Advantures in synthetic dimensions: New schemes for quantum control

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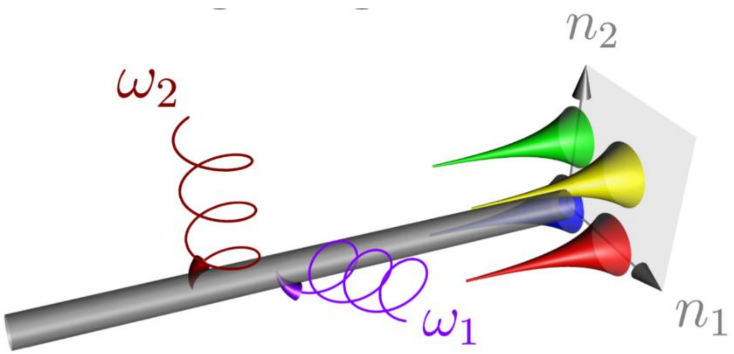

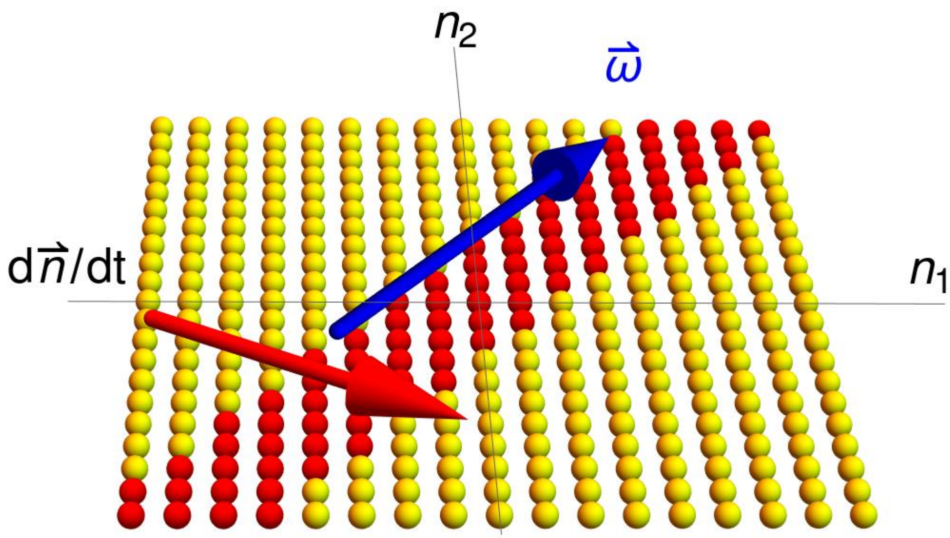

INSTITUTE FOR QUANTUM INFORMATION AND MATTER

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The potential in periodic drive

Periodic drive ⟷ **Extra dimensions**

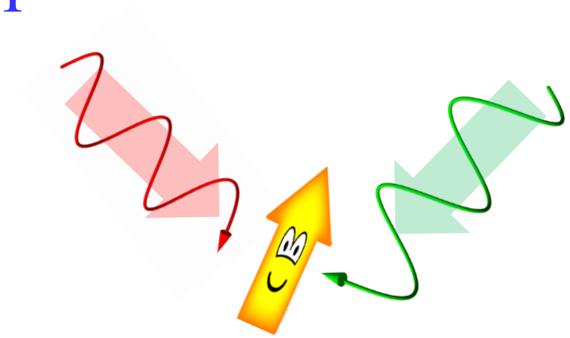
Synthetic dimensions

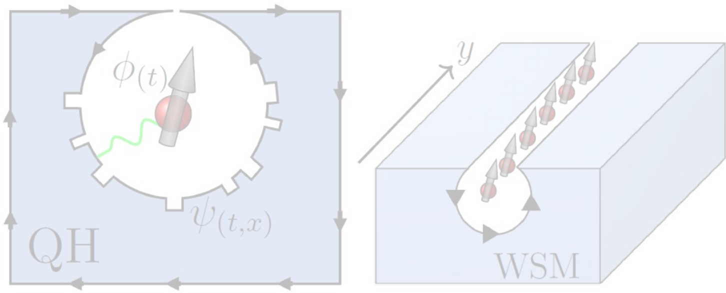
↓

New control paradigms

The Potential in periodic drive

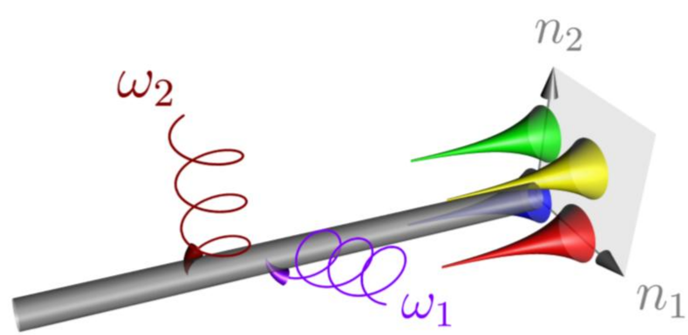
- Topological frequency conversion.





- History dependence for synthetic control

- Majorana multiplexing



New control paradigms

Periodic drives and synthetic dimensions

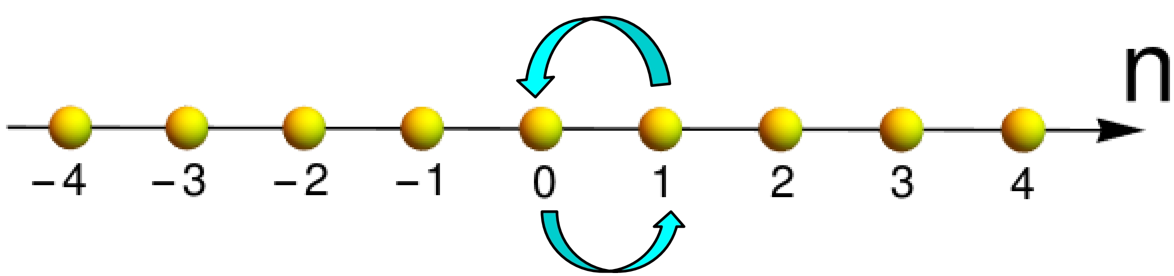
- Simple drive: $$H = \hat{H}_0 + \hat{V}\left(e^{i\omega t} + e^{-i\omega t}\right)$$

- Time evolving wave function:

$$|\psi(t)\rangle = \sum_n \exp(-in\omega t)|\psi_n(t)\rangle$$

Time dependent Schroedinger equation:

$$i\frac{\partial}{\partial t}|\psi_n(t)\rangle = \hat{H}_0|\psi_n\rangle + \hat{V}|\psi_{n+1}\rangle + \hat{V}|\psi_{n-1}\rangle - \omega n|\psi_n\rangle$$



Synthetic dimension – like a tight binding model

See also: Nathan Goldman; Shanhui Fan; Iacoppo Carussoto....

Floquet-Bloch correspondence and oscillations

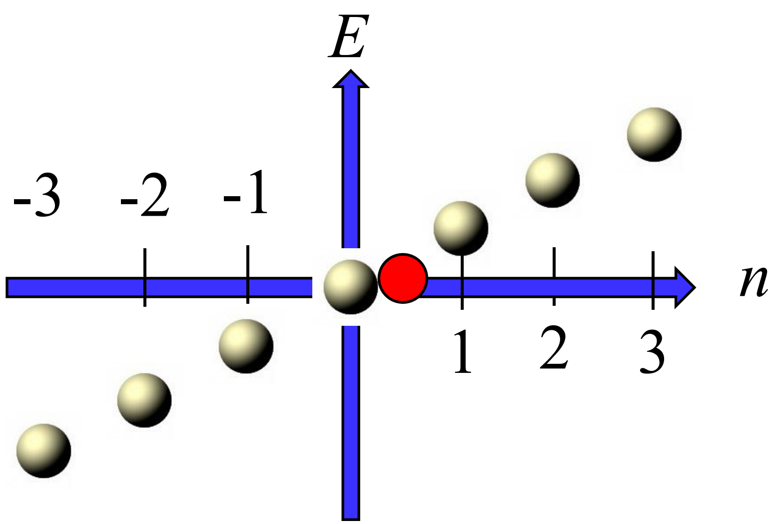
- Phase – momentum analogy:

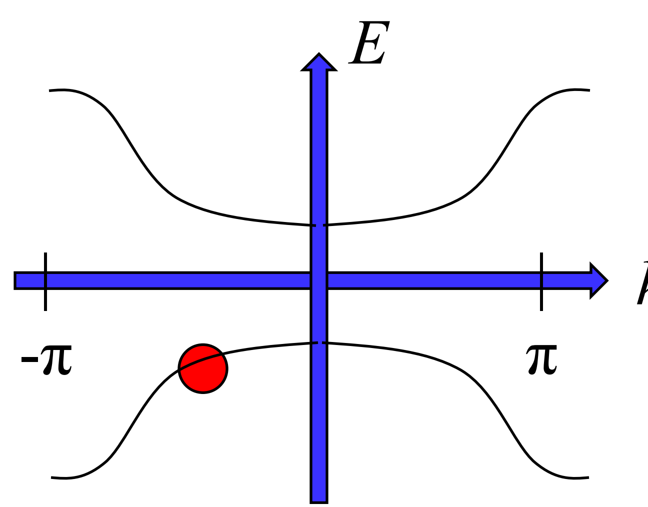
$$k(t) = \omega t \quad \Rightarrow \quad H = \hat{H}_0 + \hat{V}\left(e^{ik} + e^{-ik}\right) - \omega n$$

$$\hat{n} = -i\frac{\partial}{\partial k} \qquad \left[\hat{k}, \hat{n}\right] = -i$$

$$\omega << \| H \|$$

- Spin-1/2 example: $H = h\sigma^z + V\left(e^{ik} + e^{-ik} - 2\right)\sigma^x - \omega n$





- Energy conservation = Wannier-Stark localization

More dimensions?

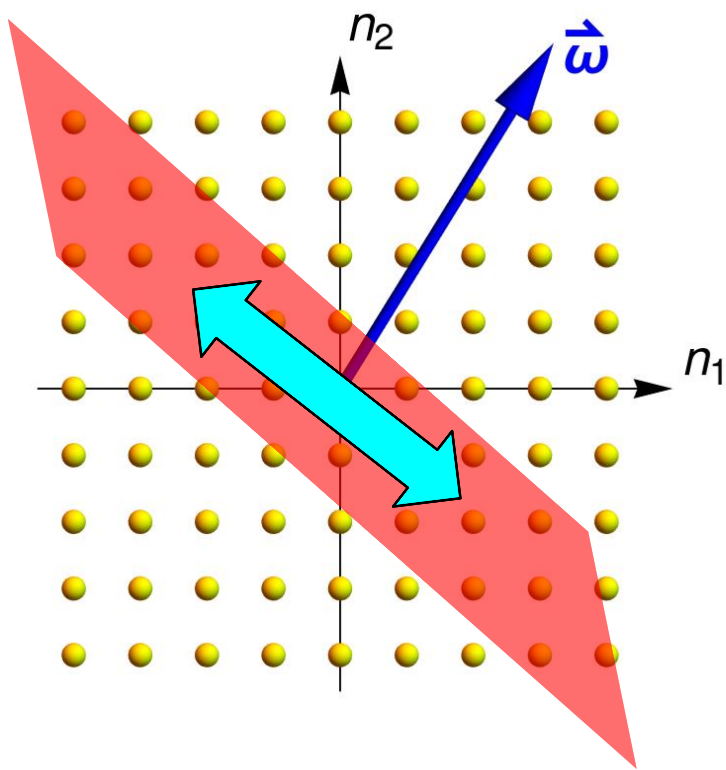
- Two ***incommensurate*** drives:

$$H = H_0 + \hat{V}_1 \cos(\omega_1 t) + \hat{V}_2 \cos(\omega_2 t)$$

$$|\psi(t)\rangle = \sum_n \exp(i n_1 \omega_1 t + i n_2 \omega_2 t) |\psi_{n_1,n_2}(t)\rangle$$

$$H = H_{hopping} - n_1\omega_1 - n_2\omega_2$$

Motion restricted by energy conservation



2D Topology in 0d system

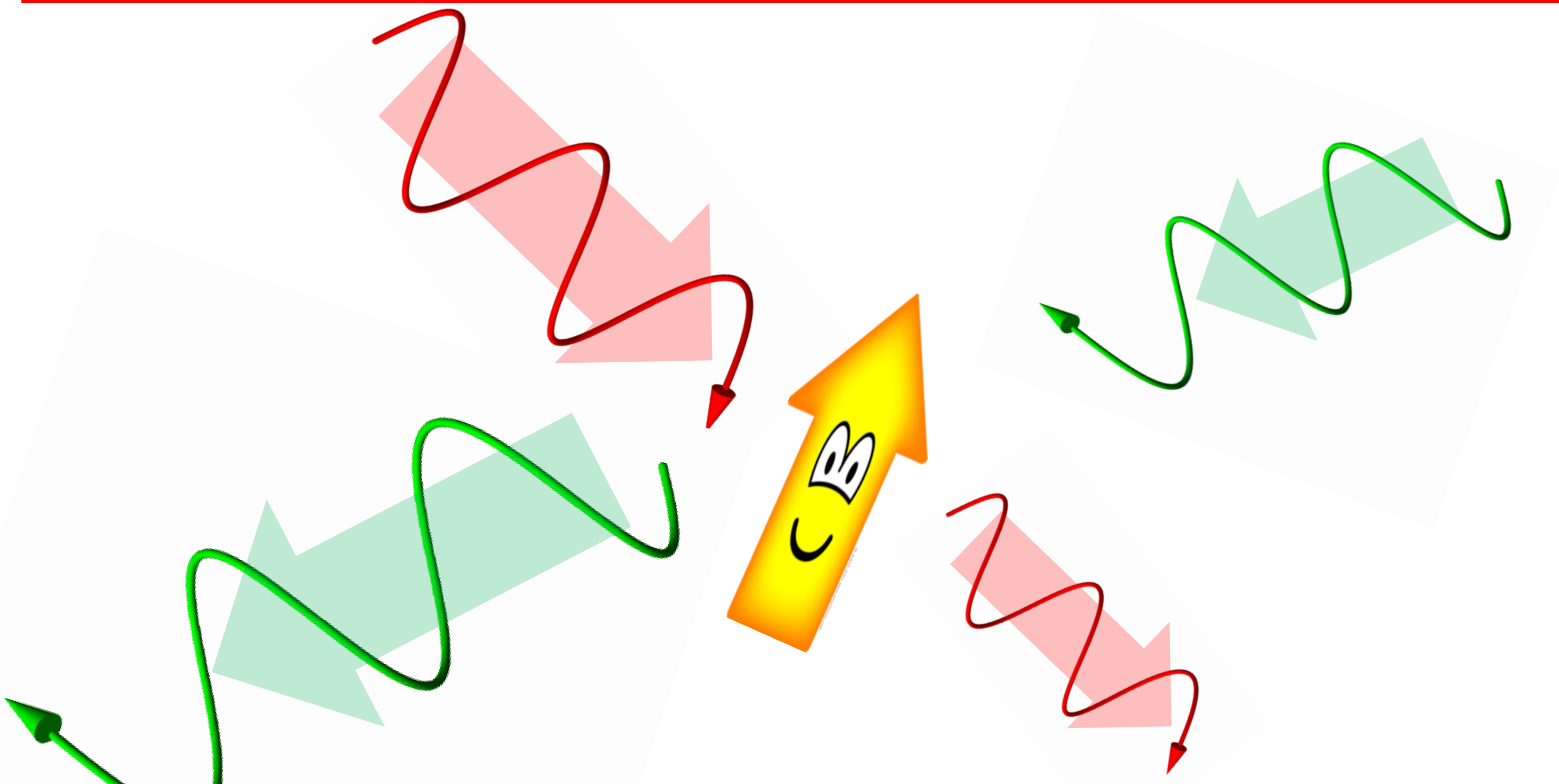
- Use BHZ band structure:

$$H = v\sigma^x \sin(k_1) + v\sigma^y \sin(k_2) + \left[m - b\cos(k_1) - b\cos(k_2)\right]\sigma^z$$

