Localization in Fractonic Random Circuits

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arXiv:1807.09776

Outline

1. Introduction to Fractons
    - Fracton Conservation Laws
    - Phenomenology

2. Introduction to Random Unitary Circuits
    - Operator Spreading, with and without Conservation Laws

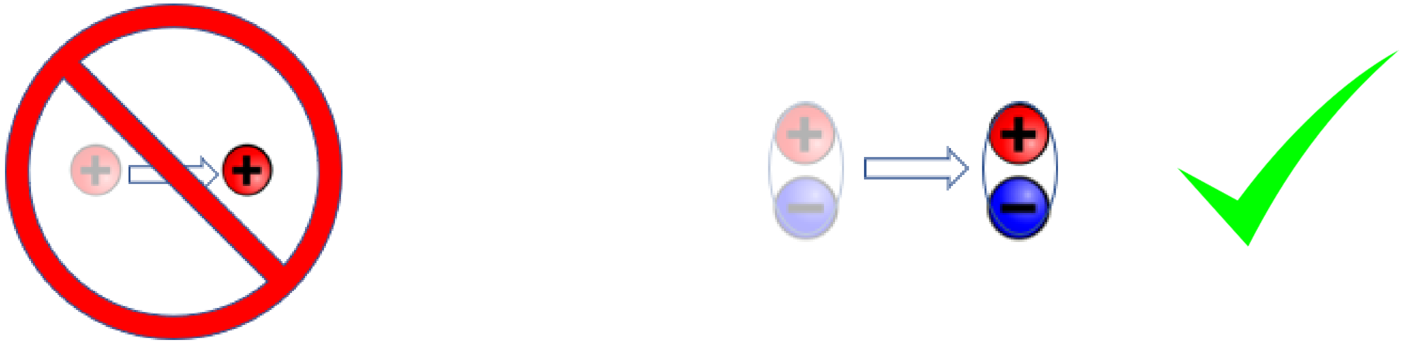
3. Fractonic Random Circuits
    - Localization of Fractons in Low Dimensions
    - New Universality Class for Operator Spreading

Part 1:

Introduction to Fractons

What are Fractons?

- Exotic type of quasiparticle excitations with severely restricted mobility
  - Strictly immobile in isolation, but can often move together in small groups



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1-dimensional particle

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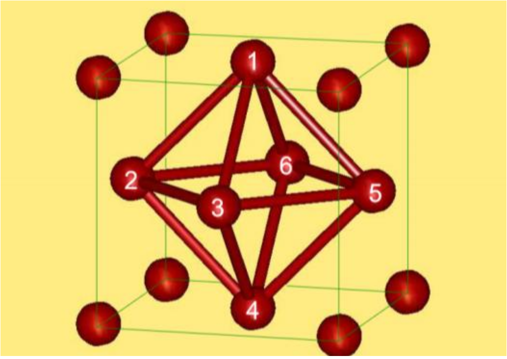
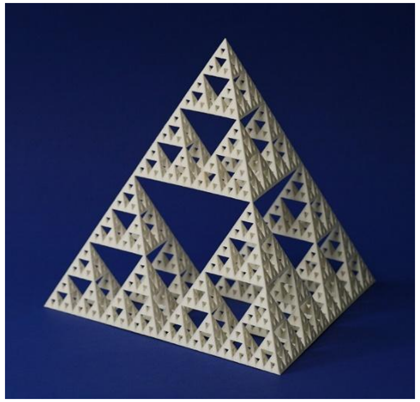
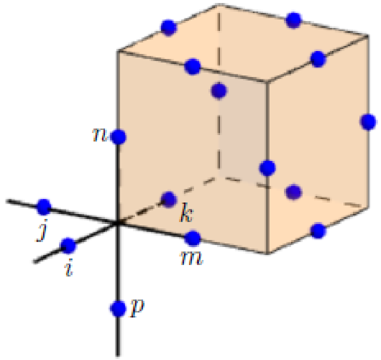


Topological Glassy Models
(Chamon, Castelnovo)
Phil. Mag. 29, 1 (2011)



Haah's Code
PRA 83, 042330 (2011)



X-Cube Model
(Vijay, Haah, Fu)
PRB 94, 235157 (2016)

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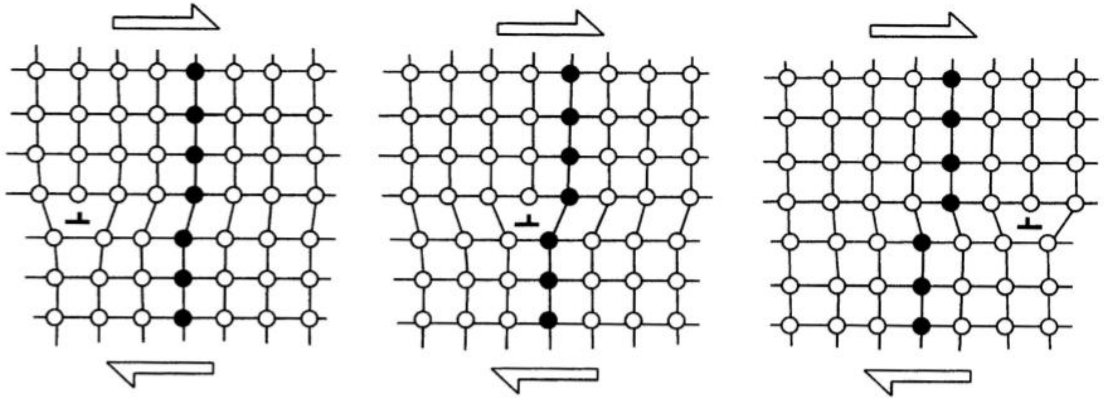
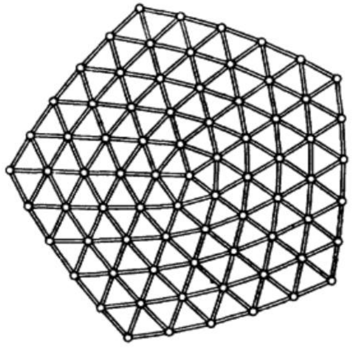
- Direct physical realization as lattice defects of quantum crystals





Fracton-Elasticity Duality
MP, Leo Radzihovsky
PRL 120, 195301

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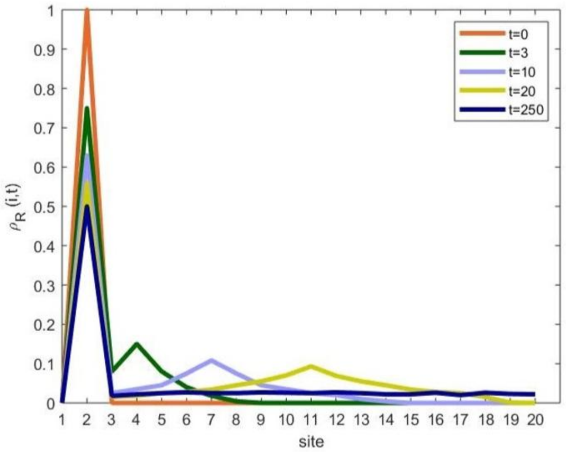
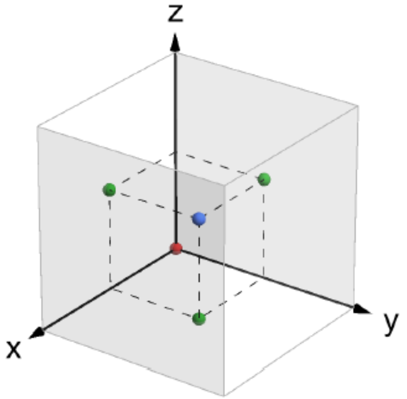
- Direct physical realization as lattice defects of quantum crystals
- Wide range of exotic phenomena
    - Pseudo-gravitational behavior
    - Generically exhibit glassy dynamics
    - In certain cases, **many-body localization**



