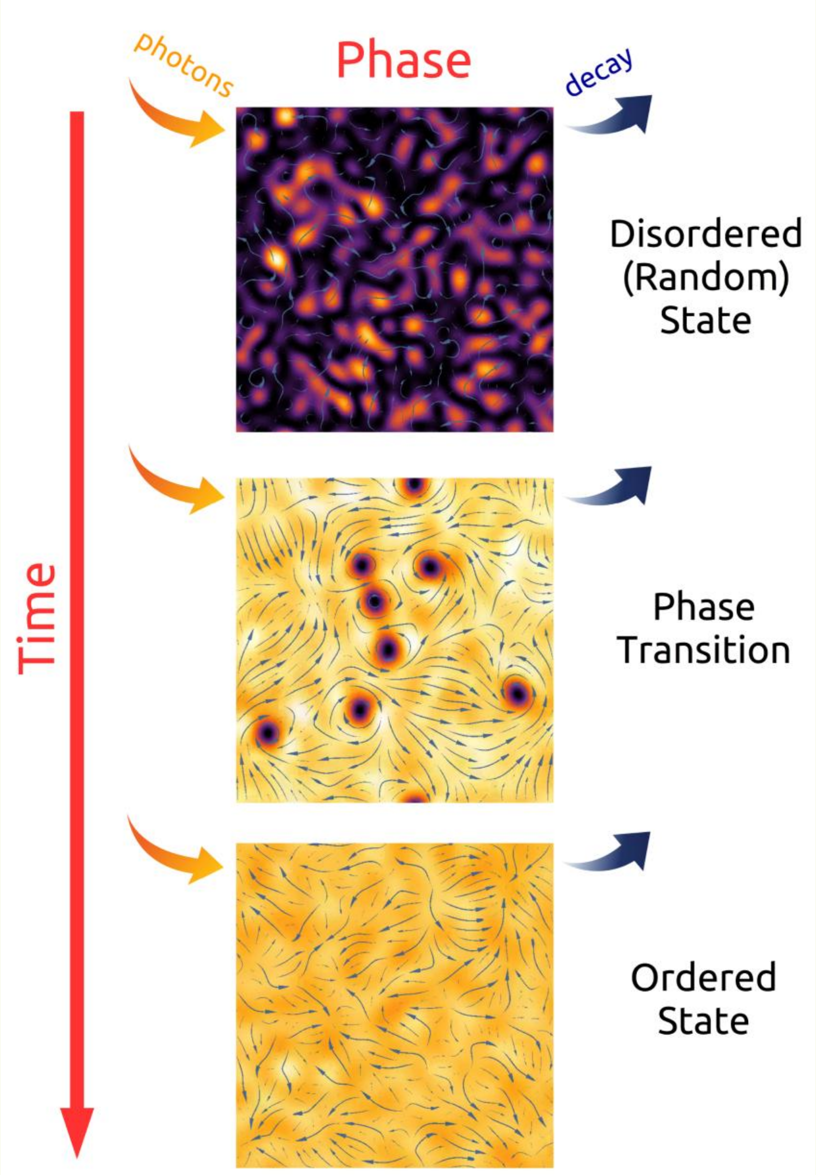
Quench Dynamics

[P. Comaron et al, PRL 2018]

Dynamics after infinitely fast quench

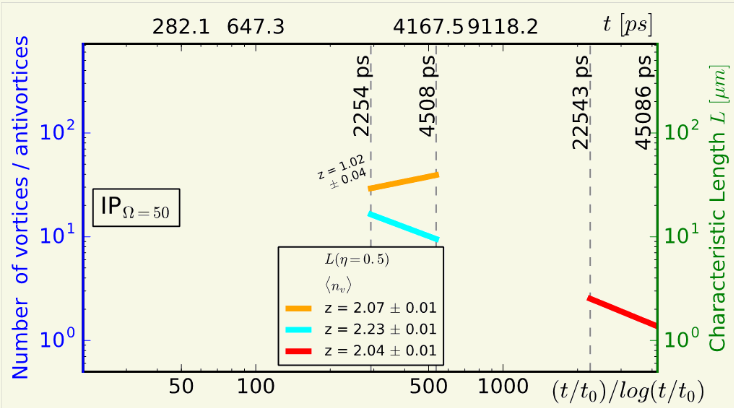
if universal can reveal the critical exponents of the transition

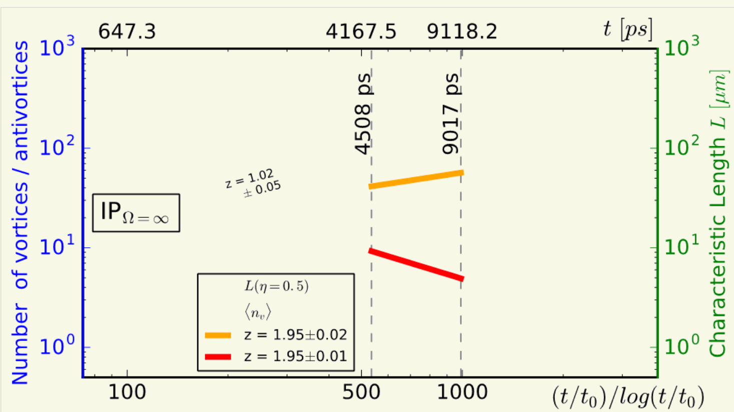
Fast change of pump strength

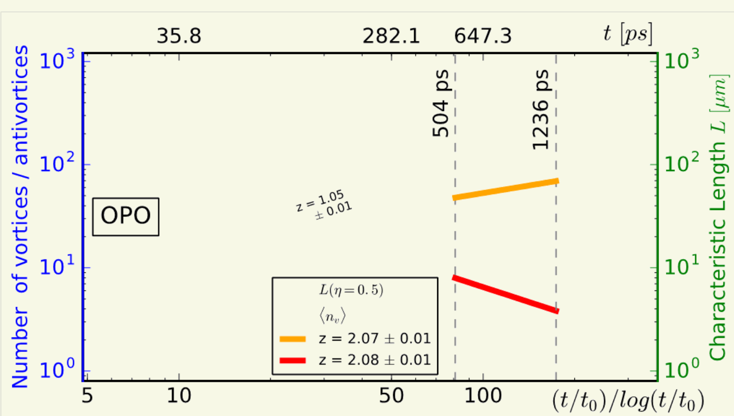


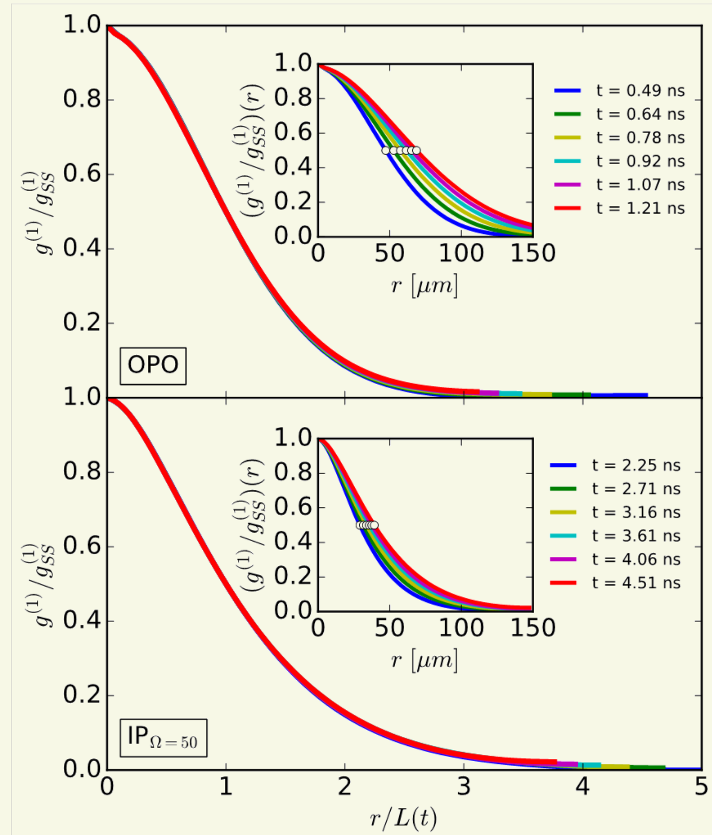
Dynamical Critical Exponents

[P. Comaron et al, PRL 2018]









