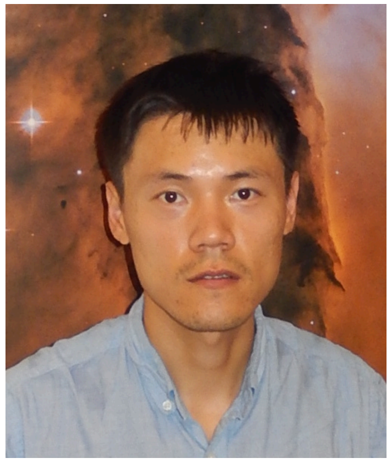
Measurement-driven entanglement transition in random circuits

Yaodong Li, UCSB
KITP Program Dynamics of Quantum Information, October 31, 2018



Xiao Chen
KITP

Matthew Fisher
UCSB

What happens when we continually measure an interacting many-body system?

What can we say about the interplay between **unitary dynamics** and **continual local measurements**?

Quantum Zeno effect: when you make measurements very frequently, the state cannot evolve.

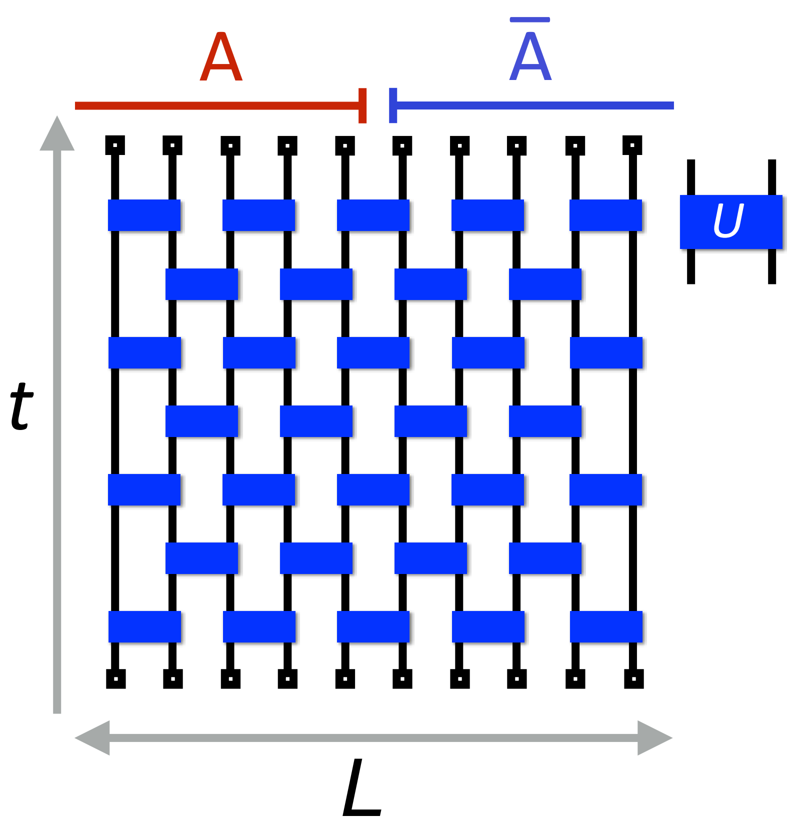
This talk:
In a prototypical model, we find a phase transition in entanglement entropy from volume-law to area-law, between *rare* and *frequent* measurements.

Random unitary circuit

Nahum, Ruhman, Vijay, Haah, 2016
Nahum, Vijay, Haah, 2017
von Keyserlingk, Rakovszky, Pollmann, Sondhi, 2017

A minimal model: random unitary circuit.
(i) unitarity
(ii) local interaction
but no other structures (chaotic, no conservation laws).

Aims to capture universal dynamics of entanglement entropy



random unitary

Focus on (1+1)D in this talk

Random non-unitary circuit

Nahum, Ruhman, Vijay, Haah, 2016
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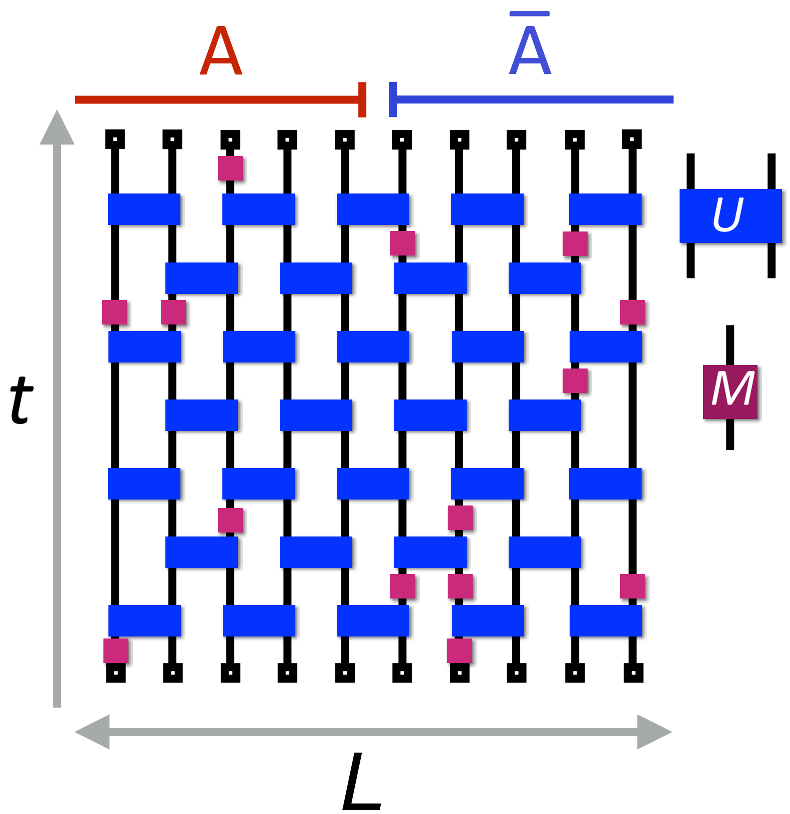


random unitary

local projective measurement

Focus on (1+1)D in this talk

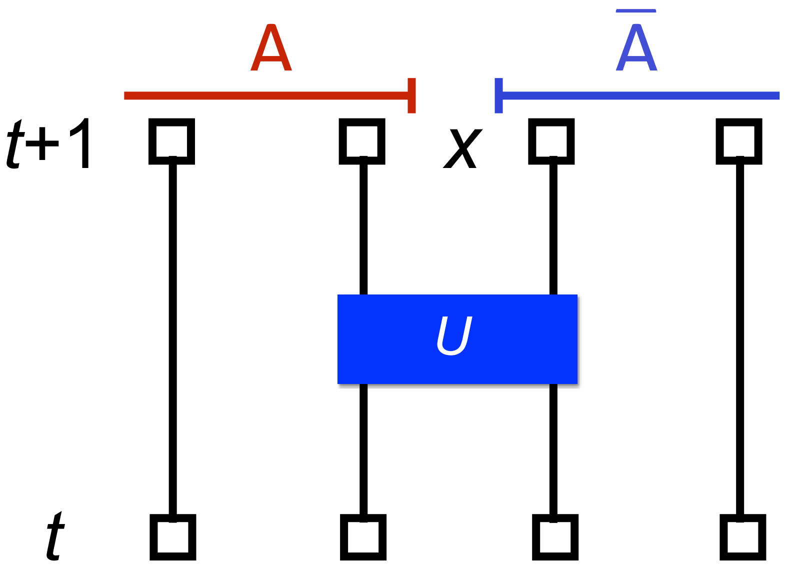
p: the measurement rate

Randomly pick a fraction *p* out of all the possible positions (*L* * *t*) to make a local measurement of *Z*.

Collapse the wave function and keep the pure state!!

