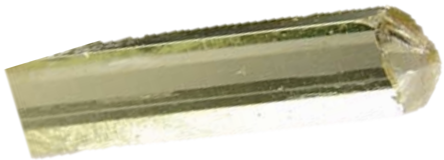
Paola Cappellaro

Localization and thermalization in nuclear spin systems

KITP,
August 28, 2018

Quantum Dynamics with Nuclear Spins

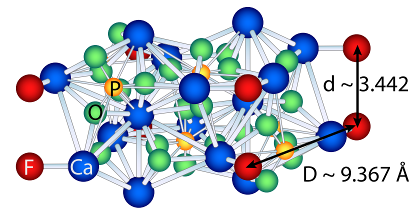
1. Buy a pendant on ebay …



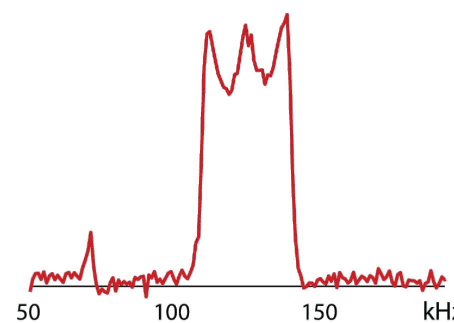
2. Use an old NMR spectrometer…

3. Observe localization & prethermalzation

1. Fluorapatite Spin system & approximate model

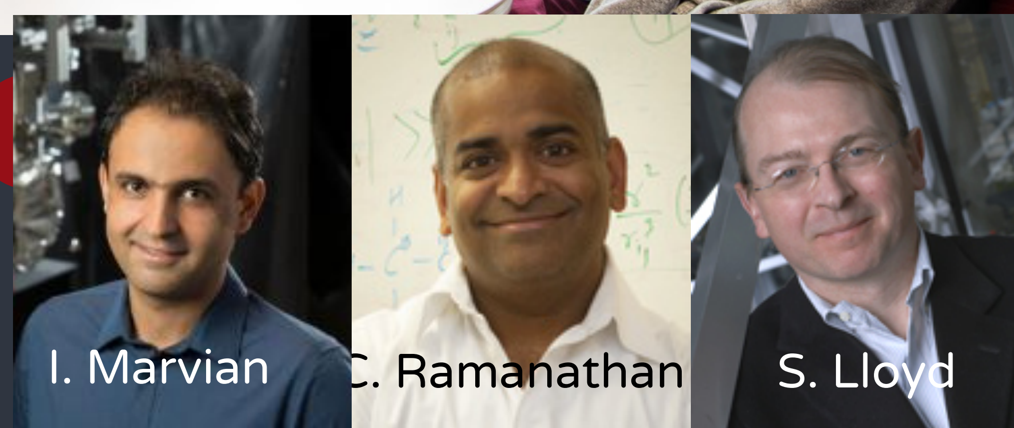


2. Hamiltonian engineering & Multiple Quantum Coherences



3. Novel metric of localization & OTOC measurement

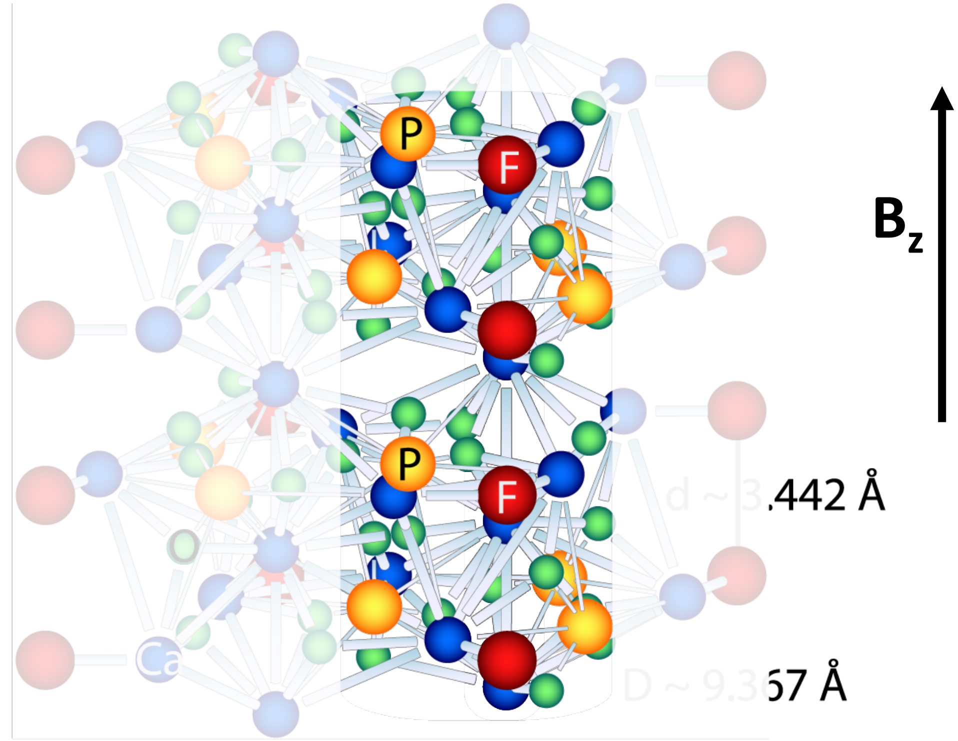
Poster this afternoon!

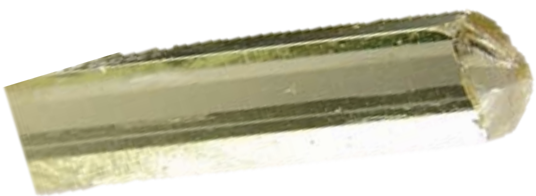


1D system: FluorApatite

- Single-crystal, $Ca_5F(PO_4)_3$
- Quasi-1D system of ^{19}F spin ½
  - Ratio of couplings:

    $C_{in}/C_x = D_x^3/d_{in}^3 \approx 40$
- Intrinsic disorder:
  - ^{31}P spin ½
  - Other defects