Fracton Conservation Laws

Fracton models all feature exotic
conservation laws restricting charge motion

Lattice Rotor Models

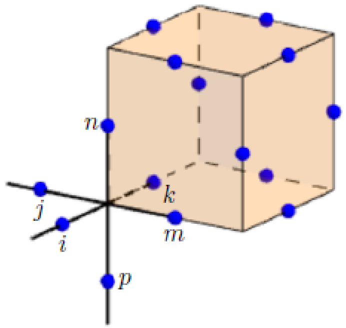


Conservation of charge
+ Conservation of dipole moment

(MP, PRB 95, 115139)

U(1) symmetric tensor gauge fields A_{ij}

X-Cube Model



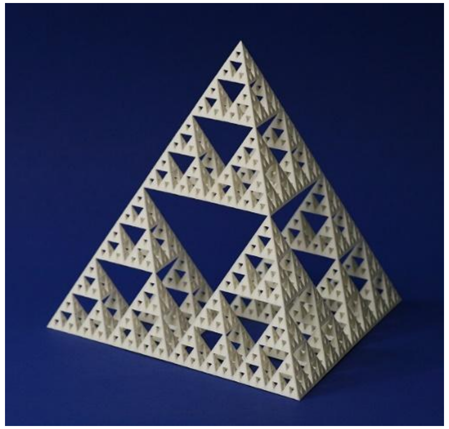
Z_2 higher moment conservation laws

Z_2 symmetric tensor gauge theory

(Ma, Chen, Hermele; Bulmash, Barkeshli)

(PRB 98, 035111) (PRB 97, 235112)

Haah’s Code



Exotic Z_2 gauge theory with highly restrictive conservation laws

(MP, PRB 95, 115139)

Example: The "Scalar Charge" Theory

U(1) symmetric tensor gauge field and electric field: A_{ij} , E_{ij}

Gauss's law: $\partial_i \partial_j E^{ij} = \rho$

Charge Conservation

$$Q = \int d^3x \; \rho = \oint \partial_i E^{ij} da_j$$

(Boundary term)

Total charge only changes by particles entering/leaving the system

Dipole Conservation

$$P^i = \int d^3x \; \rho x^i = \oint (x^i \partial_j E^{jk} - E^{ik}) da_k$$

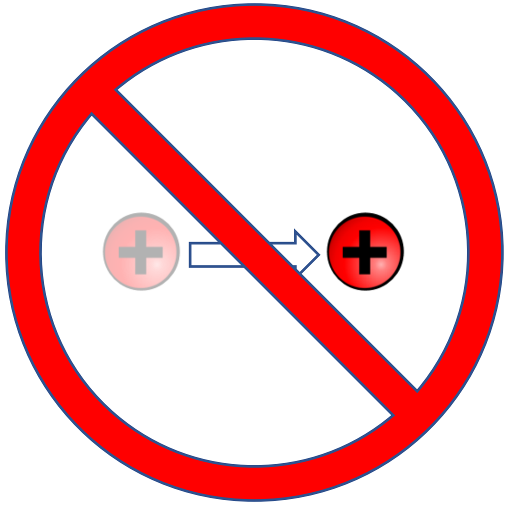
(Boundary term)

Total dipole moment only changes by particles entering/leaving the system

“Frozen” Charges

Moving a charge changes the dipole moment

Not allowed by conservation laws!

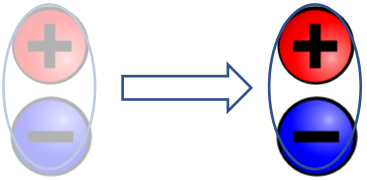


Particles are locked in place

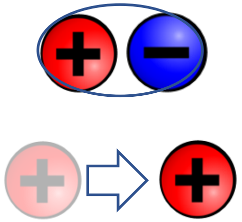
Fracton: immobile particle

Allowed processes:

Dipole motion:



Motion plus dipole creation:



Subdimensional Zoology

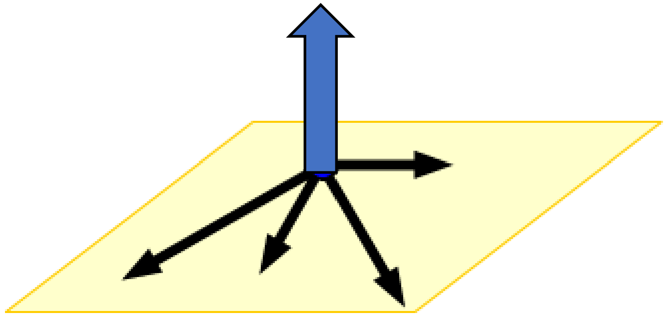
Fractons (0-dimensional particles)

Locked in place

1-dimensional particles

Motion along a line
(parallel to blue arrow)

2-dimensional particles



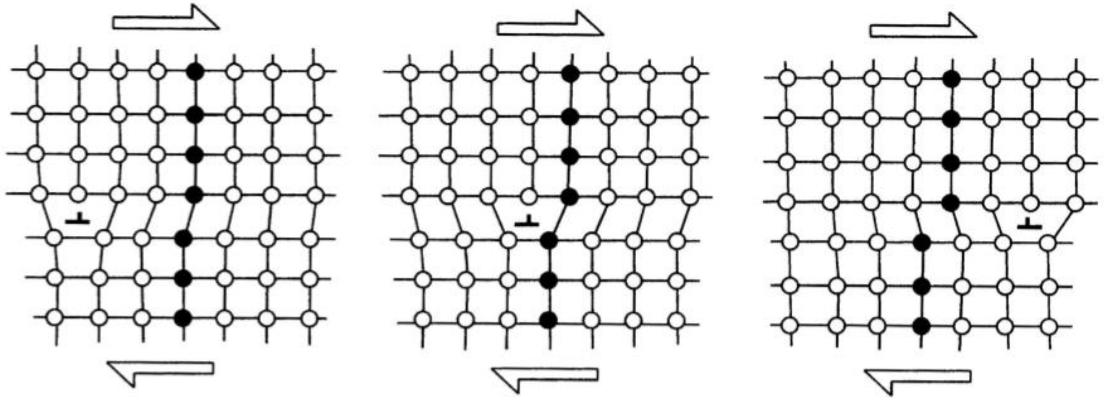
Motion within a plane
(perpendicular to blue arrow)

Physical Realization: Elasticity Theory

Fractons have already been observed as lattice defects of two-dimensional crystals

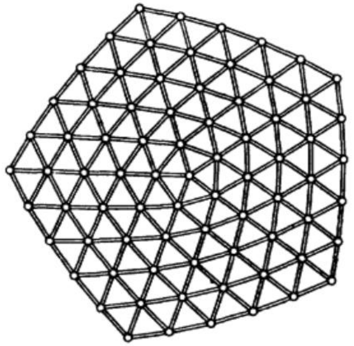
Fracton-Elasticity Duality
MP and L. Radzihovsky
PRL 120, 195301

Dislocation defects only move in the direction of their Burgers vector



1-dimensional particles

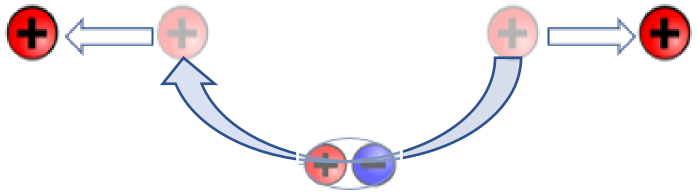
Disclination defects cannot move without creating extra dislocations



