Heating and prethermalization in driven quantum many-body systems

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Aim

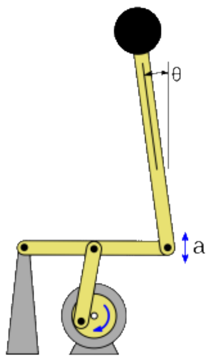
- Review some previous rigorous results pertaining to slow heating and prethermalization in periodically-driven (Floquet) quantum many-body systems

- Present some newer results regarding bounds on heating in periodically-driven quantum many-body systems with **long-range interactions** (some new technical tools required)

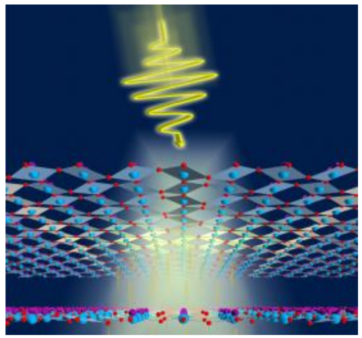
- More generally, highlight the tools and techniques used that might be illuminating in understanding thermalization processes / finding new dynamical regimes and new dynamical phases in other kinds of systems (not necessarily only driven)

Motivation: why periodic driving?

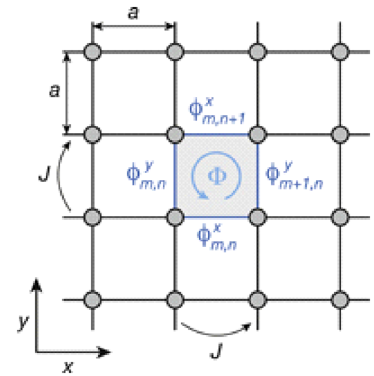
Floquet Engineering



Kapitza Pendulum
Kapitza



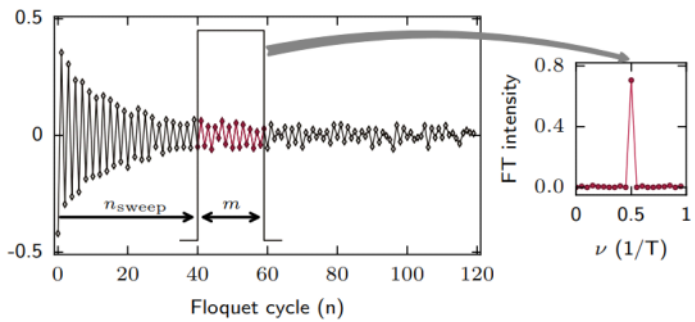
Light induced superconductivity
Fausti et al, Mitrano et al, …



Artificial Gauge Fields (Cold atoms)
Jotzu et al, Aidelsburger et al, …

Floquet TIs
Floquet SPTs
Floquet FCIs
Floquet FQH..
Lindner, Refael, Moessner, Galitski, Rudner, Kitagawa, Grushin, Lee, etc…

Novel dynamical phases of matter



Floquet Time Crystals
Else, Bauer, Nayak, Khemani, Sondhi, von Keyserlingk, Choi et al, Zhang et al, etc…

However, a challenge:

- Systems are generically interacting. In fact some of the desired Floquet physics require interactions

- Upon breaking of energy conservation + interactions, "heat death"?

$$H(t) = H(t+T)$$
$$U(t) = \mathcal{T} e^{-\int_0^t dt' H(t')}$$

$$\rho_A(t) = \lim_{t\to\infty} Tr_B\left[U(t)^\dagger \rho_0 U(t)\right] = \frac{\mathbb{I}_A}{\mathcal{D}_A} \ (??)$$

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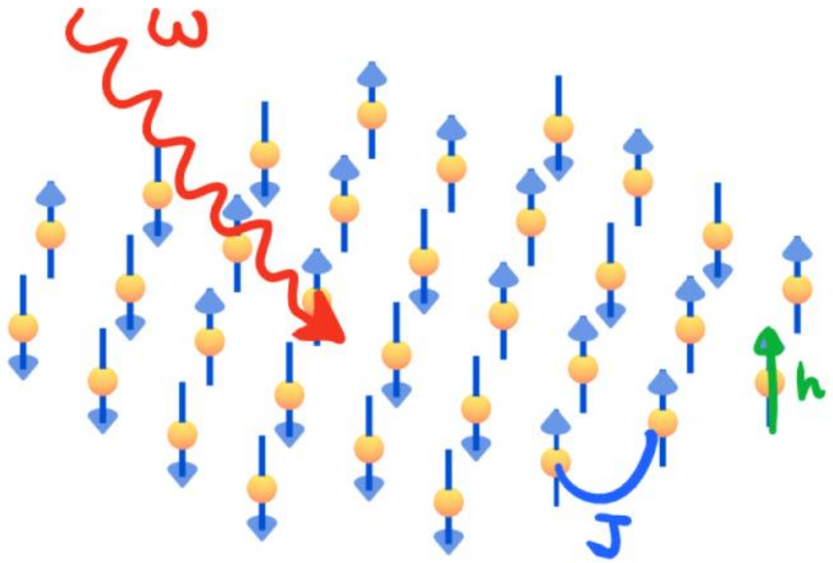
- Is this always true? No, strong disorder can prevent it (many-body localization)
- More generally, what are the heating **rates** in a driven system? What are the timescales of thermalization?
- How long-lived are transient dynamical phenomena?