Chan, Nandkishore, Pretko, Smith, 1808.05949
Skinner, Ruhman, Nahum, 1808.05953
YL, Chen, Fisher, 1808.06134

Competition between *U* & *M*



Unitary gates at *x*: generates entanglement locally, as random crystal growth

$$S(x, t+1) = \min(S(x-1, t), S(x+1, t)) + 1$$

p -> 0, maximally entangled in the bulk of the circuit (*t* >> *L*)

Nahum, Ruhman, Vijay, Haah, 2016

Competition between U & M

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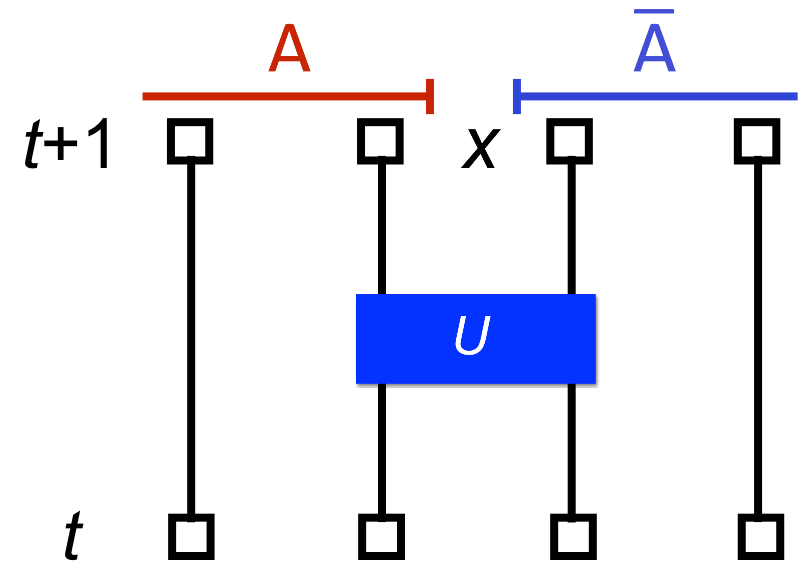
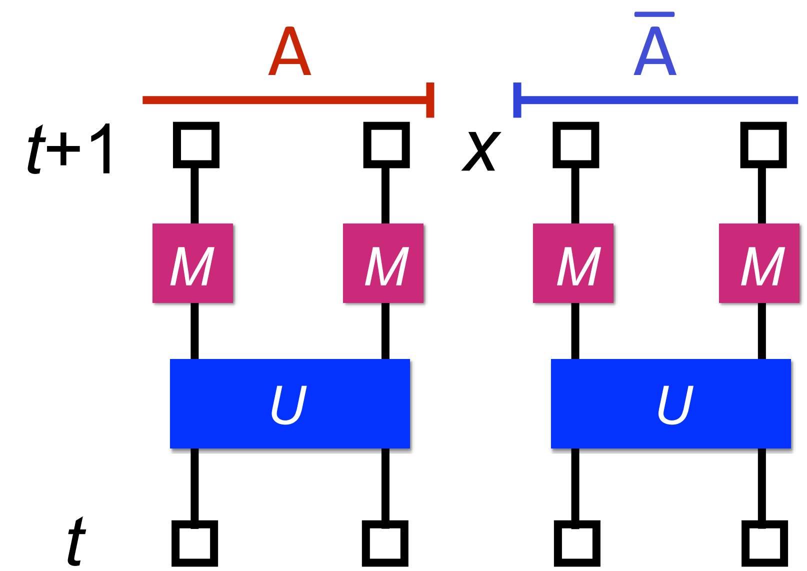


Measurements: extracting information from the system, and trying to disentangle the wavefunction.

In the Zeno limit (p -> 1), measurement on every qubit, a product state in local Z basis, fully disentangled.



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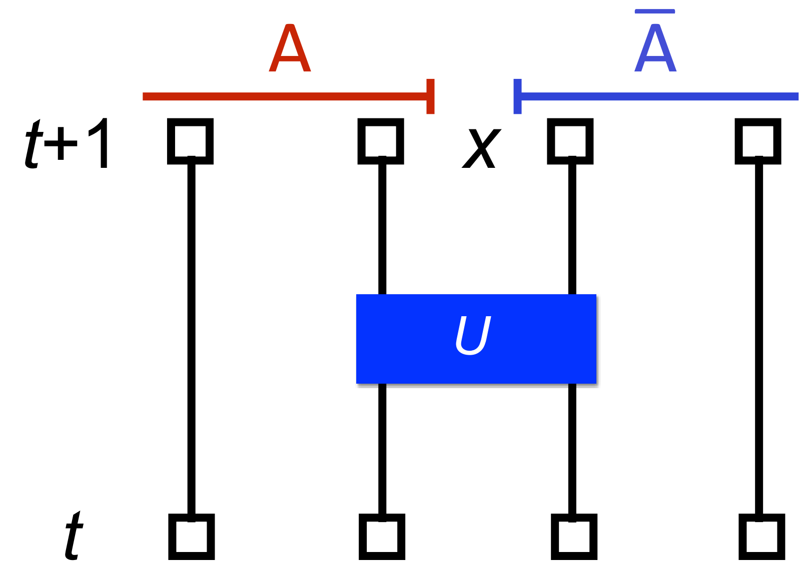

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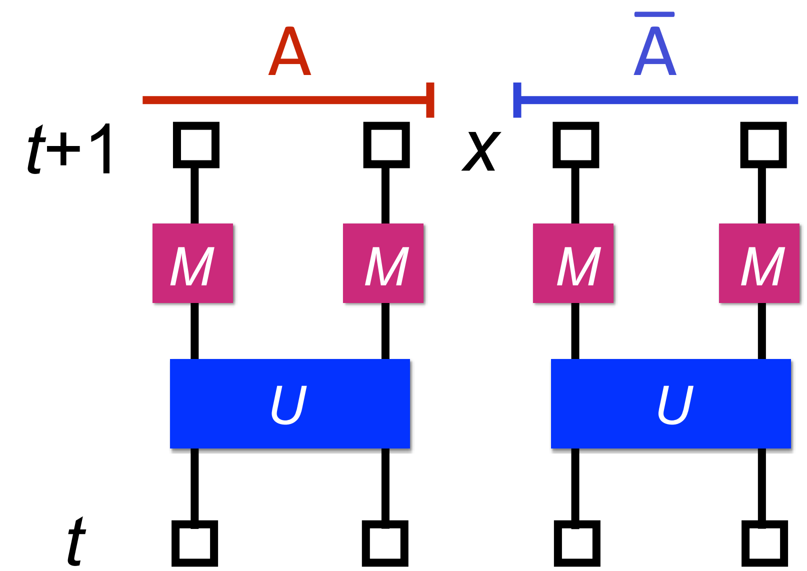

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For any ***p***, as ***t* >> *L***, the entangling effect of unitaries and the disentangling effects of the measurements presumably cancel out, thus reaching steady states with some characteristic value of entanglement entropy.

Steady state entanglement: Phase diagram?

Unitary gates at *x*: generates entanglement locally, as random crystal growth

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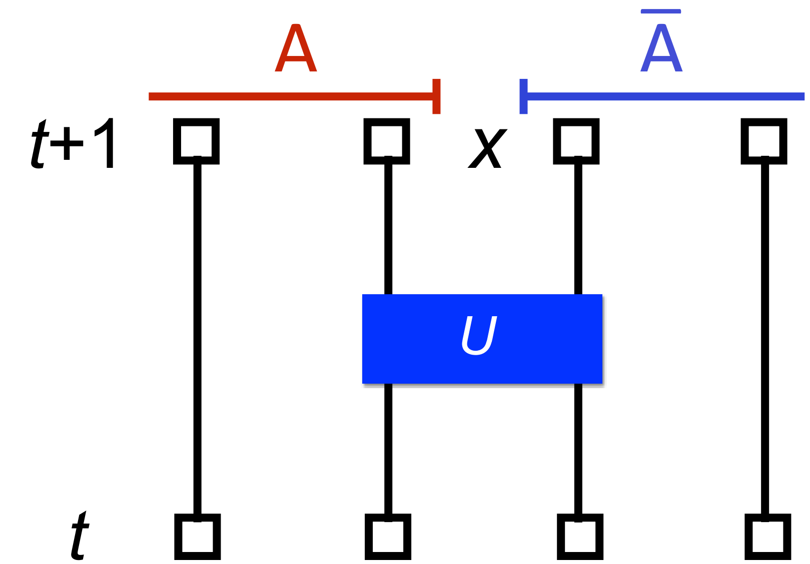

