“Full Counting Statistics” after Quantum Quenches

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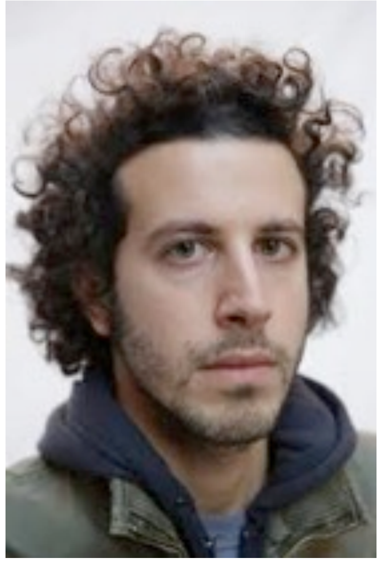
EPSRC Engineering and Physical Sciences Research Council

KITP, August 2018

Joint work with



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Probability Distributions of Observables

In many-body systems we typically focus on **expectation values** in some state or density matrix, e.g.

$$\langle\Psi|\mathcal{O}(x,t)|\Psi\rangle \qquad \langle\Psi|\mathcal{O}_1(x,t)\mathcal{O}(x',t')|\Psi\rangle$$

Correspond to averages over many measurements.

Cold atoms: access to **probability distributions** of observables $\mathcal{O}$

$$\mathcal{O}|n\rangle = \lambda_n|n\rangle$$

eigenstates of $\mathcal{O}$

$$P_{\mathcal{O},|\Psi\rangle}(m) = \sum_n |\langle n|\Psi\rangle|^2\delta(m-\lambda_n) = \langle\Psi|\delta(\mathcal{O}-m)|\Psi\rangle$$

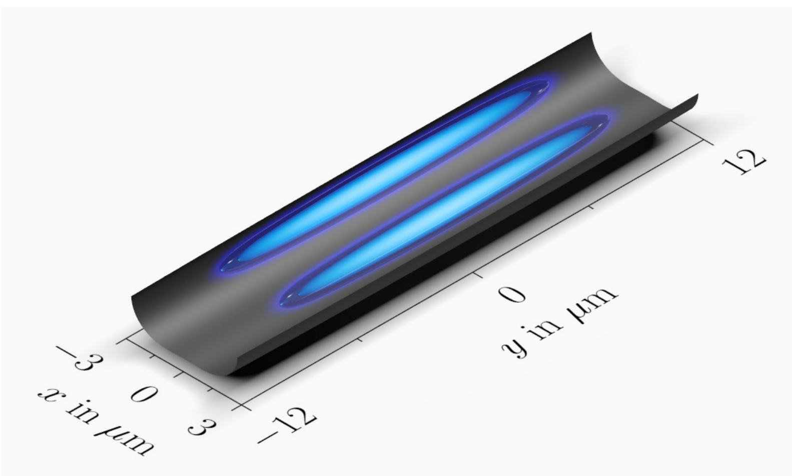
Probability for measuring $\mathcal{O}$ in $|\psi\rangle$ returning value m

Probability distributions after quantum quenches

Schmiedmayer group '10-'17

split Bose gases

$$H = \sum_{a=1}^{2} \int dx \left[\frac{1}{2m} \partial_x \Psi_a^\dagger(x) \partial_x \Psi_a(x) + g \Psi_a^\dagger(x) \Psi_a^\dagger(x) \Psi_a(x) \Psi_a(x) \right]$$



$t = 6\,\mathrm{ms}$

initial state $|\Psi(0)\rangle$

Low energies: $\Psi_{1,2}(x) \propto e^{i\Phi_s(x) \pm i\Phi_a(x)}$

$$\mathscr{H} = \sum_{j=a,s} \frac{v}{2\pi} \int dx \left[K\big(\partial_x \Phi_j(x)\big)^2 + \frac{1}{K}\big(\partial_x \Theta_j(x)\big)^2 \right]$$

In weak interaction limit measurements allow to extract

$$P_{\mathcal{O},|\Psi(t)\rangle}(m) = \langle \Psi(t) | \, \delta\Big(\int_0^{\ell} dx \; e^{i\phi_a(x)} - m \Big) \, | \Psi(t) \rangle$$

Gritsev et al '06
Kitagawa et al '10,...

Very few other results either in, or out of, equilibrium…

A. Can we find situations where probability distributions give insights significantly beyond expectations values/variances ?

B. Can probability distributions be calculated analytically ?

Consider **lattice spin models** $\Longrightarrow$ natural observables are operators $\mathcal{O}$ (quantized eigenvalues) that act on sub-systems of linear size ℓ, e.g. **sub-system magnetization**;

When do we expect (non) trivial prob. distr.?

In states with finite correlation length ξ and $\xi \ll \ell$ usual "thermodynamic" arguments apply

$$P_{\mathcal{O}}(m) = \langle \Psi | \delta(\mathcal{O} - m) | \Psi \rangle = \sum_r P_w(r) \delta\big(m - r\big)$$

$\approx$ Gaussian for large ℓ

Cases with (i) $\xi \to \infty$ or (ii) $\xi \gtrsim \ell$ will be most interesting.

(i) D=1: quantum critical GS ($\to$ equilibrium) or long-range int.
(ii) Energy density after QQ should not be too large.

A. Melting of LRO after a "Quantum Quench"

M. Collura, FHLE
in preparation

- Consider a spin-1/2 chain with Hamiltonian

$$H = J \sum_{j=1}^{L} S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta\, S_j^z S_{j+1}^z$$

integrable, but essence of what follows has nothing to do with it.

