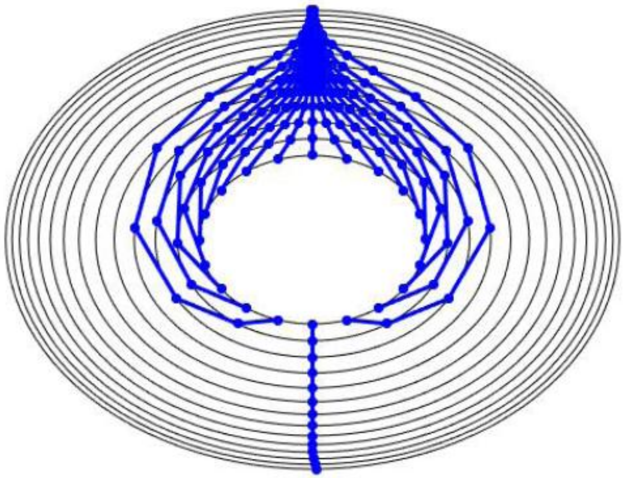
Black holes and overcooled vacuum in driven cavities

Ivar Martin



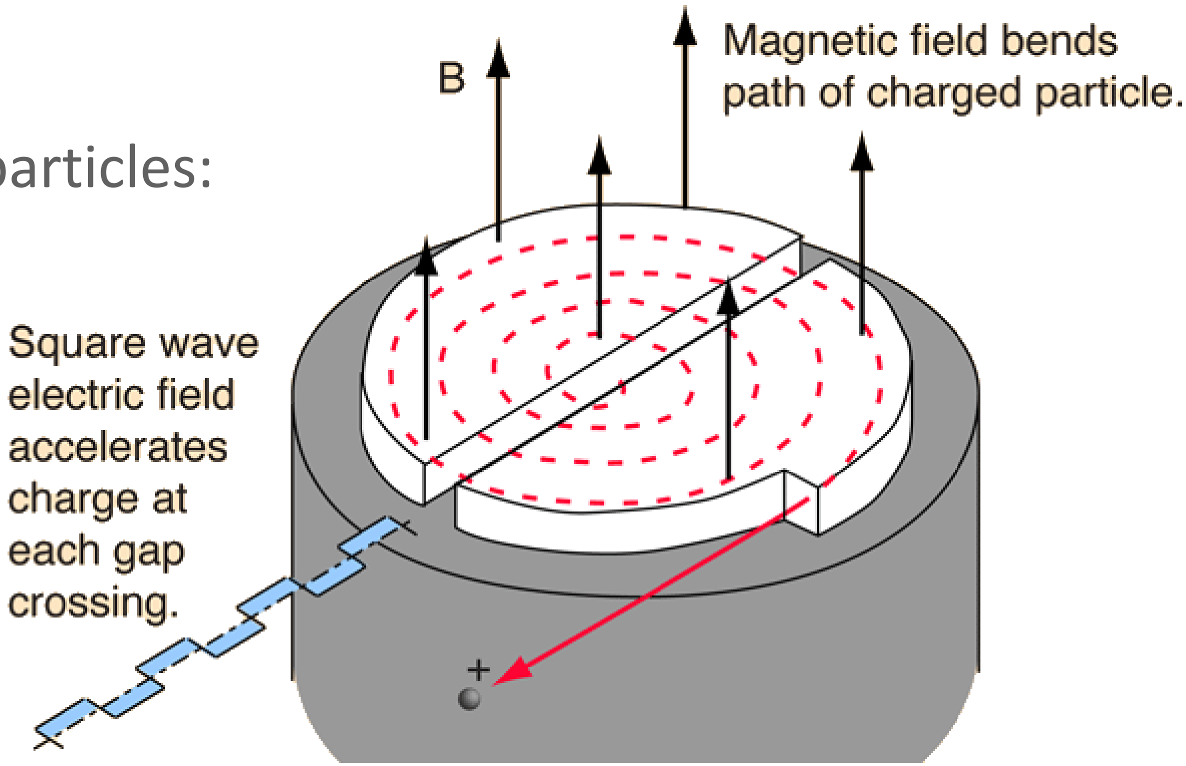
Floquet dynamics of classical and quantum cavity fields, arXiv:1809.02621

Outline

- Photon accelerator
- Maps, Fixed points, Chaos
- Solving wave equation by ***Floquet map***
- Energy singularities
- Dynamical Casimir and “supercooled” vacuum
- Signal compression/decompression
- GR analogy

Cyclotron Accelerator

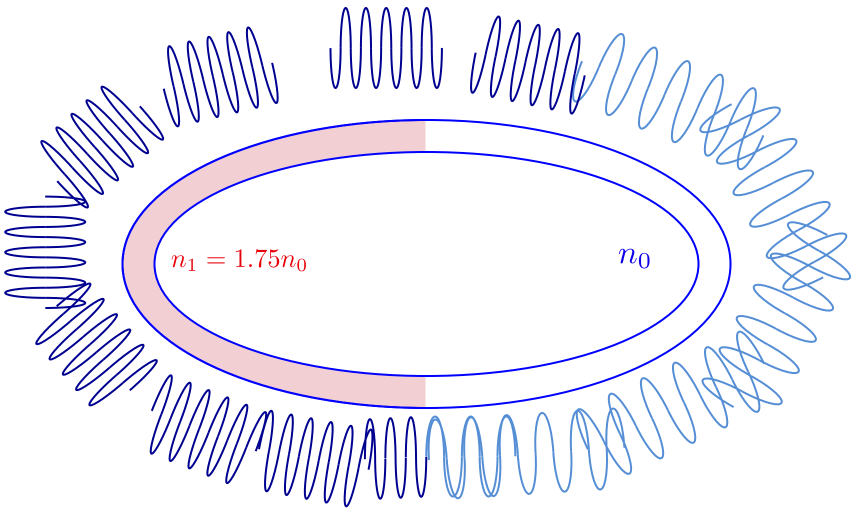
- Charged particles:



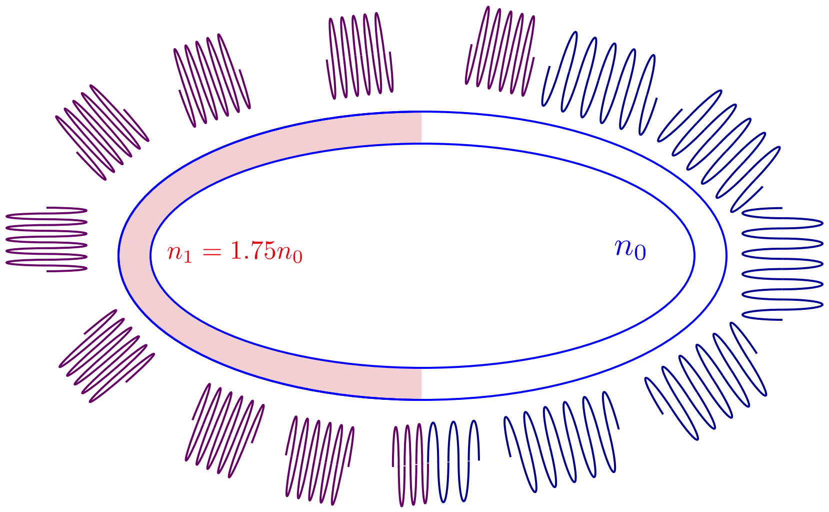
http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/cyclot.html

- Same for photons?