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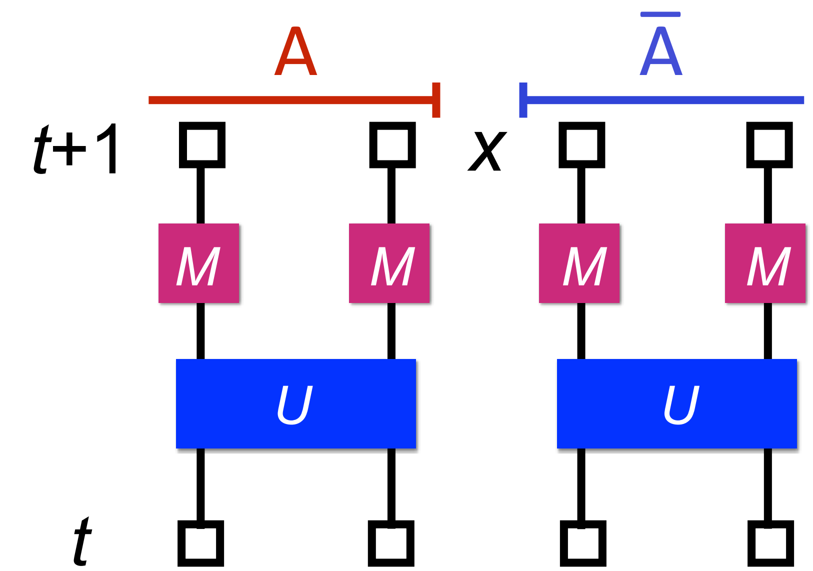
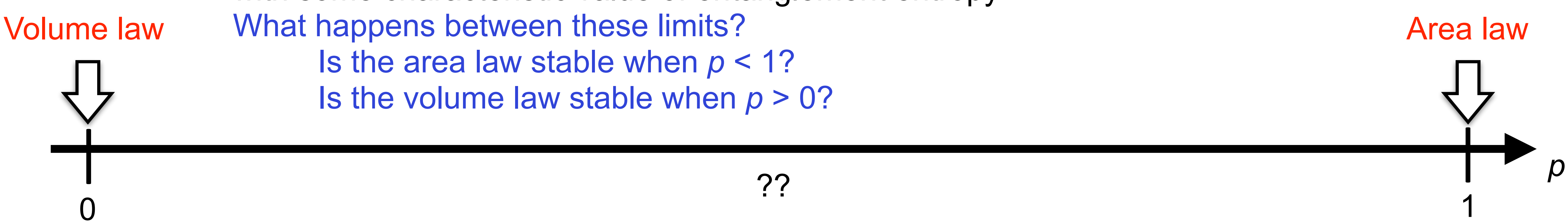


For any ***p***, as ***t* >> *L***, the entangling effect of unitaries and the disentangling effects of the measurements presumably cancel out, thus reaching steady states with some characteristic value of entanglement entropy.

What happens between these limits?

Is the area law stable when *p* < 1?

Is the volume law stable when *p* > 0?



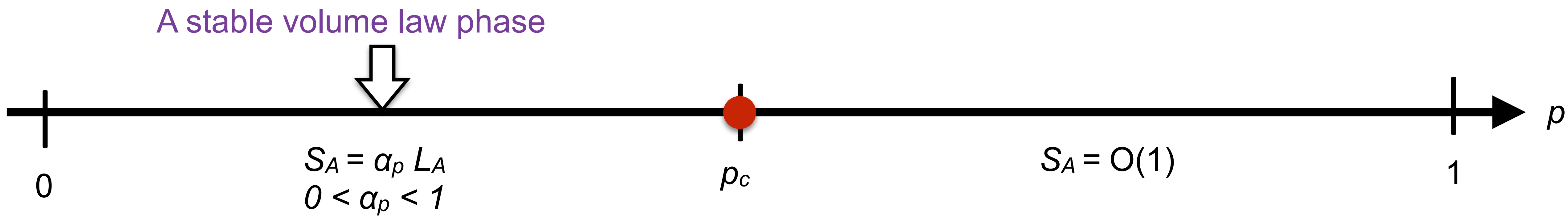
Phase transition in entanglement entropy

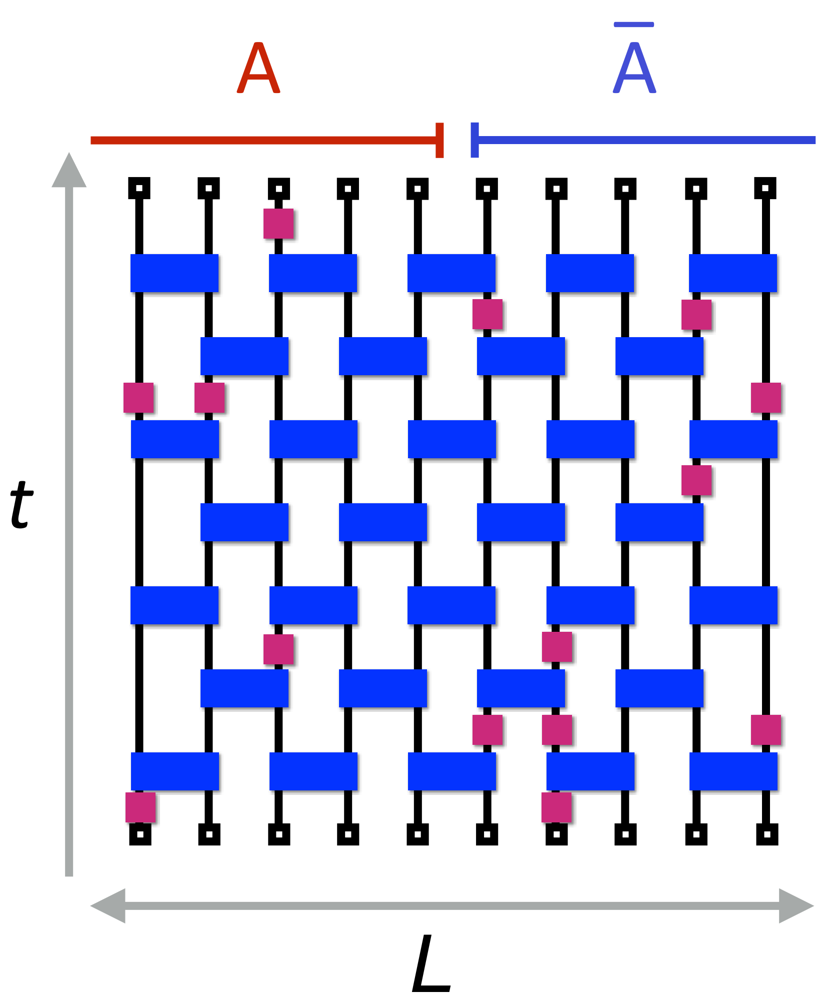
Is the area law stable when $p < 1$?
Is the volume law stable when $p > 0$?

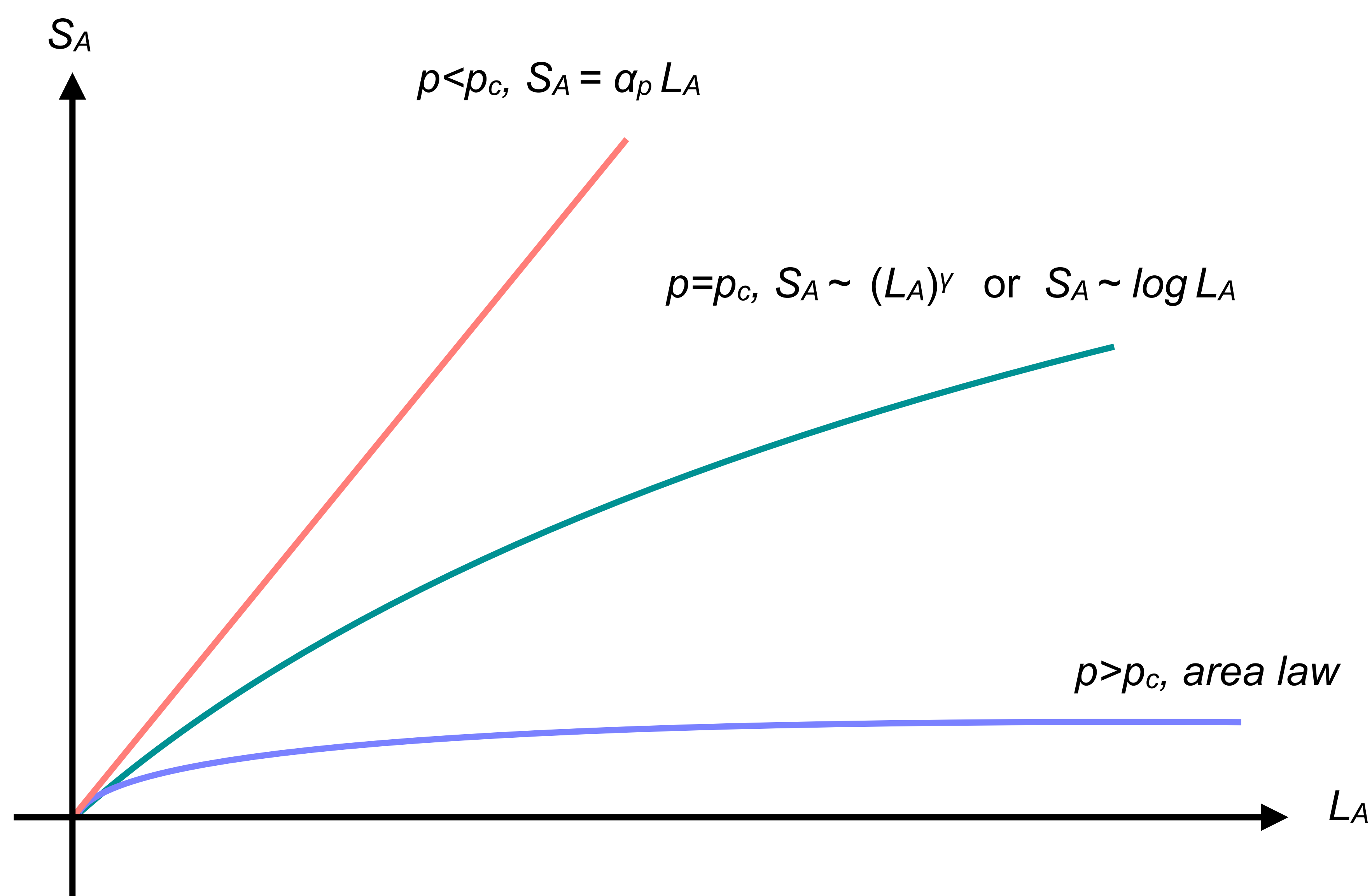
Yes and yes.