Set-up (Short-range for now)



- Consider periodically driven many-body lattice system
  $$H(t) = H_0 + g\,V(t)$$
- **Bounded** local Hilbert space e.g. spins, fermions
- **Local** interactions e.g. $J\sigma_i^z\sigma_{i+1}^z, h\sigma_i^x, g\sigma_i^z f(t)$, …
- Also consider the regime $\omega \gg J, g, h$

Warm-up
(Linear Response, Fermi's Golden Rule)

- $H(t) = H_0 + g\, V(t)$
- Consider beginning from a single eigenstate $|n\rangle$ of H_0
- $V(t)$ has harmonics $\pm\omega$
- Transition rate:

$$\Gamma(\omega) = g^2 \sum_m |\langle n|V|m\rangle|^2 \delta(\omega - E_n + E_m)$$

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Trick: Insert commutator with H_0:

Abanin, De Roeck, Huveneers, PRL

$$\Gamma(\omega) = g^2 \sum_m \frac{\left|\langle n|[H_0, V]|m\rangle\right|^2}{\omega^2} \delta(\omega - E_n + E_m)$$

$$= g^2 \sum_m \frac{\left|\langle n|[[H_0, [H_0, V]]|m\rangle\right|^2}{\omega^{2\times 2}} \delta(\omega - E_n + E_m)$$

$$= \cdots = g^2 \sum_m \frac{\left|\left\langle n\left|[[H_0, [H_0, \cdots [H_0, V]]]^{(p)}\right|m\right\rangle\right|^2}{\omega^{2p}} \delta(\omega - E_n + E_m)$$

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ω: Largest scale, **suppression**$\sim \frac{1}{\omega^{2p}}$

Nested commutators:

1. $[h_{i,i+1} + h_{i,i-1}, v_i] = \widetilde{h_{i,i+1}} + \widetilde{h_{i,i-1}}$
2. $[h_{i,i+1} + h_{i,i-1}, \widetilde{h_{i,i+1}} + \widetilde{h_{i,i-1}}] = h'_{i,i+1} + h'_{i-1,i,i+1} + h'_{i,i-1} + h''_{i-1,i,i+1}$
3. Many **local** terms generated…

…

p. # Terms $\sim p!$

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(Linear Response, Fermi's Golden Rule)

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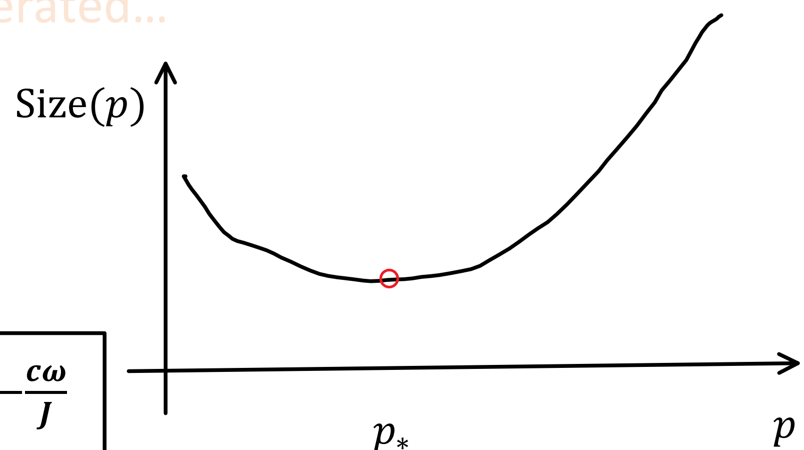
p. # Local terms $\sim p!$

Size of term $\leq \left(\frac{p!}{\omega^p}\right)^2$

Optimal $p_* \Rightarrow \Gamma(\omega) \leq e^{-\frac{c\omega}{J}}$



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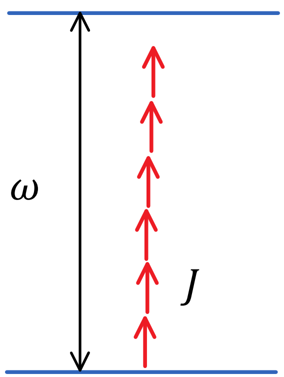
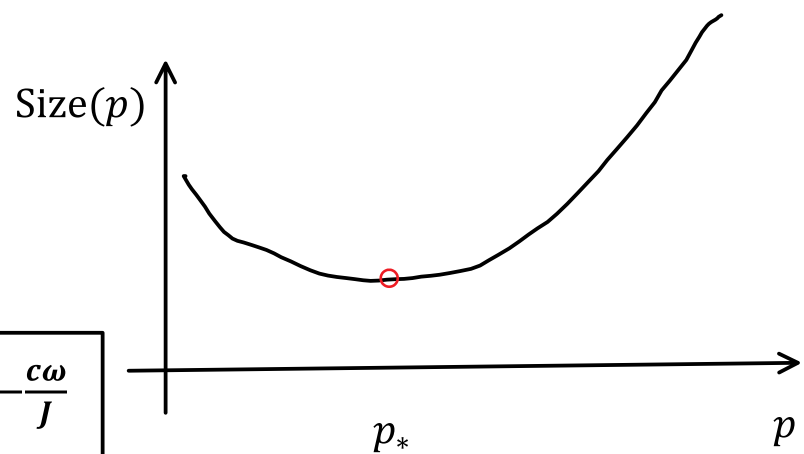
$$\Gamma(\omega) = g^2 \sum_m \frac{\left|\left\langle n \left| [[H_0,[H_0,\cdots[H_0,V]]]^{(p)} \right| m \right\rangle\right|^2}{\omega^{2p}} \delta(\omega - E_n + E_m)$$

Intuition:





Size of term $\leq \left(\frac{p!}{\omega^p}\right)^2$

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An aside: Off-diagonal matrix elements in ETH

Srednicki's ansatz:

$$\langle n|V|m\rangle = e^{-\frac{S}{2}} f_V(\omega) R_{nm}$$

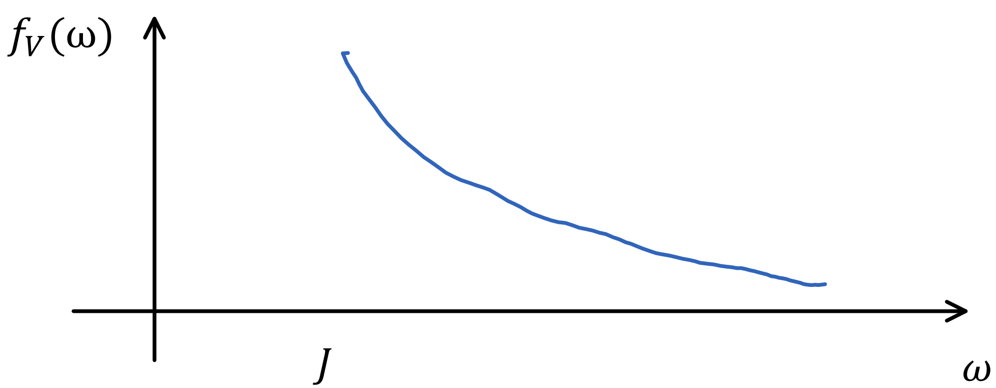
How does $f_V(\omega)$ behave?