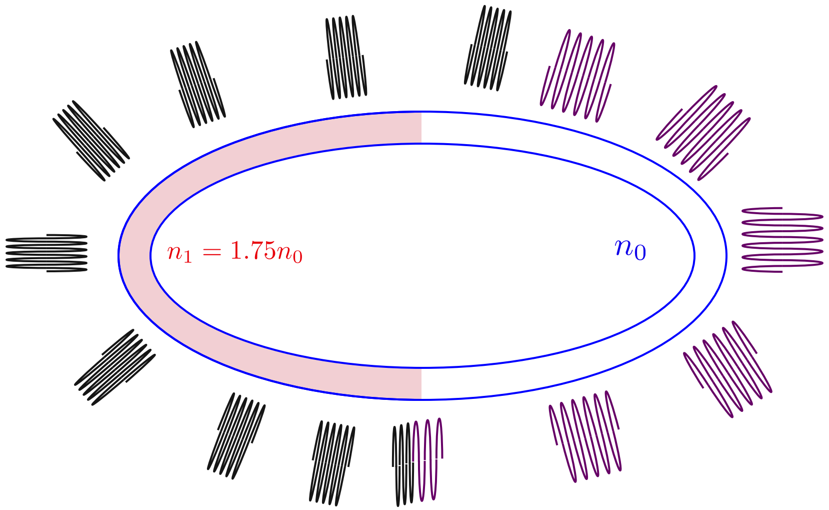
Photon accelerator 1



Photon accelerator 2

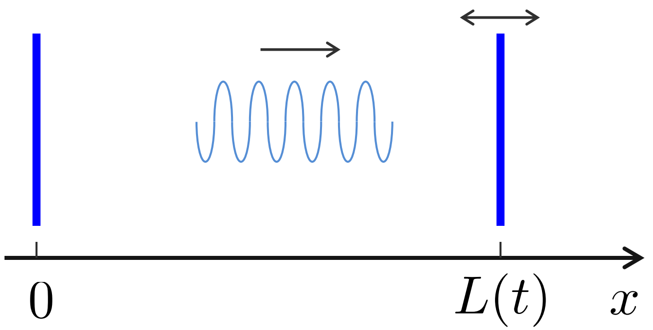


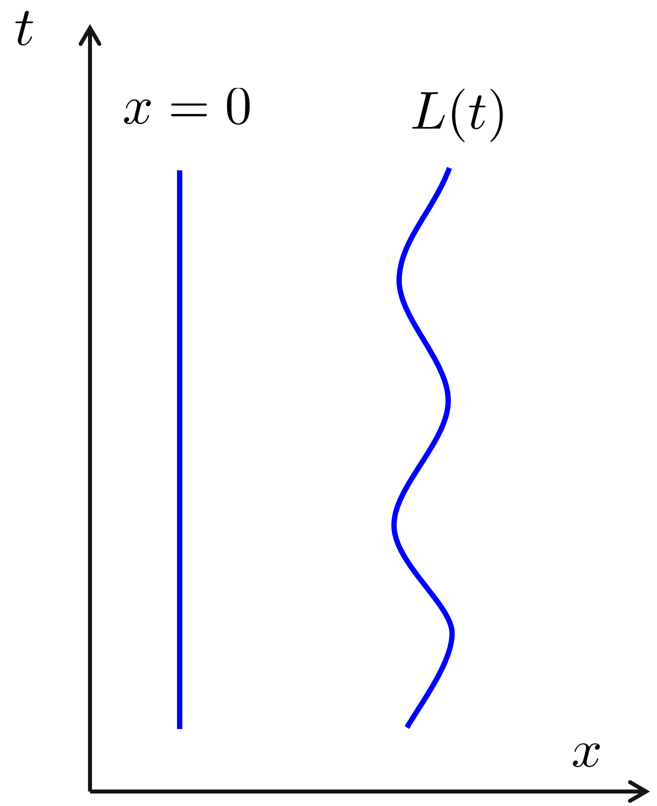
Photon accelerator 3



Alternative: Modulation of physical length

- Cavity with a moving mirror – dynamical Casimir effect





- Wave equation

$$\frac{\partial^2 \mathcal{A}}{\partial t^2} - \frac{\partial^2 \mathcal{A}}{\partial x^2} = 0.$$

- Boundary conditions

$$\mathcal{A}(t, 0) = 0 \qquad \mathcal{A}(t, L(t)) = 0$$

JOURNAL OF MATHEMATICAL PHYSICS VOLUME 11, NUMBER 9 SEPTEMBER 1970

Quantum Theory of the Electromagnetic Field in a Variable-Length One-Dimensional Cavity*†

GERALD T. MOORE
Department of Physics, Brandeis University, Waltham, Massachusetts 02154

- Mapping to a static cavity problem

$$t + x = F(s + w),$$
$$t - x = F(s - w).$$

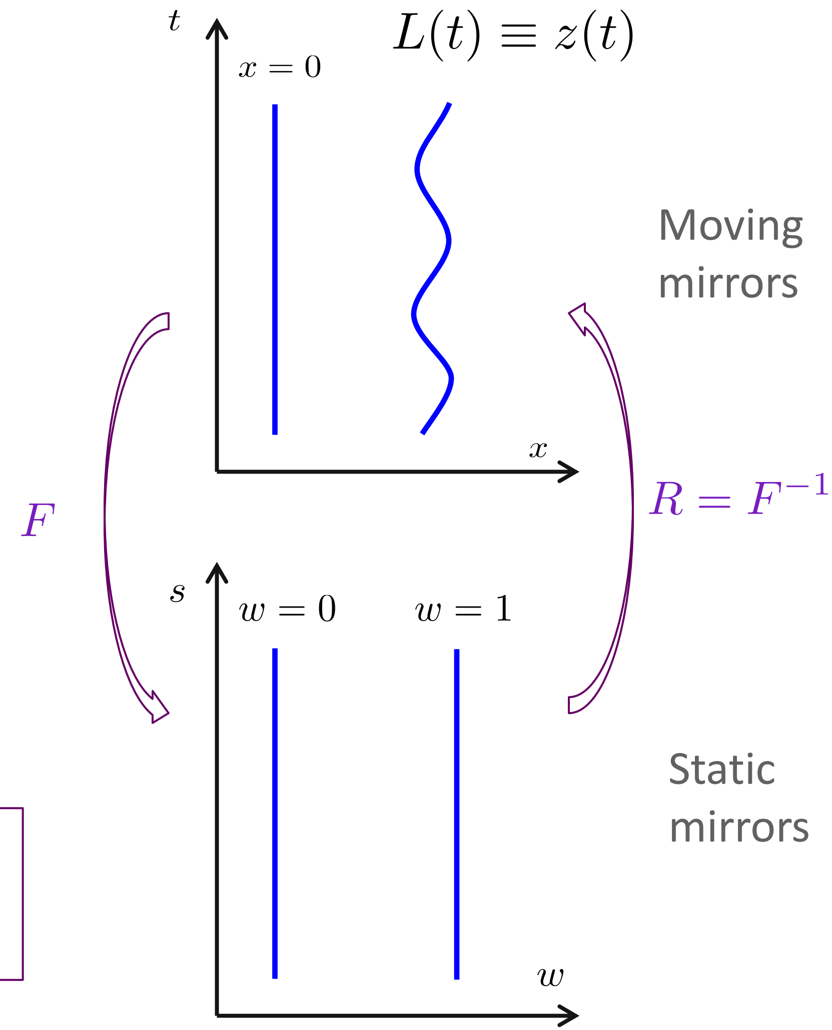
$$\frac{\partial^2 \mathcal{A}}{\partial s^2} - \frac{\partial^2 \mathcal{A}}{\partial w^2} = 0.$$

$$t = F(s),$$
$$t + z(t) = F(s + 1),$$
$$t - z(t) = F(s - 1).$$

$$\boxed{R(t + z(t)) - R(t - z(t)) = 2}$$

- Solution:

$$\boxed{\begin{gathered} A(s, w) \rightarrow \\ \mathcal{A}_+ (R(t - x)) - \mathcal{A}_+ (R(t + x)) \equiv \tilde{\mathcal{A}}(t, x). \end{gathered}}$$



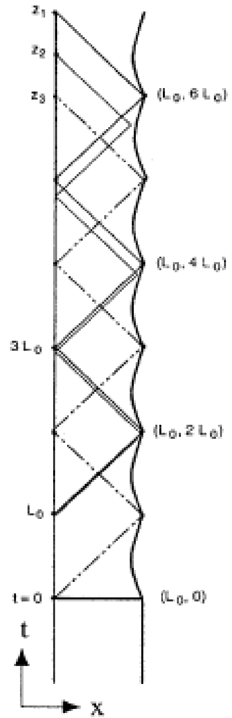
Need conformal map function; near resonance becomes singular!