Skinner, Ruhman, Nahum, 1808.05953
YL, Chen, Fisher, 1808.06134

The phase diagram we found:



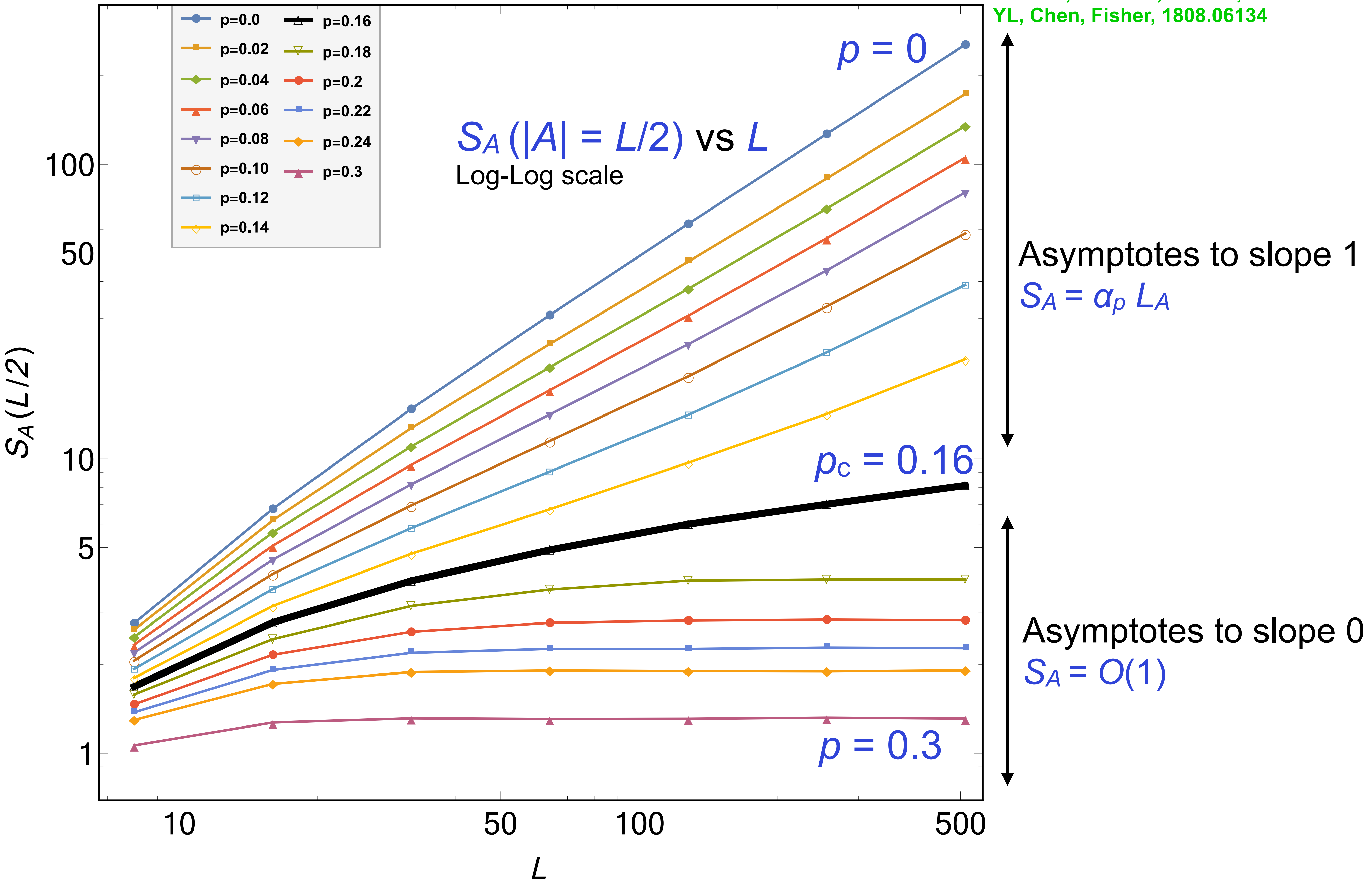




Phase transition in entanglement entropy

random Clifford circuit

Skinner, Ruhman, Nahum, 1808.05953
YL, Chen, Fisher, 1808.06134



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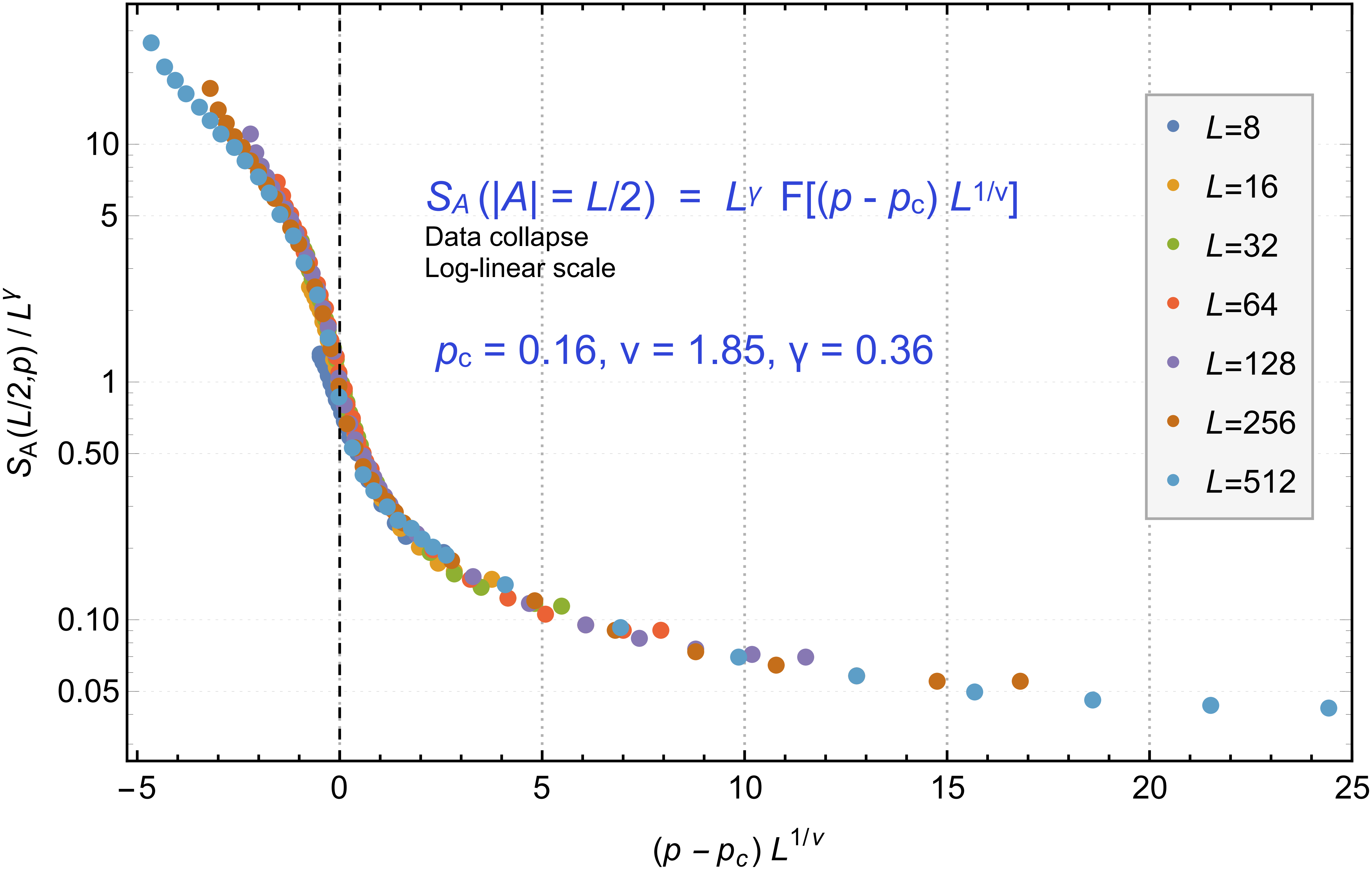
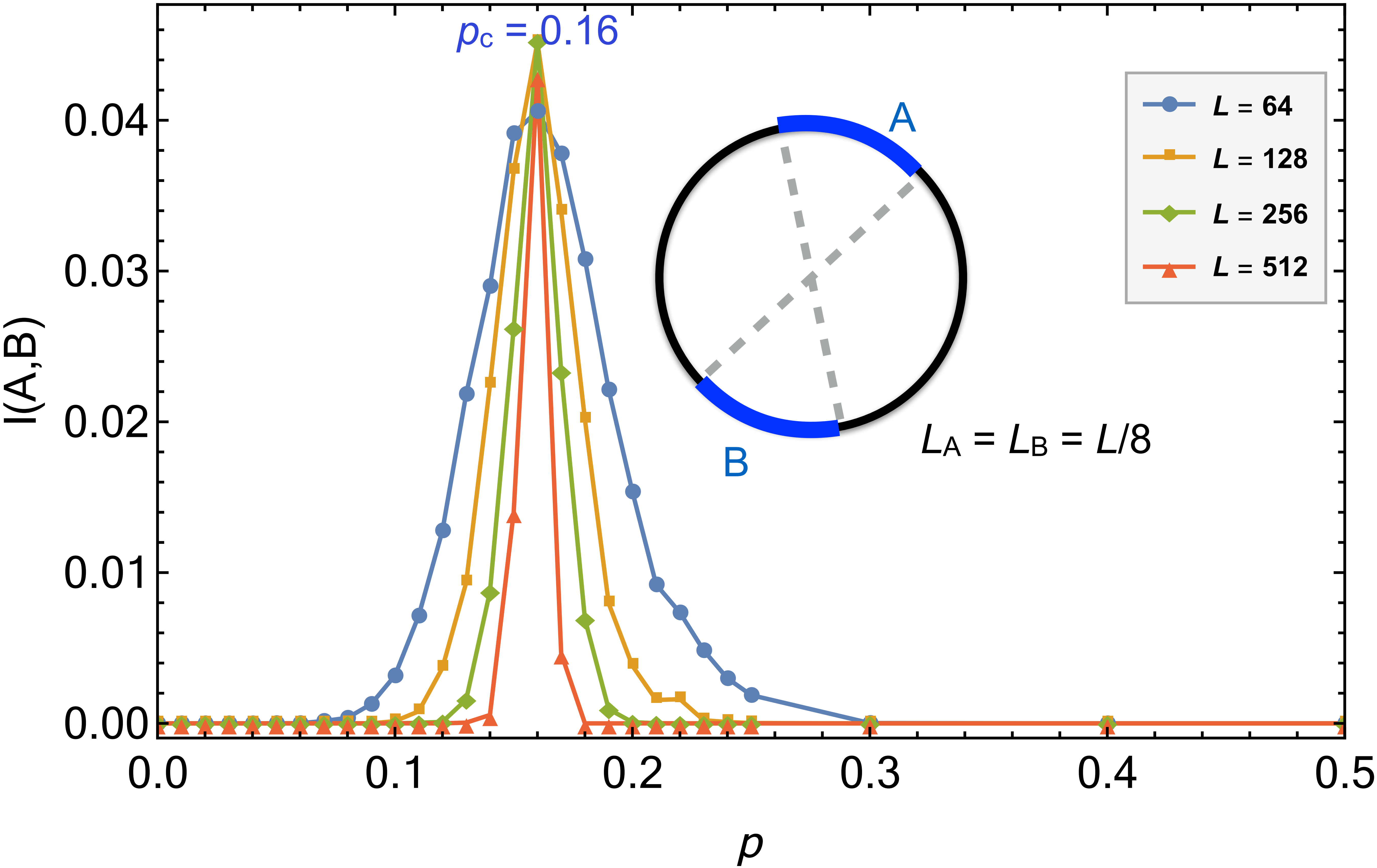


Phase transition seen in mutual information

random Clifford circuit

$I(A, B) = S_A + S_B - S_{A \cup B}$ measures the correlation between A and B

Skinner, Ruhman, Nahum, 1808.05953
YL, Chen, Fisher, in preparation



Quantum trajectory v.s. quantum channel

1) After each measurement, we "record" the result of measurement.
Effectively looking at the **pure state wavefunction** as $t \to \infty$

