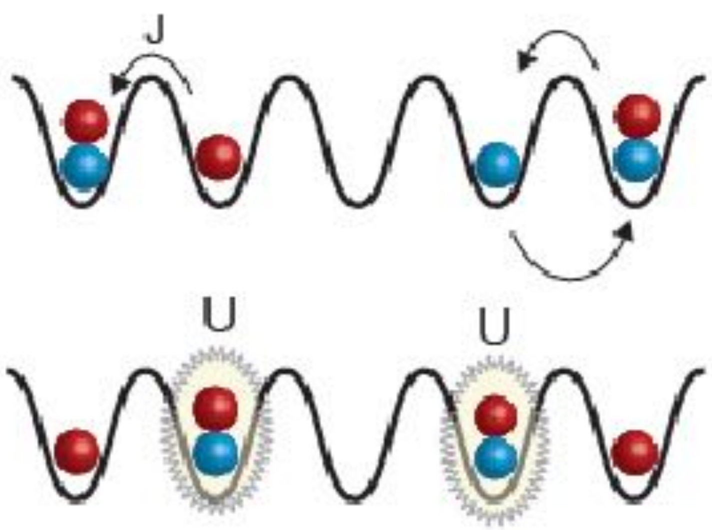
Nonequilibrium dynamics and transport in the 1d Fermi-Hubbard model



Fabian Heidrich-Meisner
University of Göttingen

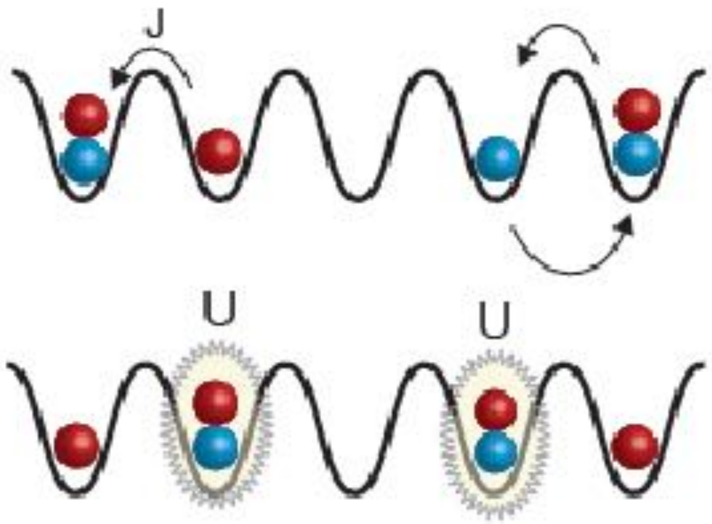
***The Dynamics of Quantum Information*, KITP, September 13 (2018)**

GEORG-AUGUST-UNIVERSITÄT
GÖTTINGEN

$[q,p]=i\hbar$

(Nonequilibrium) transport in the 1d Fermi-Hubbard model

(and how all this can be studied with quantum gases)



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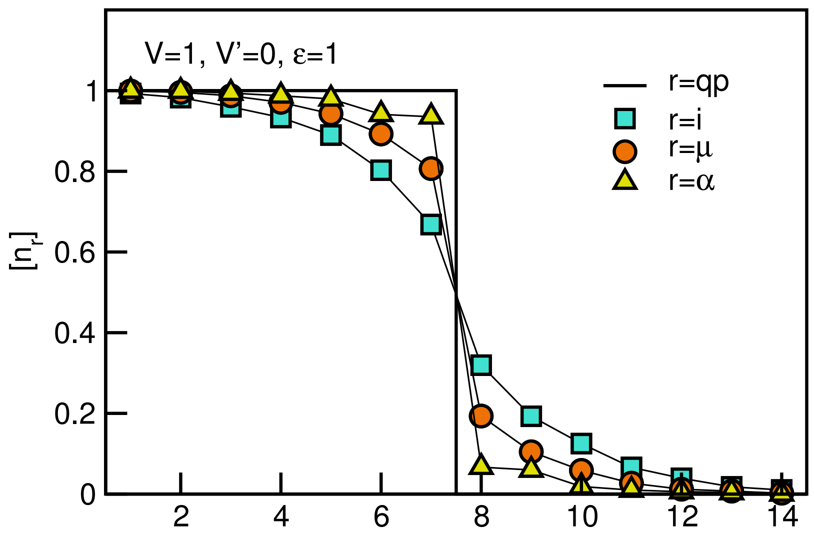
GÖTTINGEN

[q,p]=iħ

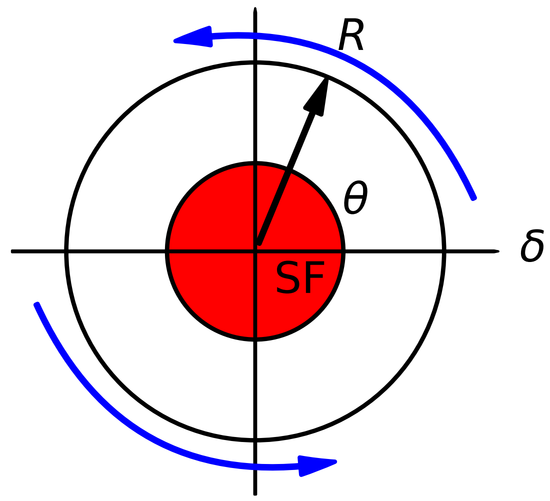
What's *not* in this talk

MBL: One-particle density matrix



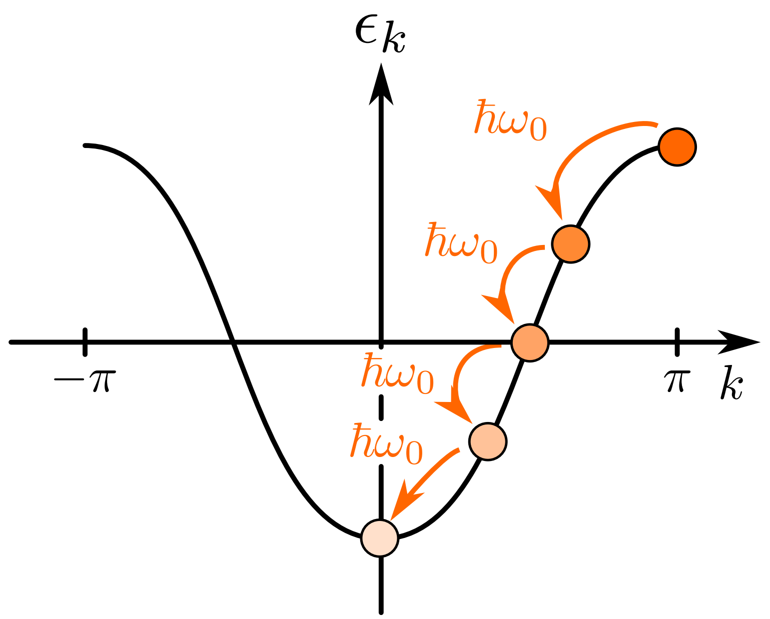
Bera, Schomerus, FHM, Bardarson, Phys. Rev. Lett. 115, 046603 (2015)
Bera, Martynec, Schomerus, FHM, Bardarson, Annalen der Physik (2017)

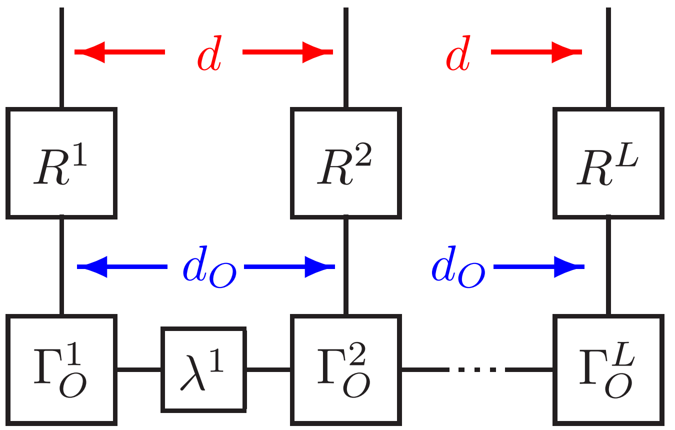
Topological charge pumps



Hayward, Schweizer, Lohse, Aidelsburger, Bloch, FHM, in preparation
(started @ KITP in 2016)

Relaxation & Thermalization in Electron-Phonon models

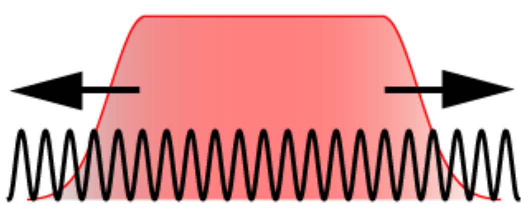




Dorfner, Vidmar, Brockt, Jeckelmann, FHM Phys. Rev. B 91, 104302 (2015)
Brockt, Dorfner, Vidmar, FHM, Jeckelmann, Phys. Rev. B 92, 241106(R) (2015)
Jansen, Stolpp, Vidmar, FHM, in preparation

Two topics for today's talk

Nonequilibrium mass transport

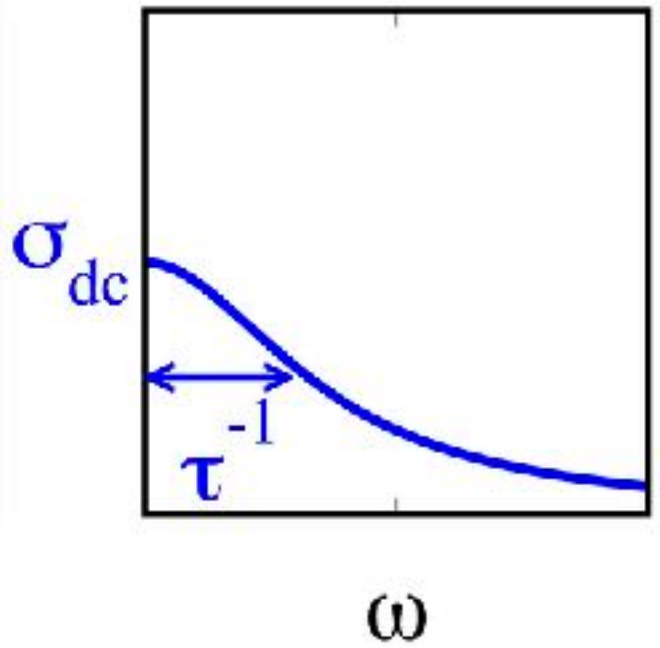


"Cooling a quantum gas
by letting it expand"

Dynamical dilution in 1d:
Access to Bethe root densities

Scherg, Kohlert, Herbrych, Stolpp,
Schneider, FHM, Aidelsburger, Bloch, PRL, in press, arXiv:1805.10990
Mei, Vidmar, Bolech, FHM, PRA ***93****, 021607(R) (2016)*

Transport in linear response



Coexistence
of ballistic & diffusive transport

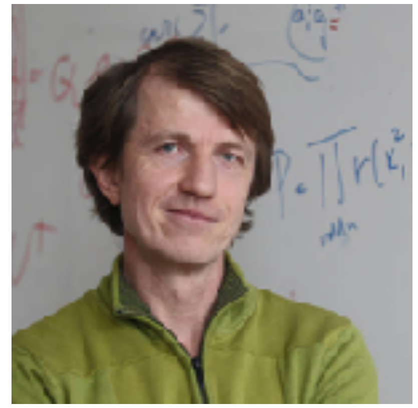
Proposal: Measure Drude weights

Karrasch, Kennes, FHM PRL 117, 116401 (2016)
Karrasch, Prosen, FHM PRB 95, 060406(R) (2017)
Fin, Steinigeweg, FHM, Michielsen, de Raedt PRB 92, 205103 (2015)

In collaboration with

Christoph Karrasch
FU Berlin

Dante Kennes



Tomaz Prosen
U Ljubljana

Jan Stolpp
U Goe

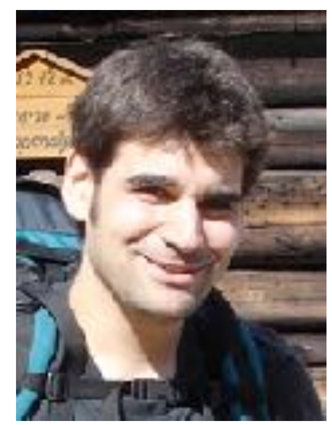
Jacek Herbrych
U Tennessee

Sebastian Scherg

Thomas Kohlert

Pranjal Bordia

LMU Munich & MPQ Garching



Ulrich Schneider
Cambridge, UK

Immanuel Bloch
LMU & MPQ

Monika Aidelsburger

Zhangtao Mei, Carlos Bolech
U Cinncinati

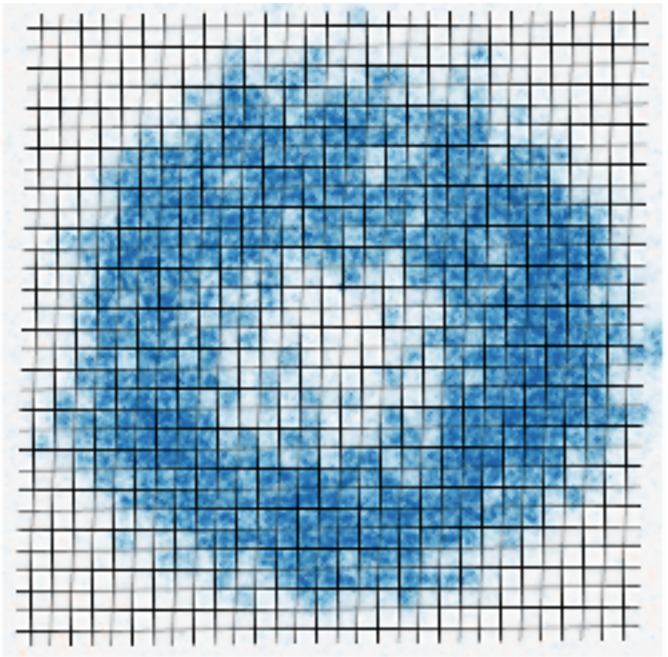
Giuliano Orso, U Paris-Diderot
Marcos Rigol, PSU
Stephan Langer, (free man)

Deutsche Forschungsgemeinschaft
DFG

1d Fermi-Hubbard

$$H = -J \sum_{\langle i,j \rangle} (c^\dagger_{i\sigma} c_{j\sigma} + h.c.) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

... realized in deep optical lattices ...