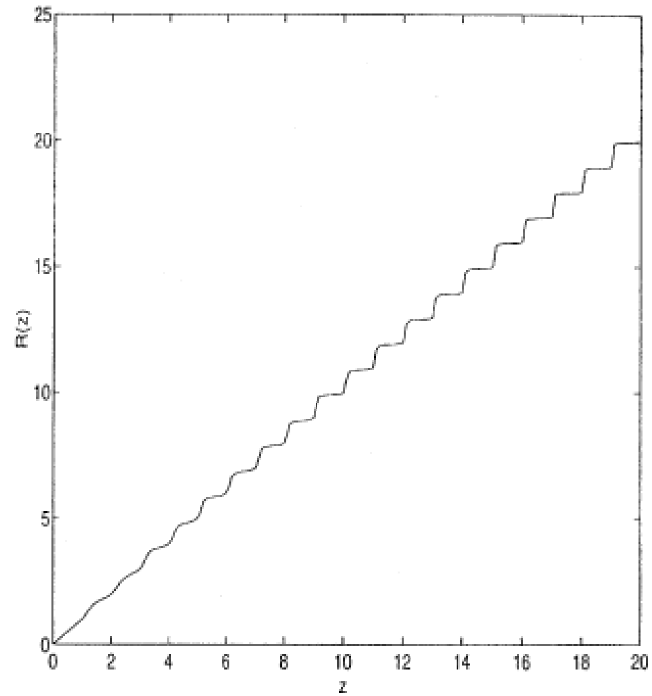
PHYSICAL REVIEW A　　VOLUME 52, NUMBER 6　　DECEMBER 1995

Radiation modes of a cavity with a moving boundary

Carl K. Cole and William C. Schieve
Center for Studies in Statistical Mechanics and Complex Systems and Physics Department, The University of Texas at Austin, Austin, Texas 78712

(Received 1 March 1995; revised manuscript received 8 June 1995)



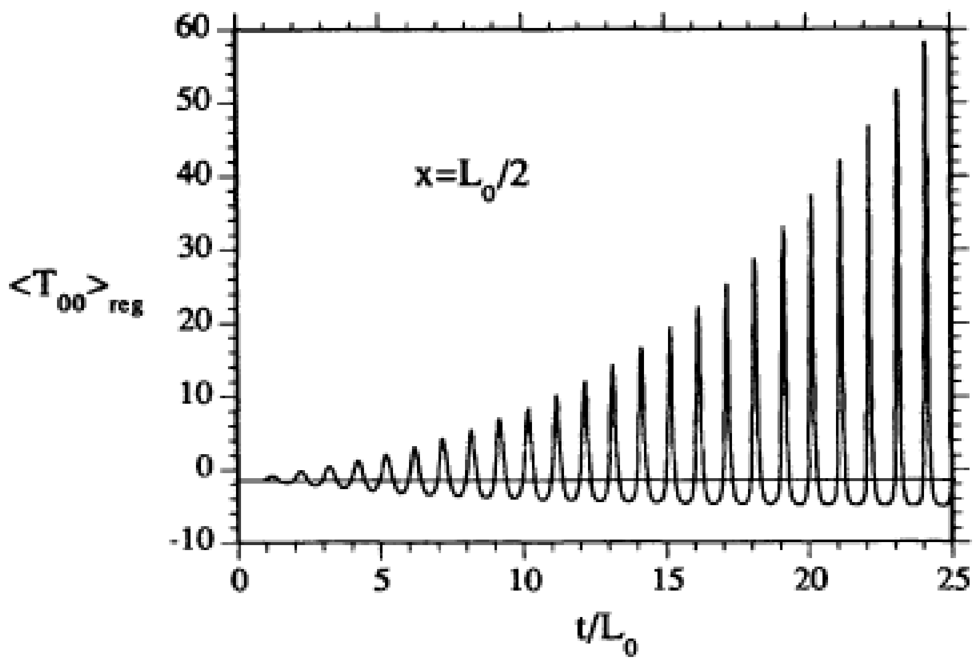


Consequences of singularity: energy density peaks

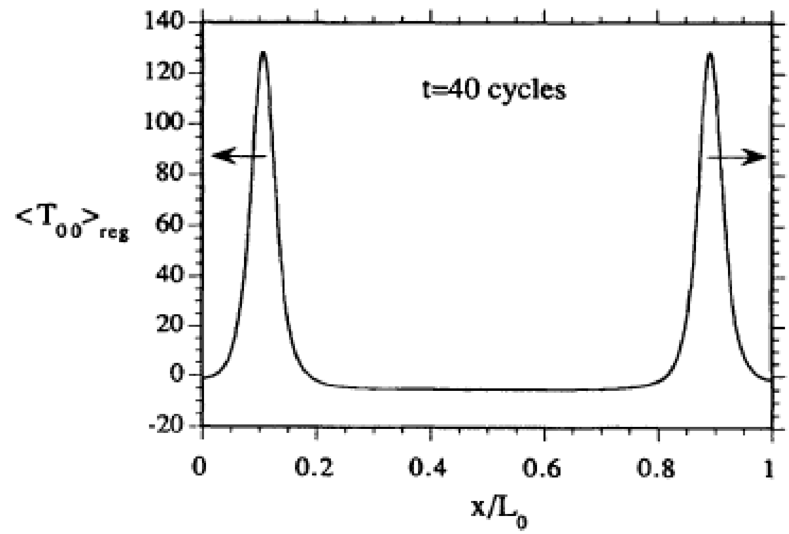
VOLUME 73, NUMBER 14 PHYSICAL REVIEW LETTERS 3 OCTOBER 1994

Resonance Response of the Quantum Vacuum to an Oscillating Boundary

C. K. Law
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627
(Received 20 December 1993)



Energy density as a function of t at $x = L_0/2$.



$$\langle T_{00}\rangle = -(2\pi\epsilon^2)^{-1} - [f(v)+f(u)],$$

$$\langle T_{01}\rangle = f(u) - f(v),$$

$$f = (24\pi)^{-1}\left[\frac{R'''}{R'} - \frac{3}{2}\left(\frac{R''}{R'}\right)^2 + \tfrac{1}{2}\pi^2(R')^2\right].$$

Outline

- Photon accelerator
- **Maps, Fixed points, Chaos**
- Solving wave equation by ***Floquet map***
- Energy singularities
- Dynamical Casimir and “supercooled” vacuum
- Signal compression/decompression
- GR analogy

A different kind of map: dynamical systems

Quantitative Universality for Nonlinear Transformations

Mitchell J. Feigenbaum[1]