Quantum trajectory

$$p_\alpha = \langle\psi| P_\alpha |\psi\rangle$$

$$|\psi\rangle \to \frac{P_\alpha |\psi\rangle}{\| P_\alpha |\psi\rangle \|}$$

Allows us to look at the average entanglement entropy:

$$S_A(t) = \overline{\mathrm{Tr}_A \rho_A(t) \log \rho_A(t)}, \quad \rho_A(t) = \mathrm{Tr}_{\overline{A}} |\psi(t)\rangle \langle\psi(t)|$$

2) After each measurement, we "forget" the result of measurement.
Effectively looking at the **mixed state density matrix** as $t \to \infty$

Quantum channel

$$\rho \to \mathcal{E}[\rho] = \sum_{\alpha=0}^{m-1} P_\alpha \rho P_\alpha^\dagger$$

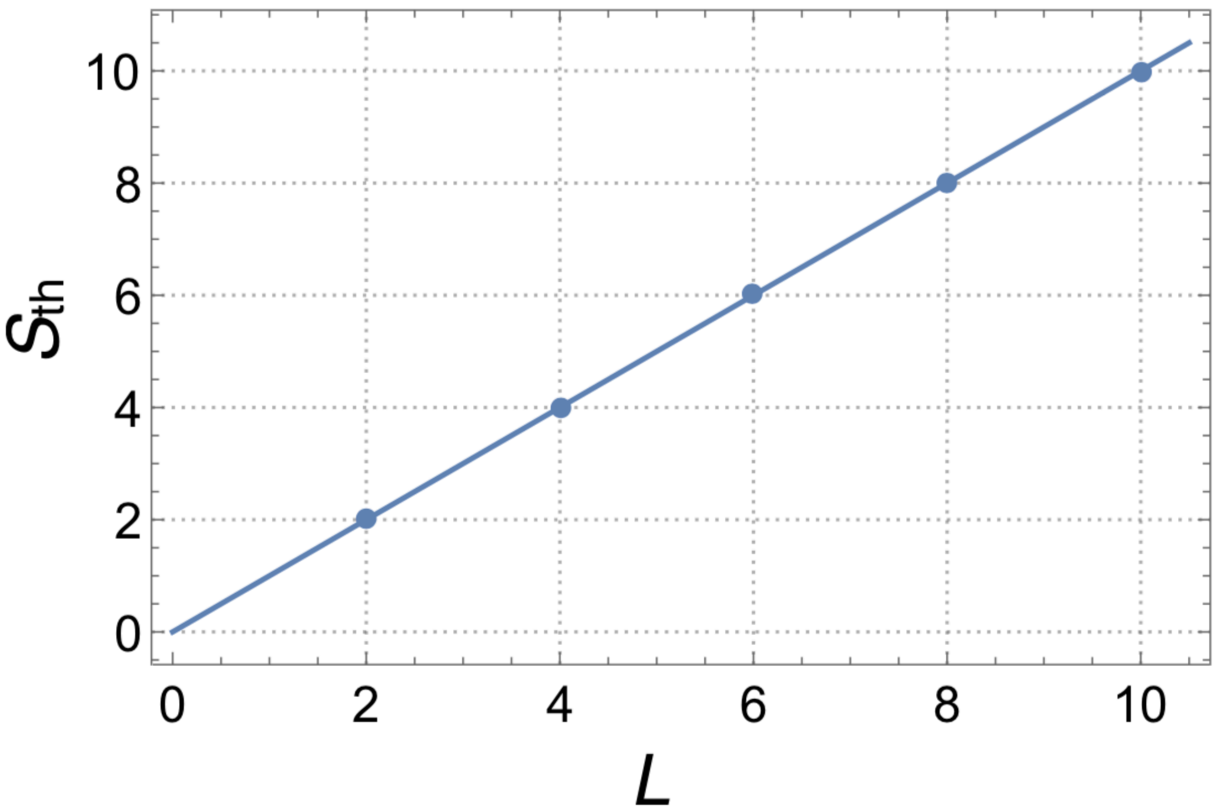
Allows us to look at the thermal entropy:

$$S_{\mathrm{th}} = \mathrm{Tr} \rho \log \rho$$

Mixed state: always (infinitely) thermal for any $p > 0$!