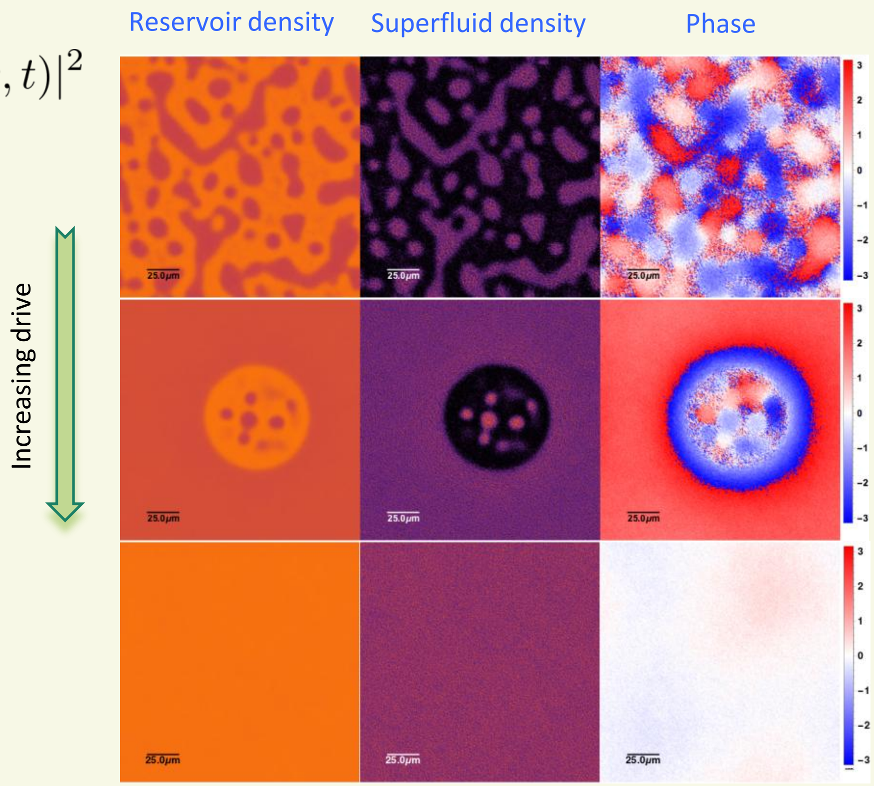
Critical exponent z=2

as in equilibrium XY model
i.e. equilibrium BKT

System with Reservoir

$$id\psi(\mathbf{r},t) = \left[-\frac{\nabla^2}{2m} + g|\psi(\mathbf{r},t)|^2 + g_R n_R(\mathbf{r},t) + i(R\left[n_R(\mathbf{r},t)\right] - \kappa)\right]\psi(\mathbf{r},t)dt + dW$$

$$n_{\mathrm{R}}(\mathbf{r},t)t = P(\mathbf{r}) - \gamma_R n_R(\mathbf{r},t) - R[n_R(\mathbf{r},t)]|\psi(\mathbf{r},t)|^2$$



Weak drive – disordered phase, g1(r) exponential

Medium drive – inhomogeneous superfluid, vortices pushed to low density regions, g1(r) exponential

Strong drive – homogeneous superfluid, g1(r) algebraic

1st order phase transition, phase separation, sudden jump in number of vortices and correlations – different to BKT

System with Reservoir

$$id\psi(\mathbf{r},t) = \left[-\frac{\nabla^2}{2m} + g|\psi(\mathbf{r},t)|^2 + g_R n_R(\mathbf{r},t) + i(R\left[n_R(\mathbf{r},t)\right] - \kappa)\right]\psi(\mathbf{r},t)dt + dW$$

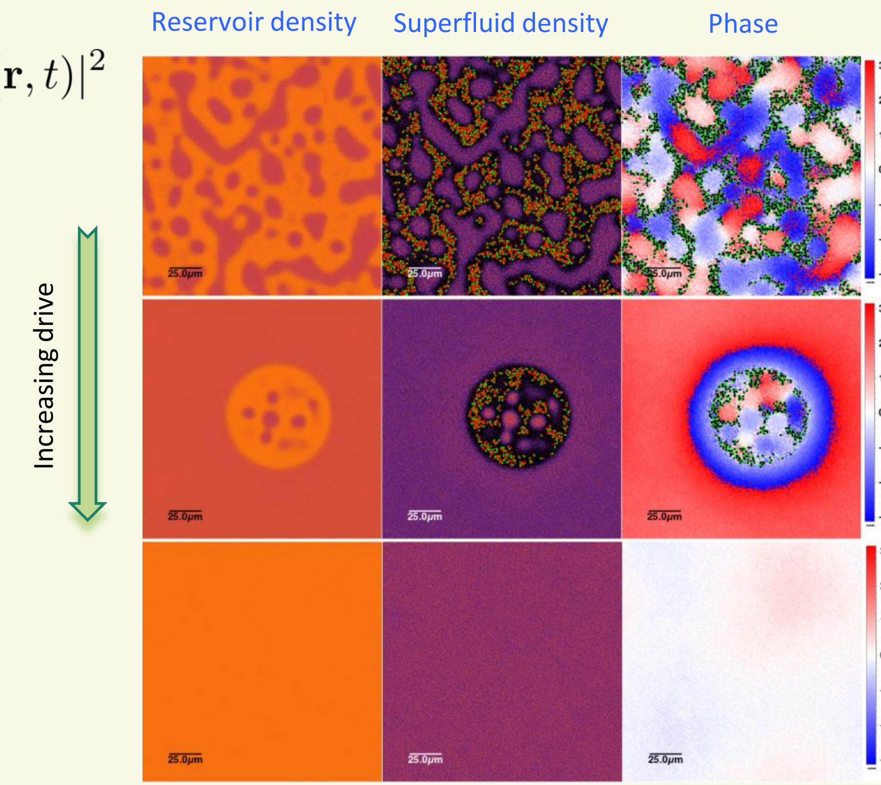
$$n_{\mathrm{R}}(\mathbf{r},t)t = P(\mathbf{r}) - \gamma_R n_R(\mathbf{r},t) - R[n_R(\mathbf{r},t)]|\psi(\mathbf{r},t)|^2$$

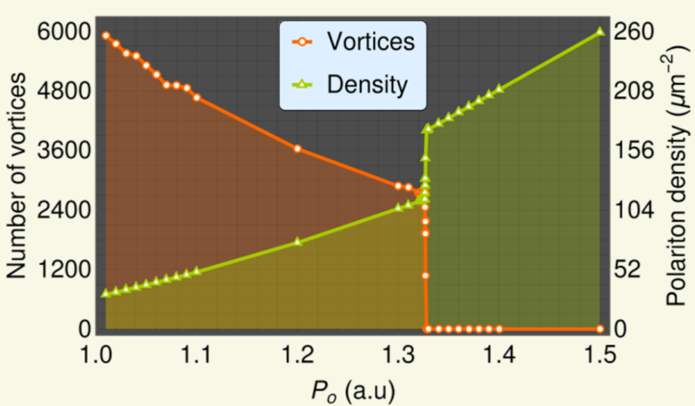
Weak drive – disordered phase, g1(r) exponential

Medium drive – inhomogeneous superfluid, vortices pushed to low density regions, g1(r) exponential

Strong drive – homogeneous superfluid, g1(r) algebraic

1st order phase transition, phase separation, sudden jump in number of vortices and correlations – different to BKT



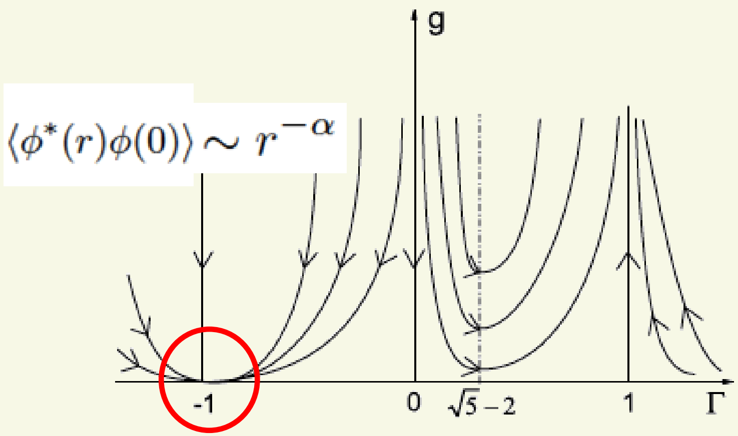


Possible BKT transition within superfluid paddles between weak and medium drives

Anisotropic KPZ

$$\partial_t\theta = D_x\partial_x^2\theta + D_y\partial_y^2\theta + \boxed{\frac{\lambda_x}{2}(\partial_x\theta)^2 + \frac{\lambda_y}{2}(\partial_y\theta)^2} + \bar{\zeta}(\mathbf{x},t)$$

KPZ non-linearity



$$\Gamma = \frac{\lambda_y D_x}{\lambda_x D_y}$$

$$g = \frac{\lambda_x^2\Delta}{D_x^2\sqrt{D_xD_y}}$$

Spatial anisotropy could help

Anisotropic system flows to a Gaussian fixed point:
power-law at any scale
g which defines KPZ lengthscale depends on anisotropy:
perhaps we can make L_1 smaller?