$$x_{n+1} = f(x_n)$$

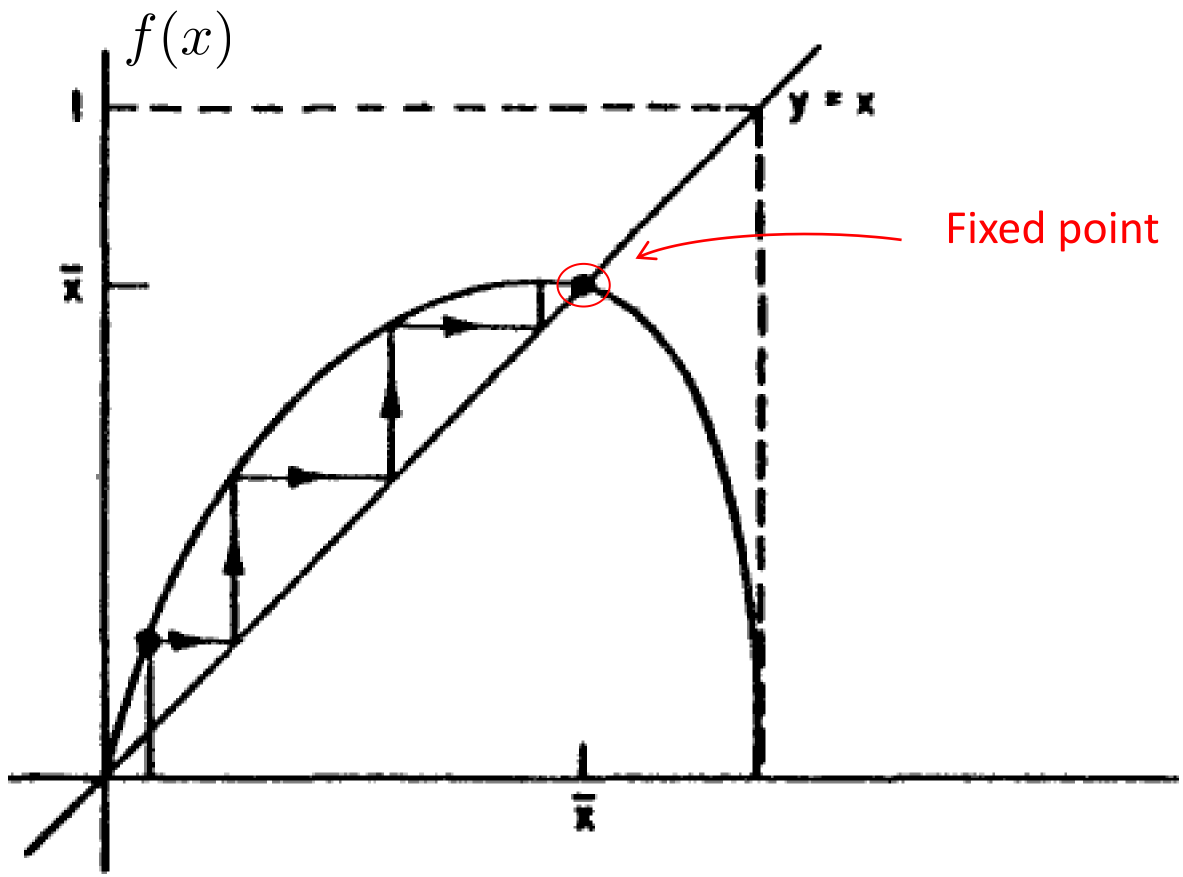


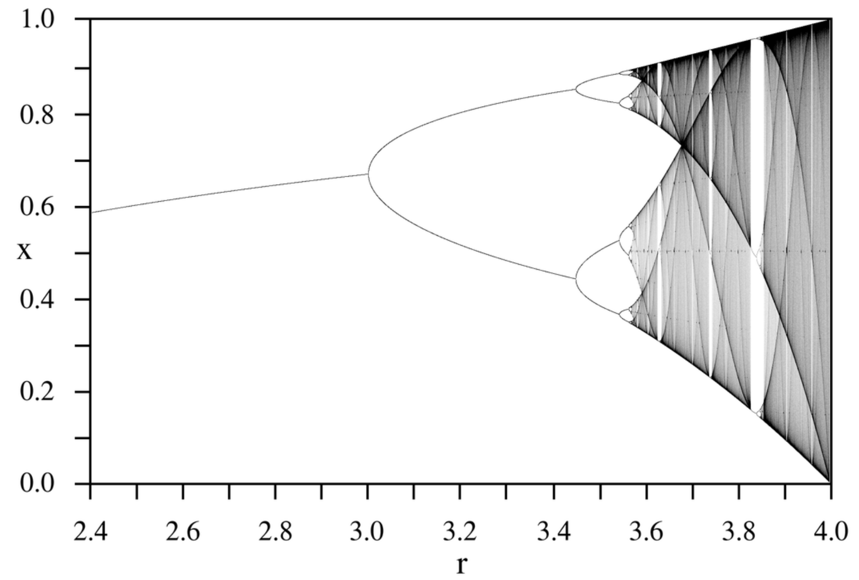
Fig. 2

Journal of Statistical Physics, Vol. 19, No. 1, 1978

Bifurcations, chaos, fractals

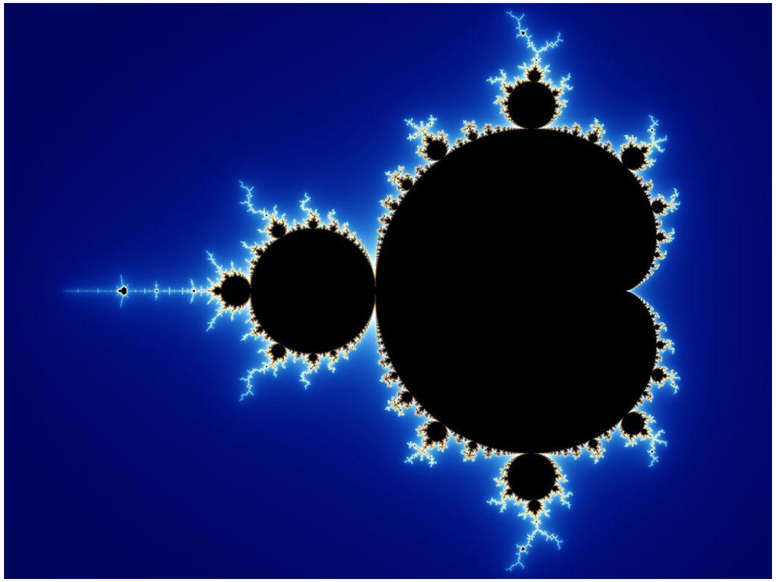
- Real logistic map

$$x_{n+1} = rx_n(1 - x_n).$$



- Complex logistic map (Mandelbrot set)

$$z_{n+1} = z_n^2 + c$$



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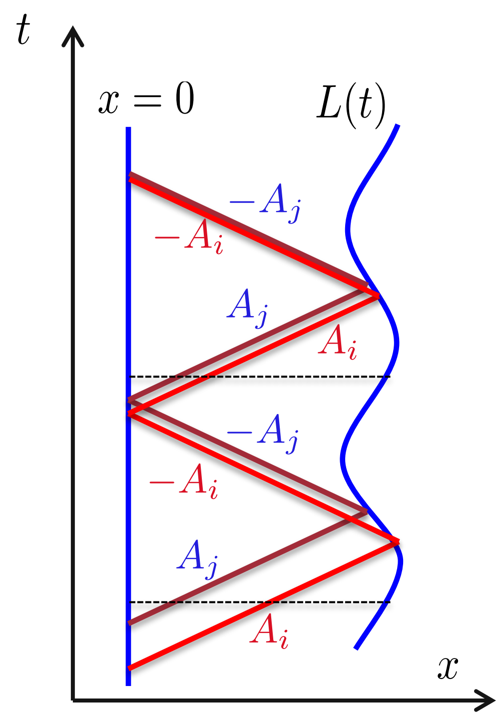
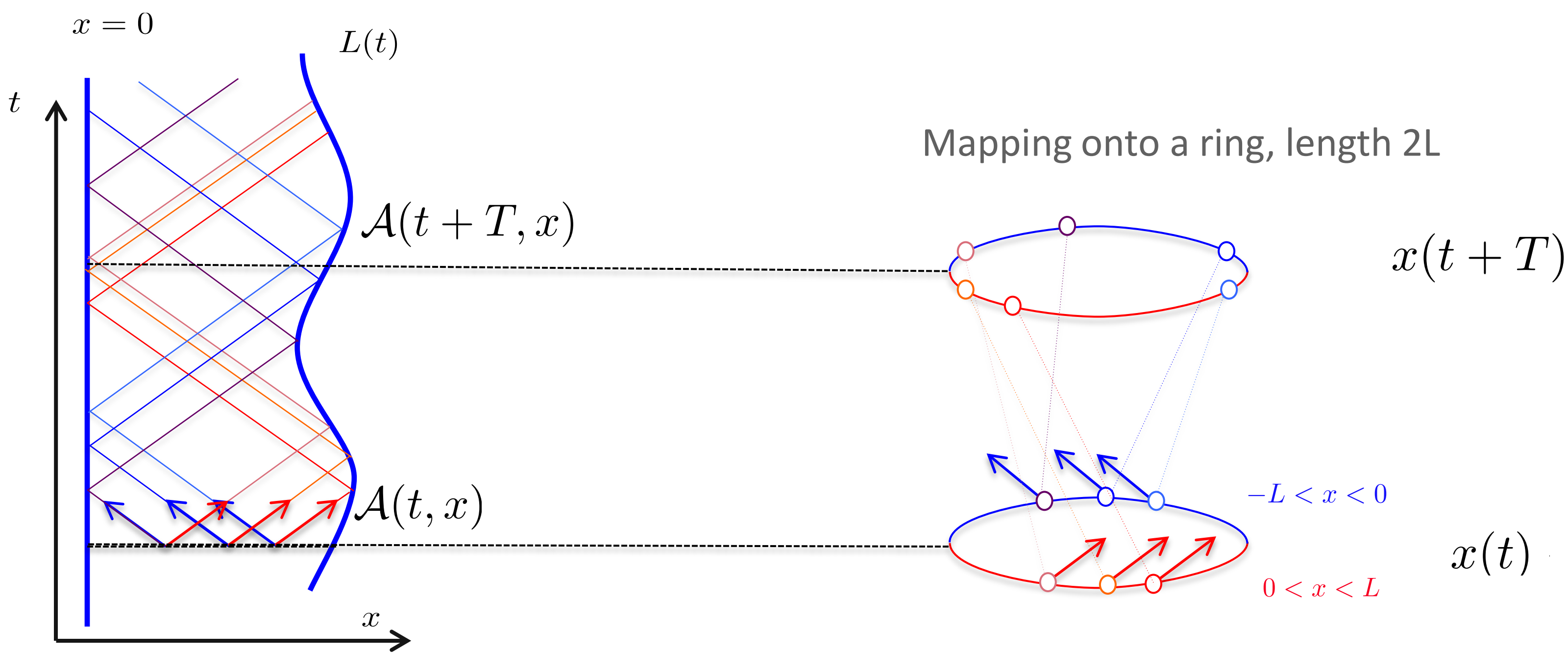
Transport of A (vector potential) along characteristics