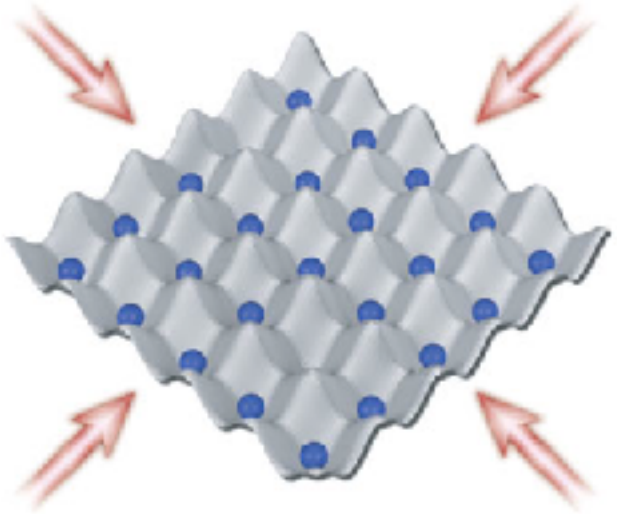


Fermionic Quantum Gas Microscope

Greiner (Harvard), Bloch/Gross (MPQ), Zwierlein (MIT), Kuhr (Strathclyde), Thywissen (Toronto), Bakr (Princeton), ...
1D: Boll et al, Science 353, 1257 (2016)

Hubbard model in optical lattice

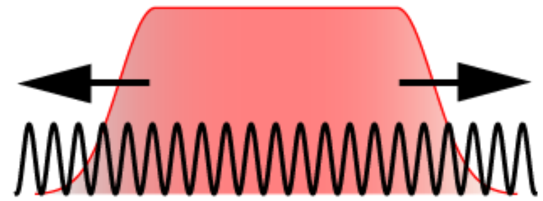
Schneider et al. (2008), Jördans et al. (2008)
Hart et al. (2015), Greif et al. (2014)



Nonequilibrium mass transport in optical lattices

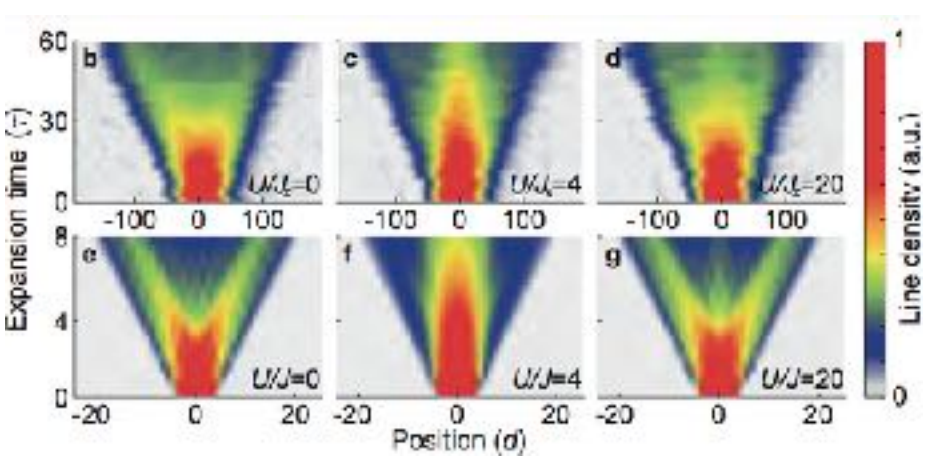
Quench: trap removal, expansion in flat lattice

$$H_{\mathrm{FHM}} + H_{\mathrm{trap}} \rightarrow H_{\mathrm{FHM}}$$



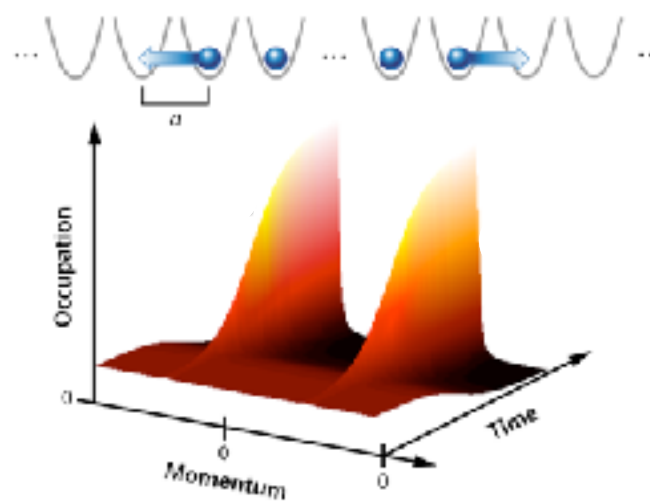
Experiments (LMU, see also D. Weiss group @ PSU):

Ballistic non-equilibrium mass transport (bosons)



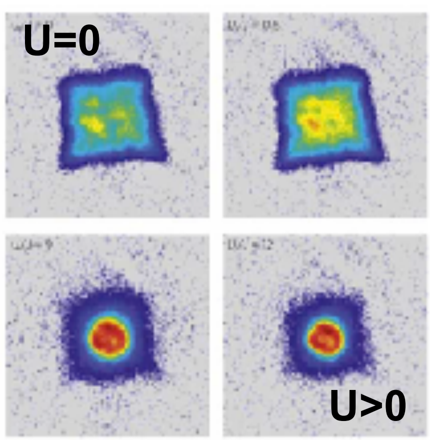
Ronzheimer, FHM, Bloch et al. PRL 110, 205301 (2013)
Vidmar et al. PRB 88, 235117 (2013)

(Quasi-) BEC in nonequilibrium (bosons)



Vidmar, FHM, Bloch, Schneider et al. PRL (2015)
Rigol, Muramatsu PRL (2004)
Full theory: Vidmar, Iyer, Rigiol PRX 2017

Breakdown of diffusion (2D, fermions)

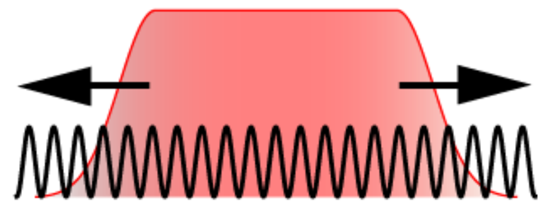


Schneider, Bloch et al.
Nature Physics 8, 213 (2012)

Nonequilibrium mass transport in the 1d Hubbard model

Quench: trap removal, expansion in flat lattice

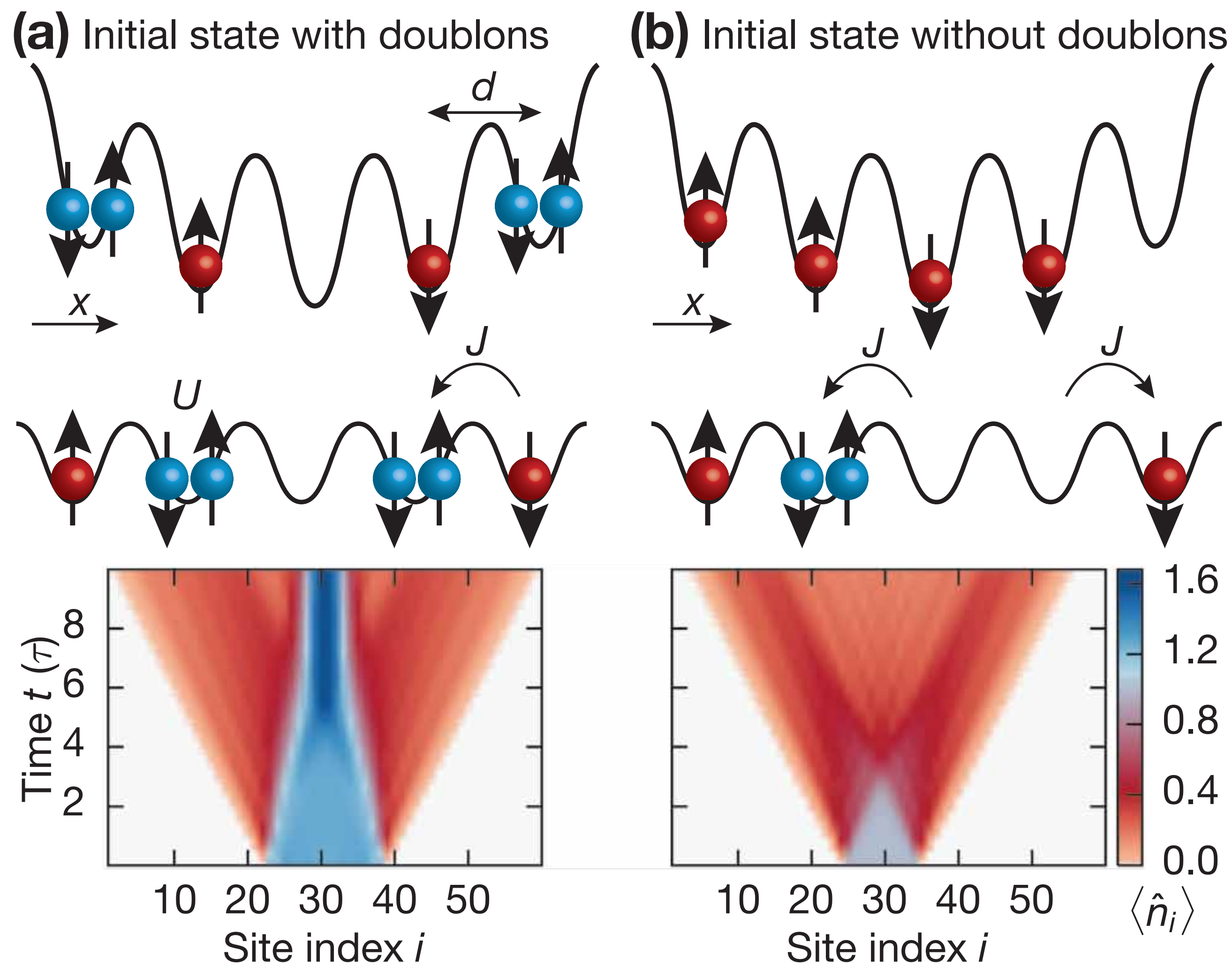
$$H_{\mathrm{FHM}} + H_{\mathrm{trap}} \rightarrow H_{\mathrm{FHM}}$$



Part I:

Transient dynamics

Quantum distillation

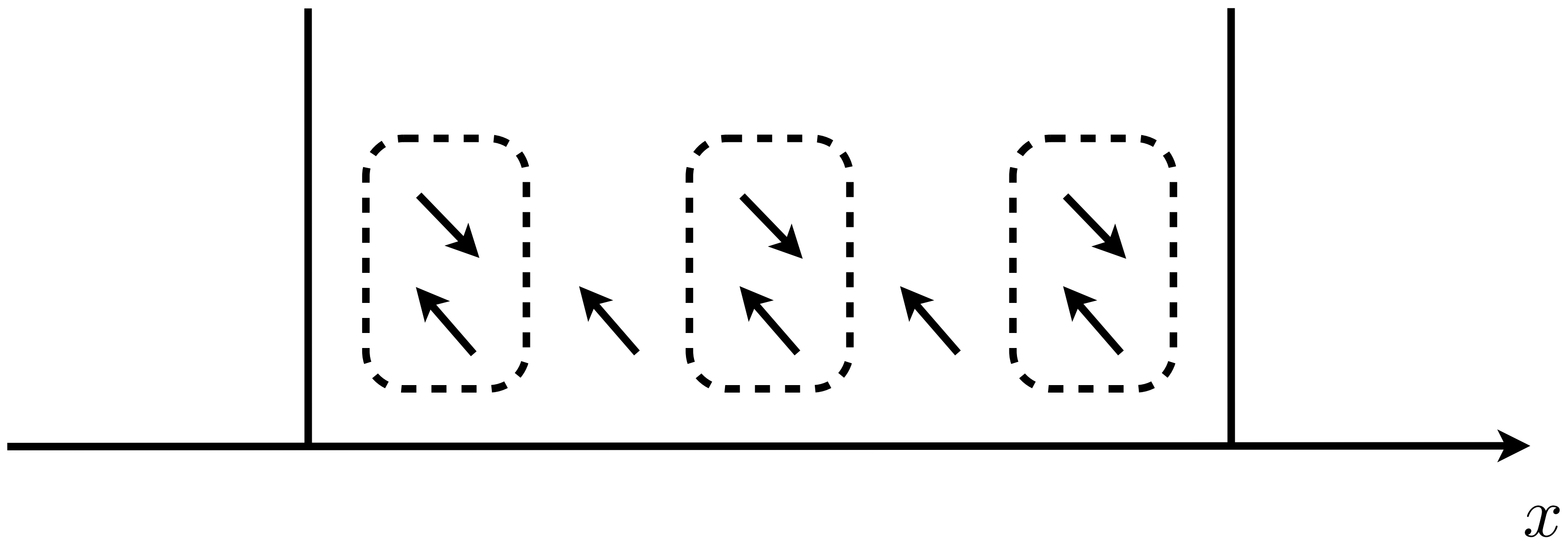


Part II:

Asymptotic dynamics

Scherg, Kohlert, Herbrych, Stolpp, Schneider, FHM, Aidelsburger, Bloch, Phys. Rev. Lett., in press; arXiv:1805.10990

Dynamics of doublons at higher densities: Quantum distillation

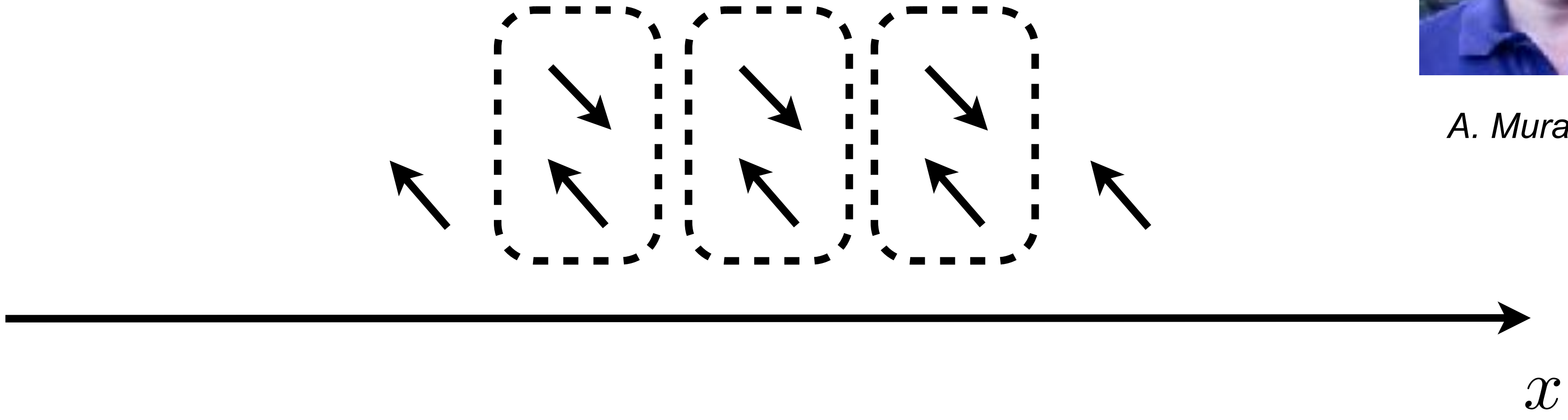


$$E = \text{const} \qquad\qquad J_{\text{pair}} \sim \frac{J^2}{U} \ll J$$

Expansion blocked by slow pairs/doublons?