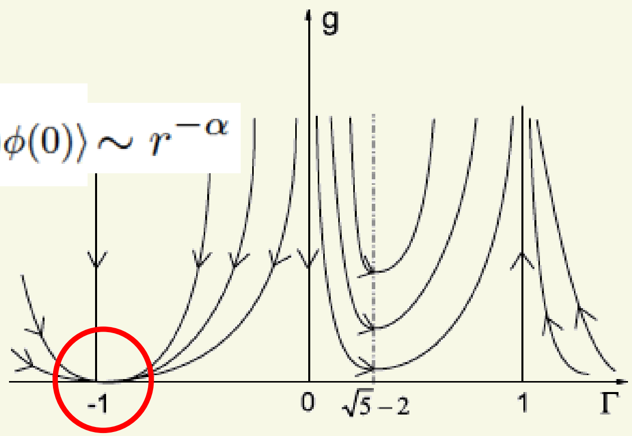
How to realise in experiment?

Anisotropic KPZ

$$\partial_t\theta = D_x\partial_x^2\theta + D_y\partial_y^2\theta + \boxed{\frac{\lambda_x}{2}(\partial_x\theta)^2 + \frac{\lambda_y}{2}(\partial_y\theta)^2} + \bar{\zeta}(\mathbf{x},t)$$

KPZ non-linearity

$$\langle\phi^*(r)\phi(0)\rangle \sim r^{-\alpha}$$

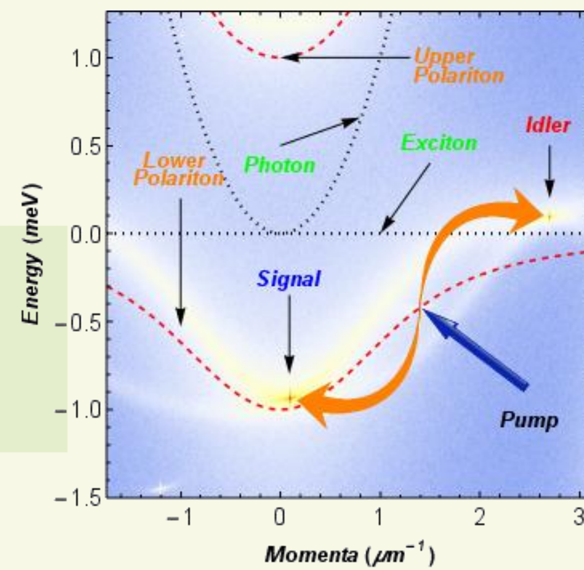


$$\Gamma = \frac{\lambda_y D_x}{\lambda_x D_y}$$

$$g = \frac{\lambda_x^2\Delta}{D_x^2\sqrt{D_x D_y}}$$

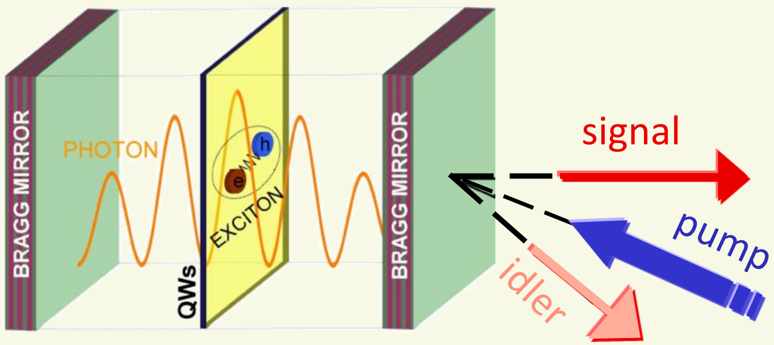
Spatial anisotropy could help

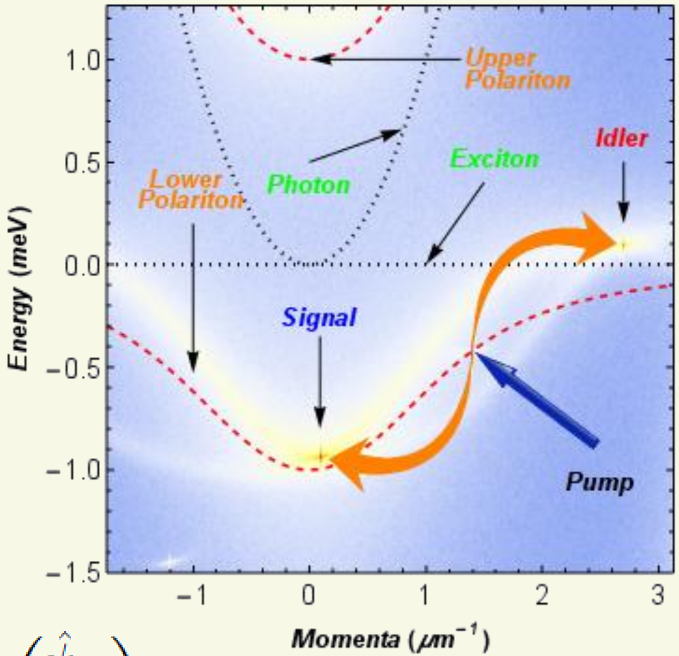
Anisotropic system flows to a Gaussian fixed point:
power-law at any scale
g which defines KPZ lengthscale depends on anisotropy:
perhaps we can make L_1 smaller?



ent?

Playing with Spatial Anisotropy: OPO





$$\hat{H}_S = \int d\mathbf{r} \, (\hat{\psi}_X^\dagger \;\; \hat{\psi}_C^\dagger) \begin{pmatrix} \frac{-\nabla^2}{2m_X} + \frac{g_X}{2}|\hat{\psi}_X|^2 & \frac{\Omega_R}{2} \\ \frac{\Omega_R}{2} & \frac{-\nabla^2}{2m_C} \end{pmatrix} \begin{pmatrix} \hat{\psi}_X \\ \hat{\psi}_C \end{pmatrix}$$

$$\hat{H}_{SB} = \int d\mathbf{r} \left[F(\mathbf{r},t)\hat{\psi}_C^\dagger(\mathbf{r},t) + \text{H.c.} \right]$$

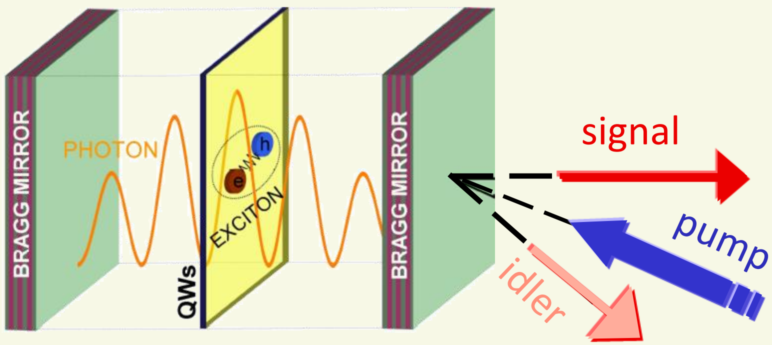
$$+ \sum_{\mathbf{k}} \sum_{l=X,C} \left\{ \zeta_{\mathbf{k}}^l \left[\hat{\psi}_{l,\mathbf{k}}^\dagger(t) \hat{B}_{l,\mathbf{k}} + \text{H.c.} \right] + \omega_{l,\mathbf{k}} \hat{B}_{l,\mathbf{k}}^\dagger \hat{B}_{l,\mathbf{k}} \right\}$$