Look at weight contained in high frequencies:

$$\sum_{m:E_n - E_m > \omega} |\langle n|V|m\rangle|^2 = \int_\omega^\infty d\omega\, [\rho\, \cancel{e^{-S}}]\, |f_V(\omega)|^2 = \int_\omega^\infty d\omega\, |f_V(\omega)|^2$$

Insert commutators

$$\boxed{\int_\omega^\infty d\omega\, |f_V(\omega)|^2 \lesssim e^{-c\frac{\omega}{J}}}$$



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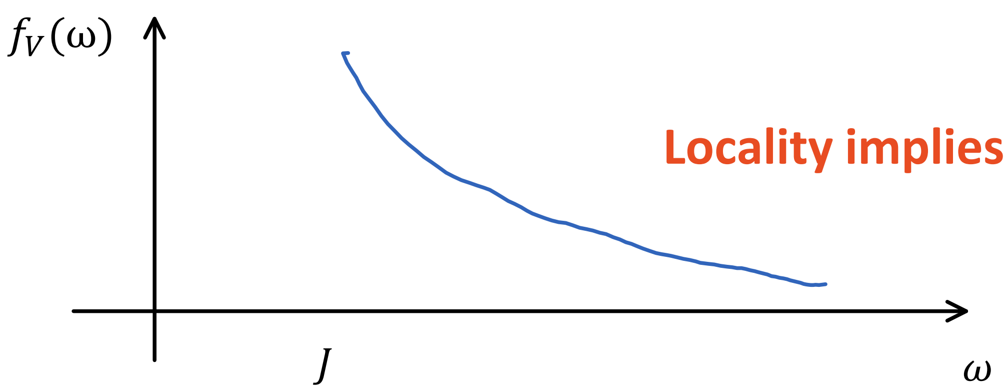
Insert commutators

$$\boxed{\int_{\omega}^{\infty} d\omega \left|f_V(\omega)\right|^2 \leq\sim e^{-c\frac{\omega}{J}}}$$



Locality implies exponential decay

Can result be made non-perturbative?

(i.e. finite driving amplitude g)

Effective Hamiltonians

$$H(t) = H_0 + V(t)$$

Can we understand dynamics in terms of a static Hamiltonian H_{eff}?
Floquet Theorem + Magnus expansion:

$$H_{eff} = \frac{1}{T}\int_0^T H(t)dt + \frac{1}{2T}\int_0^T dt_1 \int_0^{t_1} dt_2 [H(t_1), H(t_2)] + \cdots$$

Presumably works well for small T or large ω.

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Presumably works well for small T or large ω.

Do they matter?

If we can ignore them, then this is an avoidance of Floquet thermalization to infinite temperature. These terms **must** matter generically.

Analysis of series: Mori, Kuwahara, Saito, PRL (2016), AOP (2016)

Effective Hamiltonians

$$H(t) = H_0 + V(t)$$

Go into a suitably chosen rotating frame

$$H'(t) = Q(t)^\dagger \big(H_0 + V(t) - i\,\partial_t\big) Q(t)$$

Choose $Q(t) = \exp\left[\big(S_1(t) + S_2(t) + S_3(t) + \cdots + S_n(t)\big)\right]$

Book-keeping: $\sim \frac{1}{\omega}$ $\sim \frac{1}{\omega^2}$ $\sim \frac{1}{\omega^3}$ $\sim \frac{1}{\omega^n}$

Abanin, De Roeck, **WWH**, Huveneers, CMP (2017)
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Expand $H'(t)$ and group terms in powers of $\frac{1}{\omega}$

(0)-th order: $H_0 + \bar{V}$

(1)-st order: $\tilde{V}(t) - i\,\partial_t S_1$

(2)-nd order: $[S_1, H_0 + V(t)] - \frac{i}{2}[S_1, \partial_t S_1] - i\,\partial_t S_2$

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Extract time-indept piece, **renormalizes** Hamiltonian

Solve $S_2(t)$ to absorb oscillating piece. Sum of local but longer range terms.

Abanin, De Roeck, **WWH**, Huveneers, CMP (2017)

Abanin, De Roeck, **WWH**, Huveneers, PRB (2017)

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(2)-nd order: $[S_1, H_0 + V(t)] - \frac{i}{2}[S_1, \partial_t S_1] - i\,\partial_t S_2$

Higher orders involve more and nested commutators.

Number of local terms grows as $n!$ again. Suppression as ω^n

Abanin, De Roeck, **WWH**, Huveneers, CMP (2017)

Abanin, De Roeck, **WWH**, Huveneers, PRB (2017)

Effective Hamiltonians

$$H(t) = H_0 + V(t)$$

Go into **'best'** rotating frame:

$$H'(t) = H_{eff} + \delta V(t)$$

$H_0 + \bar{V}$ + corrections
Quasi-local

$\sim e^{-c\frac{\omega}{J}}$
Quasi-local

Abanin, De Roeck, **WWH**, Huveneers, CMP (2017)
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Quasi-local

Dynamics of local observables given by $U(t) = e^{-iH_{eff}t}$ for $t_* \leq \sim e^{c\frac{\omega}{J}}$

(with Lieb-Robinson type arguments)

Implies prethermalization to H_{eff} for long times.

