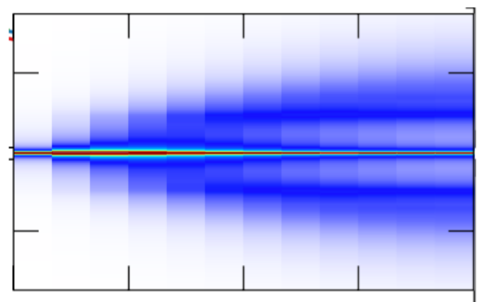
Many – body localization in disordered Hubbard chains



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KITP, 27. 9. 2018

Outline

Characteristic features of MBL systems:

- some results for the 'standard' model of MBL

MBL in 1D disordered Hubbard model with potential disorder

- relation to experiments on cold atoms
- at large disorder charge correlations non-ergodic and spin corr. ergodic ?
- power-law growth of the entanglement entropy

Local integrals of motion - disordered Heisenberg: Hubbard

- counting of LIOM in the Heisenberg model in magnetic field
- missing LIOM in the potentially disordered Hubbard chain

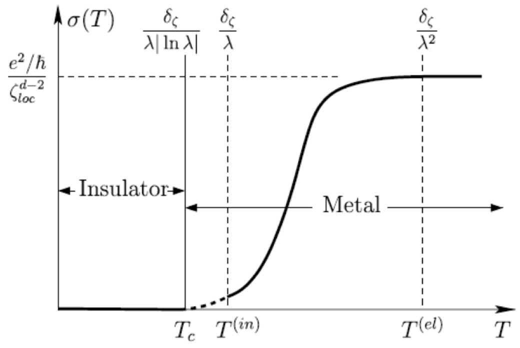
Spin subdiffusion in disordered Hubbard chain - effective spin model

- reduction of Hubbard model to effective spin model
- subdiffusion – single: multiple weak-link scenario

What is MBL and why is it so interesting ?

Nonergodic behaviour in a macroscopic many-body quantum system: T > 0

- non-interacting (NI) fermions on disordered lattice: Anderson localization
- integrable many-body models: Heisenberg chain etc…
- systems undergoing phase transition (macroscopic ordering at $T < T_c$)
- **many-body-localized systems** = correlations + large disorder ?



Basko, Aleiner, Altshuler (2006):

- MI transition at T=T* at fixed disorder W
- MI transition at $W=W_c$ even at $T = \infty$!

Does MBL exist (phase transition or crossover ..) ? qualitative or quantitative phenomenon ?

Which are properties of the ergodic and non-ergodic phase ?

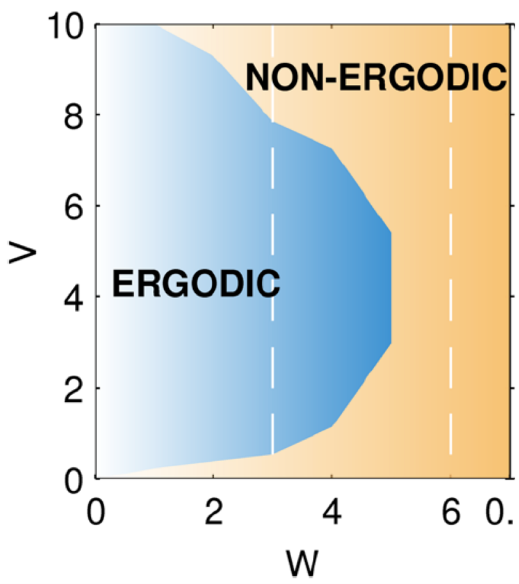
‘Standard ‘ model of MBL

1D model of interacting spinless fermions + Anderson disorder:
equivalent to anisotropic Heisenberg model + random fields: $\Delta = V/(2t)$

$$H = -t\sum_i (\hat{c}_i^\dagger \hat{c}_{i+1} + \hat{c}_{i+1}^\dagger \hat{c}_i) + V\sum_i \left(\hat{n}_i - \frac{1}{2}\right)\left(\hat{n}_{i+1} - \frac{1}{2}\right) + \sum_i h_i \left(\hat{n}_i - \frac{1}{2}\right)$$

$$H = J\sum_i \left[\frac{1}{2}(S_{i+1}^+ S_i^- + S_{i+1}^- S_i^+) + \Delta S_{i+1}^z S_i^z\right] + \sum_i h_i S_i^z. \qquad h_i \in [-W, W]$$

T ~ ∞: phase diagram (approximate)

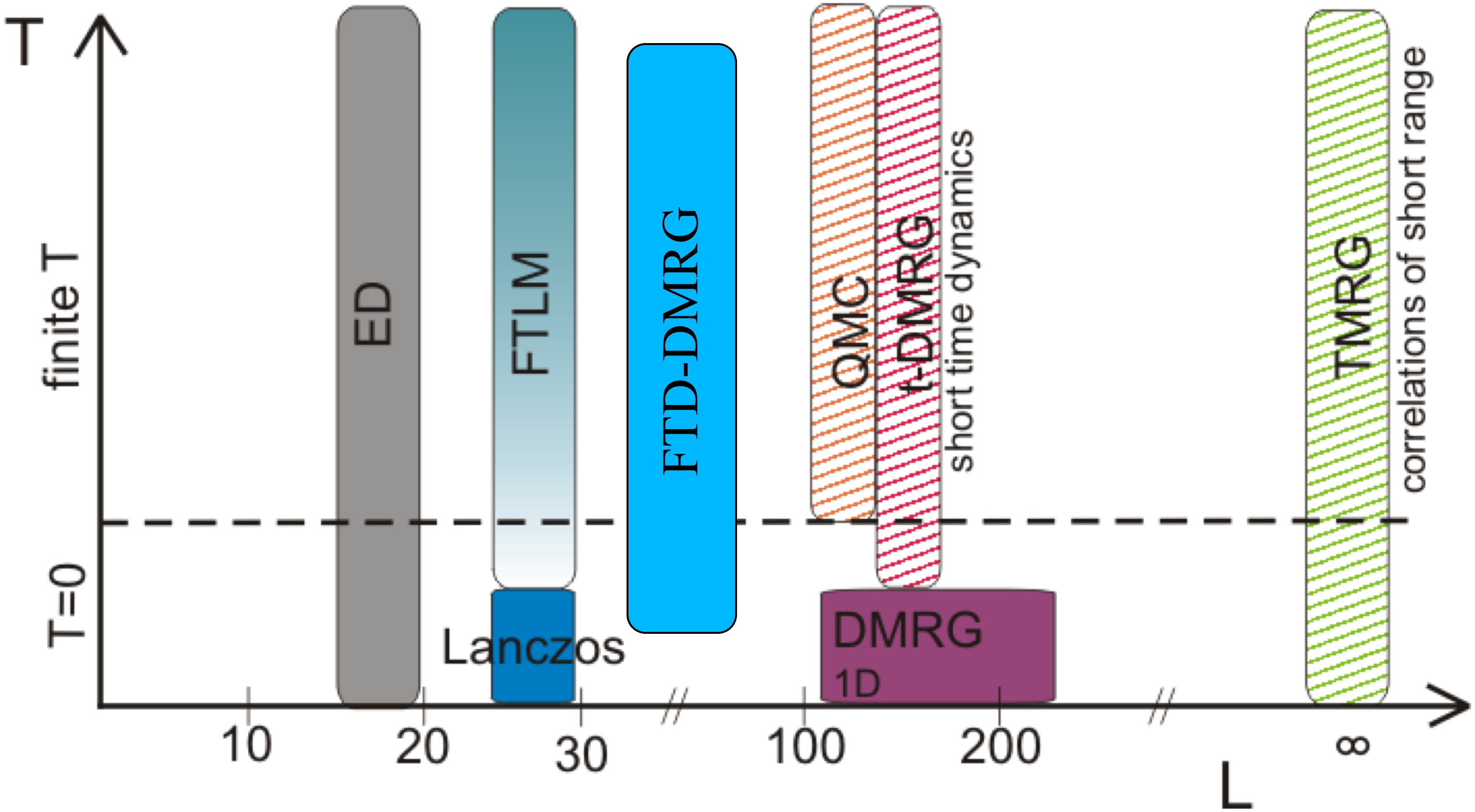


Bar Lev et al, PRL (2015)

