Spatio-temporal quenches for fast preparation of ground states of critical models

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In collaboration with: Ravin Bhatt, Shivaji Sondhi, Matteo Ipolitti, Prahar Mitra (IAS)

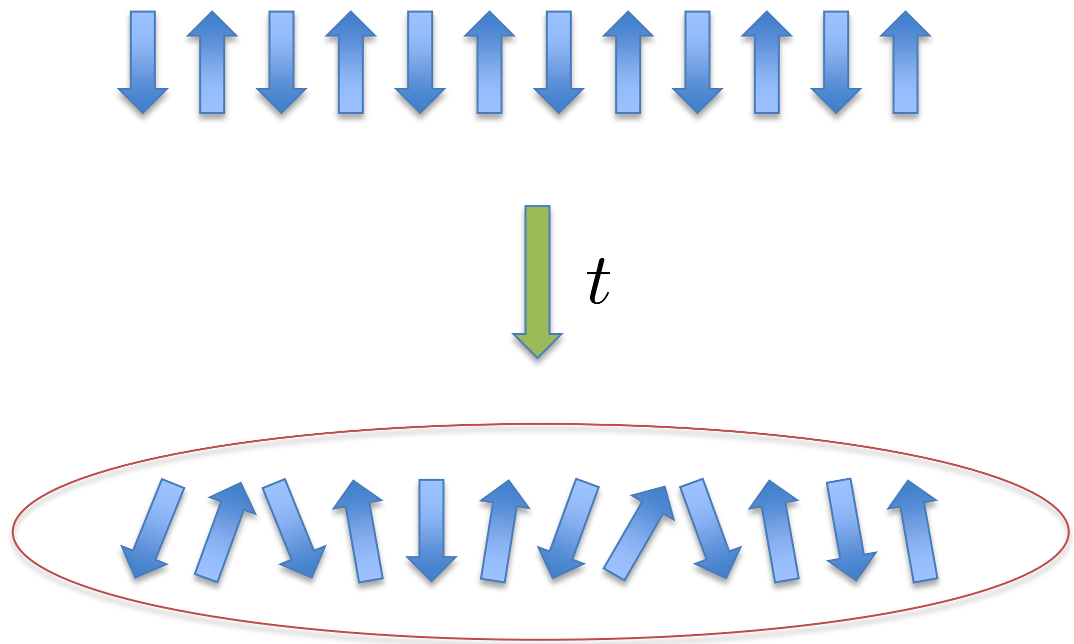
K. Agarwal, R. N. Bhatt, and S. L. Sondhi, Phys. Rev. Lett. **120**, 210604

P. Mitra, M. Ipolitti, R. N. Bhatt, S. L. Sondhi, K. Agarwal (to be out on ArXiv soon)

U.K. Foundation

Central Question

Classical Product State-ish
Easy to Prepare
Ground state of some Gapped Hamiltonian
Low entanglement



Entangled State
Ground state of some *Gapless* Hamiltonian

What is the optimal way of performing such a transformation (with local operations)?