Topological for $m<2b$

- Shift to double driven spin:

$$H = v\sigma^x \sin(\omega_1 t) + v\sigma^y \sin(\omega_2 t) + \left[m - b\cos(\omega_1 t) - b\cos(\omega_2 t)\right]\sigma^z$$



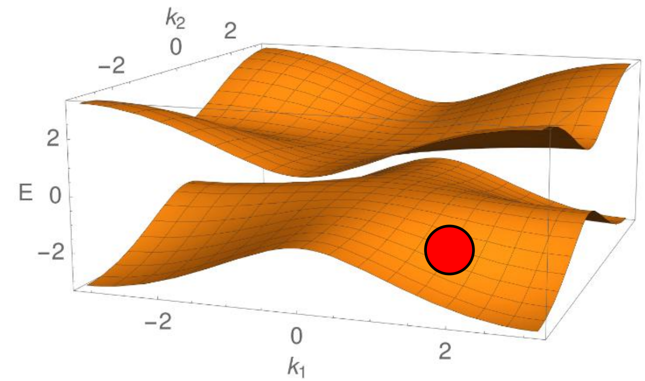
Semiclassical motion

- Equations in motion:

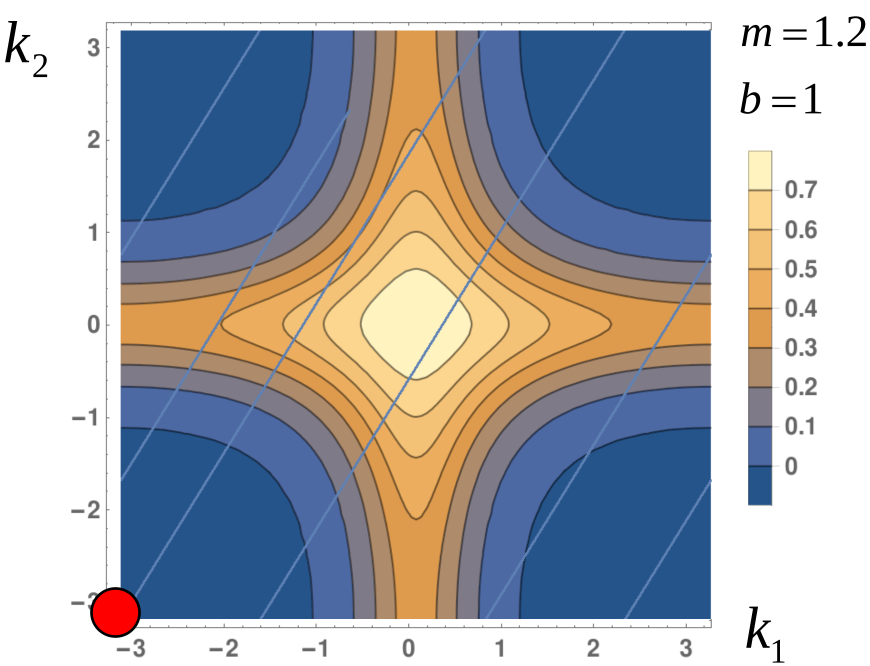
$$\Omega(k_1,k_2)=\nabla_k \times A_k$$

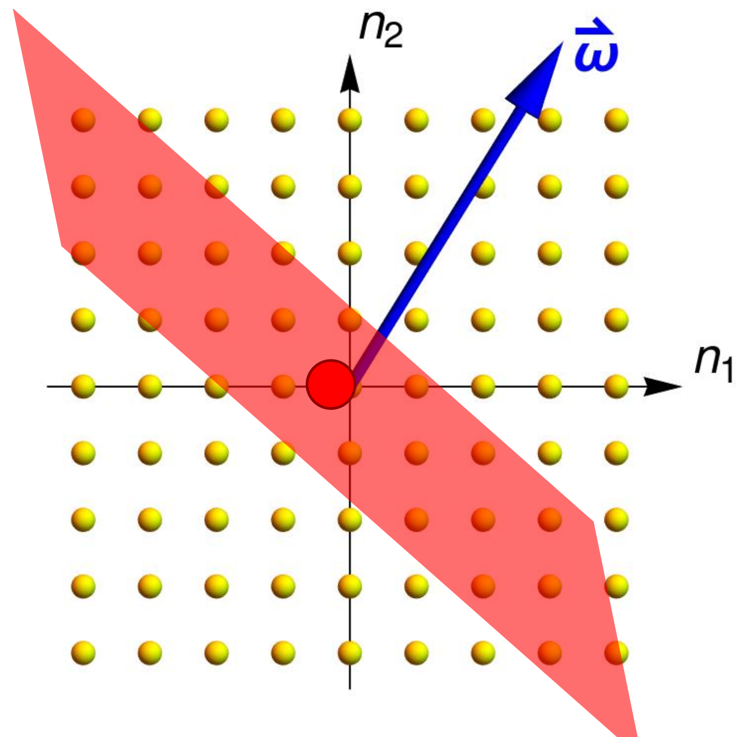
$$A_k=-i\langle\psi_k|\nabla_k|\psi_k\rangle$$

$$\frac{dn}{dt}=\langle\nabla_k\varepsilon_k\rangle+\langle\Omega_k\rangle\times\frac{dk}{dt}$$



- BHZ Berry curvature:





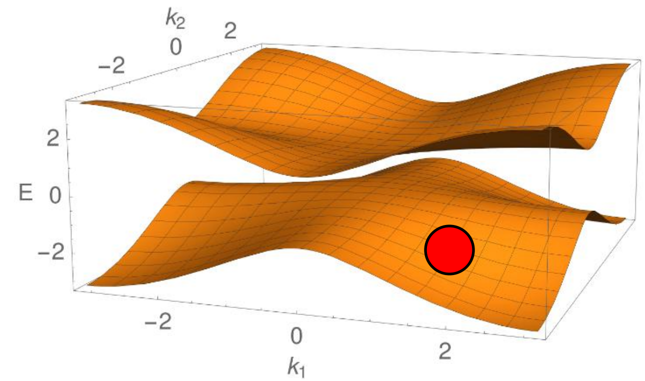
Semiclassical motion

- Equations in motion:

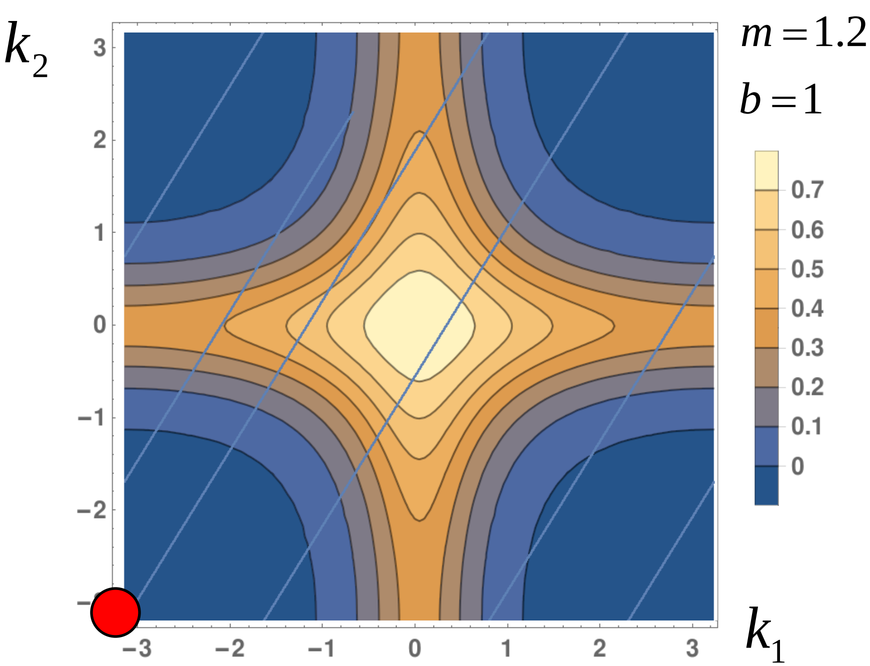
$$\Omega(k_1, k_2) = \nabla_k \times \vec{A}_k$$

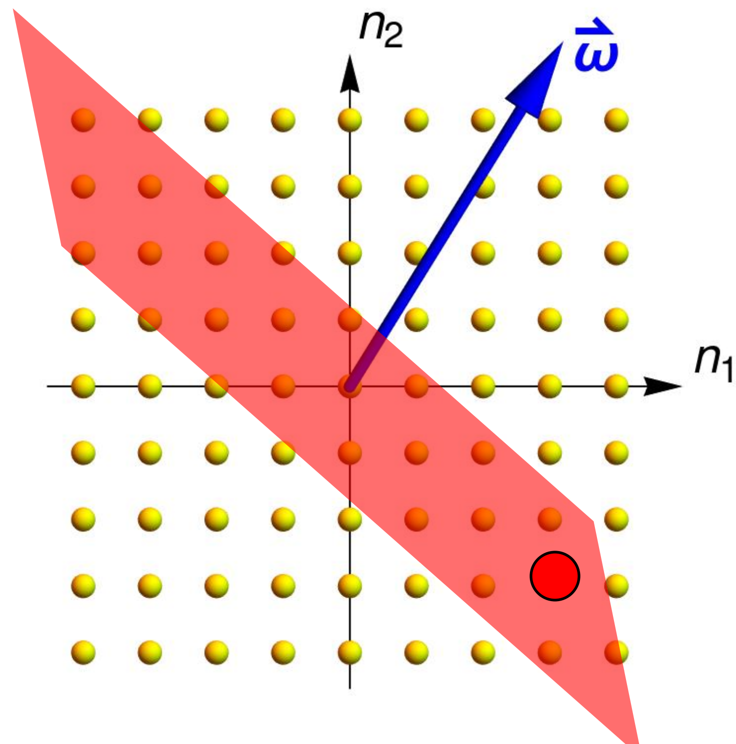
$$\vec{A}_k = -i\langle \psi_k | \nabla_{\vec{k}} | \psi_k \rangle$$

$$\left\langle \frac{d\vec{n}}{dt} \right\rangle = ~~\langle \nabla_k \varepsilon \rangle_k~~ + \langle \Omega_{\vec{k}} \rangle \times \frac{d\vec{k}}{dt}$$



- BHZ Berry curvature:





Quantized energy pumping

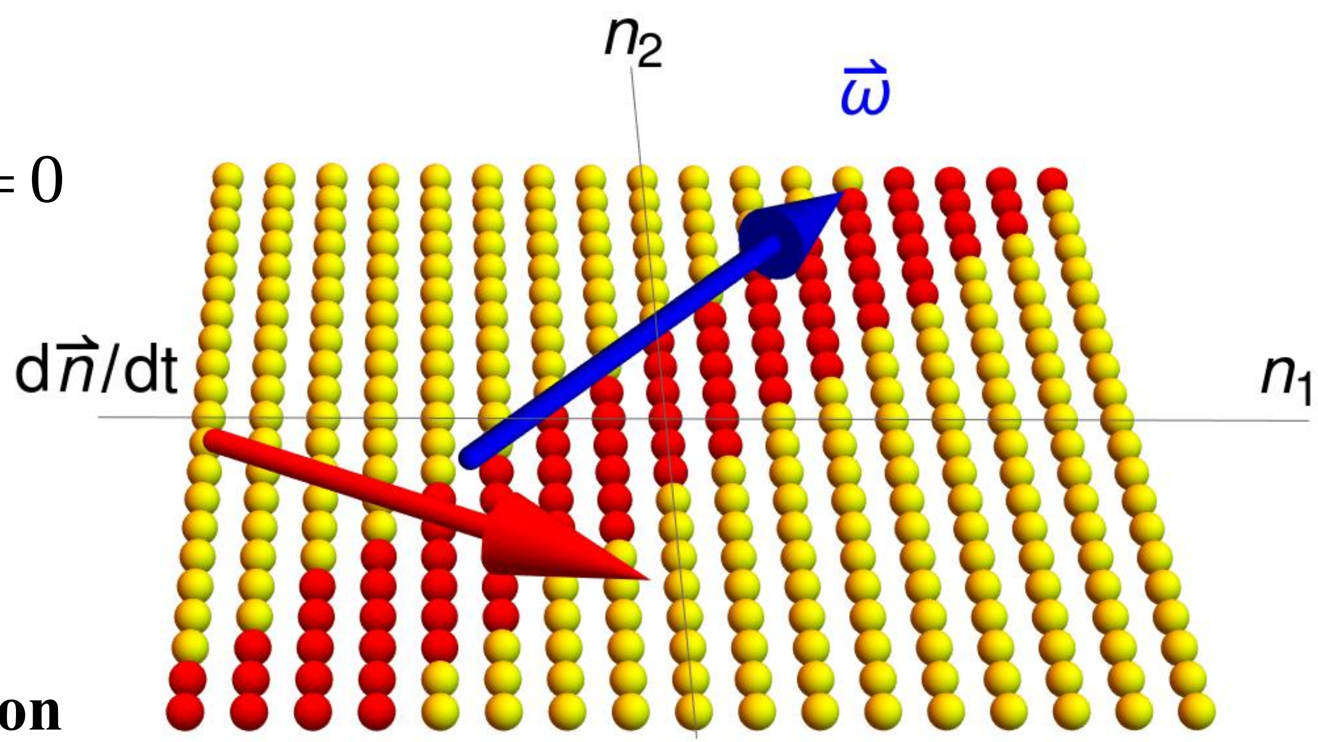
$$\overline{\Omega} = \frac{1}{2\pi} C$$

$$\frac{dE_{1,2}}{dt} = \omega_{1,2} \left\langle \frac{dn_{1,2}}{dt} \right\rangle = \pm \overline{\Omega} \omega_1 \omega_2 = \pm \frac{C}{2\pi} \omega_1 \omega_2$$

Quantized σ_{xy} → Quantized energy pumping

$$\omega_1 \frac{dn_1}{dt} + \omega_2 \frac{dn_2}{dt} = 0$$

Energy pumped between lasers: **Topological frequency conversion**



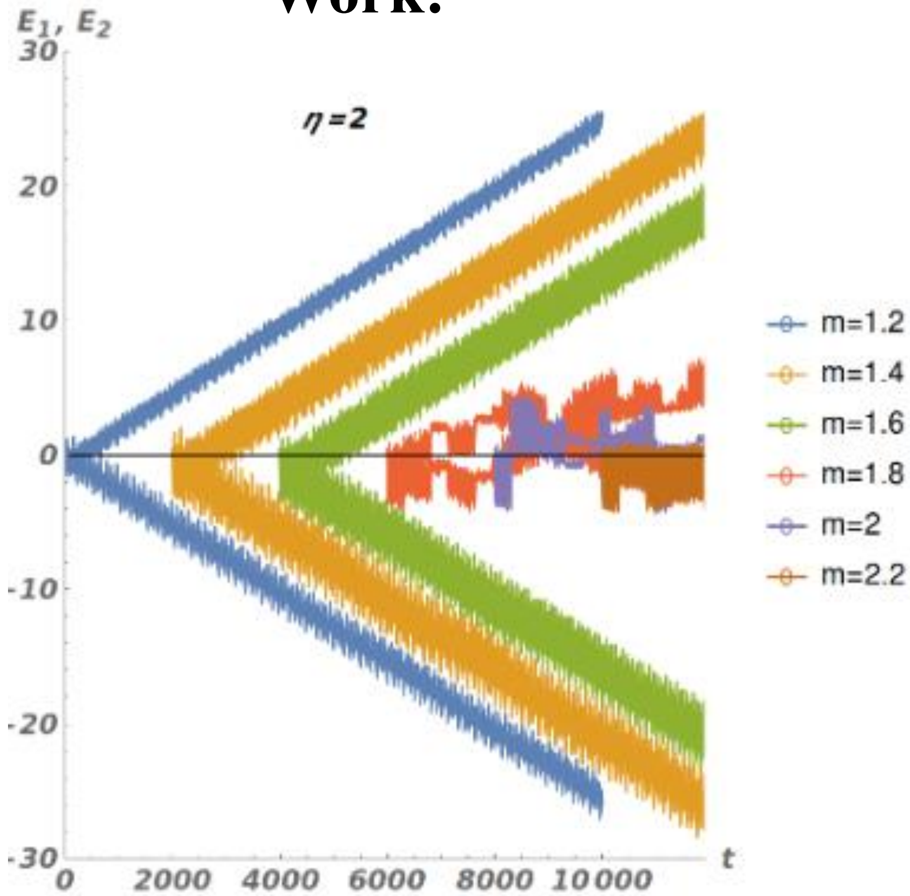
Quantization - if all BZ is covered…

Numerics I: Incommensurate Frequencies [strong coupling]

