HK Moffatt: Discussion on 16 May '08.

$$\hat{u}(\underline{k}, \omega) = -\frac{\underline{A}(\underline{k}, \omega)}{D(\underline{k}, \omega)} \hat{\theta}(\underline{k}, \omega)$$

where

$$\underline{A} = \frac{-(\underline{k}\cdot\underline{B}_0)^2\, \underline{k}\wedge(\underline{k}\wedge\underline{g})}{\eta k^2 - i\omega} - 2\underline{k}\cdot\underline{\Omega}\,(\underline{k}\wedge\underline{g})$$

$$D = 4(\underline{k}\cdot\underline{\Omega})^2 + \frac{(\underline{k}\cdot\underline{B}_0)^4 k^2}{(\eta k^2 - i\omega)^2}$$

[$D = 0$ is dispersion relation for magnetostrophic waves damped by magnetic resistivity η]

$$\hat{\underline{b}}(\underline{k}, \omega) = \frac{i(\underline{B}_0\cdot\underline{k})\,\hat{\underline{u}}}{\eta k^2 - i\omega}$$

$$i\,\underline{A}^*\wedge\underline{A} = \frac{-4(\underline{k}\cdot\underline{\Omega})\,\omega\,(\underline{k}\cdot\underline{B}_0)^2(\underline{k}\wedge\underline{g})^2\,\underline{k}}{\eta^2 k^4 + \omega^2} \quad \text{(real)}$$

Hence helicity spectrum

$$\mathcal{H}(\underline{k}, \omega) = \frac{4(\underline{k}\cdot\underline{\Omega})\,\omega\,(\underline{k}\cdot\underline{B}_0)^2 k^2}{|D|^2(\eta^2 k^4 + \omega^2)} (\underline{k}\wedge\underline{g})^2\, \Gamma(\underline{k}, \omega)$$

$\Gamma(\underline{k}, \omega)$ = spectrum fn. of θ (assumed given)

[Break up-down symmetry]

and hence

$$\alpha = \tfrac{1}{3}\alpha_{ii} = -\frac{\eta}{3}\iint \frac{k^2\,\mathcal{H}(\underline{k}, \omega)}{\omega^2 + \eta^2 k^4}\, d\underline{k}\, d\omega \quad (R_m \leq 1) \text{ FOS}$$

α-quenching is contained in this expression

Buoyancy Flux

$$F_z = \langle \underline{u}\,\theta \rangle_z = \iint \frac{(\underline{k}\cdot\underline{B}_0)^2 \eta k^2 [\underline{k}\wedge(\underline{k}\wedge\underline{g})]_z\, \Gamma(\underline{k}, \omega)\, d\underline{k}\, d\omega}{4(\underline{k}\cdot\underline{\Omega})^2(\eta^2k^4 + \omega^2) + (\underline{k}\cdot\underline{B}_0)^4(\eta^2 k^4 - \omega^2)/k^2}$$

Reynolds stress

$$\tau_{ij} = \langle u_i u_j \rangle = \iint \frac{\mathrm{Re}[A_i^* A_j]}{|D|^2}\, \Gamma(\underline{k}, \omega)\, d\underline{k}\, d\omega$$

$$\tau_{ij}^M = \langle b_i b_j \rangle = \cdots$$

These drive differential rotation and meridional circulation

$$\frac{\partial \theta}{\partial t} + \underset{\sim}{u}(\theta) \cdot \nabla \theta = \kappa \nabla^2 \theta$$

linear functional of θ'

$$Pe = \frac{v \ell}{\kappa} \gg 1 \qquad (\sim 10^8 \text{ in core})$$

So this nonlinearity dominates.

Example $\quad \underset{\sim}{u} = \nabla \wedge \underset{\sim}{A} \qquad (\text{so } \nabla \cdot \underset{\sim}{u} = 0)$

let $\underset{\sim}{A} = \underset{\sim}{a}(\underset{\sim}{x})\, \theta$

$$\underset{\sim}{u} = \nabla \wedge \underset{\sim}{A} = -\underset{\sim}{a} \wedge \nabla \theta + \underset{\sim}{b}\, \theta \qquad (\underset{\sim}{b} = \nabla \wedge \underset{\sim}{a})$$

$$\underset{\sim}{u} \cdot \nabla \theta = \theta\, \underset{\sim}{b} \cdot \nabla \theta$$

Suppose $\underset{\sim}{b} = (0, 0, b)$ with b constant

Then we get

$$\frac{\partial \theta}{\partial t} + b\, \theta \frac{\partial \theta}{\partial x} = \kappa \nabla^2 \theta$$

Burger's equation; sharp fronts form in a finite time!

So not Kolmogorov type turbulence ??