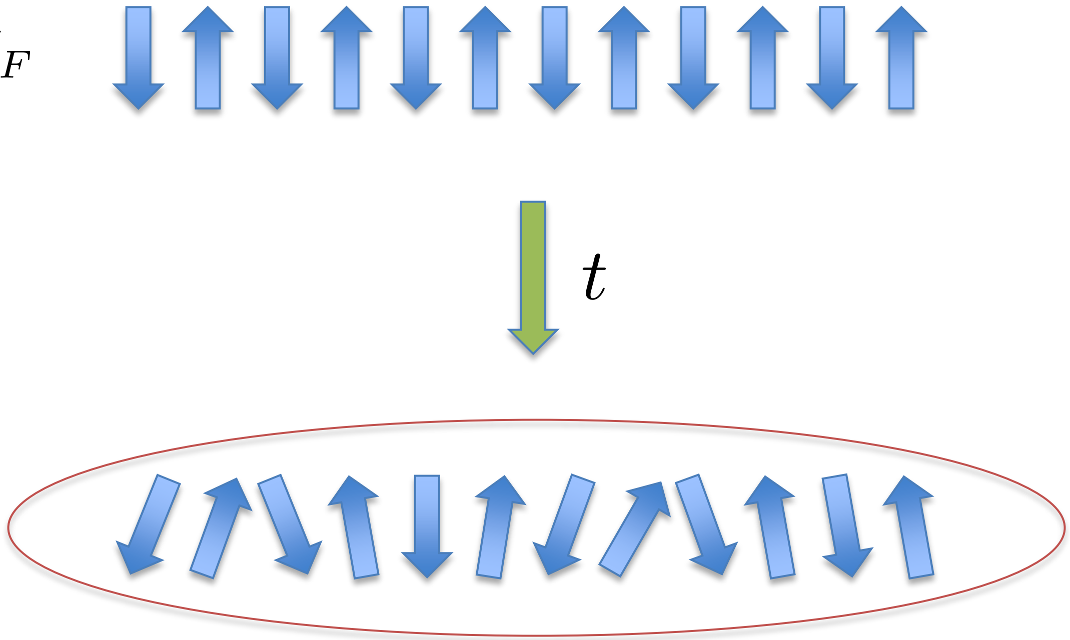
Adiabatic Approach

$$H = (1 - \lambda(t))\, H_I + \lambda(t) H_F$$

$$\lambda(t) : 0 \to 1$$

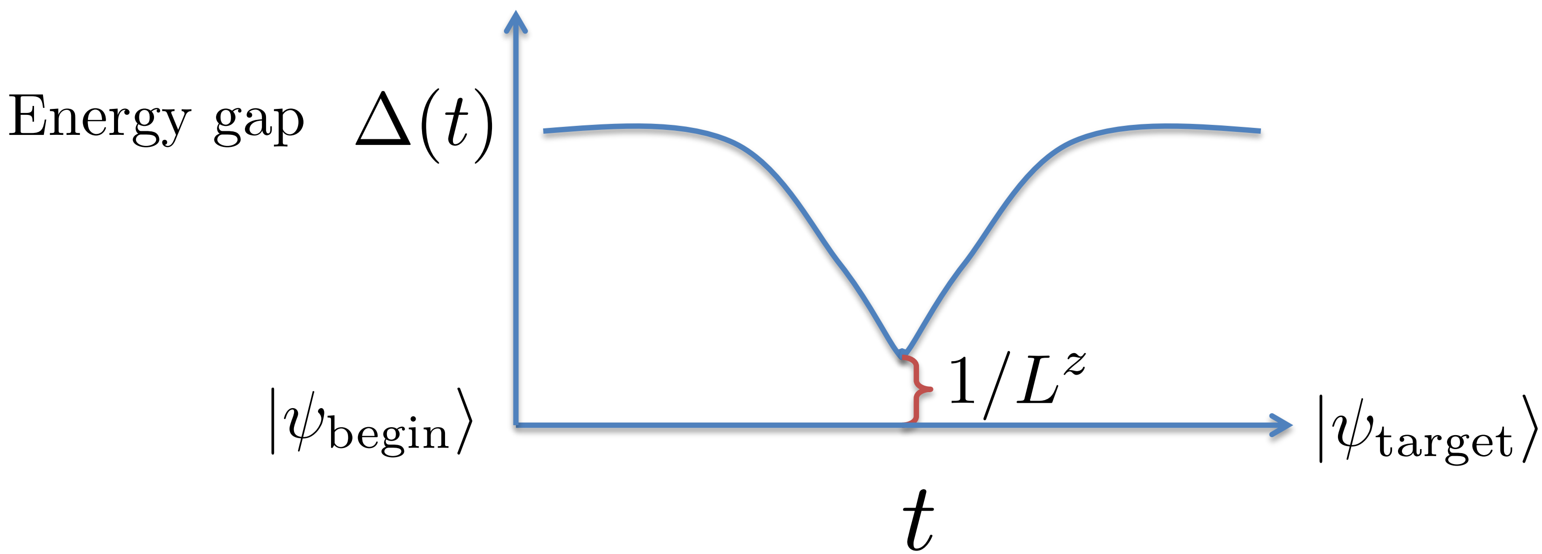
$$H_I = \sum_i (-1)^i S_z^i$$

$$H_F = J \sum_i \vec{S}_z^i \cdot \vec{S}_z^{i+1}$$



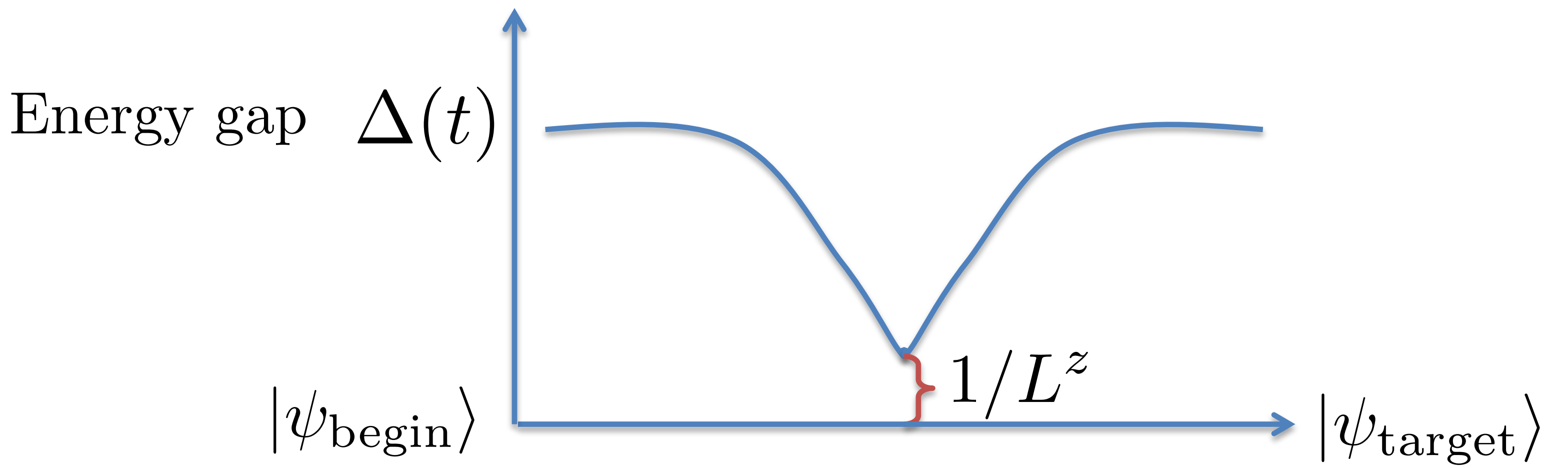
Adiabatic Approach is slow

$$H = (1 - \lambda(t))\, H_I + \lambda(t) H_F \qquad \lambda(t) : 0 \to 1$$



To maintain Adiabaticity: $\frac{d\Delta}{dt} \ll \Delta^2$

Adiabatic Approach is slow



"Quantum" Kibble-Zurek : $\frac{d\Delta}{dt} \ll \Delta^2 \qquad \Delta(t) = -vt$

$$t_{\text{out-of-eq.}} \sim 1/\sqrt{v}, \quad \Delta_{\text{out-of-eq.}} \sim \sqrt{v} \sim \frac{1}{L^z}$$

$$T_{\text{protocol}} \sim L^{2z} \rightarrow L^2$$

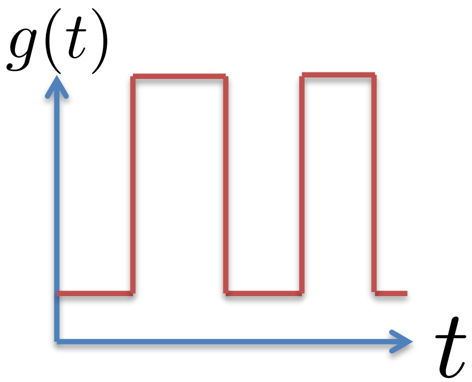
Non-adiabatic protocols: I

- Bang-Bang Protocols (Pontryagin Theorem)

Initial State: $|\psi(0)\rangle$

Given: $H_g(t) = \sum_\alpha g_\alpha(t)\, H_\alpha \quad , \quad g_\alpha^{\min} \leqslant g_\alpha(t) \leqslant g_\alpha^{\max}$

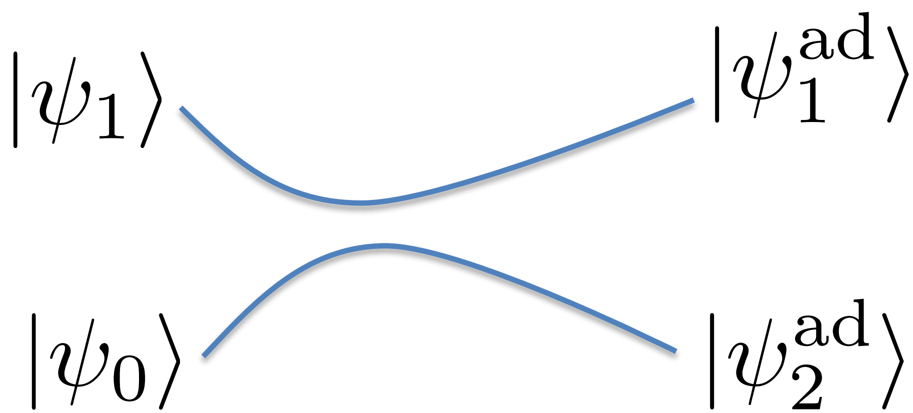
Minimize: $\langle\psi(T)|O|\psi(T)\rangle$



- If Hamiltonian is linear in g, solution is bang-bang. (Yang et al., PRX **7**, 021027)
  - Greatly constraints the search for optimal protocols, which is nice.
  - The bang-bang is particularly useful for small systems. (moving majoranas, Gmon qubits, a few spins…)
  - But the protocols have to be determined numerically and often provide little intuition as to their guiding principles.

Non-adiabatic protocols: II

- Counter-diabatic or transitionless driving



$$\tilde{H} = U^\dagger H U + i U^\dagger \partial_t U$$

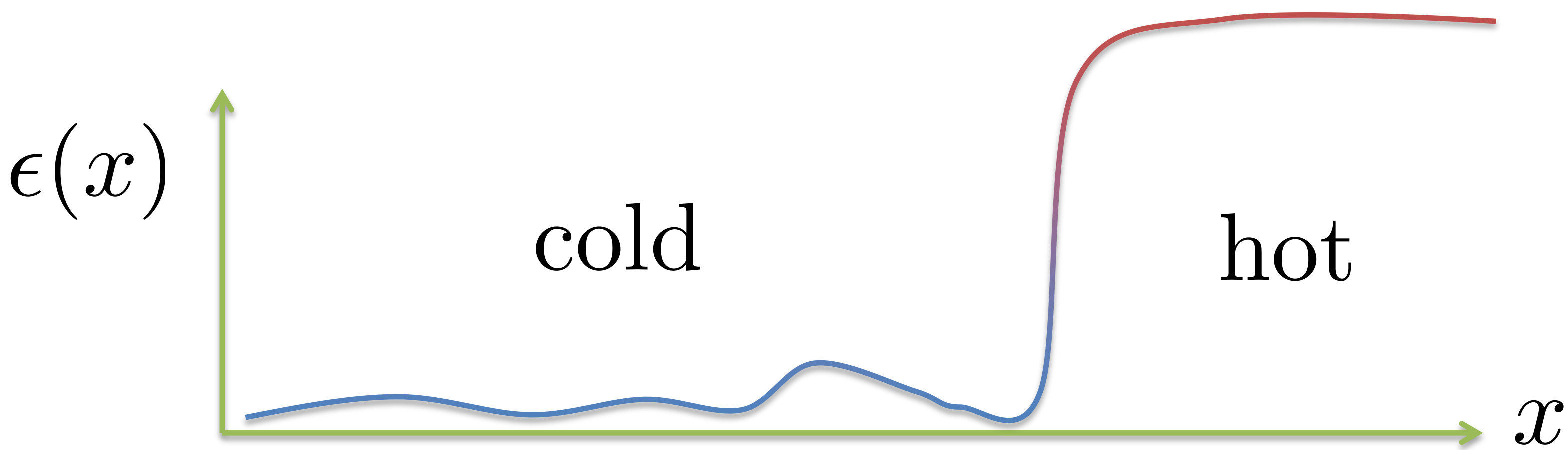
$$\Gamma \sim \left| \langle \psi_1^{\rm ad} | U^\dagger \partial_t U | \psi_2^{\rm ad} \rangle \right|^2$$

$$\boxed{H_{\rm cd} = -i U \partial_t U^\dagger}$$

Variational ansatz/ local CD driving:
D. Sels, A. Polkovnikov , PNAS, Volume 114, 20 (2017)

Non-Adiabatic Protocols: An alternative approach.