- Prepare the system at time t=0 in a classical Néel state

$$|\Psi(0)\rangle = |\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\ldots\rangle$$

- AFM Long-range order $\langle\Psi(0)|\sum_j (-1)^j S_j^z |\Psi(0)\rangle \neq 0$

- time-evolve with H $|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle$

Consider the PD of AFM short-range order

$$N_\ell^z = \sum_{j=1}^{\ell} (-1)^j S_j^z$$

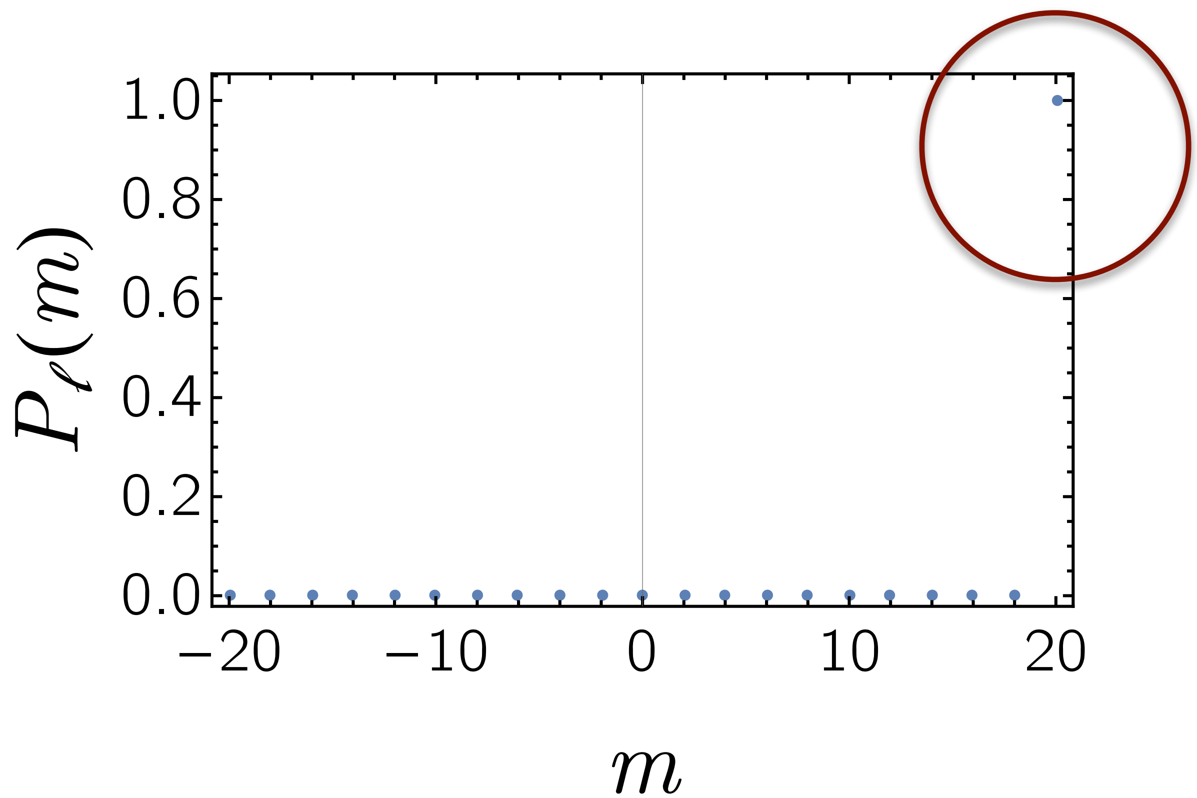
$$P_{N_\ell^z, |\Psi(t)\rangle}(\mu) = \langle \Psi(t) | \delta(N_\ell^z - \mu) | \Psi(t) \rangle = \int_{-\infty}^{\infty} \frac{d\theta}{2\pi} e^{-i\mu\theta} \langle \Psi(t) | e^{i\theta N_\ell^z} | \Psi(t) \rangle$$

$$\equiv \sum_{m \in \mathbb{Z}} P_\ell(m)\, \delta(\mu - m) \qquad (\ell \text{ even})$$

→ require $\langle \Psi(t) | e^{i\theta N_\ell^z} | \Psi(t) \rangle$

Initially depends on time, but eventually relaxes to a stationary value (**"local relaxation"**) as $e^{i\theta N_\ell^z}$ is a local operator.

Probability distribution in **initial state (t=0):**



→ delta-function at m=ℓ/2

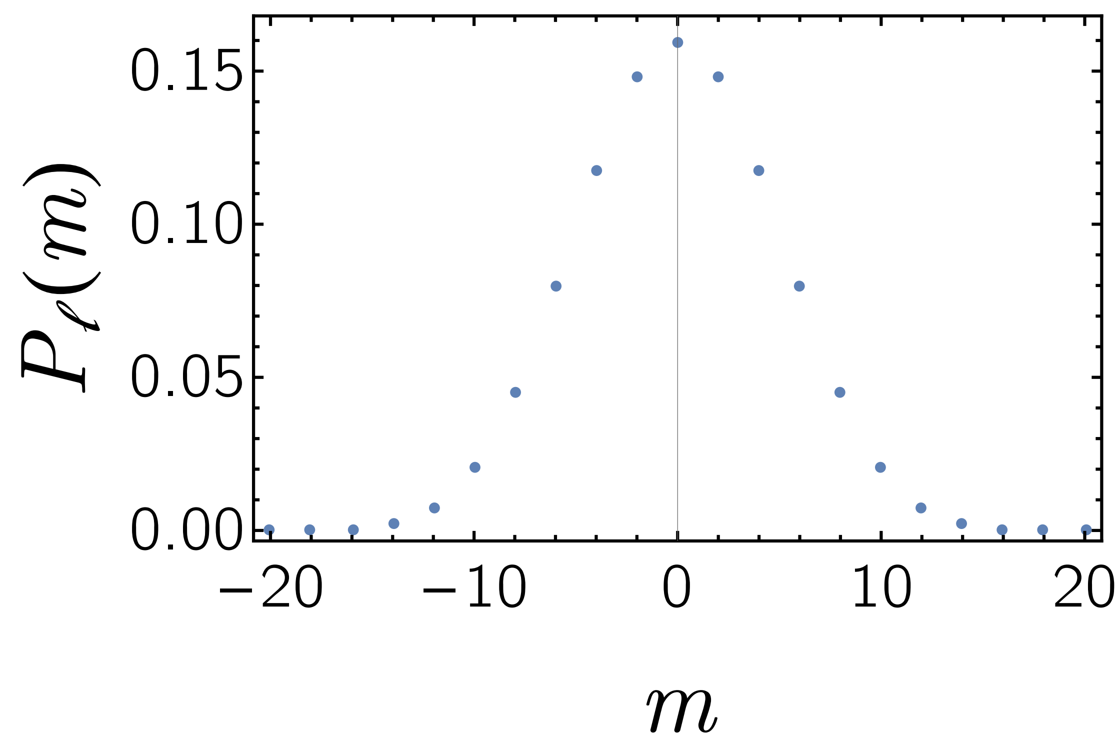
What do we expect in the stationary state?