Period of modulation

Cavity field problem as a map



$$\frac{\partial^2 \mathcal{A}}{\partial t^2} - \frac{\partial^2 \mathcal{A}}{\partial x^2} = 0.$$

$$\frac{\partial \mathcal{A}}{\partial t} - \frac{\partial \mathcal{A}}{\partial x} = 0$$

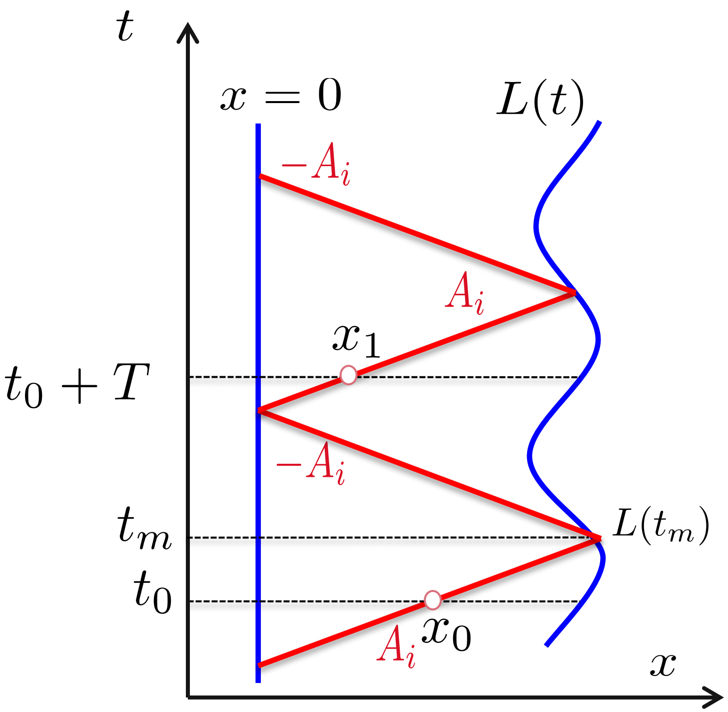
Vector potential remains constant on characteristics (= light rays = null lines)

One period map

$$t_m - t_0 = \frac{L(t_m) - x_0}{c}$$

$$L(t_m) - x_0 + L(t_m) + x_1 = cT$$

$$x_{n+1} = f(x_n)$$



Iterative map

Can relate $f : x_0 \to x_1 \to x_2 \to \dots$

Vector potential is invariant under this map:

$$\mathcal{A}(x_0, t_0) = (\pm)\mathcal{A}(x_1, t_0 + T) = (\pm)\mathcal{A}(x_2, t_0 + 2T) = \dots$$

Weak modulation - explicit map

Approximation:

$$t_m - t_0 = \frac{L(t_m) - x_0}{c} \longrightarrow t_m \approx t_0 + \frac{L_0 - x_0}{c}$$

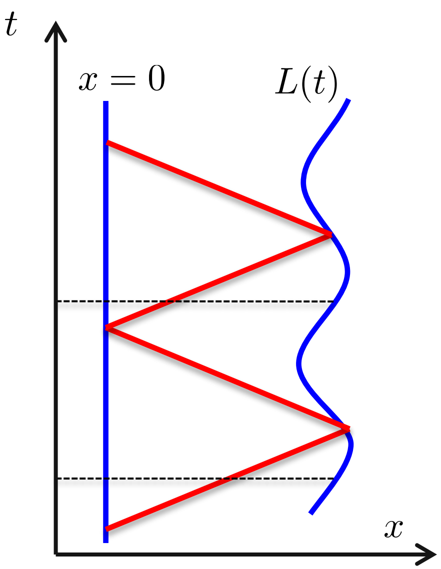
$$x_1 = x_0 + cT - 2L(t_m) \longrightarrow x_1 = x_0 + cT - 2L(\phi_0 - x_0)$$

Single-step map

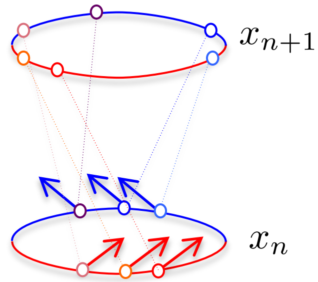
$$x_{n+1} = x_n + cT - 2L(\phi_0 - x_n)$$

Multi-step map, continuous limit

$$\frac{dx}{dn} = cT - 2L(\phi_0 - x)$$

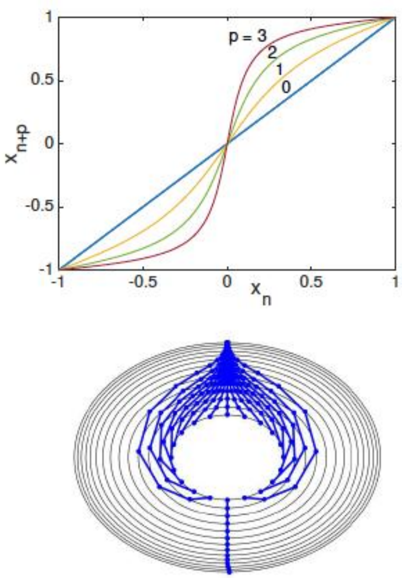


Example of a map

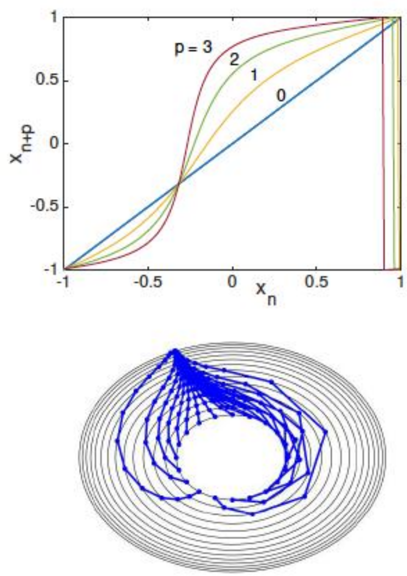


$$L(t) = L_0 + A\cos\Omega t$$

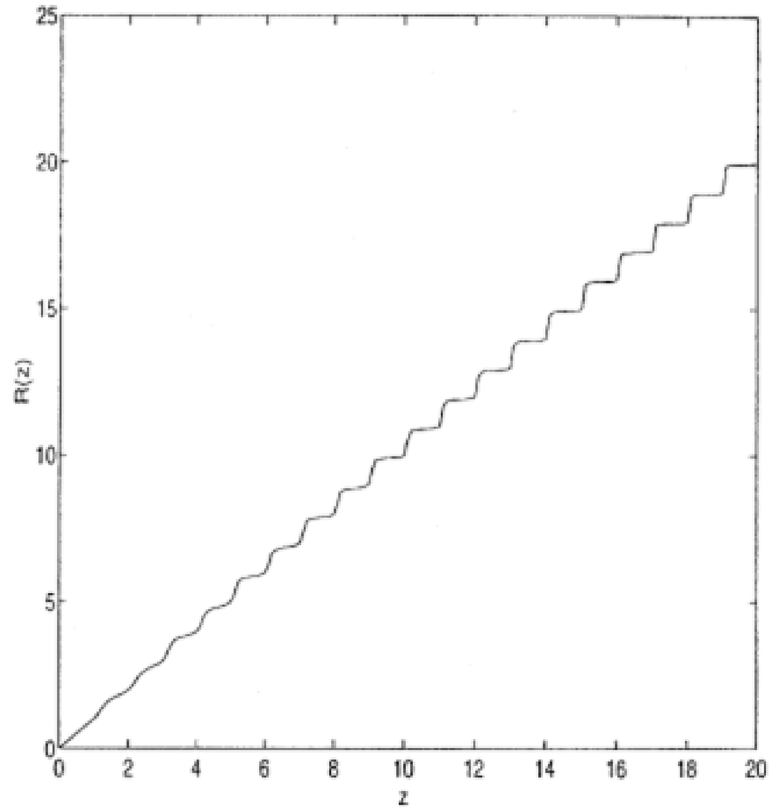
Compare to the conformal function R:
(Cole + Schieve)