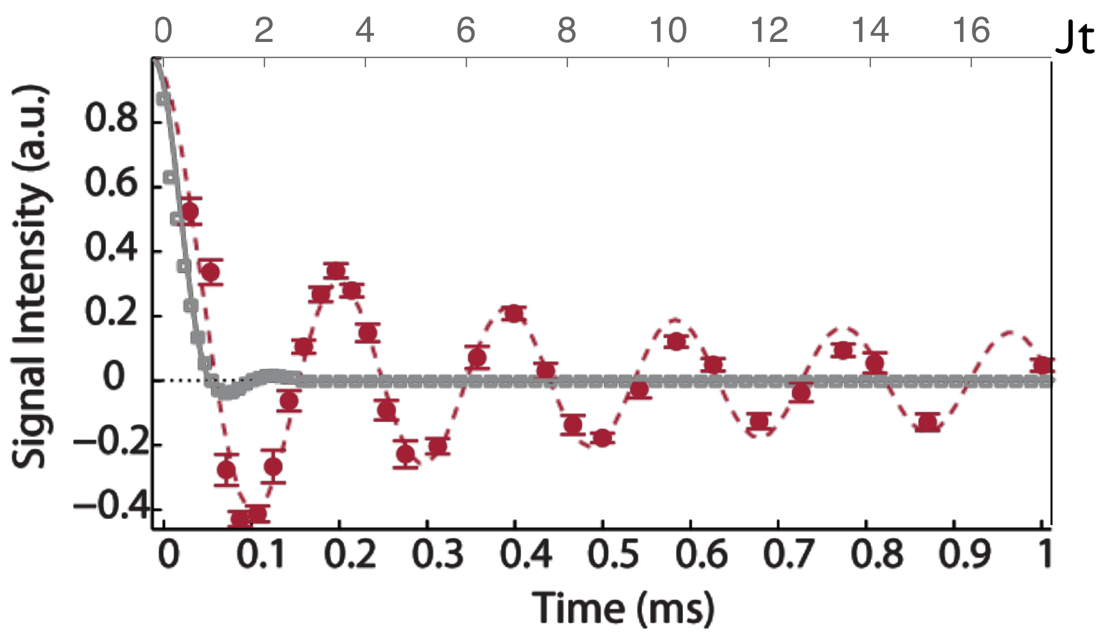




Cho, Yesinowski, J. Phys. Chem. 1993
P.C., Ramanathan, Cory, PRL 2007

^{19}F spin dynamics

- Thermal state: $\rho = \frac{e^{-\beta\mathcal{H}}}{Z} \approx \frac{\mathbb{1}}{2^N} + \epsilon \sum_i \sigma_z^i \qquad (\epsilon \approx 10^{-5})$
- Many-body evolution: apparent loss of coherence in <100us
- Control recovers the coherence for 1-2ms,
  - Signal shows distinct 1D, nearest-neighbor behavior



Van Lugt, Casper, Physica 1964, Zhang, **PC**, Viola et al. PRA 2009

Hamiltonian Engineering

- Dipolar Interaction with "long" range coupling $J \sim 1/r^3$

$$H_F = \sum_{j<k} J^F_{jk}(2\sigma^j_z\sigma^k_z - \sigma^j_x\sigma^k_x - \sigma^j_y\sigma^k_y) + \sum_{j,\kappa} h_{j\kappa}\sigma^j_z s^\kappa_z$$

- Use Average Hamiltonian* techniques to create Floquet Hamiltonian

$$H = u\sum J(\sigma^j_x\sigma^{j+1}_x + \sigma^j_y\sigma^{j+1}_y)$$

integrable

$$+v\sum J(\sigma^j_z\sigma^{j+1}_z)$$

interaction

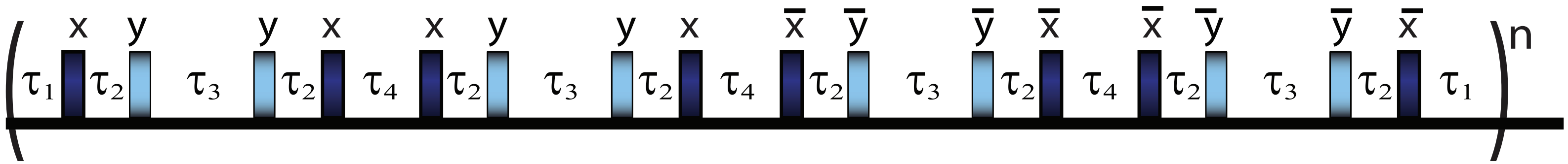
$$+g\sum h_j\sigma^j_z$$

disorder

*U. Haeberlen, 1976

Floquet Hamiltonian

- Periodic pulse sequence stroboscopically creates desired H



$$\mathcal{T}\left\{e^{-i\int_0^t [H_F + H_c(t')]dt'}\right\} \equiv e^{-iHt}$$

- Use Average Hamiltonian* techniques to create Floquet Hamiltonian

$$H = u\sum J(\sigma_x^j\sigma_x^{j+1} - \sigma_y^j\sigma_y^{j+1})$$

integrable

$$+v\sum J(\sigma_x^j\sigma_x^{j+1} + \sigma_y^j\sigma_y^{j+1} - 2\sigma_z^j\sigma_z^{j+1})$$

$$+g\sum h_j\sigma_z^j \quad +g'\sum \sigma_x^j$$

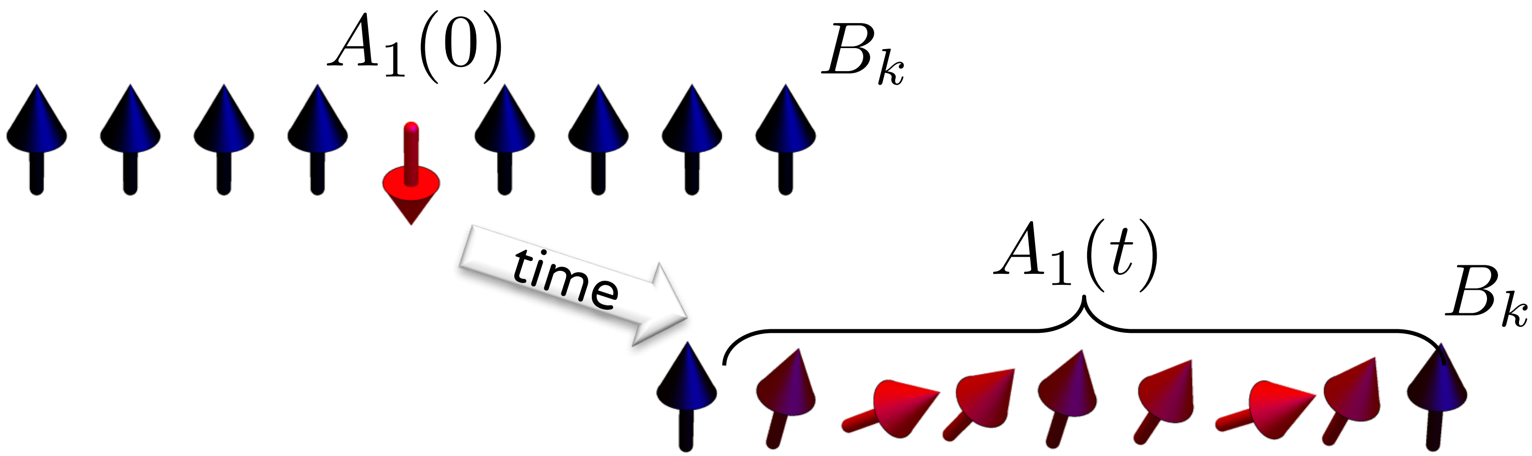
transverse field

Out-of-Time-Order Commutator

- Has a (traveling) perturbation affected a distant operator?
  - Do distant operators no longer commute, $[A_1(t), B_k(0)] \neq 0$?
  - Measure :