Stationary State:

Finite correlation length ξ

$\xi < \ell$

SRO has melted



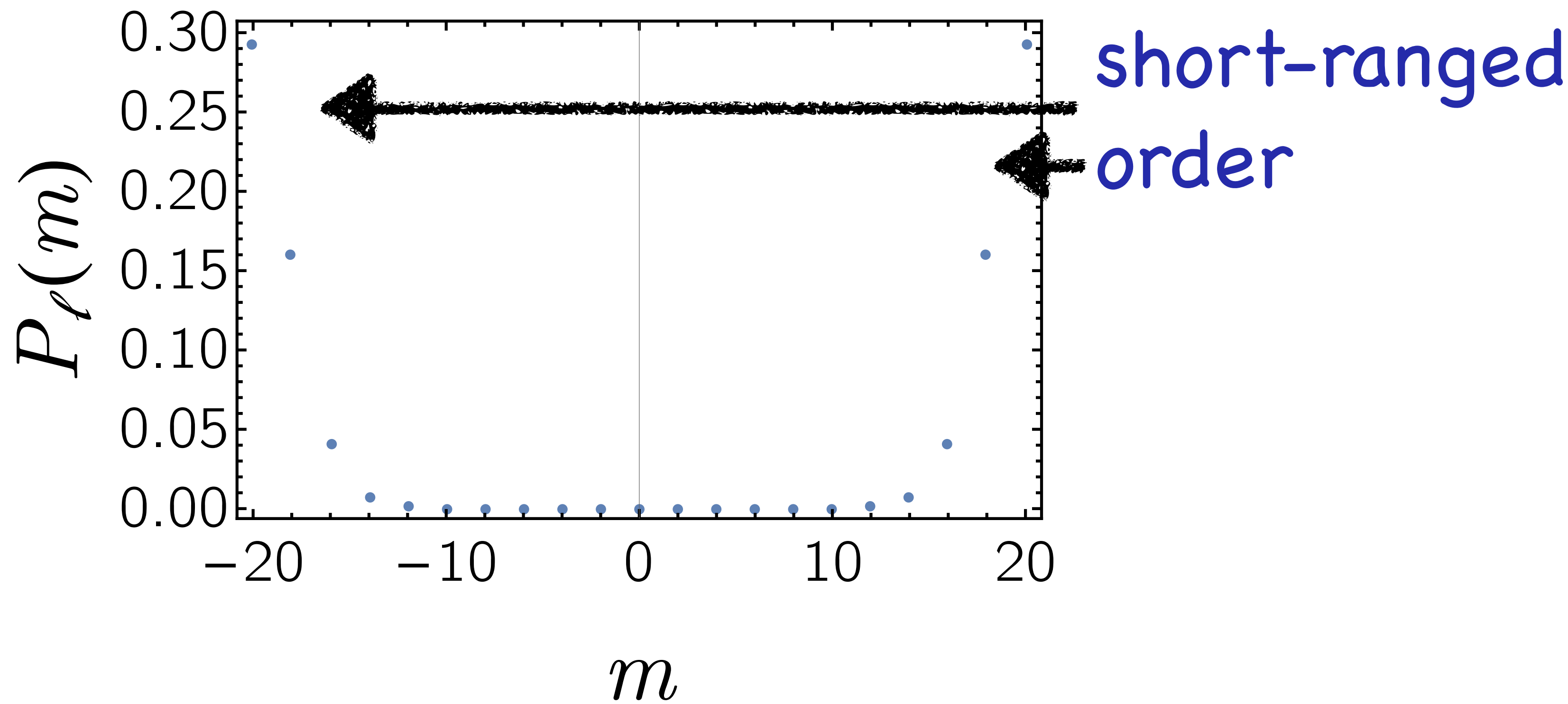
part. relevant for "large quenches"

Stationary State:

Finite correlation length ξ

$\xi > \ell$

SRO remains, but spin-flip symmetry should be restored.

