

# A model for the Fat pathways role in cellular polarity and growth regulation

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- Growth and form:

*"An organism is so complex a thing, and growth so complex a phenomenon, that for growth to be so uniform and constant in all the parts as to keep the whole shape unchanged would indeed be an unlikely and an unusual circumstance. Rates vary; proportions change, and the whole configuration alters accordingly: "* - D'Arcy Thompson

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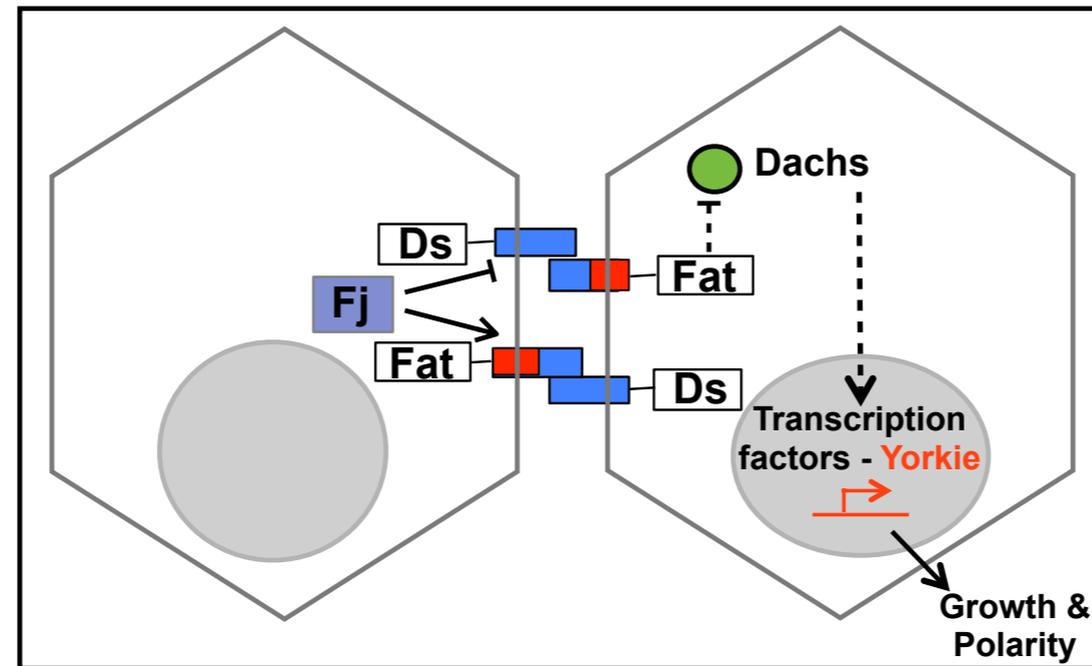
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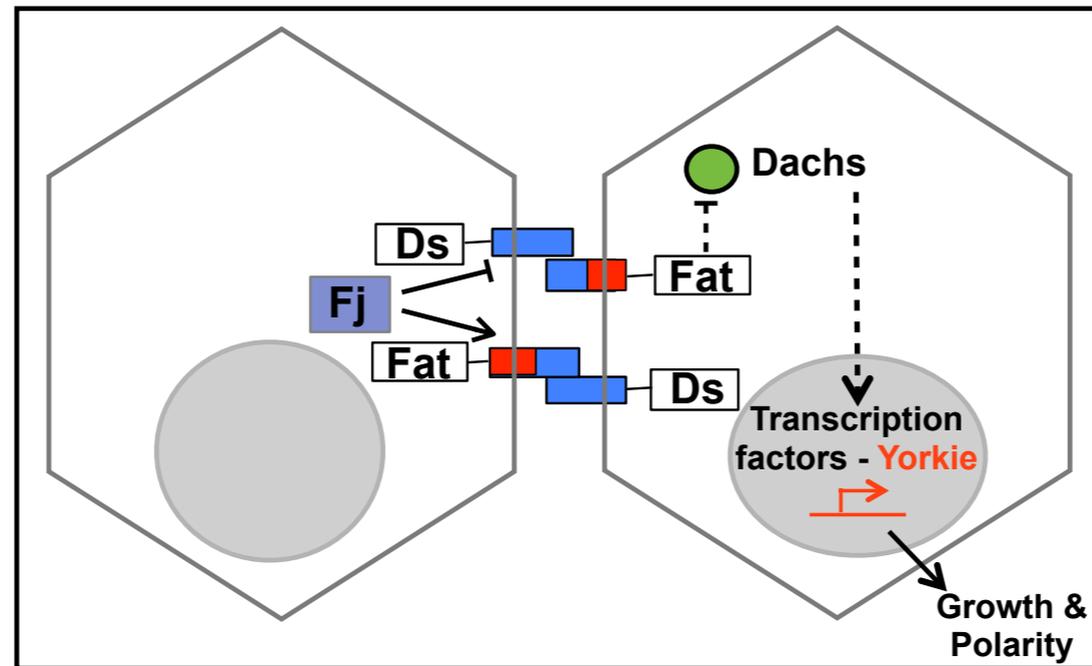
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# Observations - Who signals to whom? With what molecules?

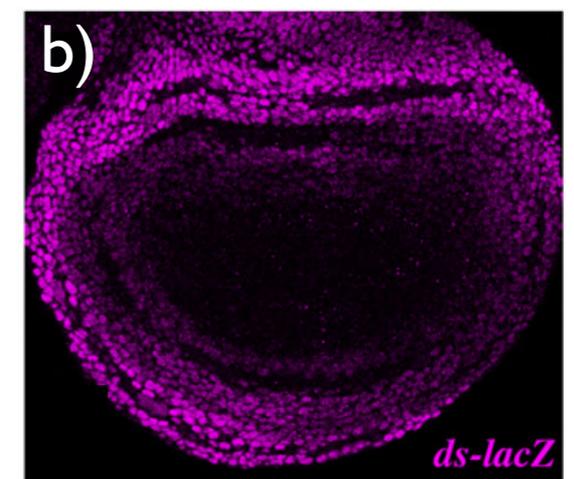
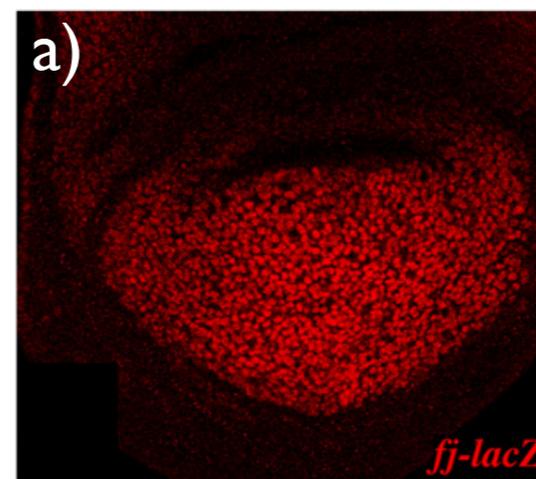
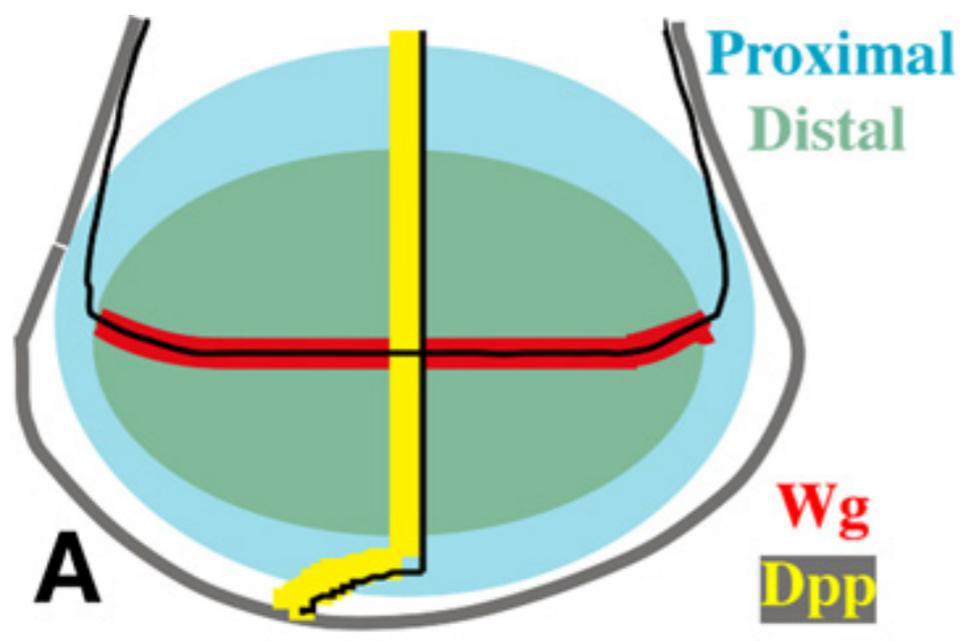