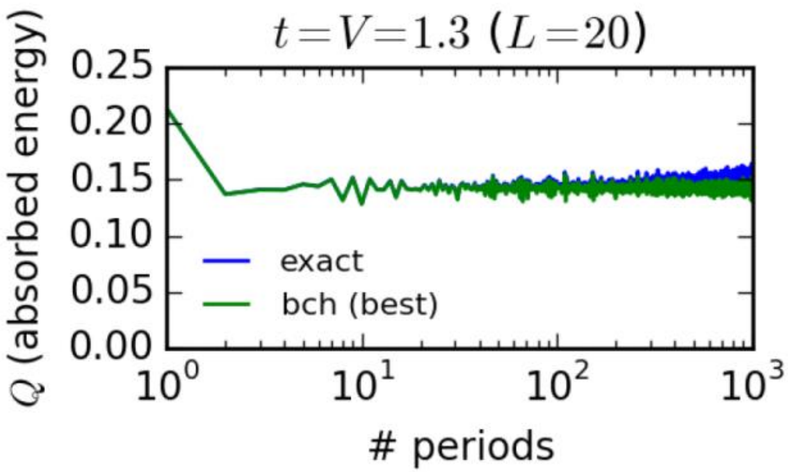
Results from locality + high frequency

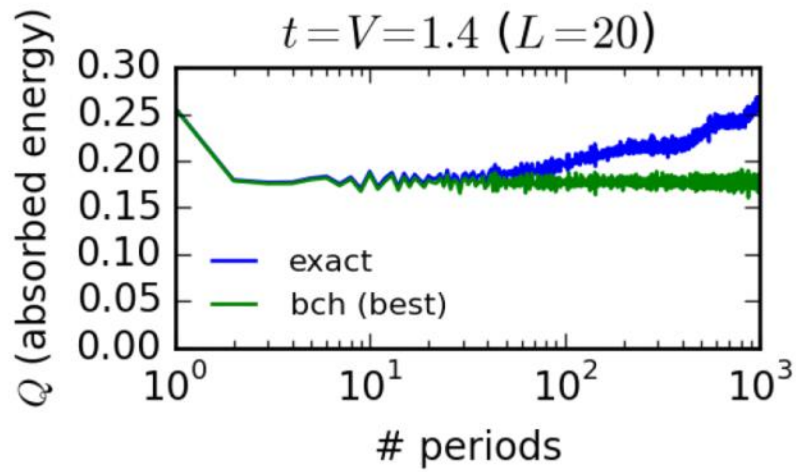
Abanin, De Roeck, **WWH**, Huveneers, CMP (2017)
Abanin, De Roeck, **WWH**, Huveneers, PRB (2017)

Some numerics (unpublished)

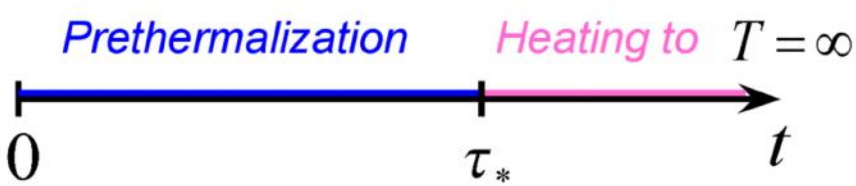
$$U_F = e^{-iH_0 t} e^{-iH_1 V}, \qquad H_0 = \sum_i h s_i^z + J s_i^z s_{i+1}^z, \qquad H_1 = \sum_i J s_i^x s_{i+1}^x$$

$U_{\rm bch} = e^{-iH_* T}$ eff. Hamiltonian



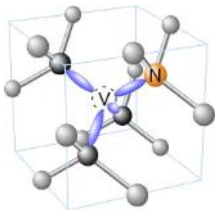


Absorbed heat: $Q = \frac{E(t)-E_0}{E(\infty)-E_0}$

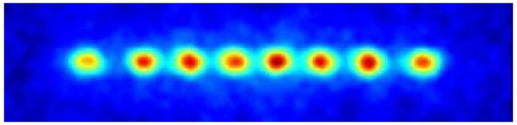


How about systems with long-range interactions $\sim \frac{1}{r^{\alpha}}$?

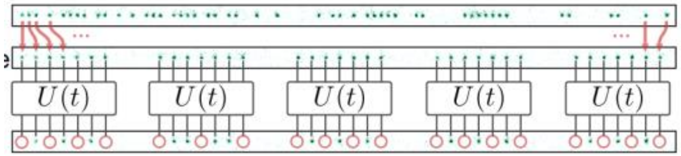
Many synthetic quantum systems have long-range interactions



NV-centers, Polar molecules $\sim \frac{1}{r^3}, d = 3$



Trapped ions $\sim \frac{1}{r^\alpha}, 0 < \alpha < 3, d = 1$



Rydberg atoms (van der Waals) $\sim \frac{1}{r^6}, d = 1,2$

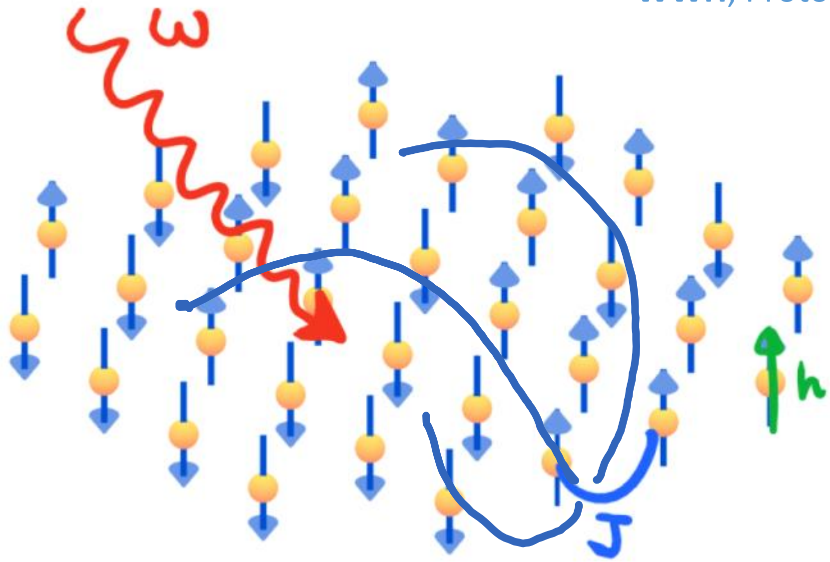
Long-range interactions: faster entanglement generation, faster preparation of states, possible new phases of matter (with or without driving).