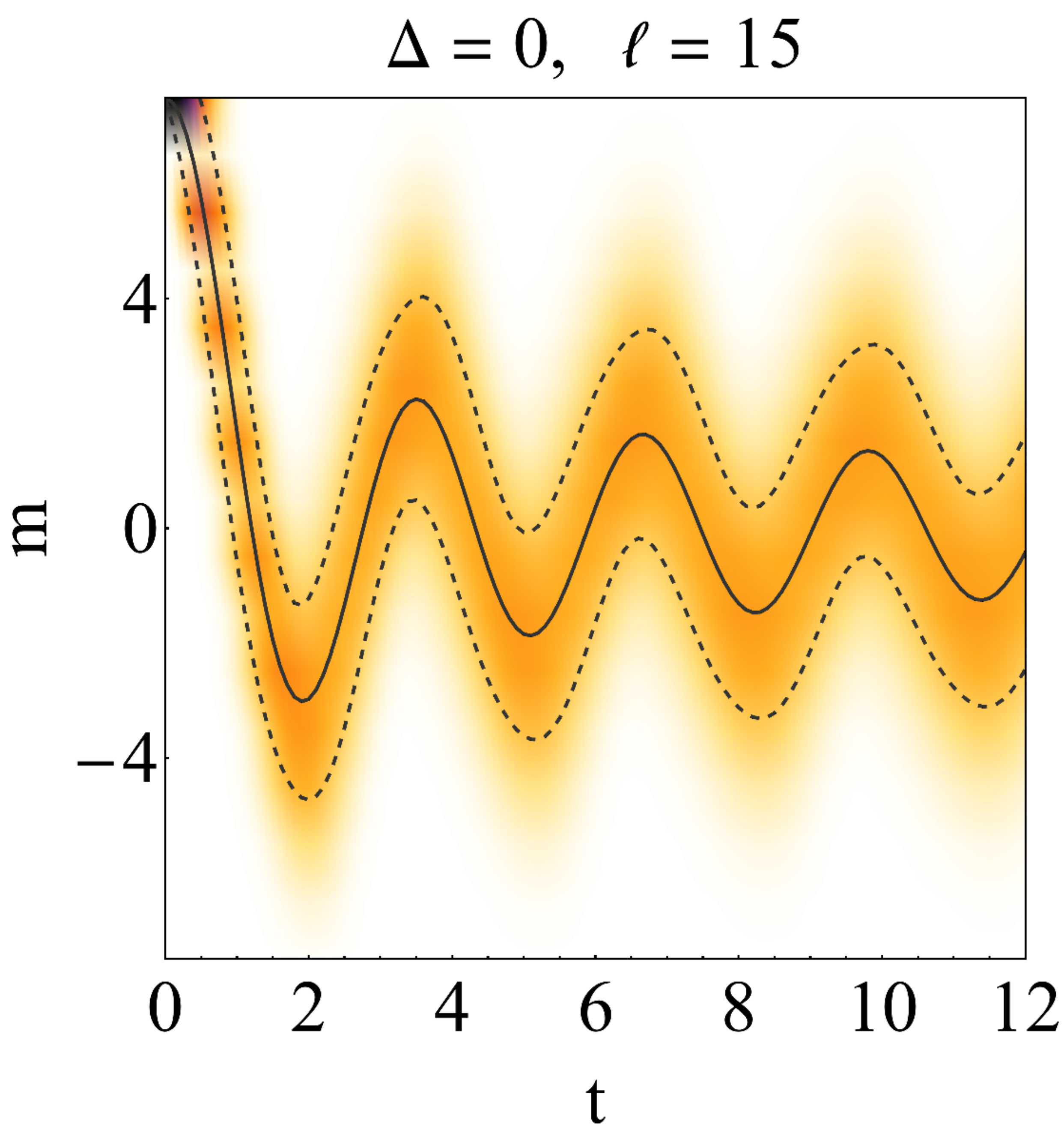


part. relevant for "small quenches"

Analytic understanding for large Δ.

Time evolution for a "large quench"

(obtained from iTEBD)



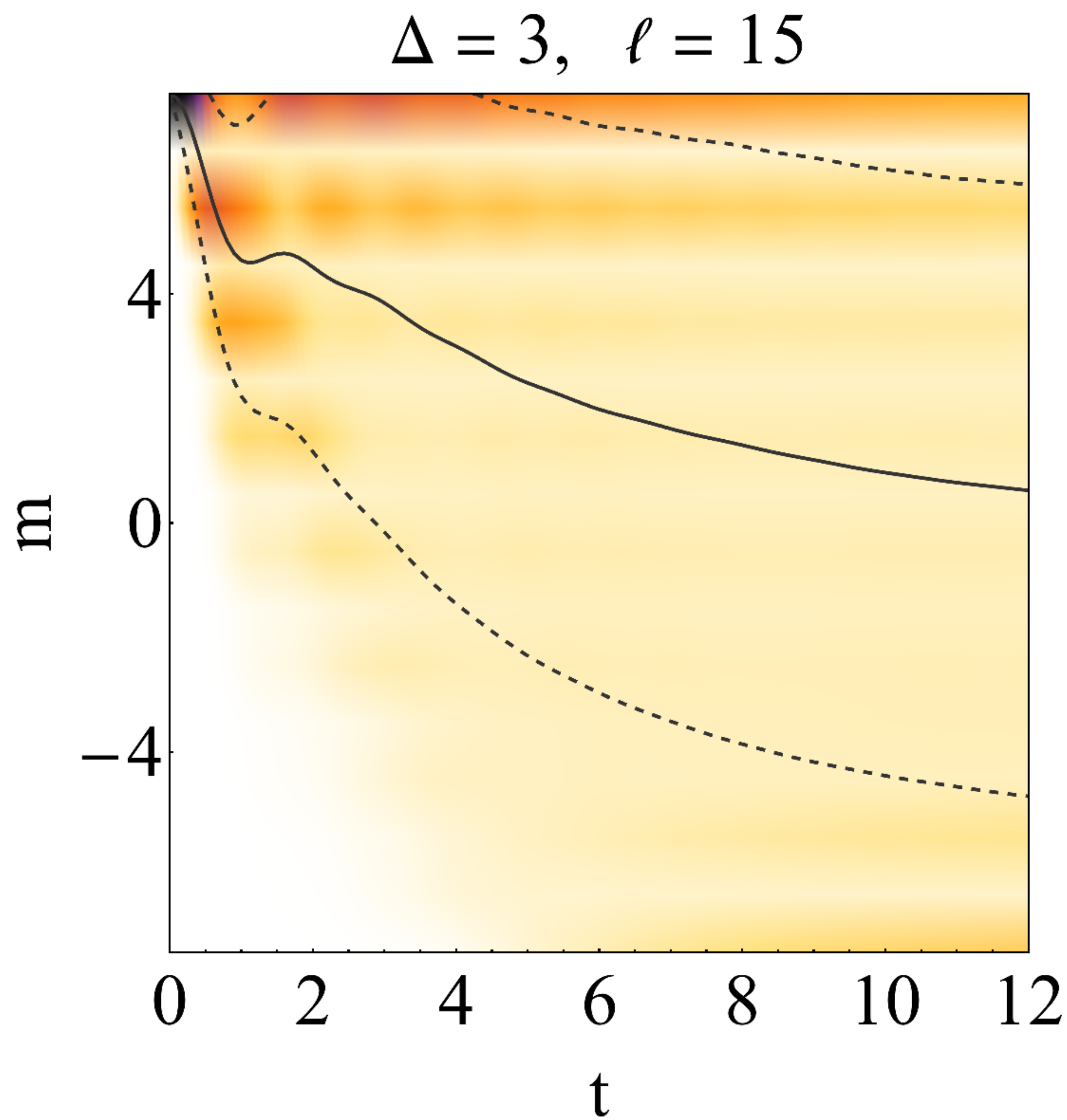
Prob. dist. = narrow Gaussian

← average

Here the prob. dist. does not give a lot of extra info (except at short times)...