Dynamics of doublons at higher densities: Quantum distillation

A. Muramatsu



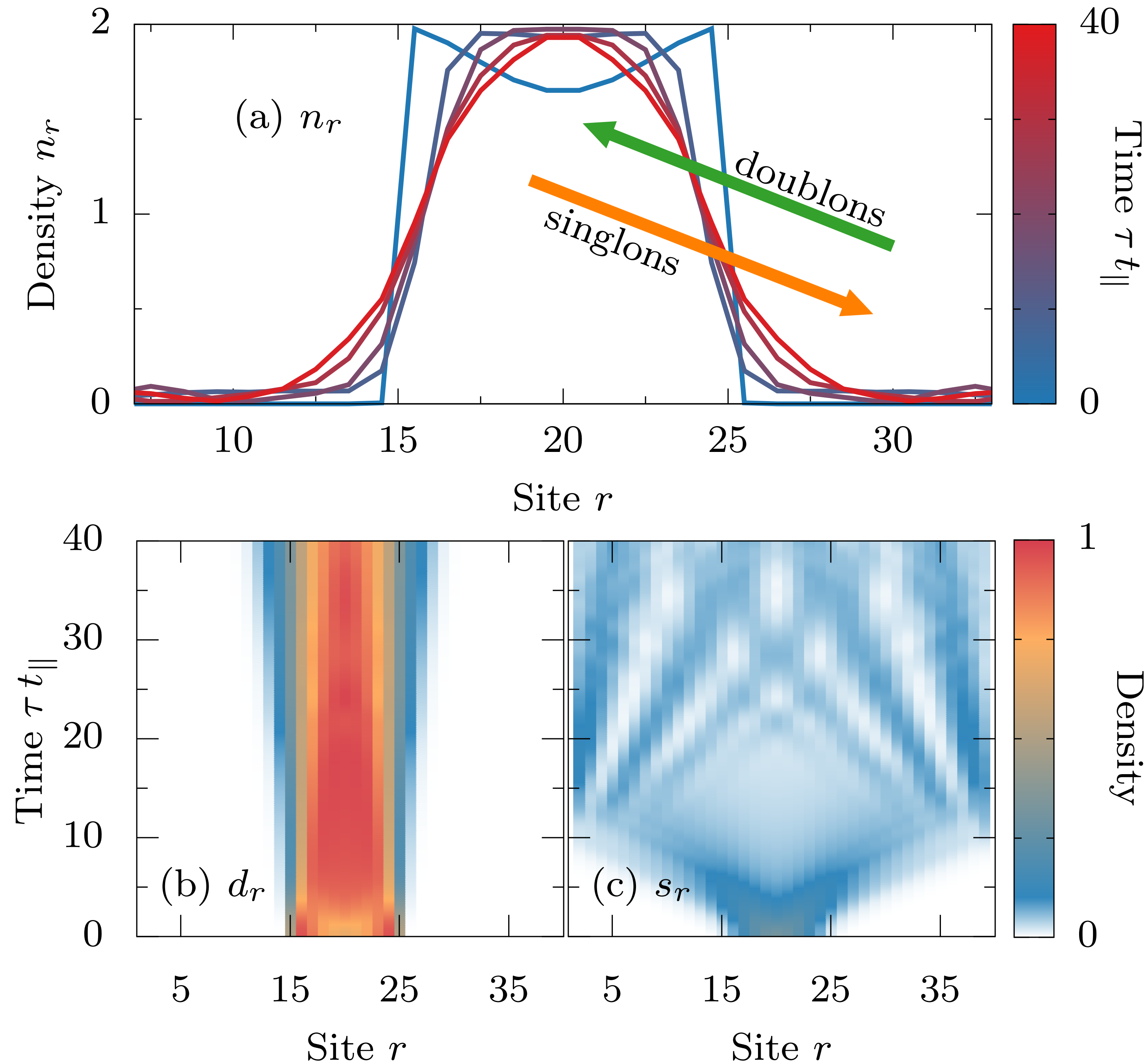
$$E = \text{const} \qquad\qquad J_{\text{pair}} \sim \frac{J^2}{U} \ll J$$

**Doublons/ Pairs move towards center,
"single" atoms evaporate:"Quantum distillation"**

Purification of an imperfect fermionic band insulator!

FHM, Manmana, Rigol, Muramatsu, Feiguin, Dagotto PRA 80, 041603(R) (2009)
Bosons: Muth, Petroysan, Fleischhauer PRA 85, 013615 (2012)

Dynamics of doublons at higher densities: Quantum distillation



1D, U/J=40

Doublons move to center fast followed by slow dynamics

Singlons evaporate - just as if the doublons were not there

FHM, Manmana, Rigol, Muramatsu, Feiguin, Dagotto PRA 80, 041603(R) (2009)
Herbrych, Feiguin, Dagotto, FHM, Phys. Rev. A 96, 033617 (2017)

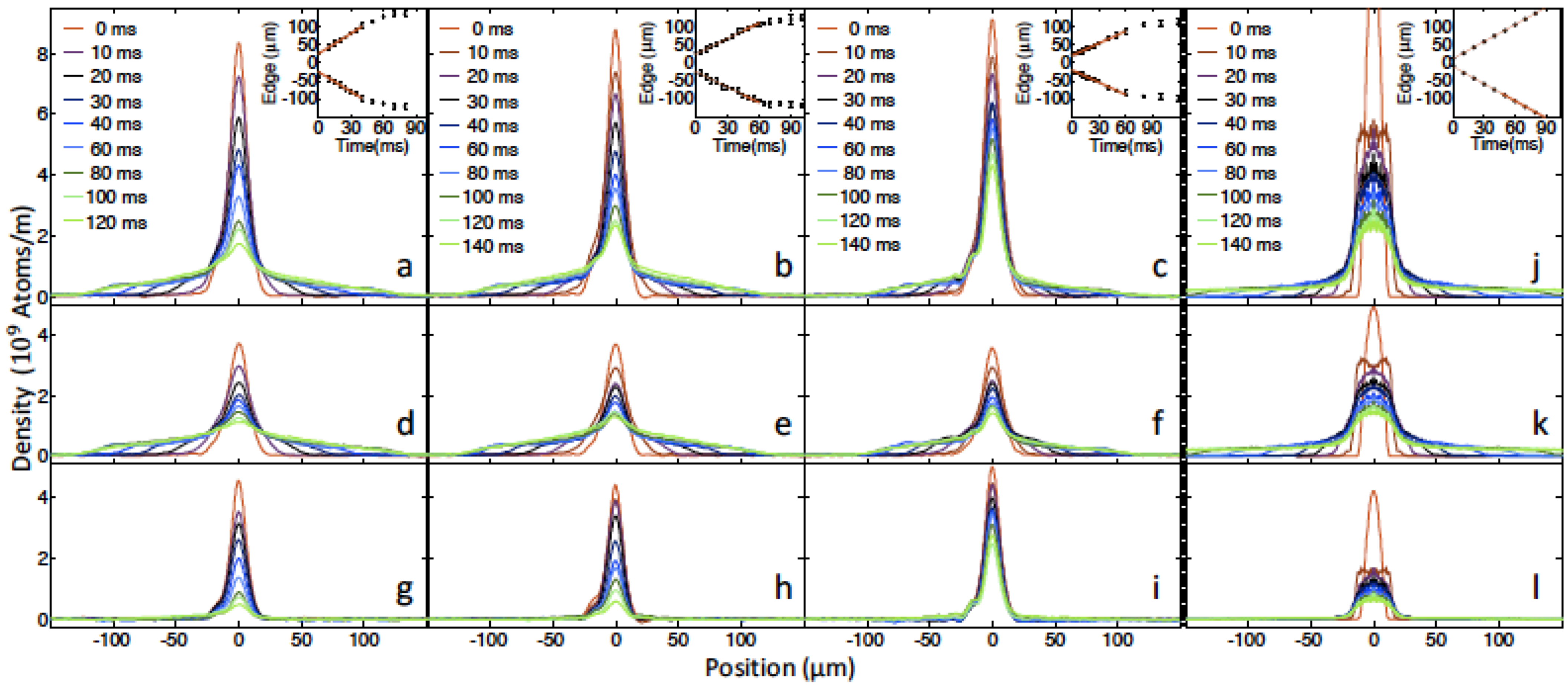
Quantum distillation: Observed for bosons!

$$U/J \lesssim 10$$

Full density

Singlons

Doublons



Bosons cluster in the center!

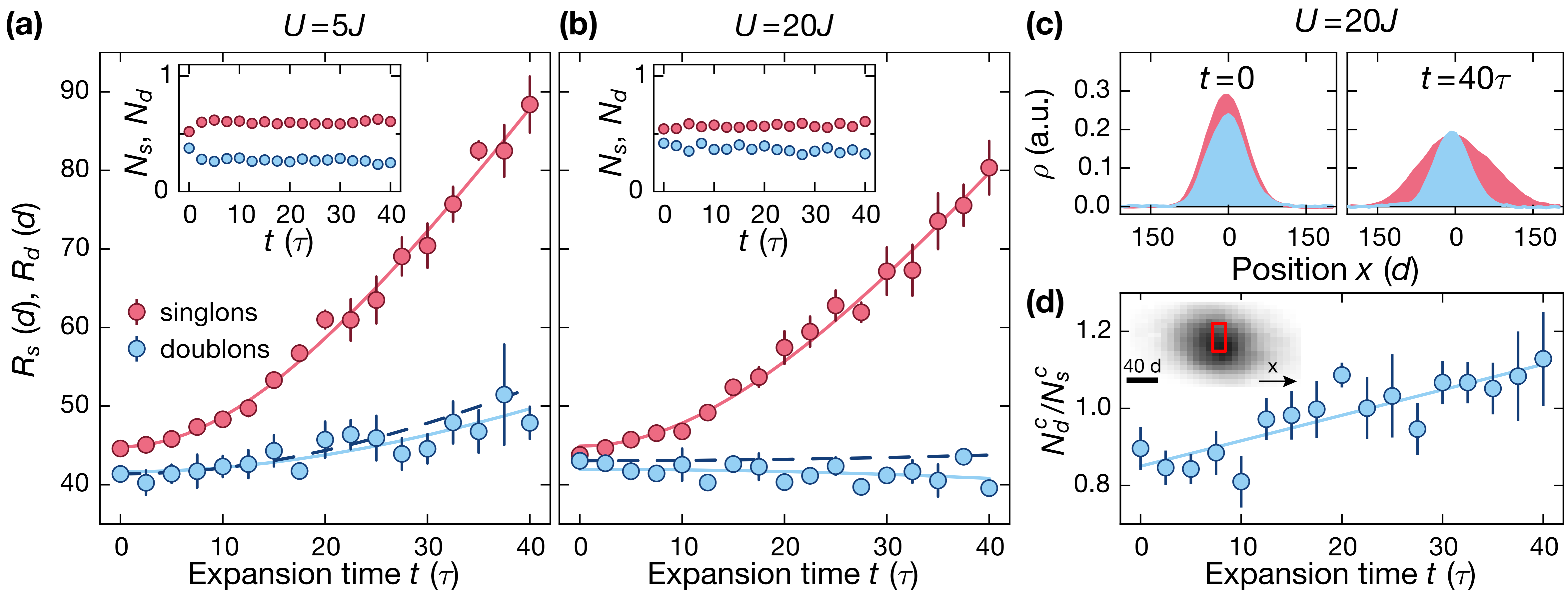
Experiment from PSU (D. Weiss' group)

Xia et al. Nature Physics 11, 316 (2015)

Open: Does this work in 2d? Experiments for fermions?

New LMU experiment: Quantum distillation with fermions

1d Fermi-Hubbard $$H = -J \sum_{\langle i,j \rangle} (c^\dagger_{i\sigma} c_{j\sigma} + h.c.) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



Dynamical separation! Singlons move out of center

Limitations: hole defects, average over many 1d tubes, t_{max}, ….

Scherg, Kohlert, Herbrych, Stolpp, Schneider, FHM, Aidelsburger, Bloch, Phys. Rev. Lett., in press; arXiv:1805.10990

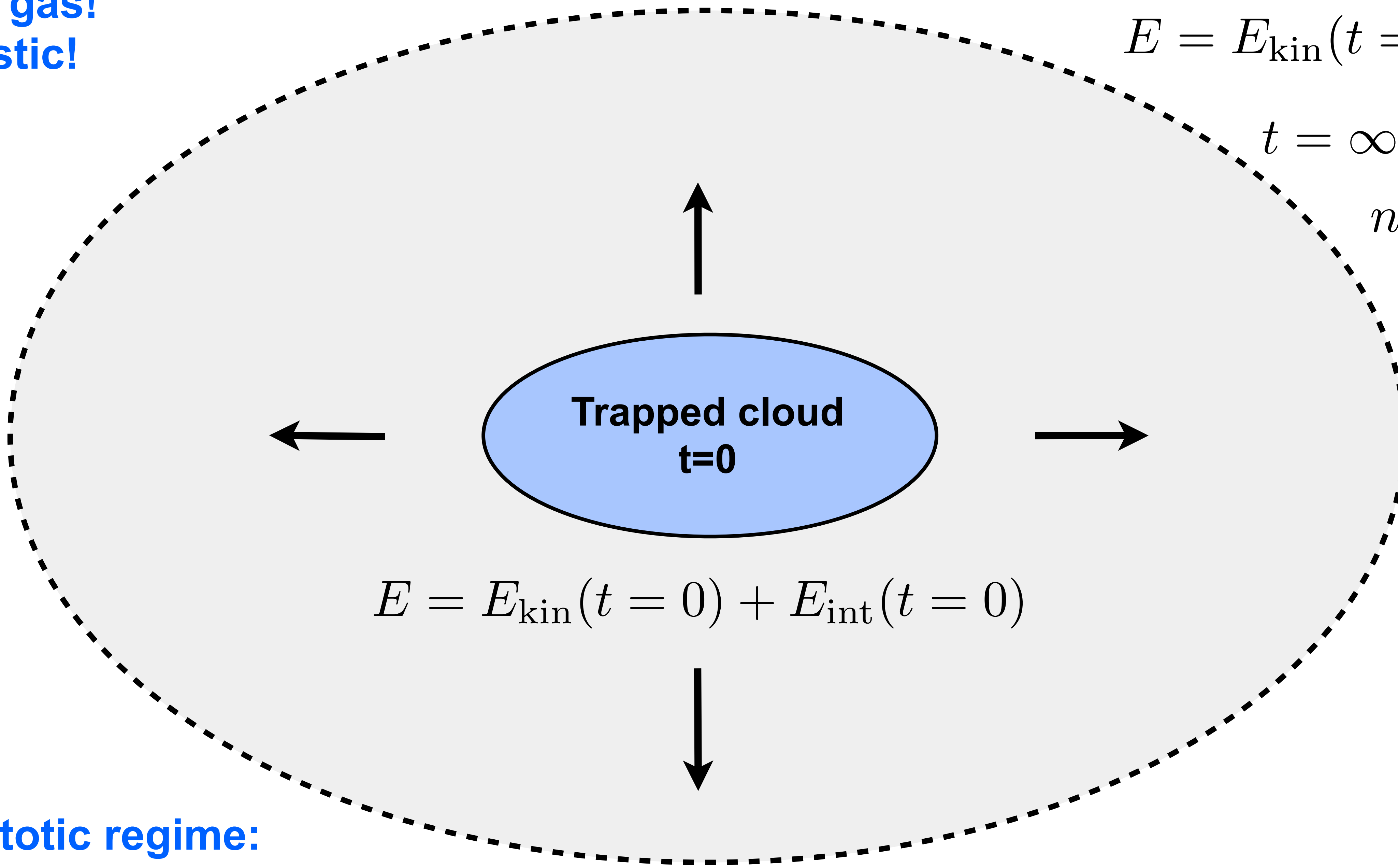
Sudden expansion: Asymptotic properties

Dilute gas!
Ballistic!

$$E = E_{\mathrm{kin}}(t=\infty)$$

$$t = \infty$$

$$n \to 0$$



Asymptotic regime:

$$H \to \sum_k \epsilon_k n_k(t=\infty)$$

$$n_k(t=\infty) = f(E/N, \dots)$$

Role of conservation laws/ integrability?