MBL phase $W > W_c$:
- Poisson level statistics
- vanishing d.c. transport
- area (log) law for entanglement entropy increase
- non-ergodic behaviour of correlation functions
- no thermalization
- local integrals of motion (LIOM)

Numerical methods for dynamics of correlated systems

Example: 1D (random) Heisenberg model



MBL problem: dynamics - long times = very low frequencies !

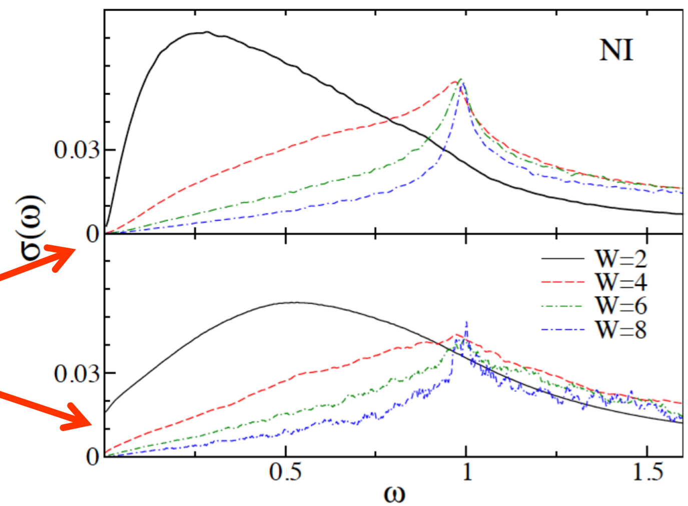
+ large sizes ?

Characteristic features: dynamical conductivity and d.c. transport

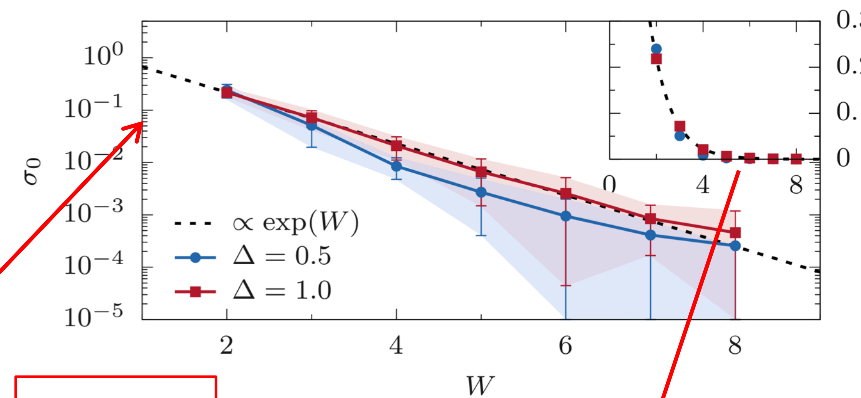
Barišić et al, PRB (2010, 2016)

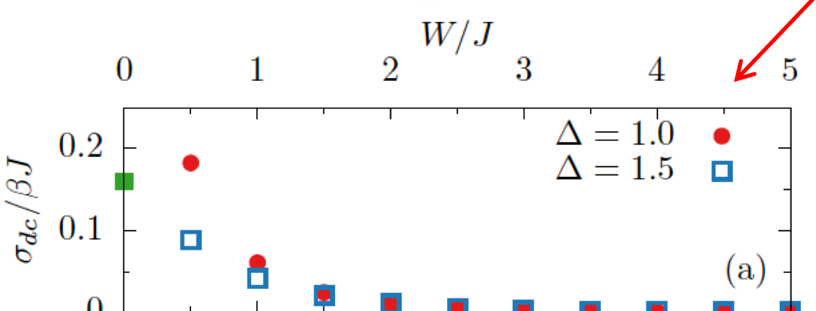
vanishing d.c. transport σ_0 for $W > W_c \sim 5$

$$\sigma(\omega) \sim \sigma_0 + \zeta|\omega|^\alpha \qquad \alpha \sim 1$$



log !





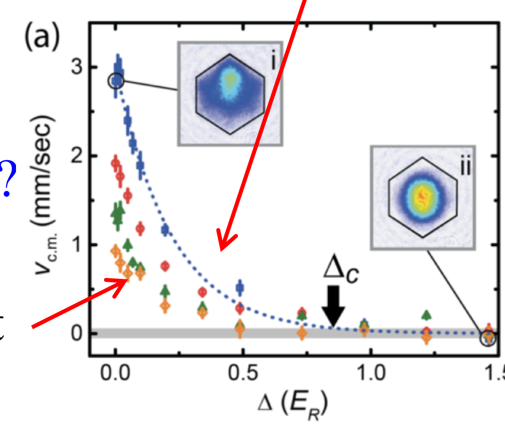
T >> 1:

transition :

sharp crossover ??

Kondov, 2015

theory : experiment



Steinigeweg et al, PRB (2016)

subdiffusion ??

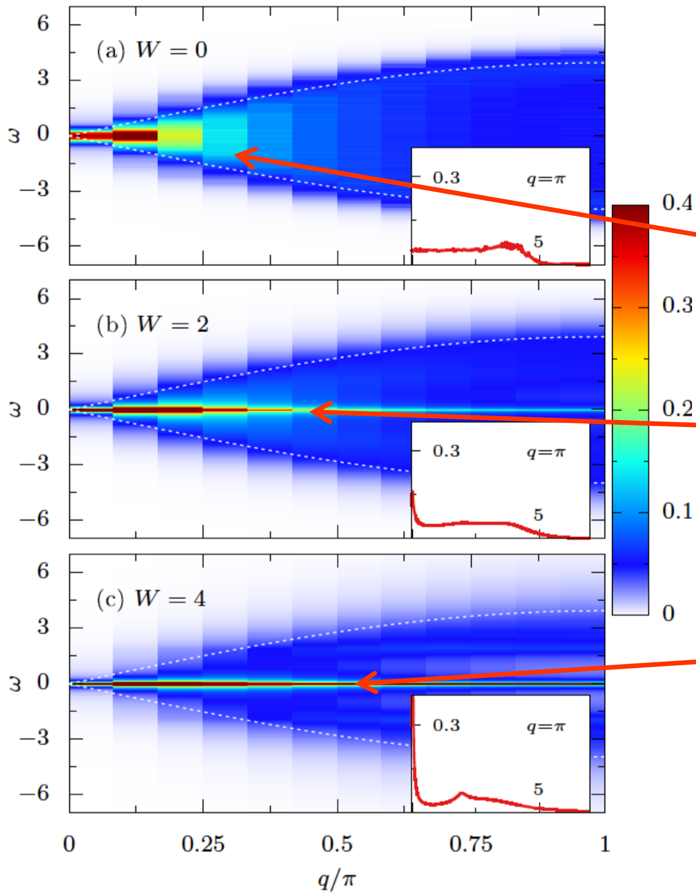
Dynamical structure factor

PP, Herbrych, PRB (2017)