On resonance

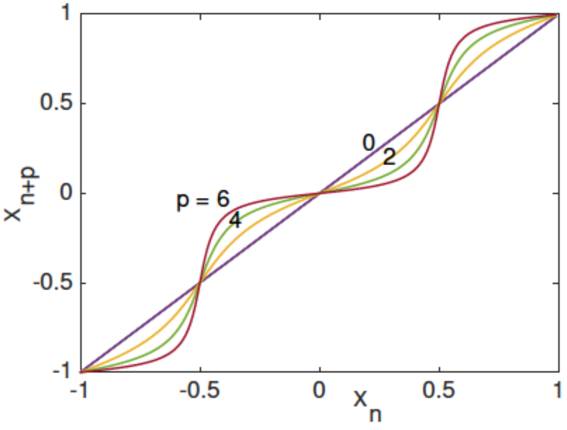


Bit away

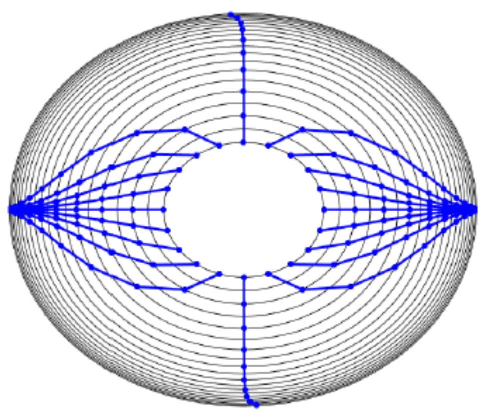


Driving near higher cavity resonances

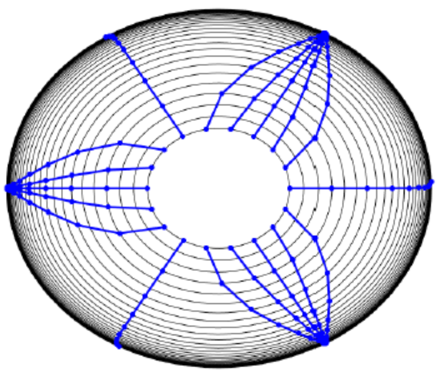
$$L(t) = L_0 + A\cos 2\Omega t$$



a



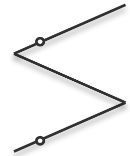
$$L(t) = L_0 + A\cos 3\Omega t$$

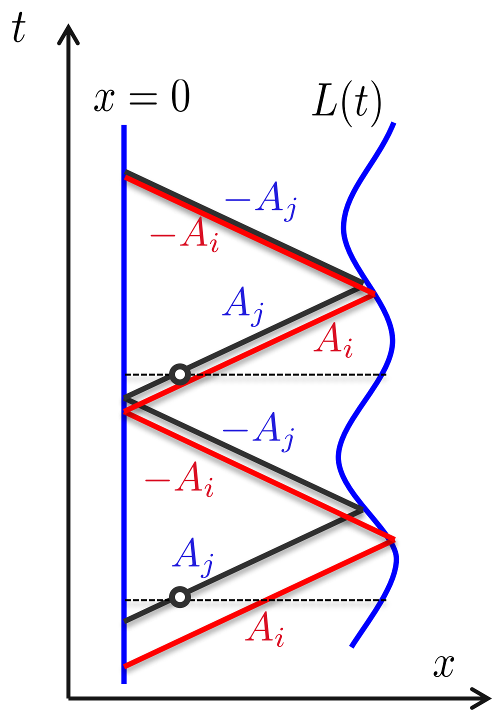


Back to continuous time

Stroboscopic observation -> Fixed points

Continuous observation ->
"fixed point trajectories"





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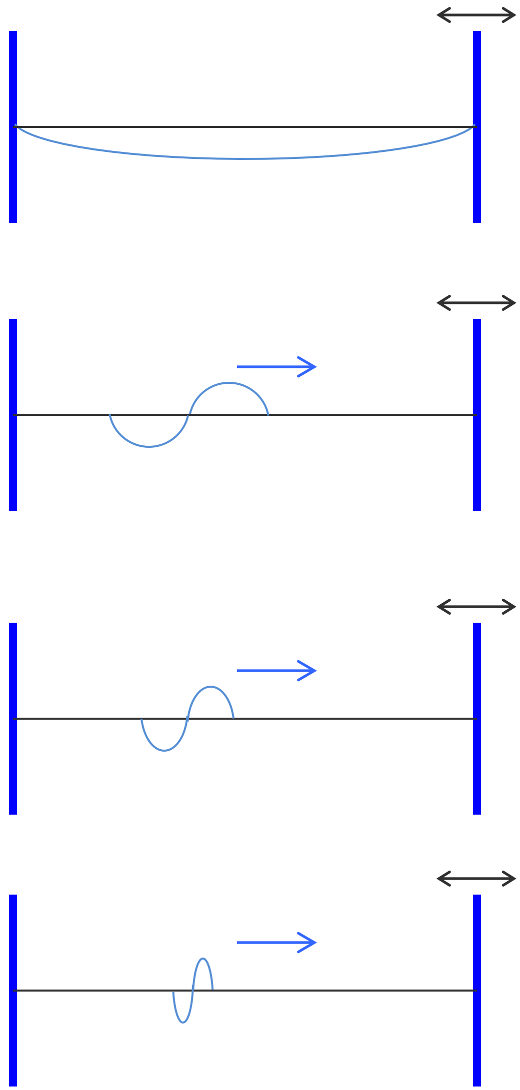
Qualitative picture



$$k(t) \sim k_0 \left(\frac{c+v}{c-v}\right)^{t/T} \approx k_0 e^{\frac{2vt}{cT}}$$

v Mirror velocity at fixed point

Energy Density

$$(\partial_x \mathcal{A})^2 + (\partial_t \mathcal{A})^2 \sim k^2 \mathcal{A}^2$$

Total Energy

$$[(\partial_x \mathcal{A})^2 + (\partial_t \mathcal{A})^2]/k \sim k \mathcal{A}^2$$

Both grow exponentially with time

A known effect…

http://en.rfi.fr/20180612-frances-queen-accordeon-yvette-horner-dies/

Energy and energy density

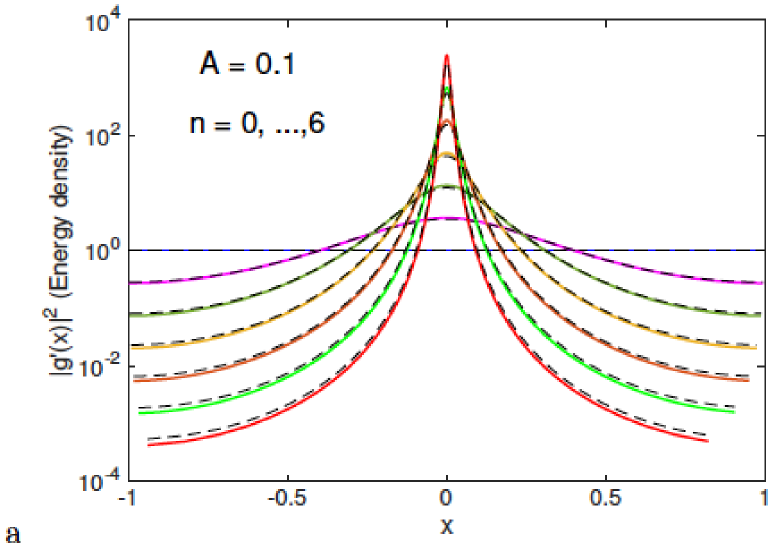
$$
\begin{aligned}
E(t) &= \int dx_t T_{00}(x_t, t) \\
&= \frac{1}{2}\int dx_t \left[\left(\frac{\partial \mathcal{A}(x_t,t)}{\partial x_t}\right)^2 + \left(\frac{\partial \mathcal{A}(x_t,t)}{\partial t}\right)^2\right] \\
&= \int dx_t \left[\frac{\partial g^{(t)}(x_t)}{\partial x_t}\right]^2 \left[\mathcal{A}_0'(g^{(t)}(x_t))\right]^2 \\
&= \int dx_0 \frac{\partial g^{(t)}(x_t)}{\partial x_t} \left[\mathcal{A}_0'(x_0)\right]^2 \\
&= \int dx_0 \frac{1}{\partial f^{(t)}(x_0)/\partial x_0} \left[\mathcal{A}_0'(x_0)\right]^2
\end{aligned}
$$