$$\mathcal{C}(t) = \left\langle \| [A_1(t), B_k(0)] \|^2 \right\rangle_\beta$$

- Metric for localization, quantum information scrambling, quantum criticality, quantum chaos, …



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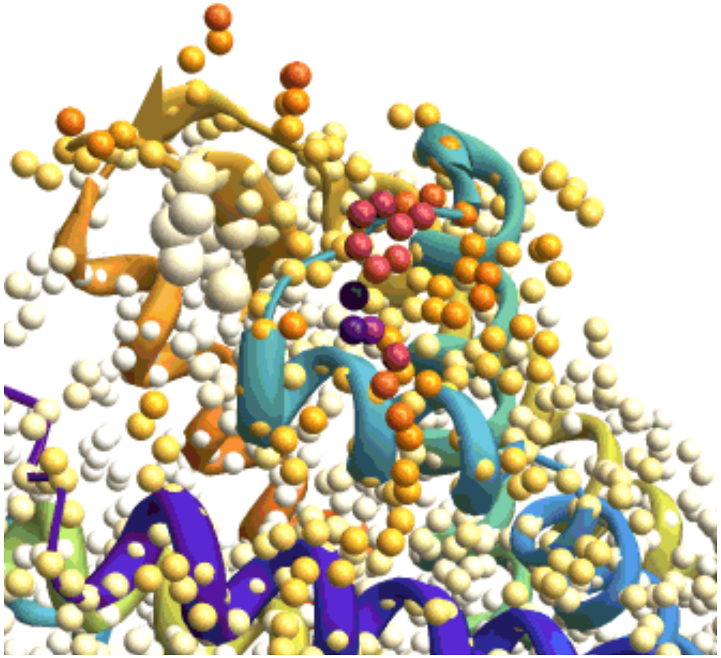
- Metric for localization, quantum information scrambling, quantum criticality, quantum chaos, …
- Extensive quantities, $B = \Sigma_k B_k$, reveal same effect

How to measure OTOc?

- "Spin counting" experiments in NMR:
  - How many correlated spins are there?
  - Gives information about sample geometry/structure
- Tool: Multiple Quantum Coherences (MQC)



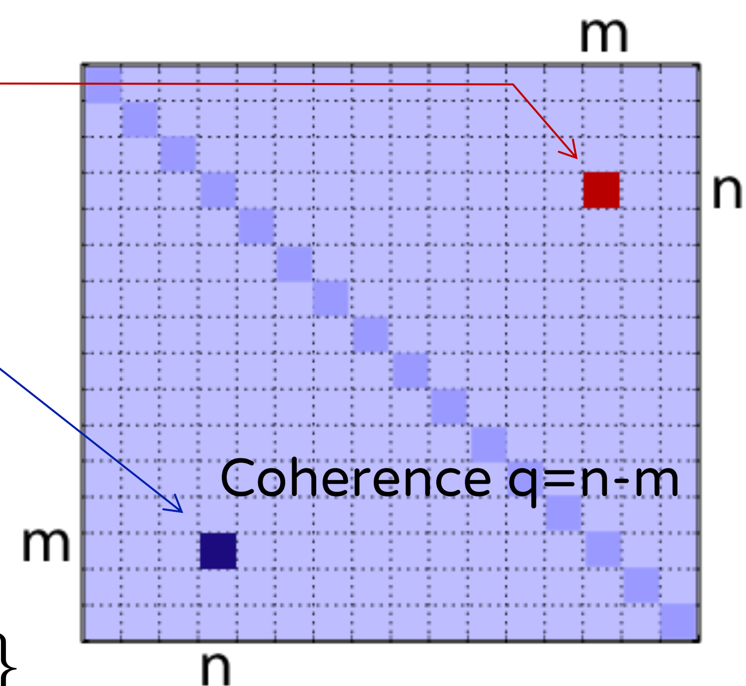
Baum, Pines, JACS 1986, Munowitz, Pines, Mehring, JCP 1987

Multiple Quantum Coherences

- Tool: Multiple Quantum Coherences
  - Coherent superposition: $|\psi\rangle = \frac{|m\rangle + |n\rangle}{\sqrt{2}} \xrightarrow{\sigma_z} \frac{|m\rangle + e^{i\varphi(n-m)}\,|n\rangle}{\sqrt{2}}$

$$\rho = |\psi\rangle\langle\psi| = \frac{1}{2}(|m\rangle\langle m| + |n\rangle\langle n| + \boxed{|m\rangle\langle n|} + \boxed{|n\rangle\langle m|})$$

$$\rho = \sum_{q=-L}^{L} \rho_q \qquad \rho_q \xrightarrow{\sigma_z} \rho_q e^{i\varphi q}$$



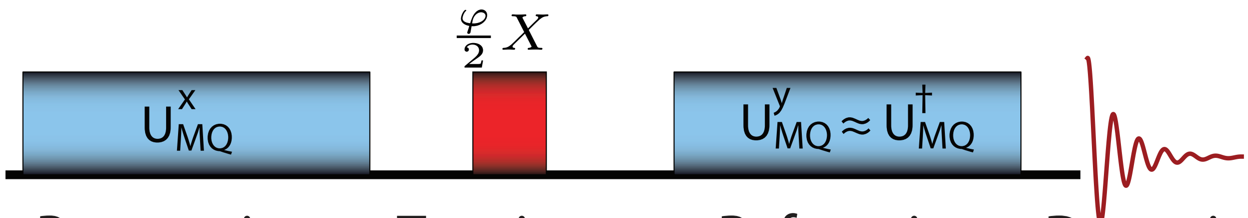
- MQC intensities: $I_q = \mathrm{Tr}\left\{\rho_q \rho_{-q}\right\}$

OTOC & MQC

- We can measure OTOC:

  -> NMR has been doing it for the past 40 years!