W=0

W=2

W=4



$$S(q,\omega) = \frac{1}{\pi}\mathrm{Re}\int_0^\infty dt e^{i\omega t}\langle n_q(t) n_{-q}\rangle$$

MCLM: L = 26 $\quad T = \infty,\ \Delta = 1$

NI particles: response for W = 0

normal metal:

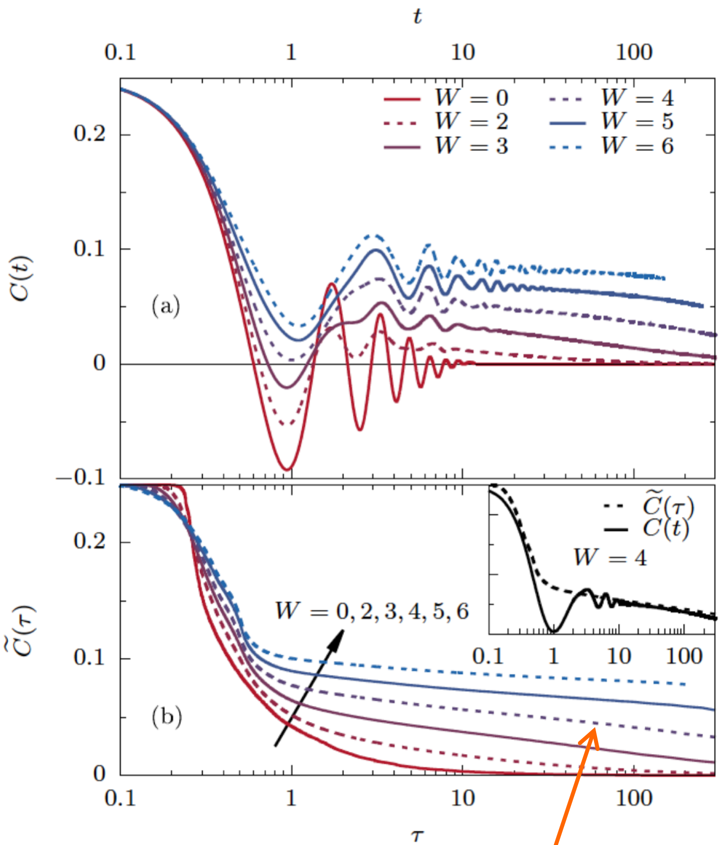
diffusion q ~ 0 pole for $W < W_c$

MBL:

$\delta(\omega)$ peak for $W > W_c$ at all q !

Characteristic features: non-ergodic behaviour

Mierzejewski et al., PRB (2016)



$$\widetilde{C}(\tau) \sim \log \tau / \tau^{*}$$

density-wave (imbalance) correlation function: $T = \infty$, $V = t$ (Δ=0.5), ED, L = 16

$$C(\omega) = \frac{1}{\pi} \mathrm{Re} \int_0^{\infty} \mathrm{d}t\, e^{\imath \omega^{+} t} \langle n_{q=\pi}(t)\, n^{\dagger}_{q=\pi} \rangle$$

a) real-time dynamics: $C(t) = \int_{-\infty}^{\infty} \mathrm{d}\omega\, e^{-\imath \omega t} C(\omega)$

oscillations emerging from NI physics

b) ‘quasi’-time dynamics: $\widetilde{C}(\tau) = \int_{-1/\tau}^{1/\tau} \mathrm{d}\omega\, C(\omega)$

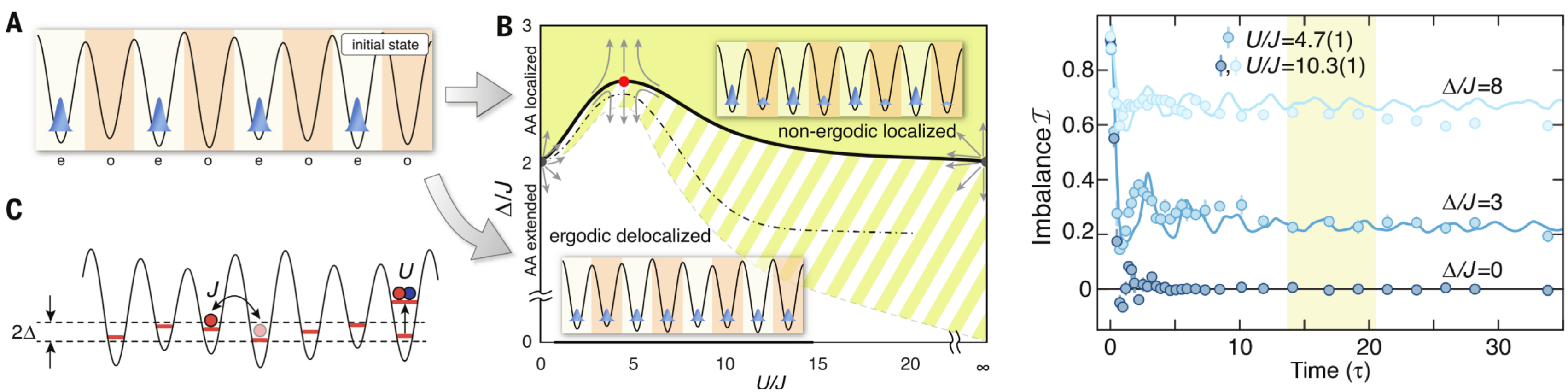
the same long-time variation

nonergodic (MBL) phase: $W > W^{*} \sim 4$

$C_0 = C(t=\infty) > 0$ + anomalous time dependence $C(\omega) = C_0 \delta(\omega) + C_{\mathrm{reg}}(\omega)$

Cold atoms (fermions) on 1D optical lattice

Schreiber et al, Science (2015): K^{40} atoms on 1D optical lattice
+ quasi-periodic (Andre-Aubry) disorder



$$\hat{H} = -J\sum_{i,\sigma}(\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{i+1,\sigma} + \text{h.c.}) + \Delta\sum_{i,\sigma}\cos(2\pi\beta i + \phi)\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{i,\sigma} + U\sum_{i}\hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow}$$

(charge) density imbalance I(t):
non-ergodic for large disorder Δ

model: **1D Hubbard model** with quasi-periodic (random) potential

MBL in 1D Hubbard chain

PP, Barišić, Žnidarič, PRB (2016)

potential disorder only

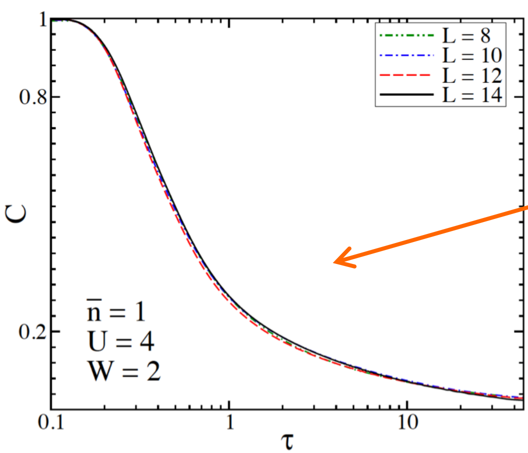
$$H = -t\sum_{is}(c^\dagger_{i+1,s}c_{is} + c^\dagger_{is}c_{i+i,s}) + U\sum_i n_{i\uparrow}n_{i\downarrow} + \sum_i h_i n_i \qquad -W < h_i < W$$