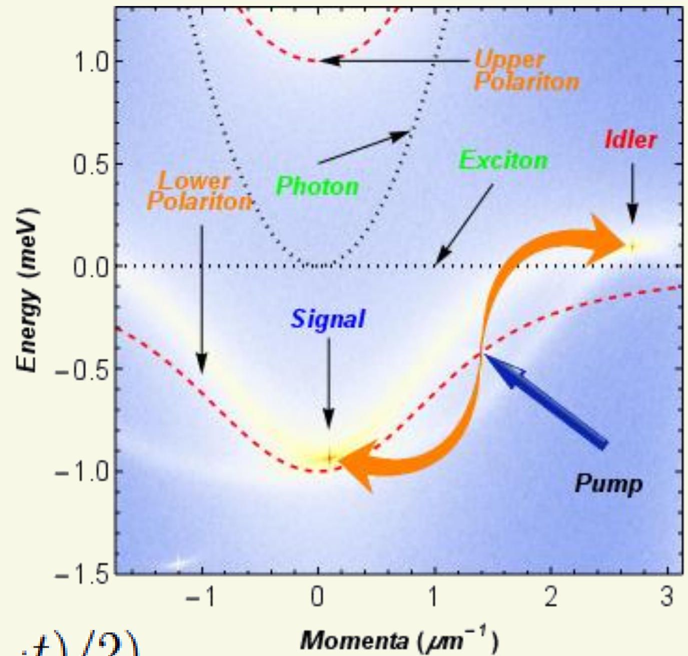
- ✧ Non-thermal occupation
- ✧ Signal phase is completely free and idler phase locked to signal via pump

$$2\varphi_p = \varphi_s + \varphi_i$$

- ✧ Spontaneous U(1) symmetry breaking gapless and diffusive Goldstone mode

Playing with Spatial Anisotropy: OPO

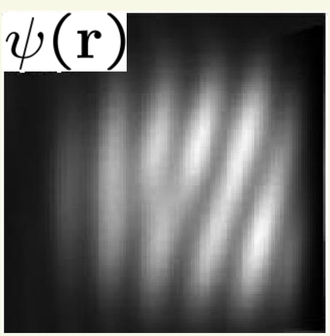


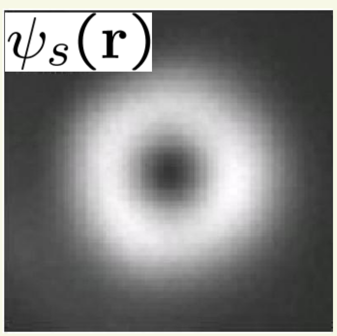


$$
\begin{aligned}
|\psi_{\mathrm{LP}}(\mathbf{r},t)|^2 &= \sum_j \rho_j^2 \\
&+ 2\left(\sqrt{\rho_s\rho_p}\cos(\phi_s - \phi_p + (\mathbf{k}_{si}\cdot\mathbf{r} - \omega_{si}t)/2)\right. \\
&\quad + \sqrt{\rho_p\rho_i}\cos(\phi_i - \phi_p - (\mathbf{k}_{si}\cdot\mathbf{r} - \omega_{si}t)/2) \\
&\quad\quad \left. + \sqrt{\rho_s\rho_i}\cos(\phi_s - \phi_i + \mathbf{k}_{si}\cdot\mathbf{r} - \omega_{si}t)\right)
\end{aligned}
$$

- ✧ Time crystal
- ✧ Vortices: dislocations in density wave and time crystal

- ✧ After filtering in momentum: usual vortices

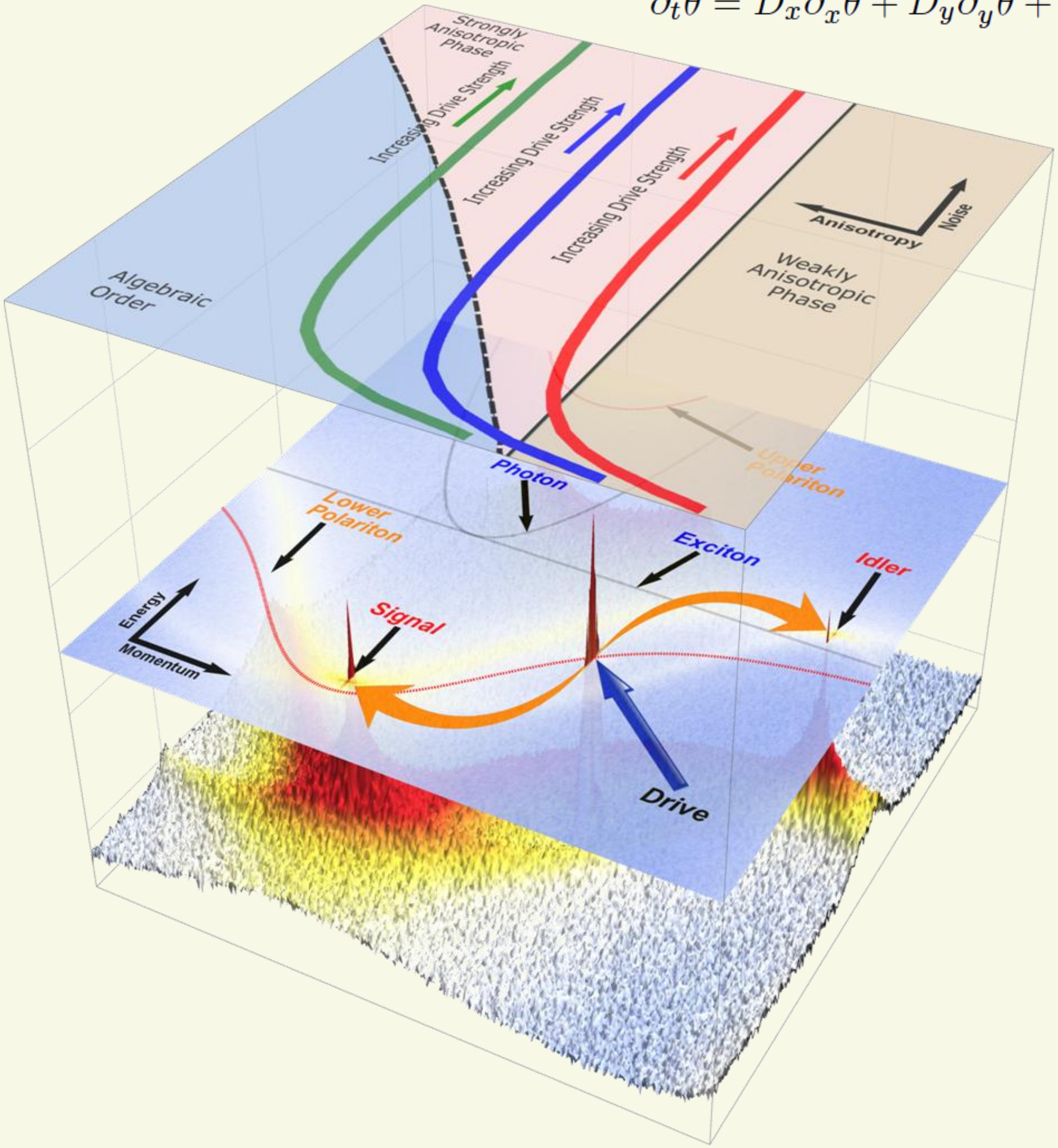




Tunning Across Universalities with OPO

[A. Zamora et al., PRX, 2017]

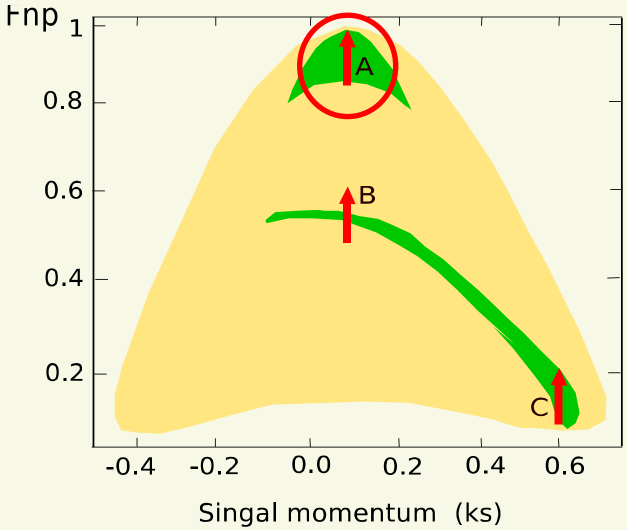
$$\partial_t\theta = D_x\partial_x^2\theta + D_y\partial_y^2\theta + \frac{\lambda_x}{2}(\partial_x\theta)^2 + \frac{\lambda_y}{2}(\partial_y\theta)^2 + \bar{\zeta}(\mathbf{x},t)$$