But also, curse: more heating?

Can we understand heating rates in such driven systems? Is there prethermalization?

Set-up (with Long-range interactions)

WWH, Protopopov, Abanin, PRL (2018)



- Consider periodically driven many-body lattice system with $\omega \gg J, h, \ldots$
- **Bounded** local Hilbert space e.g. spins, fermions in $\boldsymbol{d}$**-dim**
- **Long range** interactions

$$H = \sum_{ij} \frac{J_{ij}}{r_{ij}^{\alpha}} O_{ij} + \kappa \sum_i \vec{h}_i \cdot \vec{\sigma_i} + g \sum_x V_x \cos(\omega t)$$

Consider $\frac{\boldsymbol{d}}{\boldsymbol{2}} < \boldsymbol{\alpha} \leq \boldsymbol{d}$. To ensure thermodynamic stability, assume J_{ij} **random variable** with **zero mean** and **bounded** higher moments.

Set-up (with Long-range interactions)

$$H = \sum_{ij} \frac{J_{ij}}{r_{ij}^{\alpha}} O_{ij} + \kappa \sum_{i} \vec{h}_i \cdot \vec{\sigma_i} + g \sum_{x} V_x \cos(\omega t) \qquad \frac{d}{2} < \alpha \leq d$$

Look at energy absorption at **high temps (low $\boldsymbol{\beta}$)**

$$\frac{dE}{dt} = 2g^2\omega^2\sigma(\omega,\beta)$$

$$\sigma(\omega,\beta) = \sum_{nm} \frac{\pi\beta\omega}{Z_0} |\langle n|V|m\rangle|^2 \delta(E_n - E_m - \omega)$$

Weight of **disorder averaged** spectral function at high frequencies

$$\sigma([\omega]) \equiv \langle \int_{-\omega}^{\infty} d\omega' \sigma(\omega',\beta) \rangle$$

Tricks

$$\sigma([\omega]) = \langle \int_{\omega}^{\infty} \sum_{nm} \frac{\pi\beta\omega}{Z_0} |\langle n|V|m\rangle|^2 \delta(E_n - E_m - \omega)\rangle$$

1) Insert in nested commutators of H_0

2) Delta function picks out subset of states.
Lift restriction to let all pairs contribute;
becomes trace:

$$\leq \frac{\pi\beta\omega}{Z_0\omega^{2p}} \left\langle \mathrm{Tr}\left([[[V,H],\cdots],H]^{(p)}[[[V,H],\cdots],H]^{(p)}\right)\right\rangle$$

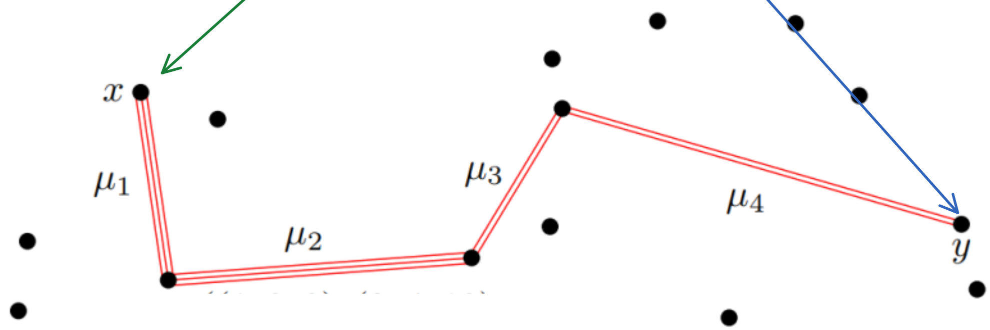
$$= \frac{\pi\beta\omega}{Z_0\omega^{2p}} \left|\left\langle \mathrm{Tr}\left(V[[[V,H],\cdots],H]^{(2p)}\right)\right\rangle\right|$$

3. Use cyclicity of trace

WWH, Protopopov, Abanin, PRL (2018)

Result

$$\sigma([\omega]) \leq \frac{\pi\beta\omega}{Z_0\omega^{2p}} \sum_{x,y} \sum_{\vec{\mu}} \left|\mathrm{Tr}(V_y[[V_x, O_{\mu_1}], \cdots], O_{\mu_{2p}}])\right| \times \frac{\left|\langle J_{\mu_1} J_{\mu_2} \cdots J_{\mu_{2p}} \rangle\right|}{r^{\alpha}_{\mu_1} r^{\alpha}_{\mu_2} \cdots r^{\alpha}_{\mu_{2p}}} \qquad (36)$$



Disorder averaging forces links μ to be **at least paired**. This makes relevant terms have denominators

$$\sim \frac{1}{r_\mu^{n\alpha}} \text{ where } n \geq 2$$

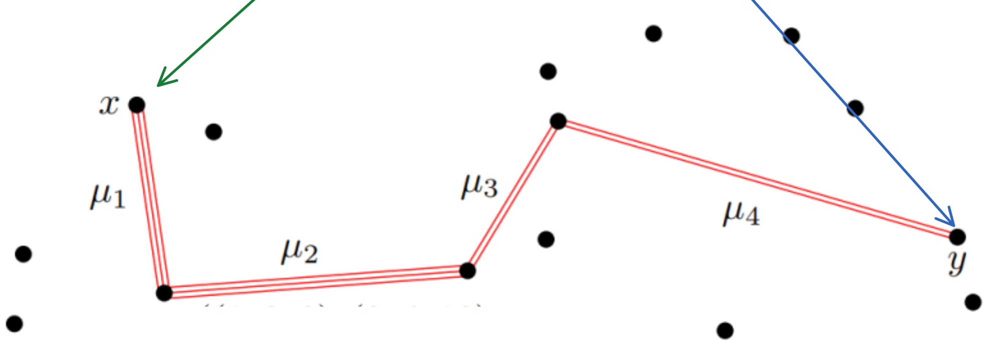
effectively "longer-ranged". Clever resummation of the infinite terms gives

$$\boxed{\sigma([\omega]) \leq N\pi\beta\omega e^{-\frac{\omega}{B}}}$$

WWH, Protopopov, Abanin, PRL (2018)