Hubbard model: more degrees of freedom – charge + spin

$n_i = n_{i\uparrow} + n_{i\downarrow}$ $\quad m_i = n_{i\uparrow} - n_{i\downarrow}$

numerical calculation of imbalance correlations: MCLM

charge (CDW) correlations: $C(t) = \frac{\alpha}{L}\langle n_{q=\pi}(t) n_{q=\pi}\rangle$

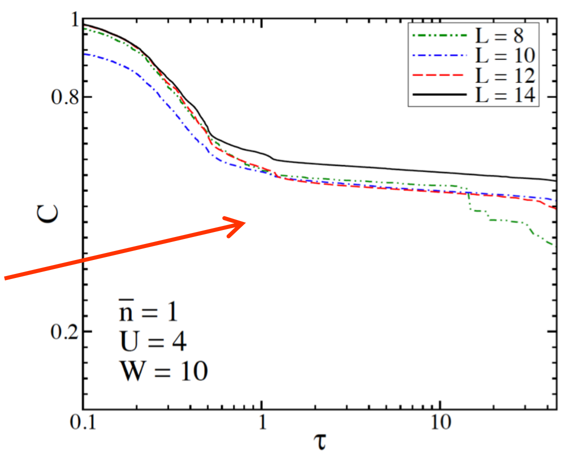


U=4, n=1, L= 8 - 14

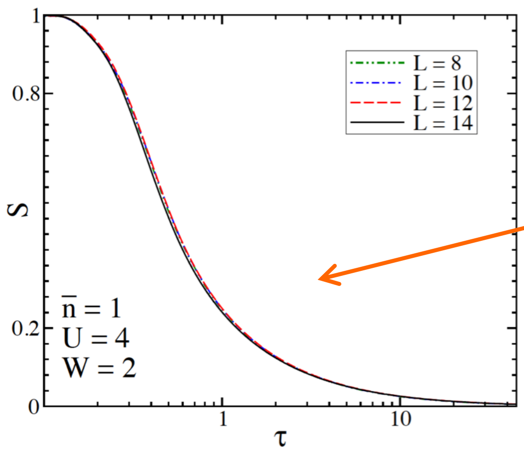
W=2: ergodic

W=10: non-ergodic

expected ?



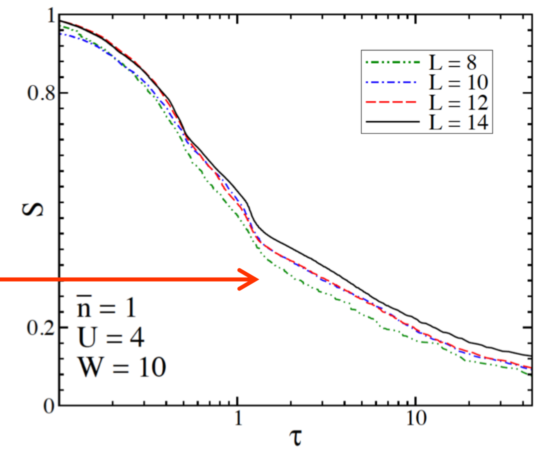
spin (SDW) correlations: $S(t) \;=\; \frac{\alpha}{L}\langle m_{q=\pi}(t)m_{q=\pi}\rangle$

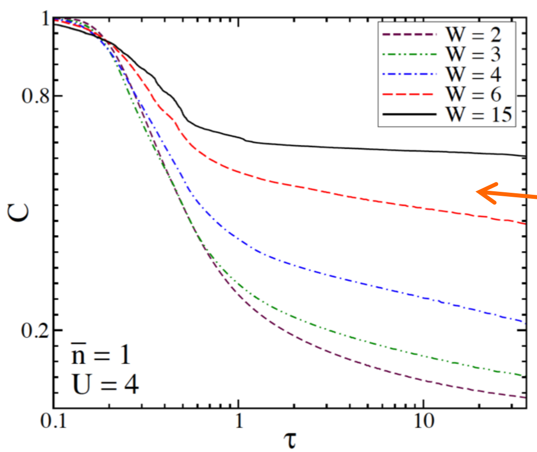


U=4, n=1, L=8-14

W=2: ergodic

W=10: ergodic ?



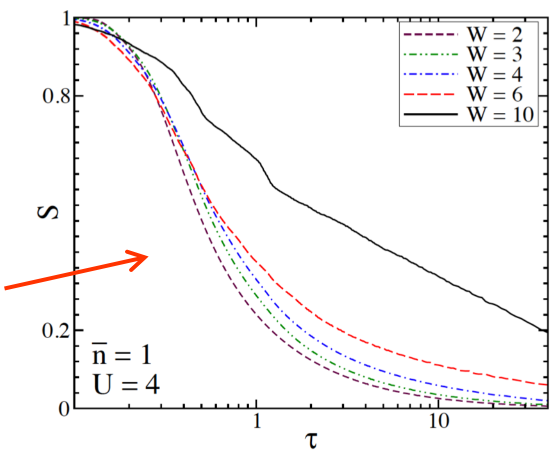


U=4, L=14

charge: $W_c \sim$ 4-6

spin: no transition

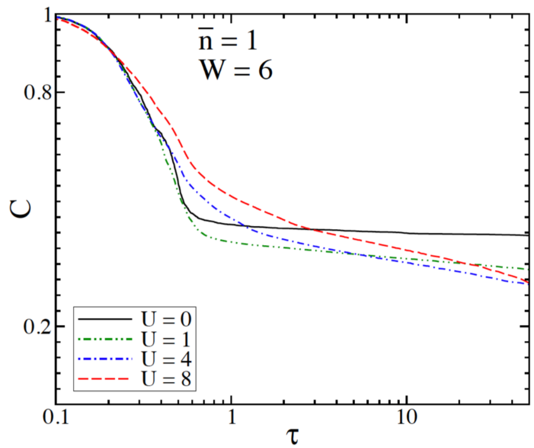
no full MBL !



varying U: from Anderson localization to MBL ?