Negative detuning

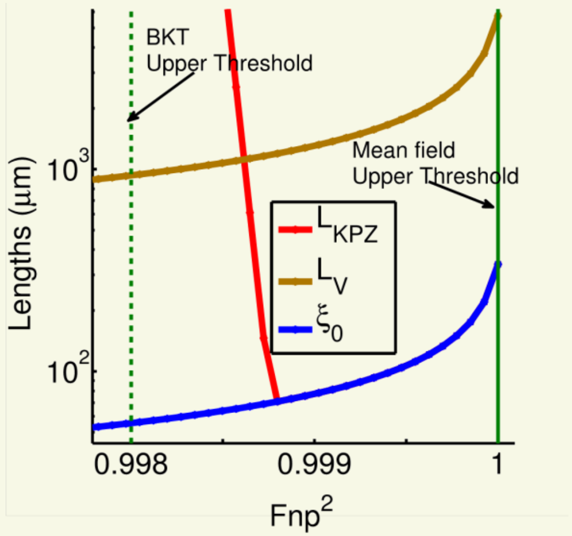
By increasing drive we move from non-equilibrium to equilibrium fixed point

Two different universality classes as the drive is increased



Searching for the KPZ Phase

[A. Zamora et al., PRX, 2017]



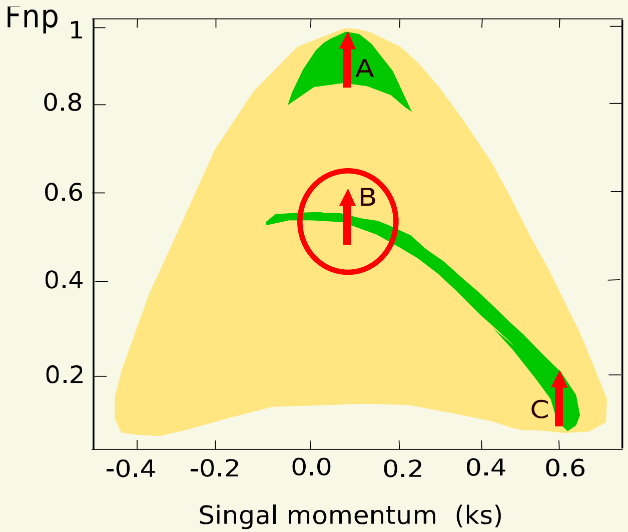
Away from threshold:

L_{KPZ} astronomical

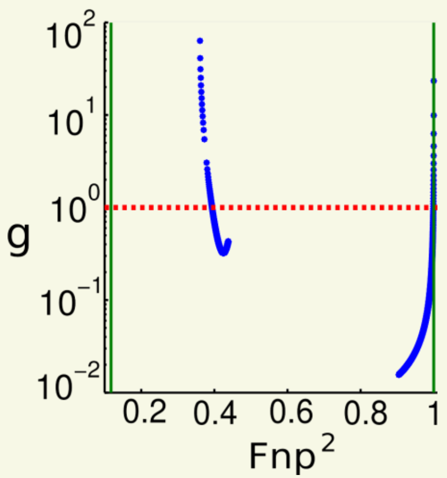
Very close to threshold

L_{KPZ} reasonable and $L_{KPZ} < L_V$ only extremely close to threshold i.e. below BKT transition

Note: analytics not valid in this regime



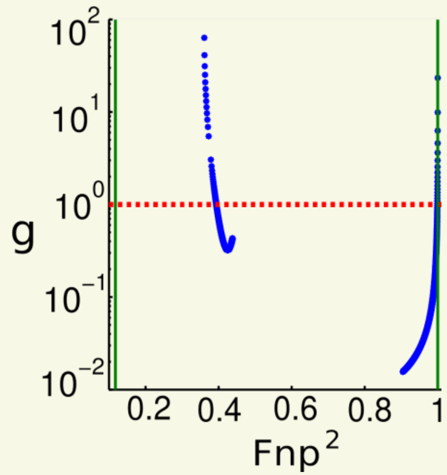
Middle of the OPO regime



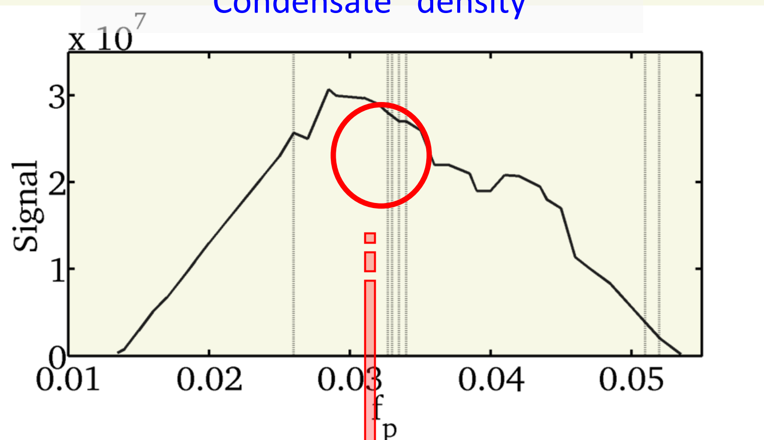
Large g, even > 1

KPZ at all length-scales?

Searching for the KPZ Phase

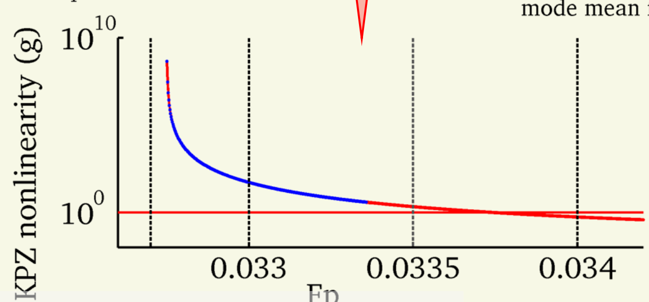


"Condensate" density

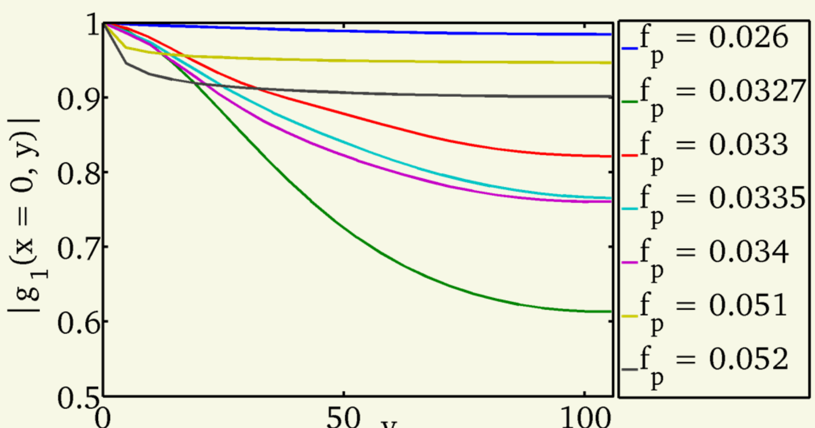


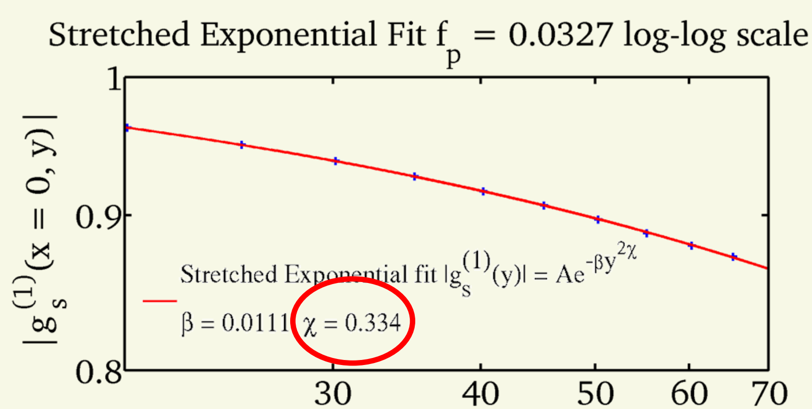
kp=1.4, ks=0.1191, γ =0.045, $\hbar\Omega_R$= 4.4meV,
Hopfield coefficients=1 for all the modes

Red: stable KPZ
Blue: stable 3 mode mean field

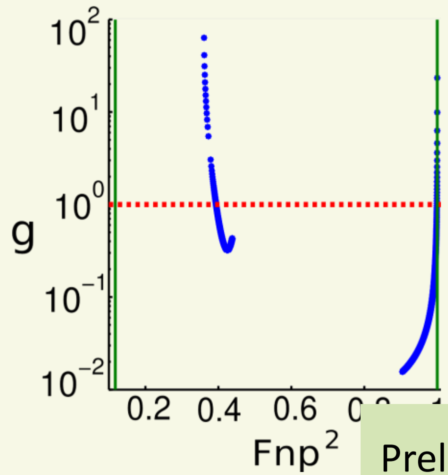


First order spatial coherence g1(r)