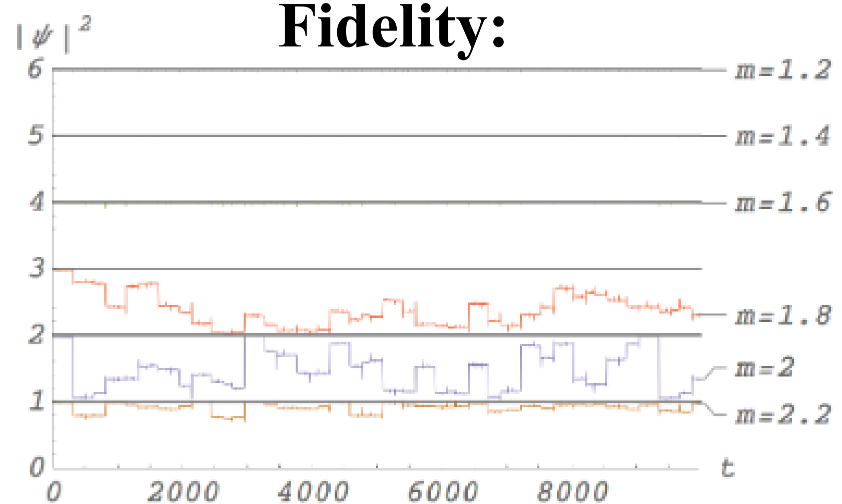
- $\omega_1 / \omega_2 \neq p / q$:

$$\omega_1 = 0.1,\ \omega_2 = \omega_1 \frac{\sqrt{5}+1}{2},\ b = 1$$

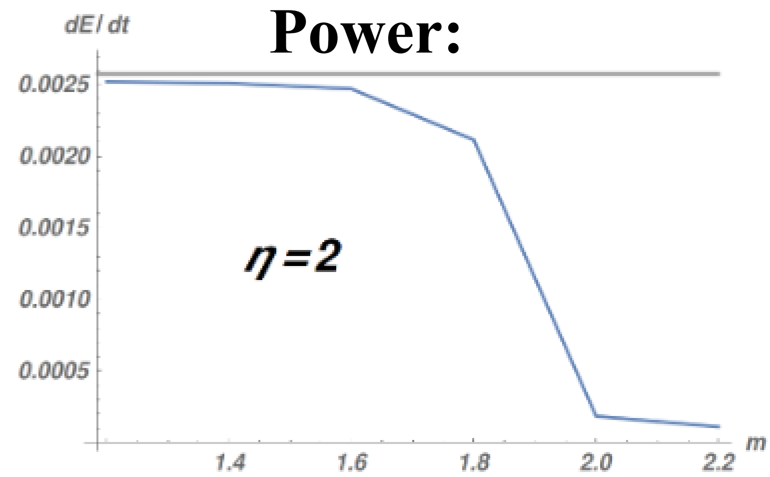
Work:



Fidelity:



Power:



History-dependence And Synthetic dimension control

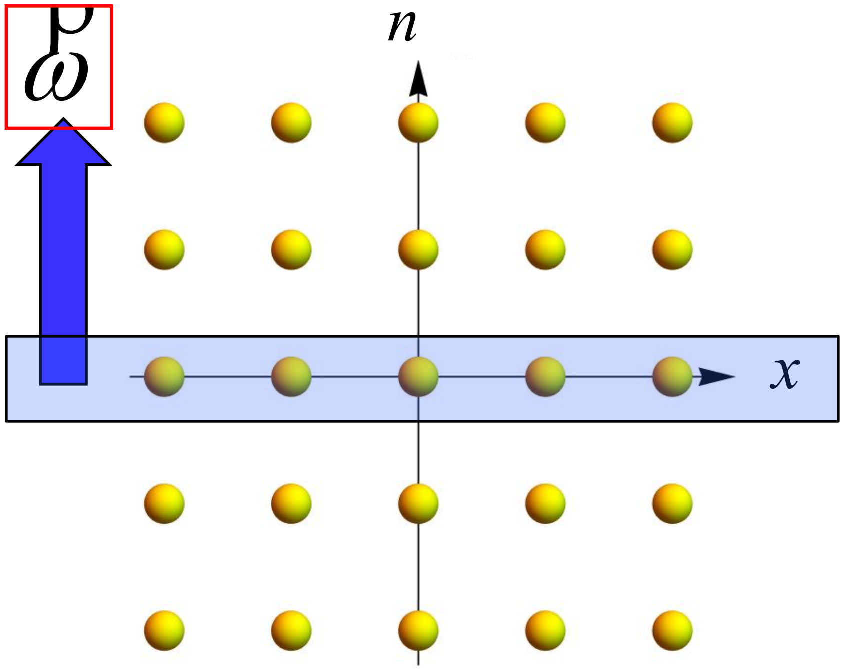
Phys. Rev. Lett. 120, 106402 (2018)

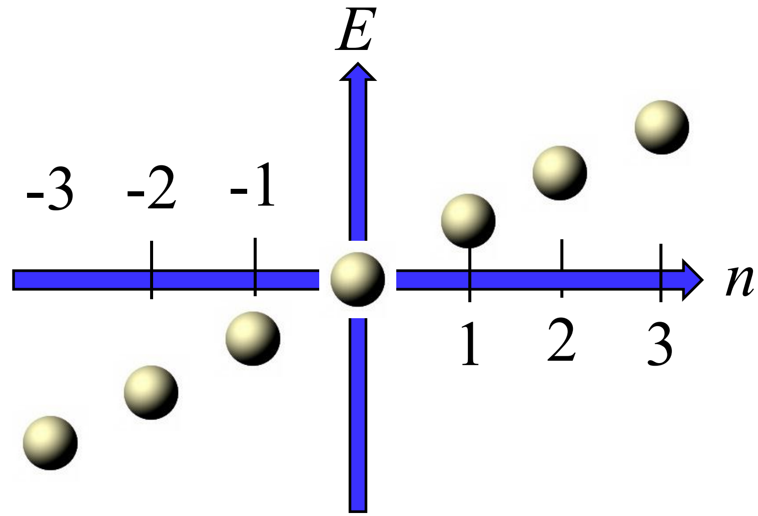
w/ Yuval Baum

1D+1SD topological insulator?

$$H = v_1\sigma^x \sin(\omega_1 t) + v_2\sigma^y \sin(\omega_2 t) + \left[m - b_1\cos(\omega_1 t) - b_2\cos(\omega_2 t)\right]\sigma^z$$

$$H = v_1\sigma^x \sin(\omega_1 t) + v_2\sigma^y \sin(k_x) + \left[m - b_1\cos(\omega_1 t) - b_2\cos(k_x)\right]\sigma^z$$





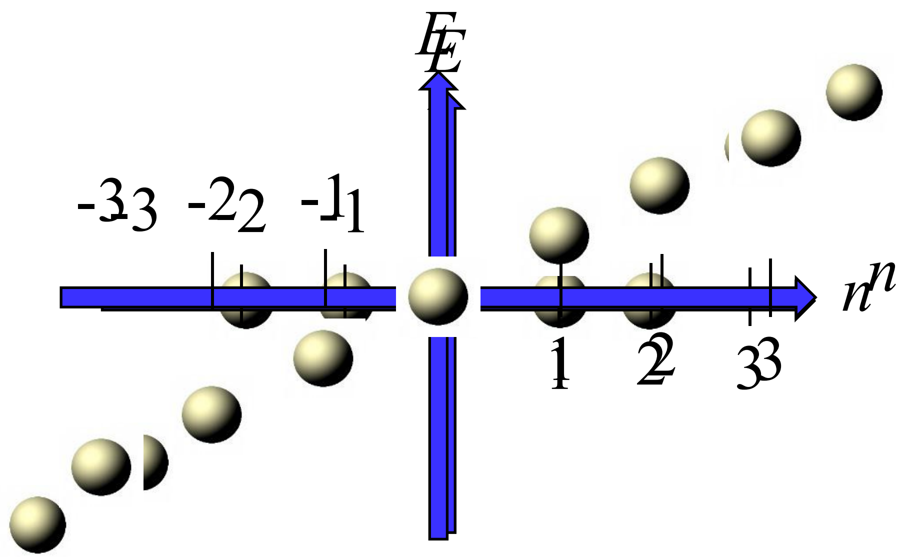
- What about Edge states?

The potential in periodic drive

- Synthetic dimension construction:

$$H = H_{hopping}(n,x) - \omega n + U(n)$$

$$U(n) = -\omega n + U_{ext}(i\partial_t)$$ ⇒ *History dependence!*



Tailoring the potential

- Augment SE with history kernel:

$$i\frac{\partial \psi(t)}{\partial t} = \hat{H}\,\psi(t) + \int_0^T dt'\,\hat{U}(t')\,\psi(t-t')$$

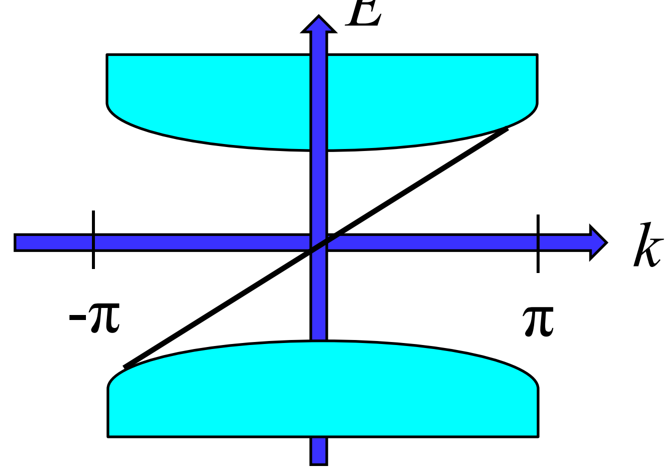
$$E\psi_n \equiv \sum_m \left(H_0 + n\omega\right)\psi_n + \int_0^T dt'\,\hat{U}(t')\,e^{-i(E+\omega n)t'}\psi_m$$

$$U_{E,n} = \int_0^T dt'\,\hat{U}(t')\,e^{-i(E+\omega n)t'}$$

- Edge states: near *E=0*

$$U_{0,n} = \int_0^T dt'\,\hat{U}(t')\,e^{-i\omega n t'} = \tilde{U}_n$$

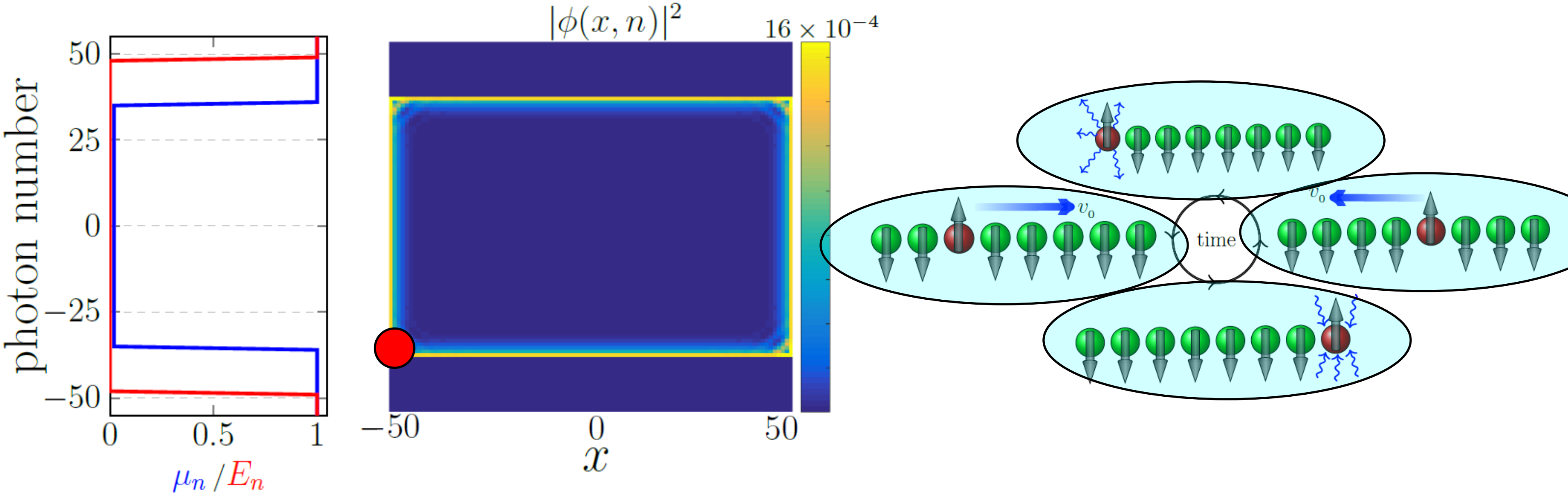
Fourier series component



1D+1SD topological insulator?