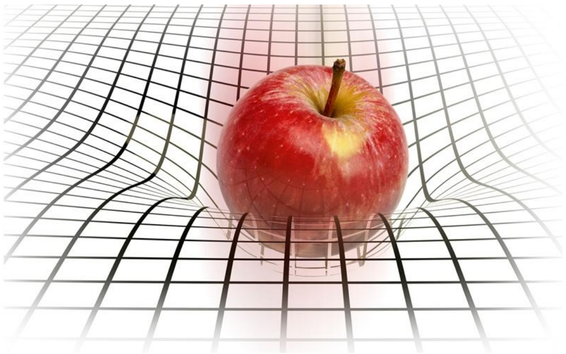
Fractons

Pseudo-Gravitational Behavior

- Fractons can push and pull each other through virtual processes



- Leads to a net attractive force between fractons
  - Motion along geodesic-like curves
  - Emergent gravitational behavior
  - Manifestation of Mach’s principle



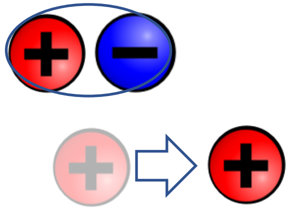
MP, PRD 96, 024051 (2017)

Fractons: The Central Science

- Fractons have deep connections with:
  - Elasticity theory (MP, Radzihovsky; Pai, MP)
  - Gravitation (MP)
  - Holography (Yan)
  - Deconfined quantum criticality (Han Ma, MP)
  - Subsystem symmetry protected topological phases (You, Devakul, Burnell, Sondhi)
  - Quantum Hall physics (Prem, MP, Nandkishore)
  - **Many-body localization** (Pai, MP, Nandkishore; Prem, Nandkishore, Haah)
  - …

Warm-Up: Glassy Dynamics

- Dipole conservation severely restricts motion of particles. However, fractons can move through interactions with thermal dipoles



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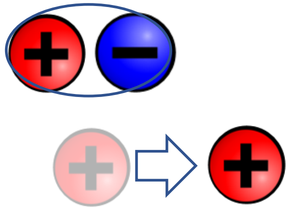
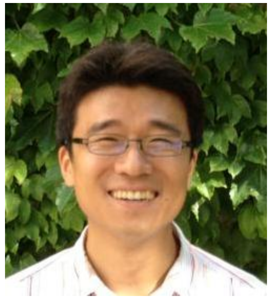


- In 3d, interactions eventually cause the system to thermalize, BUT:
  - Logarithmic relaxation to equilibrium
  - Glassy dynamics/asymptotic MBL in a translation invariant system
  - In certain systems (e.g. Haah’s code), relaxation time is <u>superexponential</u> in the inverse temperature

PRB 95, 155133 (2017)

Abhinav Prem

Rahul Nandkishore



Jeongwan Haah

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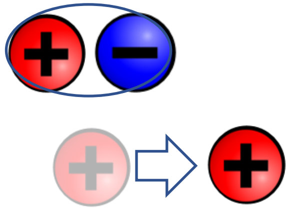

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  - Logarithmic relaxation to equilibrium
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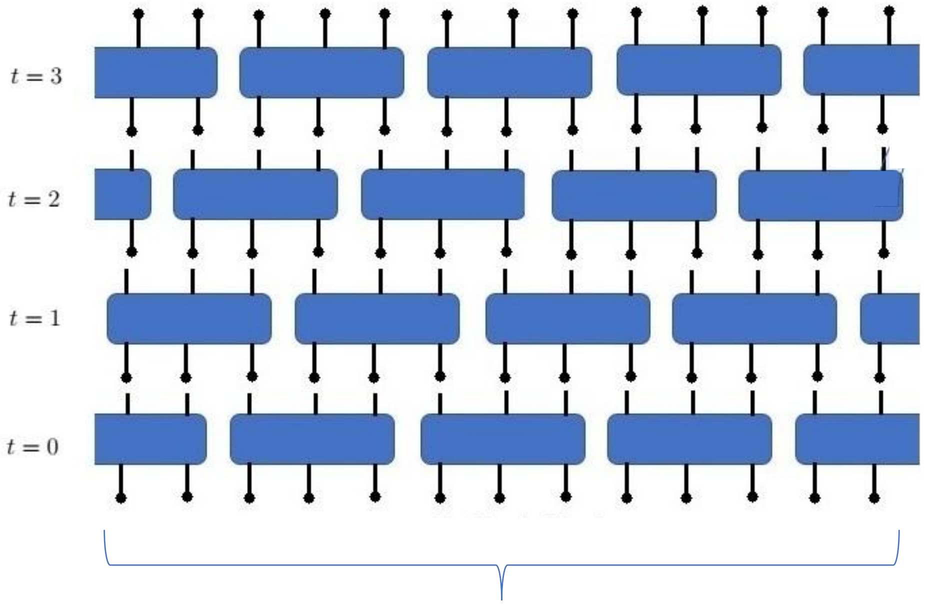
  - In certain systems (e.g. Haah's code), relaxation time is superexponential in the inverse temperature
- In low dimensions, the story changes substantially
  - Most easily studied in the context of random unitary circuits

Part 2:
Introduction to Random Unitary Circuits

Random Unitary Circuits



Input state

Nahum, Ruhman, Vijay, Haah
PRX 7, 031016 (2017) and PRX 8, 021014 (2018)

von Keyserlingk, Rakovszky, Pollman, Sondhi
PRX 8, 021013 (2018)

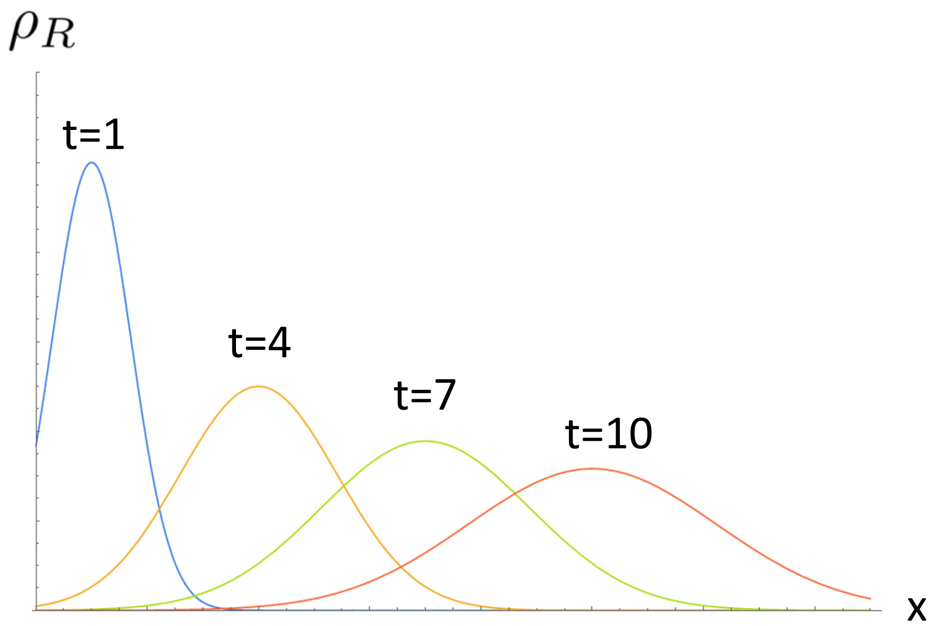
- Consider time-evolving a quantum state by acting with a circuit of randomly chosen local unitary gates
- Least constrained form of local unitary time evolution, without any conservation laws (even energy)
- Numerically (and in certain cases analytically) tractable setting for studying entanglement growth, operator spreading, and quantum chaos