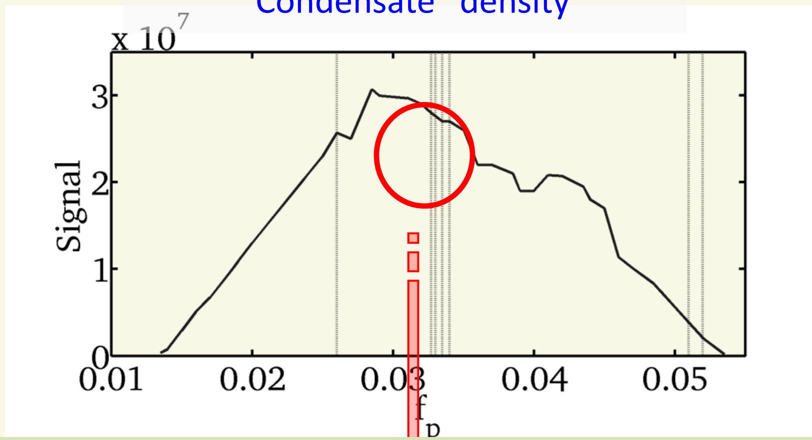




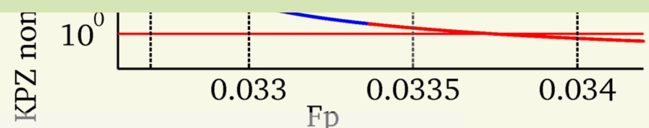
Searching for the KPZ Phase



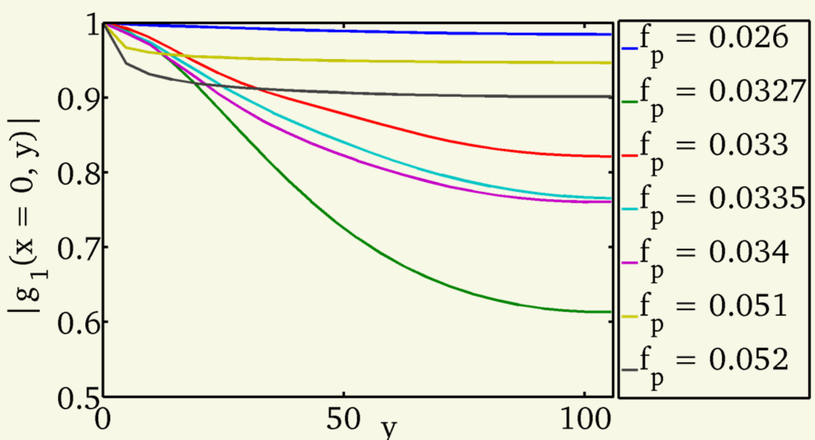
"Condensate" density

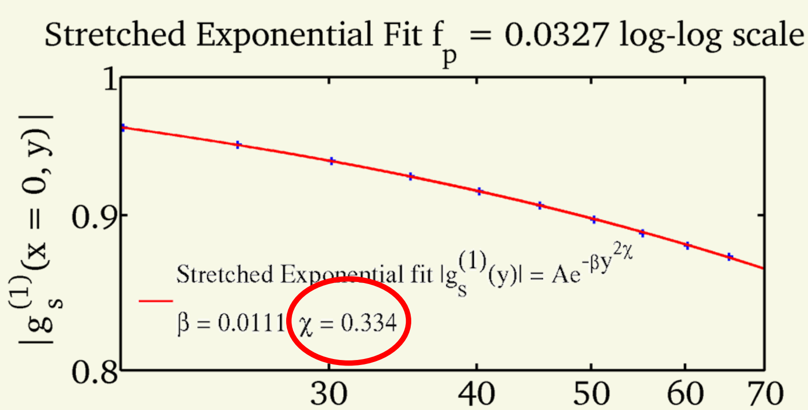


Preliminary numerical analysis of the microscopic equations for the OPO at intermediate drives are consistent with the KPZ scaling



First order spatial coherence g1(r)



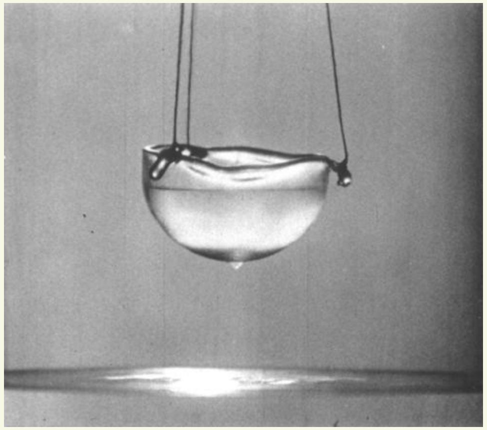


Flow Properties of Quantum Fluids

What is a superfluid?

Defined by flow properties:

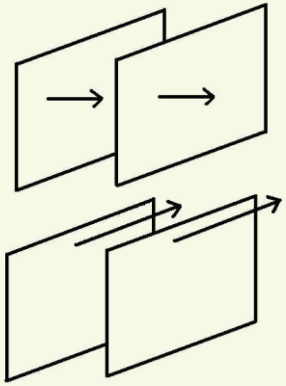
- No viscosity
- No transverse response
- Quantised vortices
- Metastable persistent flow



Current-current response function: $\delta j_i(\boldsymbol{q}) = \chi_{ij}(\boldsymbol{q})\delta f_j(\boldsymbol{q})$

Long wavelength limit:

- Transverse direction first: longitudinal response
- Longitudinal direction first: transverse response

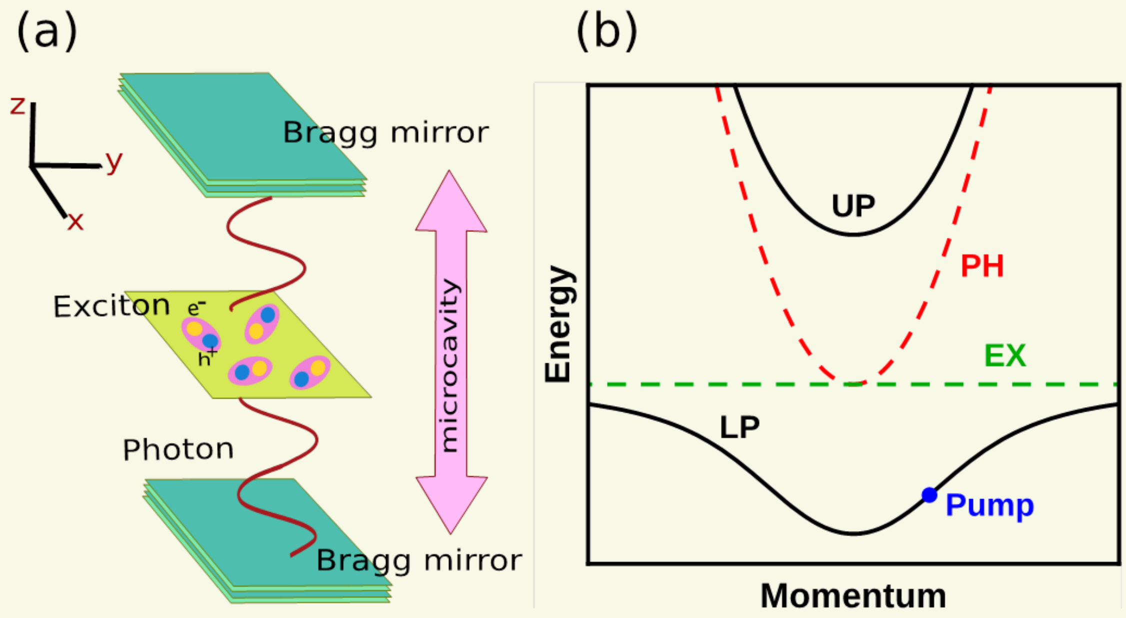


Superfluid component responds to longitudinal but not transverse perturbations

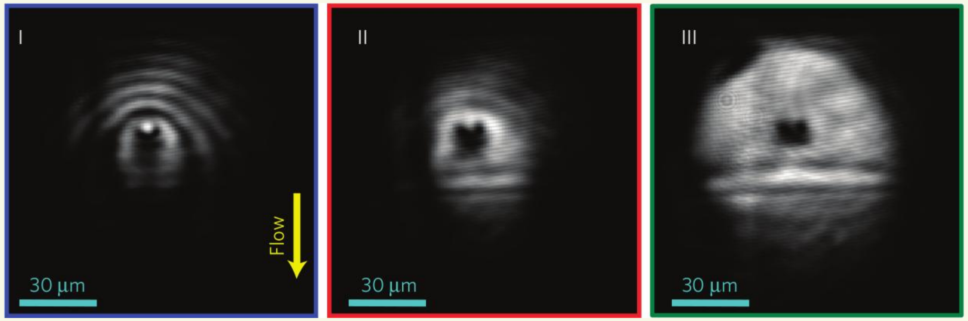
Superfluid density:

$$\rho_S = m \lim_{\boldsymbol{q}\to\boldsymbol{0}} \left(\chi_L(\boldsymbol{q}) - \chi_T(\boldsymbol{q})\right)$$

