Unwinding Short-Range Entangled Phases of Matter

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- Work in collaboration with Tzu-Chieh Wei (Stony Brook) and Juven Wang (IAS)

- Talk based on *Unwinding Short-Range Entanglement*, Phys. Rev. B 98, 125108 (2018), arXiv:1804.11236 [quant-ph]

- We are interested in *classifying* and *characterizing* various phases of matter.
- Focus on:
    - Quantum systems in *thermodynamic limit*,
    - Dynamics by *local* Hamiltonians.
    - Zero temperature *quantum phase* [1].
- "Two systems are said to be in the same *gapped phase* if their Hamiltonians can be interpolated without closing the gap."
- Q1: Why do we focus on gapped systems?
- Q2: Why is interpolation a good notion of equivalence?

[1] S. Sachdev, "*Quantum Phase Transitions*",Cambridge University Press

Phases of matter

Reasonable (?) expectations:

- Expect: systems are in the same phase if they share some common *rigid* [2](possibly unknown) properties.
- Expect: systems in same *quantum phase* if their *ground space* shares same rigid properties.
- Expect: ground states share same properties if they can be smoothly deformed into each other.
- Claim: Ability to interpolate Hamiltonians $\Longrightarrow$ ability to smoothly deform ground states *quantum adiabatic theorem*.

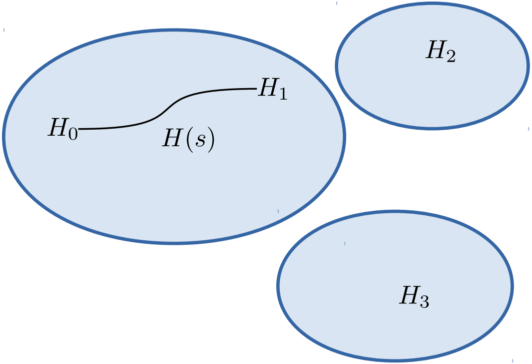
[2]solidness of a solid, liquidness of a liquid

Proof

Claim: Ability to interpolate Hamiltonians $\implies$ ability to smoothly deform ground states

- Consider a family of local Hamiltonians $\mathcal{B}$.
- Let H_0 and $H_1 \in \mathcal{B}$ with spectral gap $\Delta_{0/1} \neq 0$.
- Let H_s be a gapped interpolation. $\Delta_s \neq 0 \; \forall \; s$
- Quantum adiabatic theorem (QAT): interpret s as time, change the Hamiltonian *slowly* at a rate $\nu(s) << \Delta(s) \implies$ *always* stay in the ground space.
- If gap closes at s^* in the path, $\Delta(s) \to 0$ as $s \to s^*$, for QAT to hold, the rate also vanishes $\nu(s) \to 0$ and it takes infinitely long to reach H_1.

In other words, if the subspace of *gapped* Hamiltonians in $\mathcal{B}$ is disconnected, the different disconnected pieces correspond to different gapped quantum phases of matter.



Long-Range and Short-Range Entangled Phases

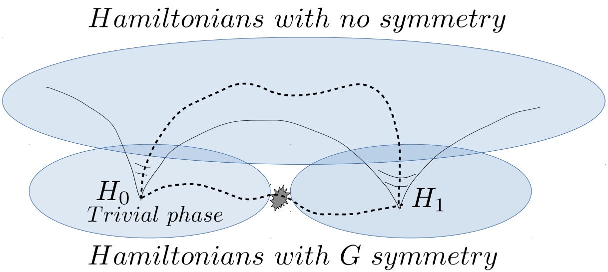
- Two Hamiltonians are *definitely* in different phases if their ground state degeneracy (GSD) are different. Called long-range entangled (LRE) phases.
- This happens if the Hamiltonians in consideration have a global symmetry G that is spontaneously broken (SSB) to a H [3]. GSD $= |G/H|$.
- Most well known, observed, studied mechanism for phases.
- Topological order: GSD without symmetries or SSB.
- Phases with no broken symmetries, unique ground state (GSD $= 1$) are called *short-range entangled* phases (SRE).

[3] Actually an orbit of isomorphic subgroups $\{gHg^{-1}\}$

Some facts about SRE phases:

- Ground states are unique on any closed manifold.
- *Invertible phases*. Given a SRE phase A, there exists an inverse phase A_{inv} whose stacking belongs to the trivial phase.
- A system belongs to a trivial phase if its ground state can be adiabatically interpolated to a product state. $|\Phi\rangle = |\varphi_1\rangle \otimes |\varphi_2\rangle \dots$

- SRE phases of Hamiltonians with some global symmetry G.
- Partial classification: d+1 dim SPT phases are classified by $H^{d+1}(G, U(1))$. Focus on these
- Can be connected to trivial phase in a larger space of Hamiltonians without symmetries i.e. by explicitly breaking symmetry.



Q) How much symmetry should be explicitly broken to connect a non-trivial G SPT phase to a trivial one [4]?