Maxim: Be not afraid to create excitations. But shepherd them wisely.



Intuition: Use symmetries exhibited by states

Focus in this talk: Relativistic/Lorentz Symmetry

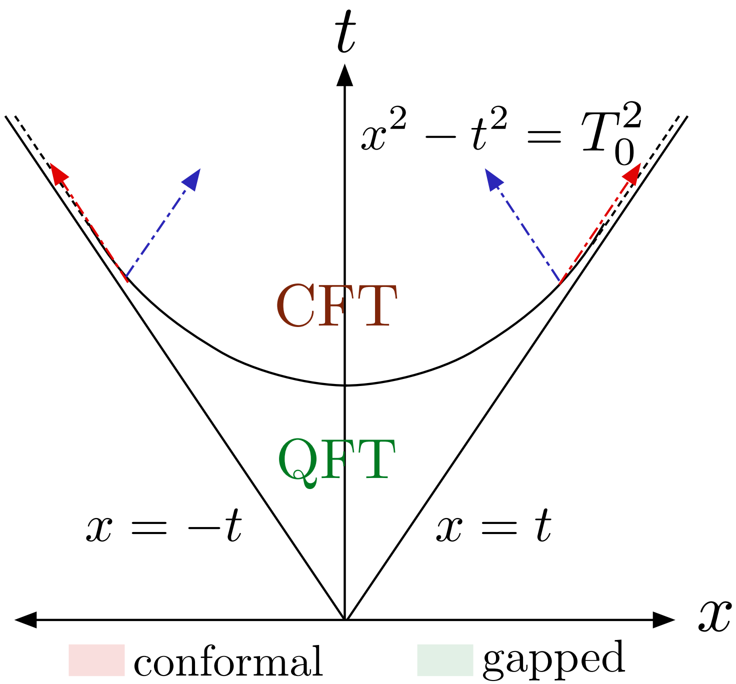
Why relativistic models?

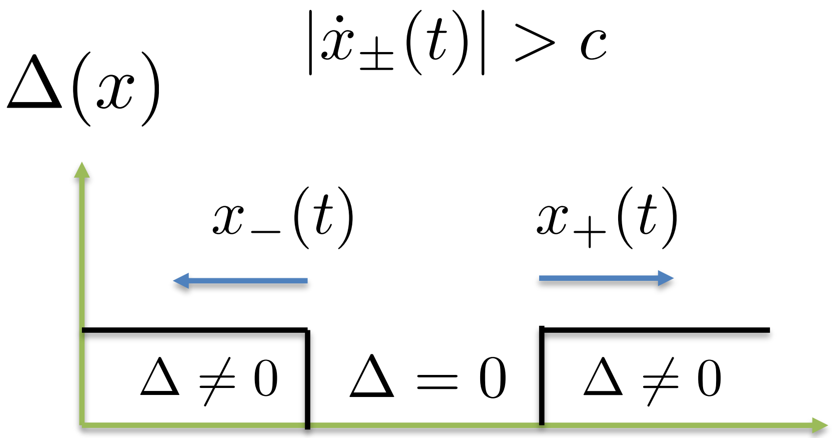
- Quantum Criticality in 1+1D or 2D: Conformal Field Theory which includes relativistic symmetry
  - One dimensional Systems of interacting fermions, bosons, gapless spin chains---Luttinger Liquids.

- Criticality in higher D: often associated with Goldstone modes described by $z = 1$
  - Example of interest: Half-filled Hubbard Model

Effective speed of light `c’, nothing to do with the real speed of light

Solution: Optimal Procotol

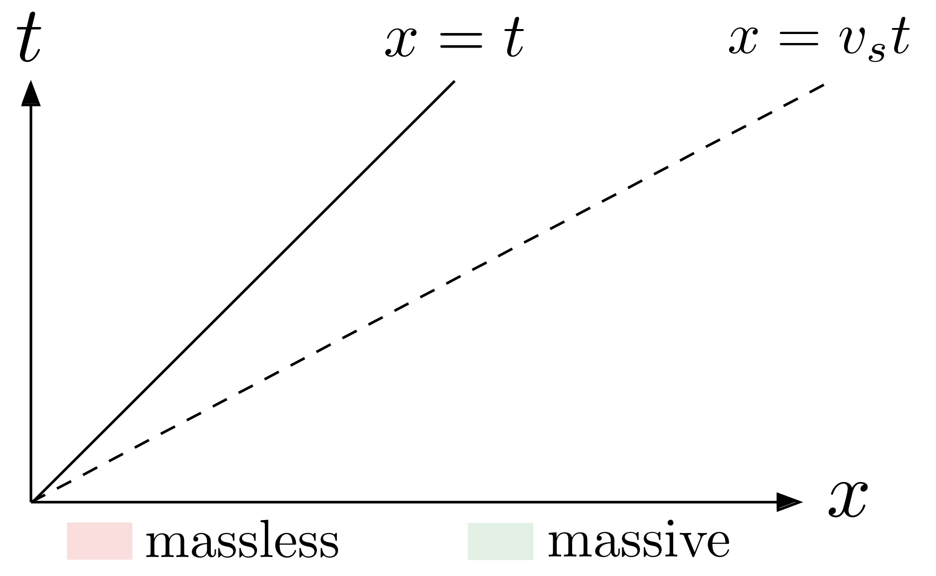




This is an entire class of quench protocols (delineated by T_0) for which the system relaxes to the ground state after the quench in time $t \sim \mathcal{O}[L]$.

The limit $T_0 \to 0$ is the most optimal among these. At any fixed time, the system is in the ground state of the CFT in the "red" space-time region.

Superluminal Fronts



$$S_m = \frac{K}{2}\int d^d x dt \left[(\partial_t \phi)^2 - (c\nabla\phi)^2 - m^2(x,t)\phi^2\right]$$