Note: lacZ

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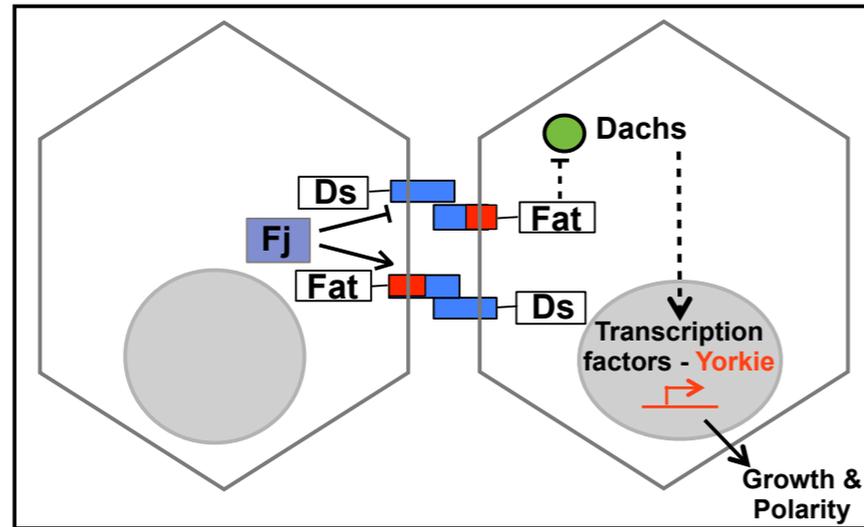
## Expression of core components



Note: lacZ

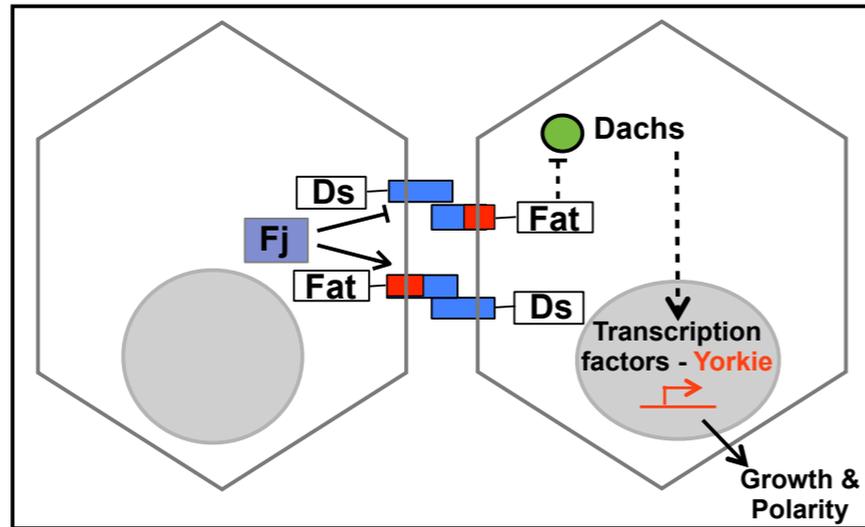


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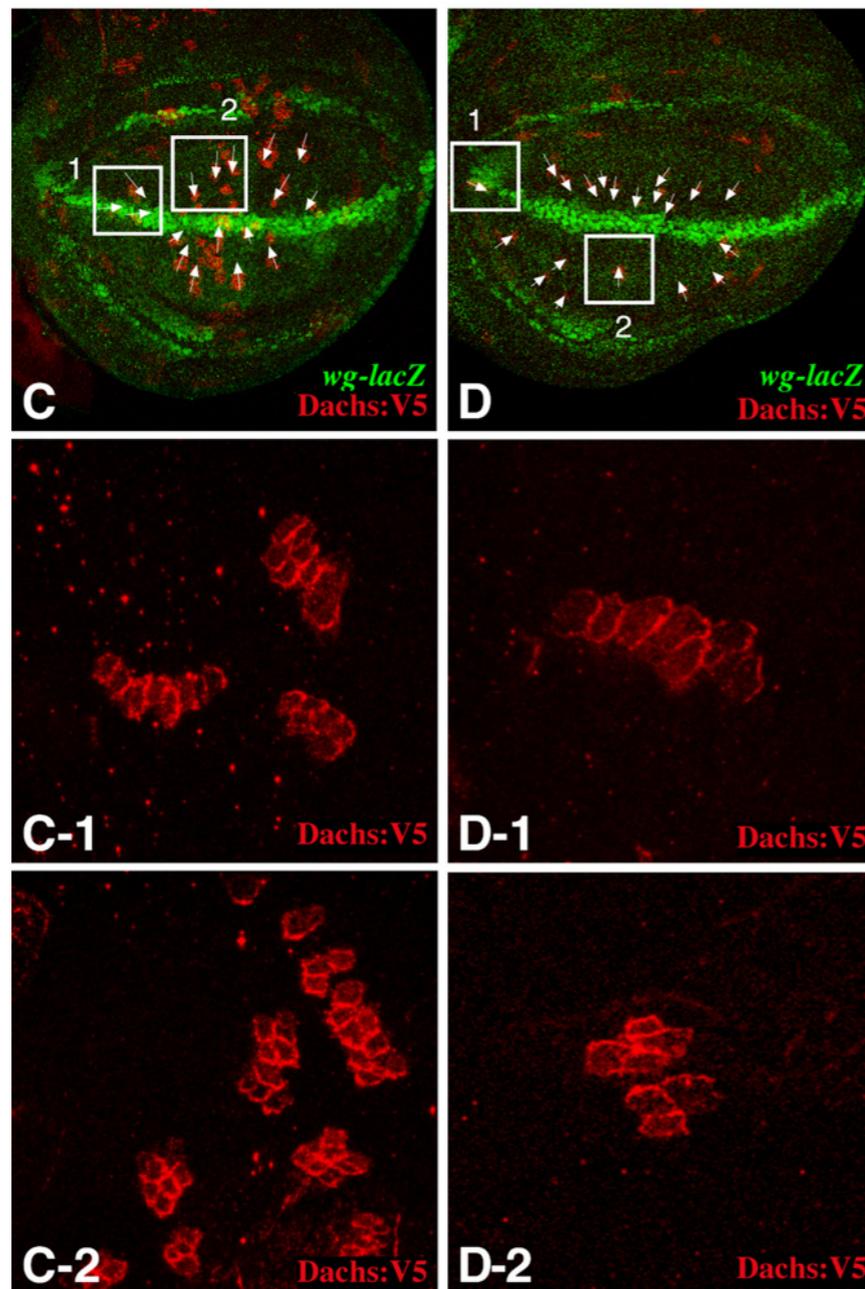