1. Tweaking the pulse sequence we achieve time reversal
2. Exploiting initial mixed state $\rho_0 = \mathbb{1} + \epsilon Z$ we measure OTOC at T = ∞



Preparation Tagging Refocusing Detection

- The signal is $S_\varphi(t) = \mathrm{Tr}[\Phi^\dagger(t)\rho_0\Phi(t)Z]$

$$S_\varphi(t) = \mathrm{Tr}[\Phi^\dagger(t)Z\Phi(t)Z]$$

with $\Phi(t) = U(t)e^{i\frac{\varphi}{2}X}U^\dagger(t)$

- OTO correlator for Φ at $\beta = 0$

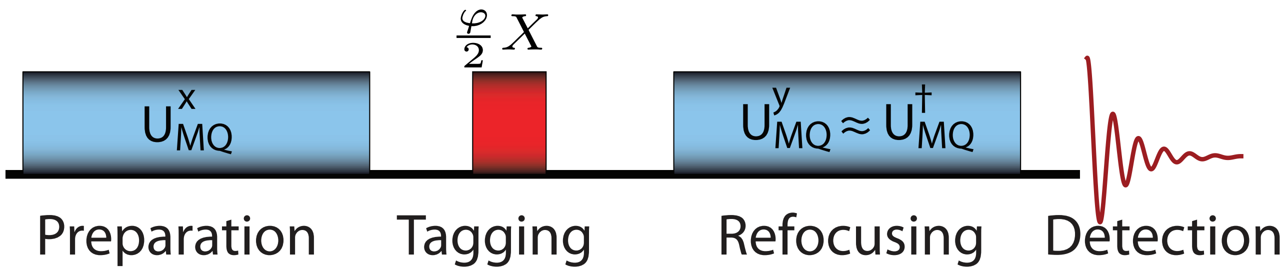
Munowitz, Pines, Mehring, JCP 1987

OTOC & MQC

- We can better measure OTOC:

  -> by detecting NMR multiple quantum coherence distribution

1. Repeat the measurement varying φ
2. Fourier transform the signal to get MQC intensities, $I_q = \sum_\varphi S_\varphi e^{i\varphi q}$



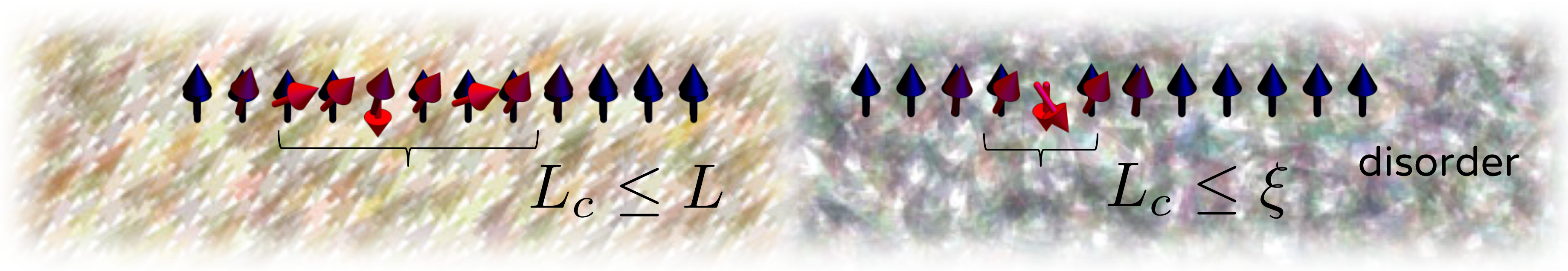
- Expanding in power of φ, $S_\varphi \sim Z^2 + \|[Z(t), X]^2\| \varphi^2 + \ldots$

  which can be reliably extracted from MQC 2nd moment

$$\langle \|[Z(t), X]^2\| \rangle_{\beta=0} = 4 \sum_q q^2 I_q(X)$$

Correlation Length

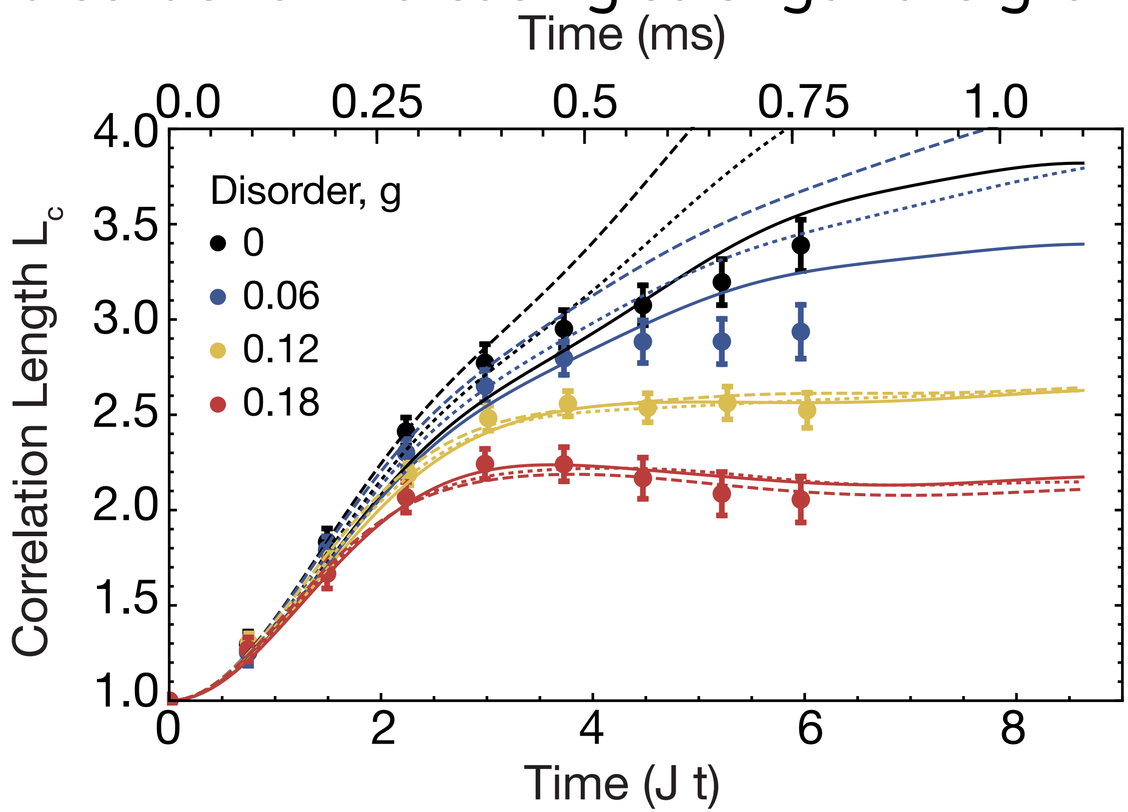
- For 1D systems, close to non-interacting, we can even relate MQC to the length over which correlations spread