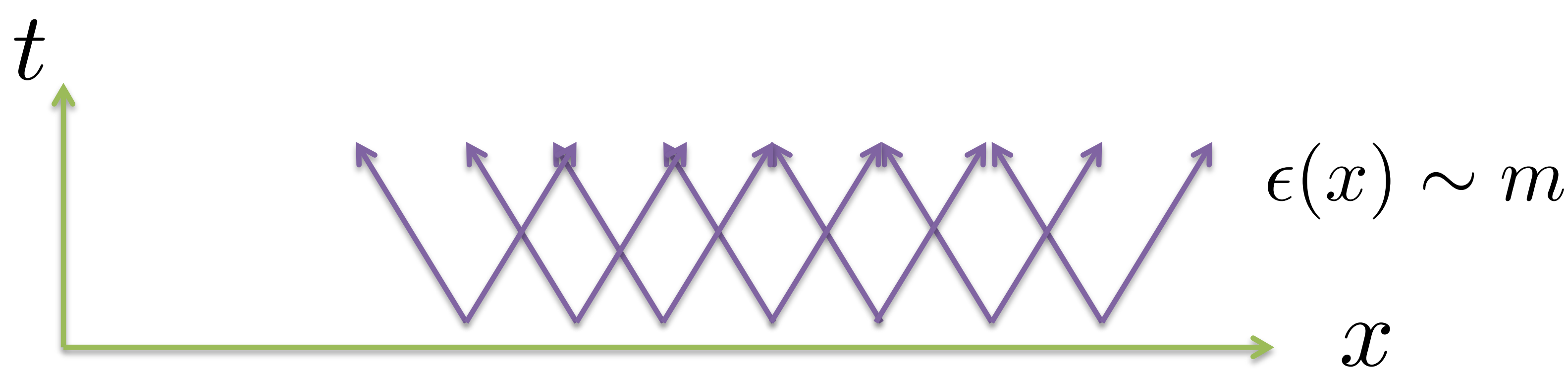
$$m(x,t) = m\,\theta(x_1 - v_s t)$$

Basis for solution: Intuitive analysis of a moving front

$$S(\phi) \;=\; S_{\text{gapless}}(\phi) \;-\; m^2(x,t)\phi^2$$

Initial state: $\left|\Omega\right\rangle$ vacuum of the massive theory

$$m(t) = m \to 0$$



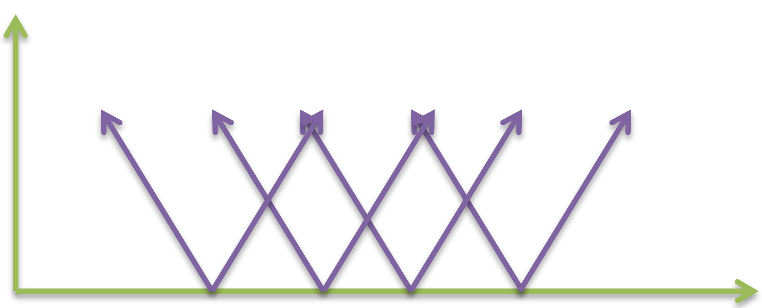
Basis for solution

The Shepherding of Excitations

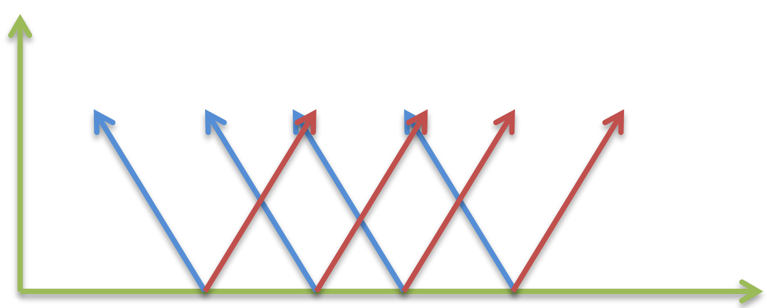
1. The vacuum state is invariant under Lorentz transformations

2. The quench front occurs at a fixed time in a Lorentz Boosted frame. This frame is moving at speed $u_s \equiv 1/v_s$ with respect to the lab frame.

Boosted Frame



Laboratory Frame



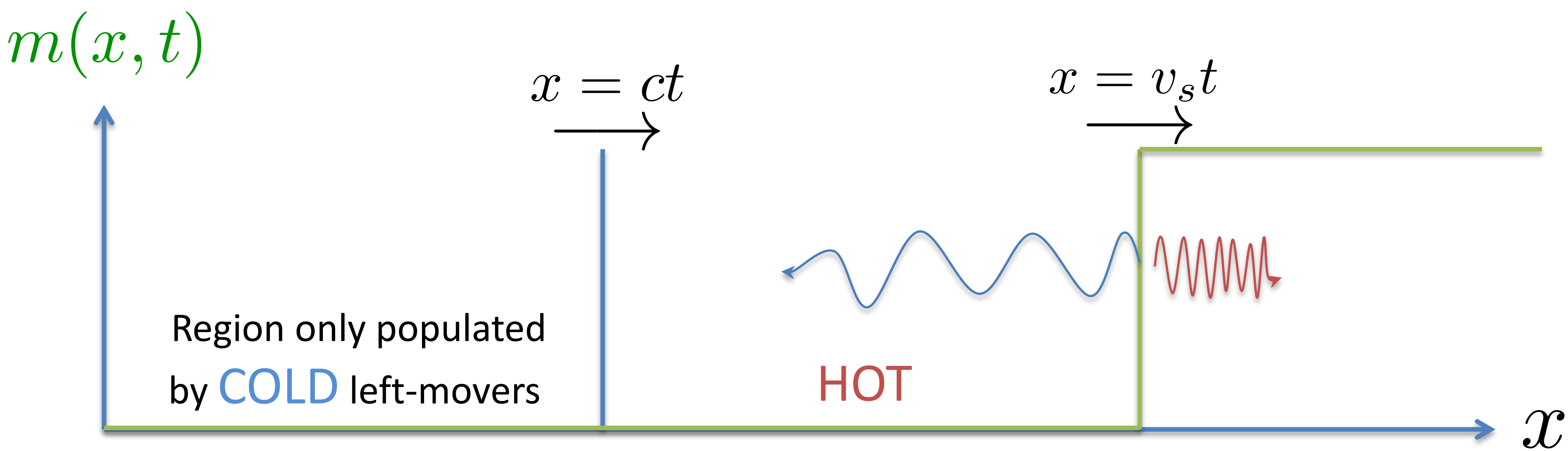
By itself, this chirality does not do anything: if any point point in space, we have hot and cold movers, it will not lead to physical separation of hot and cold regions.

Basis of solution

The Shepherding of Excitations

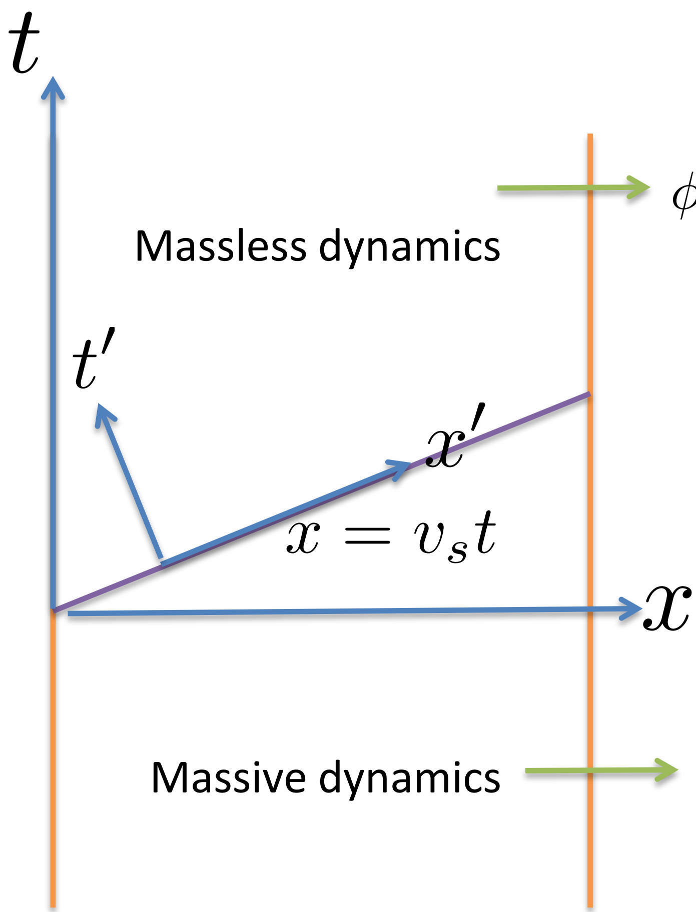
By itself, this chirality does not do anything: if any point point in space, we have hot and cold movers, it will not lead to physical separation of hot and cold regions.

But! A boundary changes everything.