Dachs polarity and build up of nuclear Yorkie

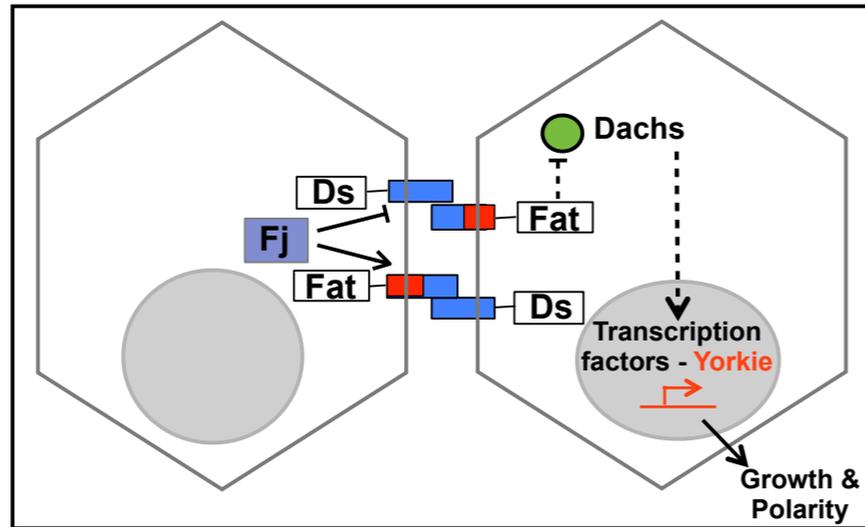
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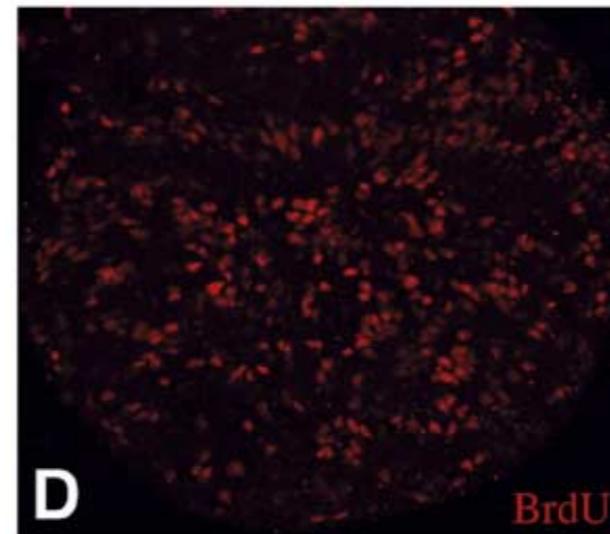
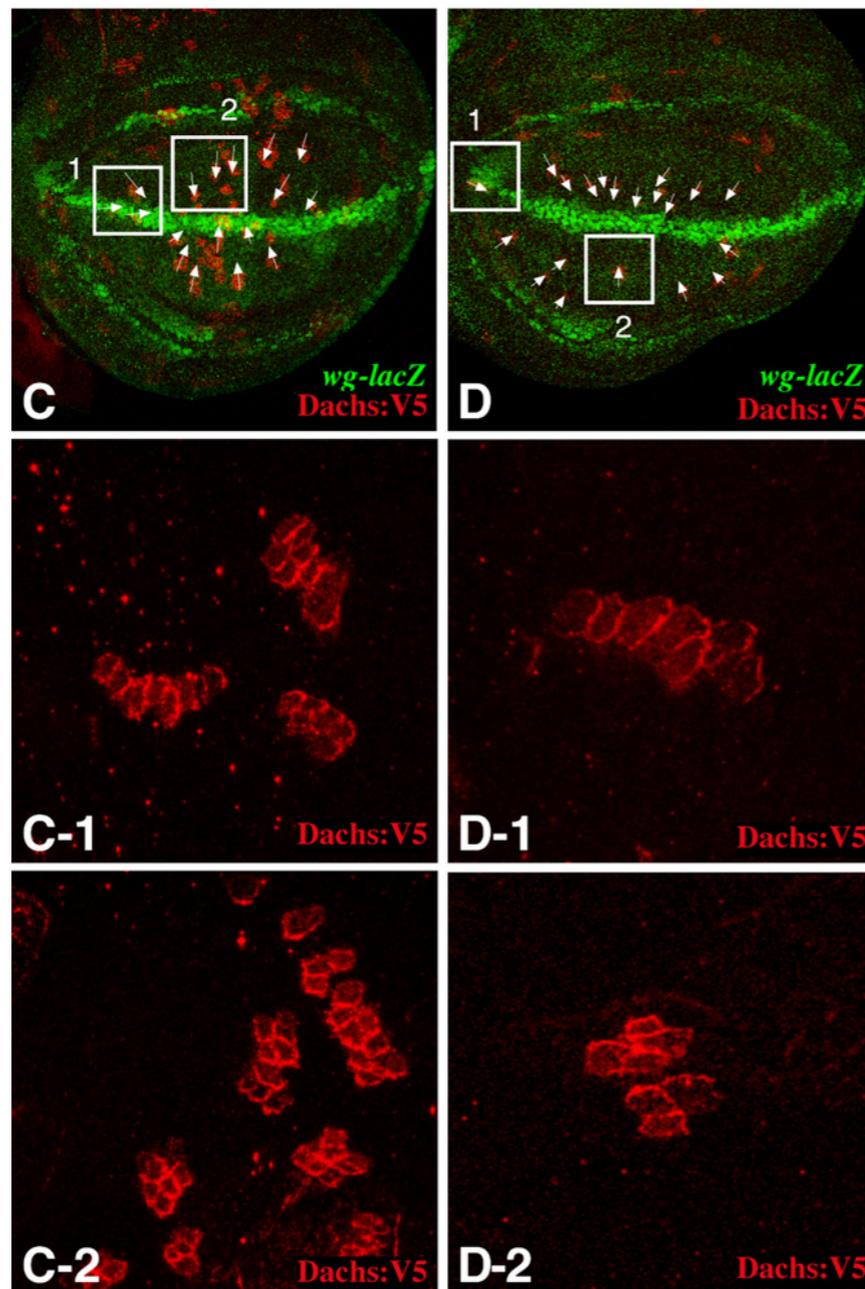
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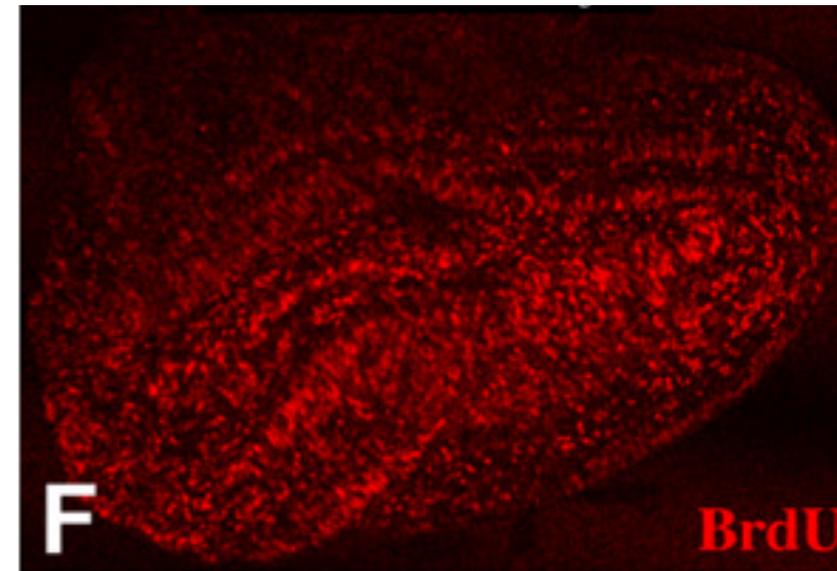
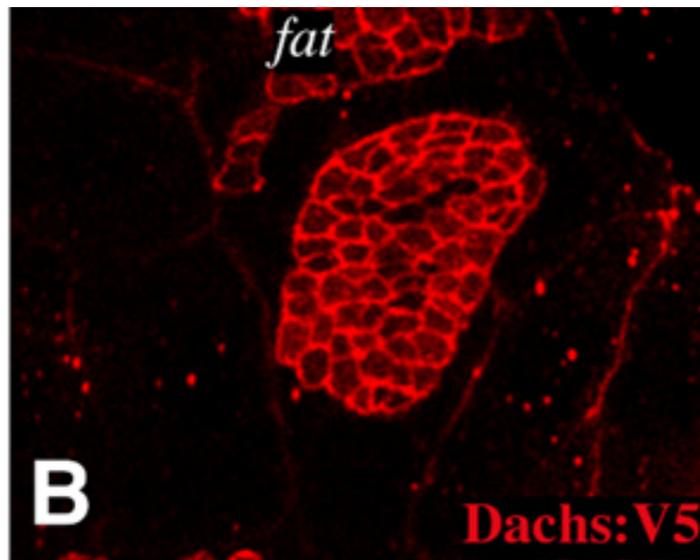
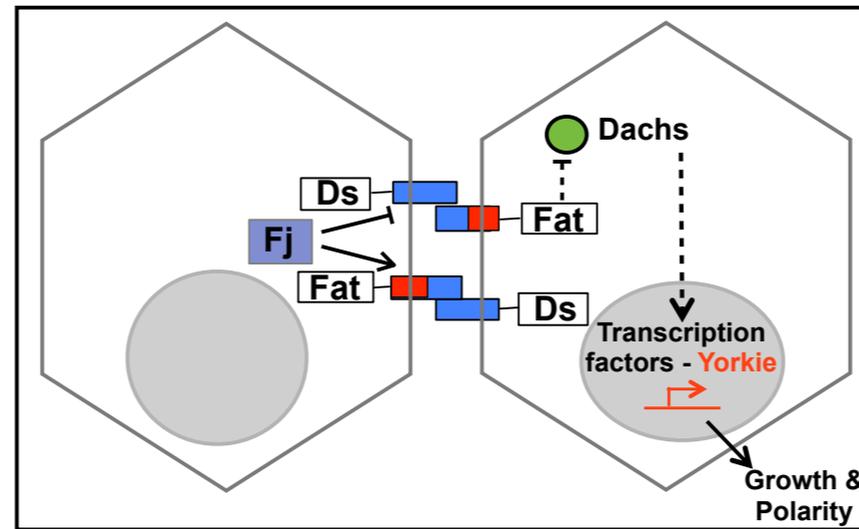
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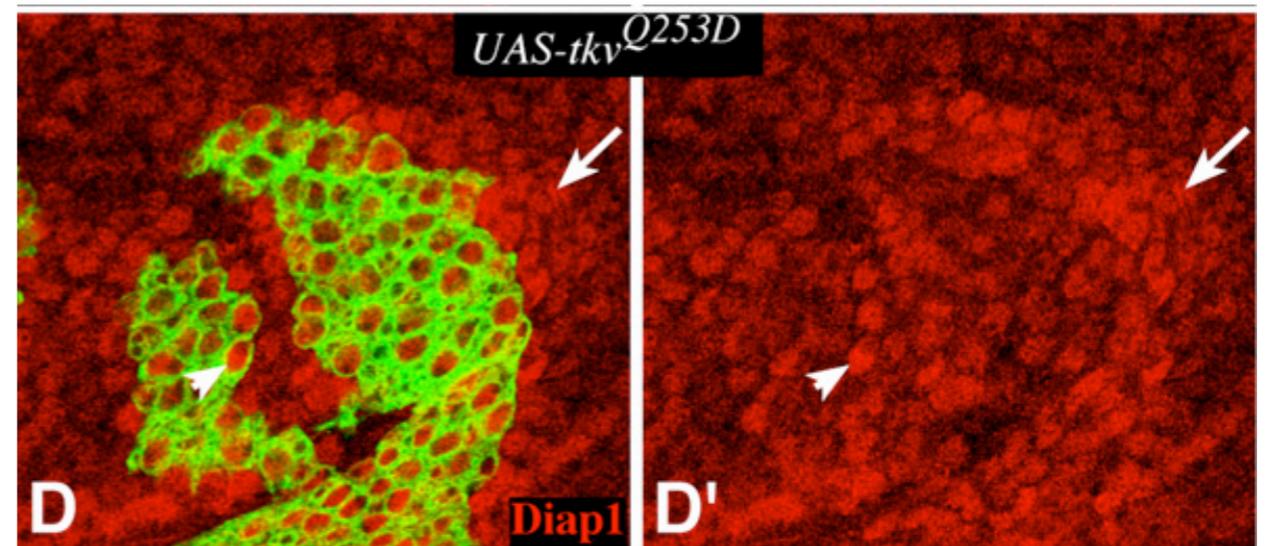
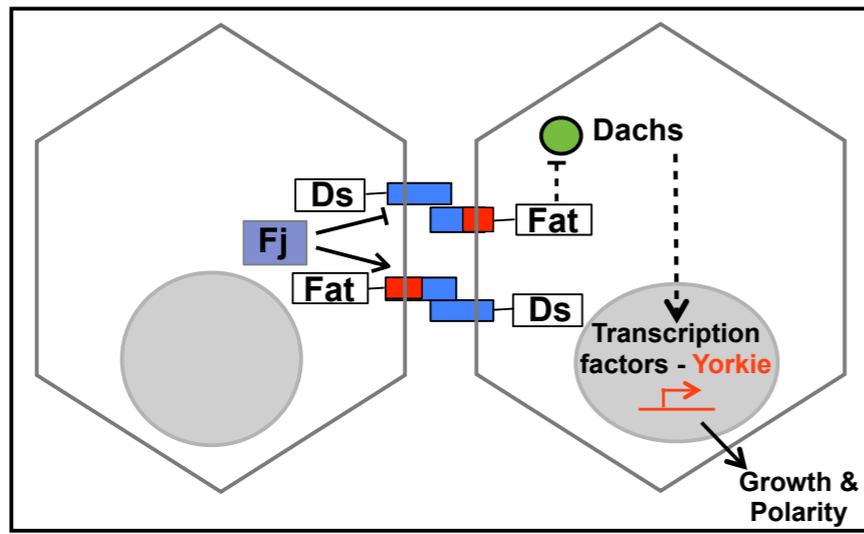
# Observations - To what end?