$$H = v_1\sigma^x \sin(\omega_1 t) + v_2\sigma^y \sin(k_x) + \left[m - b_1\cos(\omega_1 t) - b_2\cos(k_x)\right]\sigma^z$$

$$\tilde{U}_n = \left[n\omega + V\sigma^z\right]\Theta(|n| - n_0)$$



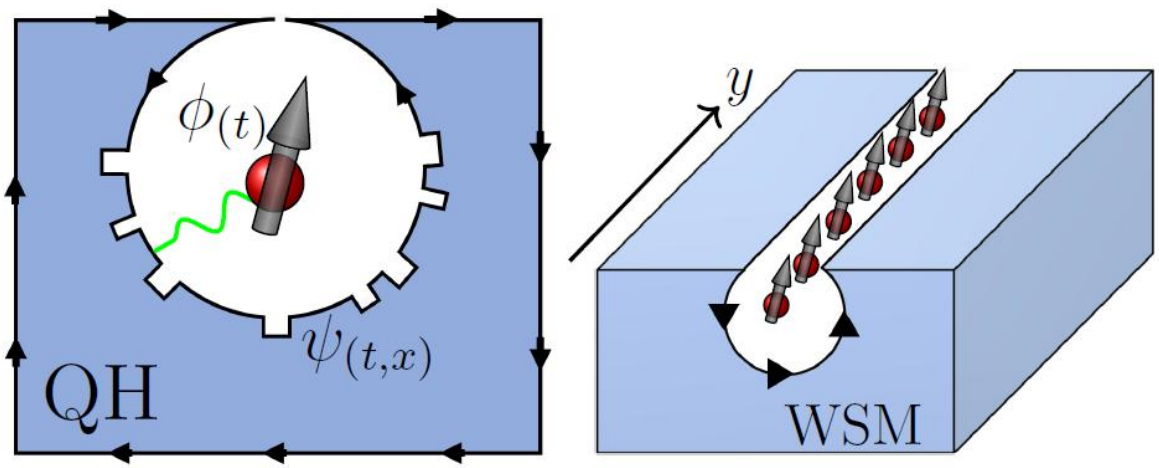
Realizations

- Chiral surface states = memory register:

$$\Big(\mathrm{i}\partial_t - H_\phi(t)\Big)\phi(t) = -\int \mathrm{d}x\, \lambda(x)\psi(x,t)$$

$$\Big(\mathrm{i}\partial_t - H_\psi(\hat{x})\Big)\psi(x,t) = -\lambda^\dagger(x)\phi(t)$$

$$\hat{U}_n = \sum_m \frac{\hat{\lambda}_n \hat{\lambda}_m}{|m-n|}$$

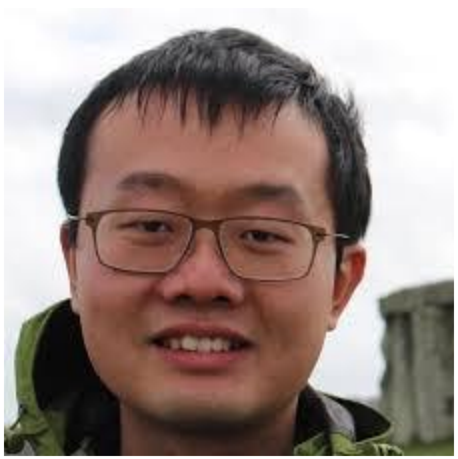


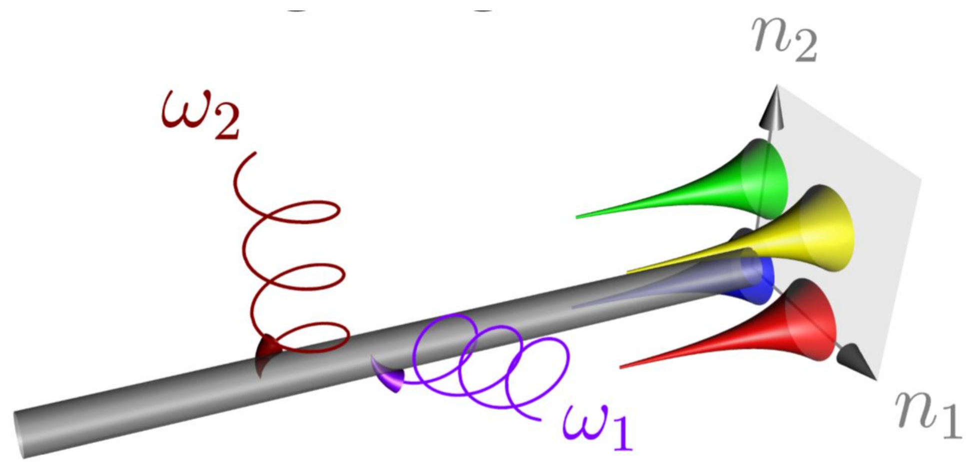
0D *1D*

Majorana-Multiplexing

Arxiv:1805.01896

w/ Yang Peng

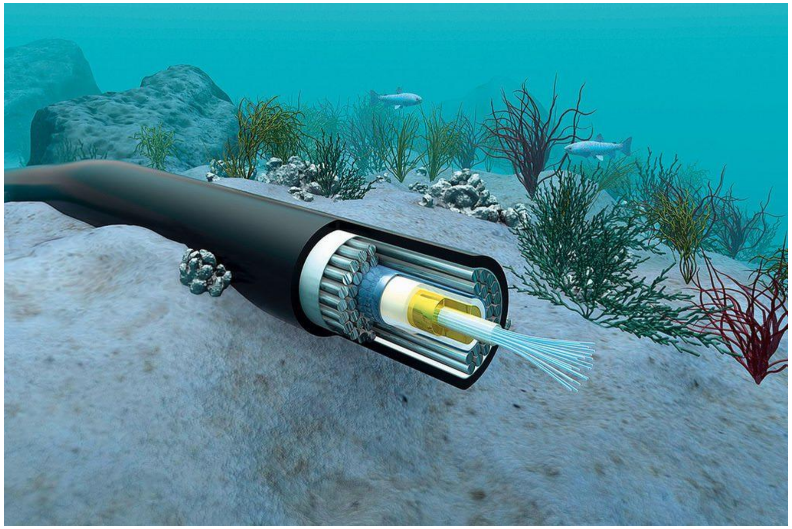




Multiplexing

- Optical fiber limits? One time-series signal line per fiber?

Newly installed Internet Underwater cable
[Saigoneer]

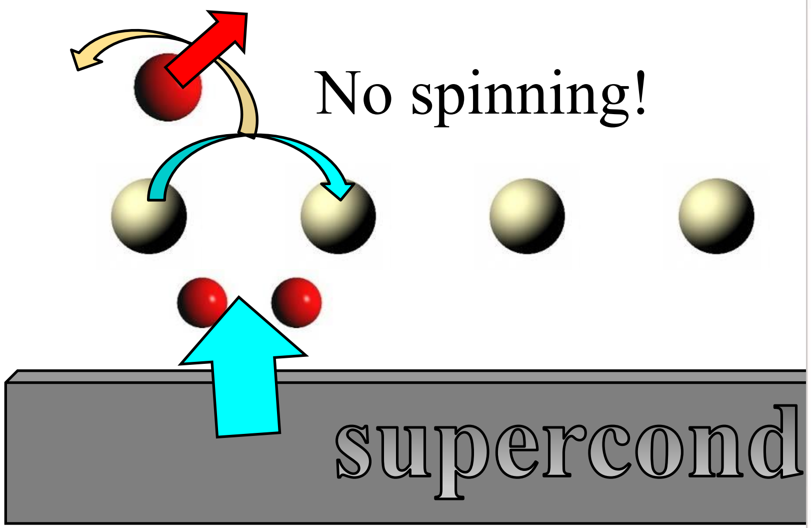


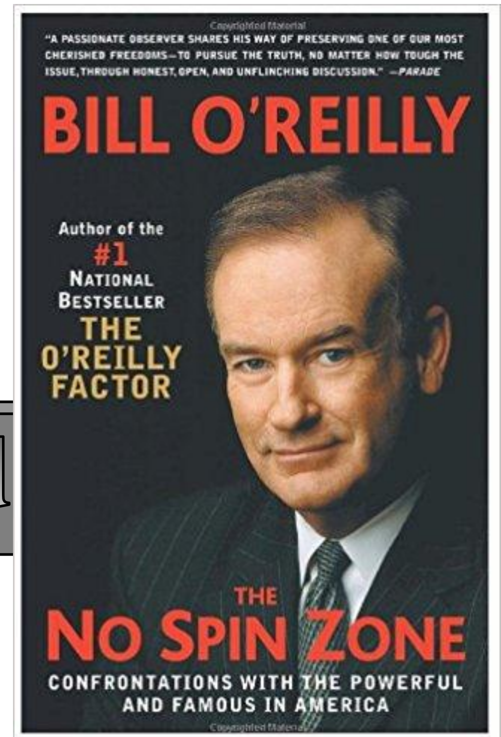
- Much more – multiplexing!

Kitaev Model for Majoranas

- 1d p-wave superconductor:

$$H = -\sum_i \left(2Jc_i^+ c_{i+1} - \mu c_i^+ c_i + 2\Delta c_i^+ c_{i+1}^+\right) + h.c.$$



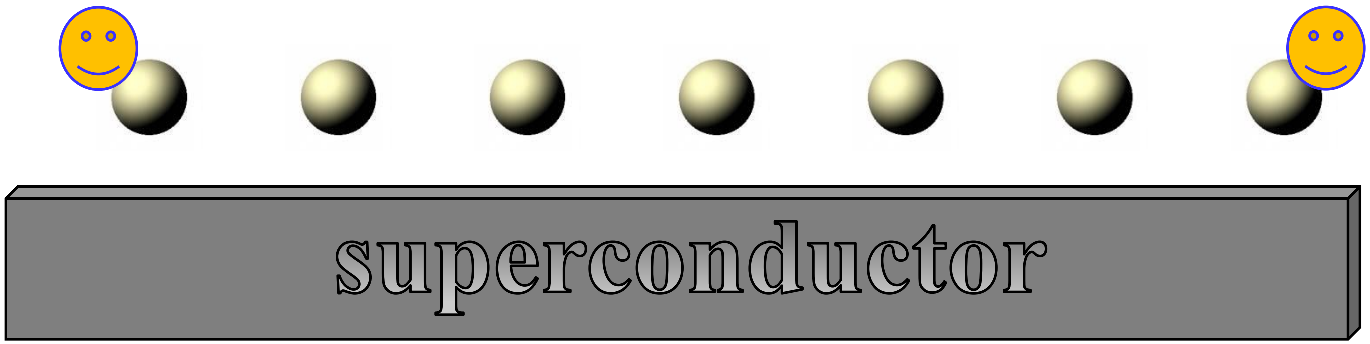


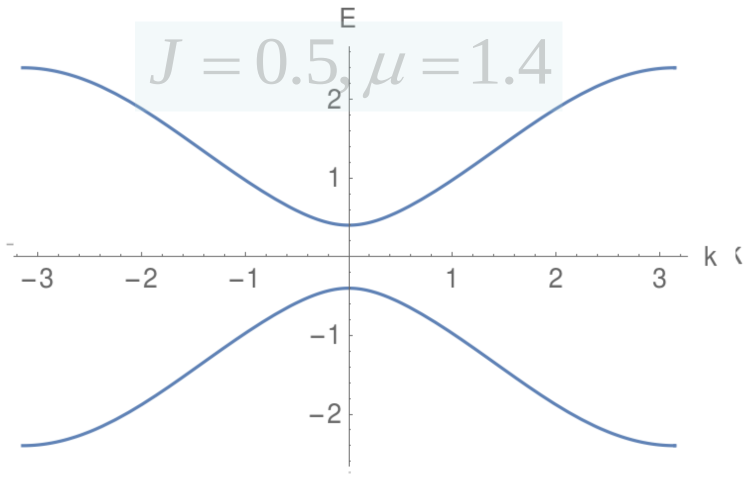
Kitaev Model for Majoranas

- 1d p-wave superconductor:

$$H = -\sum_i \left(2J c_i^+ c_{i+1} - \mu c_i^+ c_i + 2\Delta c_i^+ c_{i+1}^+\right) + h.c.$$

$$H_k^{BdG} = (-2J\cos k - \mu)\tau^z + 2\Delta\tau^x \sin k$$





$$J = 0.5, \mu = 1.4$$

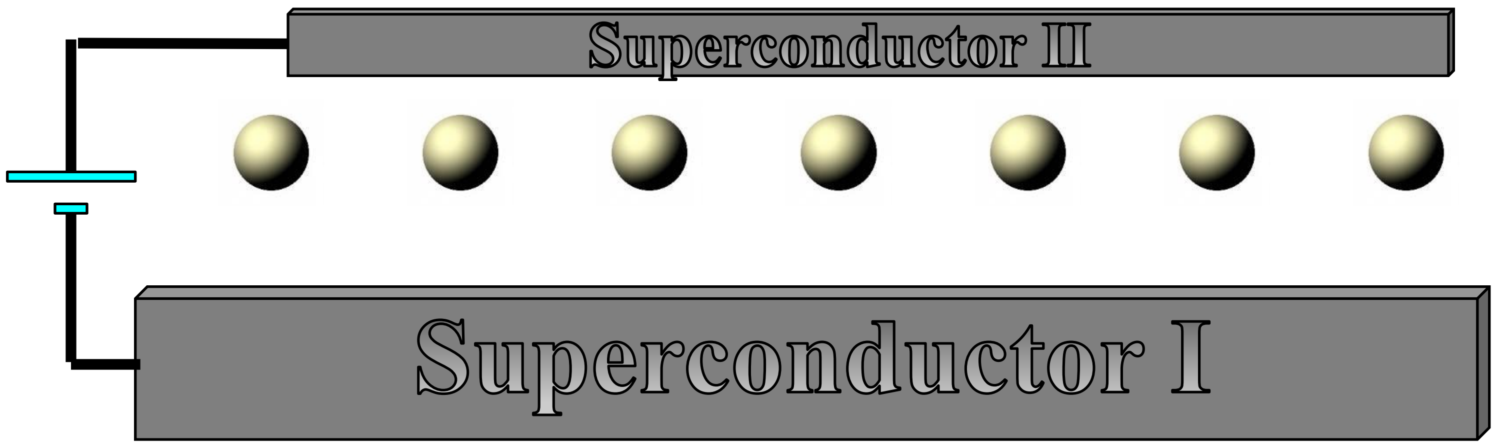
Floquetified Kitaev Model

- Add periodic time dependence:

$$H = -\sum_i [2Jc_i^+ c_{i+1} - \mu c_i^+ c_i + 2\Delta c_i^+ c_{i+1}^+ + 2\Delta_1 e^{i\omega t} c_i^+ c_{i+1}^+ + h.c.]$$

$$H_k^{BdG} = (-2J\cos k - \mu)\tau^z + 2\Delta\tau^x \sin k + 2\Delta_1 \exp(i\omega\tau^z t)\tau^x \sin k$$