Time evolution for a "small quench"

(obtained from iTEBD)



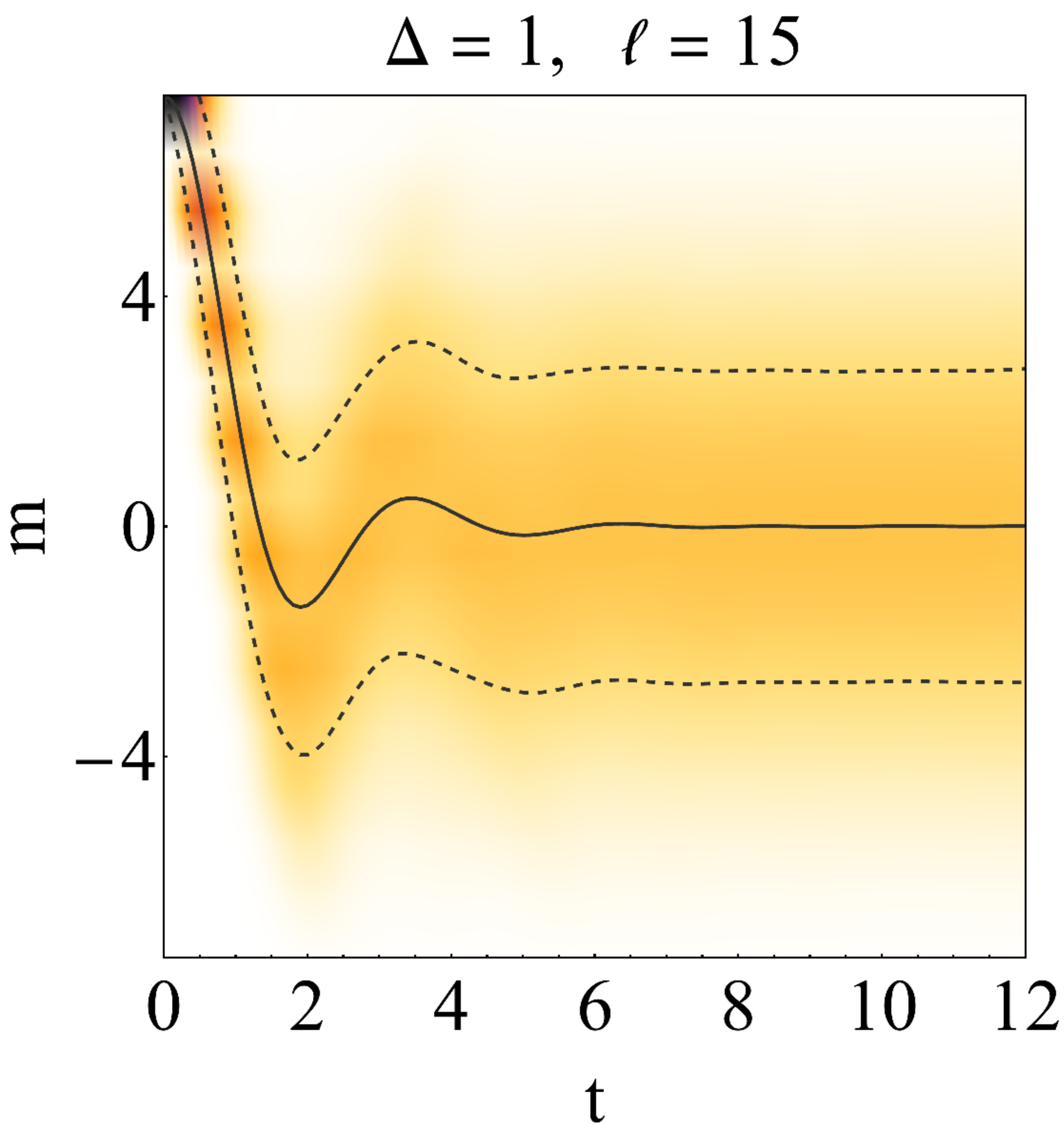
peak at $m=\ell/2$ remains

average

peak at $m=-\ell/2$ develops

Prob. dist. reveals a lot of physics beyond the average!

Time evolution for an "intermediate quench"



Very broad prob. dist.

average

B. Analytic results: Transverse-Field Ising Chain

S. Groha, FHLE & P. Calabrese '18

$$H(h) = -\sum_{j=-\infty}^{\infty} \left[\sigma_j^x \sigma_{j+1}^x + h\sigma_j^z\right].$$

Consider QQs e.g. from **ground states** of H(h_0) and determine PD of transverse subsystem magnetisation:

$$S_u^z(\ell) = \sum_{j=1}^{\ell} \sigma_j^z$$

local in fermions

$$P_{S_u^z(\ell),|\Psi(t)\rangle}(\mu) = \langle \Psi(t) | \delta\big(S_u^z(\ell) - \mu\big) | \Psi(t) \rangle = \int_{-\infty}^{\infty} d\lambda\ e^{-i\mu\lambda} \underbrace{\langle \Psi(t) | e^{i\lambda S_u^z(\ell)} | \Psi(t) \rangle}_{\chi_u(\lambda,\ell)}$$

$$= 2\sum_{r\in\mathbb{Z}} P_w^{(u)}(r,t)\ \delta(m-2r) \quad (\ell \text{ even})$$

How to obtain analytic results after QQ: generalize

Fagotti & Calabrese'08

Step 1: exact determinant representation for generating function

$$\chi^{(u)}(\lambda,\ell) = (2\cos\lambda)^{\ell}\sqrt{\det\left(\frac{1-\tan(\lambda)\Gamma'}{2}\right)},$$

← known 2ℓx2ℓ matrix

Step 2: multiple integral representation

$$\ln\chi^{(u)}(\lambda,\ell,t) = \ell\ln(\cos\lambda) - \frac{1}{2}\sum_{n=1}^{\infty}\frac{\big(\tan(\lambda)\big)^n}{n}\,\mathrm{Tr}\big[(\Gamma')^n\big]$$

$$\mathrm{Tr}\big[(\Gamma')^n\big] = \left(\frac{\ell}{2}\right)^n \int_{-\pi}^{\pi}\frac{dk_1\ldots dk_n}{(2\pi)^n}\int_{-1}^{1} d\zeta_1\ldots d\zeta_{n-1}\;\mu(\vec{\zeta})\;C(\vec{k})\;F(\vec{k})\;\exp\left(-i\ell\sum_{j=1}^{n-1}\frac{\zeta_j}{2}(k_j-k_0)\right)$$

Step 3: asymptotics from multi-dim stationary phase approx and summing result over all n

difficult.