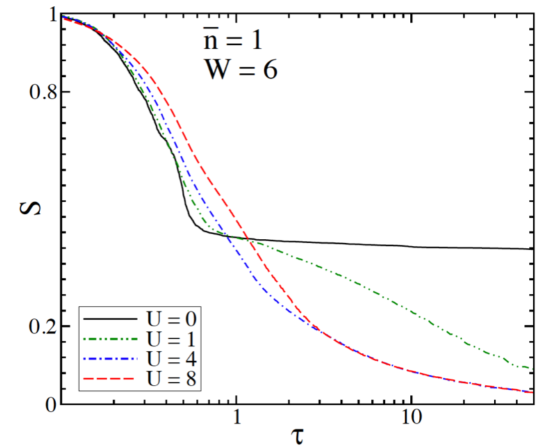
$W > W_c$: large disorder

charge



W = 6

spin



U > 0 induces weak decay of CDW,

charge localization remains

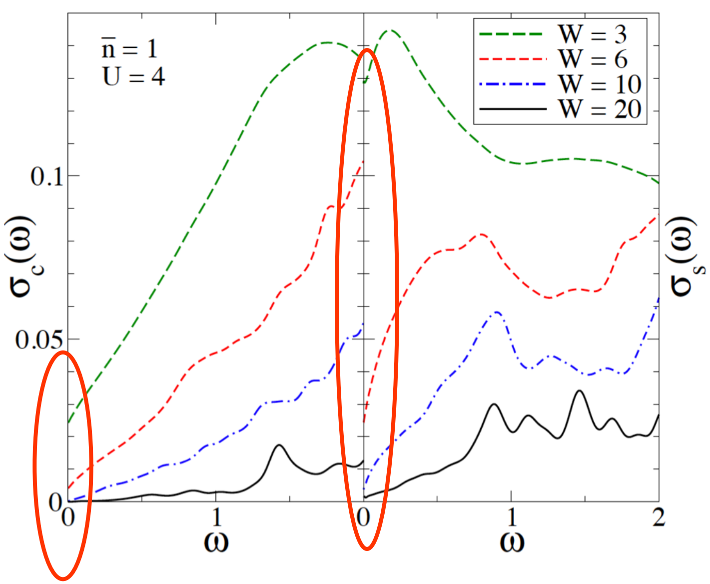
U > 0 leads to decay of SDW

spin behaves ergodic

disorder induced charge – spin separation !?

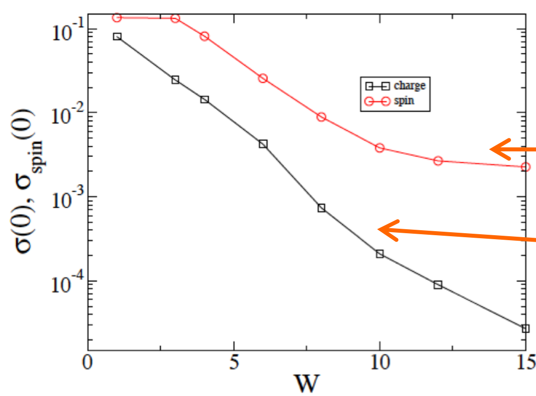
Dynamical charge and spin conductivities:

charge



spin

log !

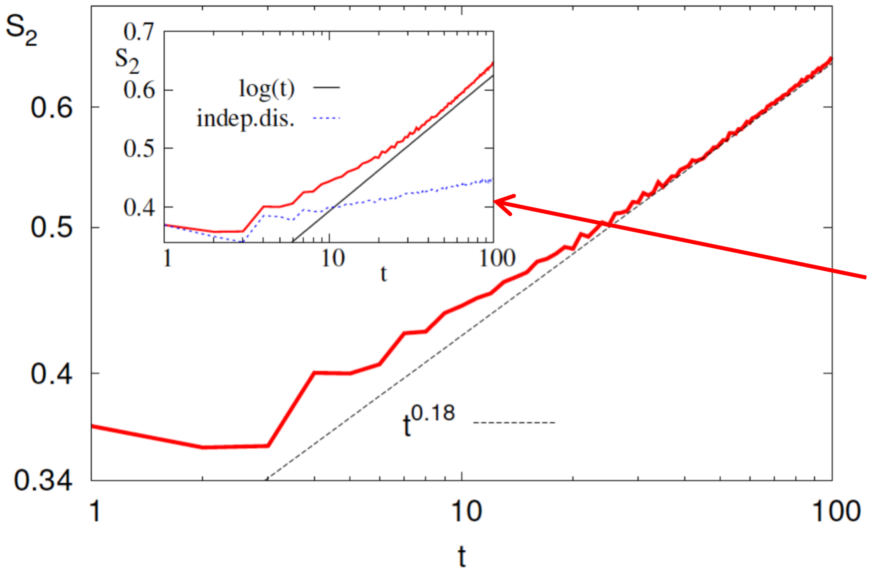


d.c. charge and spin conductivity:

- $\sigma_{spin}(0)$ always finite ?

- $\sigma_{charge}(0)$ – at W ~ 4 transition or crossover ?

Entanglement entropy:



$$S_2(t) = -\mathrm{tr}[\rho_A(t) \log_2 \rho_A(t)]$$

full MBL: $S_2(t) \sim \log(t)$

the case with random field disorder

Hubbard chain – potential disorder

$$S_2(t) \sim t^{0.18}$$

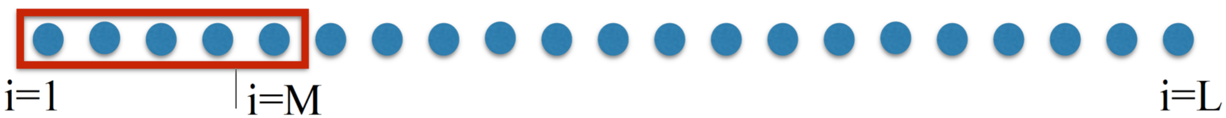
Conclusion: no full MBL - due to remaining SU(2) symmetry ?
is this explanation enough ?

Local integrals of motion

Mierzejewski, PP, Kozarczewski, PRB (2018)

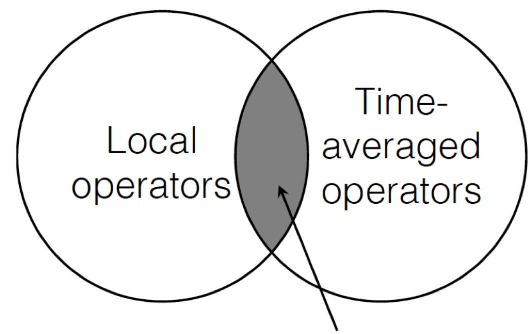
Integrals of motion: $[A, H] = 0 \qquad H = \sum_m E_m |m\rangle\langle m|$

Constructing IOM, starting from **general local operator *A*** on $M << L$ sites:



$$\bar{A} = \lim_{\tau \to \infty} \frac{1}{\tau} \int_0^{\tau} dt' \exp(iHt') A \exp(-iHt')$$

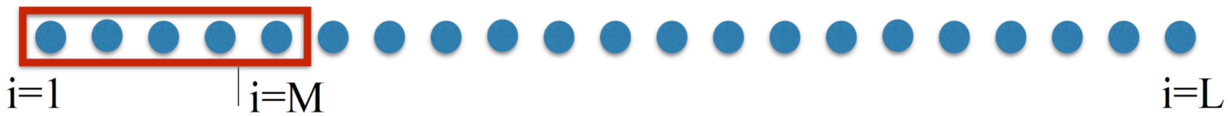
$$= \sum_m |m\rangle\langle m|A|m\rangle\langle m| \quad = \text{IOM, typically nonlocal}$$



LIOM: $\bar{A} = A$

if overlap finite for $L >> M$

Constructing and counting (L)IOM



1) Define an orthogonal set of operators out of traceless local *{A}* on *M* sites

$$\langle O_a O_b \rangle = \delta_{ab}$$

2) Find the time-averaged (L)IOM by diagonalizing *H* on *L* sites: $O_a \Longrightarrow \bar{O}_a$