$$V = \frac{\eta\omega}{2e}$$



L. Jiang, T. Kitagawa,…(PRL, 2011)

Floquetified Kitaev Model

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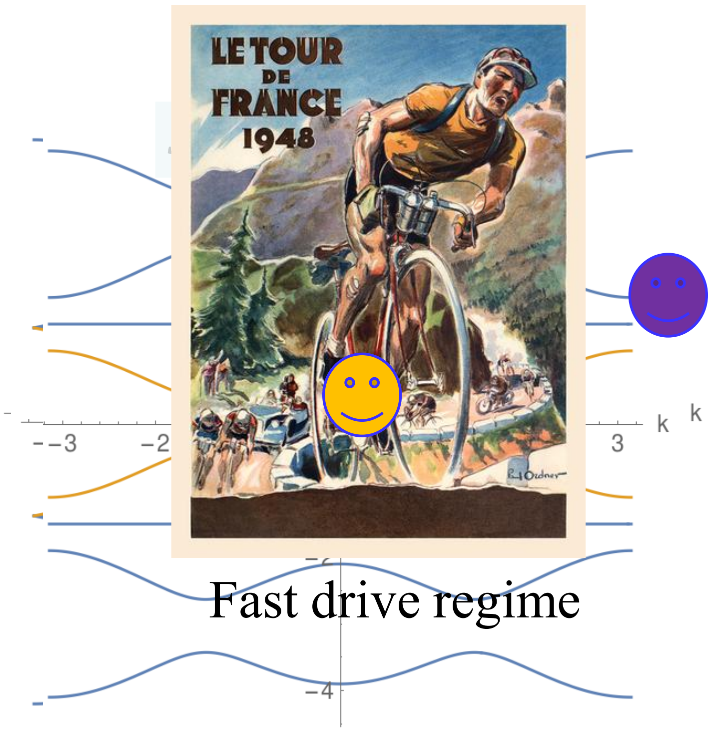
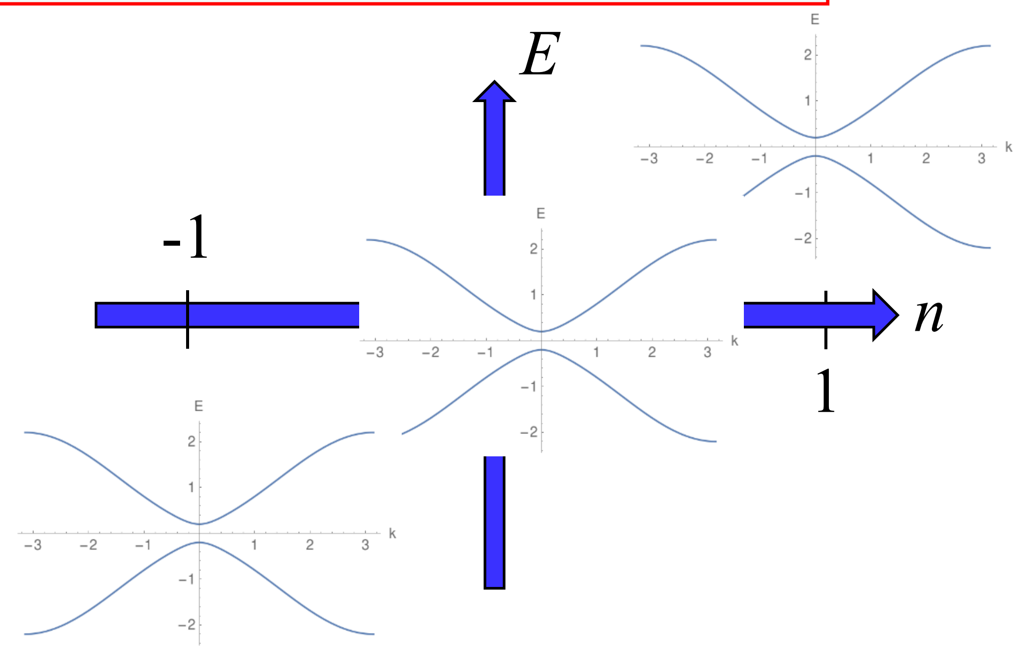
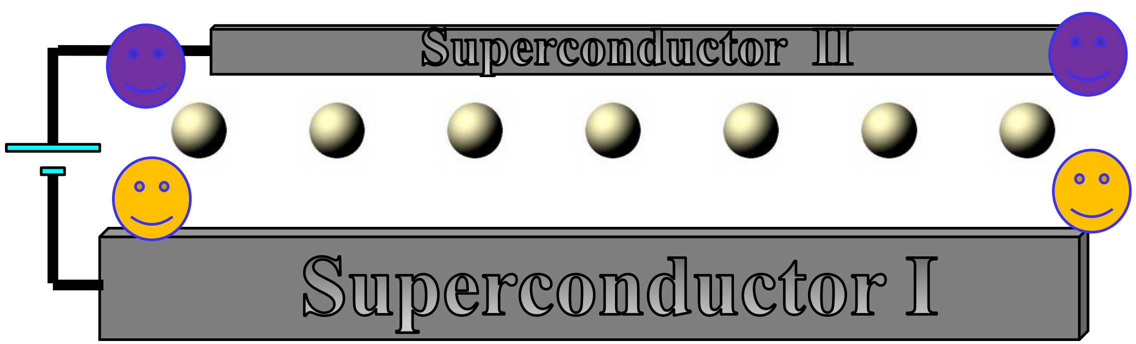
$$H_k^{BdG} = (-2J\cos k - \mu)\tau^z + 2\Delta\tau^x \sin k$$
$$+ 2\Delta_1 \exp(i\omega\tau^z t)\tau^x \sin k$$



Fast drive regime

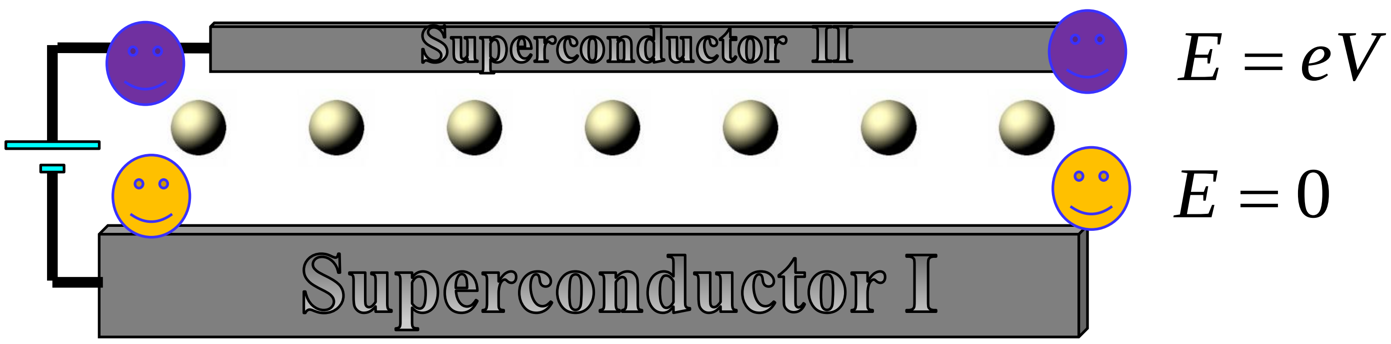




Shameless commerce department:

Topologically protected braiding in a single wire using Floquet Majorana modes

Bela Bauer,[1] T. Pereg-Barnea,[2,3] Torsten Karzig,[1] Maria-Theresa Rieder,[3] Gil Refael,[4,5] Erez Berg,[3,6] and Yuval Oreg[3]



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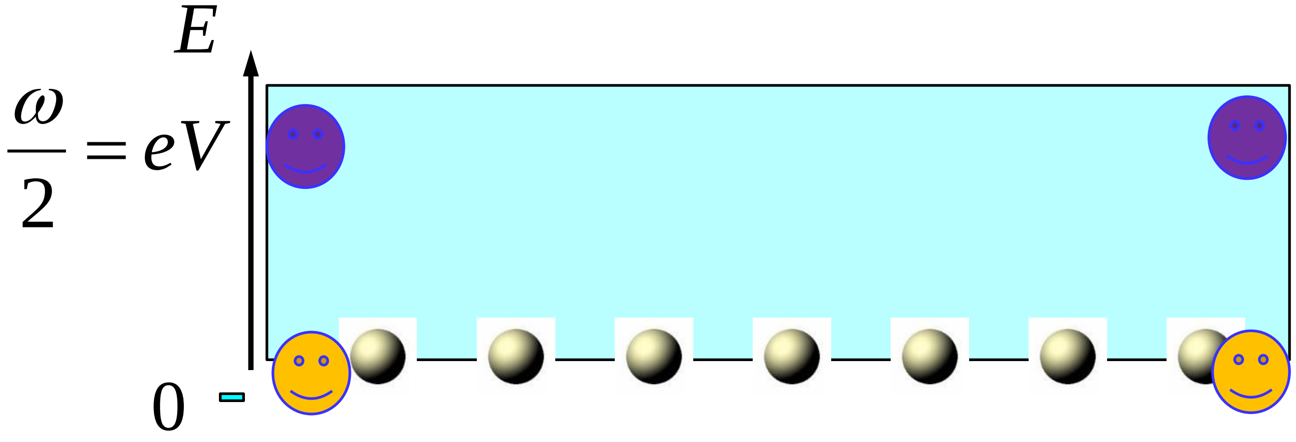

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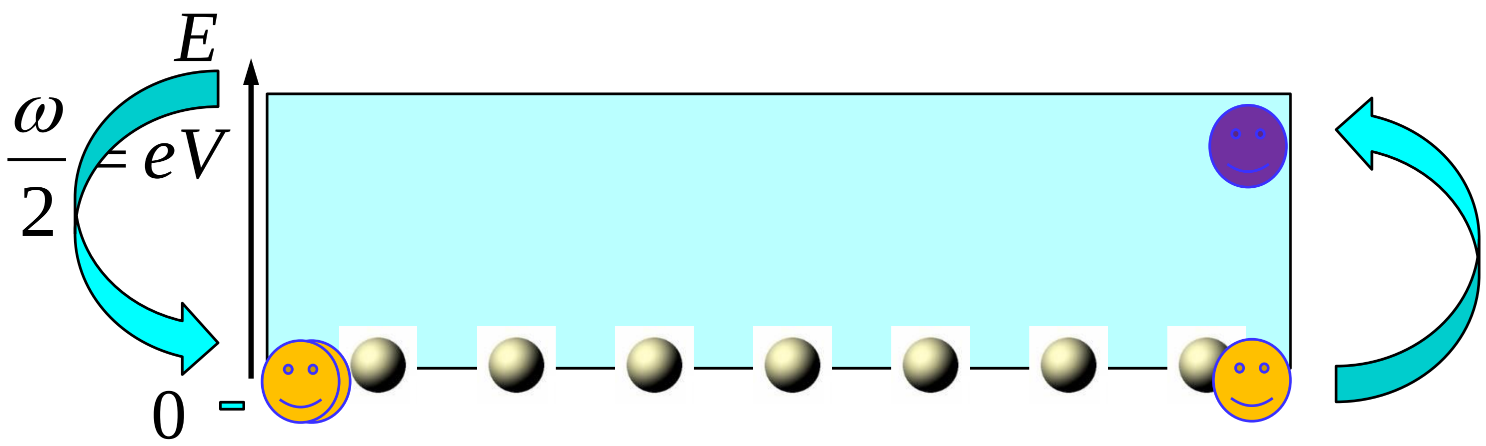


Multi-dimensional Floquet states

- Multi frequency floquet:

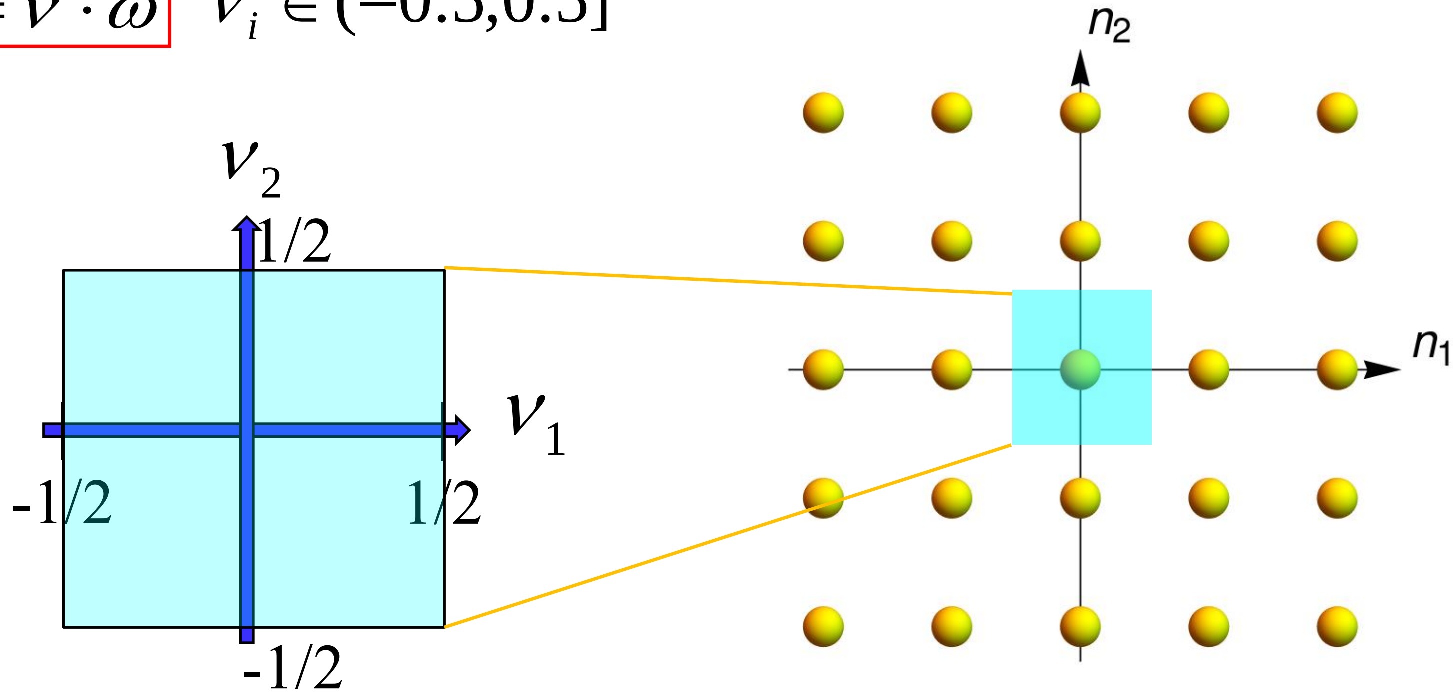
$$H = \hat{H}_0 + \hat{V}(e^{i\omega_1 t}, e^{i\omega_2 t}, e^{i\omega_3 t}, \ldots)$$

$$|\varepsilon\rangle = e^{i\varepsilon t} \sum_{n \in Z} \exp\left(-i(\vec{n} \cdot \vec{\omega})t\right) |\psi_n^\varepsilon\rangle$$

$$\varepsilon \Leftrightarrow \varepsilon + \vec{n} \cdot \vec{\omega}$$

- Multi dimensional Floquet Zone:

$$\varepsilon = \vec{\nu} \cdot \vec{\omega} \quad \nu_i \in (-0.5, 0.5]$$



Multi-dimensional Floquet and PH symmetry

- Majoranas – invariant under particle-hole symmetry:

$$K^{+}\hat{H}K = -\hat{H}$$

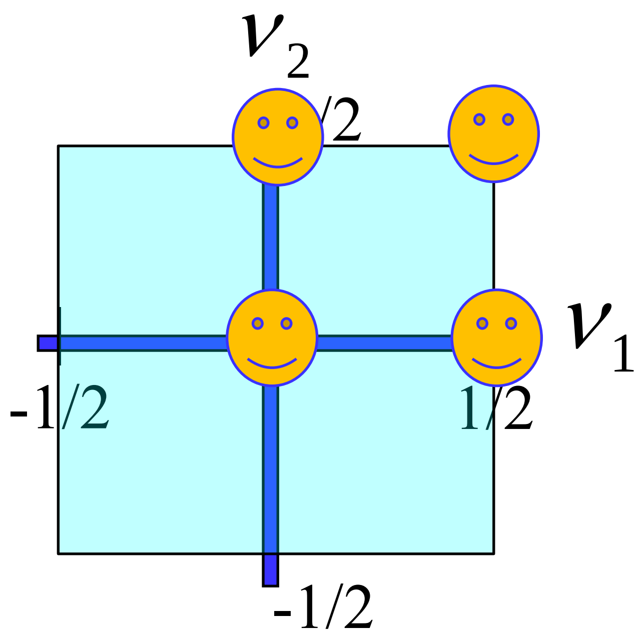
$$K^{+}\hat{\gamma}K = \hat{\gamma}$$

- Majorana Energy:

$$E_{\gamma} = -E_{\gamma} = 0$$

- Fooquet Majorana Energy:

$$\varepsilon_{\gamma} = -\varepsilon_{\gamma} + \vec{p}\cdot\vec{\omega} = \vec{\nu}\cdot\vec{\omega}$$