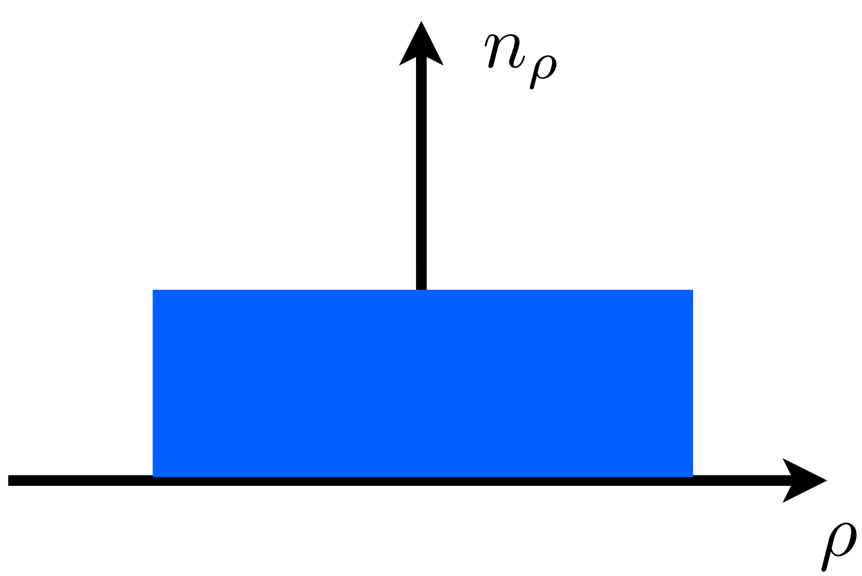
Predicting the asymptotic MDF from "first principles"

Generalization of dynamical fermionization of HCBs for other *integrable* 1D models

Distribution of *rapidities:*

Quantum numbers in Bethe ansatz
Defined by initial state

$$E = \int d\rho \, n_\rho \epsilon_\rho$$



$$\rho = \rho(N, U, \dots)$$

Sutherland's interpretation:
Rapidities = Asymptotic momenta

$$n_k^{\text{physical}}(t \to \infty) \to n_\rho$$

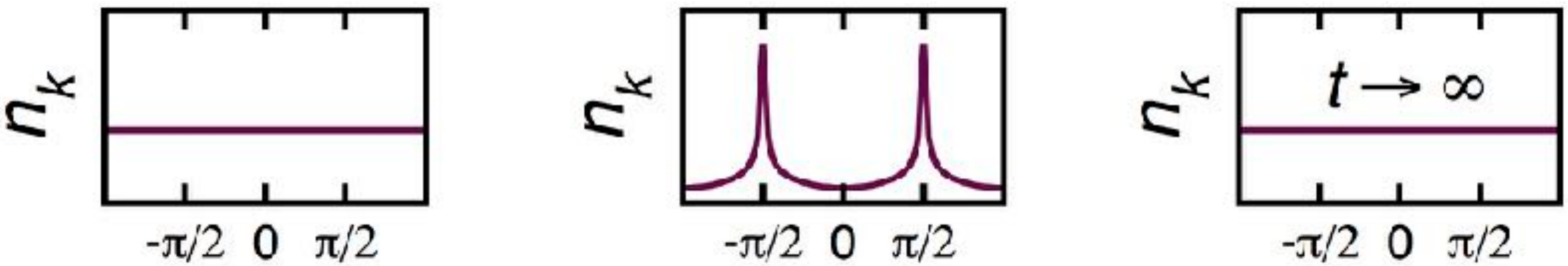
Sutherland PRL 80, 3678 (1988)
Sutherland: "Beautiful models"

Asymptotic regime: Hard-core bosons

$$H = -J\sum_i (b_i^\dagger b_{i+1} + h.c.) \quad [b_i, b_i^\dagger] = 1$$

1D Bose-Hubbard model at n=1: $U/J = \infty \quad H = -2J\sum_k \cos(k) n_k^f$

Initial state | **Dynamical quasi-condensation** | **Dynamical fermionization**



Momentum distribution of *physical* particles becomes identical to the one of *underlying free fermions*

$$n_k^{HCB}(t \rightarrow \infty) \rightarrow n_k^f$$

Rigol, Muramatsu, PRL 93, 230403 (2005); Minguzzi, Gangardt PRL 94, 240404 (2005)

Predicting the asymptotic MDF from "first principles"

Here: Fermi-Hubbard model, U<0

$$U < 0; \quad N_\uparrow > N_\downarrow$$

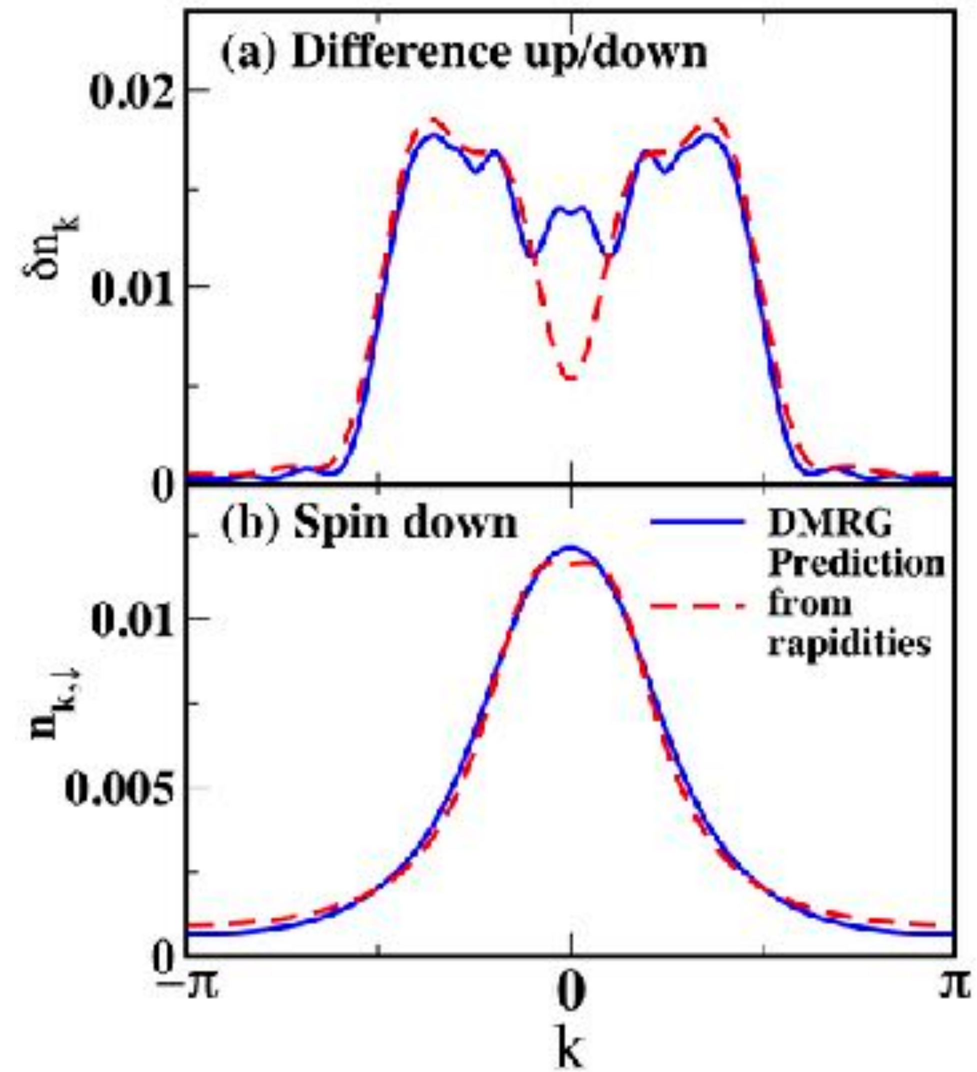
$N_\downarrow$ pairs **(FFLO!)**

$$N_\uparrow - N_\downarrow \text{ unpaired fermions}$$

$$\delta n_k = n_{k,\uparrow} - n_{k,\downarrow} \to n_{\rho_{\text{unpaired}}}$$

$$n_{k,\downarrow}(t \to \infty) \to n_{\rho_{\text{pair}}}$$

Asymptotic form of MDF
U=-8J



Long-time limit of MDFs: Determined by distribution of Bethe-ansatz rapidities of initial state

Bolech, FHM, Langer, McCulloch, Orso, Rigol PRL 109, 110602 (2012)
For the continuum: Campbell, Gangardt, Kheruntsyan PRL 114, 125302 (2015)

Predicting expansion velocities

Repulsive interactions: Slow approach of MDF to asymptotic regime

Expansion velocities converge fast - average over: $\sin^2(k)$

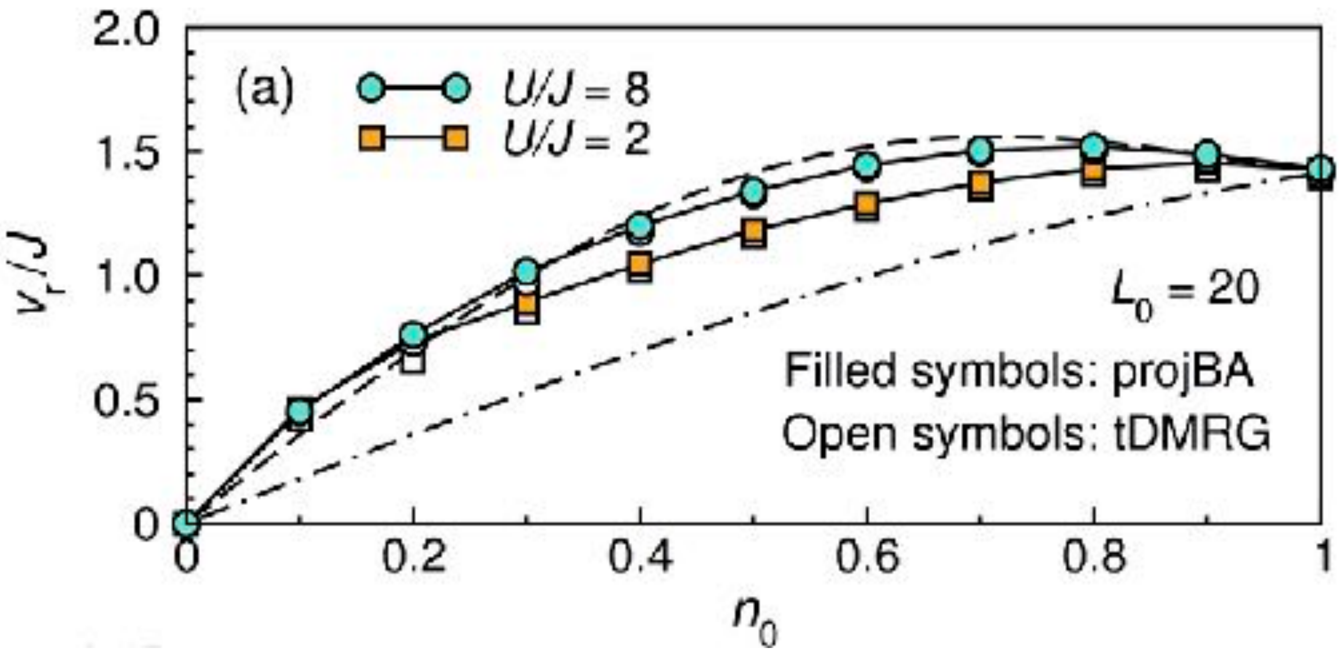
Consequence $R = v_r t$

$$v_r^2 = \frac{1}{N} \sum_\rho v_\rho^2 n_\rho$$

↑

Defined by initial condition
Obtained from Bethe ansatz

DMRG vs Bethe ansatz - fermions
Expansion from ground states



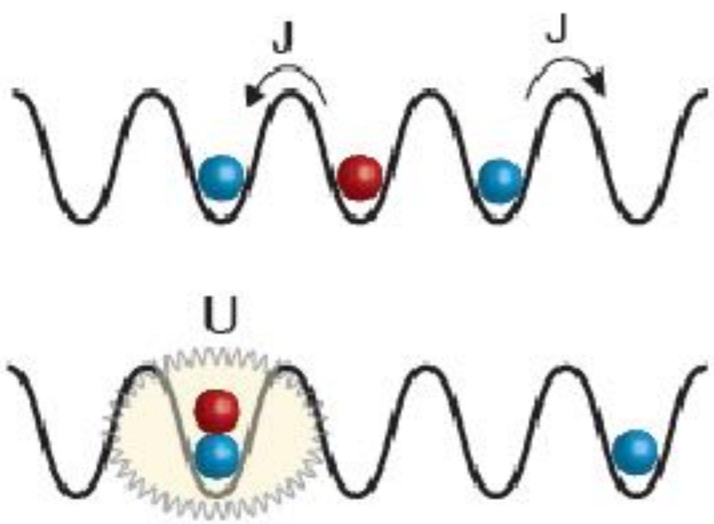
Mei, Vidmar, FHM, Bolech PRA 93, 021607(R) (2016)
Schuetz, Langer, McCulloch, Schollwöck, FHM PRA 85, 043618 (2012)
Sort of similar, two-body: Ganahl et al. PRL (2012), Fukuhara, Gross, Bloch et al. Nature (2013)