Random Unitary Circuits

- Ballistic spreading of initially local operators under Heisenberg evolution:

$$O(t) = U^\dagger(t) O U(t)$$

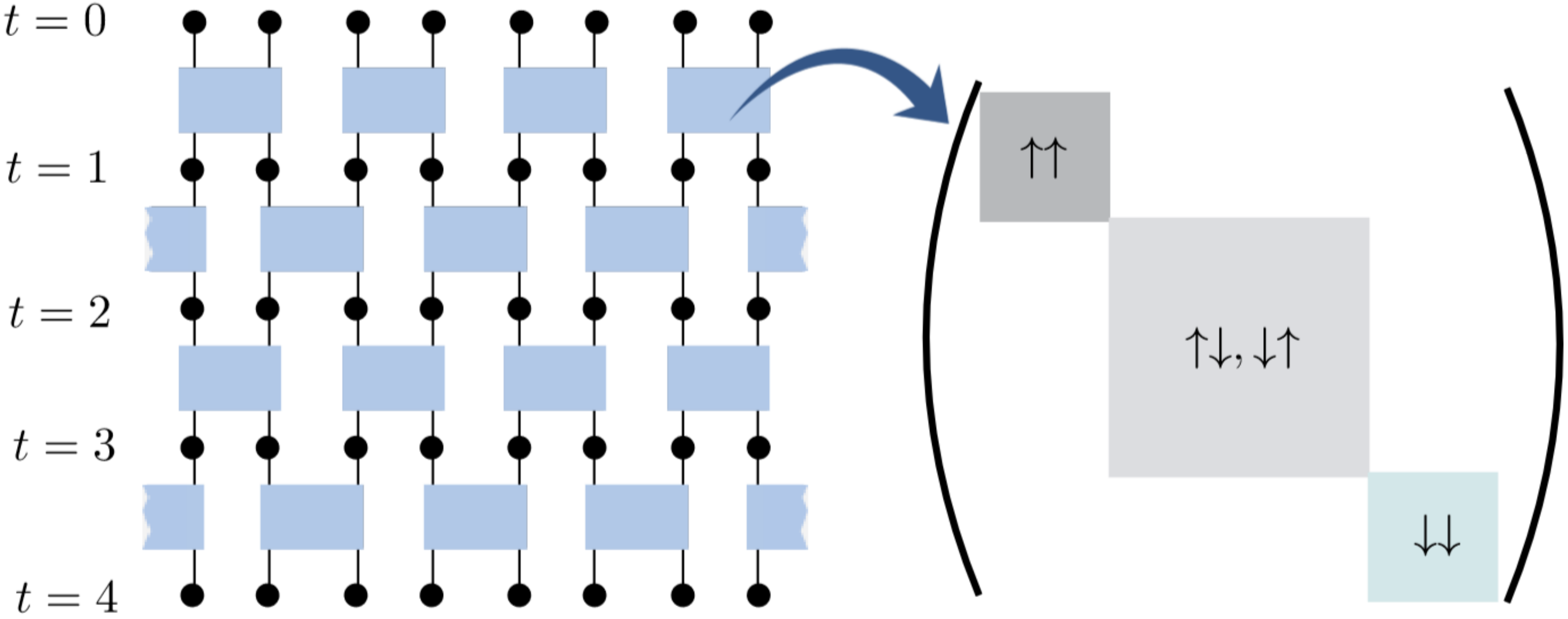
- Simple hydrodynamic description in terms of biased diffusion equation



“Right-weight” profile of an initially local operator

Constrained Random Unitary Circuits

Now consider a random circuit where the unitary gates are restricted to obey a conservation law

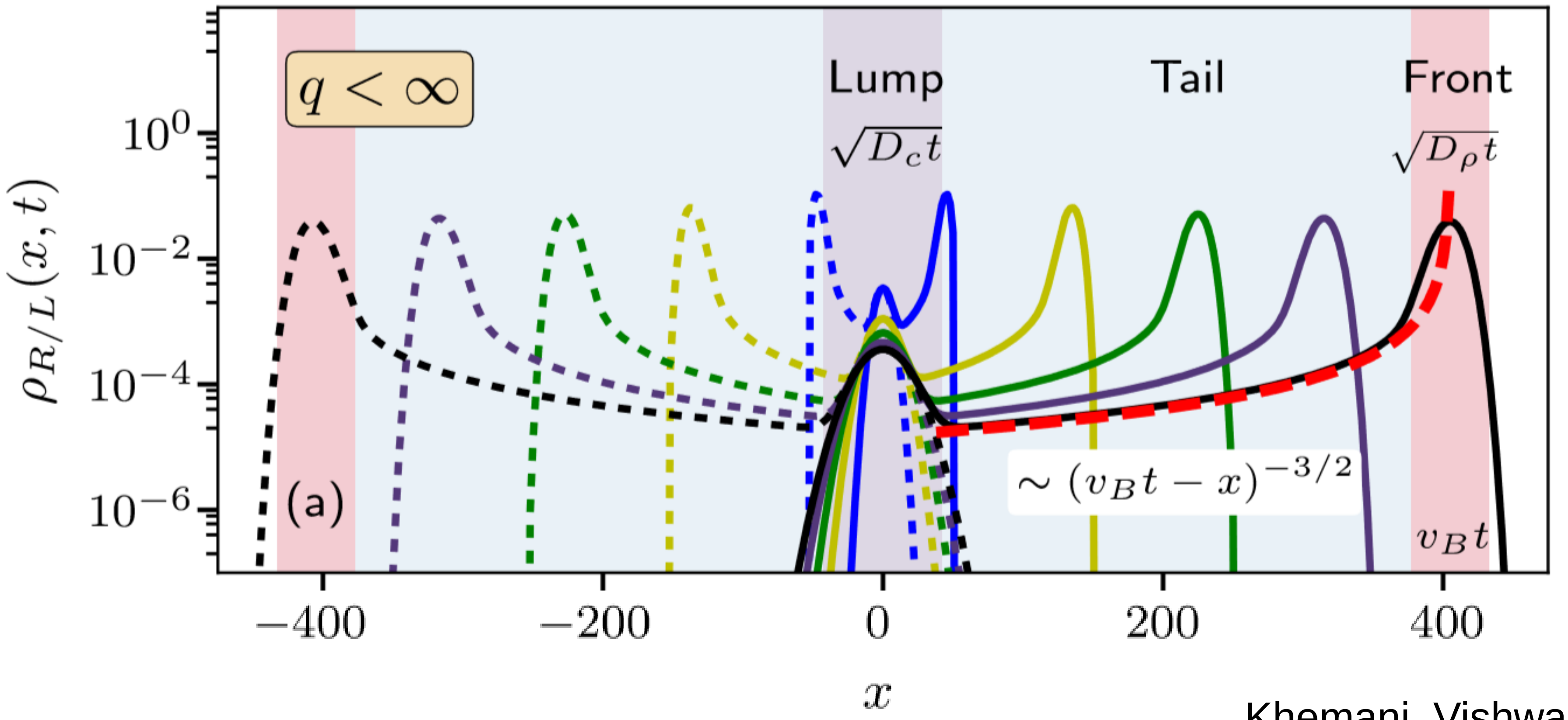


Khemani, Vishwanath, Huse: arXiv:1710.09835

Rakovszky, Pollman, von Keyserlingk: arXiv:1710.09827

Constrained Random Unitary Circuits

Operator spreading profiles feature both a ballistic peak (with power law tail) and a decaying stationary “lump”



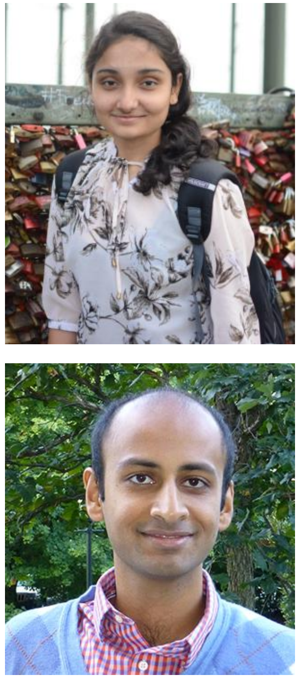
Khemani, Vishwanath, Huse:

Part 3:

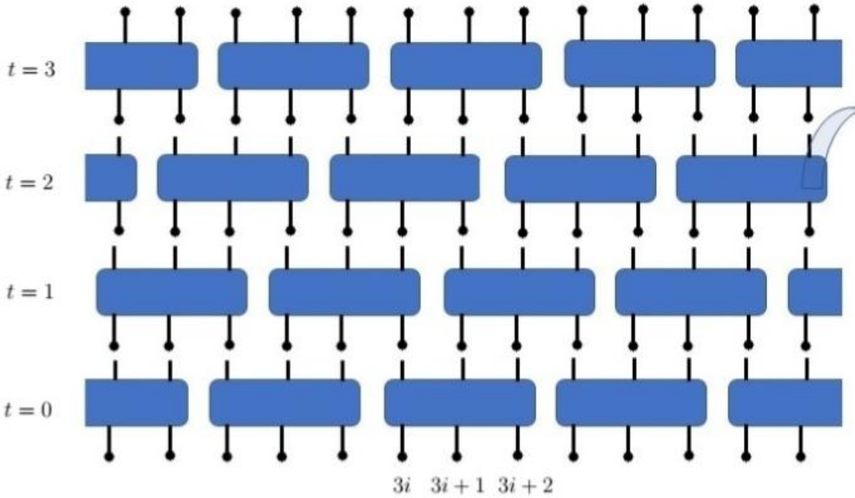
Fractonic Random Circuits

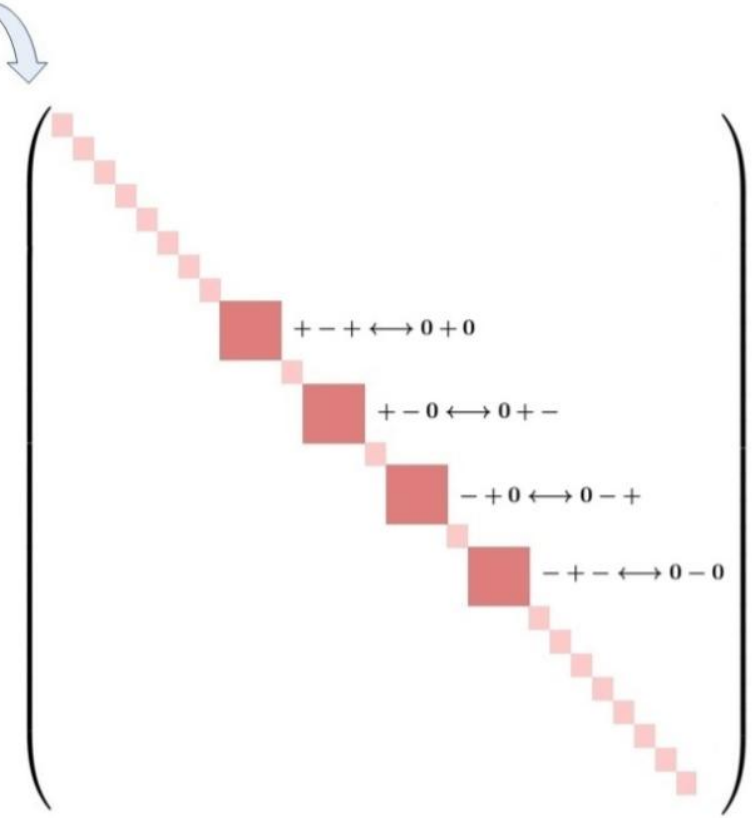
Fractonic Random Circuits

Consider a random unitary circuit with fracton conservation laws
(i.e. conservation of dipole moment)



Shriya Pai, MP, and
Rahul Nandkishore
(arXiv:1807.09776)





- Can be efficiently implemented in an S=1 spin chain
- Impose conservation of “charge” (S_z) and its dipole moment

A Few Technical Details

- Expand time-evolved operator in terms of “strings” of Gell-Mann matrices:

$$O(t) = \sum_S a_S(t) S \qquad S = \prod_i \Sigma_i^{\mu_i}$$

(9^L total strings)

- Define “right weight” as density of right endpoints of strings:

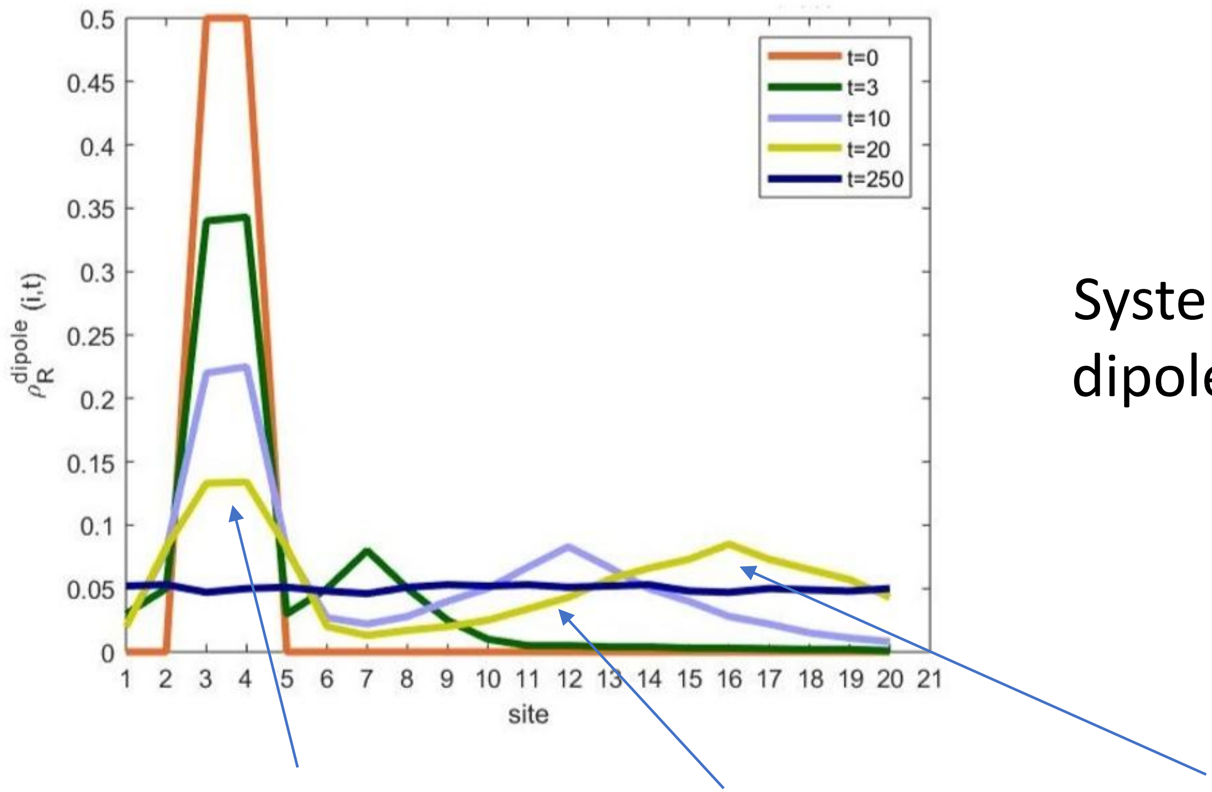
$$\rho_R(i,t) = \sum_{\substack{\text{strings } S \text{ with} \\ \text{rightmost non-identity} \\ \text{at site } i}} |a_S(t)|^2$$

Unitarity $\Rightarrow$ $\sum_S |a_S(t)|^2 = 1$ $\Rightarrow$ $\sum_i \rho_R(i,t) = 1$

(Conserved density)

Diffusive Spreading of Dipole Operators

Dipole operator acts like an ordinary conserved charge,
eventually reaching thermal equilibrium