Result

$$\sigma([\omega]) \leq \frac{\pi\beta\omega}{Z_0\omega^{2p}} \sum_{x,y} \sum_{\vec{\mu}} \left| \mathrm{Tr}(V_y[[V_x, O_{\mu_1}], \cdots], O_{\mu_{2p}}]) \right| \times \frac{\left| \langle J_{\mu_1} J_{\mu_2} \cdots J_{\mu_{2p}} \rangle \right|}{r^{\alpha}_{\mu_1} r^{\alpha}_{\mu_2} \cdots r^{\alpha}_{\mu_{2p}}} \quad (36)$$



Disorder averaging forces links μ to be **at least paired**. This makes relevant terms have denominators

$$\sim \frac{1}{r_{\mu}^{n\alpha}} \text{ where } n \geq 2$$

effectively "longer-ranged". Clever resummation of the infinite terms gives

$$\boxed{\sigma([\omega]) \leq N\pi\beta\omega e^{-\frac{\omega}{B}}}$$

Exponentially suppressed heating rate!

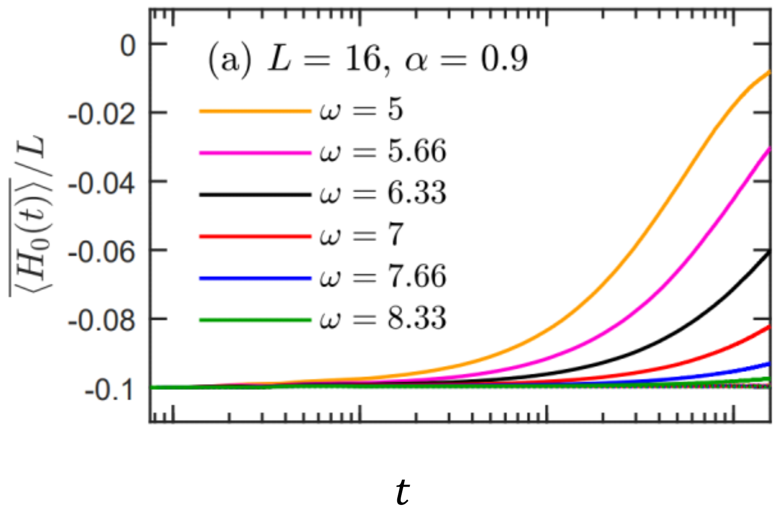
WWH, Protopopov, Abanin, PRL (2018)

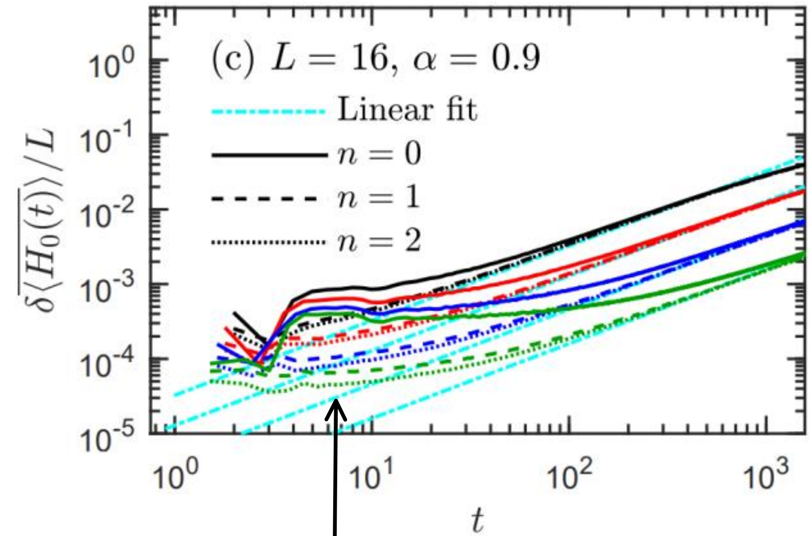
Can result be made non-perturbative?
Is there a prethermal Hamiltonian?

Numerics

$$H(t) = \sum_{ij} \frac{s_{ij}}{r_{ij}^{\alpha}} (J_{zz}\sigma_i^z \sigma_j^z + J_{xx}\sigma_i^x \sigma_j^x) + \sum_i h_x \sigma_i^x$$
$$+ g[1 - 2\theta(t - T/2)] \sum_i (\sigma_i^z + \sigma_i^y),$$

Step function drive





Long-lived prethermal plateau

See also Francisco, …, Nayak, Yao (arXiv:1708.01620)

WWH, Protopopov, Abanin, PRL (2018)