Expansion velocities from experiments

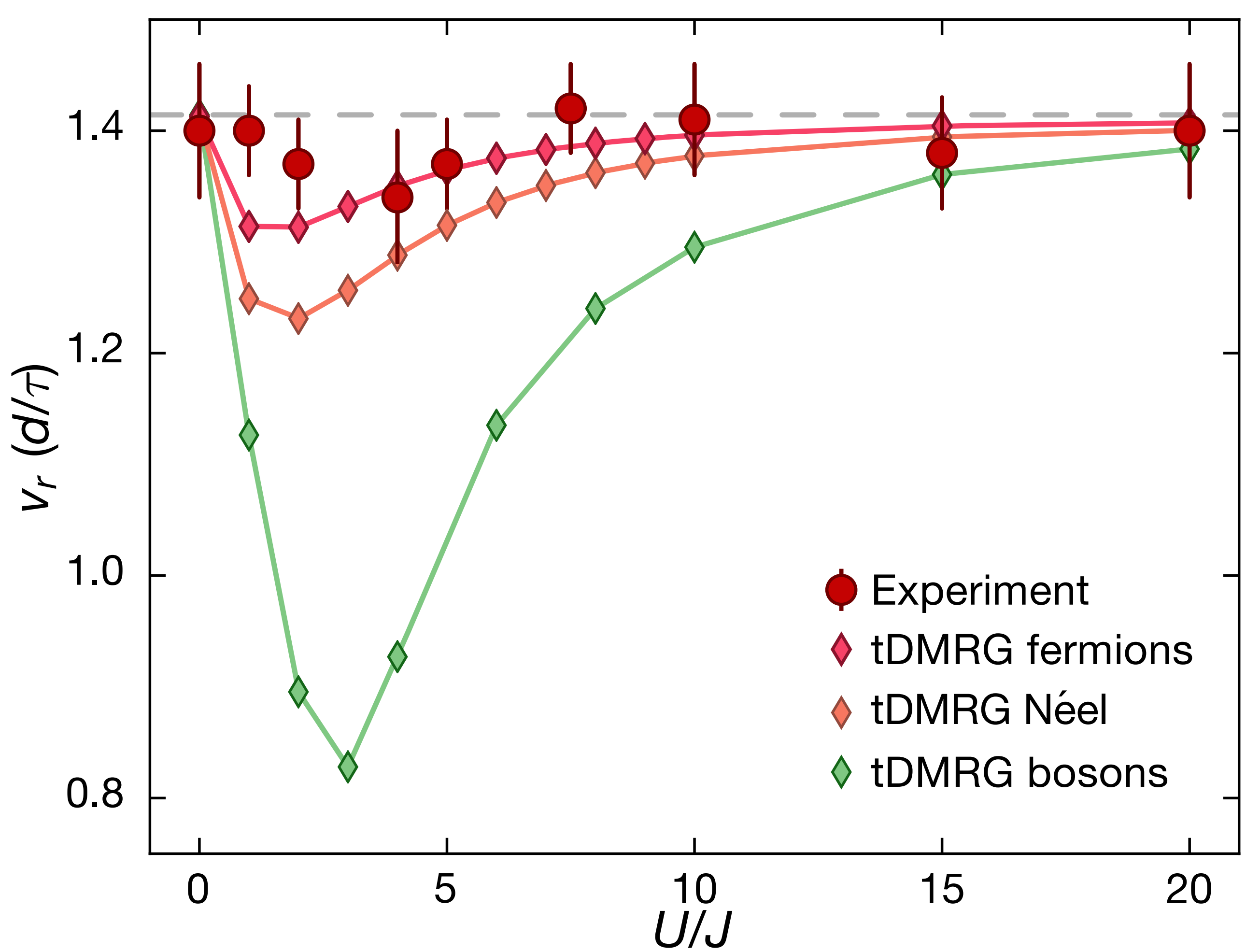
Initial state: Product state, random spin distribution

$$|\psi_0\rangle = |\uparrow, \downarrow, \downarrow, \uparrow, \downarrow, \ldots\rangle \quad N_\uparrow = N_\downarrow$$

Dynamically induced doublons



Average over many 1d systems



Why the minimum?

Dynamically induced doublons move to center, inert on experimental time scales

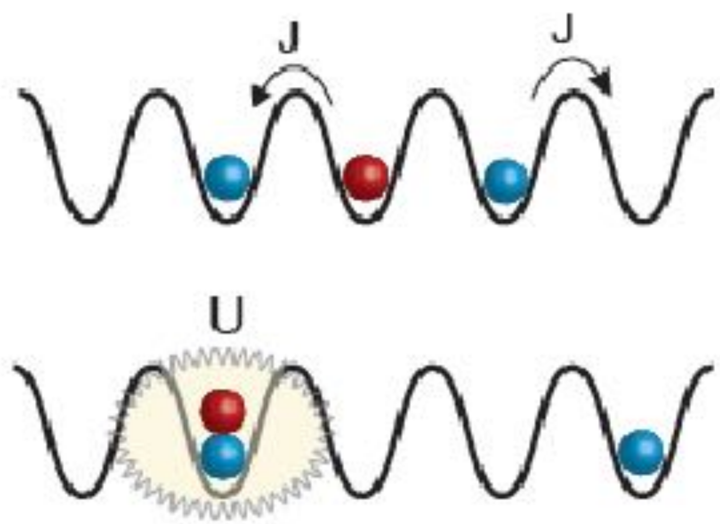
Scherg, Kohlert, Herbrych, Stolpp, Schneider, FHM, Aidelsburger, Bloch, Phys. Rev. Lett., in press, arXiv:1805.10990

Expansion velocities from experiments

Initial state: Product state, random spin distribution

$$|\psi_0\rangle = |\uparrow,\downarrow,\downarrow,\uparrow,\downarrow,\ldots\rangle \quad N_\uparrow = N_\downarrow$$

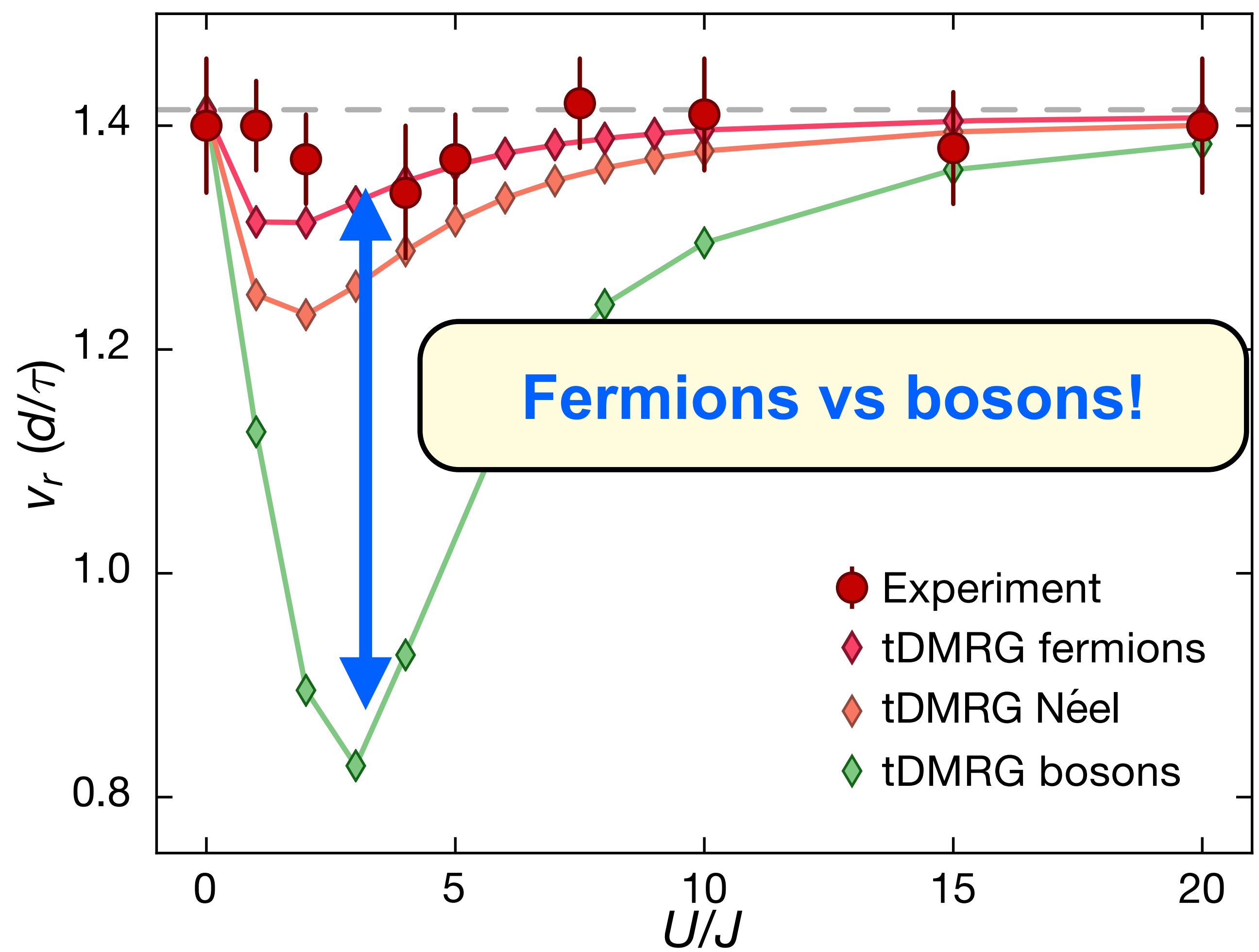
Dynamically induced doublons



Why the minimum?

Dynamically induced doublons move to center, inert on experimental time scales

Average over many 1d systems



Scherg, Kohlert, Herbrych, Stolpp, Schneider, FHM, Aidelsburger, Bloch, Phys. Rev. Lett., in press, arXiv:1805.10990
Bosons: Ronzheimer, Schreiber, Langer, Hodgman, FHM, Bloch, Schneider PRL (2013)

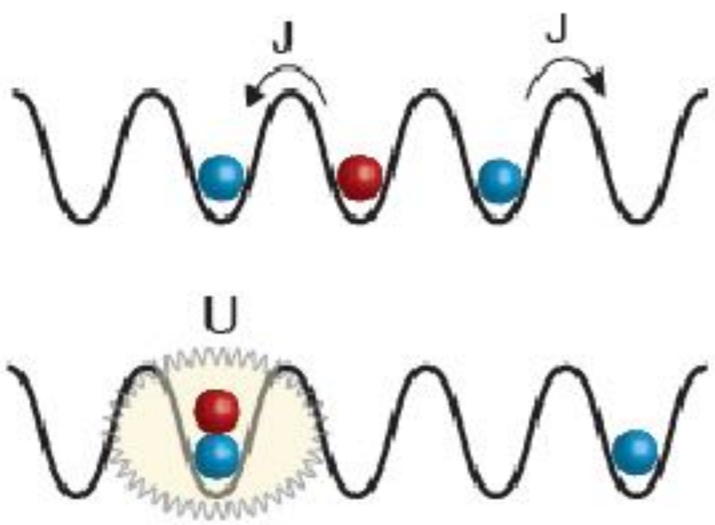
Expansion velocities from experiments

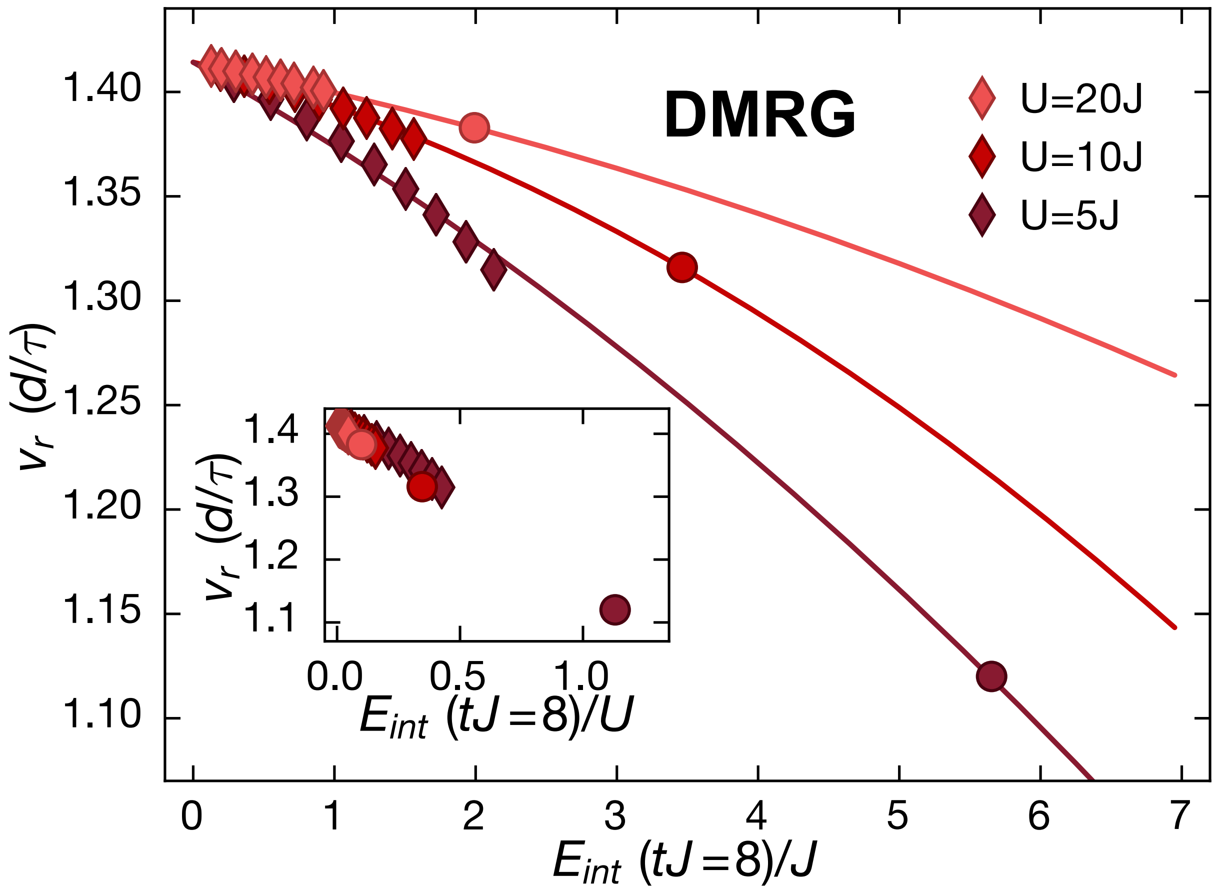
Initial state: Product state, random spin distribution

Fermions: $|\psi_0\rangle = |\uparrow,\downarrow,\downarrow,\uparrow,\downarrow,\dots\rangle \quad N_\uparrow = N_\downarrow$

Bosons: $|\psi_0\rangle = |1,1,1,1,\dots\rangle$

Dynamically induced doublons





Open: Asymptotic velocities from Bethe Ansatz for product states

Bose-Hubbard: Larger E_{int} possible!