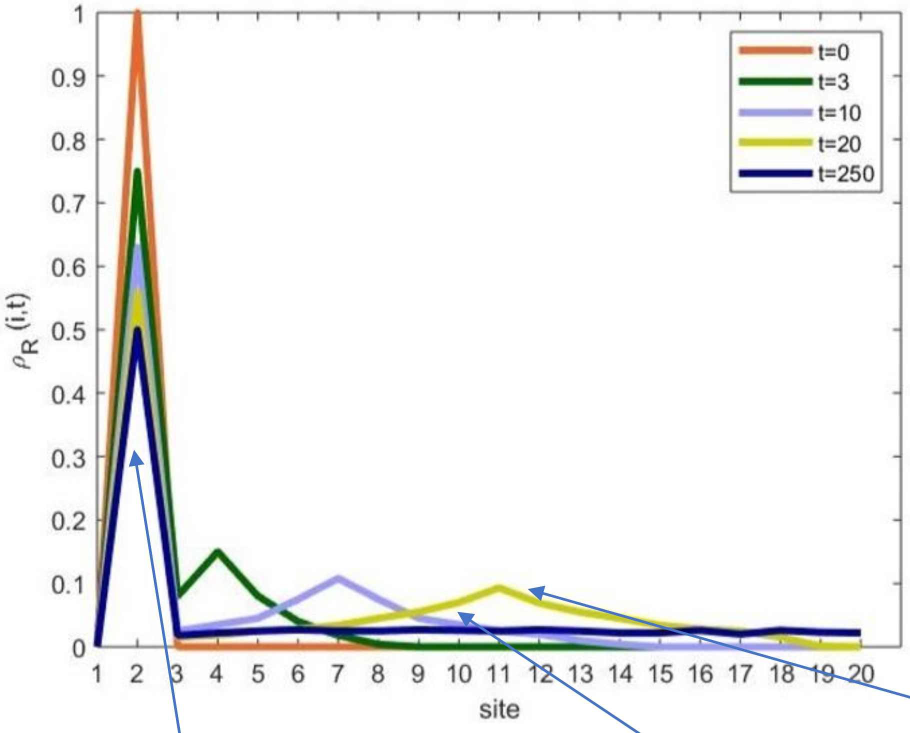


System initialized with
dipole on sites 3 and 4

Right weight features diffusive lump, power-law tail, and ballistic front

Localization of Fracton Operators

Fracton operators are more constrained, and maintain a permanent memory of initial conditions

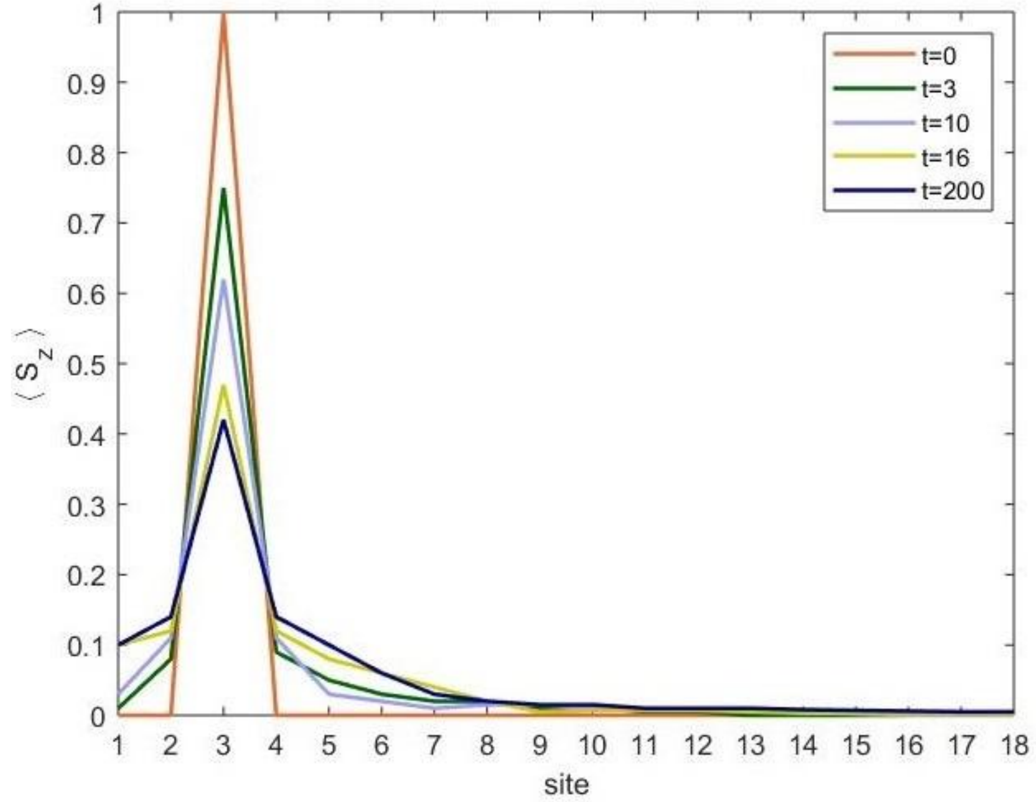


System initialized with fracton on site 2

Right weight features permanent peak, power-law tail, and ballistic front

Localization of Fracton Operators

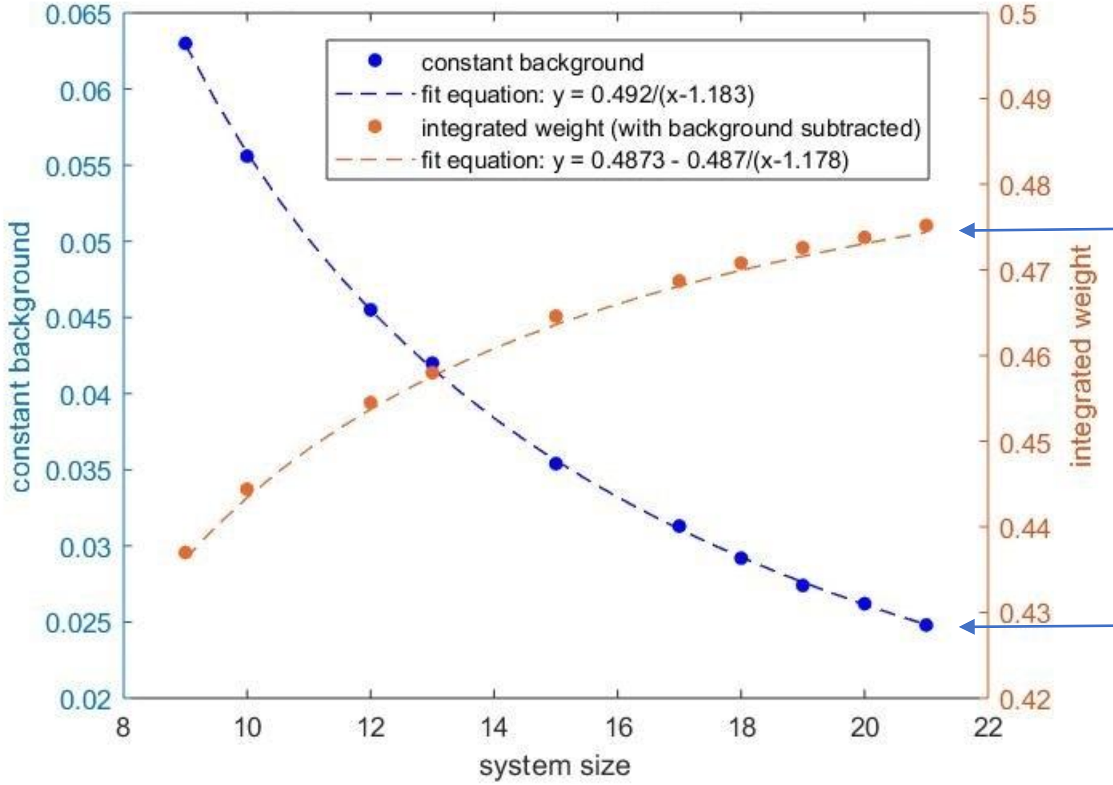
Permanent peak also present in S_z expectation values, though without a ballistic peak



Exponentially decaying distribution around initial position

Localization of Fracton Operators

Integrated weight in the permanent peak remains finite,
even in the thermodynamic limit