How to solve $m^2(x,t) = m^2\Theta(x - v_s t)$

Important: $v_s > c$



$$\phi(x, t > x/v_s) = \frac{1}{\sqrt{K}} \sum_n \left[\gamma_n(x,t) b_n + \gamma_n^*(x,t) b_n\right]$$

$$\gamma_n(x,t) = \sum_m \left[\alpha_{n,m}^* u_m(x,t) + \beta_{n,m} u_m^*(x,t)\right]$$

$$u_n \sim e^{i\boldsymbol{k}.\boldsymbol{r} - i\omega_{\boldsymbol{k}} t}$$

$$\phi(x,t) = \frac{1}{\sqrt{K}} \sum_n \left[b_n v_n(x,t) + b_n^\dagger v_n^*(x,t)\right]$$

$$\forall n \; b_n|\Omega\rangle = 0 \qquad v_n \sim e^{i\boldsymbol{k}.\boldsymbol{r} - i\Omega_{\boldsymbol{k}} t}$$

Invariant inner product

$$(\phi_a, \phi_b) = -i \int d^d\boldsymbol{r}\sqrt{g}n_\mu J^\mu_{(a,b)}(\boldsymbol{r}) \quad \text{where, } J^\mu_{(a,b)} = (\phi_a \nabla^\mu \phi_b^* - \phi_b^* \nabla^\mu \phi_a).$$

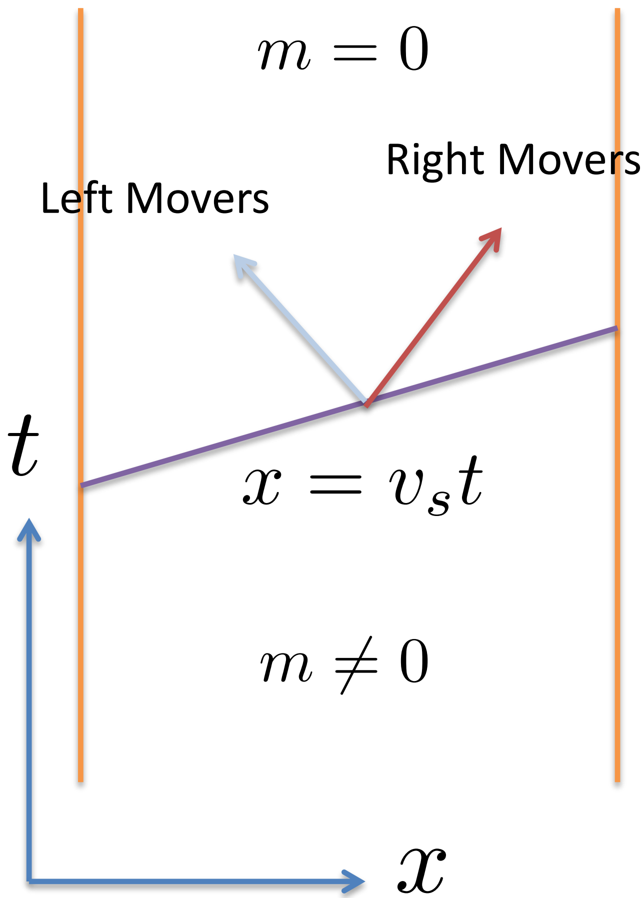
$$\nabla_\mu J^\mu_{(a,b)} = 0$$

Can be evaluated at any space-like hypersurface:

$$(\phi_a, \phi_b) = -i \int_{-L/2\gamma_s}^{L/2\gamma_s} dx' \left[\phi_a \partial_{t'} \phi_b^* - \phi_b^* \partial_{t'} \phi_a\right]\Big|_{x',t'=0}$$

1. Normalizing a complete set of modes according to the Klein-Gordon inner product assures that the field operators satisfy the correct commutation relations
2. One can expanding a given set of modes in terms of another set of modes using this inner product. Importantly, the modes will automatically satisfy the continuity conditions on the Amplitude and its space/time derivates

Chiral Excitation: Result for finite v_s and $\tau = 0$



$$N_k \omega_k \xrightarrow{k \to 0} \frac{m}{4\eta(\theta)}$$

$$\eta(\theta) \equiv \gamma_s \left(1 - u_s \cos\theta\right) \qquad \gamma_s = \frac{1}{\sqrt{1 - u_s^2}}$$

Instantaneous Case:

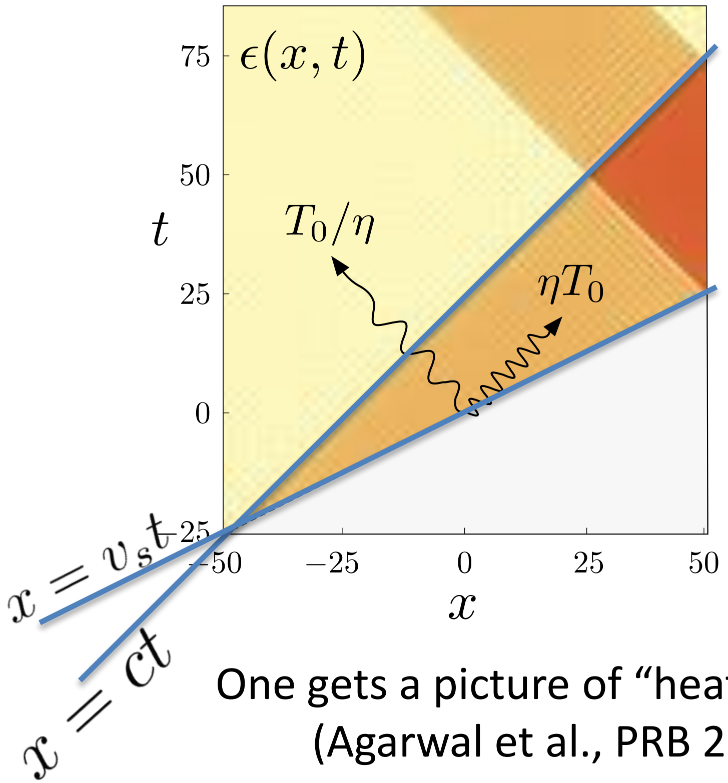
$$u_s = 1/v_s = 0 \;\; ; \;\; \eta(\theta) = 1$$

1D finite u_s Case:

Right Mover: $\eta(\pi) = \sqrt{\dfrac{1 - u_s}{1 + u_s}} \equiv \dfrac{1}{\eta_0} < 1$

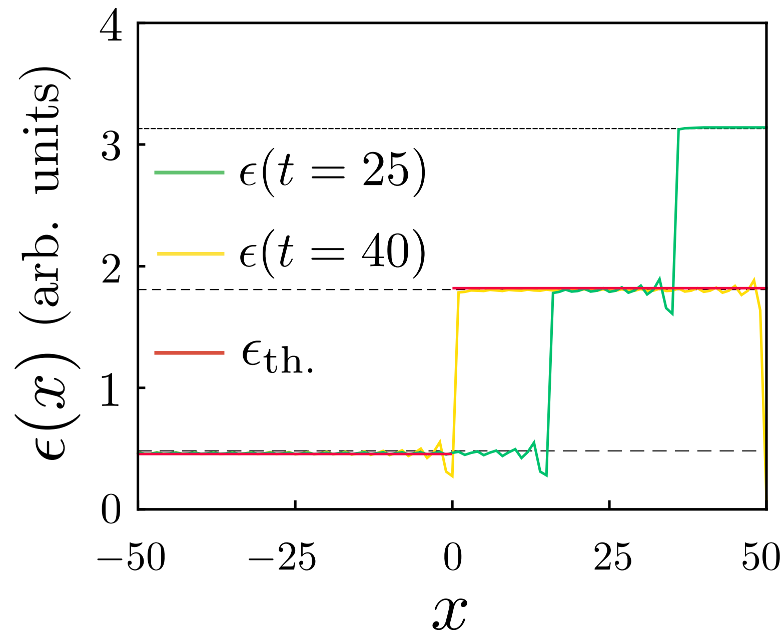
Left Mover: $\eta(0) = \sqrt{\dfrac{1 + u_s}{1 - u_s}} = \eta_0 > 1$

Cooling in dimension d=1



One gets a picture of "heat waves".
(Agarwal et al., PRB 2017)

These waves travel left (x = -t), right (x = t) with energies that are different in perpetual non-equilibrium motion/steady state.