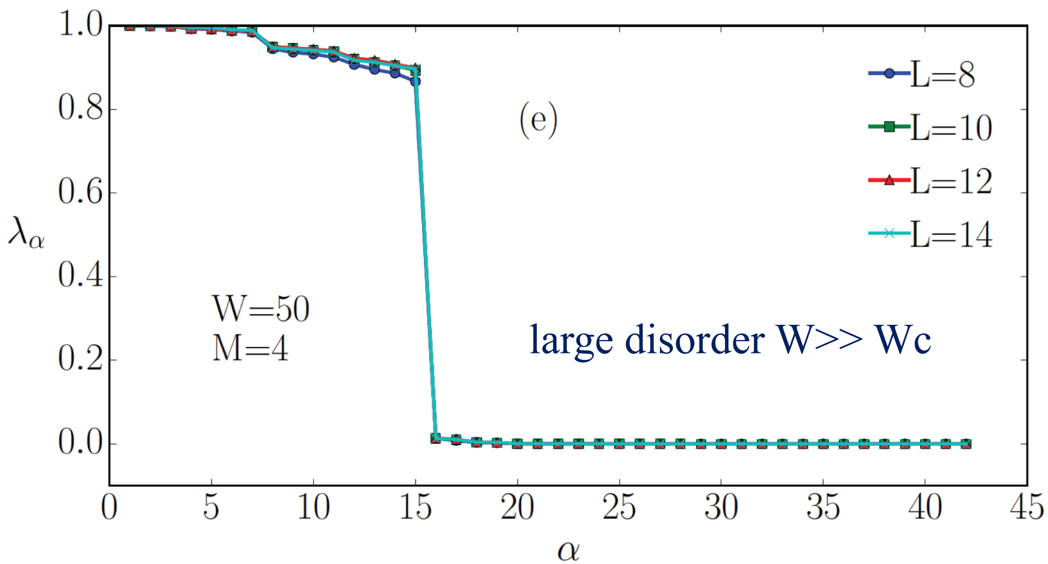
3) Diagonalize the new matrix of (L)IOM

$$\sum_{a,b} U^T_{\alpha a} \langle \bar{O}_a \bar{O}_b \rangle U_{b\beta} = \lambda_\alpha \delta_{\alpha\beta}$$

4) Express (L)IOM in terms of local and nonlocal operators

$$\boxed{\bar{Q}_\alpha = \lambda_\alpha Q_\alpha + Q^\perp_\alpha} \qquad \bar{Q}_\alpha = \sum_a U_{a\alpha} \bar{O}_a \qquad Q_\alpha = \sum_a U_{a\alpha} O_a$$

LIOM in ‘standard’ MBL model



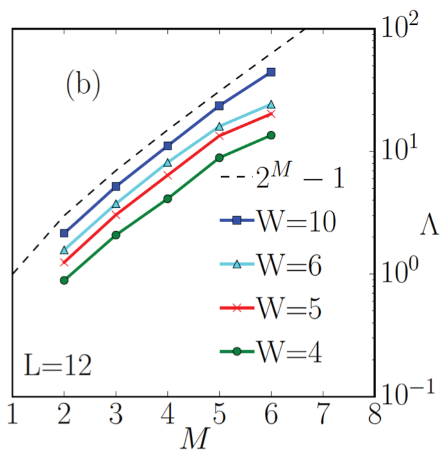
number of LIOM

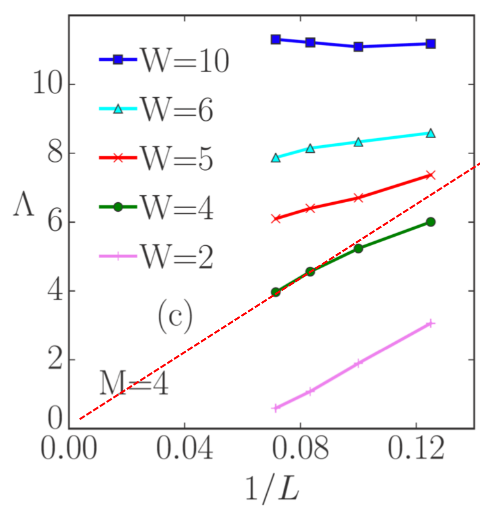
$$N_M = 2^M - 1$$

identity

$$\Lambda = \sum_\alpha \lambda_\alpha$$

measure of locality





V = 2Δ =1

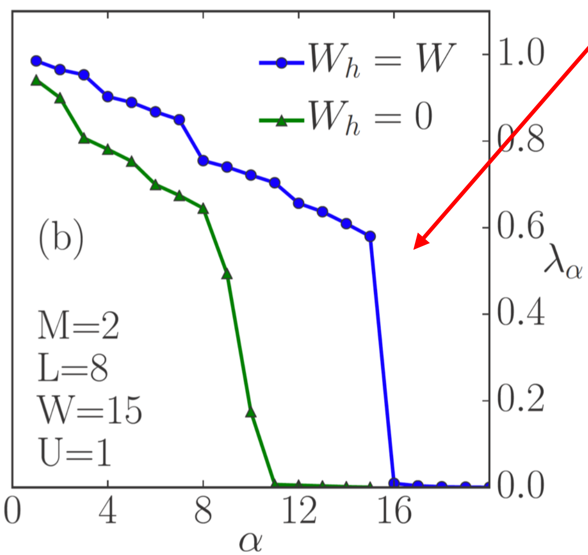
MBL:

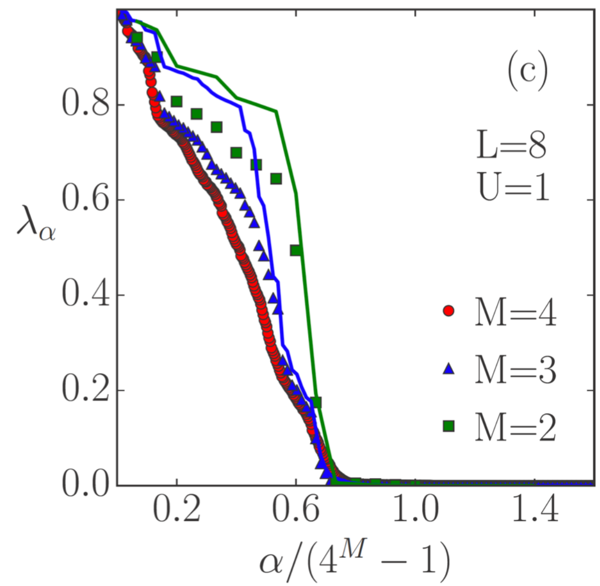
L >> 1: finite Λ

LIOM in disordered Hubbard chain

$N_M = 4^M - 1$

adding (strong) random field





$$3^M - 1 < N_M \lesssim 4^M/2$$

not enough LIOM for full MBL !

Spin subdiffusion in disordered Hubbard model

Kozarczewski, PP, Mierzejewski, PRL (2018)

$$H = -t\sum_{is}(c^\dagger_{i+1,s}c_{is} + c^\dagger_{is}c_{i+i,s}) + U\sum_i n_{i\uparrow}n_{i\downarrow} + \sum_i h_i n_i \qquad -W < h_i < W$$

solve U = 0 (Anderson) problem + rewrite Hubbard in Anderson basis +

$$\phi_{ia} = \langle i|a\rangle$$

$$H = \sum_{a\sigma}\epsilon_a c^\dagger_{a\sigma}c_{a\sigma} + \frac{U}{2}\sum_{aa'bb'\sigma}\chi^{ab}_{a'b'}c^\dagger_{a\sigma}c^\dagger_{b\bar{\sigma}}c_{b'\bar{\sigma}}c_{a'\sigma} \qquad \chi^{ab}_{a'b'} = \sum_i \phi^*_{ia}\phi^*_{ib}\phi_{ib'}\phi_{ia'}$$