Numerics

$$H(t) = \sum_{ij} \frac{s_{ij}}{r_{ij}^{\alpha}} (J_{zz}\sigma_i^z \sigma_j^z + J_{xx}\sigma_i^x \sigma_j^x) + \sum_i h_x \sigma_i^x$$
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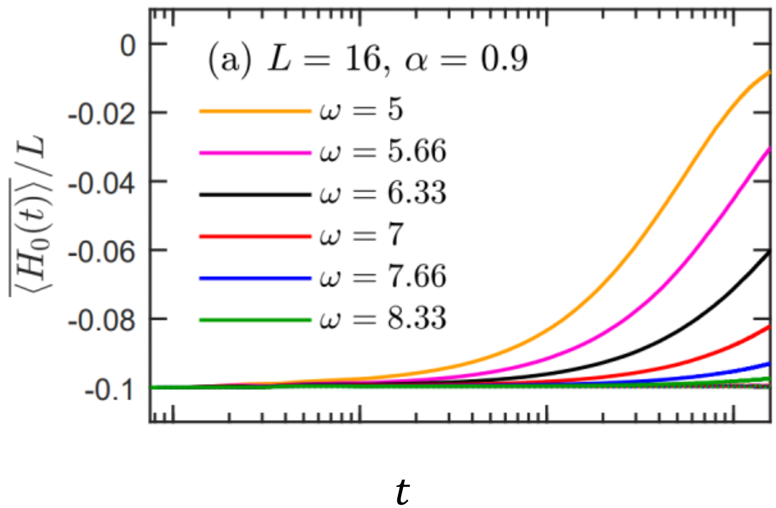
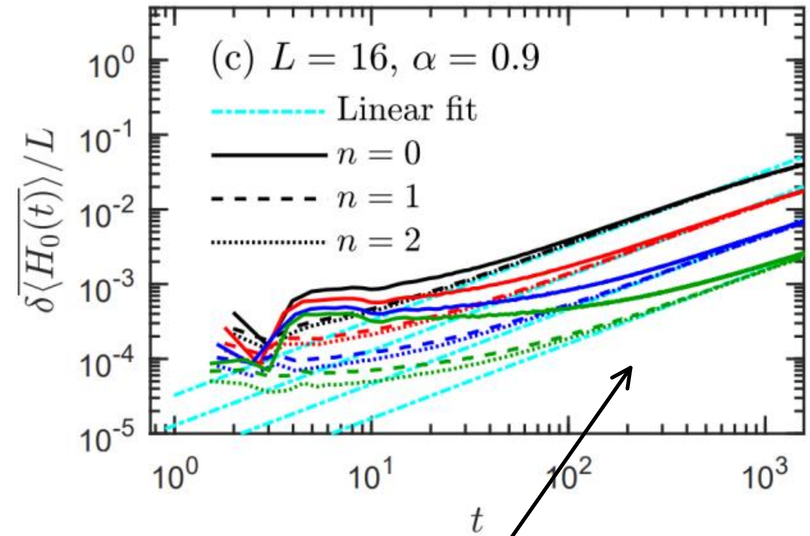
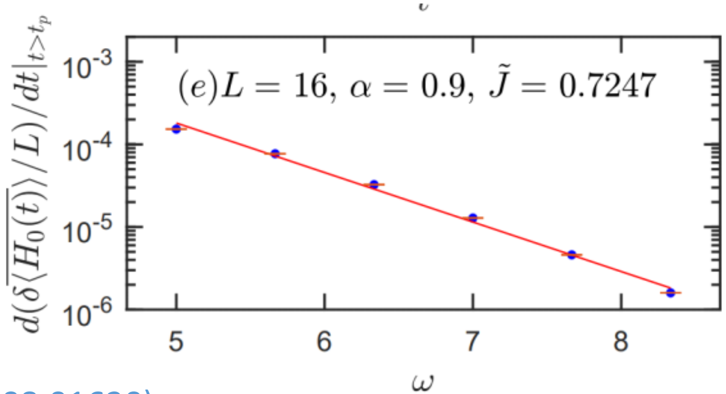
Step function drive







Heating rate consistent with $\sim e^{-\omega/J_{\mathrm{eff}}}$

See also Francisco, …, Nayak, Yao (arXiv:1708.01620)

WWH, Protopopov, Abanin, PRL (2018)

Lessons learnt so far

Locality and **large energy scale** (driving frequency) leads to **exponentially suppressed heating rate** and **prethermalization** in periodically driven many-body systems, with short or long-range interactions. This requires choosing an appropriate **frame of reference** to view the system from. Note that the "large" scale does not mean large compared to the unphysical many-body bandwidth.

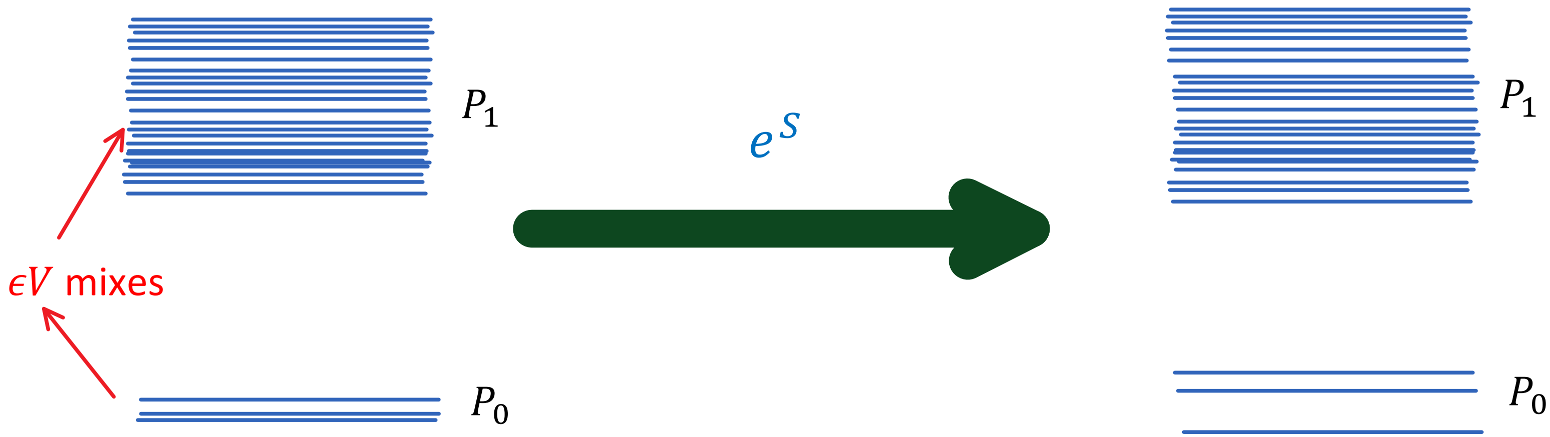
This story is actually a lot more general.

Generically, one can consider different iterative frame transformations that might yield different effective Hamiltonians and effective dynamical behavior.

Schrieffer-Wolff transformation

If H_0 has a low energy sector separated by a spectral gap, can find a rotated frame of reference where $H_0 + \epsilon V$ has same low energy sector with some effective dynamics

$$e^{S}\left[H_0 + \epsilon V\right]e^{-S} = P_0 H_{eff} P_0 + \cdots$$



$$H_{eff} = H_0 + \epsilon P_0 V P_0 + \cdots$$

Near "integrable" systems

Say H has an almost conserved charge e.g. U(1) charge $\sum_i S_i^z$

$$H = J \sum_i S_i^z + \eta D + \epsilon V \qquad J \gg \eta, \epsilon$$

Here D commutes with charge, V does not.