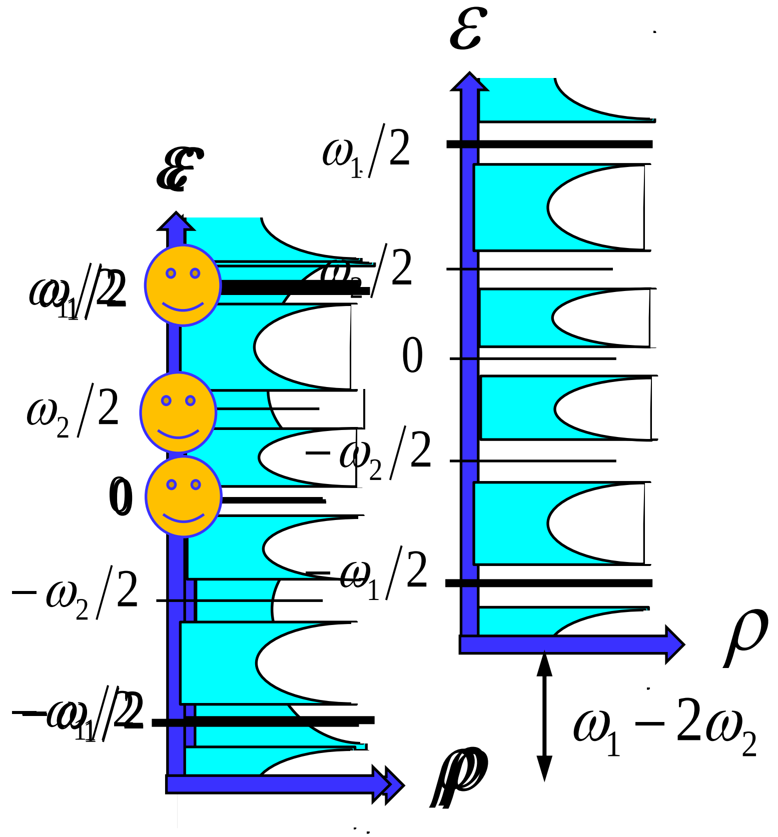


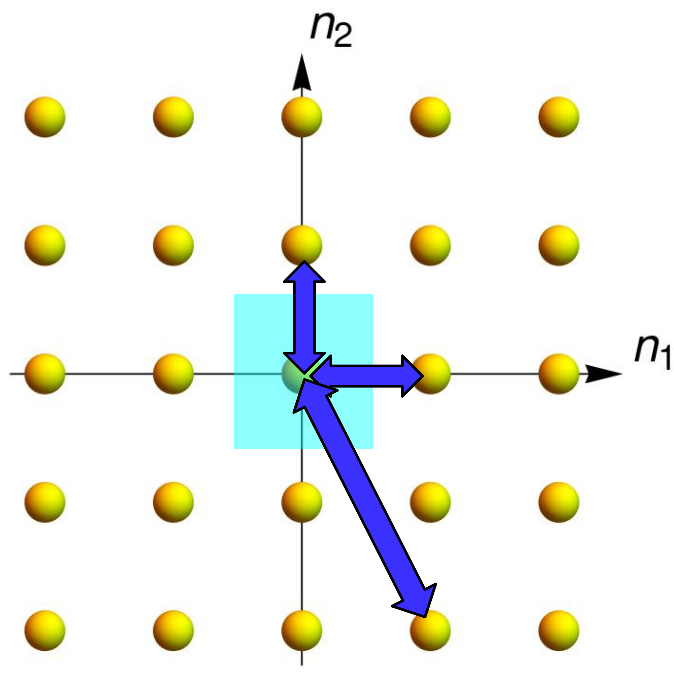
$$\nu_i = 0, 0.5$$

Majorana multiplexing

- Multiply driven Kitaev model:

$$H_k^{BdG} = (-2J\cos k - \mu)\tau^z + 2\Delta\tau^x \sin k$$
$$+ 2\Delta_1 \exp(i\omega_1\tau^z t)\tau^x \sin k + \Delta_2 \exp(i\omega_2\tau^z t)\tau^x \sin k \ldots$$





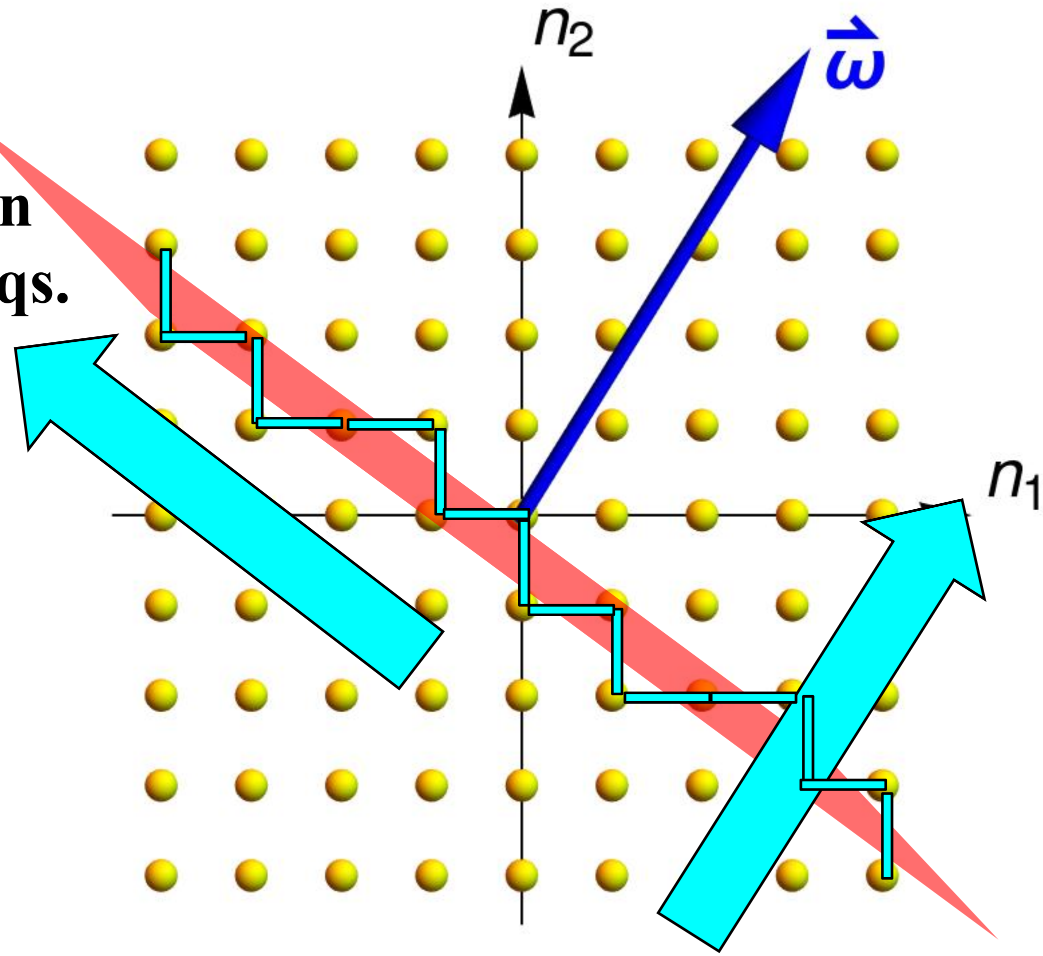
Dense spectrum!

Localization in synthetic dimensions

- Floquet eigenstates draw from all energy-conserving combinations

Quasi-periodic potential:
Aubry-Andre localization
For incommensurate freqs.

Majoranas survive!

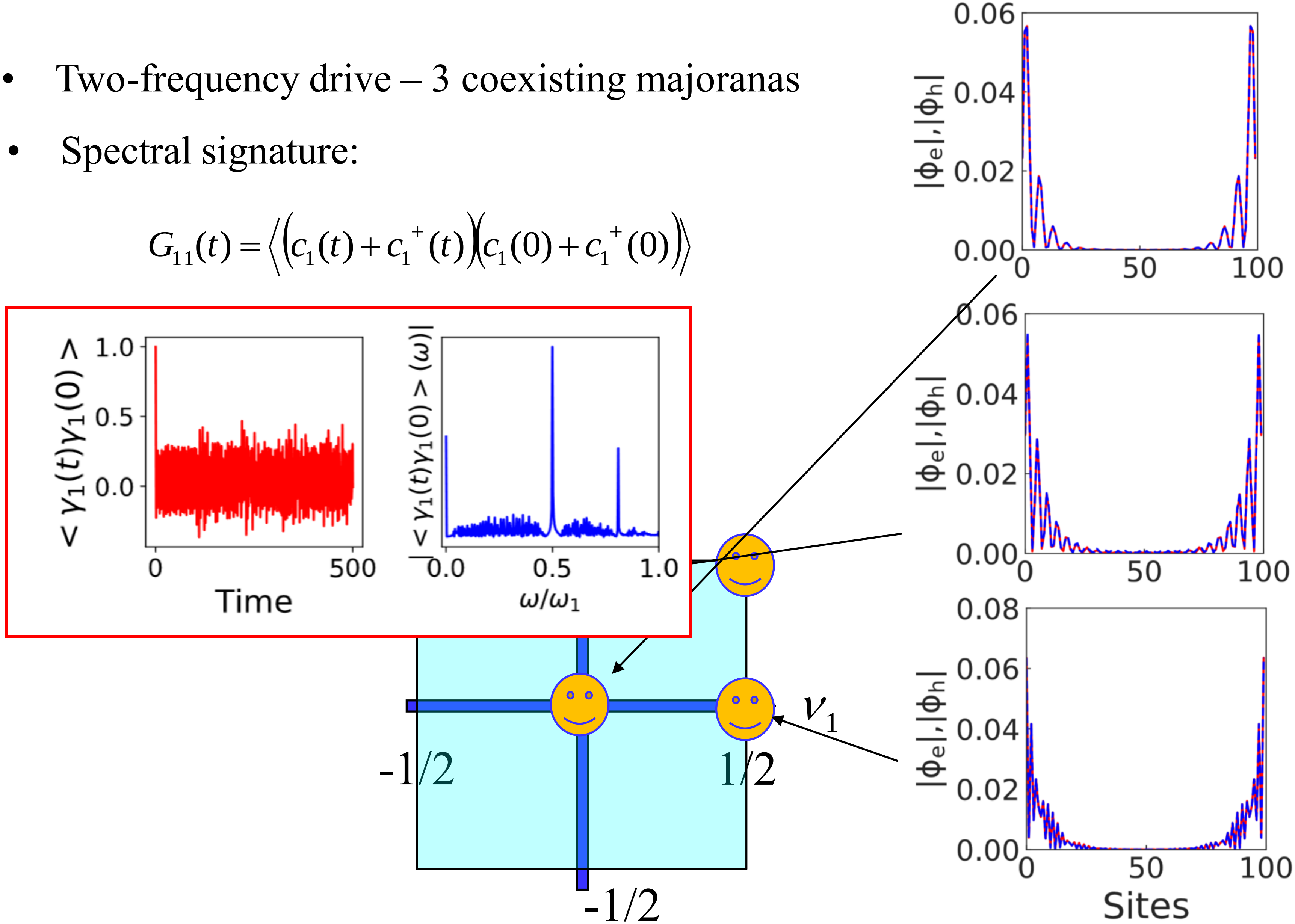


Suppressed by
Stark Localization

Numerics

- Two-frequency drive – 3 coexisting majoranas
- Spectral signature:

$$G_{11}(t) = \left\langle \left(c_1(t) + c_1^{+}(t)\right)\left(c_1(0) + c_1^{+}(0)\right)\right\rangle$$



Summary and conclusions

- Periodic drives induce additional dimensionality.

- Topological frequency conversion,
  quantized energy pumping, optical amplifier.

- History dependence can control synthetic dimensions.

- Coexisitng majoranas at several frequencies.
  [connection to quasi-periodic time crystals]

Energy pumping measurement

- How to measure n?

$$\frac{dn_i}{dt} = \left\langle \frac{\partial H}{\partial k_i} \right\rangle = \left\langle \psi(t) \middle| \frac{\partial H}{\partial k_i} \middle| \psi(t) \right\rangle$$

- Rate of work done:

$$\frac{dW_i}{dt} = \frac{dn_i}{dt}\omega_i = \omega_i \left\langle \frac{\partial H}{\partial k_i} \right\rangle$$

- Measurement:

$$H = H_1(\omega_1 t + k_1) + H_2(\omega_2 t + k_2)$$

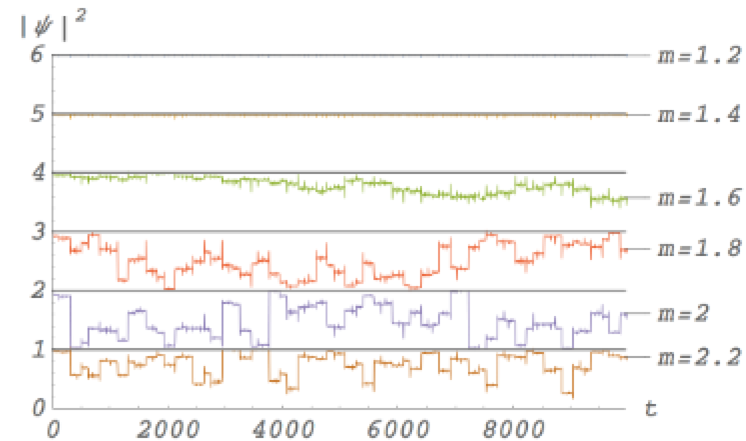
$$\Rightarrow \quad \frac{dW_i}{dt} = \omega_i \left\langle \frac{\partial H}{\partial k_i} \right\rangle = \left\langle \frac{\partial H_i}{\partial t} \right\rangle$$

Numerics I: Incommensurate Frequencies [weak coupling]

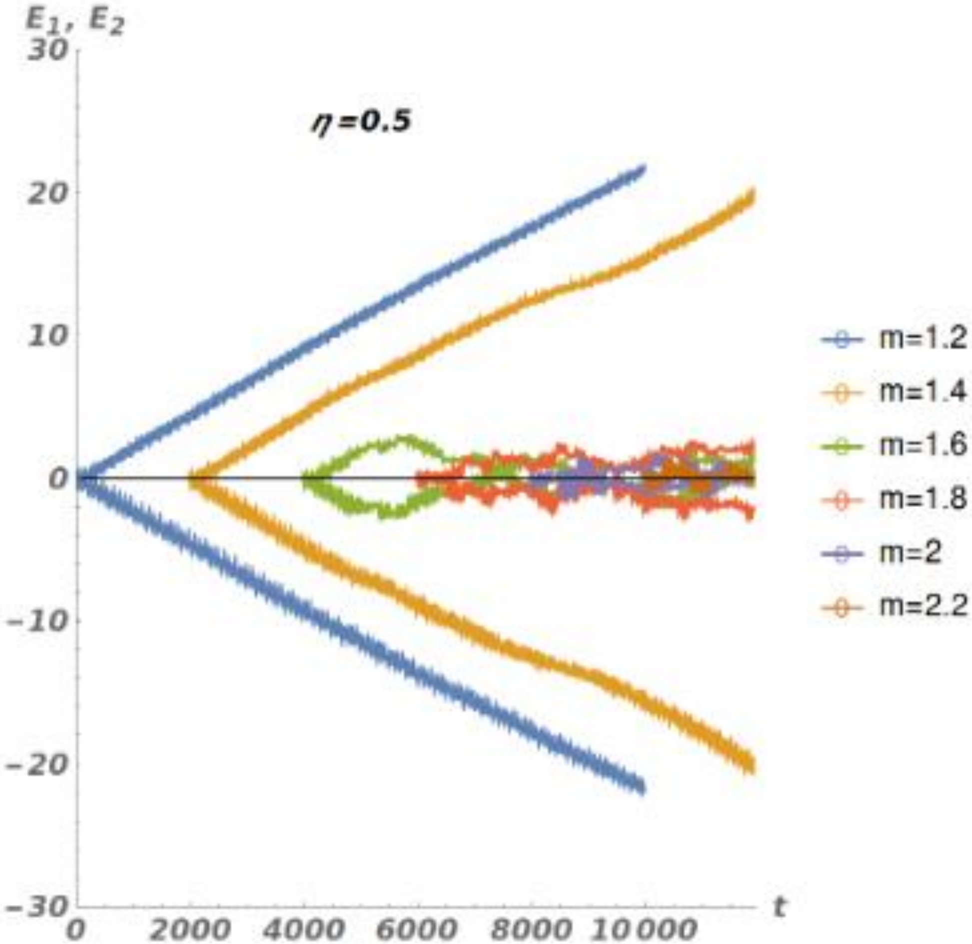
- $\omega_1 / \omega_2 \neq p / q$:

$$\omega_1 = 0.2,\ \omega_2 = \omega_1 \frac{\sqrt{5}+1}{2},\ b = 1$$

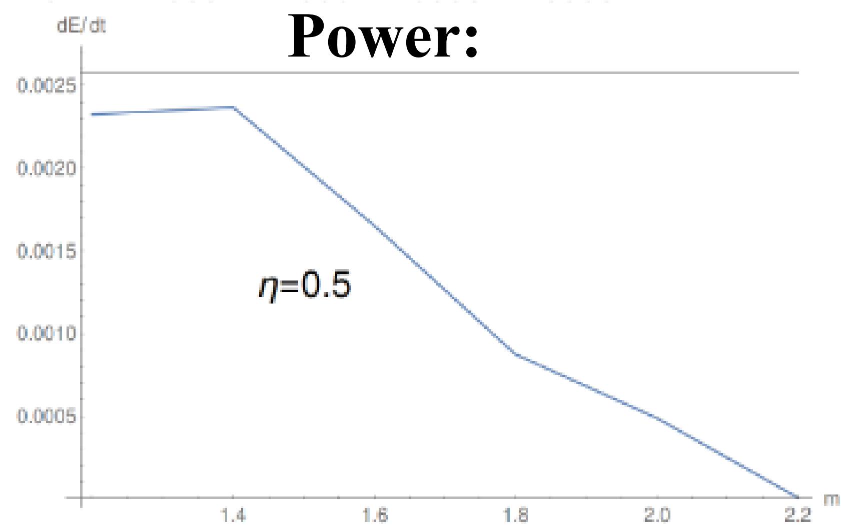
Fidelity:



Work:



Power:



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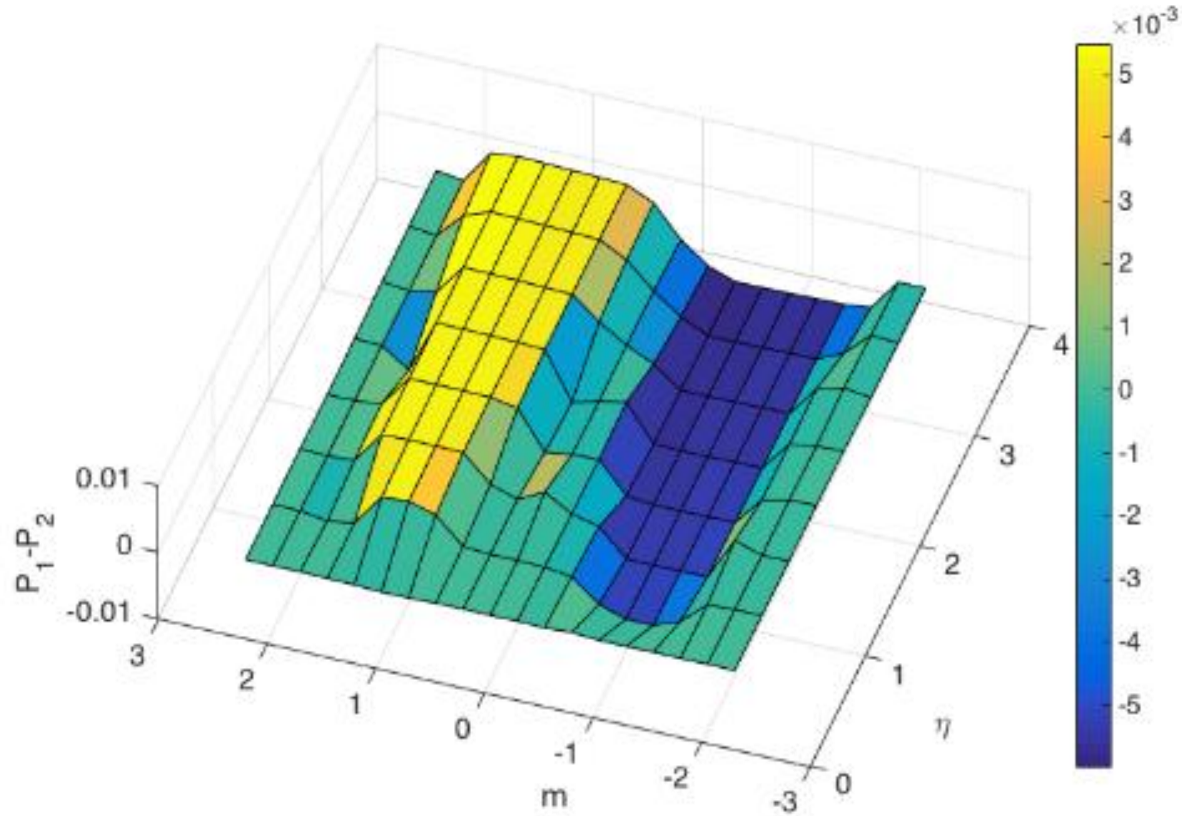
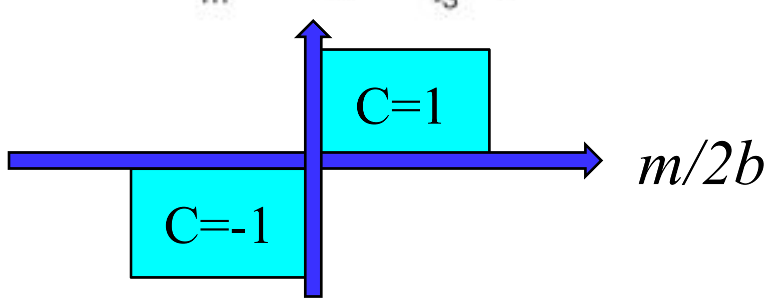


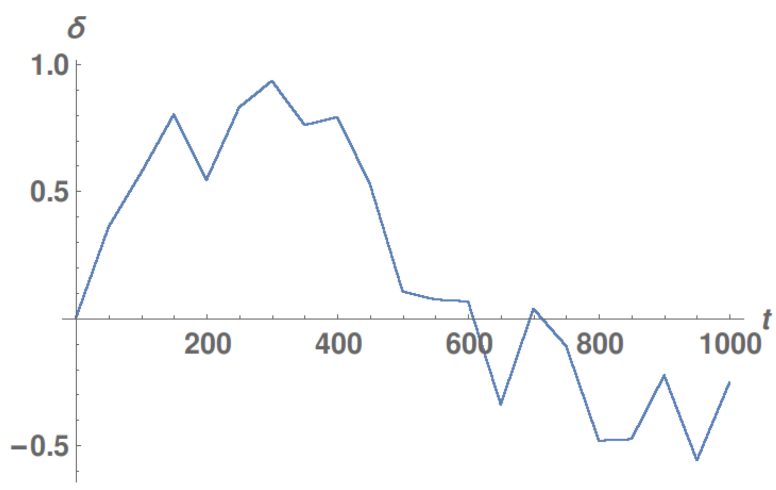


Disorder effects

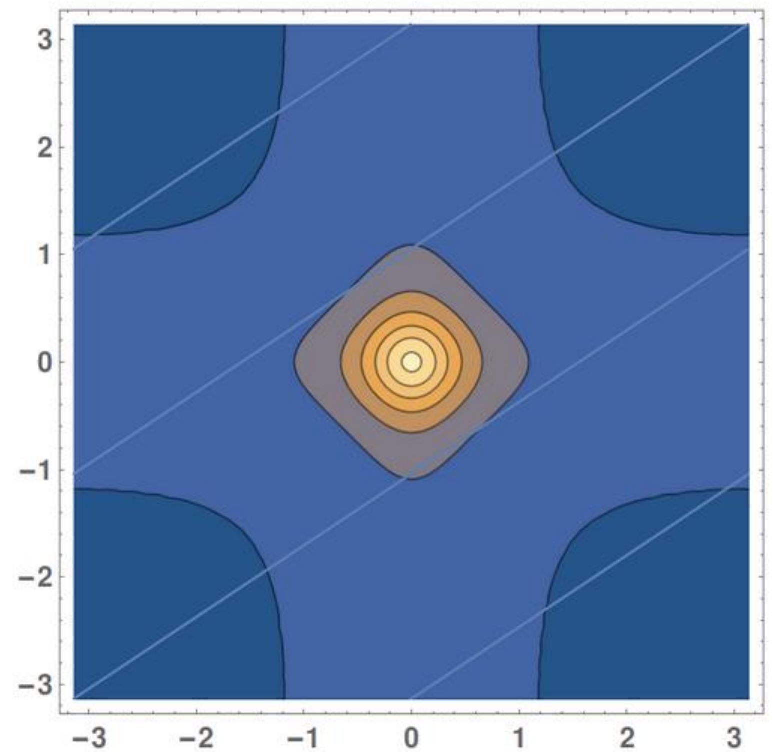
- Consider temporal noise:

$$\omega_1 t + \phi_1 + \delta(t) \qquad \left\langle \frac{d\delta(t)}{dt} \cdot \frac{d\delta}{dt}(0) \right\rangle = D\exp\left(-t^2/\tau^2\right)$$

- Rational frequencies and the Floquet zone:

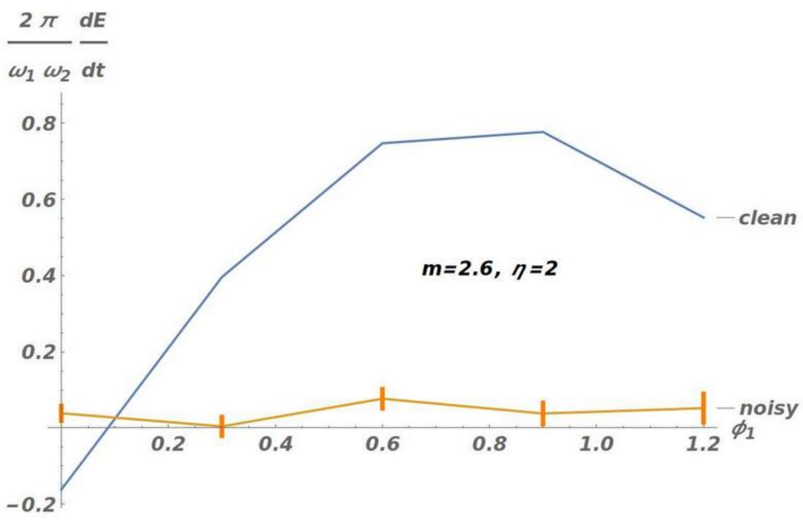


Entire Floquet zone averaged

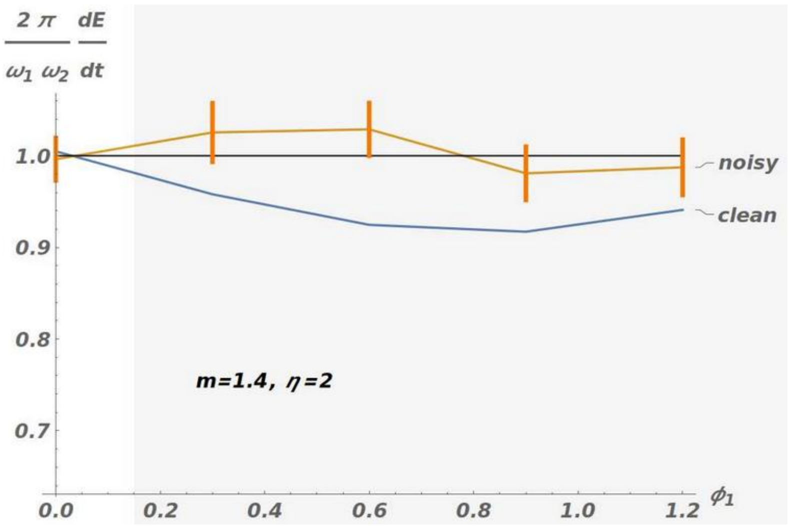


Disorder effects

- Non-topological phase:



- Topological phase:



Topology protects against Disorder
&
Disorder leads to quantized pumping

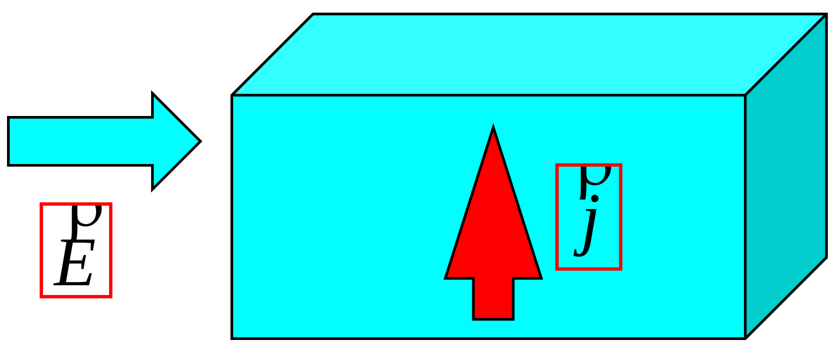
3D Topological insulator

- Use BHZ band structure:

$$H = \tau^z \left[v\vec{\sigma} \cdot \sin(\vec{k}) \right] + \tau^x \left[m - \sum_{\alpha=x,y,z} \cos(k_\alpha) \right] \cos\theta + \tau^y \sin\theta$$

- Magneto electric effect:

$$L \sim \frac{\theta}{4\pi^2} \vec{E} \cdot \vec{B} \quad \Rightarrow \quad \vec{j} \sim \frac{1}{4\pi^2} \left(\dot{\theta} \left(\nabla \times \vec{A} \right) - \left(\nabla \theta \right) \times \dot{\vec{A}} \right)$$



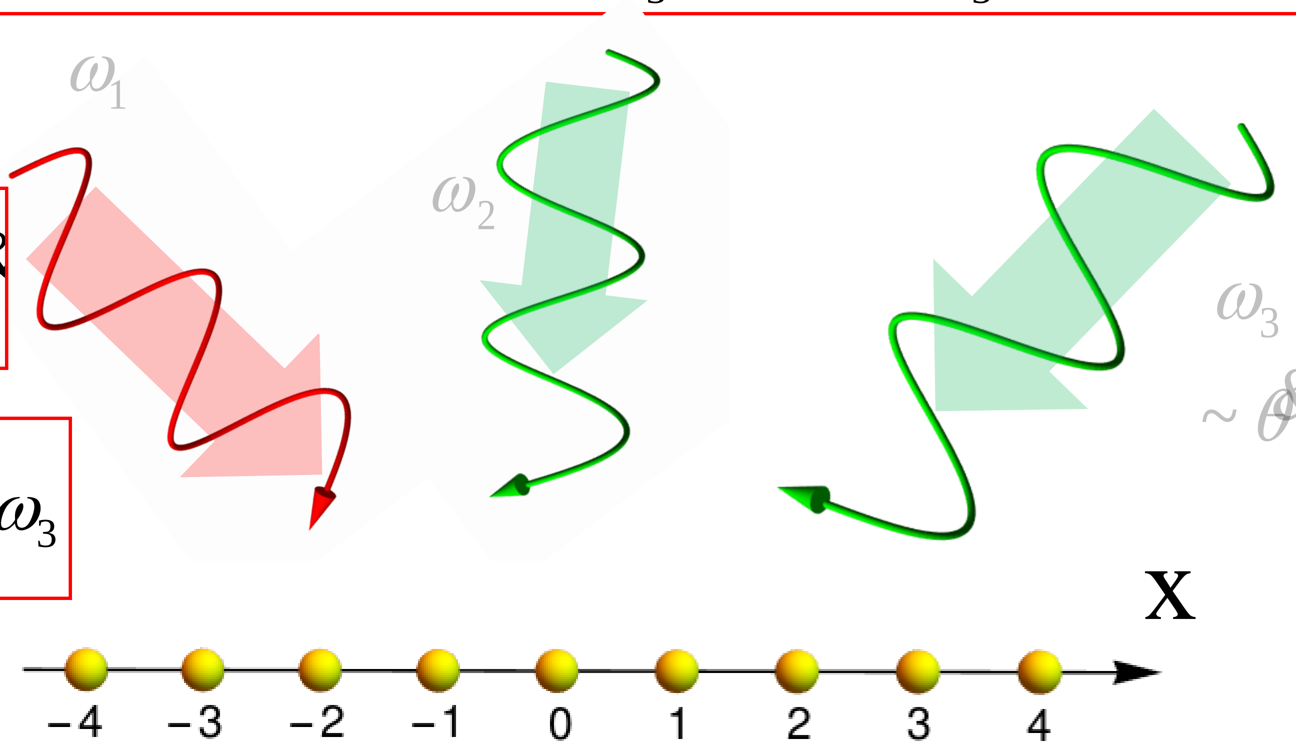
3D Magneto-electric effect or 2nd Chern number