The exact energies agree with the theoretical prediction

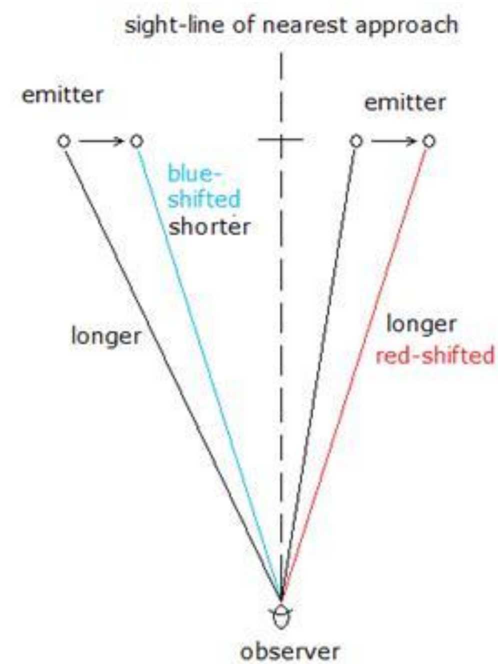
Cooling in higher dimensions

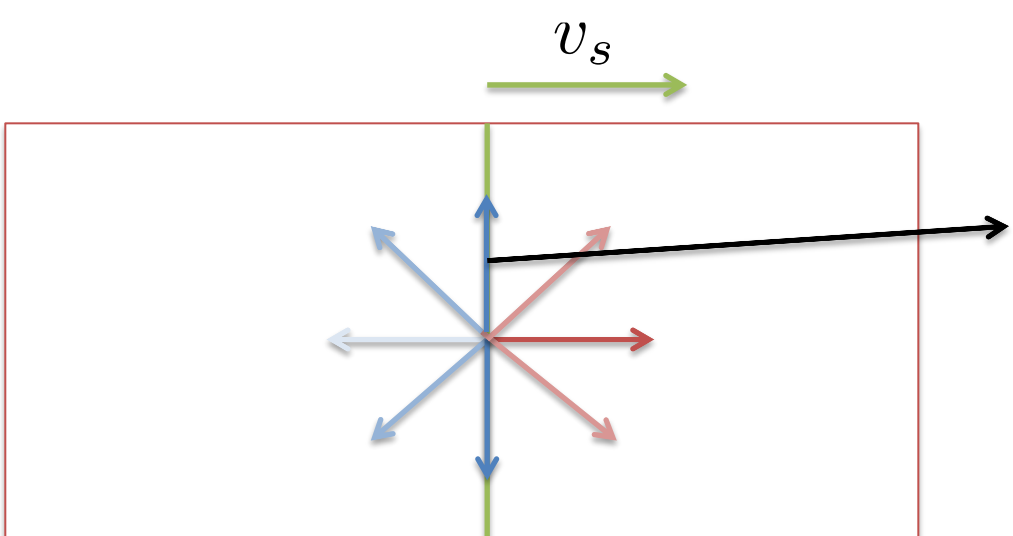
No distinction between “hot” and “cold” regions.

Classical Doppler Effect: No rare-faction/dilation for perp. light.

Relativistic Doppler Effect: A factor of γ_s red-shifts all light regardless of directionality.

$$\eta(\pi/2) = \gamma_s$$

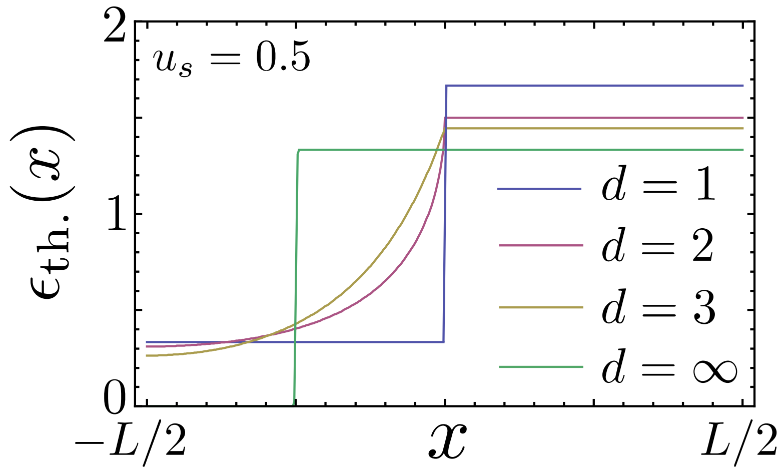


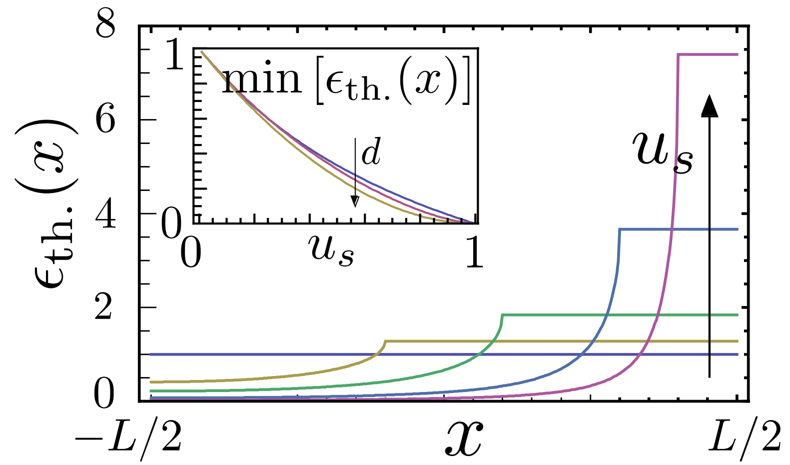


The bulk of the radiation is perpendicular, in higher dimensions.

But due to relativistic effects, it is cold too.

Cooling for Bosons: higher dimensions



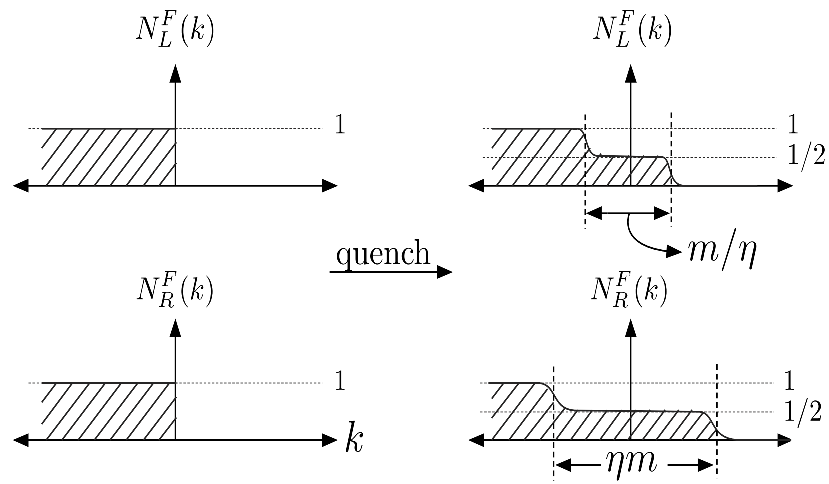


d > 1: No sharp distinction between odd and even waves.

d = 2:
But as v->c, cooling becomes just as good.

Some other points:

Cooling protocol in free fermions manifests as chirally controlled "chemical potential" as opposed to an effective temperature.

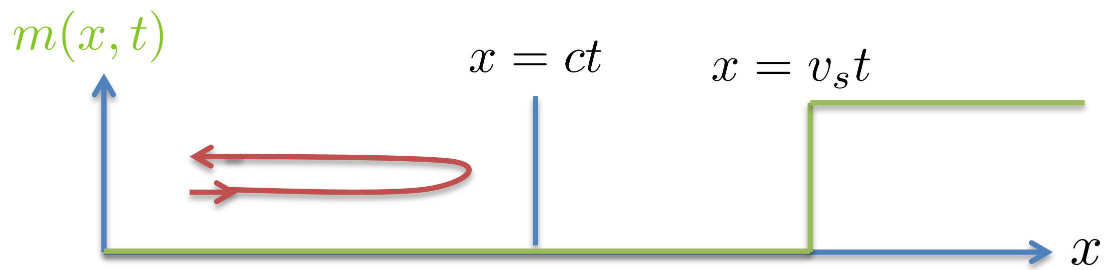


One can also solve exactly for the adiabatic Protocol in the Gaussian theory for the case: $\Theta(-t) \to f\left(\frac{-t}{\tau}\right) = \frac{1}{2} - \frac{1}{2}\tanh\left(\frac{t}{\tau}\right)$

An exponentially small final energy density requires a time ~ $O(L^2)$.