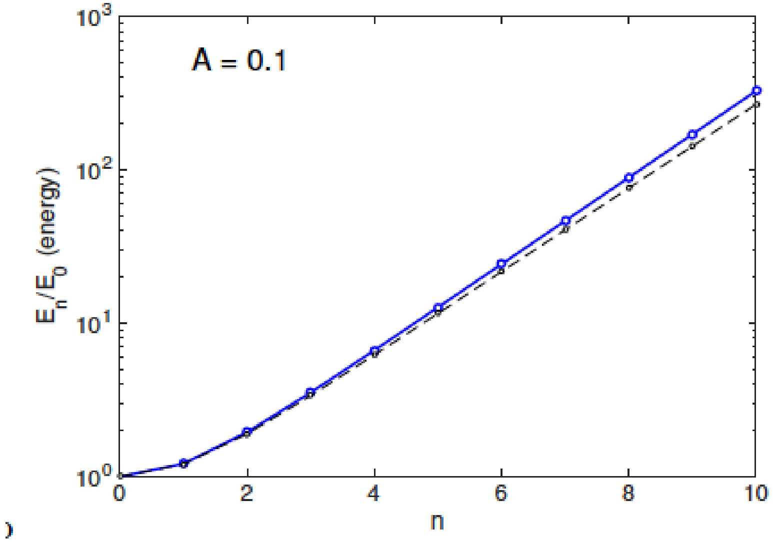
$g = f^{-1}$

Weak harmonic modulation, $\tilde{A} = \pi A/L \ll 1$

$$\tilde{x}_0 = \tilde{g}(\tilde{x}_n) = 2\arctan(e^{-2\tilde{A}n}\tan\frac{\tilde{x}_n}{2})$$

Energy and energy density, numerical





Over time, energy density increases (exponentially) near fixed point trajectories, and **decreases (exponentially) everywhere else**.

Uses:

1) pulsed energy source (similar to Mode-Locked laser)
2) Cavity cooling (Use a “dumb” Maxwell’s demon)

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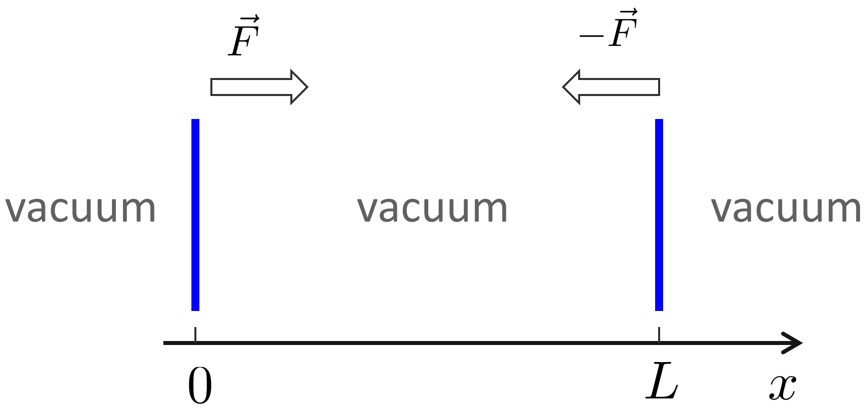
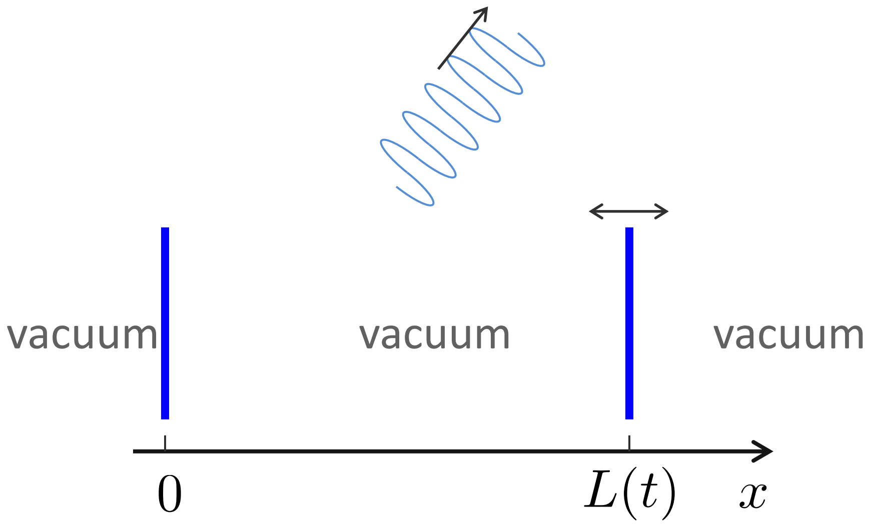
Casimir effects



Static Casimir:
Attraction between mirrors

Dynamic Casimir:
time-dependent force,
entangled photon emission



Field quantization

[Moore 1970; Filling and Davies, Proc R Soc Lond A 1976; P. D. Nation et al, RMP 2012]

- Conformal map function:

$$R(t + z(t)) - R(t - z(t)) = 2$$

- Mode function:

$$\phi_n(x, t) = (4\pi n)^{-1/2}[e^{-i\pi n R(t+x)} - e^{-i\pi n R(t-x)}]$$

- Quantized field

$$\phi(x, t) = \sum_n a_n \phi_n(x, t) + a_n^\dagger \bar{\phi}_n(x, t)$$

- Energy density

$$\langle T_{00} \rangle = -(2\pi\epsilon^2)^{-1} - [f(v) + f(u)],$$

$$\langle T_{01} \rangle = f(u) - f(v),$$

$$f = (24\pi)^{-1}\left[\frac{R'''}{R'} - \frac{3}{2}\left(\frac{R''}{R'}\right)^2 + \tfrac{1}{2}\pi^2(R')^2\right].$$

Schwarzian

Static Casimir

1D Casimir results

- Driving
  - Harmonic q
  - Mirror oscillation amplitude A

$$\langle T_{00}(x,t)\rangle = -\frac{\pi q^2}{48L^2} + \frac{\pi(q^2-1)}{48L^2}(\tilde{g}')^2$$

$$g'(x) = \frac{1}{\cosh(2qn\tilde{A}) + \sinh(2qn\tilde{A})\cos q\tilde{x}}$$

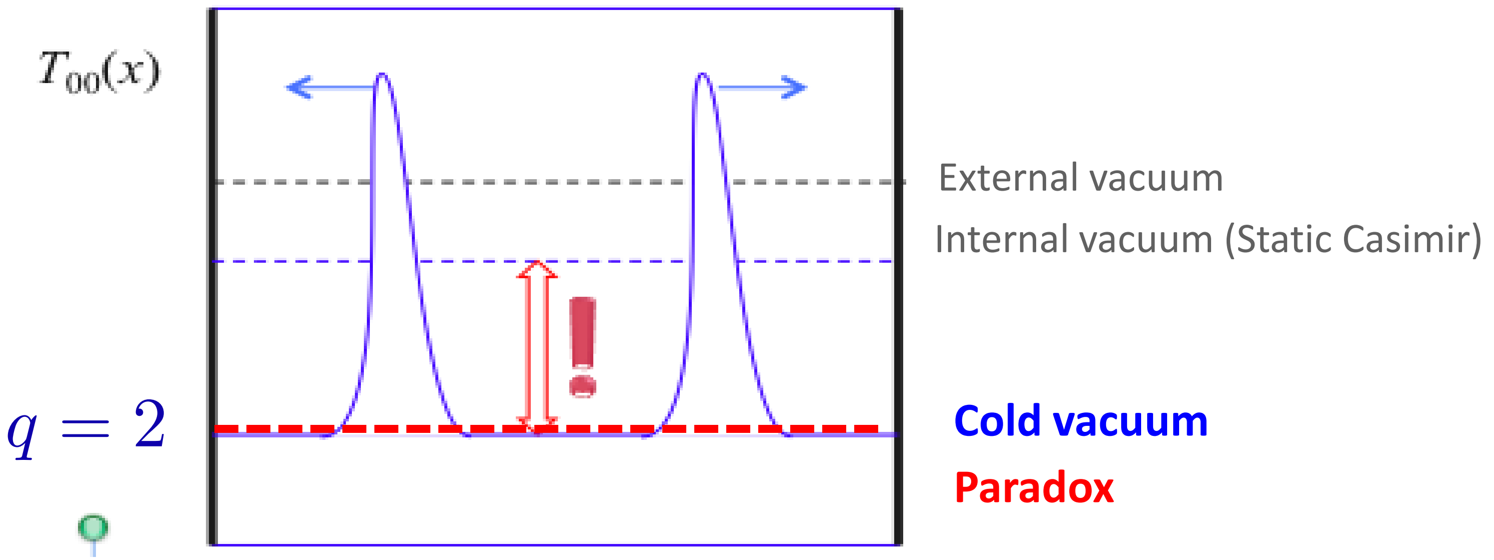
- Energy density for static (A = 0), and first hamonic ($q = 1, A \neq 0$)

$$\langle T_{00}(x,t)\rangle = -\frac{\pi}{48L^2}$$

Casimir results, 2

- Driving
  - Harmonic $q \geq 2$
  - Mirror oscillation amplitude A

$$\langle T_{00}(x,t)\rangle = -\frac{\pi q^2}{48L^2} + \frac{\pi(q^2-1)}{48L^2}(\tilde{g}')^2$$