Result:

$$\ln\chi(\lambda,\ell,t)\approx \ell\log(\cos\lambda)+\frac{\ell}{2}\sum_{n=0}^{\infty}\int_0^{2\pi}\frac{dk_0}{2\pi}\Theta(\ell-2n|v_k|t)\left[1-\frac{2n|v_k|t}{\ell}\right]\sum_{m=0}^{n+1}\cos\left(2m\varepsilon(k_0)t\right)f_{n,m}(\lambda,k_0)+\mathcal{C}$$

$$f_{0,0}(\lambda,k_0) = 2\ln\left(1+i\cos\Delta_{k_0}\tan\lambda e^{i\theta_{k_0}}\right)\ ,$$

$$f_{1,0}(\lambda,k_0) = \ln\left[1-\frac{\sin^2\Delta_{k_0}\tan^2\lambda(\cos\theta_{k_0}+i\cos\Delta_{k_0}\tan\lambda)^2}{(\sin^2\theta_{k_0}+(\cos\theta_{k_0}+i\cos\Delta_{k_0}\tan\lambda)^2)^2}\right]\ ,$$

$$f_{2,0}(\lambda,k_0) = \ln\left[1+\frac{\sin^4\Delta_{k_0}\tan^4\lambda\sin^2\theta_{k_0}(\cos\theta_{k_0}+i\cos\Delta_{k_0}\tan\lambda)^2}{((\sin^2\theta_{k_0}+(\cos\theta_{k_0}+i\cos\Delta_{k_0}\tan\lambda)^2)^2-\sin^2\Delta_{k_0}\tan^2\lambda(\cos\theta_{k_0}+i\cos\Delta_{k_0}\tan\lambda)^2)^2}\right]$$

$$f_{0,1} = -i\tan\Delta_{k_0}\ln\left[\frac{1+ie^{i\theta_{k_0}}\cos\Delta_{k_0}\tan\lambda}{1+ie^{-i\theta_{k_0}}\cos\Delta_{k_0}\tan\lambda}\right]\ ,$$

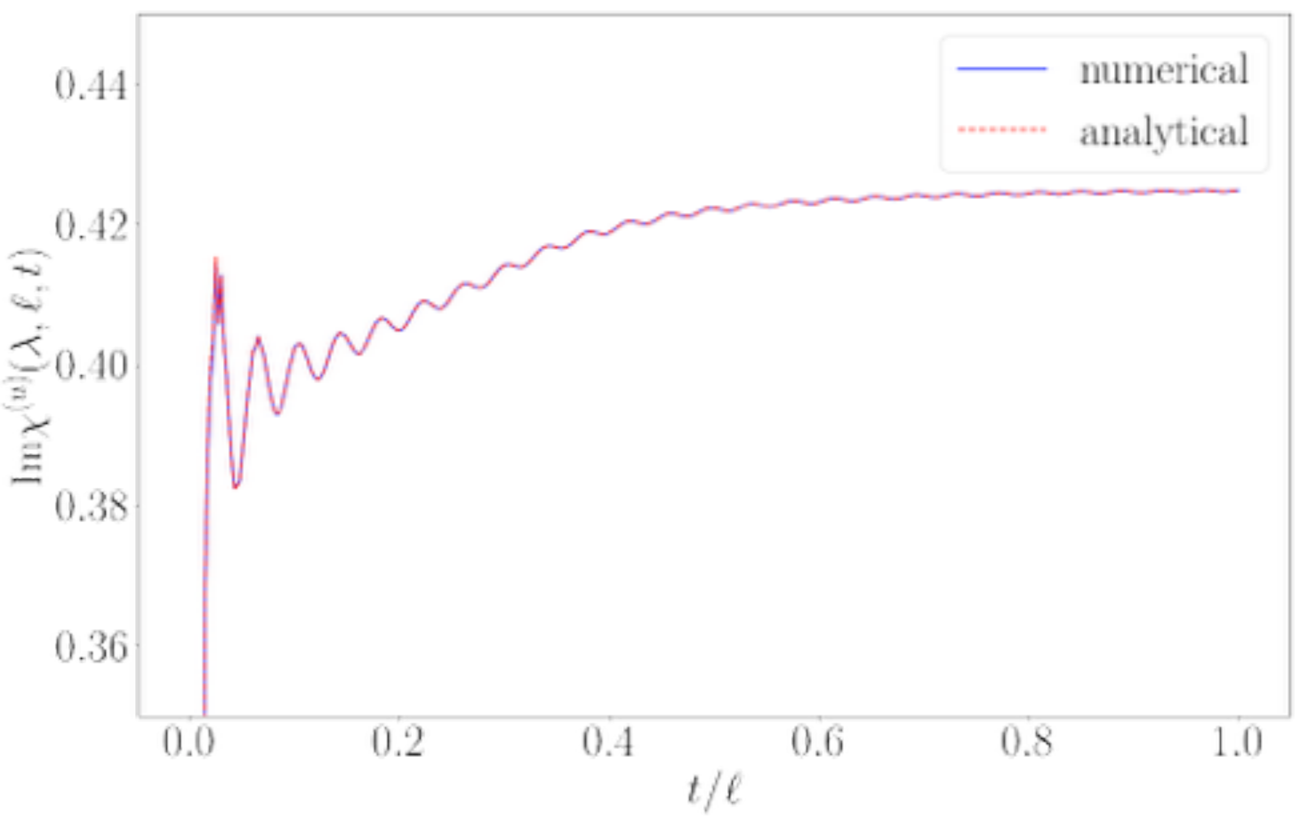
$$f_{1,1} = \tan\Delta_{k_0}\left(i\log\left[\frac{1+ie^{i\theta_{k_0}}\cos\Delta_{k_0}\tan\lambda}{1+ie^{-i\theta_{k_0}}\cos\Delta_{k_0}\tan\lambda}\right]-\frac{4\cos\Delta_{k_0}\tan\lambda\sin\theta_{k_0}}{\sin^2\theta_{k_0}+(\cos\theta_{k_0}+i\cos\Delta_{k_0}\tan\lambda)^2}\right)+\mathcal{O}\left(\sin^3(\Delta_{k_0})\right)$$