- Defined average correlation length L_c for mixed states
- Explored differences in behavior between Anderson and Many-Body localization

Experimental Results: AL

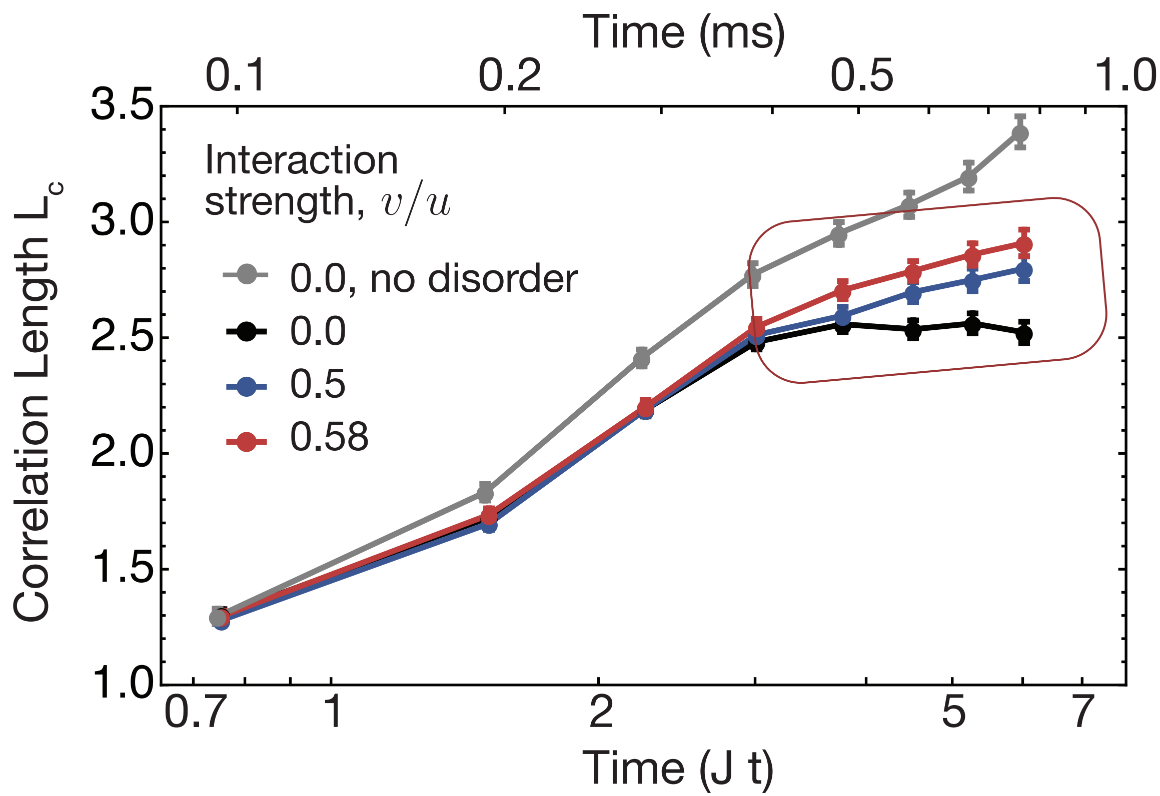
- L_c grows as a function of time.
- With disorder of increasing strength the growth is quenched



X. Wei, C. Ramanathan, **PC**, PRL 2018

Experimental Results: MBL

- Now add the interaction term, $\sim v\sigma_z\sigma_z$:
- The correlation length keeps growing, but very slowly



X. Wei, C. Ramanathan, **PC**, PRL 2018

Thermalization

- Dipolar Hamiltonian (XXZ, $J \sim r^{-3}$) leads to thermalization
  – Can a transverse field protect quantum information?

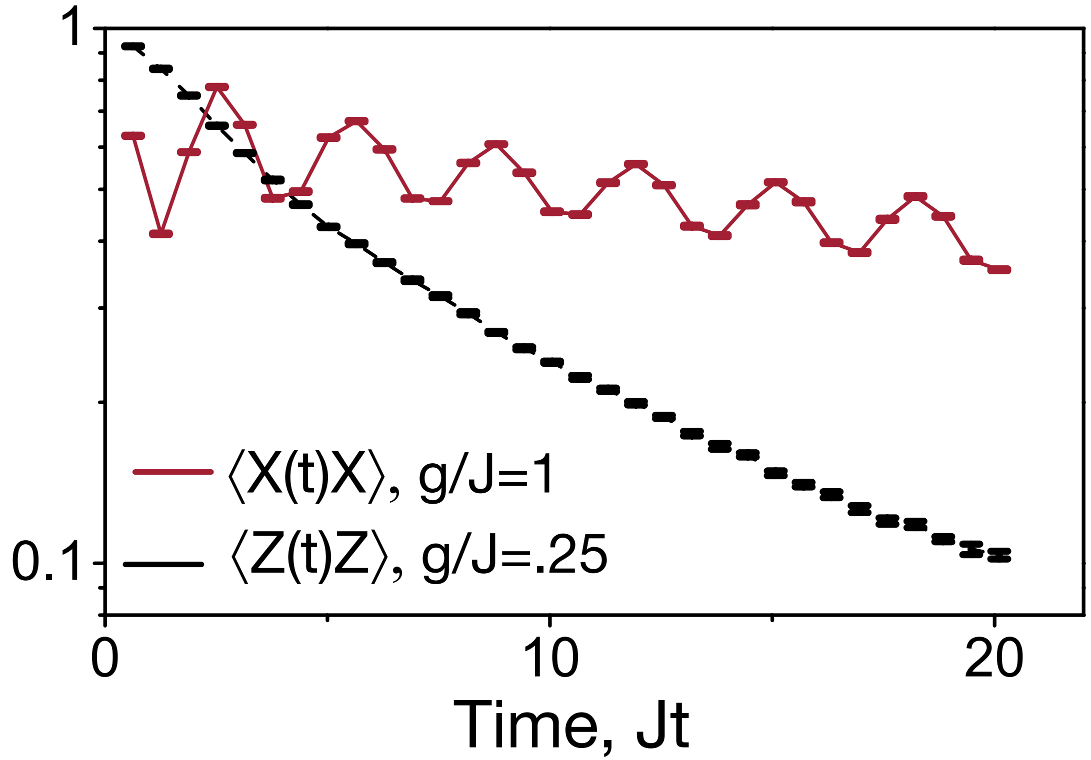
$$\mathcal{H}_{\mathrm{TFD}} = \sum_{j,k} J_{jk} \left[S_z^j S_z^k - \frac{1}{2}(S_y^j S_y^k + S_x^j S_x^k) \right] + g \underbrace{\sum S_x^j}_{X}$$