$\rho \propto 1$ is a obvious fixed point of the quantum channel $\rho \to \mathcal{E}[\rho] = \sum_{\alpha=0}^{m-1} P_\alpha \rho P_\alpha^\dagger$

random unitaries from the Haar measure



Indeed, with ***any*** initial state, we find maximal thermal entropy ***for any finite rate of measurement***, so that

$$\rho(t \to \infty) \sim \lim_{\beta \to 0} e^{-\beta H} \propto 1$$

Constantly measuring & quenching the system **always** drives it to infinite temperature!

YL, Chen, Fisher, 1808.06134

Random Clifford circuit: the stabilizer formalism

Stabilizer state (stabilizer code):

Gottesman, 1997
Nielsen, Chuang, 2000
Aaronson, Gottesman, 2004

Given a subset $G = \{g_1, \ldots, g_L\}$ of the Pauli group on L quits P_L such that
1. $[g_i, g_j] = 0$ for all pairs (i, j)
2. $(g_i)^2 = I$
3. G is independent

there is a *unique* wavefunction $|\psi>$ (on L qubits) such that

$$g_i |\psi> = |\psi> \text{ for all } i$$

We say that **$|\psi>$ is stabilized by G**.

Examples:

1. The Bell pair state
$|\psi> = (1/\sqrt{2})(|00>+|11>)$
$G = \{X_1 X_2, Z_1 Z_2\}$
$S = <G> = \{I, X_1 X_2, Z_1 Z_2, -Y_1 Y_2\}$

2. The GHZ state
$|\psi> = (1/\sqrt{2})(|000>+|111>)$
$G = \{X_1 X_2 X_3, Z_1 Z_2, Z_2 Z_3\}$
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Output = U (Input) U†

Operation	Input	Output
controlled-NOT	X_1	$X_1 X_2$
	X_2	X_2
	Z_1	Z_1
	Z_2	$Z_1 Z_2$
H	X	Z
	Z	X
$S = \sqrt{Z}$	X	Y
	Z	Z

Clifford unitaries: takes one Pauli string operator to another, thus preserves stabilizer states

Pauli measurements: for $G = \{g_1, \ldots, g_m, g_{m+1}, g_{m+2}, \ldots, g_{m+n}\}$, where $L = m+n$, and

$[g, g_1] = [g, g_2] = \ldots =[g, g_m] = 0$

$\{g, g_{m+1}\}= \{g, g_{m+2}\} = \ldots =\{g, g_{m+n}\} = 0$

after measuring in the eigenbasis of $g \in P_L$, G becomes

$$G_{after} = \{g_1, \ldots, g_m, g_{m+1} * g_{m+2}, g_{m+2} * g_{m+3}, \ldots, g_{m+n-2} * g_{m+n-1}, g_{m+n-1} * g_{m+n}, g\}$$

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Examples:

Gottesman-Knill theorem: a circuit with **Clifford unitary gates** and **Pauli measurements** can be efficiently simulated.

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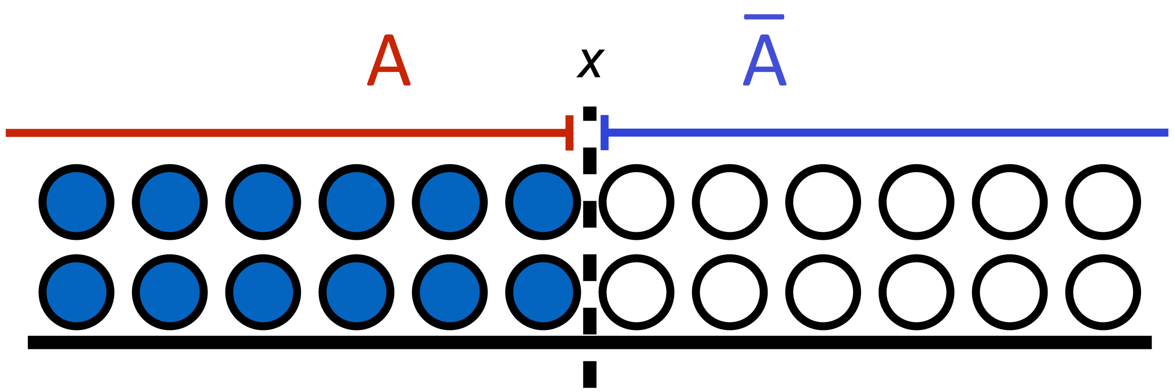
Stabilizers in the clipped gauge

Nahum, Ruhman, Vijay, Haah, 2016
YL, Chen, Fisher, in preparation

Clipped gauge: on each site *x*, there are exactly two stabilizer **endpoints** (can be either **L**eft or **R**ight endpoints),

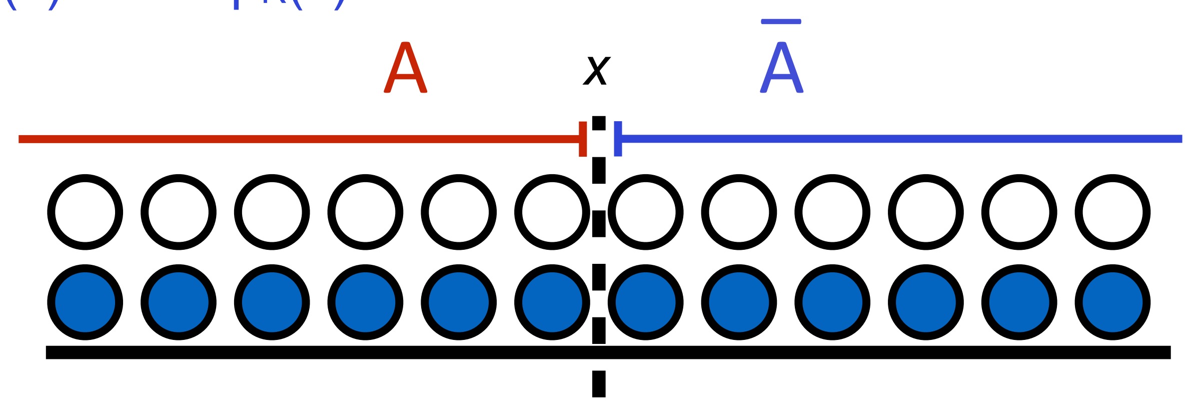
$\rho_L(x) + \rho_R(x) = 2$, for all x

which are required to be independent when $\rho_L(x) = 2$ or $\rho_R(x) = 2$.



Maximally entangled state

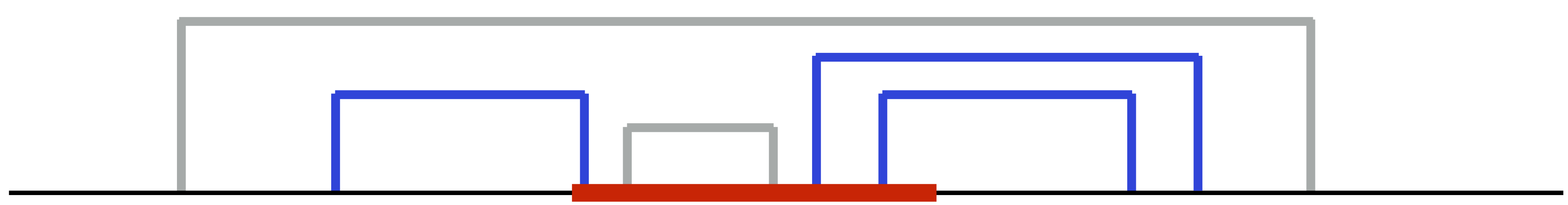
$$S(x) = \Sigma_{y <= x} [\rho_L(y) - 1]$$



Product state

Such a gauge fixing is always possible, and it gives a intuitive formula for the entanglement entropy:

S_A = (# of stabilizers in G that cross the boundary of A) / 2



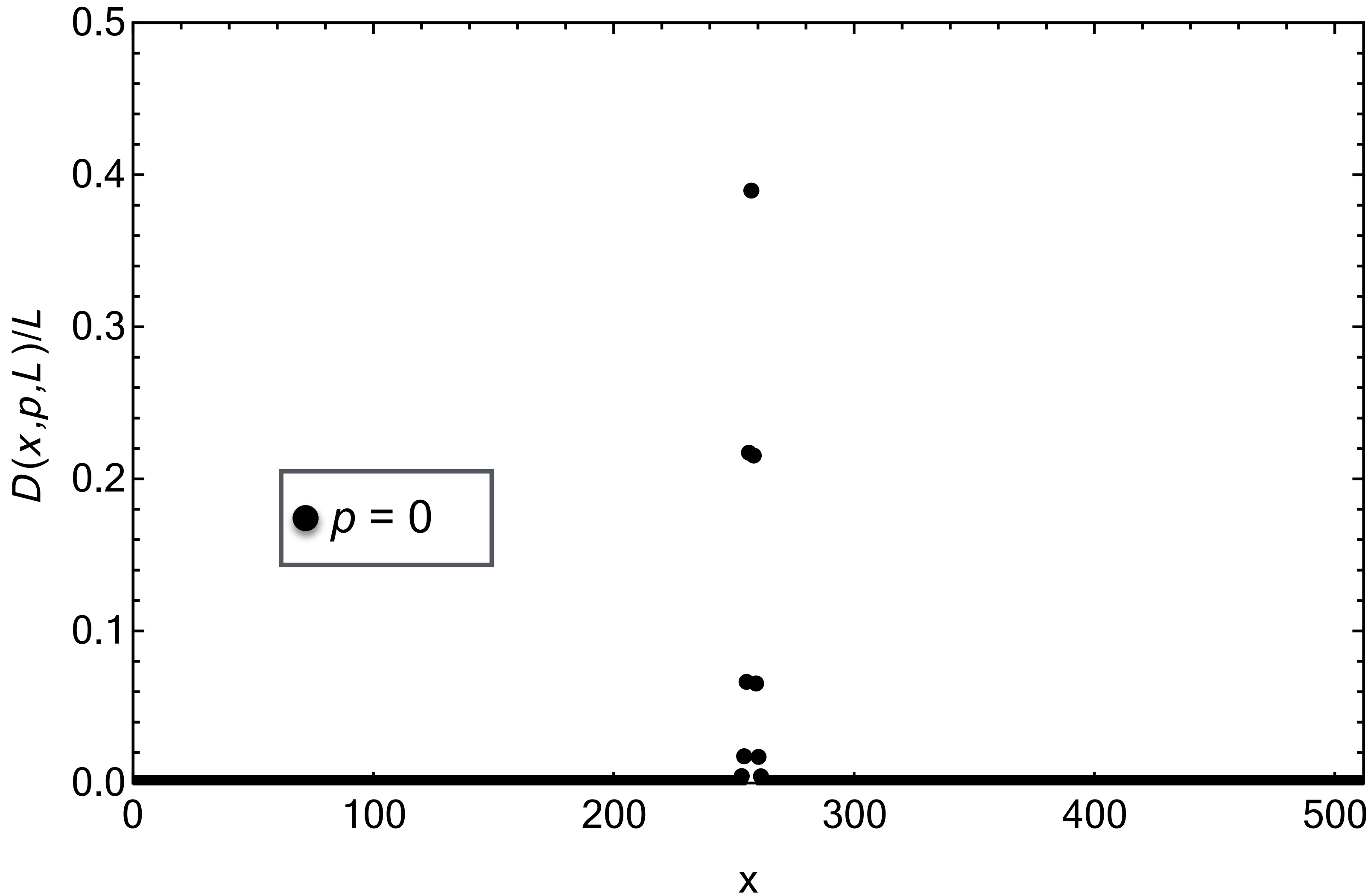
A : a consecutive segment

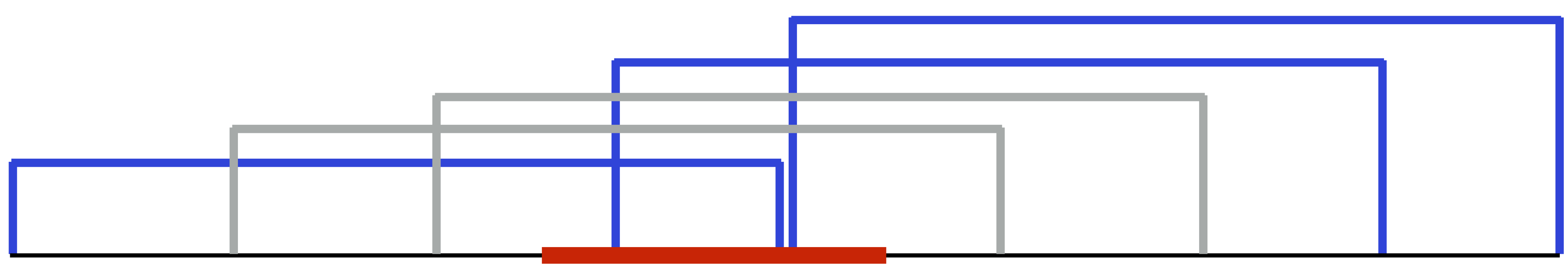
The entanglement uniquely fix the “segments” in the clipped gauge!

Stabilizer length distribution w/ no measurements

Under unitary dynamics, the stabilizers grow in their length

YL, Chen, Fisher, in preparation



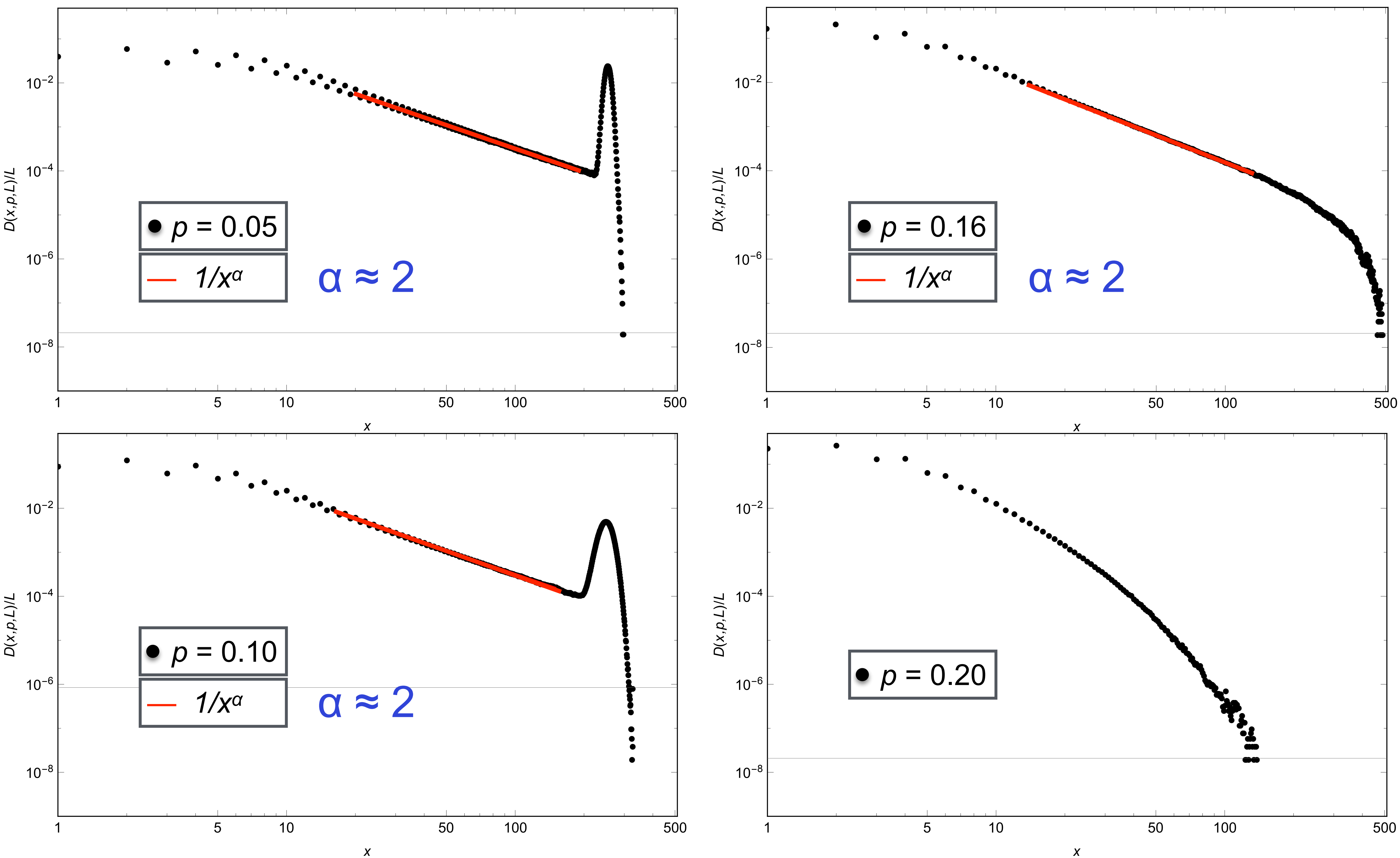


2|A| stabilizers

Stabilizer length distribution w/ measurements

YL, Chen, Fisher, in preparation

Under measurements, steady distribution have two pieces, “short” (power law) and “long” (peak at $L/2$)