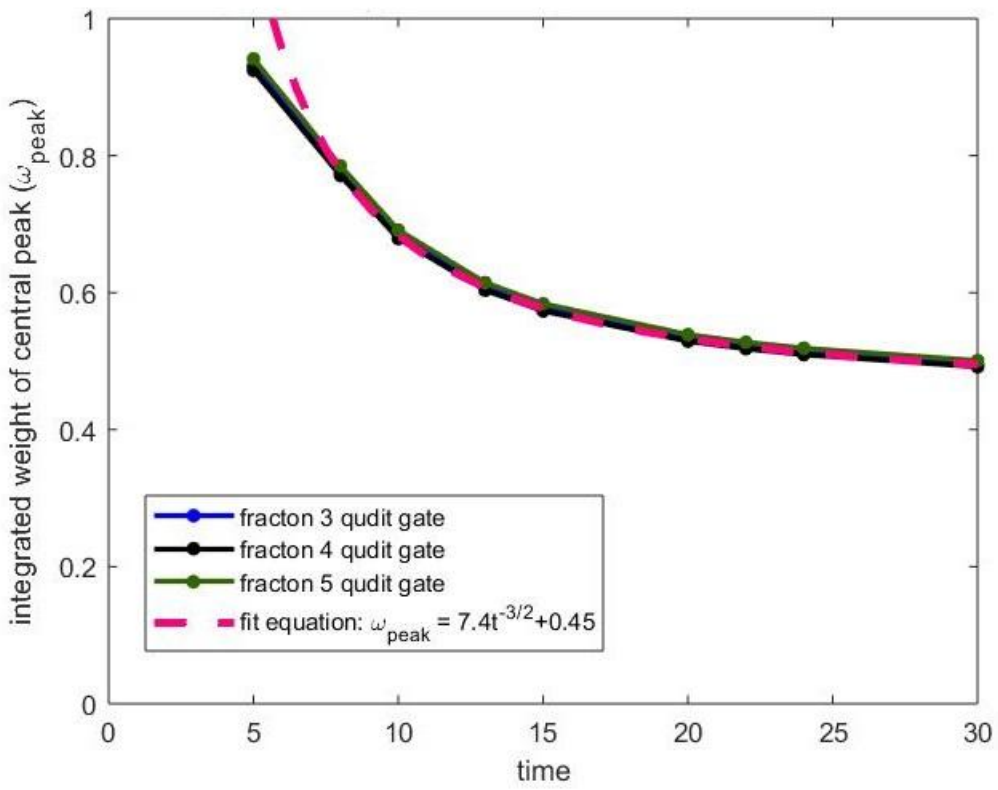


Weight in peak asymptotes
to a constant

Finite background decays to zero

Localization of Fracton Operators

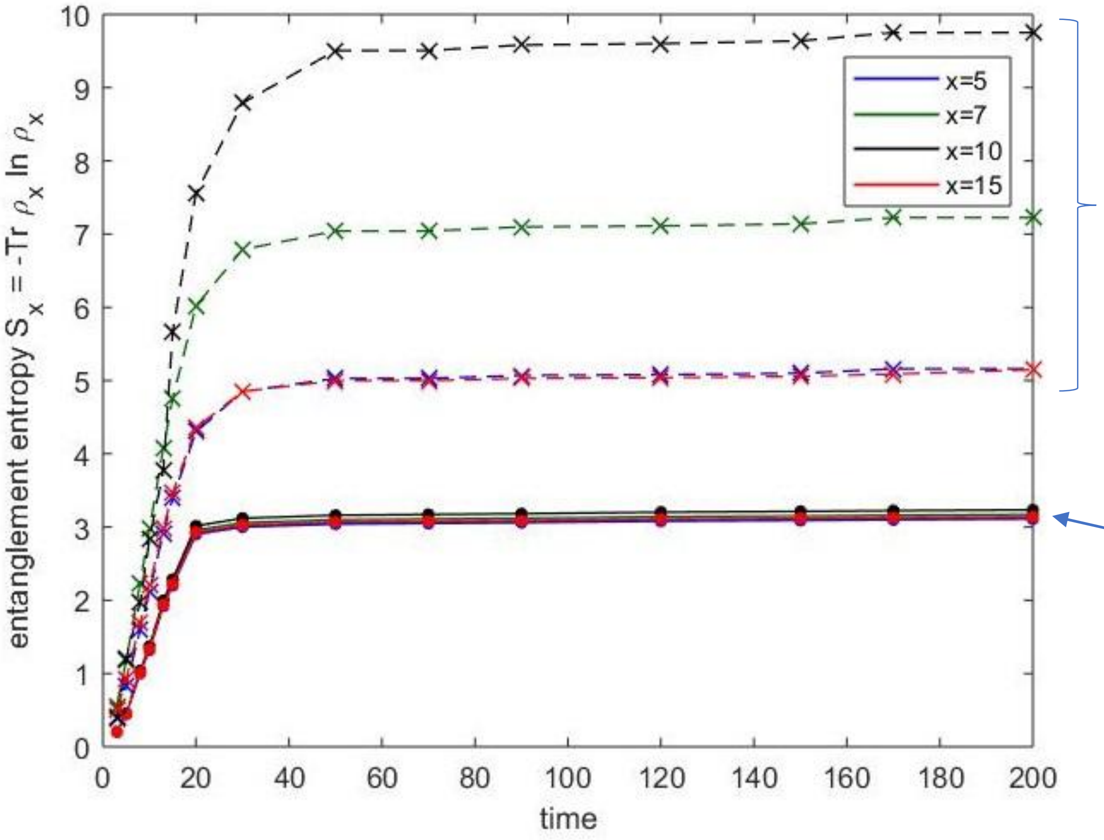
Integrated weight in the permanent peak is independent of the size of gates used in the circuit



3-site, 4-site, and 5-site gates all lead to the same integrated weight in the permanent peak at long times

Localization of Fracton Operators

Steady state exhibits area law entanglement, indicating non-thermal behavior

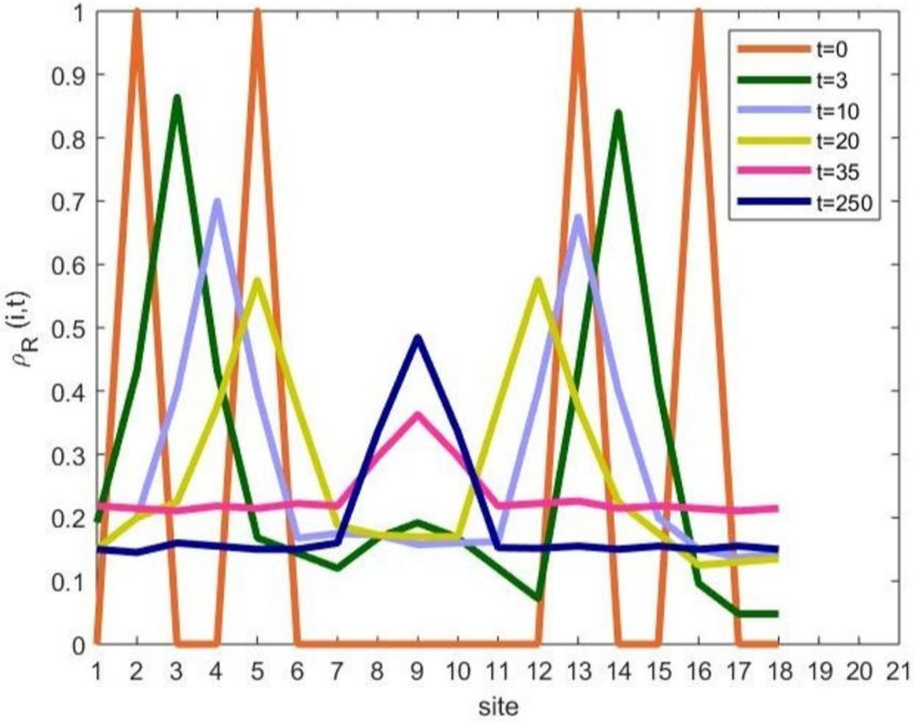


Without dipole conservation, steady state entanglement entropy grows with system size