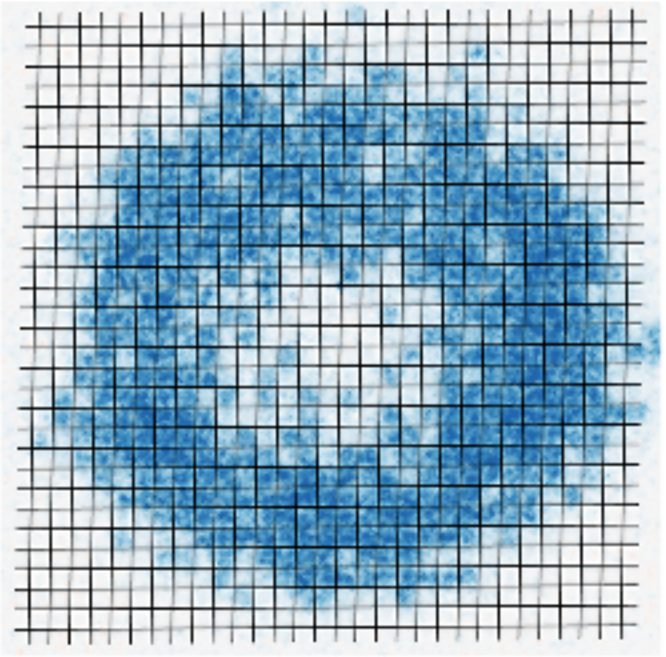
Scherg, Kohlert, Herbrych, Stolpp, Schneider, FHM, Aidelsburger, Bloch, Phys. Rev. Lett., in press, arXiv:1805.10990
Bosons: Ronzheimer, Schreiber, Langer, Hodgman, FHM, Bloch, Schneider PRL (2013)

Linear reponse transport: 1d Fermi-Hubbard

$$H = -J \sum_{\langle i,j \rangle} (c^\dagger_{i\sigma} c_{j\sigma} + h.c.) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

$$\mathrm{Re}\,\sigma(\omega) = D(T)\delta(\omega) + \sigma_{\mathrm{reg}}(\omega)$$

... and how to measure that in optical lattice experiments…

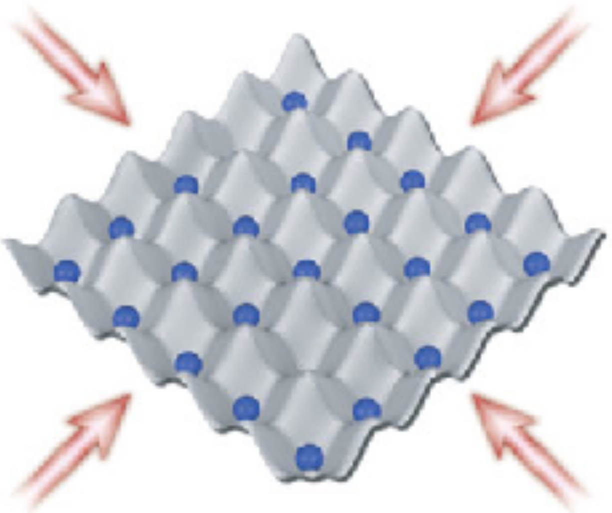


Fermionic Quantum Gas Microscope

Greiner (Harvard), Bloch/Gross (MPQ), Zwierlein (MIT), Kuhr (Strathclyde), Thywissen (Toronto), Bakr (Princeton), ...
1D: Boll et al, Science 353, 1257 (2016)

Hubbard model in optical lattice

Schneider et al. (2008), Jördans et al. (2008)
Hart et al. (2015), Greif et al. (2014)



Theoretical motivation (or obsession): Finite-temperature Drude weights

Linear response regime (Kubo): $C(t) = \langle j(t) j \rangle$

Drude weight & regular part

$$\mathrm{Re}\,\sigma(\omega) = D(T)\delta(\omega) + \sigma_{\mathrm{reg}}(\omega)$$

Exactly conserved current

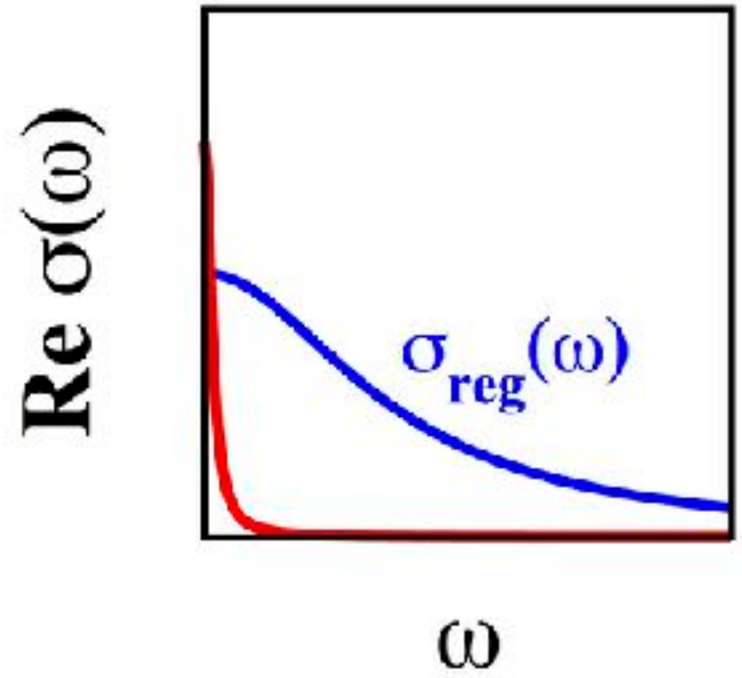
$$[H, j] = 0 \;\; \rightarrow \;\; \mathrm{Re}\;\; \sigma(\omega) = D(T)\delta(\omega)$$

Finite Drude weight:
Divergent dc conductivity
at <u>finite temperatures</u>

Zotos, Naef, Prelovšek, PRB (1997)

Integrability: Decay of currents protected by conservation law
Dissipationless transport

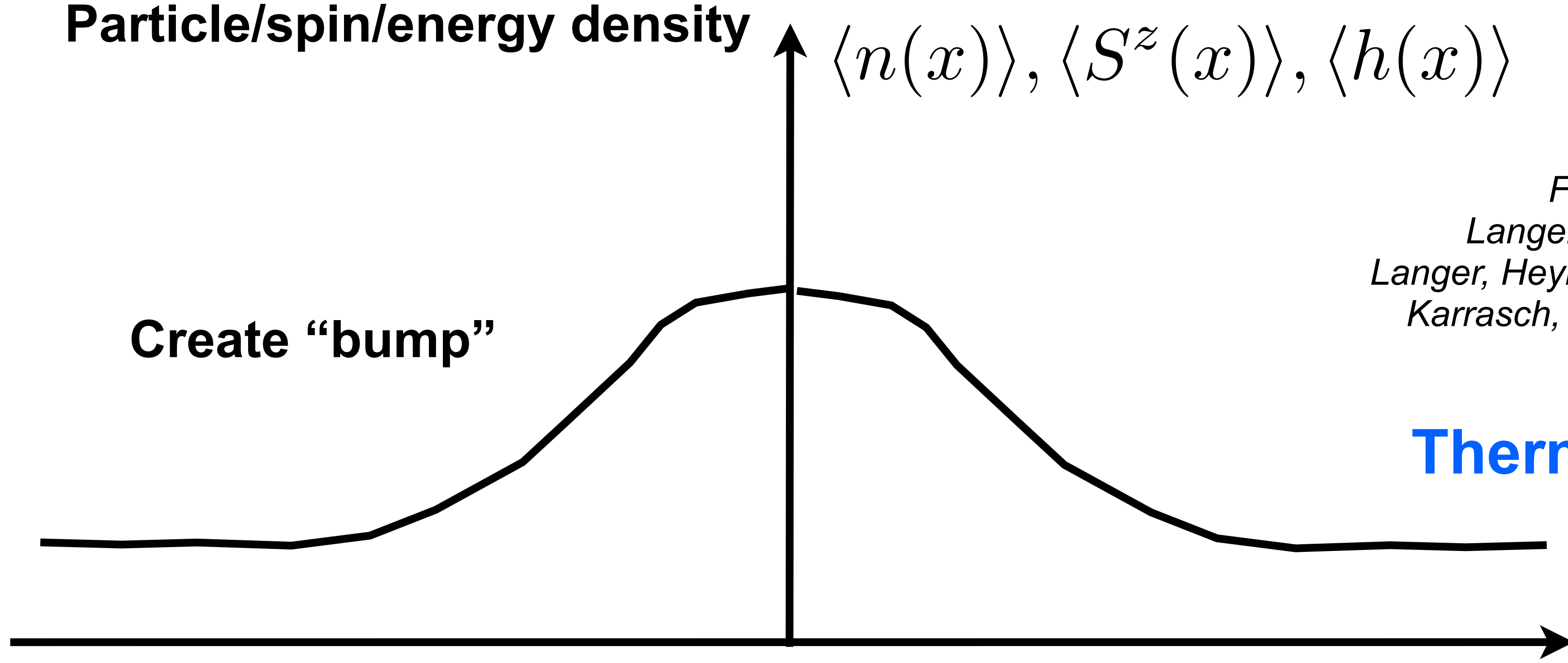
$$[H, Q_\alpha] = 0$$



Mazur inequality

$$D(T) \geq \mathrm{const}\,\frac{|\langle j Q_\alpha \rangle|^2}{\langle Q_\alpha^2 \rangle} > 0$$

Signatures in local quenches



For 1D spin systems:
Langer, FHM, et al. PRB (2009)
Langer, Heyl, McCulloch, FHM, PRB (2011)
Karrasch, Moore, FHM, PRB (2014), ...

Study width:

$$\sigma_\nu(t) \propto t^\alpha$$

$$\sigma_\nu^2(t) \sim \sum_i (i - i_0)^2 \langle S_i^z(t)\rangle$$

Steinigeweg, Wichterich, Gemmer, EPL (2009)

Generalized Einstein relation: $T = \infty$

$$\delta\sigma_\nu^2(t) = \frac{2}{L\chi_\nu} \int_0^t dt_1 \int_0^{t_1} dt_2 \langle j_\nu(t_2) j_\nu(0)\rangle_{\mathrm{eq}}$$

Diffusive case:

$$\delta\sigma_\nu^2(t) = 2\mathcal{D}_\nu t; \quad \mathcal{D}_\nu = \frac{\sigma_{dc,\nu}}{\chi_\nu}$$

Ballistic case: Drude weight!

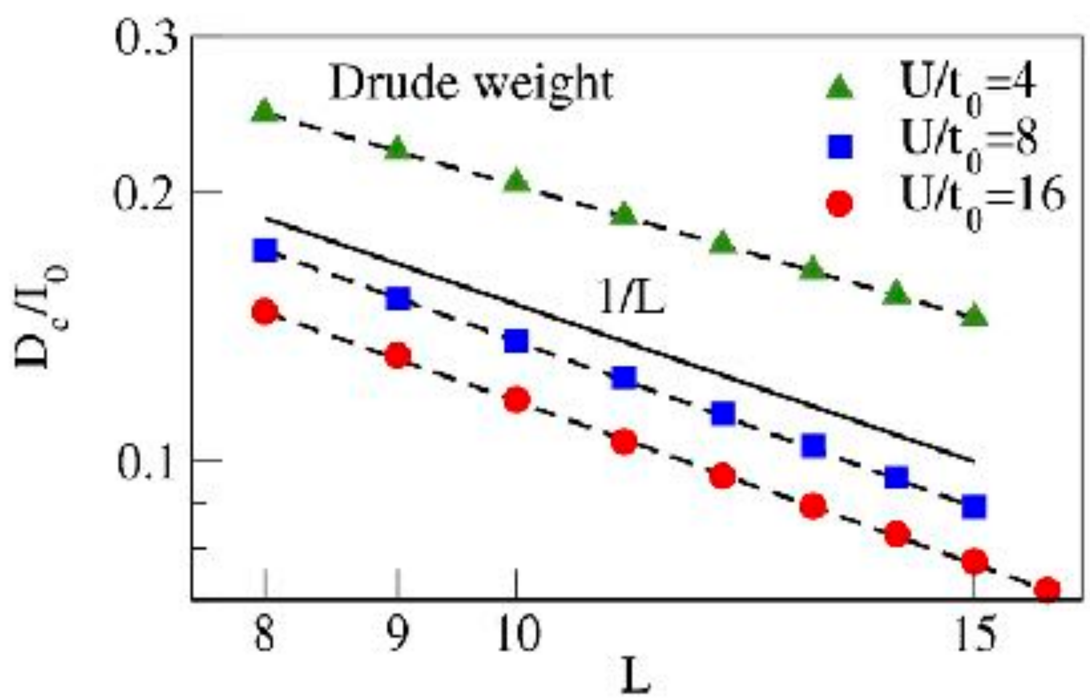
$$\delta\sigma_\nu^2(t) \propto \frac{D_\nu t^2}{\chi_\nu}$$

Integrable 1D Hubbard model

$$H = -J \sum_{\langle i,j \rangle} (c^\dagger_{i\sigma} c_{j\sigma} + h.c.) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

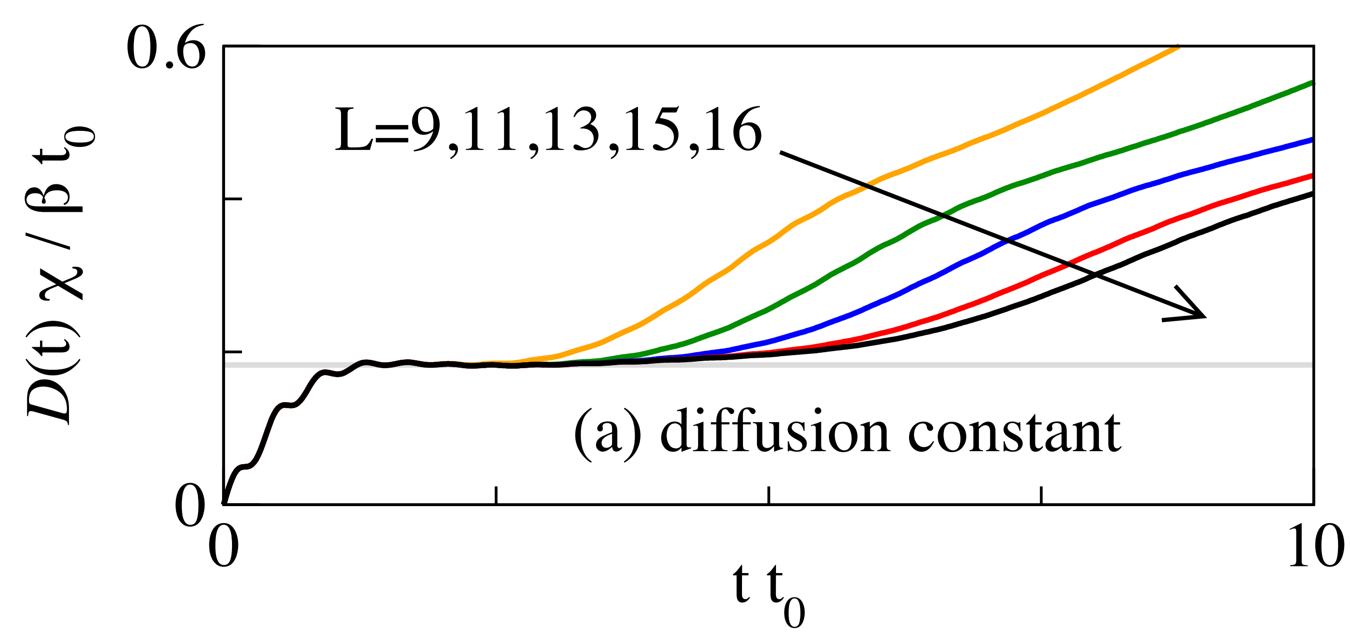
What's known about transport here (half filling)? $N_\uparrow + N_\downarrow = L$