Consider a SPT phase with symmetry G classified by the class $[\nu] \in H^{d+1}(G, U(1))$. We want to know if breaking symmetry down to subgroup $K \subset G$ makes the SPT phase trivial.

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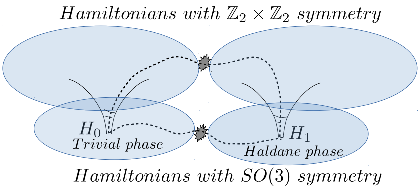
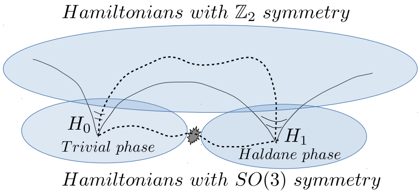
Consider the *Haldane phase*, a non-trivial SPT phase protected by $SO(3)$. Breaking $SO(3)$ to $\mathbb{Z}_2 \times \mathbb{Z}_2$ is insufficient to unwind the Haldane phase but $\mathbb{Z}_2$ is.





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A)Yes! if the class in $H^{d+1}(K, U(1))$ obtained by restricting G to K is trivial.

Given $K \subset G$, can define an injective homomorphism $i : K \to G$,

$$[i^{*}\nu] = 1 \in H^{d+1}(K, U(1))$$

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This is true when $K = 1$, the trivial group. We can *definitely* unwind any SPT phase by breaking all symmetries.

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Q) Can we unwind SPT phases without breaking symmetry but rather *extending* it?

[6] arXiv:1705.06728 [cond-mat.str-el]

Role of symmetries in SPT phases

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A) Yes! This was proven by Wang, Wen and Witten [6].

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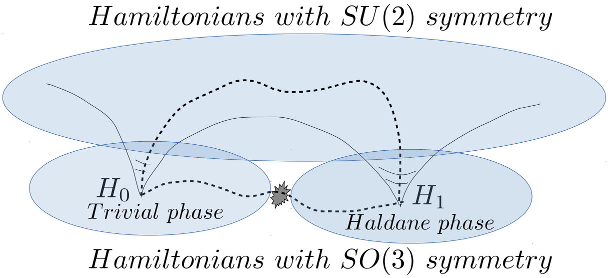
Consider the *Haldane phase* protected by $SO(3)$. Can unwind the phase by extending $SO(3)$ to $SU(2)$.



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Wang, Wen and Witten use this information to produce symmetric, gapped boundary conditions for SPT phases. (More on this later)

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Demonstration of unwinding by extension

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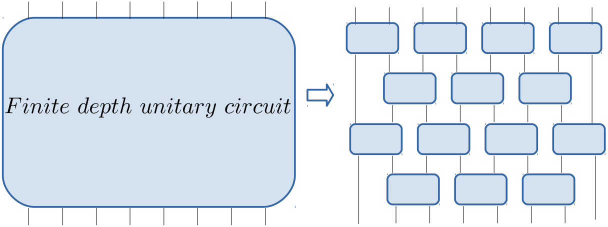
Hamiltonian can be interpolated $\implies$ ground state mapped to product state using a finite time evolution operator, $U(t) \cong \mathcal{T} \exp\left(-i \int_0^t dsH(s)\right)$

$U(t)$ can be written as a FDUC:.



Trivial phase $\implies$ FDUC : GS $\rightarrow$ product state

Three roads to unwinding

Demonstrate three roads to unwinding:

1. Inversion
2. Explicit symmetry breaking.
3. Symmetry extension

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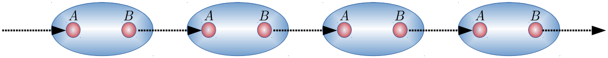
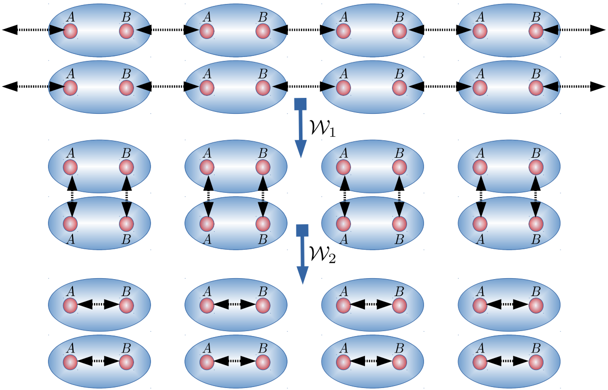
Demonstrate using AKLT-like model



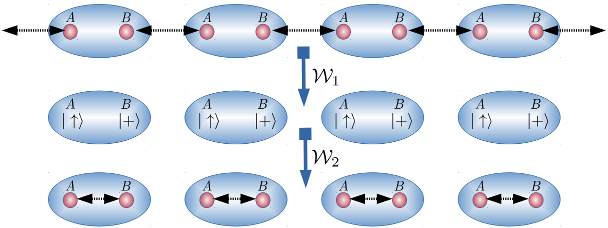
$$\frac{|\uparrow\rangle_B |\downarrow\rangle_A - |\downarrow\rangle_B |\uparrow\rangle_A}{\sqrt{2}}$$



$\mathcal{W}_i$ are products of entanglement-swap operators which are $SO(3)$ invariant.