fat mutant → Loss of polarity → overgrowth

# Observations - To what end?

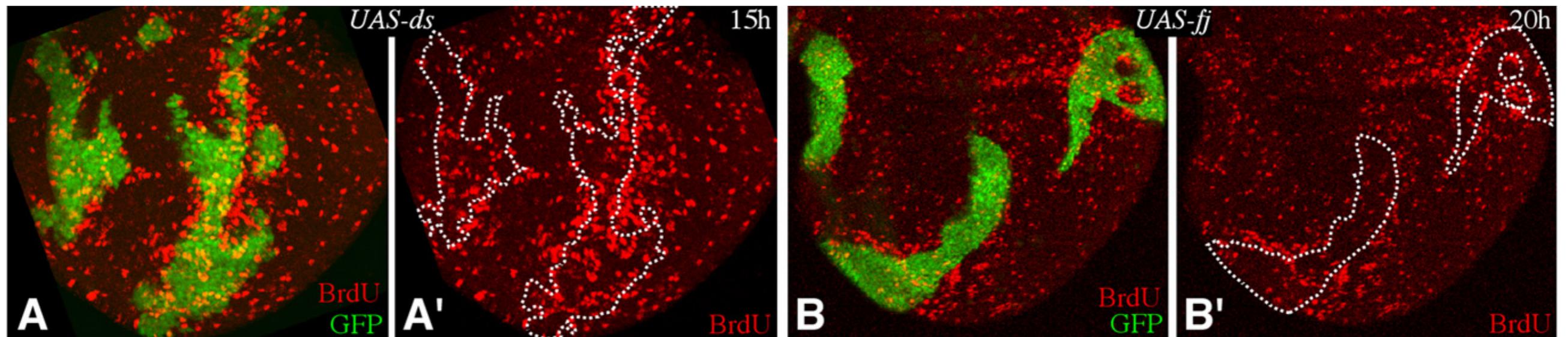
Dpp “gradients” induce growth



tkv: receptor for Dpp (ligand)

Diap I: Apoptosis inhibitor and reporter for Yorkie

“Gradients” in the Fat pathway components induce growth



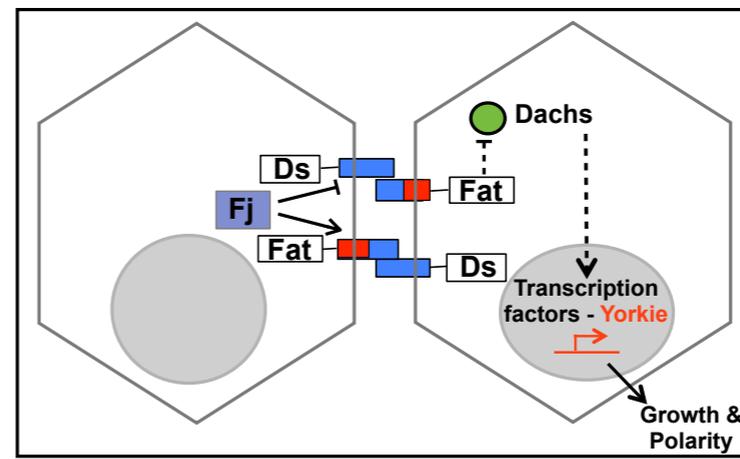
# Key Facts

- Fact 1) Transmembrane interaction & variations in level of core components signal
- Fact 2) Fat pathway is polarized in WT
- Fact 3) Gradients --> Growth