+ assume frozen charge configuration: $n_a = 0,1$

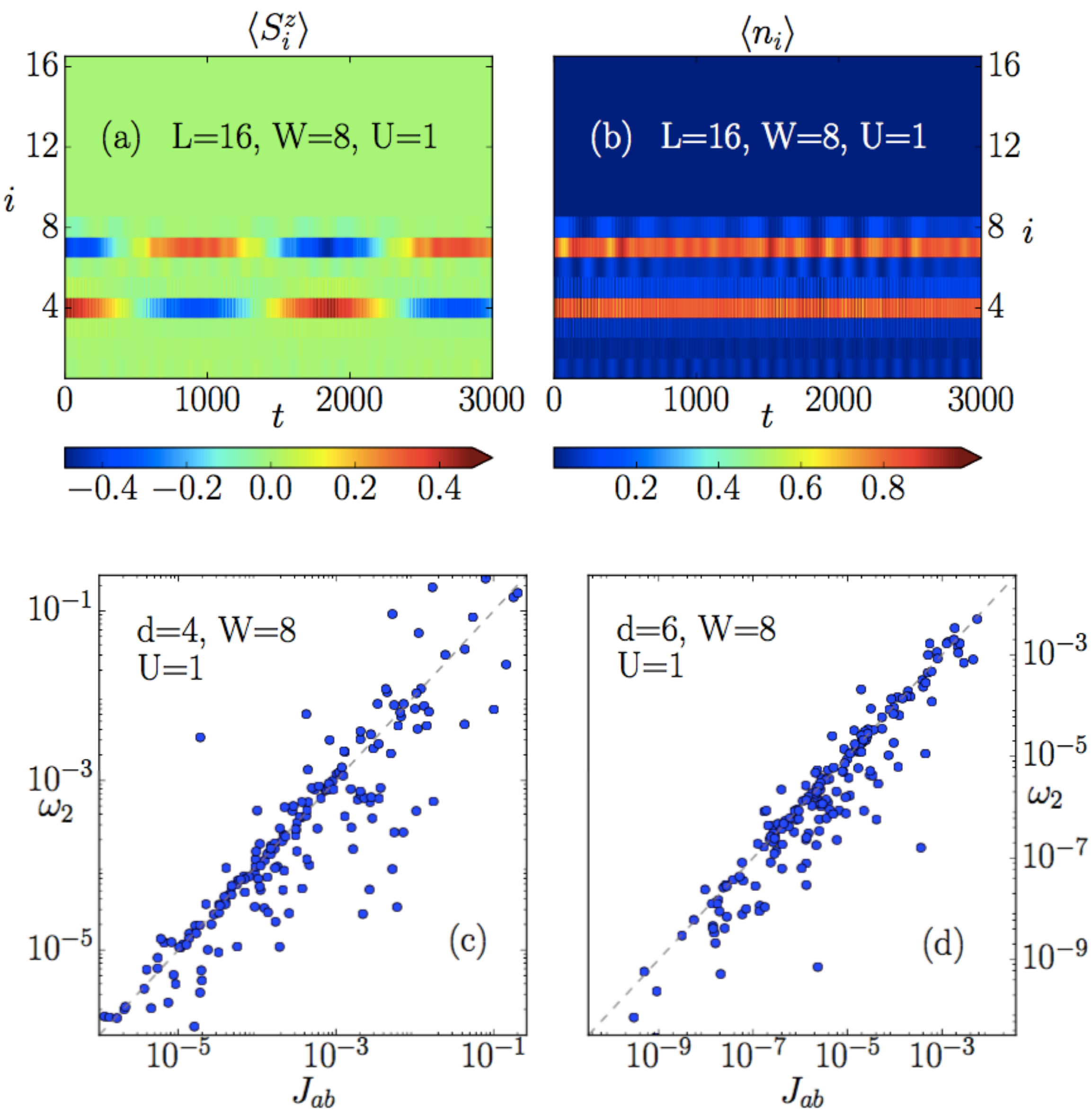
$$H_U = \frac{1}{2}\sum_{a\neq b} J_{ab}\left(\frac{1}{4}n_a n_b - \vec{S}_a\cdot\vec{S}_b\right)$$

$$J_{ab} = 2U\chi^{ab}_{ab} = 2U\sum_i |\phi_{ia}|^2|\phi_{ib}|^2$$

SU(2) invariant interaction term

- ferromagnetic !
- acting only on singly occupied sites !

N = 2 electrons - Hubbard model



$$J_{ab} = 2U \sum_i |\phi_{ia}|^2 |\phi_{ib}|^2$$

$$J_{ab} \simeq 2U \exp(-x_{ab}/\lambda)$$

$$\lambda \sim \xi$$

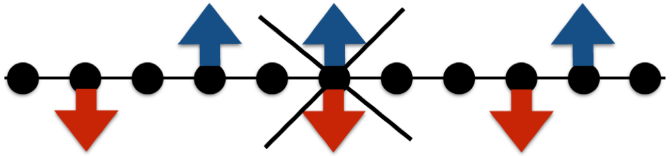
s.p. localization length

Squeezed spin model

$$H_H = -\sum_i J_i \vec{S}_i \cdot \vec{S}_{i+1}, \qquad i \in \{1, ..., \tilde{N}\} \text{ with } \tilde{N} \leqslant N$$

average spin spacing:
singly occupied sites

$$d = \frac{L}{\tilde{N}} = \frac{1}{\bar{n} - \bar{n}^2/2}$$



T >> 1: $f_d(x) = \frac{1}{d}\exp(-x/d)$ ~ probability for neighbors at distance x

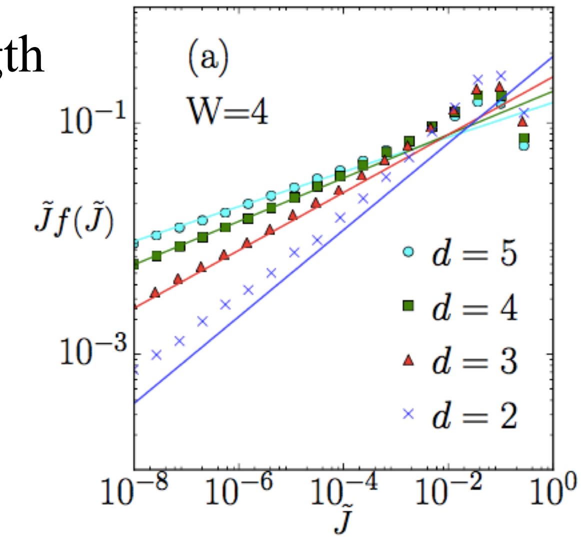
Probability distribution for effective n.n. *J*: singular !

$$f_{\tilde{J}}(\tilde{J}) = \tilde{\lambda}\tilde{J}^{\tilde{\lambda}-1}, \qquad \tilde{\lambda} = \lambda/d$$