- Replace one of the modes with a cavity:

$$H = \tau^z\left(\sigma^x \sin(k_x) + \sigma^y \sin(\omega_1 t + B\,x) + \sigma^z \sin(\omega_2 t)\right)$$
$$+\left[m - b\cos(k_x) - b\cos(\omega_1 t + bx) - b\cos(\omega_2 t)\right]\tau^x$$
$$-b\,\tau^x \cos\omega_3 t - \tau^y \cos\omega_3 t$$

$$\frac{dn_2}{dt} \sim \frac{C_2}{4\pi^2} B\dot{\theta}$$

$$W_{23} \sim \frac{C_2}{4\pi^2} b\,\omega_2\omega_3$$



Simulations

$$H = H_1(\omega_1 t + \phi_1) + H_2(\omega_2 t + \phi_2)$$

- Need to calculate:

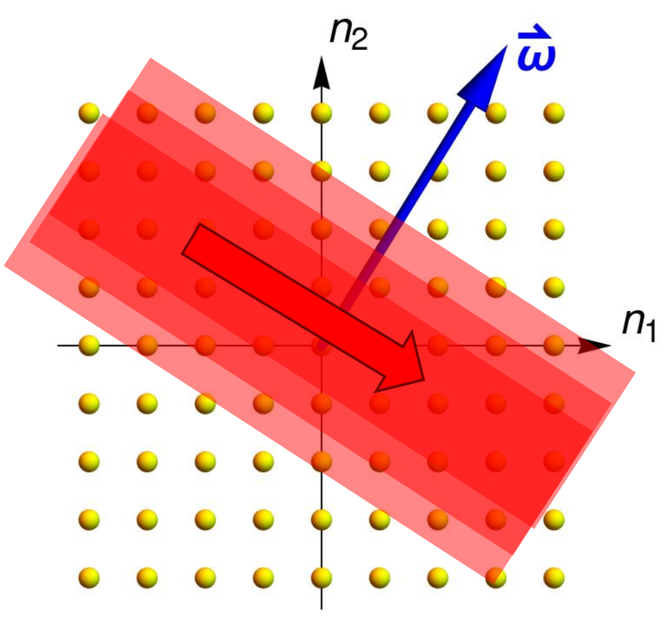
$$W_i = \int_0^t dt \langle \psi(t) | \frac{\partial H_i}{\partial t} | \psi(t) \rangle$$

$$|\psi(t)\rangle = T\left[e^{-i\int_0^t H(t)dt} \right] |\psi(0)\rangle$$

- Parameter regime?

Strong driving: $\omega_i << gap(H)$

Adiabatic evolution: $H(t)|\psi(t)\rangle = -|\psi(t)\rangle \varepsilon_{k(t)}$



- Initializing

$$H(0)|\psi(0)\rangle = -\varepsilon_{k_0} |\psi(0)\rangle$$

Synthetic 2f BHZ

- 2f Floquet BHZ:

$$H = v_1\sigma^x \sin(\omega_1 t+\phi_1) + v_2\sigma^y \sin(\omega_2 t+\phi_2) + \left[m - b_1\cos(\omega_1 t+\phi_1) - b_2\cos(\omega_2 t+\phi_2)\right]\sigma^z$$

- Semiclassical motion:

$$\vec{A}_k = -i\langle \psi_k | \nabla_{\vec{k}} | \psi_k \rangle$$

$$\Omega(k_1, k_2) = \nabla_k \times A_k$$

$$\frac{d\vec{k}}{dt} = \vec{\omega}$$

$$\frac{d\vec{n}}{dt} = \nabla_k \varepsilon_k + \Omega_{\vec{k}} \times \frac{d\vec{k}}{dt}$$

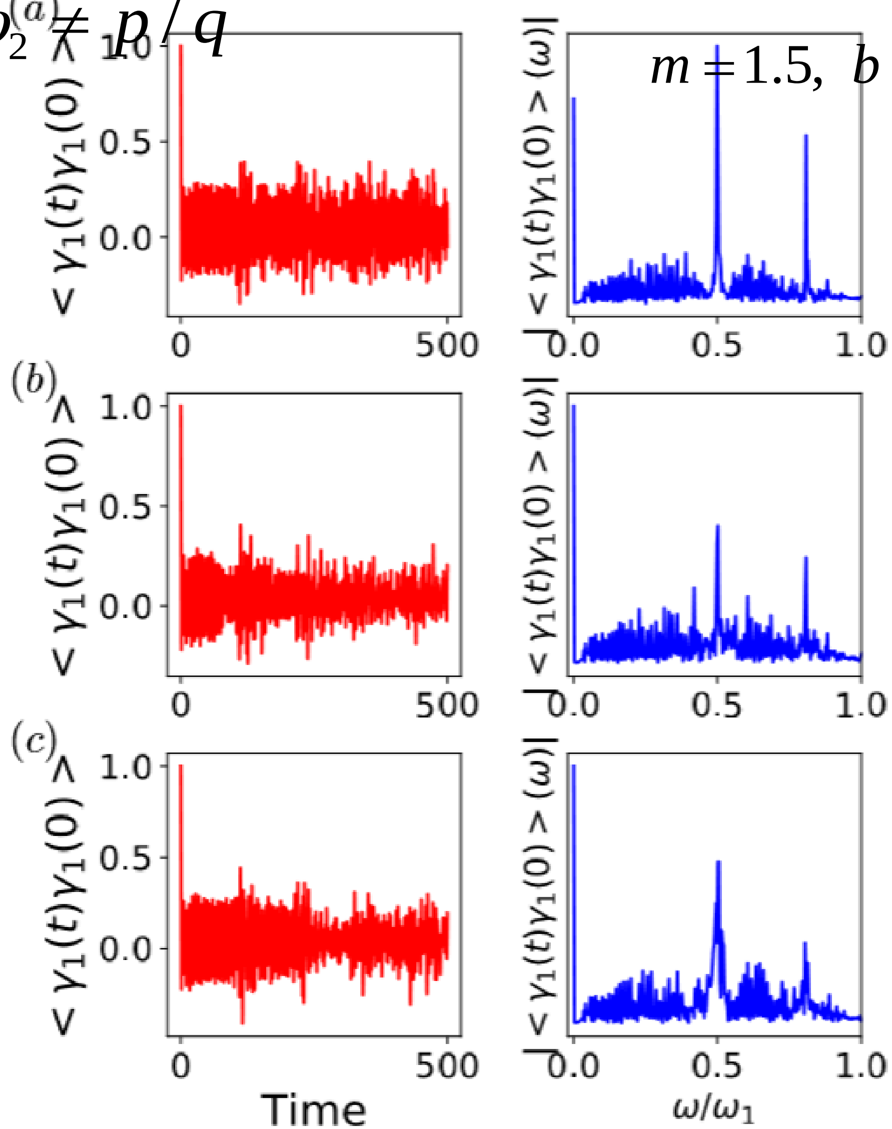
$$\left\langle \frac{dn_i}{dt} \right\rangle = \overline{\Omega}\varepsilon^{ij}\omega_j = \frac{C}{2\pi}\varepsilon^{ij}\omega_j$$

Time-dependent disorder

$$\omega_1 = 0.1,\ \ \omega_2 = 0.1\frac{\sqrt{5}+1}{2},$$

$$\omega_1 / \omega_2 \neq p / q$$

$$m = 1.5,\ \ b = 1$$



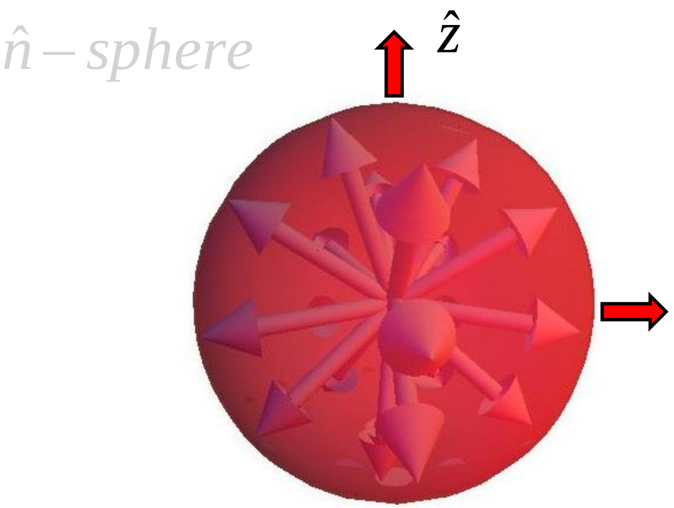
2D Topological phase

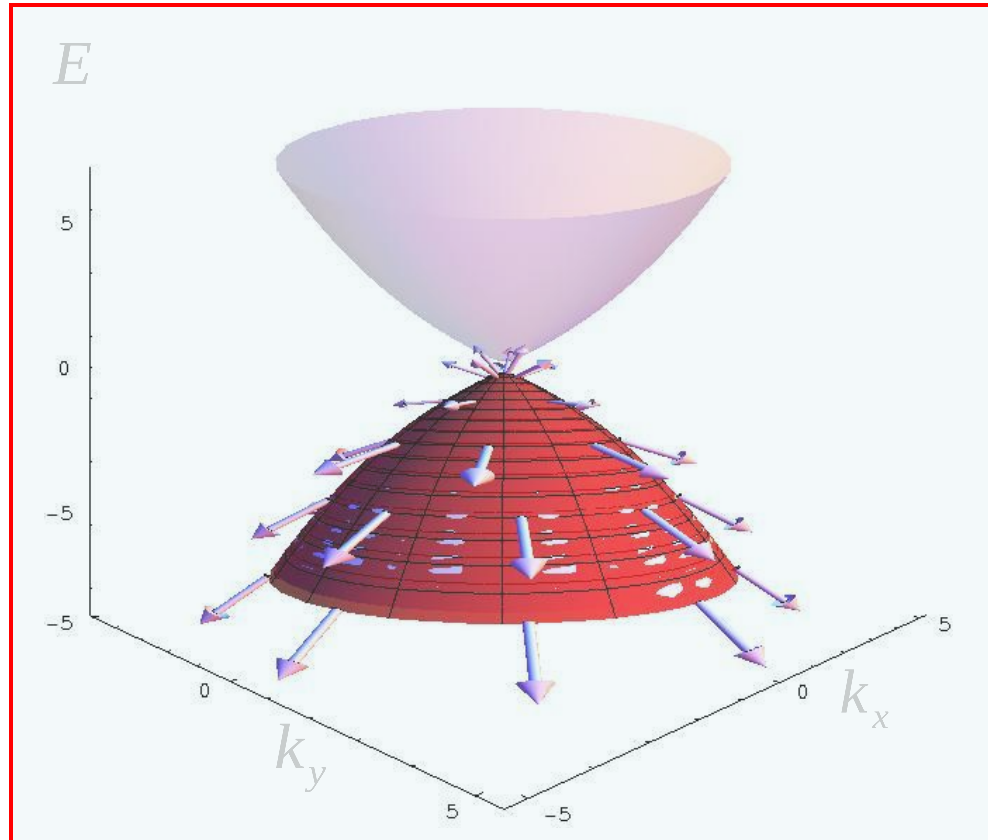
- Use BHZ band structure:

$$H = v_1 \sigma^x \sin(k_1) + v_2 \sigma^y \sin(k_2) + \left[m - b_1 \cos(k_1) - b_2 \cos(k_2)\right]\sigma^z$$

- BHZ magic: $H = \vec{\sigma} \cdot \vec{d}_{\vec{k}}$ $\qquad \hat{n} = \vec{d} / |\vec{d}|$

$$\sigma_{xy} = \frac{e^2}{4\pi h} \int_{p \in BZ} dk_x dk_y \ \hat{n} \cdot \left(\frac{\partial \hat{n}}{\partial k_x} \times \frac{\partial \hat{n}}{\partial k_y} \right)$$





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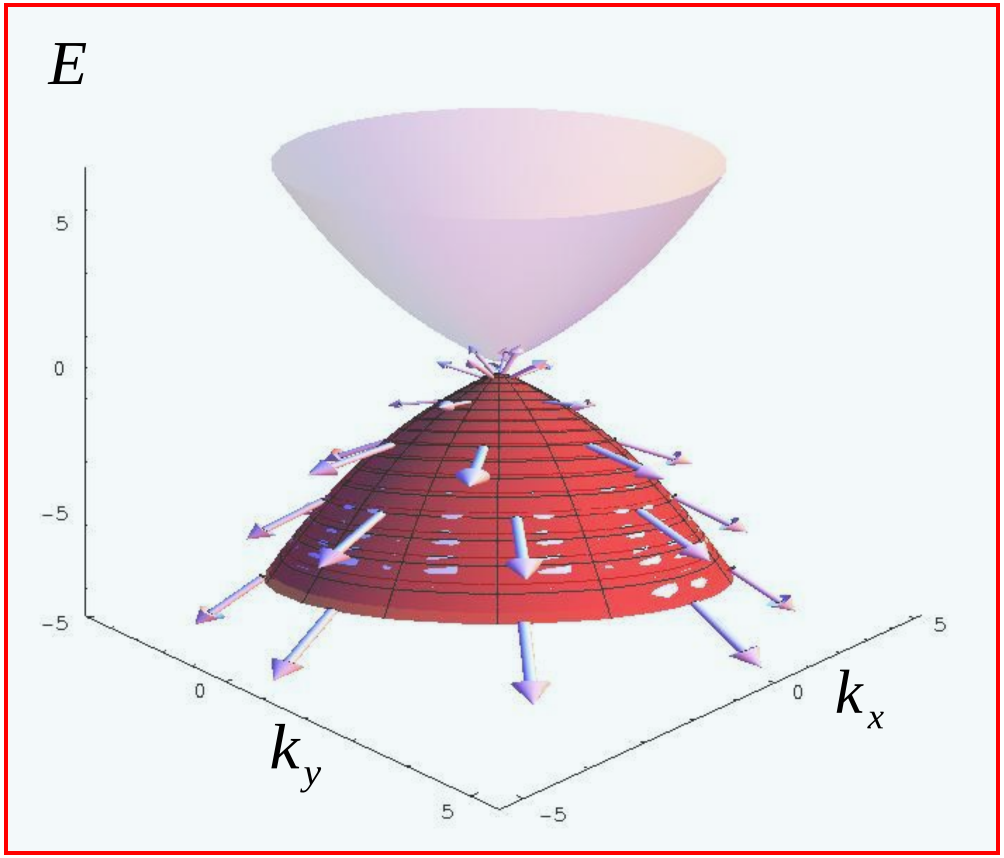
$$\Omega(k_1,k_2) = \nabla_k \times A_k$$

Berry curvature



Semiclassical motion

- Berry curvature

$$\vec{A}_k = -i\langle \psi_k | \nabla_{\vec{k}} | \psi_k \rangle \qquad \Omega(k_1, k_2) = \nabla_k \times A_k$$

$$\frac{d\vec{r}}{dt} = \nabla_k \varepsilon_k + \Omega_{\vec{k}} \times \frac{d\vec{k}}{dt}$$

- BHZ Berry curvature and motion: