School of Mathematics & Statistics, Newcastle University

Geomagnetic reversals

KITP Discussion

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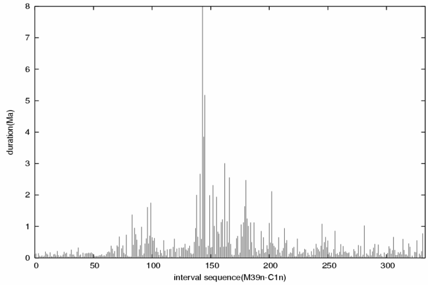
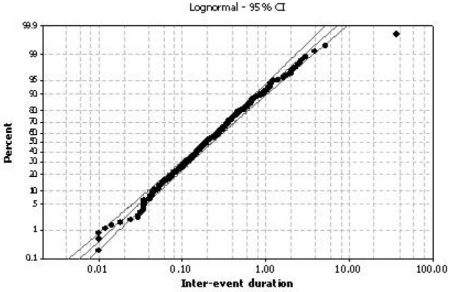
Statistics of geomagnetic reversals

The inter-reversal duration data of Gradstein & Ogg (1996) are well fit (according to Anderson–Darling, Kolmogorov–Smirnov and χ^2 tests) by a lognormal distribution.





Lognormal distributions commonly arise from processes containing multiplicative noise.

The role of multiplicative noise

Given a schematic equation for B,

$$\frac{\mathrm{d}B}{\mathrm{d}t} = \alpha B \; ,$$

$B_i = B(t_i)$ will evolve, subject to $\alpha_i = \alpha(t)$, as

$$B_{i+1} = (1 + \delta t \alpha_i)\, B_i \Longrightarrow B_N = \left(\prod_{i=0}^{N} (1 + \delta t \alpha_i) \right) B_0$$

$$\Longrightarrow \ln B_N = \left(\sum_{i=0}^{N} \ln(1 + \delta t \alpha_i) \right) + \ln B_0 \approx \left(\sum_{i=0}^{N} \delta t \alpha_i \right) + \ln B_0 \; .$$

If the α_i are small, independent random perturbations, then $\ln B$ will approach the normal distribution, and B the lognormal distribution.

A toy reversal model

This mechanism was investigated using a simple model, coupling a low order dynamo model (for S, T, ω) to a shell model of turbulence (for u_n, to produce α):

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \alpha T - \kappa S \,, \qquad \frac{\mathrm{d}T}{\mathrm{d}t} = \omega S - \kappa T \,,$$

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \Gamma - \lambda_1 ST - \lambda_2 (S^2 + T^2) - \kappa_\omega \omega \,,$$

$$\frac{\mathrm{d}u_n}{\mathrm{d}t} = -\nu k_n^2 u_n + f_n + i k_n \left(u_{n+2}^* u_{n+1}^* - \frac{1}{4} u_{n+1}^* u_{n-1}^* + \frac{1}{8} u_{n-1}^* u_{n-2}^* \right) \,,$$

$$\alpha \sim \frac{1}{3} \tau \langle \mathbf{u} \cdot \nabla \times \mathbf{u} \rangle = \xi_\alpha \sum_{n=1}^{N} (-1)^n k_n |u_n|^2 \,.$$

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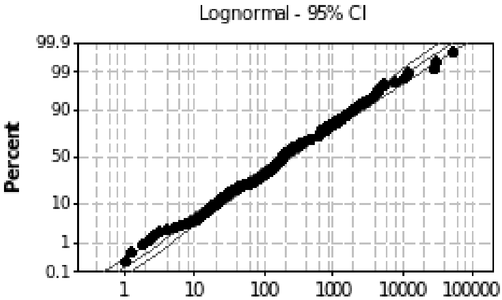
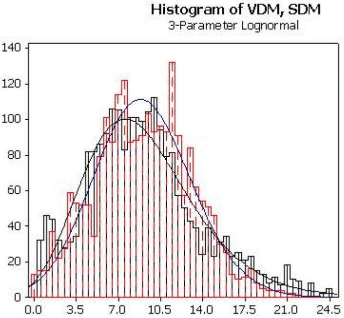
Synthetic statistics: reversals & intensity fluctuations

The model exhibits irregular reversals in the dipole field S, which are also well fit by a lognormal distribution.

Intensity fluctuations — both synthetic (S) and observed (e.g. VADMs from Valet & Meynadier, 1993) — are also reasonably fit by lognormal distributions. (Cf. McFadden & McElhinny, 1984).





Significance of superchrons?

The superchron(s) arguably remain outliers to lognormal-type fits.
They may be better fit by Pareto-Lévy tails, reflecting a modified underlying mechanism.
In our model, superchrons do correspond to a different style of dynamo action. (And similar tails can be obtained for some parameter regimes.)