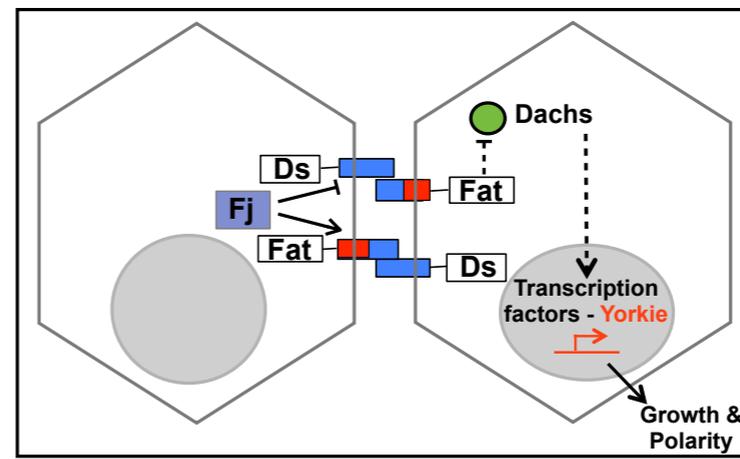
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- Fact 1) Transmembrane interaction & variations in level of core components signal
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- Input 1) Polarity generating mechanism
- Input 2) Cytosolic read out of polarity

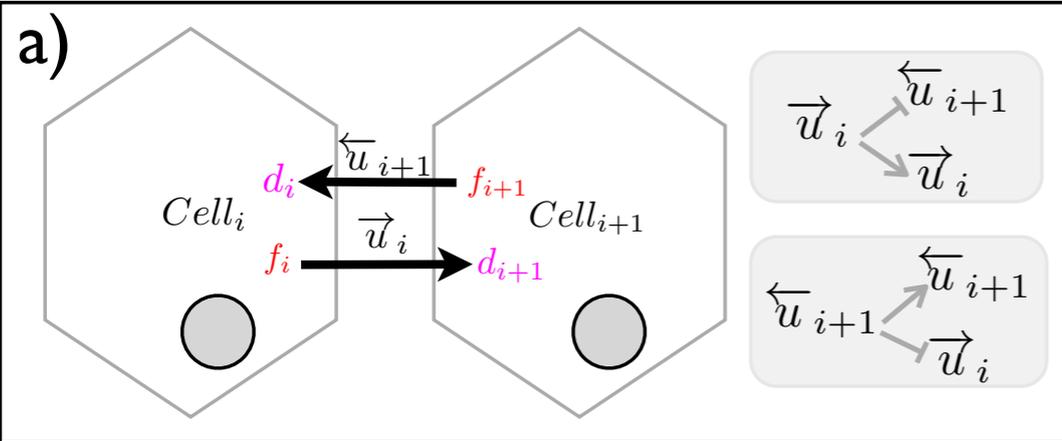
# Model



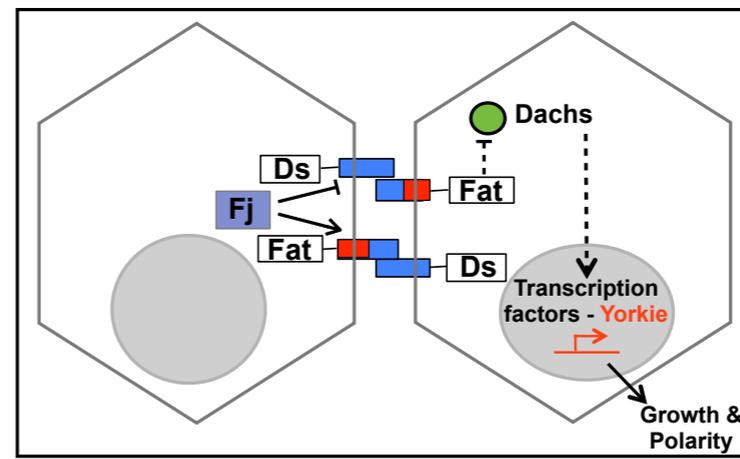
# Model



## Transmembrane Interactions



# Model

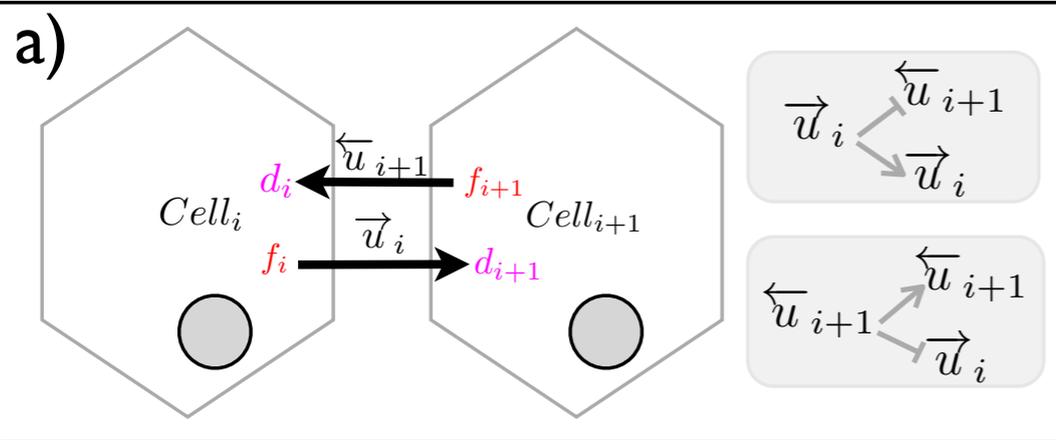


## Transmembrane Interactions

$$\frac{d\vec{u}_i}{dt} = k_u f_i d_{i+1} \text{ [red box]} - \gamma_u \vec{u}_i \text{ [red box]}$$

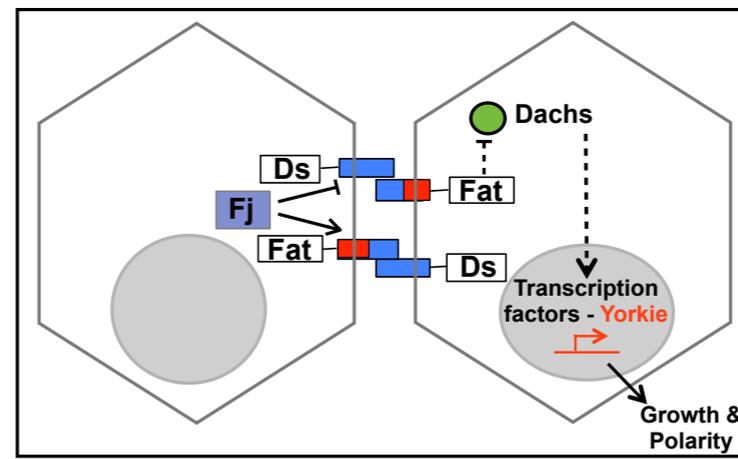
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$$f_i = f_i^0 - \vec{u}_i - \overleftarrow{u}_i \quad \& \quad d_i = d_i^0 - \vec{u}_{i-1} - \overleftarrow{u}_{i+1}$$





# Model



## Transmembrane Interactions

$$\frac{d\vec{u}_i}{dt} = k_u f_i d_{i+1} (1 + \alpha \vec{u}_i) - \gamma_u \vec{u}_i (1 + \beta \overleftarrow{u}_{i+1})$$