  – In the limit of g>>J, the transverse field becomes a quasi-conserved quantity X_{pre}, and the system enters a long quasi-equilibrium regime

under Floquet driving

Dynamics of Magnetization

- The transverse (X) and longitudinal (Z) magnetization show distinct behaviors,
  linked to the thermal and prethermal regimes



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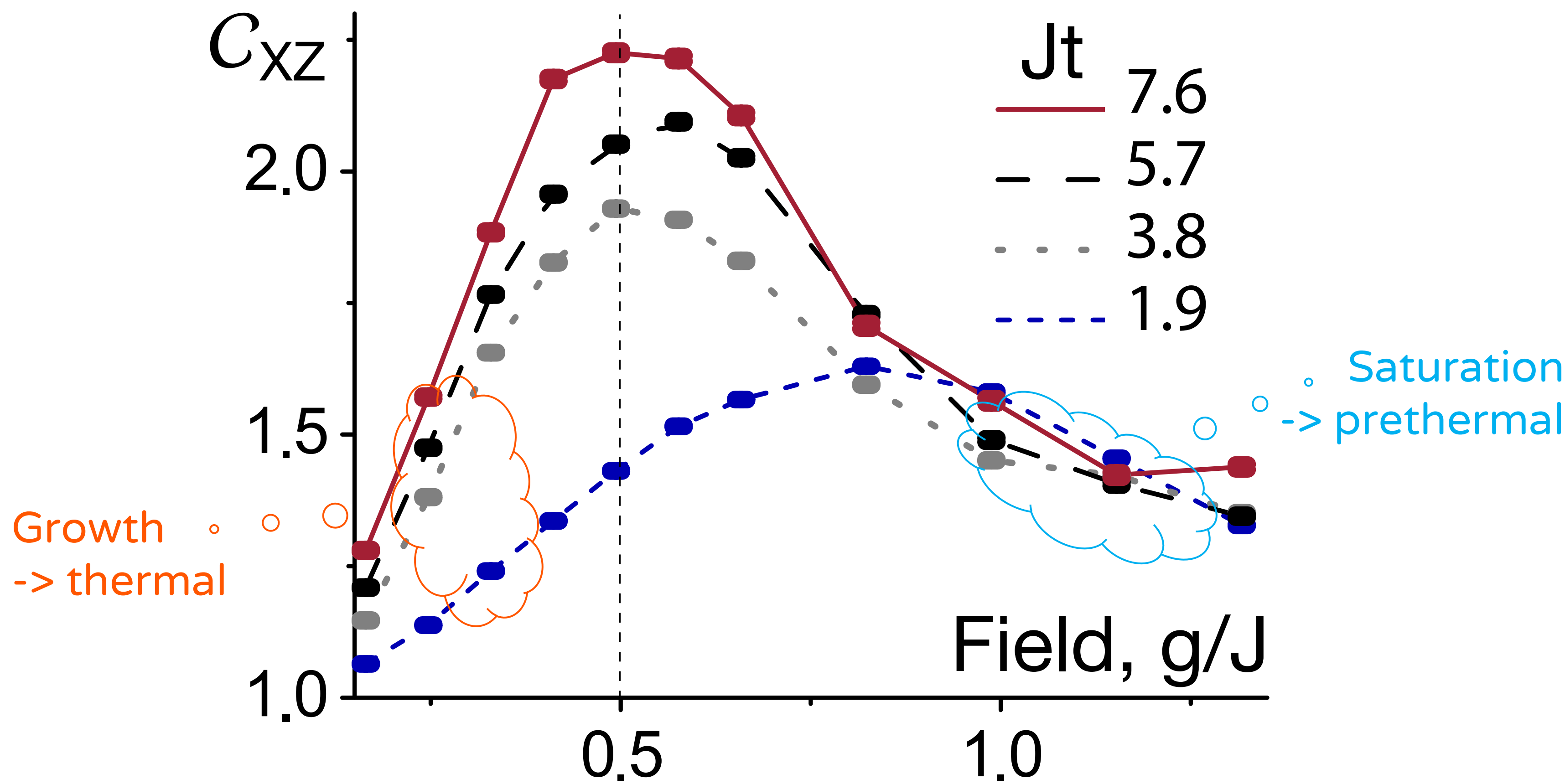
- Measuring OTOC's involving the conserved quantity, we can distinguish between thermalization and prethermal behavior

$$\mathcal{C}_{\text{XZ}} = \langle |[X(t), Z]|^2 \rangle$$

Prethermalization : Experiments

- Transverse field dipolar Hamiltonian (experiments)