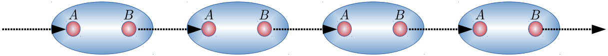
Unwinding by explicitly breaking symmetry

Break all symmetries: $1 \xrightarrow{i} SO(3)$



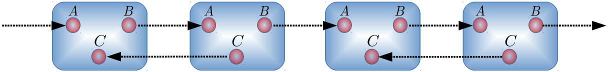
Unwinding by extension

Haldane chain example: $SU(2) \xrightarrow{s} SO(3)$.

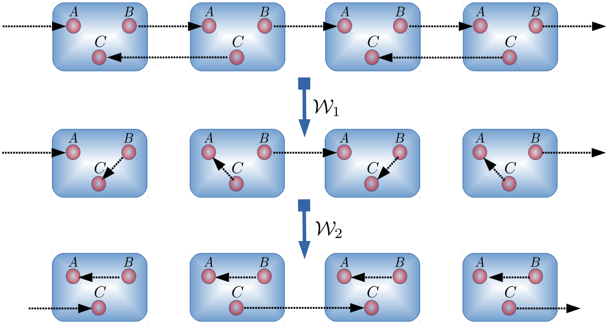


$$\frac{|\uparrow\rangle_B|\downarrow\rangle_A - |\downarrow\rangle_B|\uparrow\rangle_A}{\sqrt{2}}$$

Add ancilla to extend symmetry



Unwinding by extension



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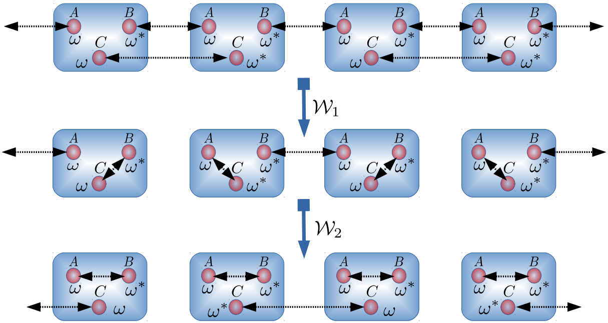
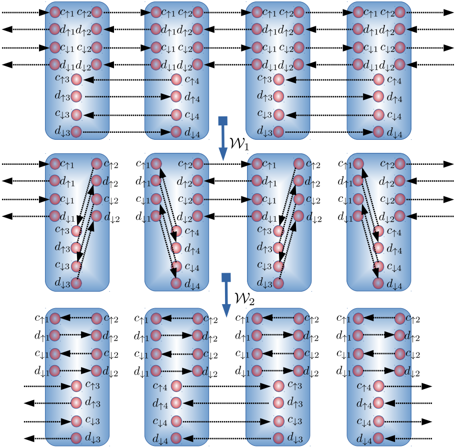
Unwinding by extension



- Can repeat this for well known representative states of *any finite G 1+1D SPT phase*.
- $H^2(G, U(1))$ which classifies 1+1D SPT phases also classifies projective representations of G.
- Theorem: Every G has atleast 1 *Schur cover* which contains both linear and projective representations of G. This is precisely the extension $\tilde{G}$ we were looking for.

Unwinding by extension

Can repeat this for specific classes of fermionic SPT phases constructed by layering Majorana chains.



Symmetric gapped boundaries of SPT phases

- SPT phases have weird boundaries because of the presence of an 't Hooft anomaly classified by $[\nu] \in H^{d+1}(G, U(1))$.
- Boundaries have 'persistent order'- gapless, SSB, topologically ordered.
- Symmetric, gapped boundary possible when topologically ordered.
- Powerful tool to classify bulk SPT phase, especially in 3d.[8]

[8] Vishwanath, Senthil (PRX, 2013), Wang, Senthil (PRB,2013), Metlitski, Kane, Fisher (PRB,2013)

Symmetry extension symmetric gapped boundary conditions

- Extended symmetries that unwind an SPT phase help construct symmetric, gapped, topologically ordered boundaries.
- Consider 3d G- SPT phase classified by $[\nu] \in H^4(G, U(1))$
- Let $\tilde{G}$ be the extension that unwinds it

$$1 \longrightarrow K \xrightarrow{i} \tilde{G} \xrightarrow{s} G \longrightarrow 1.$$

- Starting with a $\tilde{G}$ invariant 2d theory, gauge subgroup K. $\tilde{G}/K$ is the global symmetry that acts on the anyons of the K gauge theory.

Outlook

- Observation: Symmetry extension that unwinds inherently fermionic SPT phases have generators that do not commute with fermion parity.
- Can this be used to recover known boundary states in a simpler language? Newer states?
- Classifications?
- IR Dualities?
- Interesting examples of deconfined criticality?

Thank you!