Turbulence and Dynamo

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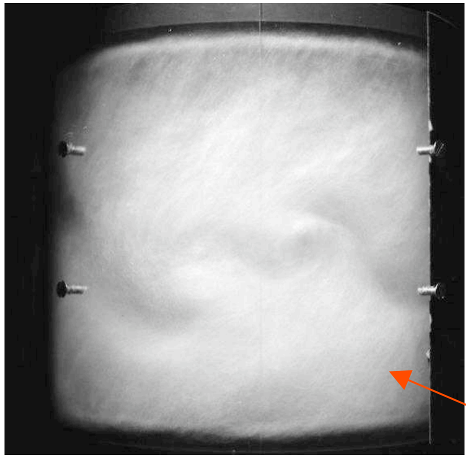
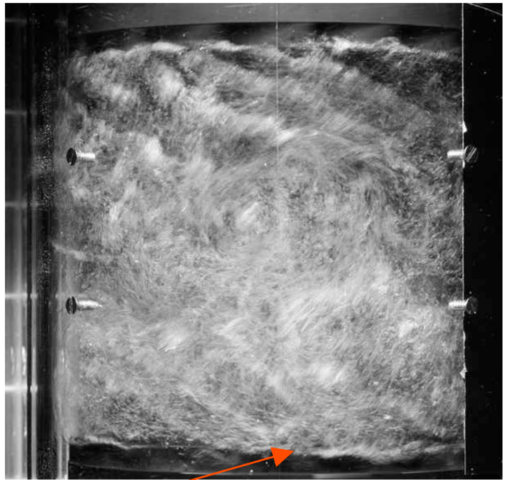
Classification of Dynamos





Turbulent flow:

$$\vec{v} = \vec{V} + \vec{v}'$$

$$\partial_t \vec{B} = curl\left(\vec{V} \times \vec{B}\right) + \eta \Delta \vec{B} + curl\left(\vec{v}' \times \vec{B}\right)$$

Mean Flow

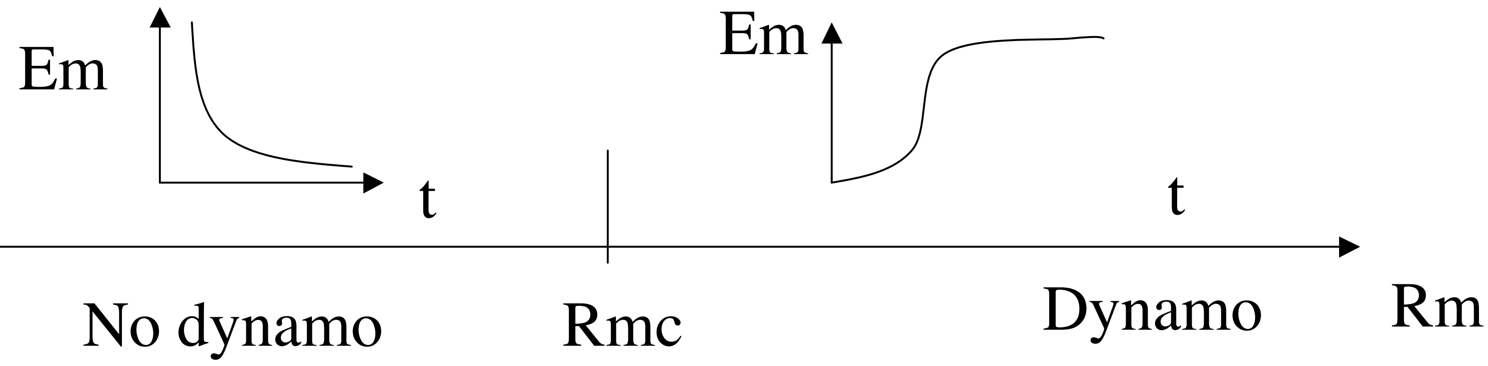
$\delta - 1$

Fluctuation

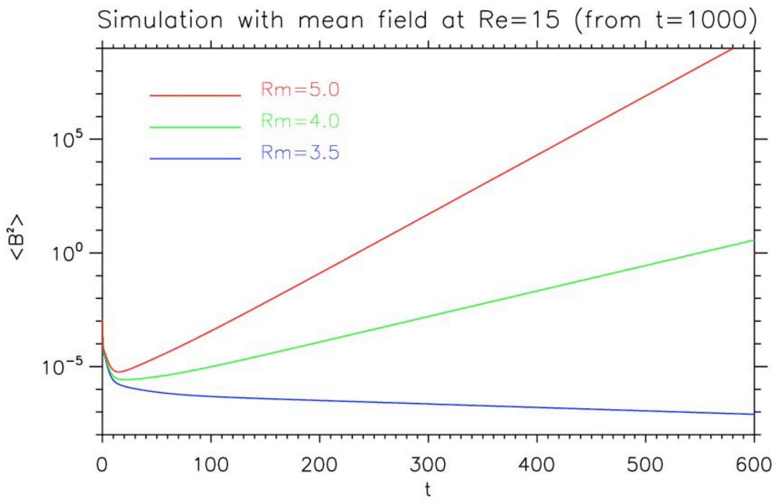
$\delta - 1 << 1$ Laminar Dynamo

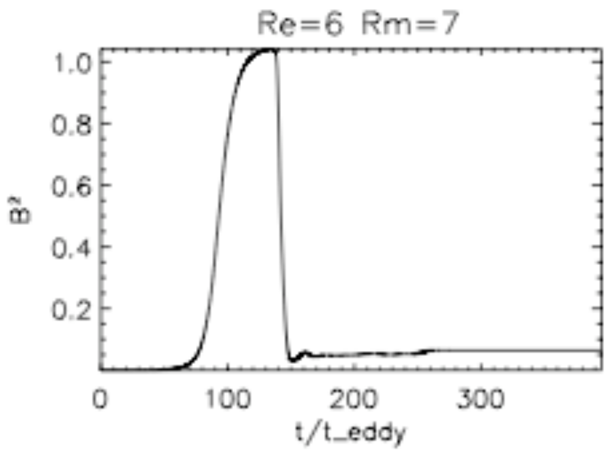
$\delta - 1 \approx O(1)$ Turbulent Dynamo

« Laminar dynamo » paradigm

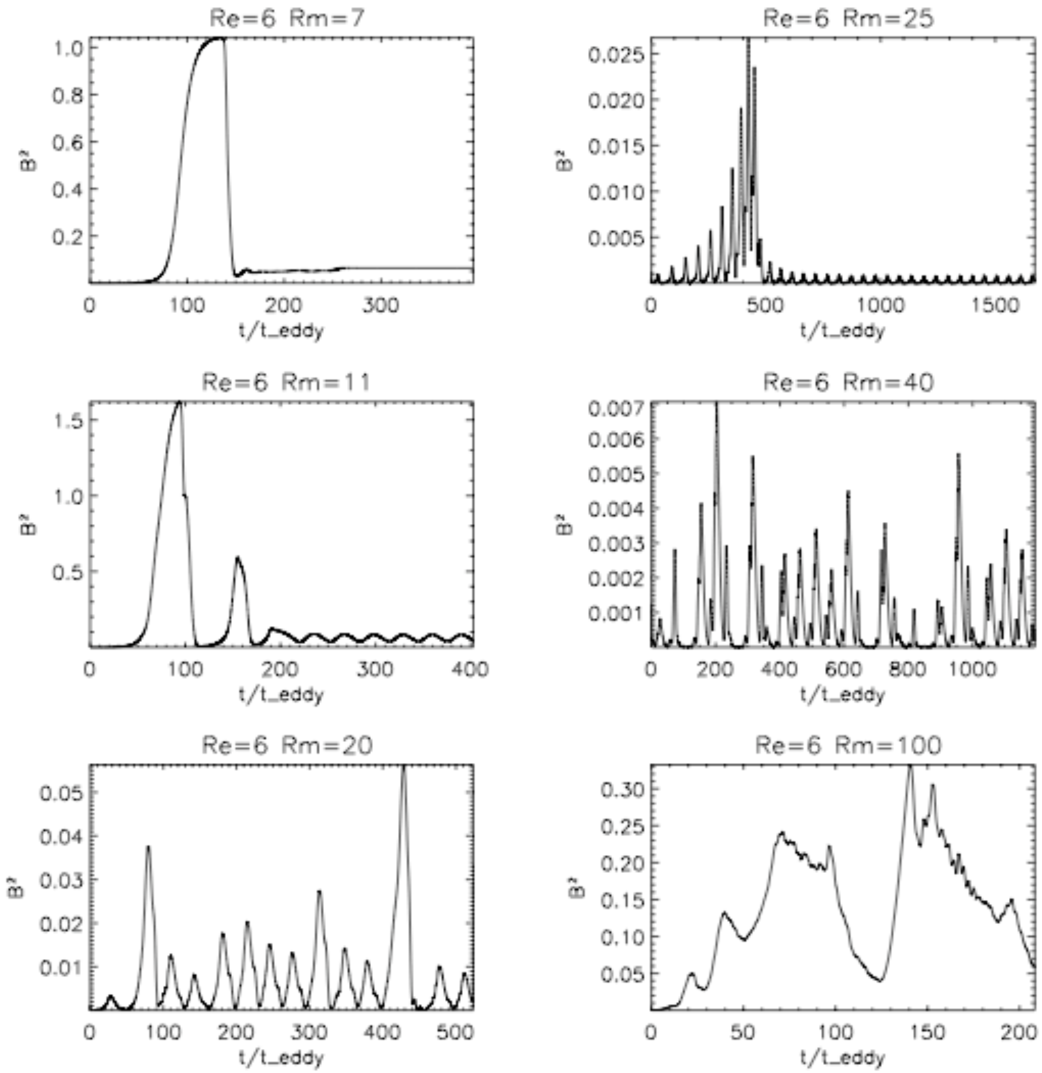


Indicator: $$2\sigma = \frac{\partial \ln\langle B^2\rangle}{\partial t}$$

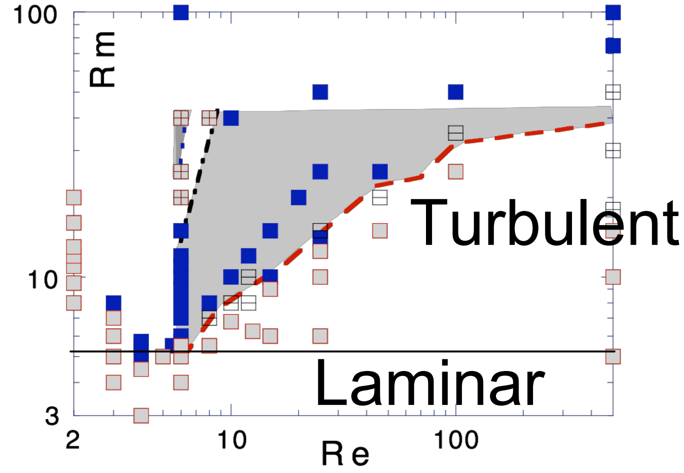




Laminar vs Turbulent Dynamos



Different Dynamos!



Different threshold!

Dubrulle et al, NJP, 9, 308 (2007)

To understand :Stochastic approach

Full equations (DNS)

$$\partial_t \vec{B} = R_m curl\left(\vec{v} \times \vec{B}\right) + \Delta\vec{B}$$

$$\partial_t \vec{v} + R_m\left(\vec{v} \bullet \vec{\nabla}\right)\vec{v} = -R_m \vec{\nabla} p + R_m curl\, \vec{B} \times \vec{B} + \frac{R_m}{\mathrm{Re}} \Delta\vec{v}$$

Model 1 (Analytical)

$$\partial_t \vec{B} = curl\left(\vec{v} \times \overleftarrow{B}\right) - KB^2 \vec{B}$$

$$\vec{v} = <\vec{V}> + \vec{v}'$$

Model 2 (SNS)

$$\partial_t \vec{B} = curl\left(\vec{V} \times \vec{B}\right) + \eta\Delta\vec{B} + curl\left(\vec{v}' \times \vec{B}\right.$$

where

$$< v'(x,t) v'(x+r, t+\tau) > = 2G(x,x')\delta(\tau)$$

Noise delta-correlated in time (Kraichnan model)

Model 1

$$\partial_t \vec{B} = curl\left(\vec{v} \times \overleftarrow{B}\right) - KB^2\vec{B}$$

Non-Linear
Kraichnan model
Multiplicative noise
Work with PDF and Λ

$$2\sigma = \frac{\partial \ln\langle B^2\rangle}{\partial t} \longrightarrow 2\Lambda = \frac{\partial\langle \ln B^2\rangle}{\partial t}$$

Model 2

$$\partial_t \vec{B} = curl\left(\vec{V} \times \vec{B}\right) + \eta\Delta\vec{B} + curl\left(\vec{v}' \times \vec{B}\right)$$

Linear
Stochastic simulation
Work with mean TG flow

$\vec{V}$ Time-average of velocity field computed through Navier-Stokes

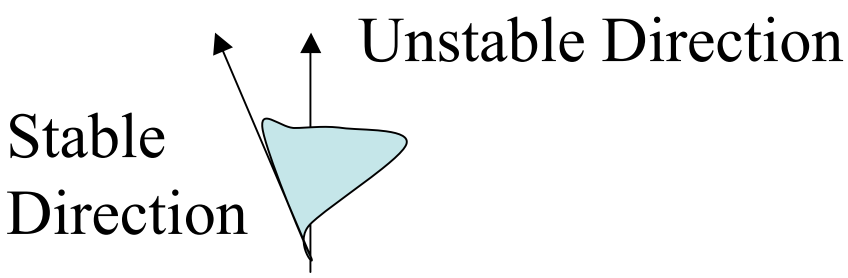
Model 1: threshold

$$a = \left\langle \mu_{ijkl} e_i e_j e_k e_l \right\rangle_G > 0$$

$$\Lambda = \left\langle \partial_k V_i e_i e_k \right\rangle_G + \left\langle \mu_{ijkl} \left(\Delta_{ik} e_i e_k + \Delta_{kj} e_i e_l \right) \right\rangle_G$$

Orientation (<0), large scale
(zero if <V>=0)

Friction, >0, small-scale
(μ effect, favourable)



$$\beta_{kl} = \left\langle v'_k v'_l \right\rangle$$

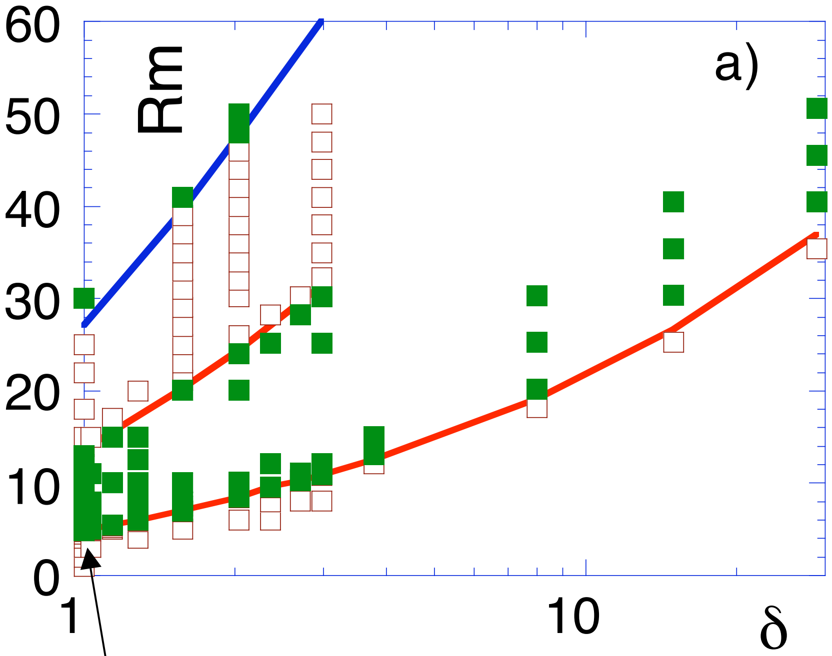
$$\alpha_{ijk} = \left\langle v'_i \partial_k v'_j \right\rangle$$

$$\mu_{ijkl} = \left\langle \partial_j v'_i \partial_l v'_k \right\rangle$$

Leprovost, Dubrulle, EPJB, 44, 395 (2005)

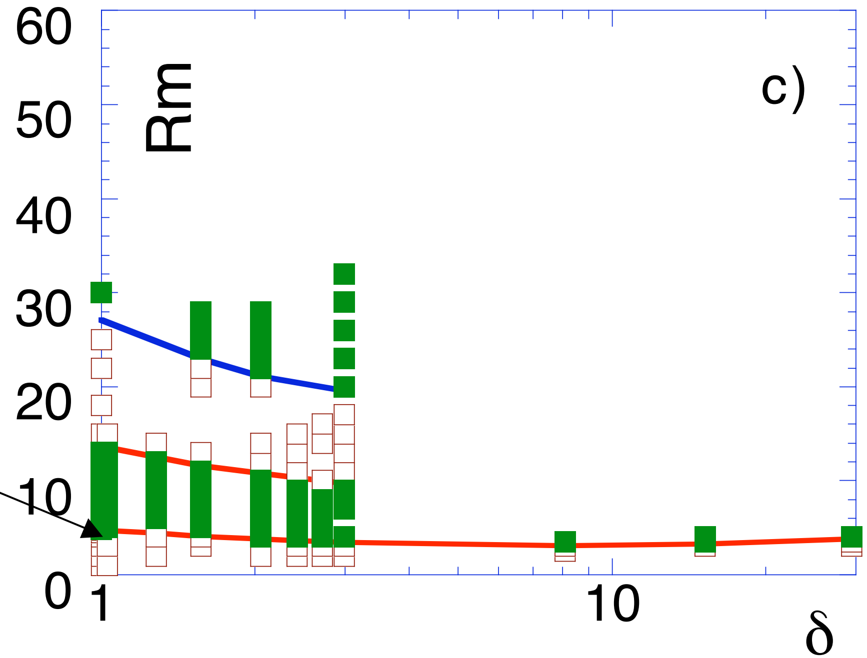
Model 2: Threshold

Forcing at ki=1



$$\vec{v} = <\vec{V}> + \vec{v}'$$

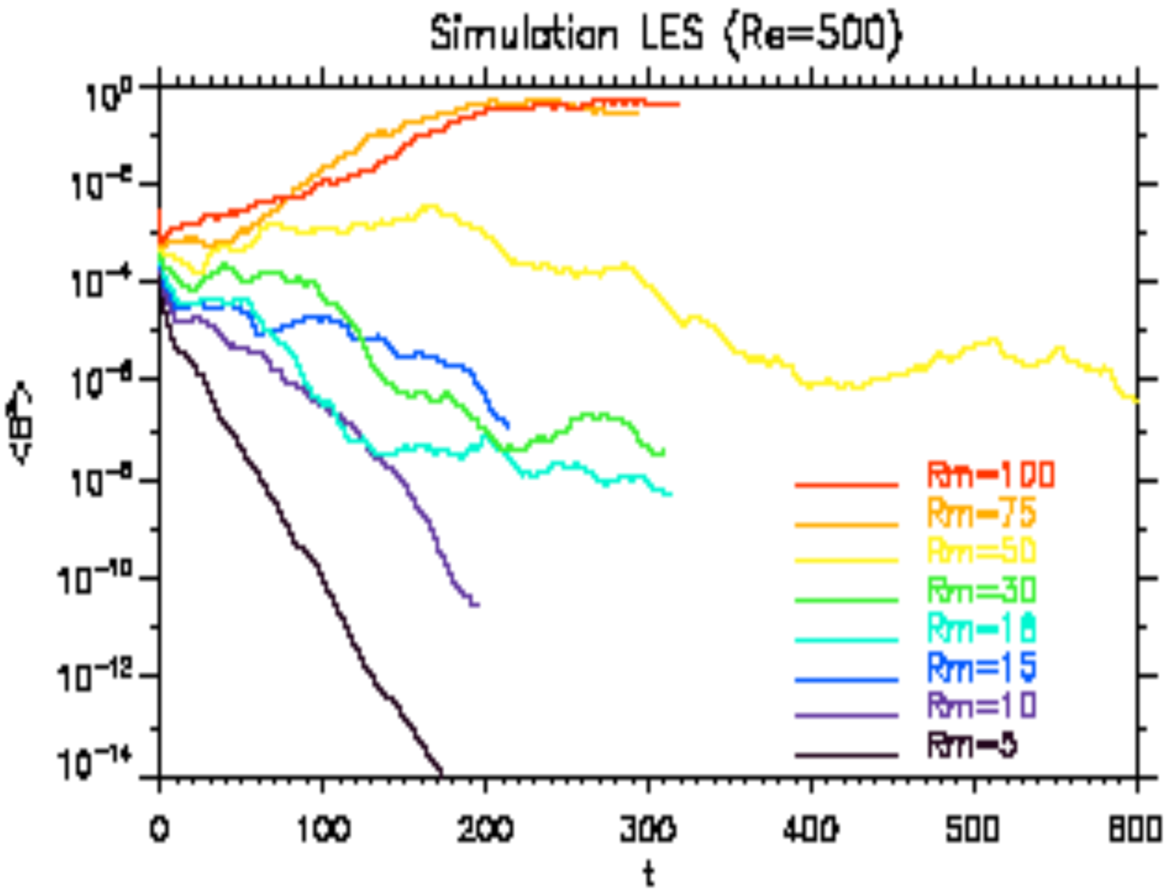
Forcing at ki=16



Linear in (δ-1)
(Fauve-Petrelis)

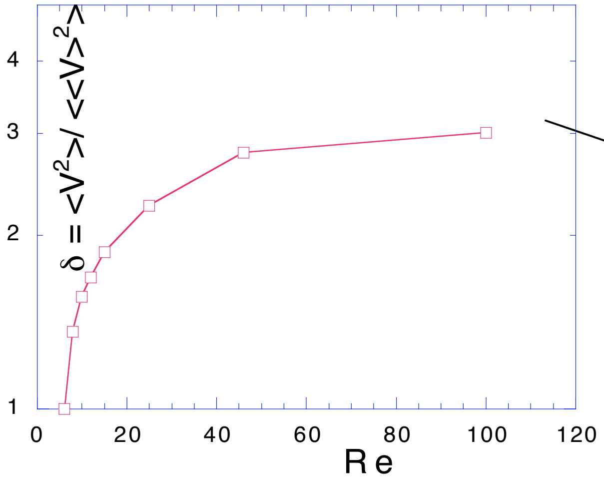
Laval et al, PRL 96, 204503 (2006)

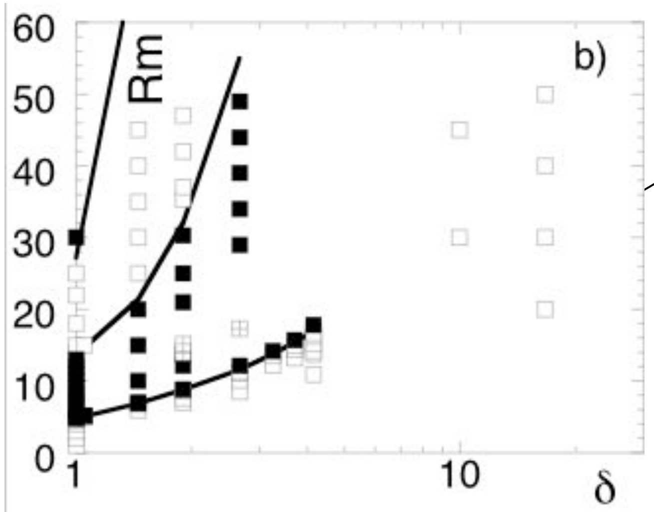
Full model:Disorientation



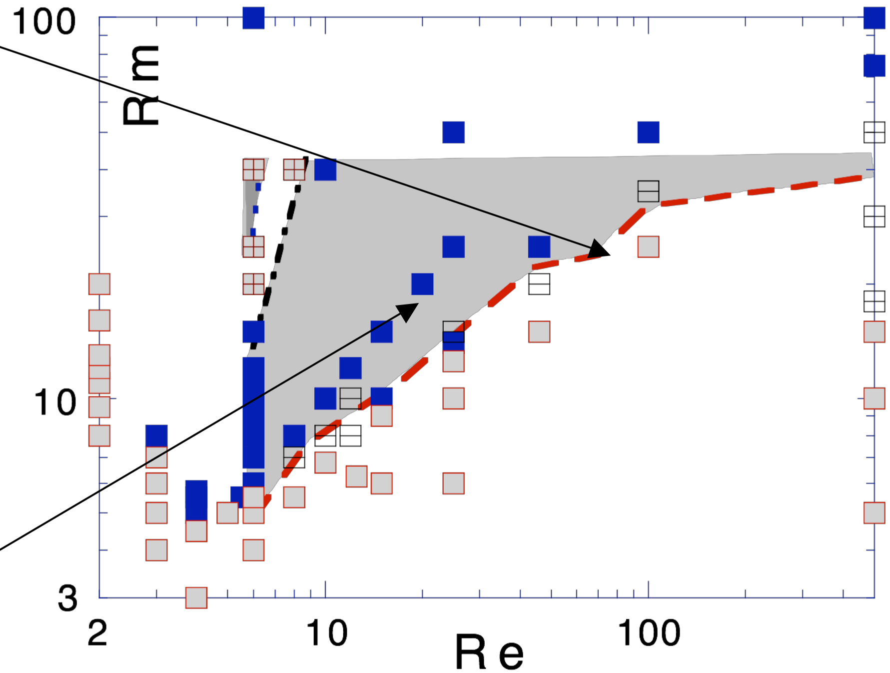
Full model: Threshold

Simulation with real velocity





Turbulence increases threshold
With respect to time-averaged!



Agreement stochastic/DNS

Laval et al, PRL 96, 204503 (2006)

Model 1: Saturation

Equation for P(B,x,t)

$$\partial_t P = -V_k \partial_k P - \left(\partial_k V_i\right)\partial_{B_i}\left[B_k P\right] + K\partial_{B_i}\left[B^2 B_i P\right]$$

$$+ \partial_k\left[\beta_{kl}\partial_l P\right] + 2\partial_{B_i}\left[B_k \alpha_{lik}\partial_l P\right] + \mu_{ijkl}\partial_{B_i}\left[B_j \partial_{B_k}\left(B_l P\right)\right]$$

with

$$\beta_{kl} = \left\langle v_k^{'} v_l^{'} \right\rangle$$

$$\alpha_{ijk} = \left\langle v_i^{'} \partial_k v_j^{'} \right\rangle$$

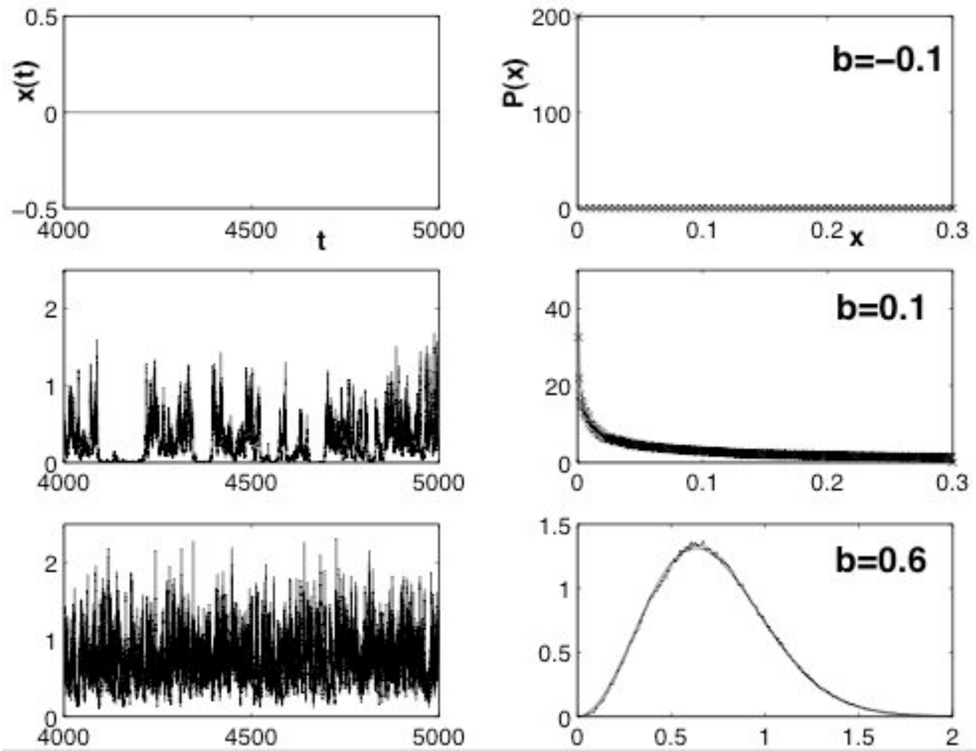
$$\mu_{ijkl} = \left\langle \partial_j v_i^{'} \partial_l v_k^{'} \right\rangle$$

Leprovost, Dubrulle, EPJB, 44, 395 (2005)

Model 1: Saturation

Non-zero Solution (normalisation)

$$a > 0 \quad et \quad \frac{\Lambda}{a} > 0$$



Most probable value

$$\Lambda > aD$$

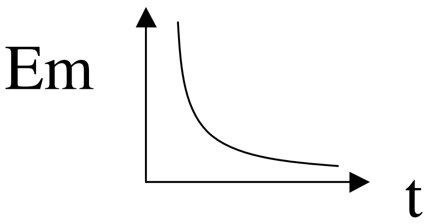
Illustration

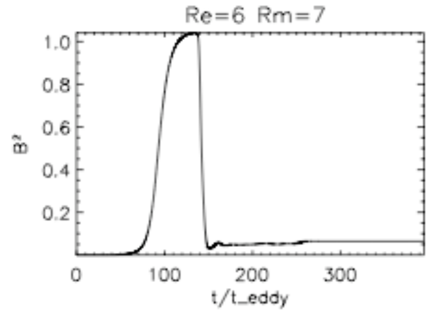
$$\dot{x} = (b + \xi)x - \gamma x^3$$

$$b = \Lambda$$

Laminar vs Turbulent Dynamos

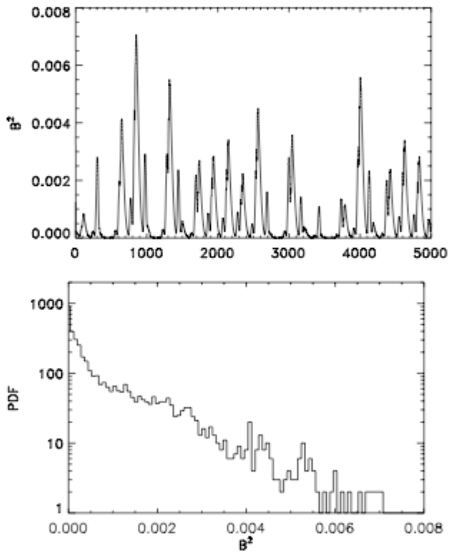
Laminar

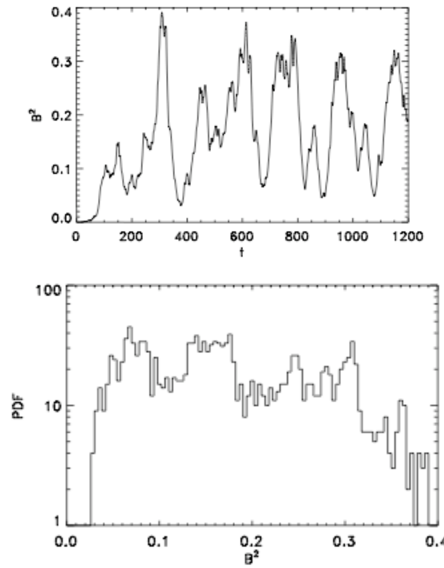




Pas dynamo | Rmc | Dynamo | Rm

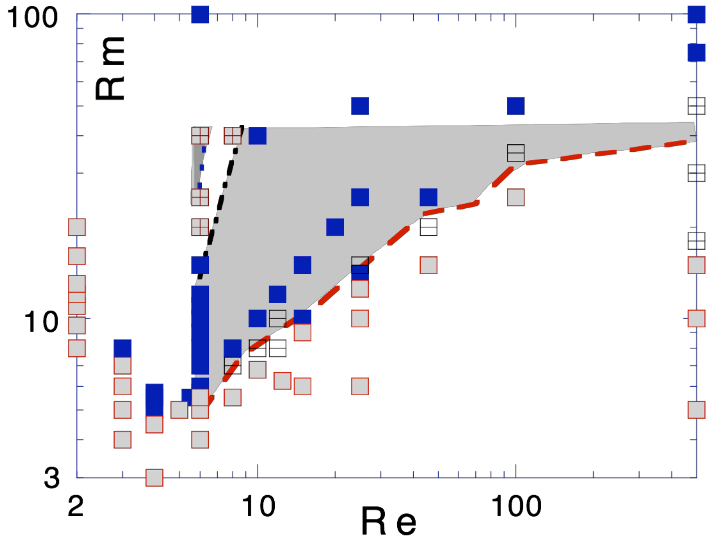
Turbulent

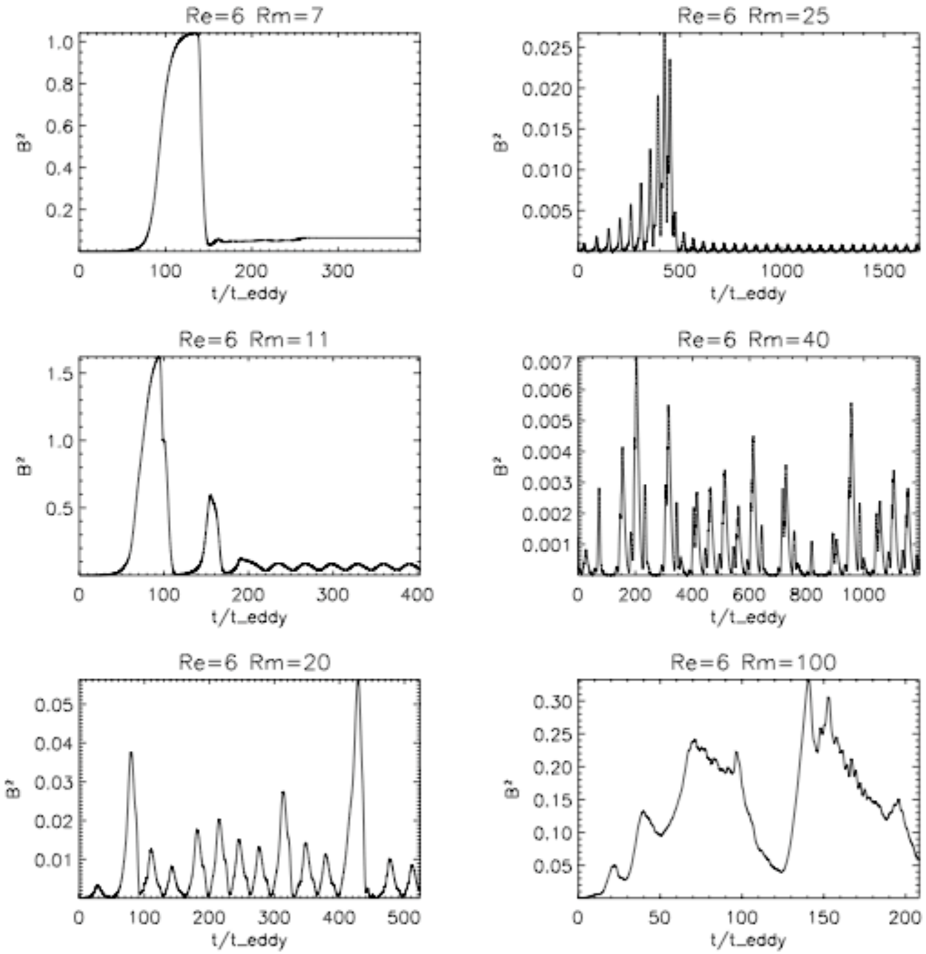




No dynamo | 0 | Intermittent Dynamo | a | Turbulent Dynamo | Λ

Full model: saturation

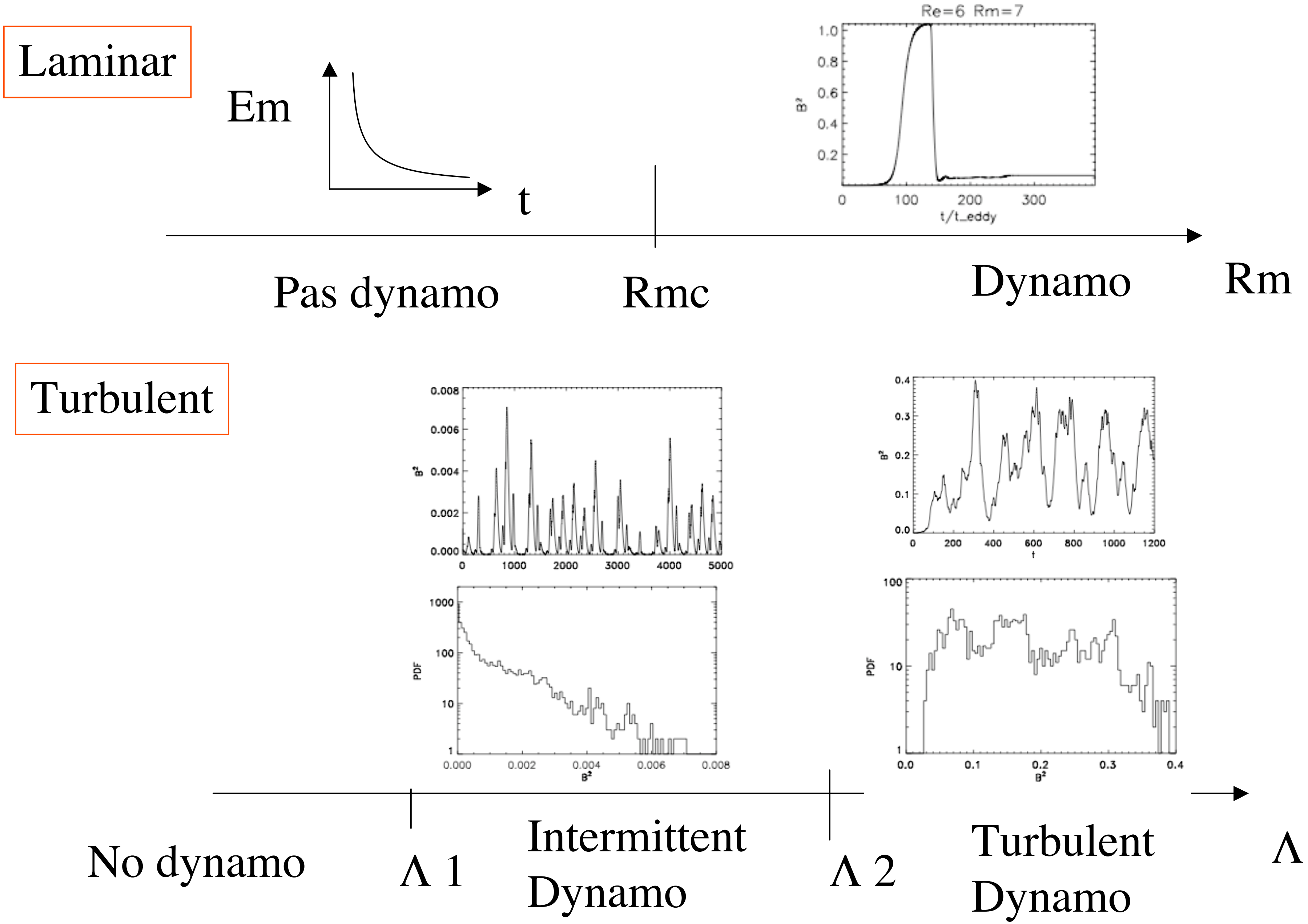




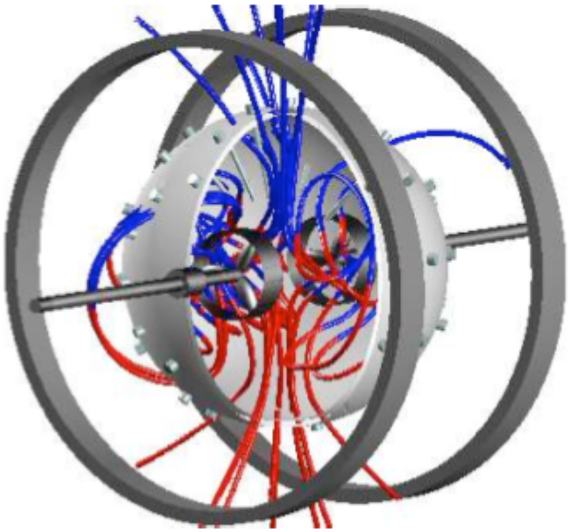
Burst with increasing Rm

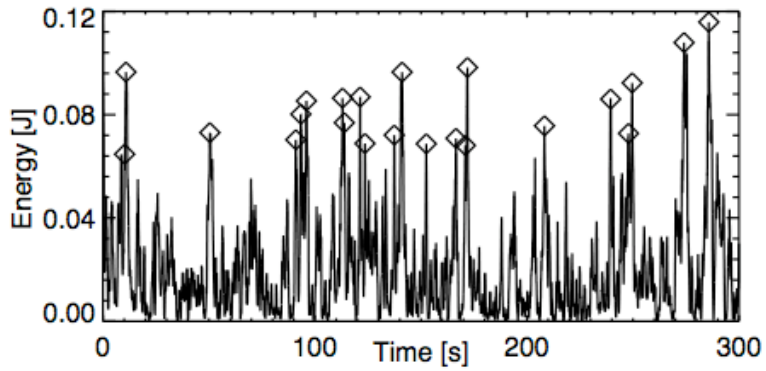
Dubrulle et al, NJP, 9, 308 (2007)

Laminar vs Turbulent Dynamos

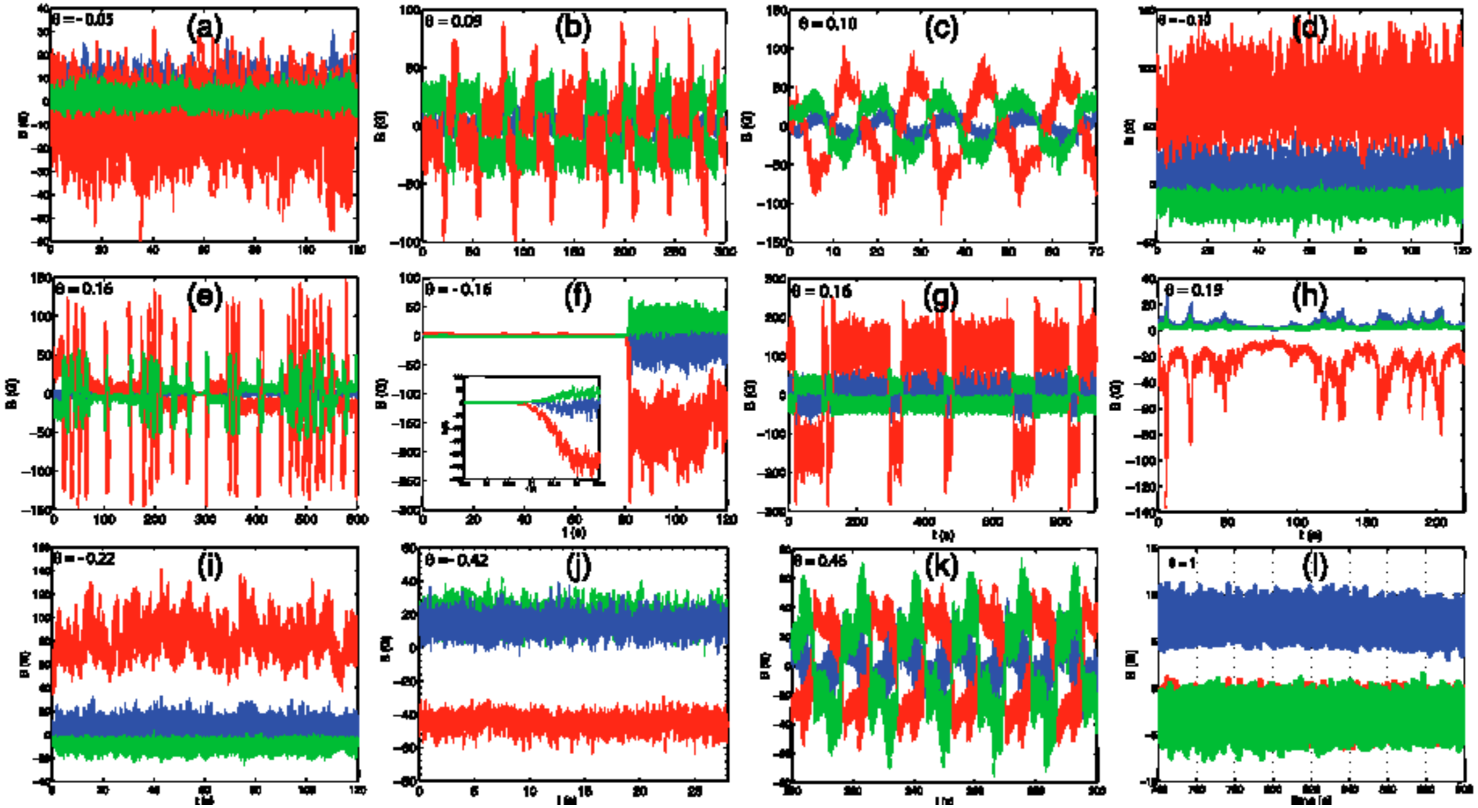


Disorientation in Wisconsin





Intermittency in VKS2



Burst seen with increasing rotation