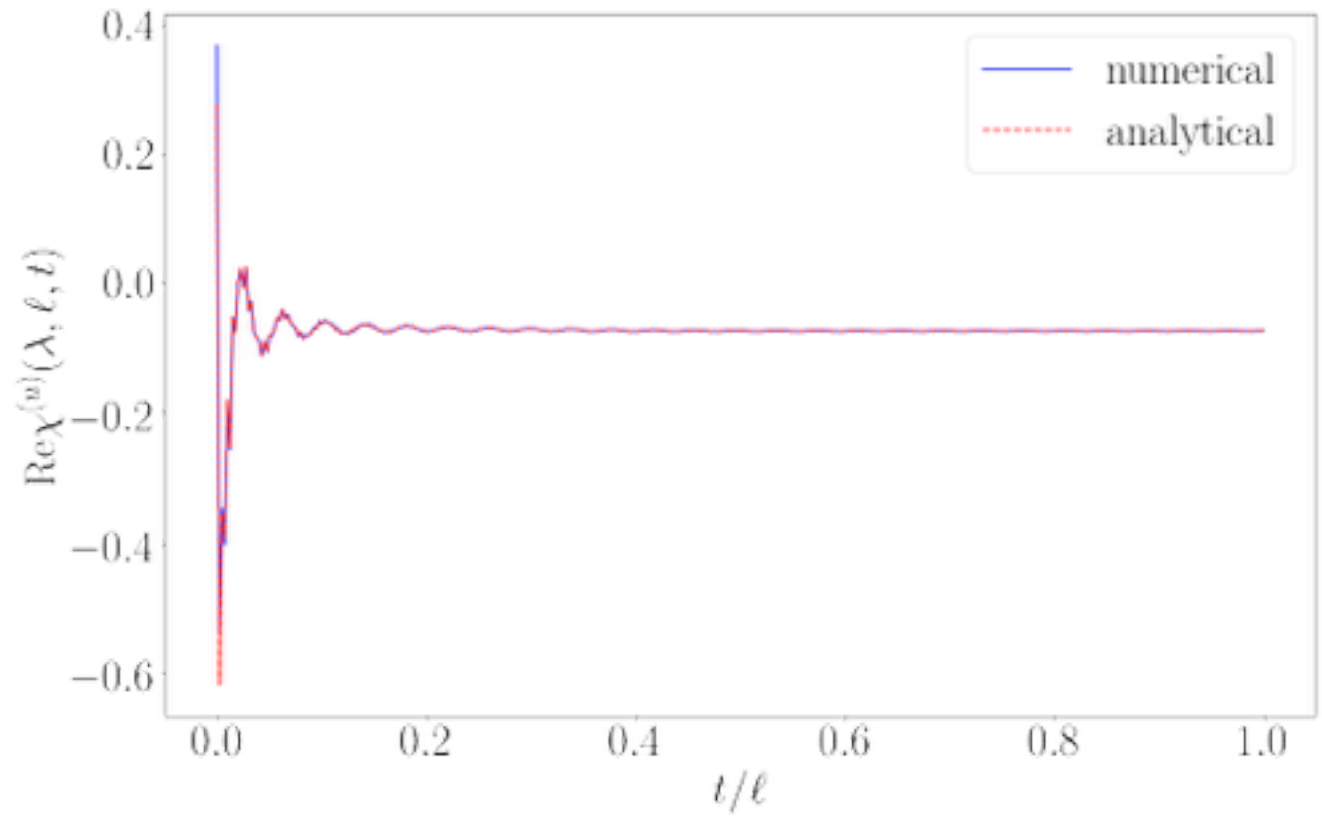
$$e^{i\theta_k}=\frac{h-e^{ik}}{\sqrt{1+h^2-2h\cos k}}\qquad \varepsilon(k)=2J\sqrt{1+h^2-2h\cos(k)}.\qquad v_k=\frac{d\epsilon(k)}{dk}$$

$$\cos\Delta_k=4\frac{hh_0-(h+h_0)\cos k+1}{\varepsilon_h(k)\varepsilon_{h_0}(k)}$$

How well does this work?

→ Compare to numerically exact results.





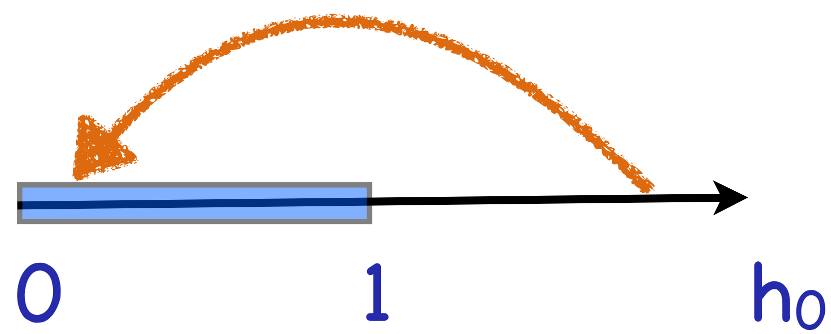
$\lambda=0.1$, $\ell=200$, $h_0=0$, $h=0.2$