With dipole conservation, entanglement entropy asymptotes to constant (i.e. 1d area law)

Finite Density of Fractons

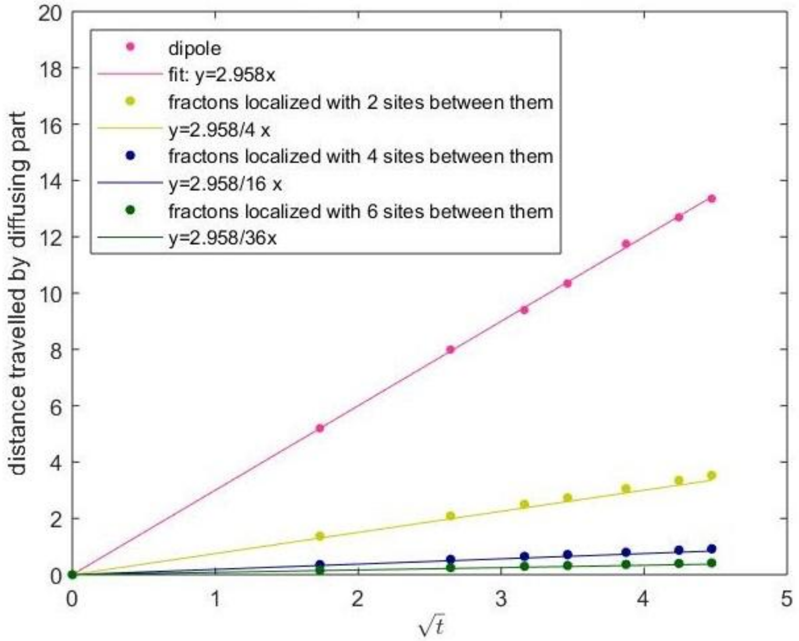
- At finite density, fractons can undergo permanent moves by exchanging dipoles
- Fractons form a cluster at their "center of mass":



Consequence of "gravitational" attraction between fractons

Finite Density of Fractons

Diffusion constant of fractons depends on the distance between them, since dipoles must propagate between two fractons



(a) Diffusion constant of fractons in a two-fracton system is lower than that of the dipoles by a factor of $1/l^2$, where l is the initial separation between the fractons.

Localization of Fracton Operators

So what’s going on?

- Fractons move via emission of dipoles
- Dipoles diffuse, undergoing a random walk

Localization of Fracton Operators

So what's going on?

- Fractons move via emission of dipoles
- Dipoles diffuse, undergoing a random walk
- In d=1 and d=2, random walks always return to the origin:

$$\int dt\ G(0,t) = \int dt \left(\frac{1}{\sqrt{4\pi D t}} \right)^d = \frac{2^{1-d} t^{1-d/2}}{(2-d)(\pi D)^{d/2}}$$

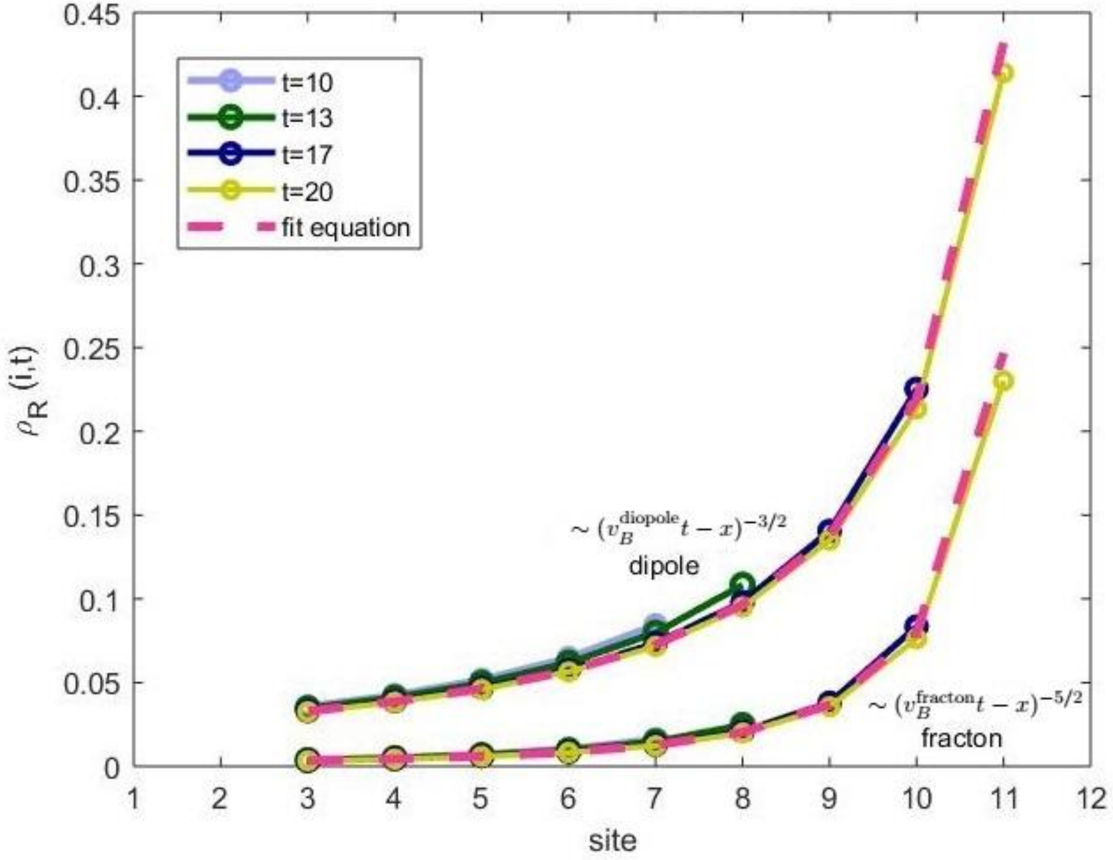
- Any motion of a fracton is eventually undone, as the emitted dipole is always reabsorbed
- Arguments generalize to Floquet and Hamiltonian dynamics, without the need for disorder, leading to many-body localization in a translationally-invariant system

A New Type of Localization

- Fracton localization does not rely on disorder
- Does not rely on locator expansion (no conserved energy)
- Robust against noise
- True localization in two dimensions
- Features only a finite number of conservation laws (non-integrable)

New Universality Class for Operator Spreading

With fractonic initial conditions, ballistic front features a modified power-law tail, with a different exponent from the dipole case



- Power of 3/2 for dipoles matches with properties of ordinary conserved charges
- Modified power of 5/2 for fracton initial conditions requires a different explanation

New Universality Class for Operator Spreading

- Can be understood through simple hydrodynamic description:

Fracton position

Dipole density

Random noise

$$\frac{dR}{dt} = -\eta + A(t)$$

$$\frac{d\eta}{dt} = D\nabla^2\eta + \frac{dR}{dt}\,\delta(x-R)$$

- Dipoles undergo diffusion in the presence of a sink at the position of the fracton
- This previously studied problem immediately gives rise to the modified 5/2 power law in the tail of the front

Conclusions

- In low dimensions (d=1,2), fracton systems maintain a permanent memory of their initial conditions
- Provides an example of true many-body localization in a translationally invariant system, even at finite energy density
- Consequence of properties of low-dimensional random walks, which does not rely on a locator expansion
- Future directions:
  - Detailed analyses on Hamiltonian and Floquet evolution in low-dimensional fracton systems
  - Three-dimensional fracton models exhibiting localization physics?
  - Long-range interactions?
  - Emergent fractonic constraints from disorder?