~ effective loc. length

dimensionless $\tilde{J} = J/2U$

Strong disorder + d >> 1: $\tilde{\lambda} \ll 1$



actual distribution of J
Hubbard: $N = 2$

Weak – link scenario

Single weakest link: on M sites

weakest link

characteristic propagation time of spin perturbation: $\tau \sim \frac{1}{2U\tilde{J}_m}$ $\tilde{J}_i\ , \quad i = 1 \cdots M$

$$f_m(\tilde{J}_0) = \tilde{\lambda} M \tilde{J}_0^{\tilde{\lambda}-1}(1-\tilde{J}_0^{\tilde{\lambda}})^{M-1}$$

probability distr.. for weakest link on M sites

$$\langle \tilde{J}_m \rangle = \int_0^1 d\tilde{J}_m f_m(\tilde{J}_m)\tilde{J}_m \simeq \Gamma\left(1+\frac{1}{\tilde{\lambda}}\right) M^{-1/\tilde{\lambda}}$$

$$\Lambda \sim Md \propto (2Ut)^{\tilde{\lambda}} \longrightarrow S_L(t) \propto \Lambda^{-1} \propto (2Ut)^{-\tilde{\lambda}} \qquad \tilde{\lambda} << 1$$

spatial extent of spin perturbation in time t

subdiffusion

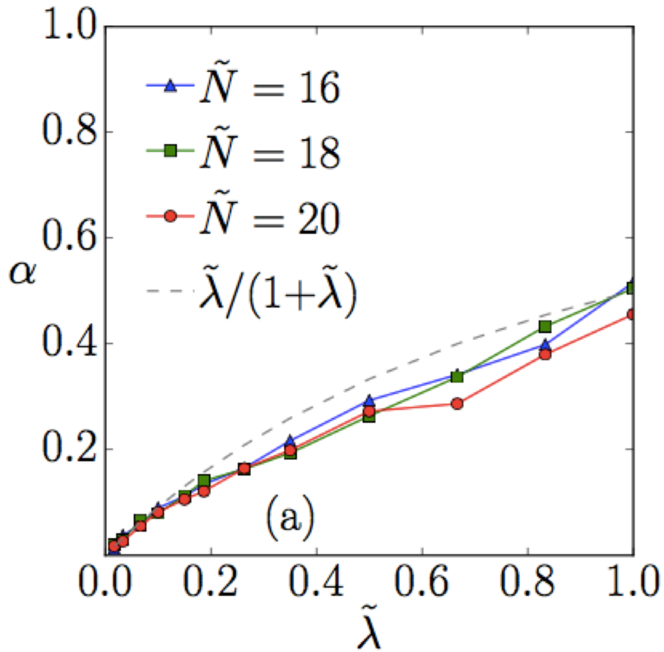
Multiple weak links: classical model of random traps: $\tau = 1/\tilde{J}$

$$f_\tau(\tau) = \tilde{\lambda}/\tau^{\tilde{\lambda}+1}$$

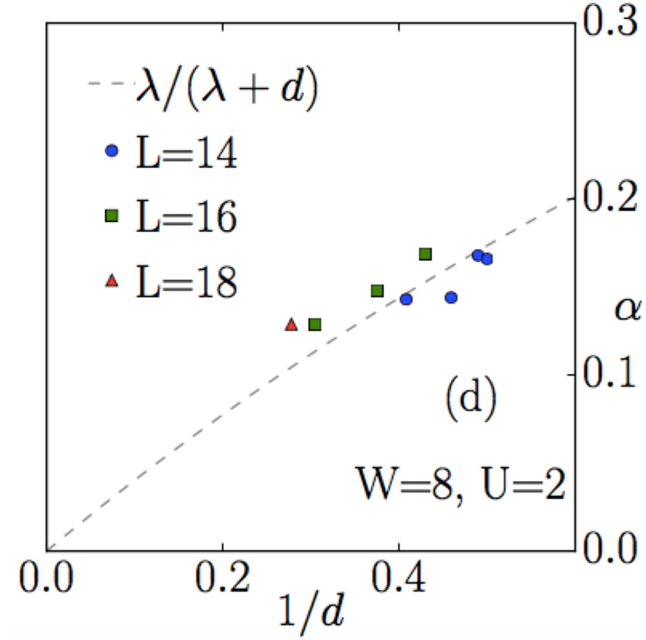
$$\Lambda \propto (2Ut)^\alpha$$

$$S_L(t) \propto (2Ut)^{-\alpha} \qquad \alpha = \frac{\tilde{\lambda}}{1+\tilde{\lambda}} = \frac{\lambda}{d+\lambda}, \quad \tilde{\lambda} < 1$$

Machta, 1985



squeezed Heisenberg

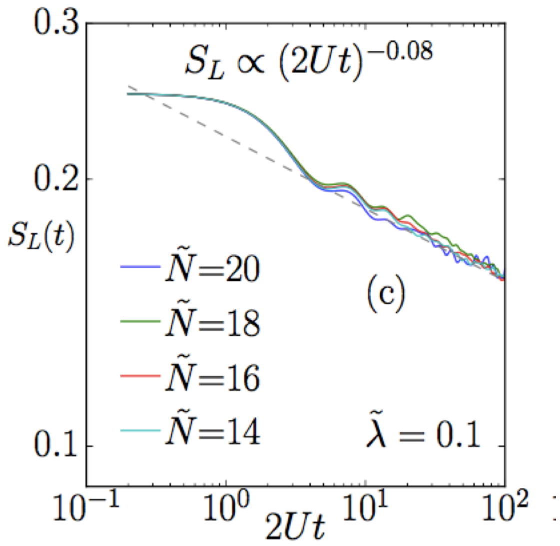


Hubbard model

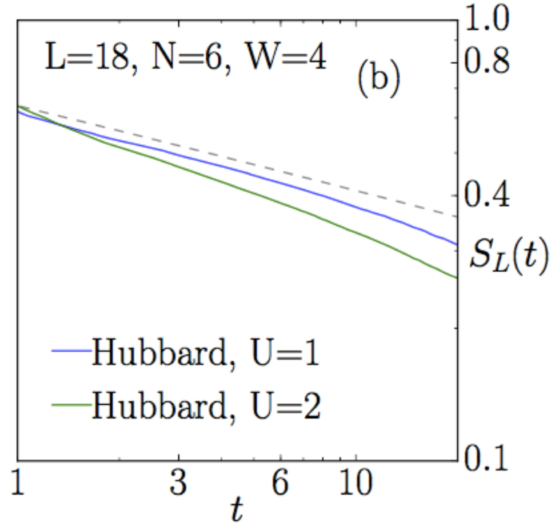
Spin subdiffusion

$$S_L(t) = \langle S_i^z S_i^z(t) \rangle = \frac{1}{\mathrm{Tr}\, 1} \langle \mathrm{Tr}\, [S_i^z(t) S_i^z] \rangle_{\mathrm{dis}}$$ local spin correlations

$$S_L(t) \propto (2Ut)^{-\alpha} \qquad \alpha < 1/2$$ subdiffusion



squeezed Heisenberg



Hubbard model

Marcin Mierzejewski,
Wroclaw

Osor Barišić,
Zagreb

Marko Žnidarič,
Ljubljana

Jacek Herbrych,
Knoxville

O.S. Barišić, J. Kokalj, I. Balog, PP, PRB **94**, 045126 (2016)

M. Mierzejewski, Herbrych, PP, PRB **94,** 224207 (2016)

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Summary

MBL in 1D disordered Hubbard chains

- potential (Anderson) disorder: CDW and SDW decay qualitatively different
- at large disorder charge nonergodic, spin ergodic
- disorder induced charge – spin separation (at all energy scales)
- there is no full MBL, but rather partial freezing

LIOM in random Heisenberg and Hubbard chain

- counting of LIOM in'standard' random Heisenberg model – full MBL
- number of LIOM in Hubbard << full MBL

Spin subdiffusion in disordered Hubbard model

- effective spin model – squeezed isotropic Heisenberg model
- subdiffusion due to singular distribution of effective exchange coupling

Open questions

- are there higher order correction ?
- effect of charge-spin coupling - does charge remain localized ??