Coherently Driven Polaritons



Nearly dissipationless flow observed



Amo et al. Nature Phys. 5 (2009)

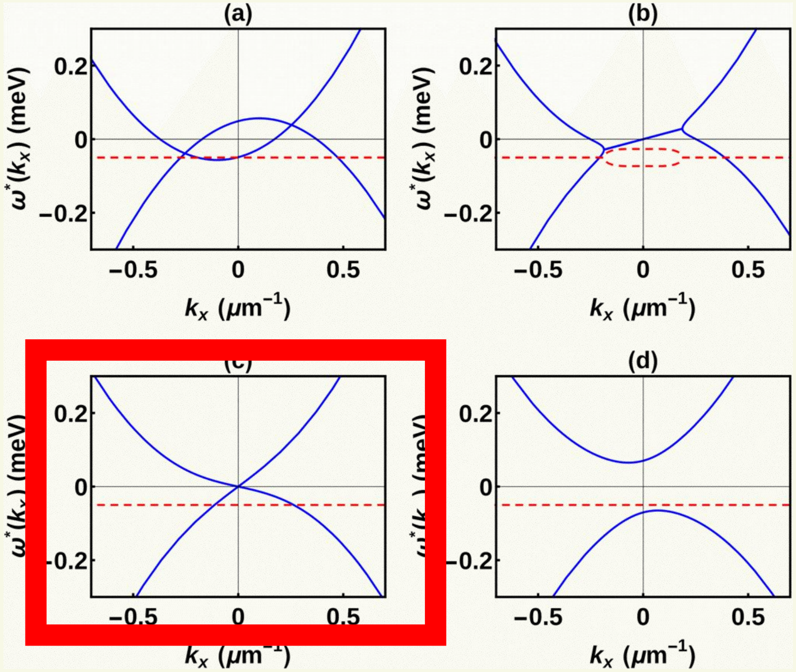
Increasing Drive

Coherently Driven Polaritons

[R. Juggins et al., Nature Comm. to appear, 2018]

$$\hat{H} = \sum_{\boldsymbol{k}} \epsilon_{\boldsymbol{k}} \hat{a}^{\dagger}_{\boldsymbol{k}} \hat{a}_{\boldsymbol{k}} + \frac{F_p}{\sqrt{2}} (\hat{a}^{\dagger}_{0} + \hat{a}_{0}) + \frac{V}{2} \sum_{\boldsymbol{k},\boldsymbol{k}',\boldsymbol{q}} \hat{a}^{\dagger}_{\boldsymbol{k}-\boldsymbol{q}} \hat{a}^{\dagger}_{\boldsymbol{k}'+\boldsymbol{q}} \hat{a}_{\boldsymbol{k}} \hat{a}_{\boldsymbol{k}'} + \sum_{\boldsymbol{p}} \omega^{A}_{\boldsymbol{p}} \hat{A}^{\dagger}_{\boldsymbol{p}} \hat{A}_{\boldsymbol{p}} + \sum_{\boldsymbol{k},\boldsymbol{p}} \zeta_{\boldsymbol{k},\boldsymbol{p}} (\hat{a}^{\dagger}_{\boldsymbol{k}} \hat{A}_{\boldsymbol{p}} + \hat{A}^{\dagger}_{\boldsymbol{p}} \hat{a}_{\boldsymbol{k}})$$

$$\omega^{*,\pm}_{\boldsymbol{k}} = \frac{\alpha^{+}_{\boldsymbol{k}} - \alpha^{-}_{\boldsymbol{k}}}{2} - i\kappa \pm \frac{1}{2}\sqrt{(\alpha^{+}_{\boldsymbol{k}} + \alpha^{-}_{\boldsymbol{k}})^2 - 4V^2 \left|\psi_0\right|^4}$$



- Phase fixed by the pump so

  spectrum always gapped

- Blue detuning $\Delta = V\left|\psi_0\right|^2 \rightarrow$ Landau criterion fulfilled in real part (c)

Coherently Driven Polaritons

[R. Juggins et al., Nature Comm. to appear, 2018]

Superfluid density from Keldysh: $\rho_S = m \lim_{\boldsymbol{q}\to\boldsymbol{0}} (\chi_L(\boldsymbol{q}) - \chi_T(\boldsymbol{q}))$ = 0

Coherently pumped polaritons are not superfluid

but …

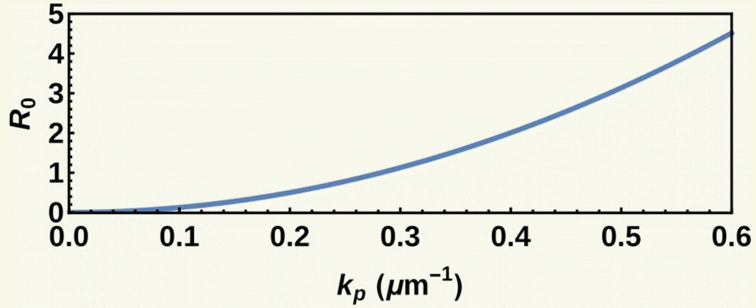
New quantum state

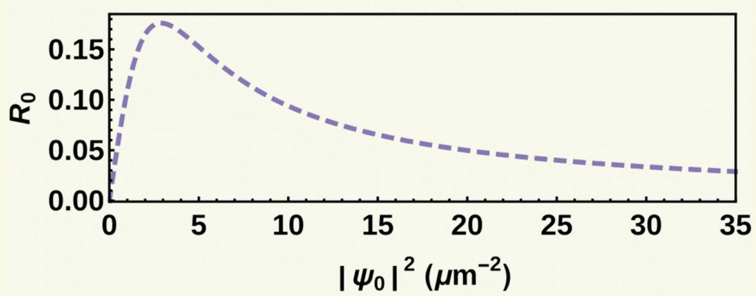
- Most of the system does not respond to neither longitudinal nor transverse forces

  no superfluid but no normal fluid either

- Rigid state fixed by the external pump
- Some normal response dependent on pump vector $\boldsymbol{k}_p$