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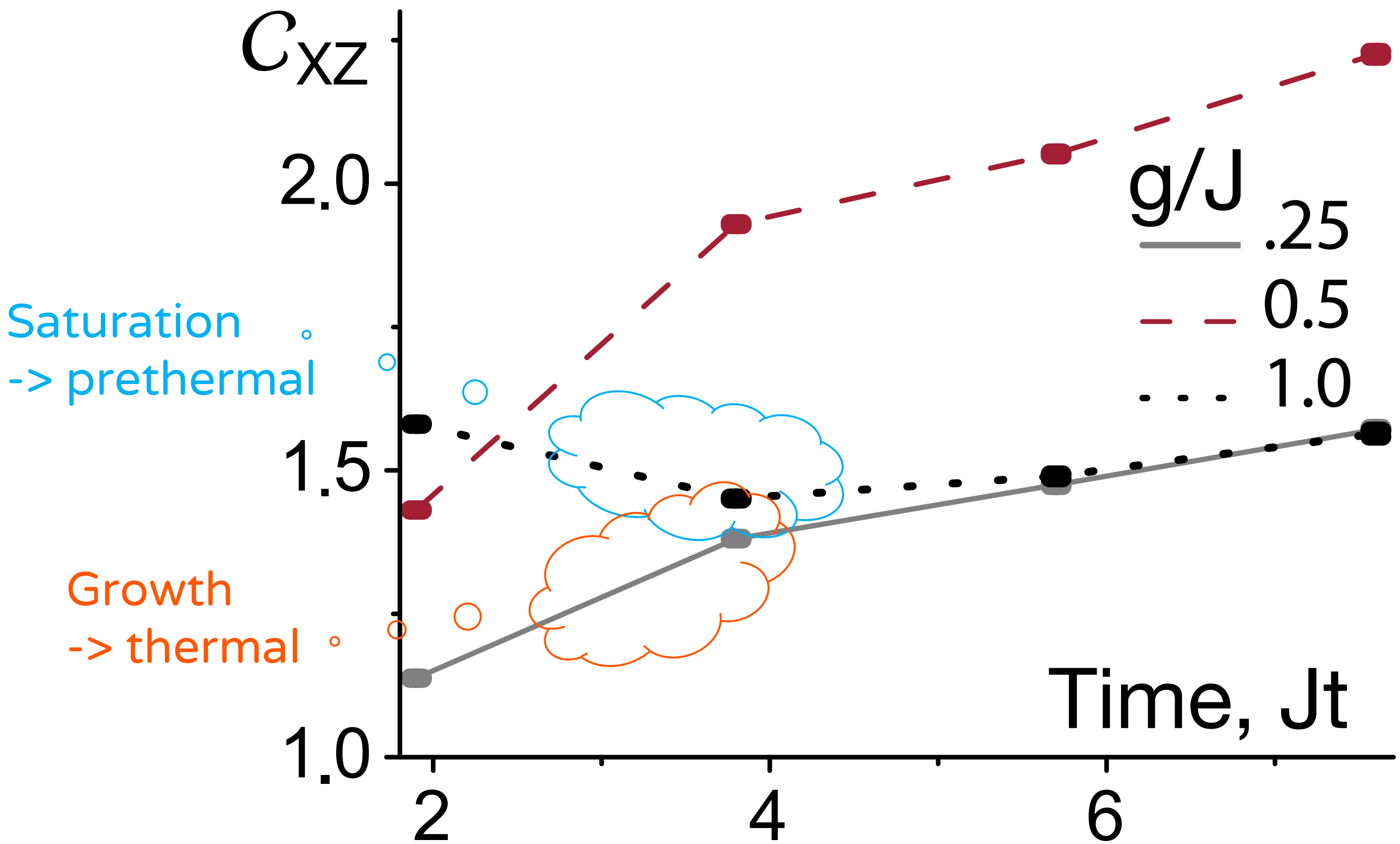

Prethermal Hamiltonian

- To check our interpretation we explicitly construct the prethermal Hamiltonian $\mathcal{H}_{\text{pre}} = \mathcal{H}' - \delta\mathcal{H}(e^{-J/g})$
  - We need to find the unitary transformation $\mathcal{H}' = U\mathcal{H}_{\text{TFD}}U^{\dagger}$
  - We use a recursive expansion in J/g: only if system is prethermal the expansion converges and $|\mathcal{H}_{\text{pre}} - \mathcal{H}'| \approx 0$