Charge transport: No Drude weight



Dynamical typicality: Diffusive

$U/t_0 = 8, T = \infty$



$$D(t) = \int_0^t dt' \, \langle j(t') j \rangle$$

Fin, Steinigeweg, FHM, Michielsen, de Raedt PRB 92, 205103 (2015)
Karrasch, Kennes, Moore PRB 90, 155104 (2014)
Also: Carmelo, Nemati Prosen, Nucl. Phys. B, 930, 418 (2018)…

Recent work: Superdiffusion in SU(2) symmetric models?
Ljubotina,Znidaric,Prosen, Nat. Comm. 8,16117 (2017), Ilievski, De Nardis, Medenjak, Prosen arXiv:1806.03288

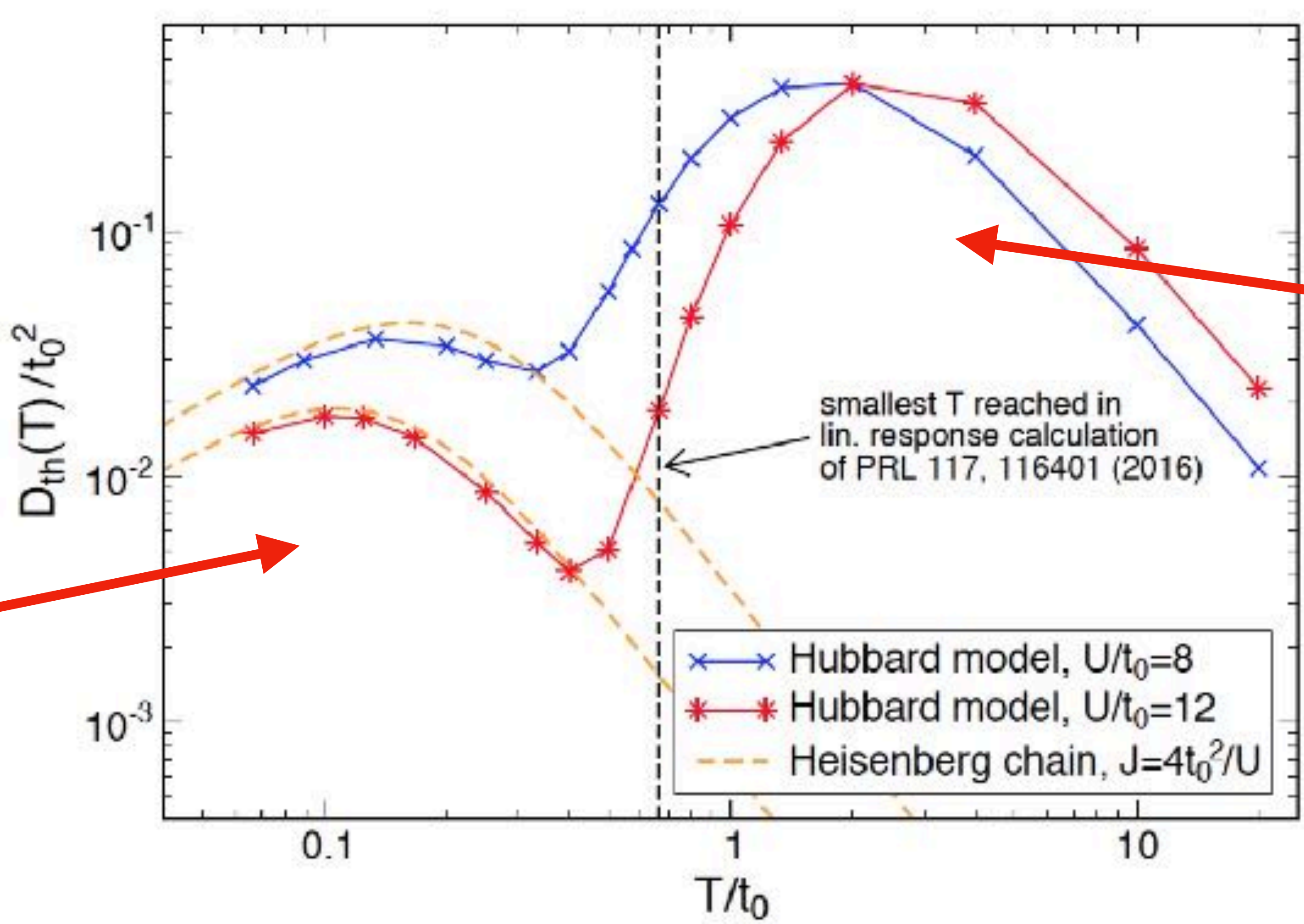
Integrable 1D Hubbard model

Charge transport: **"Diffusive"**

Thermal transport: **Ballistic**

$\langle j_E Q_3 \rangle \neq 0$

Zotos, Naef, Prelovšek, PRB (1997)



Spin dominates: Quantum magnets

Charge dominates: Optical lattices

$t_0 \to J$

Karrasch New J. Phys. 19, 033027 (2017) (using Vasseur, Karrasch, Moore PRL 115, 267201) , Karrasch, Kennes, FHM PRL 117, 116401 (2016)

Integrable 1D Hubbard model

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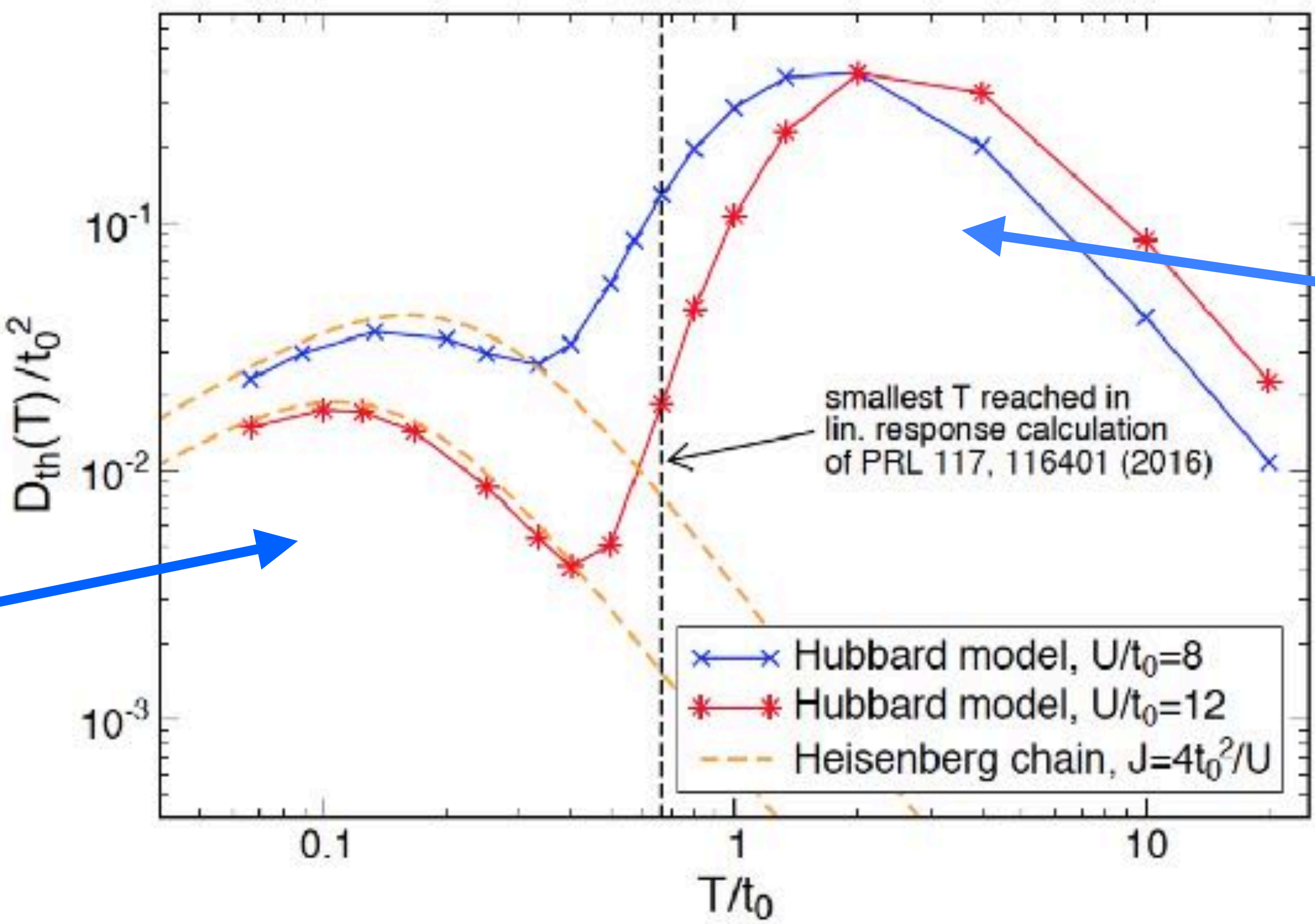

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Can this coexistence of a ballistic (energy) with a diffusive (charge) channel be observed in experiments?

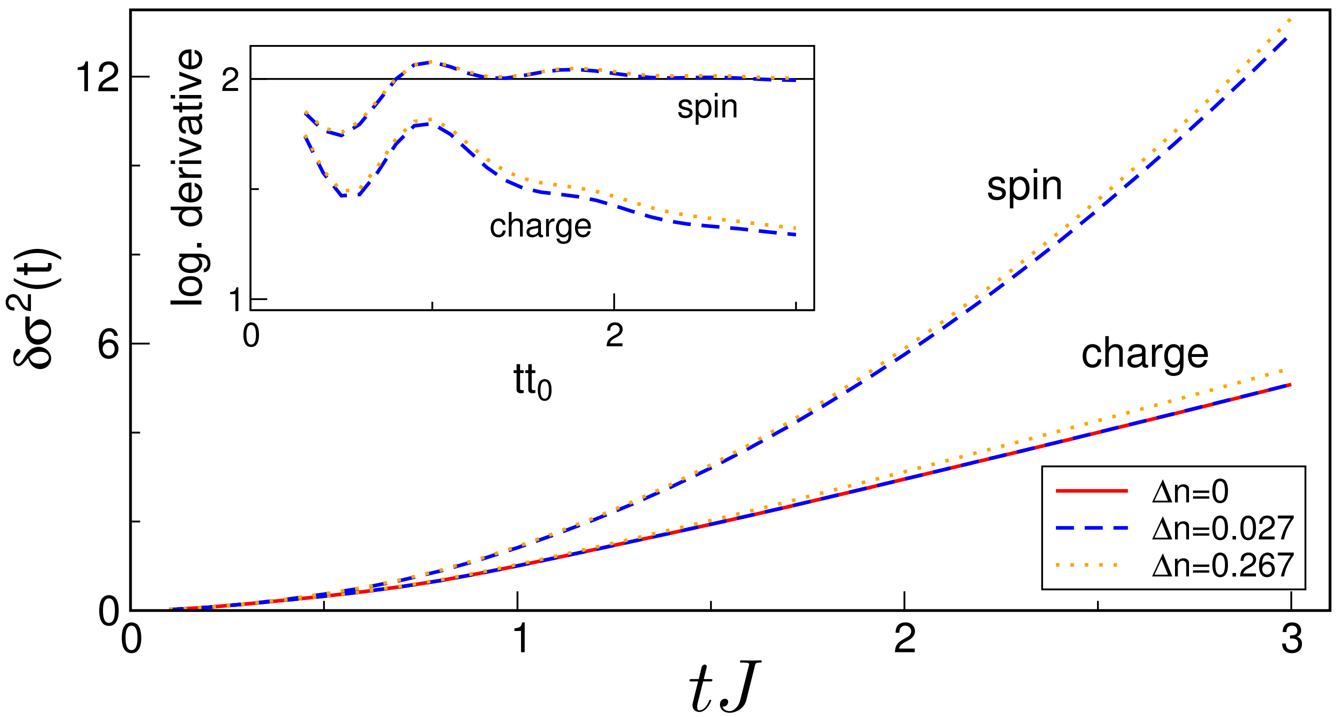
Experiments: Use integrable 1D Hubbard!

$$H = -J \sum_{\langle i,j \rangle} (c^\dagger_{i\sigma} c_{j\sigma} + h.c.) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

$$N_\uparrow \neq N_\downarrow$$

$$\Delta n = (N_\uparrow - N_\downarrow)/N$$

Spreading of density perturbation



ballistic

"diffusive"

Optical-lattice experiments do better by a factor of 2-3 !

Coexistence of *ballistic* spin & *diffusive* charge transport

Potentially better numerical approach: Time-dep. variational principle

Leviatan, Pollmann, Bardarson, Huse, Altman arXiv:1702.08894

Haegeman et al. PRL 107, 070601 (2011)

For details: See Karrasch, Prosen, FHM Phys. Rev. B 95, 060406(R) (2017)

Experiments: Use integrable 1D Hubbard!