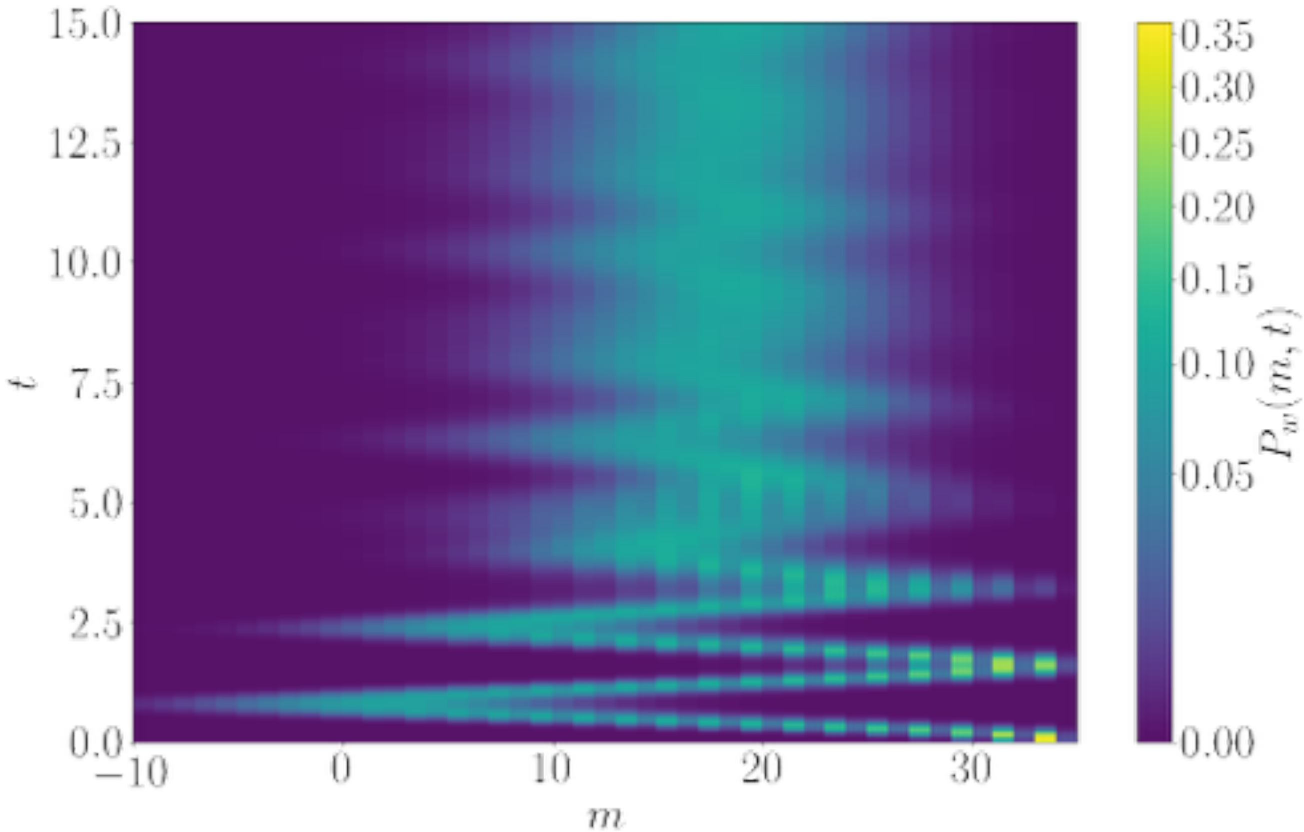
Not bad.

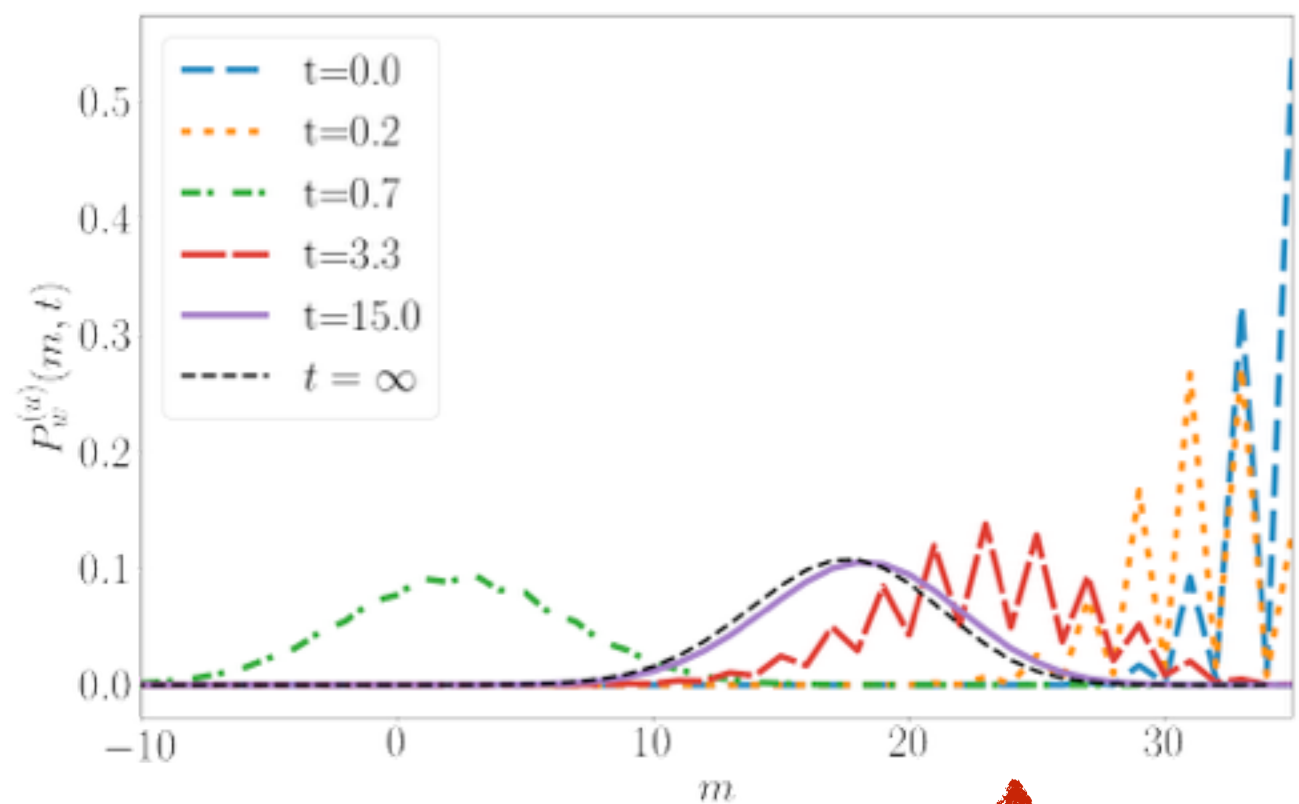
Slight caveat: when $\chi^{(u)}(\lambda, \ell, t)$ becomes very small as a fn of λ our approximation becomes poor. Not a problem for getting the PD.

"Transverse field quench": prepare system in GS of $H(h_0)$, time evolve with $H(h)$

$h_0=3$, $h=0.2$







even/odd structure that washes out over time

Summary

1. PD for subsystems can reveal interesting physics; can be **universal** at critical points.
2. PD are not easy to calculate analytically.
3. Analytic results for PD of transverse subsystem magnetisation in TFIM after QQs
4. Order parameter after Neel quench in XXZ: interesting regime after melting of LRO
5. Other results: PDs in ground states of critical XXZ chain and Hubbard model.
6. Long-range spin chains/"Kitaev models": Floquet; formation of order;...