Can find a frame of reference e^S such that

$$H' = e^S H e^{-S} = J \sum_i S_i^z + \eta D' + e^{-\frac{\epsilon}{J}} V'$$

Abanin, De Roeck, **WWH**, Huveneers, CMP (2017)

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Exponentially suppressed!

Implies dressed charge $e^{-S}\sum_i S_i^z\, e^S$ is conserved for exponentially long times

Abanin, De Roeck, **WWH**, Huveneers, CMP (2017)

A different decomposition for some Floquet drive?

Consider a periodically driven system with unitary map

$$U_F = e^{-i\frac{\pi}{2}\sum_i S_i^x} e^{-i\,\mathcal{T}\int_0^T \left(JD(t)+\epsilon V(t)\right)dt} \propto \prod_i X_i \, e^{-i\,\mathcal{T}\int_0^T \left(JD(t)+\epsilon V(t)\right)dt}$$

Z_2 symmetry

commutes

anticommutes

This form is actually pretty generic if we have a periodically-driven Hamiltonian with a large on-site $\sum_i S_i^x$ term that dominates during the period of the drive -- just go into the interacting picture.

A different decomposition for some Floquet drive?

Consider a periodically driven system with unitary map

$$U_F = \prod_i X_i \, e^{-i \mathcal{T} \int_0^T (JD(t) + \epsilon V(t)) dt}$$

Consider going into a new frame

$$e^{A_1} U_F e^{-A_1} = e^{A_1} \prod_i X_i \, e^{-i \mathcal{T} \int_0^T (JD(t) + \epsilon V(t)) dt} e^{-A_1}$$

$$= \prod_i X_i \, e^{-A_1} e^{-i \mathcal{T} \int_0^T (JD(t) + \epsilon V(t)) dt} e^{-A_1}$$

Assume A to be antisymmetric

A different decomposition for some Floquet drive?

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$$= \prod_i X_i \, e^{-A_1} e^{-i\mathcal{T}\int_0^T (JD(t)+\epsilon V(t))dt} e^{-A_1}$$

Treat as new Floquet drive;

Pick $A_1 = \frac{-i}{2}\int_0^T \epsilon V(t)dt$

$$= \prod_i X_i \, e^{-i\mathcal{T}\int_0^T (JD'(t)+(\epsilon^2 T)V'(t))dt}$$

A different decomposition for some Floquet drive?

Consider a periodically driven system with unitary map

$$U_F = \prod_i X_i \, e^{-i\,\mathcal{T}\int_0^T (JD(t)+\epsilon V(t))dt}$$

Rinse and repeat

$$e^{A_2}e^{A_1}U_F e^{-A_1}e^{-A_2} = e^{A_2}\prod_i X_i \, e^{-i\,\mathcal{T}\int_0^T (JD'(t)+(\epsilon^2 T)V'(t))dt} e^{-A_2}$$

$$e^{A_p}\ldots e^{A_2}e^{A_1}U_F e^{-A_1}e^{-A_2}\ldots e^{-A_p} = \cdots$$

A different decomposition for some Floquet drive?

Consider a periodically driven system with unitary map

$$U_F = \prod_i X_i \, e^{-i\,\mathcal{T}\,\int_0^T \left(JD(t)+\epsilon V(t)\right)dt}$$

In appropriate new frame,

$$\prod_i^p e^{A_i}\, U_F \prod_i^p e^{-A_i} \approx \prod_i X_i \, e^{-iJD'T} + O(e^{-\epsilon T})$$

If D' supports spontaneous symmetry breaking of Z_2, approximate, long-lived Floquet eigenstates are macroscopic cat states.

I.e. this is a **Prethermal Time Crystal.**

Else, Bauer, Nayak, PRX

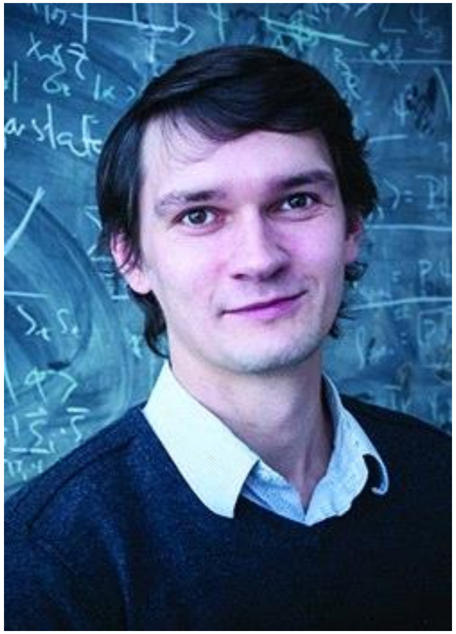
Conclusion

Combining notions of **locality**, **large energy scale**, and a **suitably chosen frame of reference** can yield bounds on dynamics, effective physics, long-lived transient dynamics, transient dynamical phases of matter etc.

Outlook:

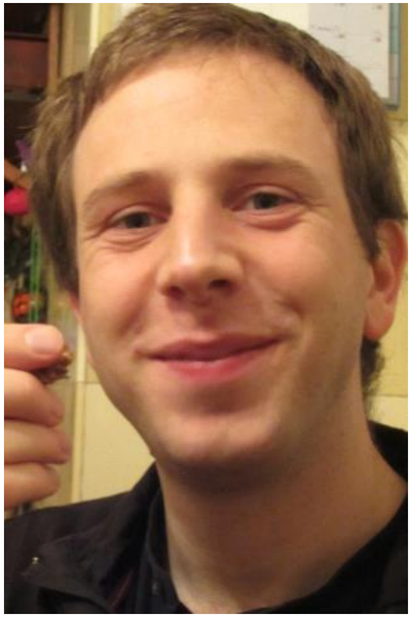
- Effective prethermal Hamiltonians for long-range systems?
- Quasi-periodically driven many-body systems?
- Quantum KAM kind of statements?
- …

Thank you!



Dima Abanin

Wojciech De Roeck



Francois Huveneers

Ivan Protopopov

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