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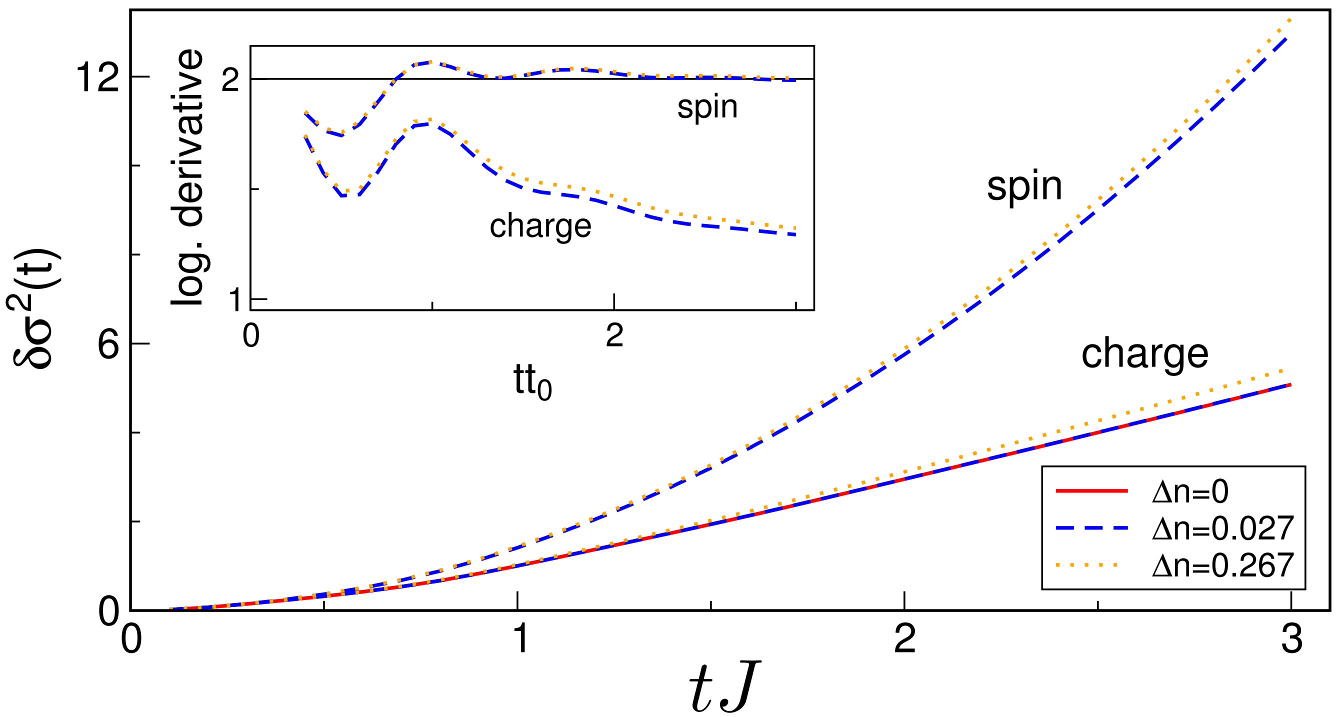
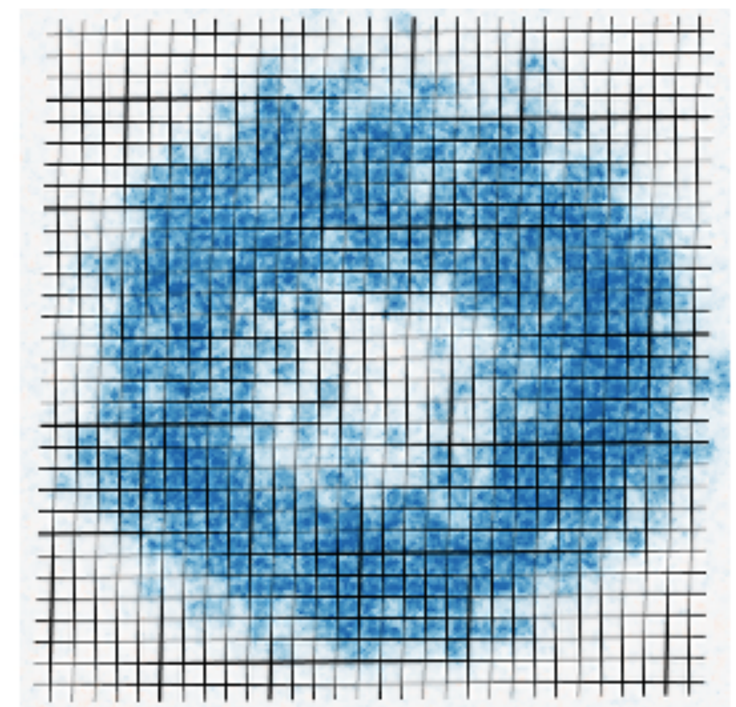


Coexistence of *ballistic* spin & *diffusive* charge transport

Fermionic quantum gas microscopes!



Greiner (Harvard), Bloch/Gross (MPQ), Zwierlein (MIT), Kuhr (Strathclyde), Thywissen (Toronto), Bakr (Princeton), ...
1D: Boll et al, Science 353, 1257 (2016)

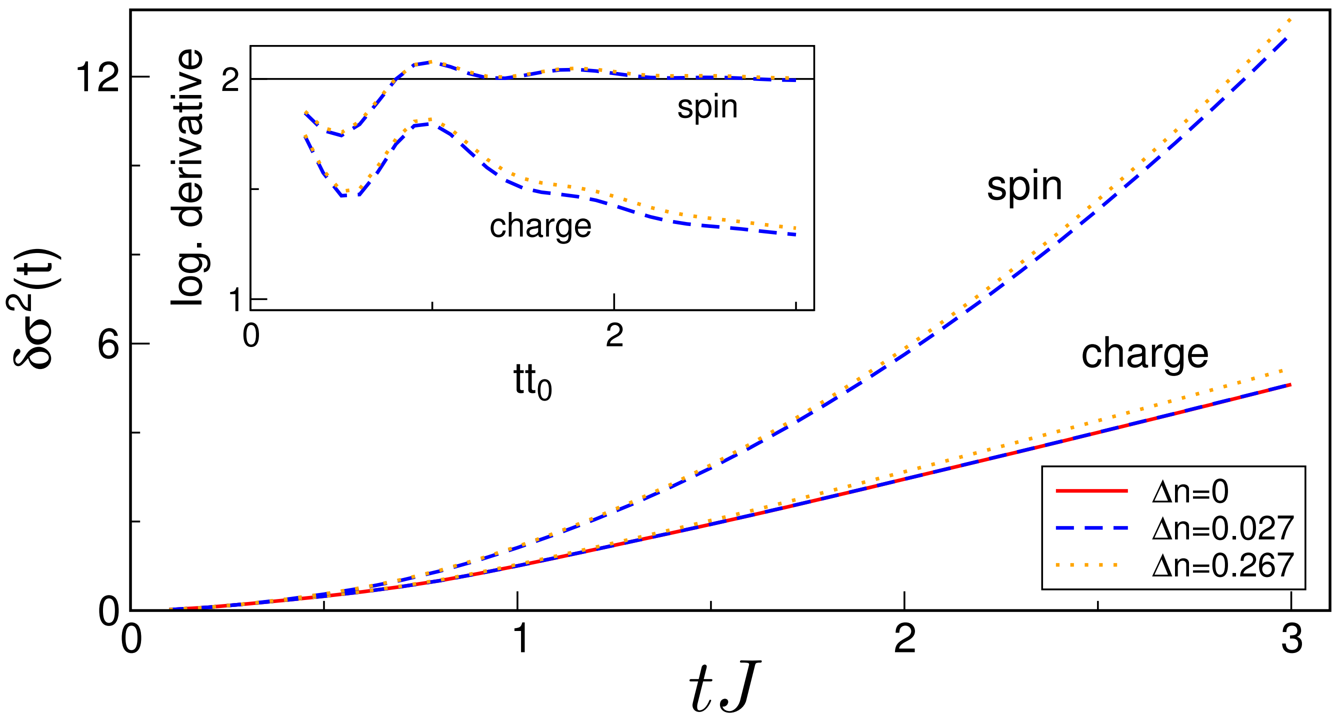
For details: See Karrasch, Prosen, FHM Phys. Rev. B 95, 060406(R) (2017)

Experiments: Use integrable 1D Hubbard!

$$H = -J \sum_{\langle i,j \rangle} (c^{\dagger}_{i\sigma} c_{j\sigma} + h.c.) + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$

$$N_{\uparrow} \neq N_{\downarrow}$$

Spreading of density perturbation



Coexistence of *ballistic* spin & *diffusive* charge transport

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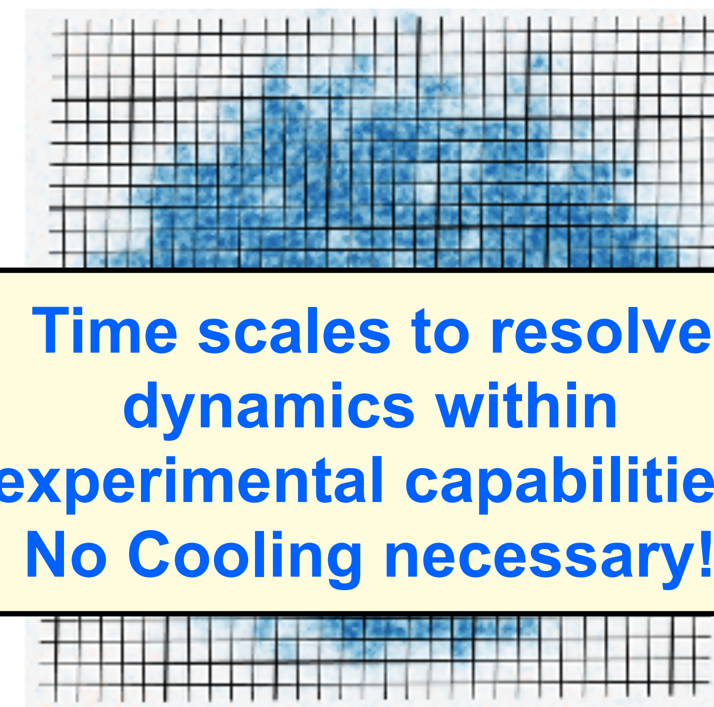


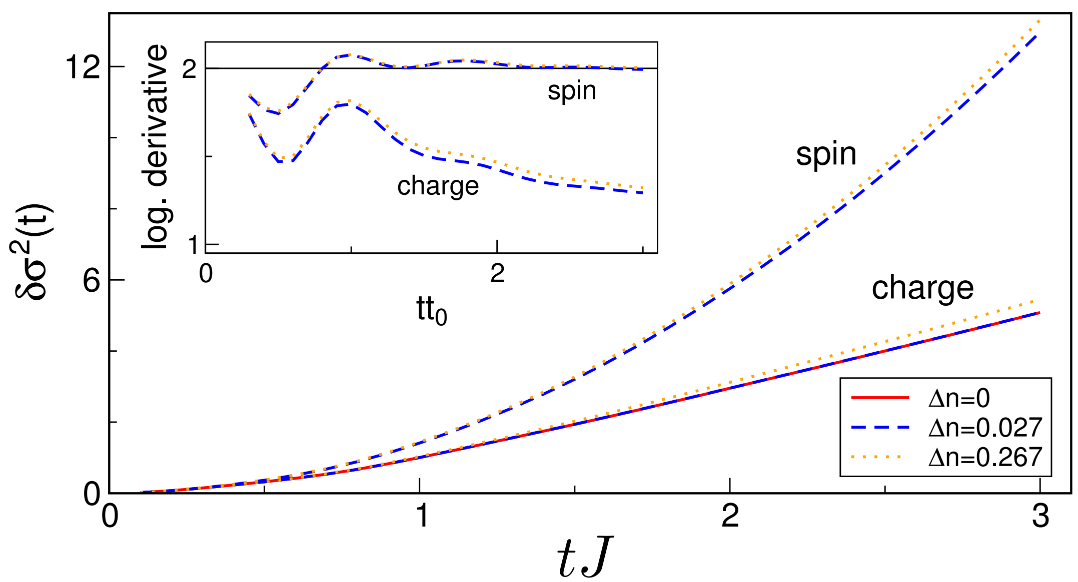
Time scales to resolve dynamics within experimental capabilities
No Cooling necessary!

Greiner (Harvard), Bloch/Gross (MPQ), Zwierlein (MIT), Kuhr (Strathclyde), Thywissen (Toronto), Bakr (Princeton), ...
1D: Boll et al, Science 353, 1257 (2016)

For details: See Karrasch, Prosen, FHM Phys. Rev. B 95, 060406(R) (2017)

Summary: Transport 1d Fermi-Hubbard

Linear response

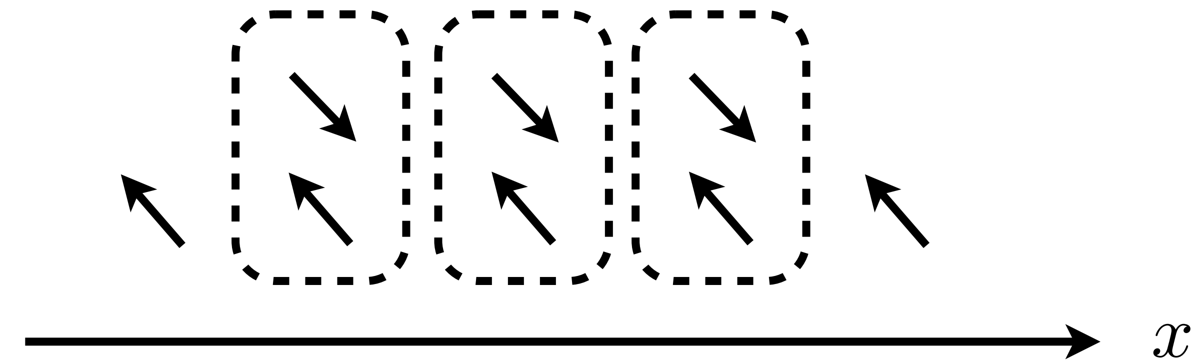


Coexistence of *ballistic* spin/ thermal & *diffusive* charge transport possible

Realistic:
Measurement of Drude weights and diffusion constants in quantum gas microscope

Karrasch, Prosen, FHM Phys. Rev. B 95, 060406(R) (2017)
Karrasch, Kennes, FHM PRL 117, 116401 (2016)

Nonequilibrium mass transport

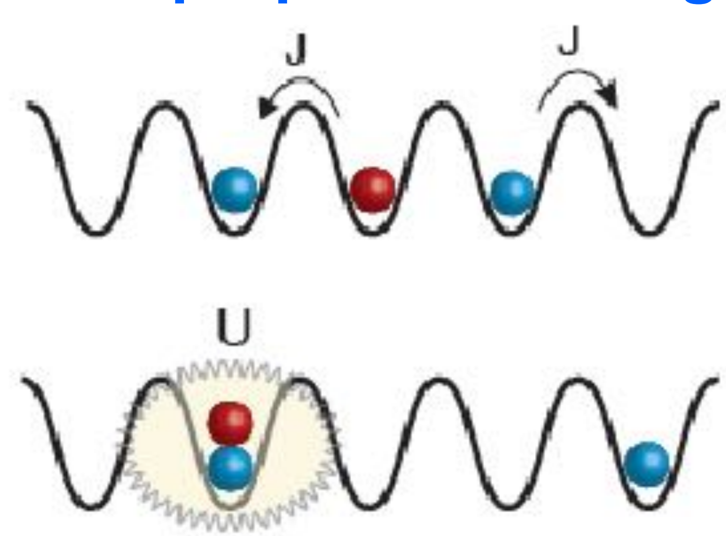


$$E = \text{const} \qquad J_{\text{pair}} \sim \frac{J^2}{U} \ll J$$

Quantum distillation with fermions

Scherg, Kohlert, Herbrych, Stolpp, Schneider, FHM, Aidelsburger, Bloch, PRL (2018) arXiv:1805.10990

Asymptotic properties & integrability



Mei, Vidmar, FHM, Bolech PRA 93, 021607(R) (2016)
Bolech, FHM, Langer, McCulloch, Orso, Rigol PRL (2012)