Application to “real” world model

- What is the role of interactions? Does it totally spoil the effect?



Alarming: Total energy generated in the quench process (1D) is

$$\propto \left[u_s \frac{1}{\eta^2} + (1 - u_s)\frac{1}{2}\left(\eta^2 + \frac{1}{\eta^2}\right)\right] = 1$$ → Independent of the quench speed!

Which means that all cooling is due to spatial segregation and could be spoiled by interactions!

- What is the role of UV cut-offs that break the Lorentz-invariance?

Solution: Adiabaticity + Supersonic Quench

New Quench Protocol: $\Theta(x - v_s t) \to \frac{1}{2} + \frac{1}{2}\tanh\left(\frac{x - v_s t}{v_s \tau}\right)$

Quench is homogeneous in boosted frame: $f\left(\frac{x - v_s t}{v_s \tau}\right) \to f\left(-\frac{t'}{\gamma_s \tau}\right)$

Effective time-scale is $\gamma_s \tau$

Again, calculate excitations in Lorentz-boosted frame: $\tilde{N}_{\boldsymbol{k}} \sim e^{-2\pi\omega_{\boldsymbol{k}}\gamma_s \tau}$

Then Doppler-shift the momenta to get populations in the laboratory frame.

Cut-off is dependent on $\gamma_s \tau$, not just τ.

$$k_c \ll \frac{1}{\gamma_s \tau} \xrightarrow{v_s \to c} 0$$

Time-Scale + Adiabatic Quench

Cut-off for excitations is controlled by γ_s

$$\epsilon_\theta(\tau) \propto \int^{\frac{m}{\gamma_s \eta(\theta)}} k^{d-1} dk\, \omega_{\boldsymbol{k}} N_{\boldsymbol{k}} \propto \frac{\tau^{-1}}{\gamma_s^d} \frac{1}{\eta(\theta)^{d+1}} \frac{1}{L_{1/\tau}^d}$$

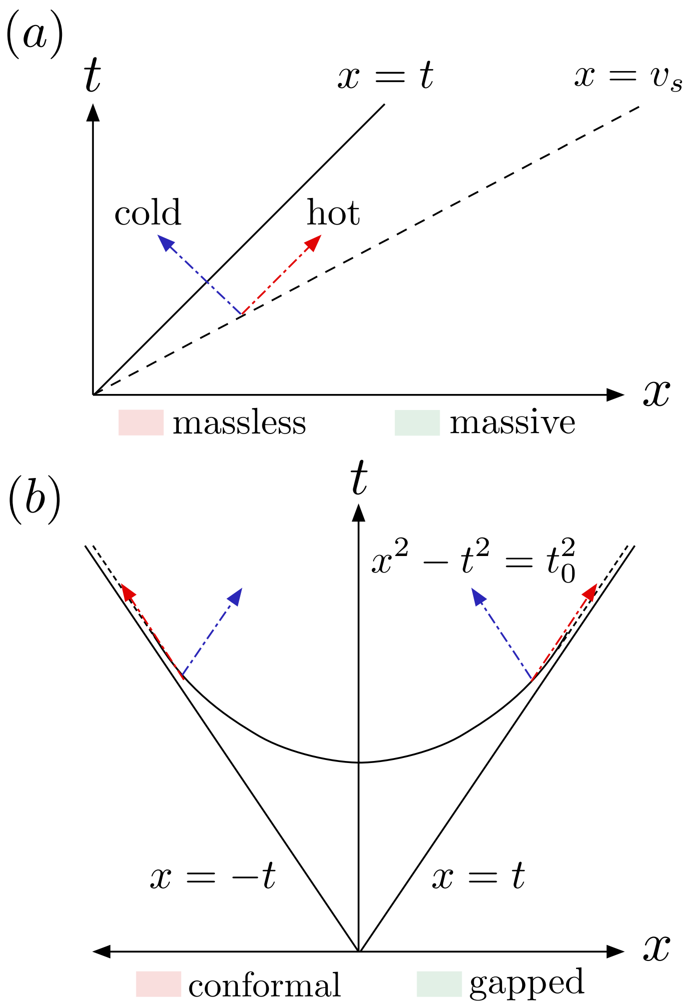
The total energy dumped in the entire quench process now scales to zero as

$$\gamma_s \to \infty$$

$$\epsilon_{\mathrm{th.},\tau=m^{-1}} \approx \frac{\epsilon_{\mathrm{th.},\tau=0}}{\gamma_s^d}$$

1. The supersonic quench *ENHANCES* adiabatic effects
2. The *TOTAL* energy dumped into the system is reduced -> good for interacting systems.
3. The *UV EFFECTS* can be addressed by simply choosing a large enough $\tau \gg \Lambda^{-1}$

Cooling extended to general CFTs. Dilation cooling



$$x = T_0 e^{\eta} \sinh(\xi)$$

$$t = T_0 e^{\eta} \cosh(\xi)$$

$$dx^2 - dt^2 = T_0^2 e^{2\eta} \left(d\eta^2 - d\xi^2\right)$$

$$g_{ab} \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$g_{\mu\nu} \equiv \begin{pmatrix} T_0^2 e^{2\eta} & 0 \\ 0 & -T_0^2 e^{2\eta} \end{pmatrix}$$

$$\text{Quench} : t^2 - x^2 = T_0^2 \Rightarrow \eta = 0$$

Cooling is a purely geometrical effect.

Translations in $\xi \to \xi + a$ is an isometry of Minkowski space-time.

Its just the Lorentz boost:

$$(t, x) \to (t \cosh a + x \sinh a, t \sinh a + x \cosh a)$$

Thus, vacuum two-point correlations must be functions of $\xi - \xi'$ and one-point correlations, such as stress-energy tensor must be independent of ξ

Thus, this is a symmetry of the pre-quench QFT, post-quench CFT, and the quench-trajectory $t^2 - x^2 = T_0^2$

Tracelessness further implies the form: $\langle T_{\mu\nu}(\eta, \xi)\rangle_{\rm CFT} = \begin{pmatrix} A & B \\ B & A \end{pmatrix}$

Where A, B are functions of η only.

Cooling is a purely geometrical effect.

$$\langle T_{\mu\nu}(\eta,\xi)\rangle_{\rm CFT} = \begin{pmatrix} A & B \\ B & A \end{pmatrix}$$

If we further assume parity symmetry, $x \to -x, \ \text{or}, \ \xi \to -\xi$

$$\langle T_{\mu\nu}(\eta,\xi)\rangle_{\rm CFT} = \begin{pmatrix} A(\eta) & 0 \\ 0 & A(\eta) \end{pmatrix}$$

Conservation further implies: $\nabla^{\mu} T_{\mu\nu} = 0, \Rightarrow \ \partial_{\eta} A(\eta) = 0$

Thus,

$$\langle T_{ab}(t,x)\rangle_{\rm CFT} = \frac{f(mT_0)}{(t^2-x^2)^2} \begin{pmatrix} t^2+x^2 & 2xt \\ 2xt & t^2+x^2 \end{pmatrix}$$

Cooling is a purely geometrical effect.

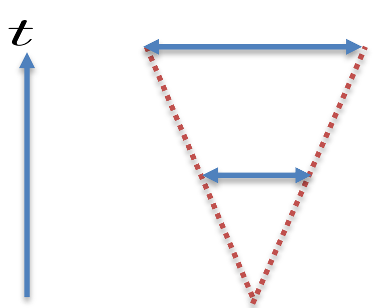
$$\langle T_{ab}(t,x)\rangle_{\mathrm{CFT}} = \frac{f(mT_0)}{(t^2-x^2)^2}\begin{pmatrix} t^2+x^2 & 2xt \\ 2xt & t^2+x^2 \end{pmatrix}$$

$$f(x) \sim x^{\epsilon}, x \to 0, \epsilon > 0$$

1. Thus, as $T_0 \to 0$, the hyperbola becomes a perfect lightcone `V’, a symmetric copy of the perfect luminal quench considered previously. The energy density goes to zero everywhere except on the light-cone! Thus, despite interactions, the previous picture holds.