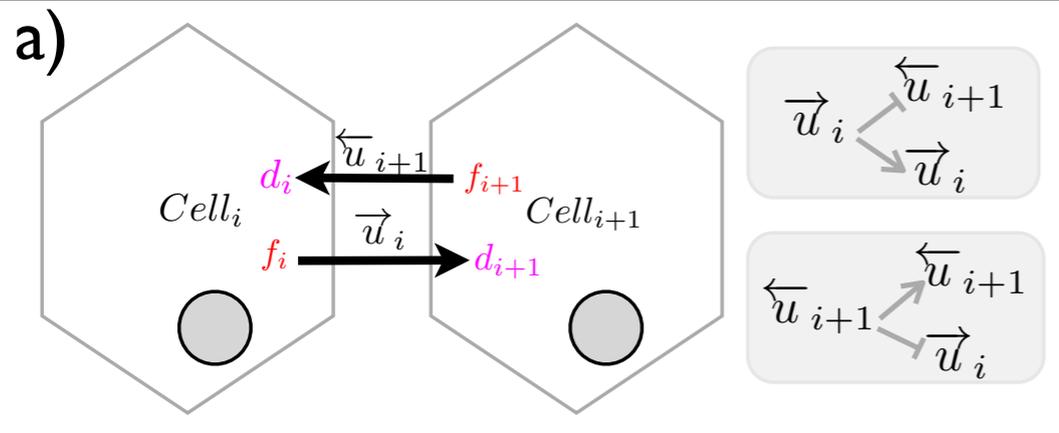
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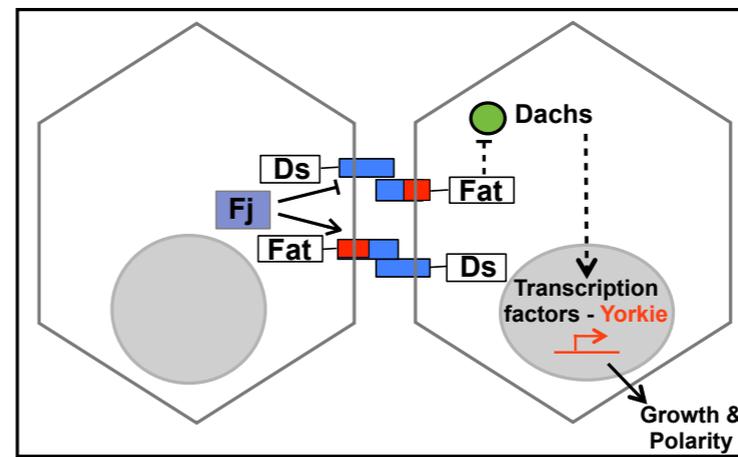
## Cytosolic Intermediate

$$\frac{d\vec{c}_i}{dt} = \sigma_c - \gamma_c \vec{c}_i - \nu_c \vec{c}_i \vec{u}_i^n$$

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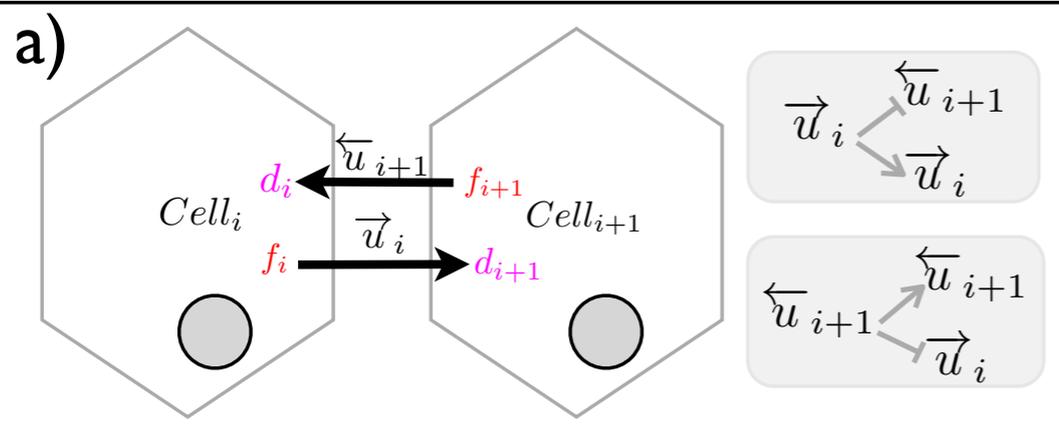
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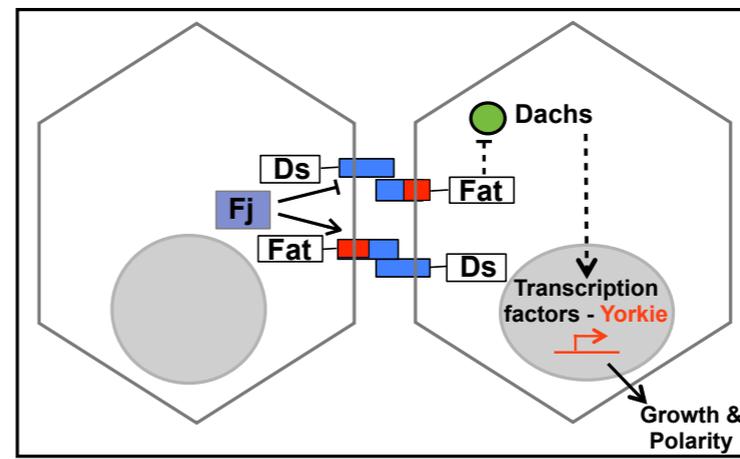
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## Nuclear Signal

$$\frac{dn_i}{dt} = k_n (\vec{c}_i + \overleftarrow{c}_i) - \gamma_n n_i$$



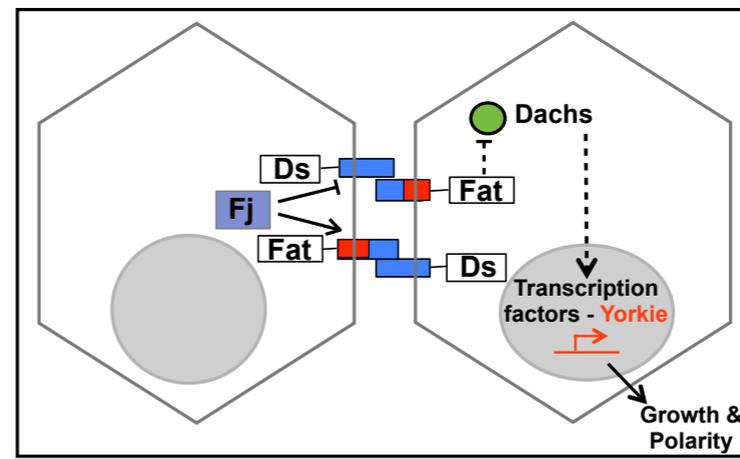
# Model - Scaled & Steady State



## Transmembrane Interactions



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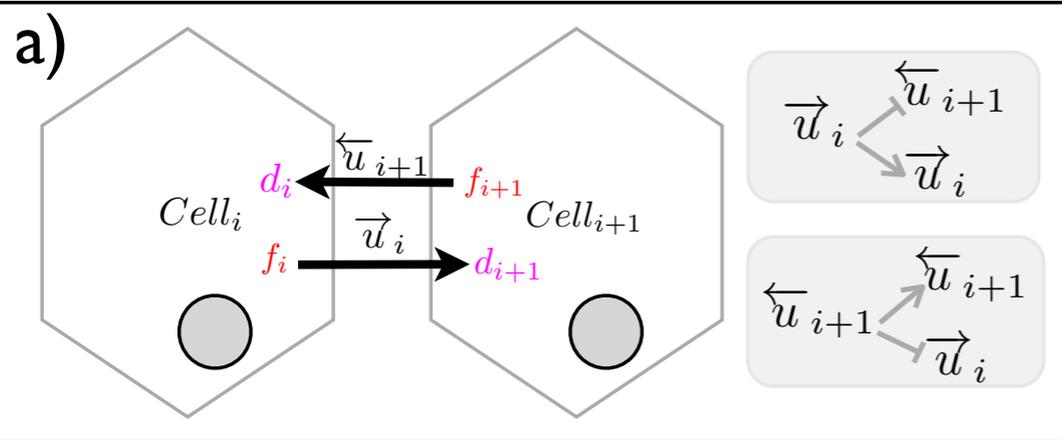