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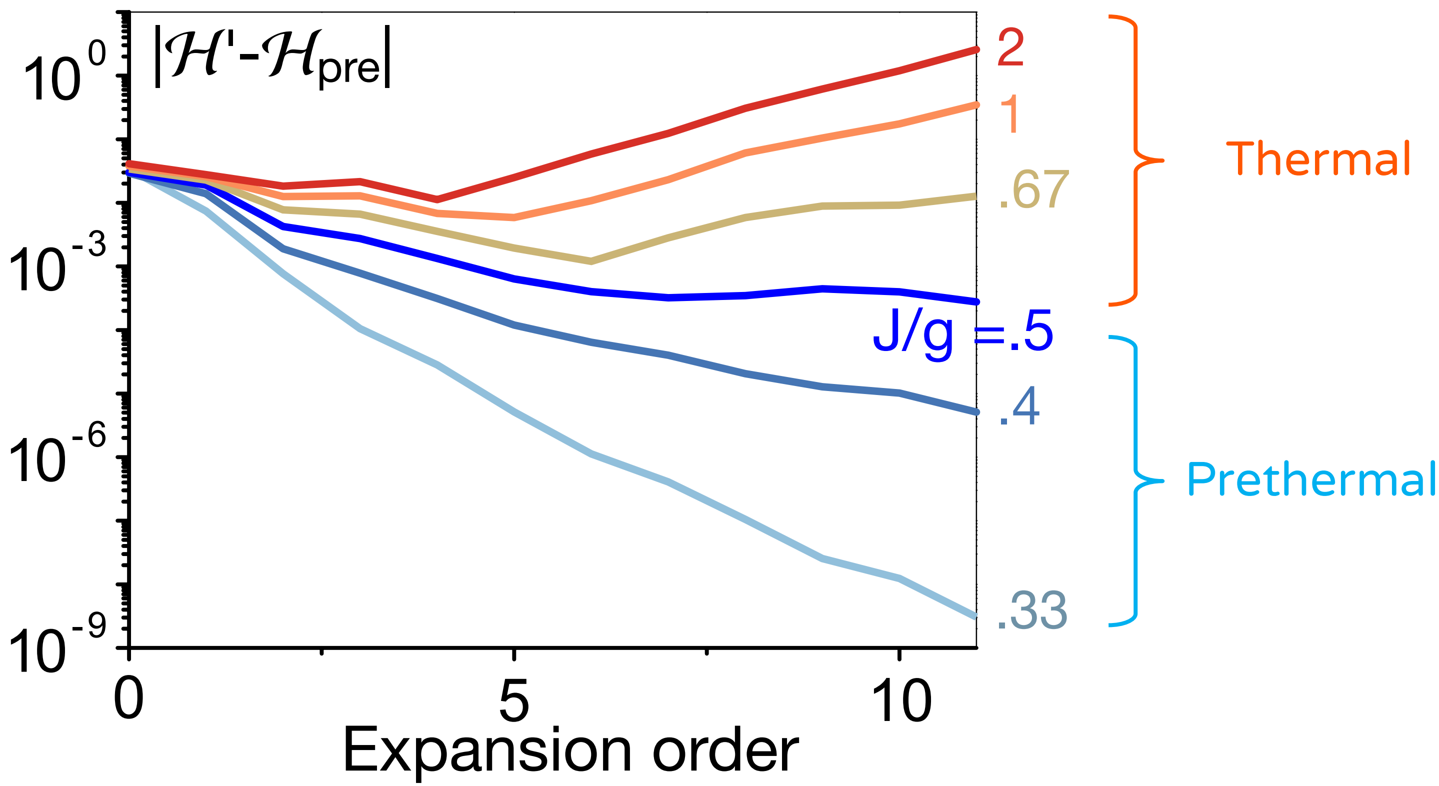
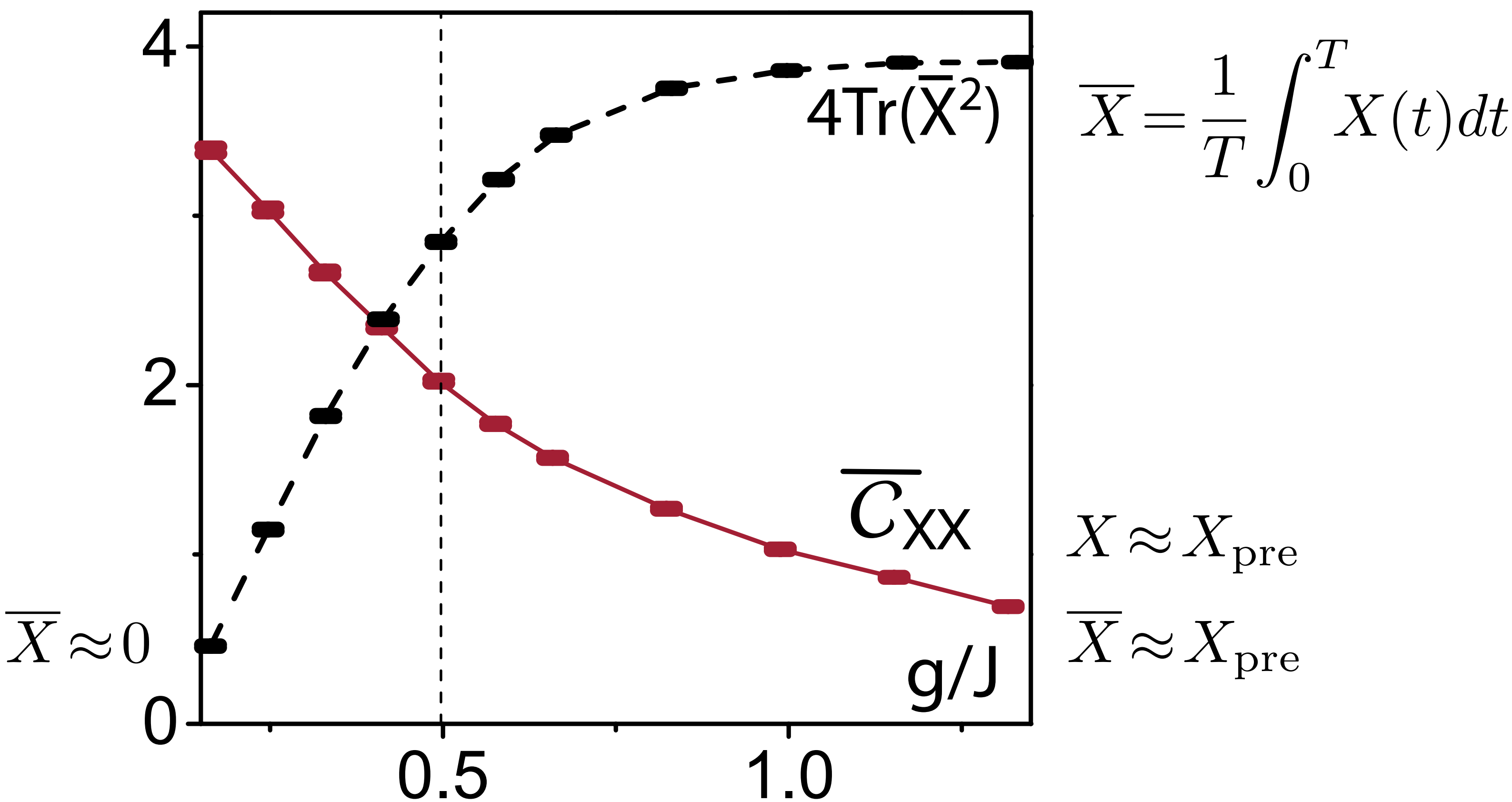


OTOC Dynamics

- Dynamics of other OTO commutators provides insight



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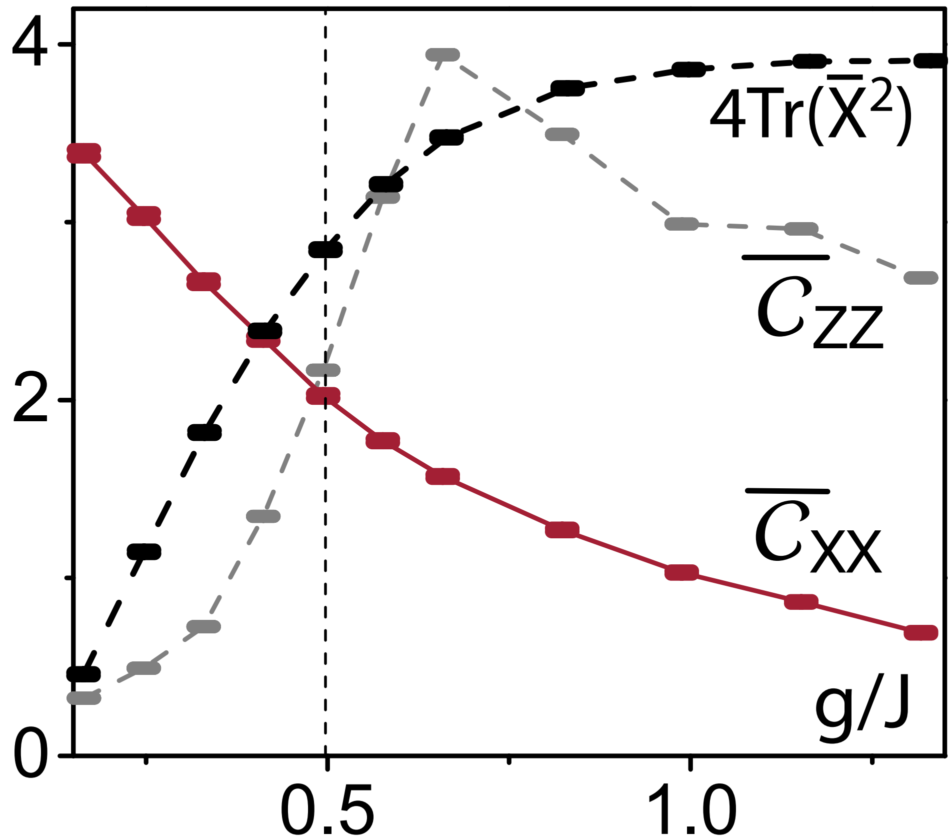

$Z \neq X_{\text{pre}}$

Poster this afternoon!