Proc. R. Soc. Lond. A. **348**, 393–414 (1976)
Printed in Great Britain

Radiation from a moving mirror in two dimensional space-time: conformal anomaly

BY S. A. FULLING AND P. C. W. DAVIES
Department of Mathematics, University of London, King's College

(*Communicated by R. Penrose, F.R.S. – Received* 13 *August* 1975)

The energy-momentum tensor is calculated in the two dimensional quantum theory of a massless scalar field influenced by the motion of a perfectly reflecting boundary (mirror). This simple model system evidently can provide insight into more sophisticated processes, such as particle production in cosmological models and exploding black holes. In spite of the conformally static nature of the problem, the vacuum expectation value of the tensor for an arbitrary mirror trajectory exhibits a non-vanishing radiation flux (which may be readily computed). The expectation value of the instantaneous energy flux is negative when the proper acceleration of the mirror is increasing, but the total energy radiated during a bounded mirror motion is positive. A uniformly accelerating mirror does not radiate; however, our quantization does not coincide with the treatment of that system as a 'static universe'. The calculation of the expectation value requires a regularization procedure of covariant separation of points (in products of field operators) along time-like geodesics; more naïve methods do not yield the same answers. A striking example involving two

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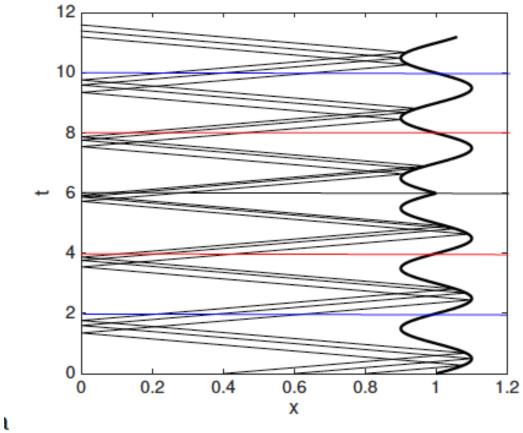
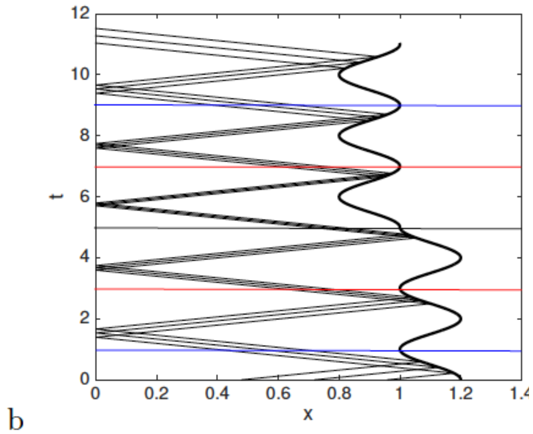
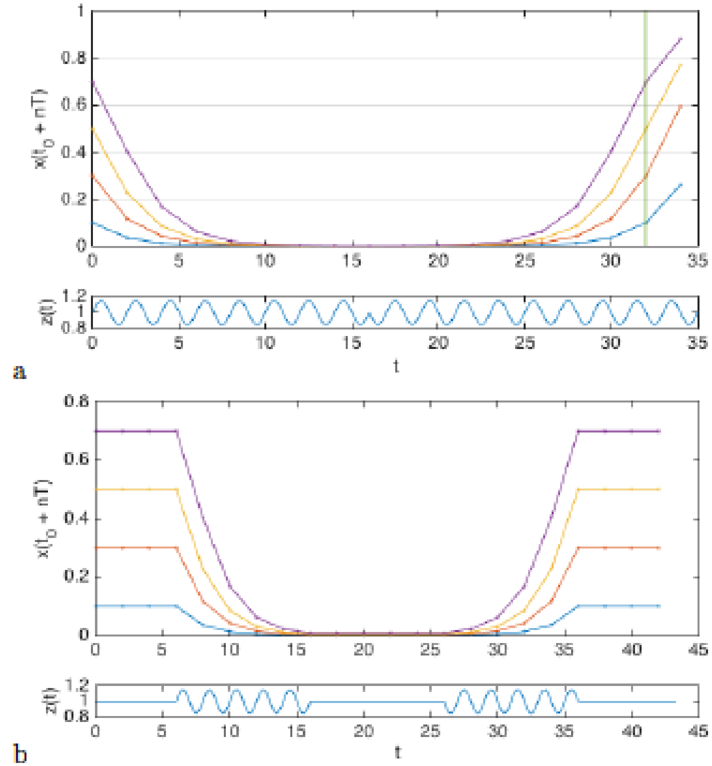
Discrete time reversal: Compression/decompression

For a fixed stroboscopic sampling, it is possible to construct discrete time-reversal protocol:

$$Z_{TR}(t) = \begin{cases} z(t), & t < t^* \\ 2L_0 - z(t), & t \geq t^*. \end{cases}$$







Signal compression/decompression

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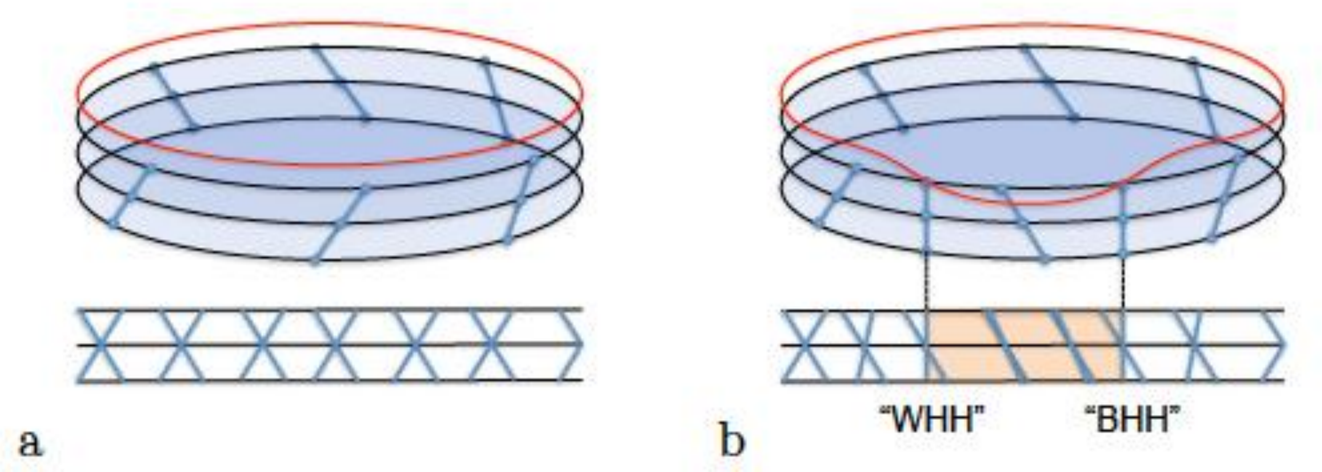
"Stroboscopic GR"

For a fixed stroboscopic sampling, the light ray evolution mimics propagation in curved spaces

(stable/unstable) fixed point <-> (white/black) hole horizon



For details see Martin, arXiv:1809.02621

Summary

- Problem of periodically modulated cavities can be solved with dynamical maps
- The effect of maps is to concentrate energy around the fixed point trajectories
- Applications: cooling, compression/coding
- Qualitative picture of dynamical Casimir effect (vacuum squeezing)
- Connections to GR

Future

- Manipulation of individual photon states
- Understand overcooled vacuum (entanglement?)
- Application to cooling tech (can use phoNons instead of phoTons)
- Other systems (BEC, etc)

Thanks

- D. Orgad
- B. I. Halperin
- E. Demler
- V. Rosenhaus
- H. Kapteyn
- K. Schwab
- KITP