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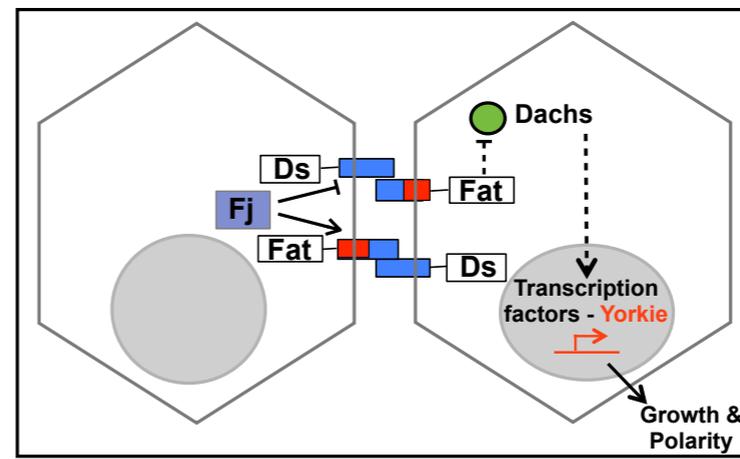
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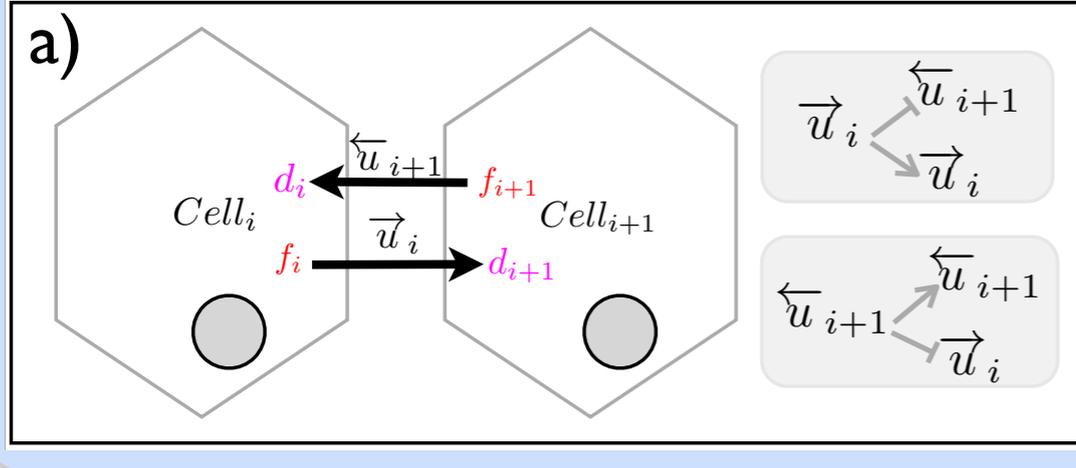
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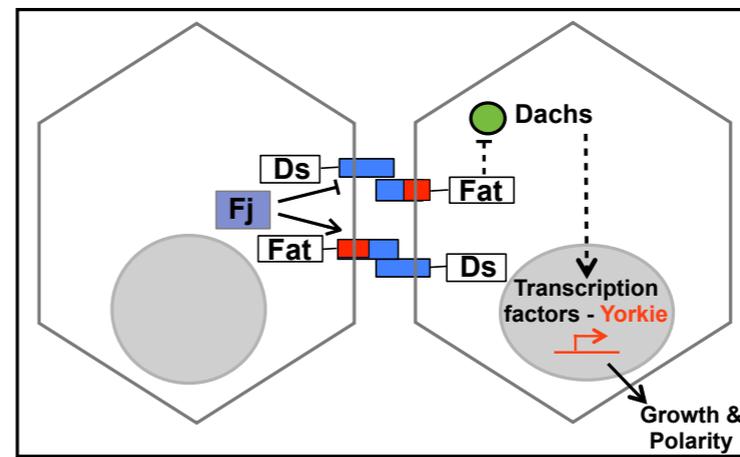
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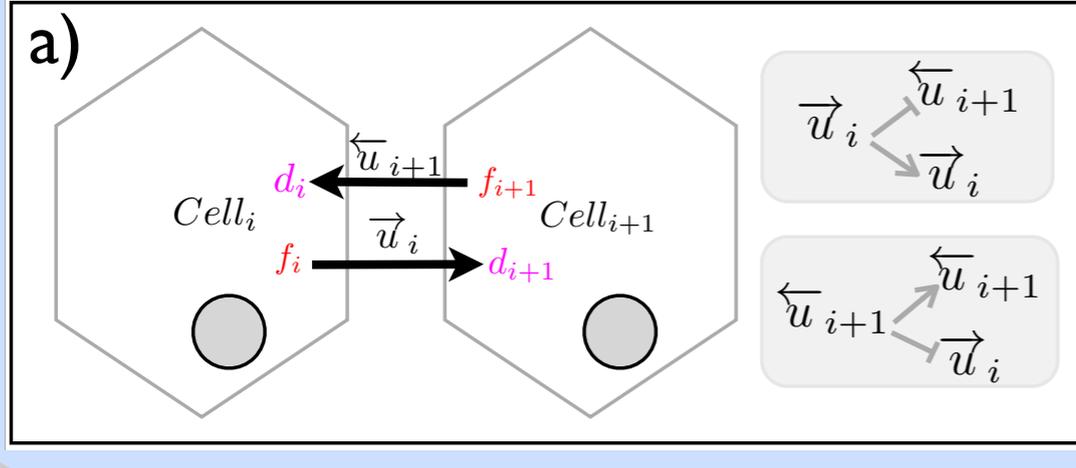


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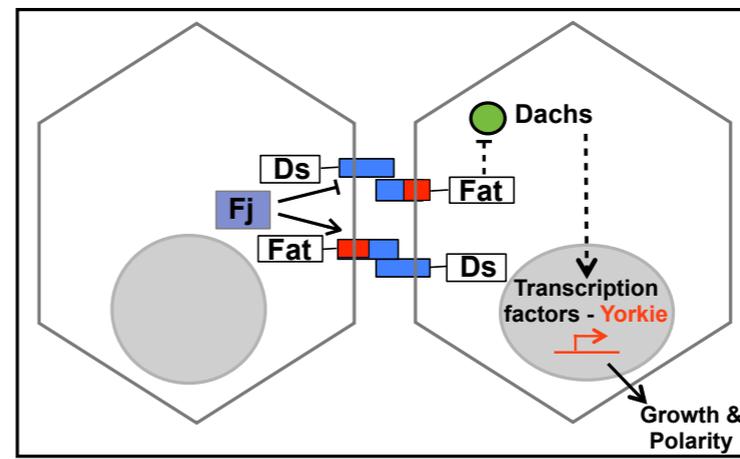
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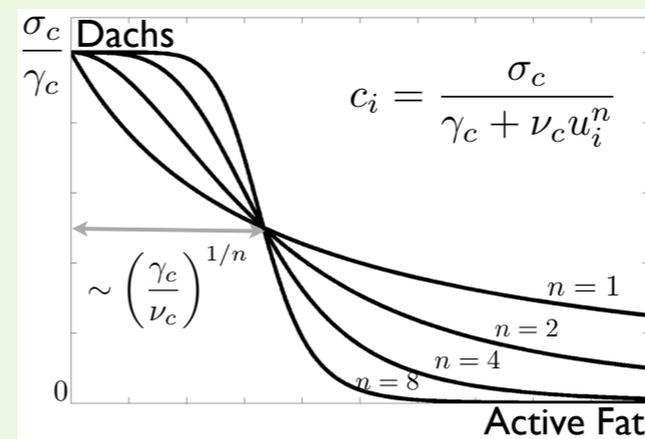
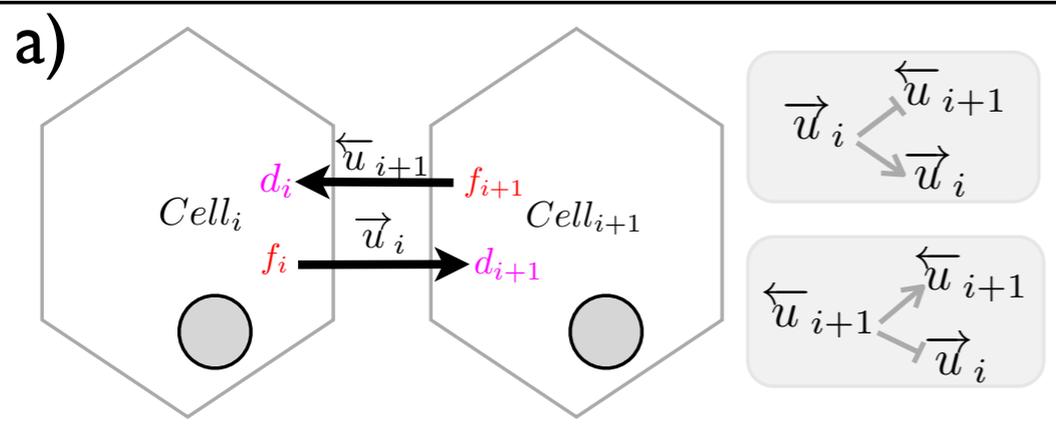
## Cytosolic Intermediate