2. Even finite T_0 is a useful protocol; because eventually, energy density for finite x tends to zero everywhere (except lightcone) as ~1/t^2.

$$d\xi \sim t dx$$



1. Momentum in `x’ direction ~ $1/t$
2. `Volume’ element in `x’ direction ~ $1/t$

3. Energy Density: d = 1: $1/t^2$

Argument for cooling in higher d

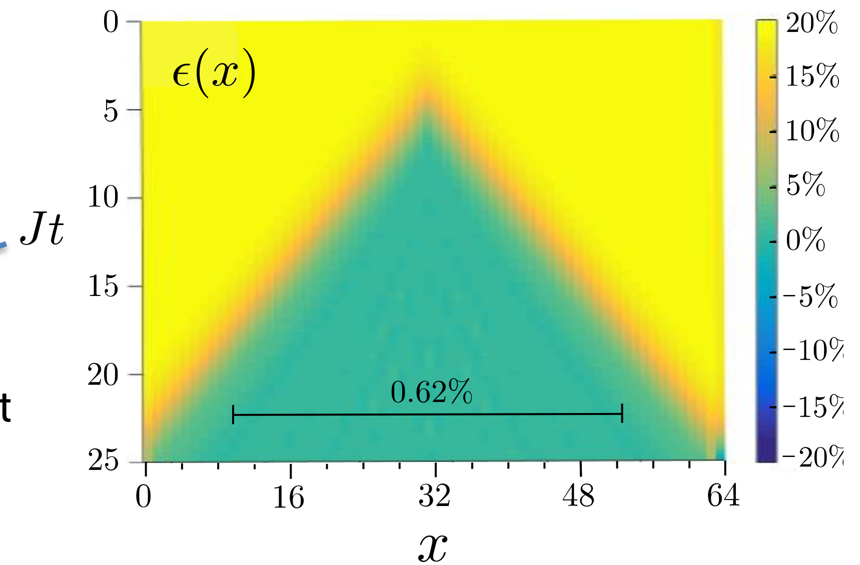
- The stress-energy tensor is unfortunately not constrained by symmetry arguments and conservation laws alone.

- If we constrain the stress-energy tensor to be isotropic in space (which is hard to assume in this setting), say due to interactions, at late times, then we can constrain the stress-energy tensor and it predicts cooling as $\sim 1/t^{1+1/d}$

- The Gaussian theory does exhibit cooling in higher d.

t-DMRG Simulations on the Heisenberg model.

$$H_{\rm XXZ} = \sum_x (-1)^x h(x,t) + J \sum_x \boldsymbol{S}_x \cdot \boldsymbol{S}_{x+1}$$

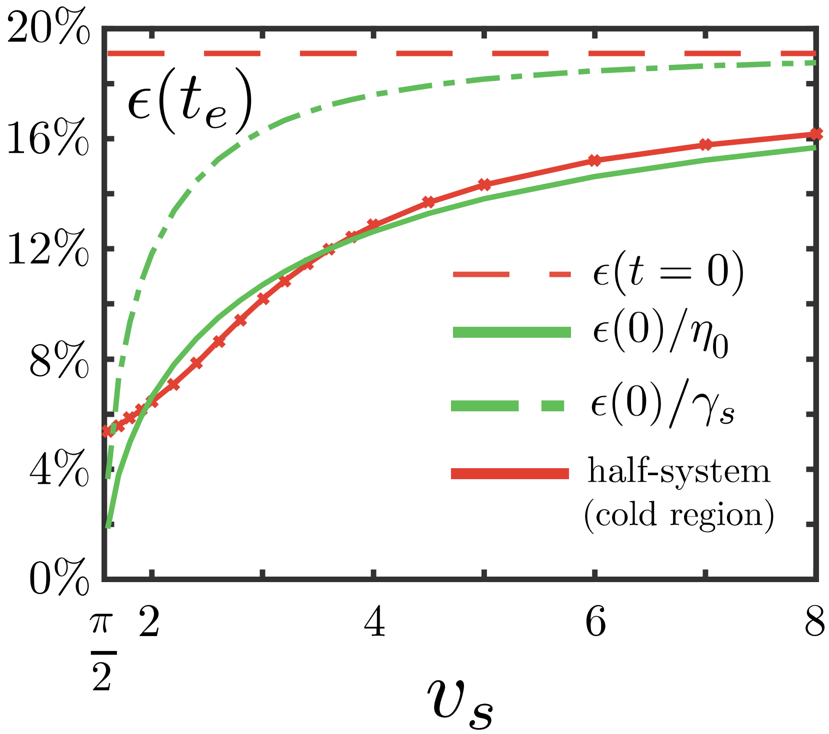


The initial state is ~20% of the band-width above t state of the gapless model. The final state is ~0.62% above G.S.

A linear `adiabatic' ramp taking the same time produces a state ~2.4% above G.S.

The competitive advantage of the Lorentz cooling protocol should get better with L

t-DMRG Simulations on the Heisenberg model.



Expectation: Avg. Energy Density

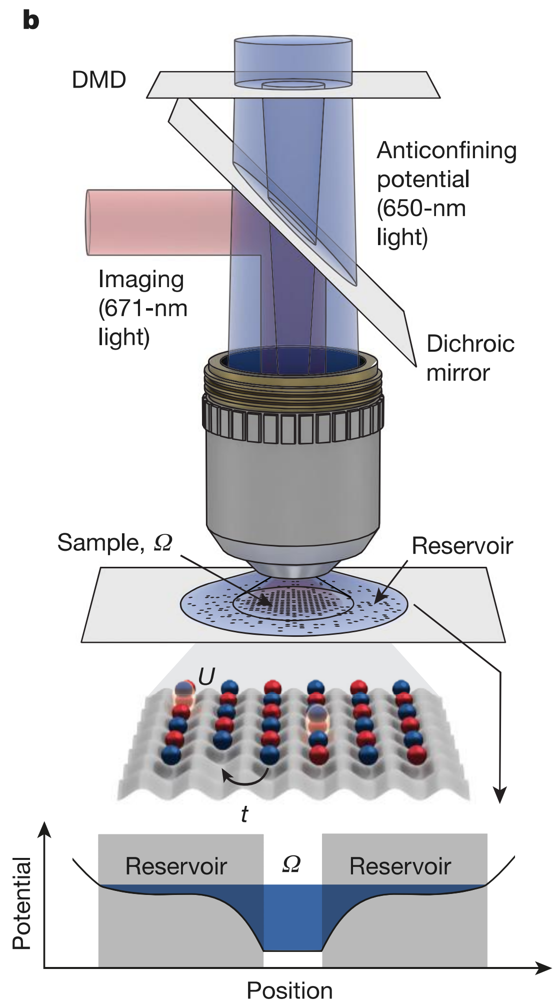
$$\epsilon(0)/\gamma_s$$

Cold region energy density

$$\epsilon(0)/\eta_0$$

Middle half system should have energy density somewhere in between

Can we use this to create low-energy states in the Hubbard model in cold atoms?



1. Create a fermionic Mott insulators in DEEP wells

Labs of T. Esslinger, W. Bakr, M. Greiner, M. Zwielein, I. Bloch

1. Single site addressing allows one to flip spins and create a classical Neel state. ~classical Neel state preparation

For Bosonic Mott insulator: Weitenberg et al., Nature 2011

3. Perform hyperbolic quench/enhance tunneling.

(Alternately, can create a local mass by dimerization…)

Conclusions

- Space-time quenches can be used to create ground states of critical theories parametrically faster than adiabatic methods.
- Argument is extremely general for $d = 1$ where hyperbolic quenches are shown to be optimal.
- One can also start from excited states (at least in the Gaussian case) and show that the quench protocol generates no entropy except at the singular lines $x = +/- t$.
- It would be great to numerically study such quenches in higher dimensions.