Rigid state fits well with inability to form vortices and solitons



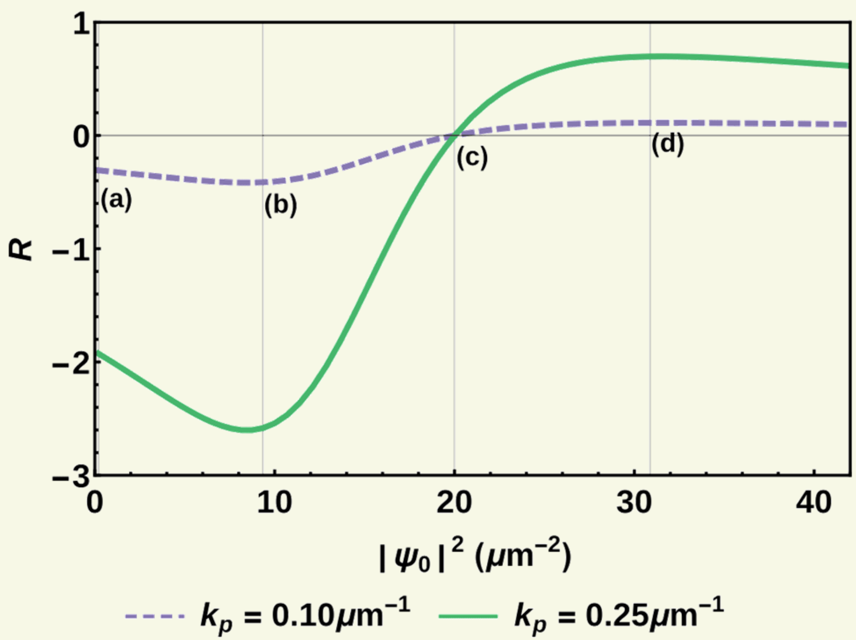


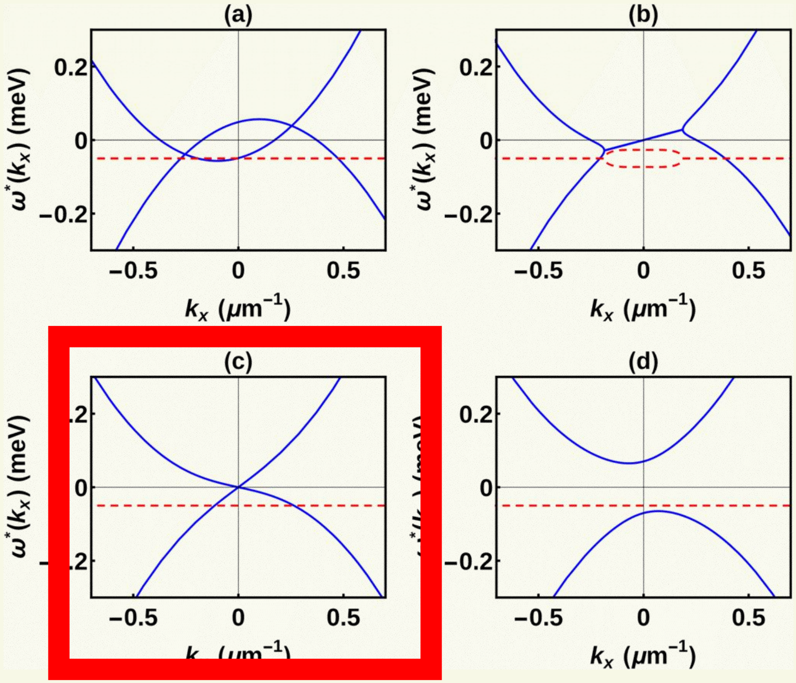
Reconciling with the Experiment

[R. Juggins et al., Nature Comm. to appear, 2018]

Detuning and the response function

- Vary pump strength for a given detuning
- Normal response goes to zero when the real spectrum fulfils the Landau criterion



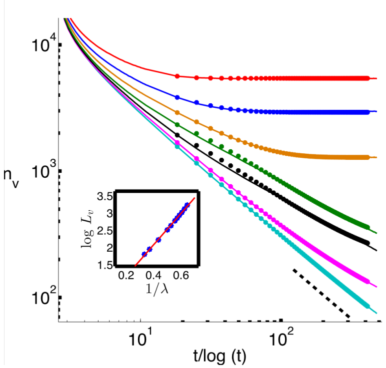


Experiment records the non-fluid rather then superfluid state

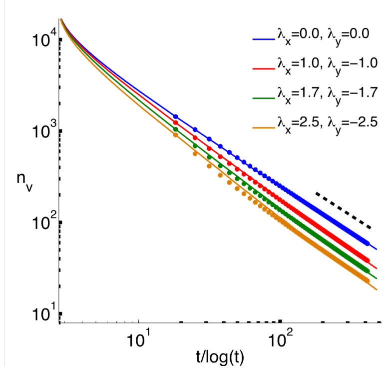
Conclusions

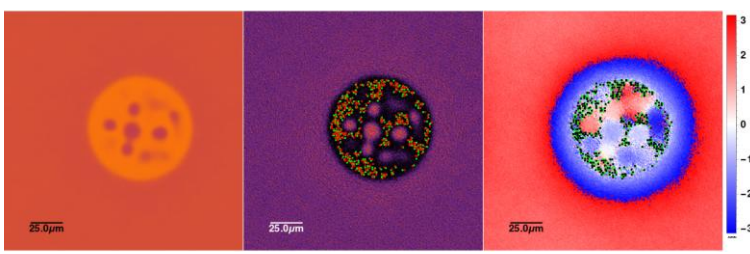
✧ Driven-dissipative superfluids: rich variety of phases depending on the kind of the drive: BKT, vortex dominated phase, 1st order transition, phase separation, etc…

Isotropic

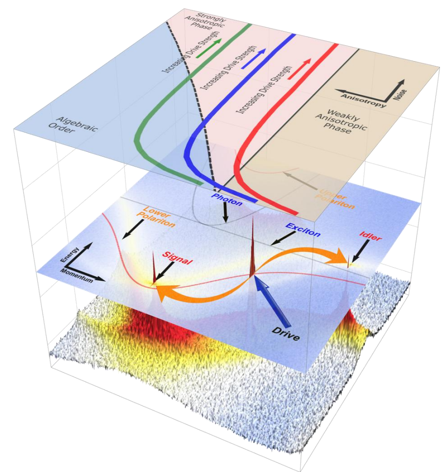


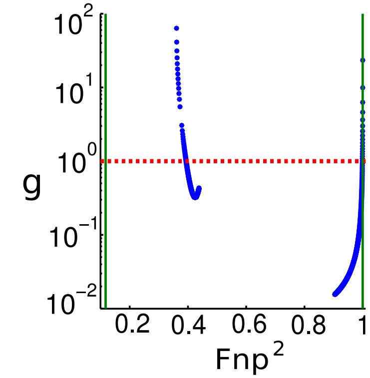
Strongly Anisotropic



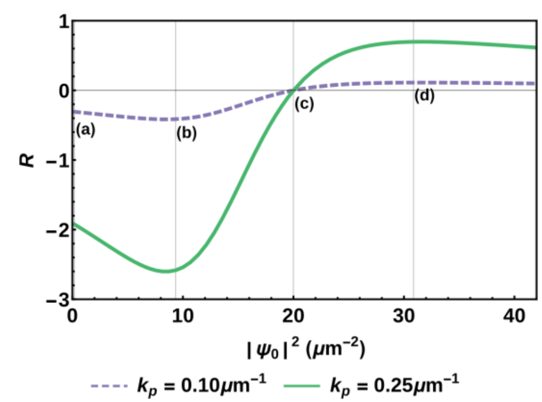


✧ Anisotropy and dissipation – as in OPO – different phases, KPZ order at all lengthscales?





✧ Coherent drive – new quantum state with novel flow properties



Acknowledgements

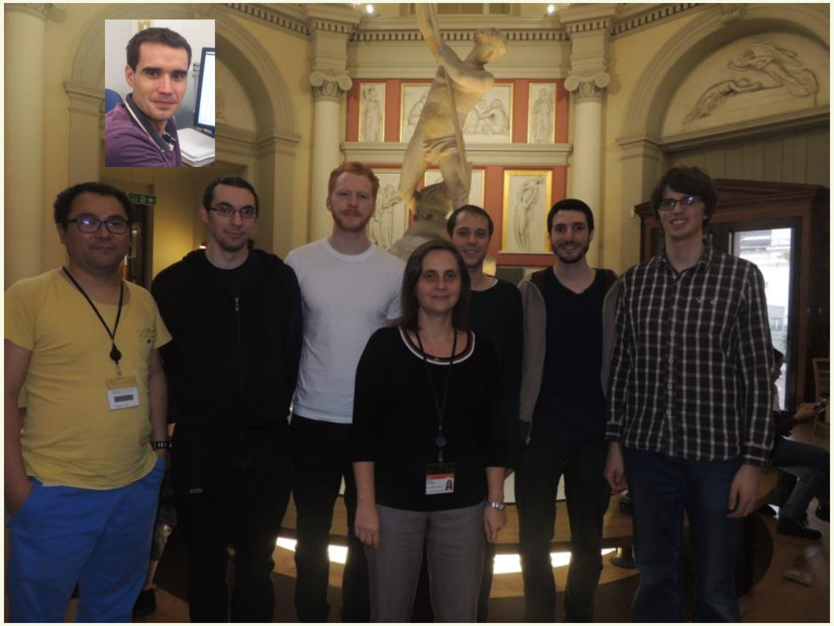
Group:


A. Zamora

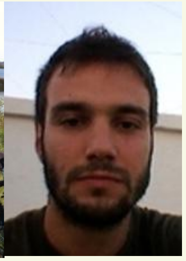

G. Dagvadorj


R. Juggins


A. Ferrier



In collaboration with:

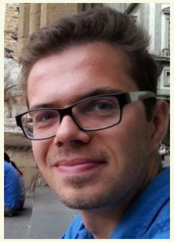



D. Sanvitto D. Ballarini
L. Dominici, D. Caputo, M. De Giorgi,
G. Gigli, K. West, L. N. Pfeifer


S. Diehl
L. Sieberer


N. Proukakis
P. Comaron


J. Keeling


M. Matuszewski

Funding:

EPSRC
Engineering and Physical Sciences
Research Council