Stabilizer length distribution w/ measurements

YL, Chen, Fisher, in preparation

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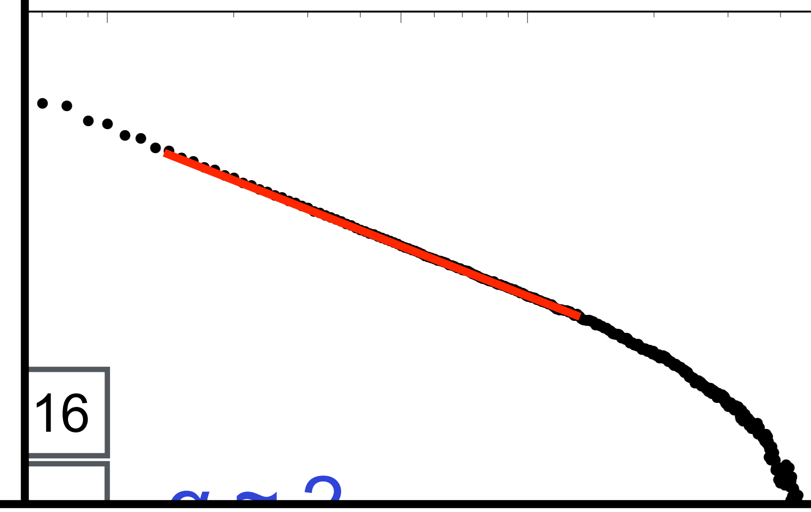
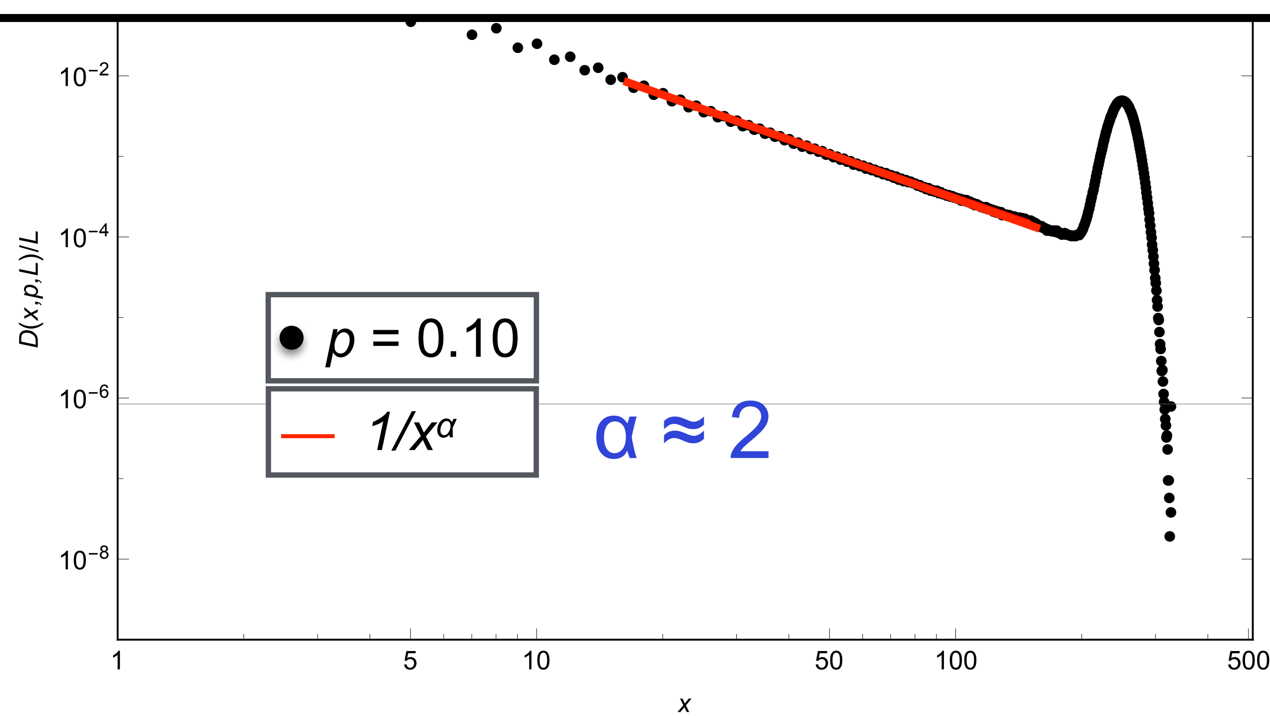
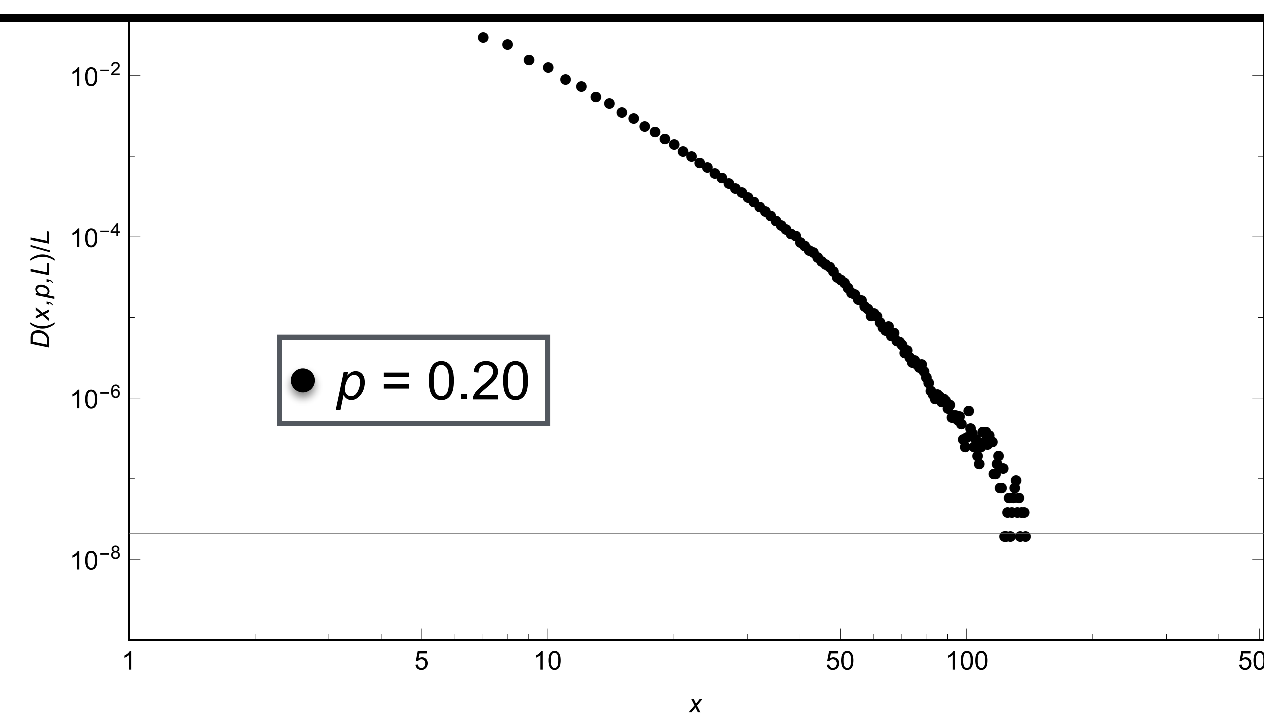
$$D(\ell, p) = \begin{cases} \frac{a_p}{\ell^2} + \frac{1}{\xi}\delta(\ell - \frac{L}{2}) & p < p_c \\ \frac{a_{p_c}}{\ell^2} & p = p_c \\ \frac{a_p}{\ell^2} e^{-\frac{\ell}{\xi}} & p > p_c \end{cases}$$



$$S_A(p) = \int_A dx_1 \int_{\bar{A}} dx_2\, D(|x_1 - x_2|, p) = \begin{cases} a_p \ \log L_A + \frac{1}{\xi} L_A & p < p_c \\ a_{p_c} \log L_A & p = p_c \\ a_p \ \log \xi & p > p_c, L_A \gg \xi \end{cases}$$





Summary

We looked at a simple model for unitary + measurement dynamics.

In the pure state, we found a phase transition from volume law to area law entanglement in the steady state. This transition is not accessible to the density matrix.

Some understanding of the transition in the Clifford circuit.

Open questions

Cao, Tilloy, De Luca, 1804.04638
Vasseur, Potter, You, Ludwig, 1807.07082
Chan, Nandkishore, Pretko, Smith, 1808.05949
Skinner, Ruhman, Nahum, 1808.05953

Existence of the transition?

Analytic treatment? A solvable model that shows transition?

Is the transition universal?

Is randomness important?

Is chaotic dynamics necessary? What about integrable dynamics? What if we put in conservation laws?

Higher dimensions?

Are the two phases equivalent to something we already know (e.g. ETH and MBL), or are they new dynamical phases?

Is the log L correction to the volume law universal?

Experimental realization?