$$\vec{C}_i = \frac{1}{1 + \left(\frac{\vec{U}_i}{\lambda}\right)^n}$$

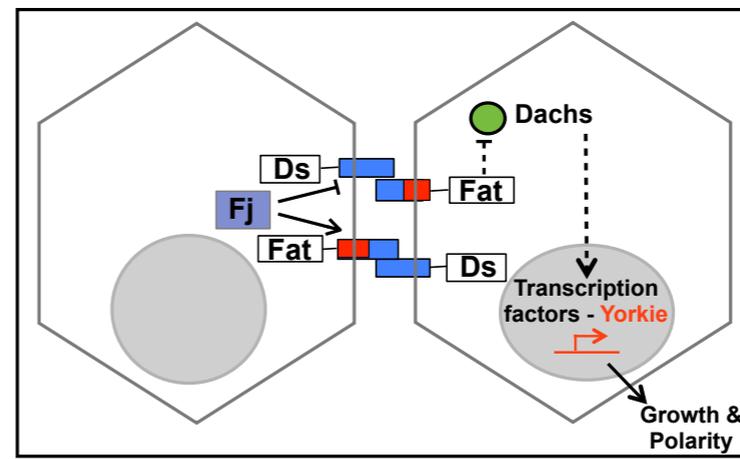
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## Nuclear Signal

$$N_i = (\vec{C}_i + \overleftarrow{C}_i)$$



# Model - Scaled & Steady State



## Transmembrane Interactions

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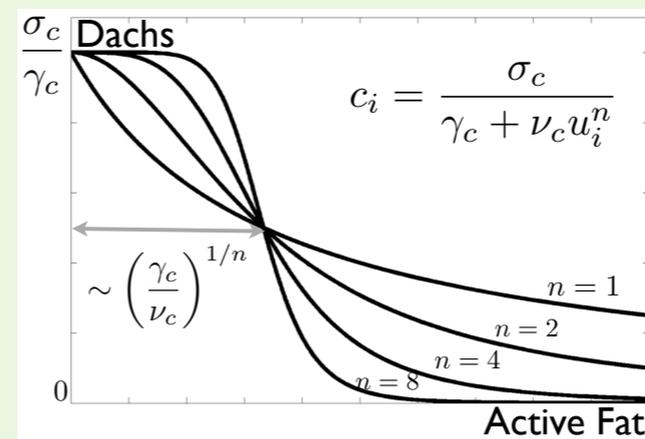
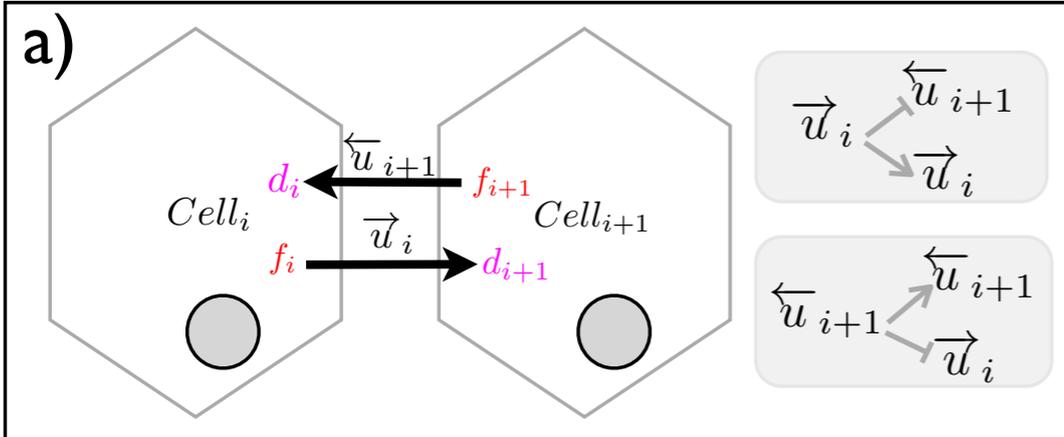
$$N_i = (\vec{C}_i + \overleftarrow{C}_i)$$

Input: Expression levels of Fat and Dachsaus

$$F_i^0 \quad \& \quad D_i^0$$

Output: Polarity & Growth

$$N_i \quad \& \quad \overleftarrow{C}_i, \overrightarrow{C}_i$$



# Analytic solution - Absolute level response

$$\frac{d\vec{u}_i}{dt} = k_u f_i d_{i+1} (1 + \alpha \vec{u}_i) - \gamma_u \vec{u}_i (1 + \beta \overleftarrow{u}_{i+1})$$

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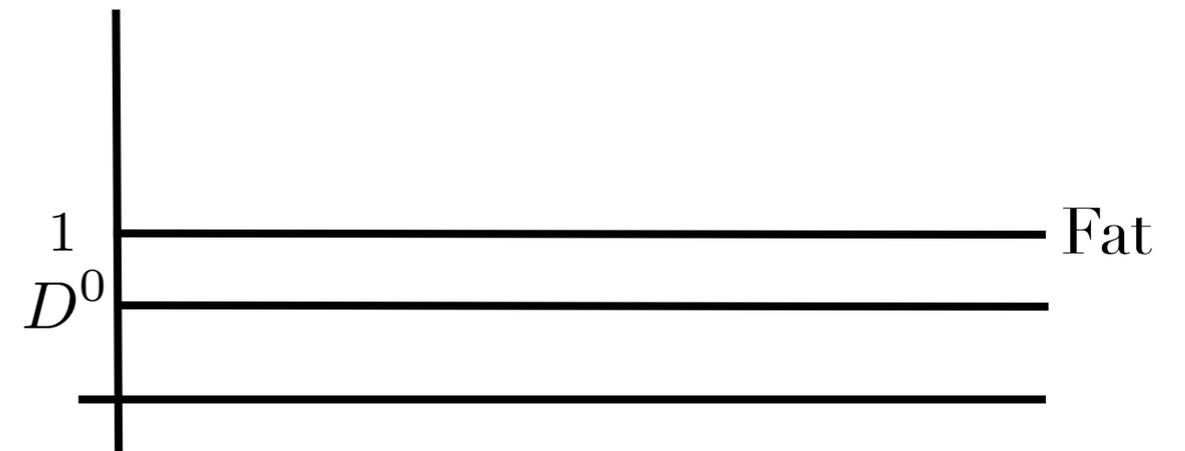
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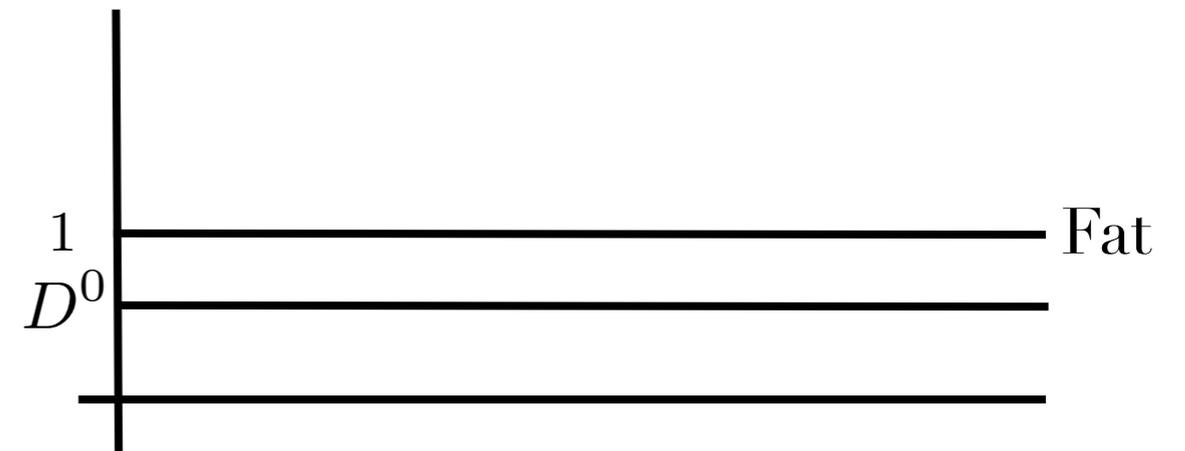


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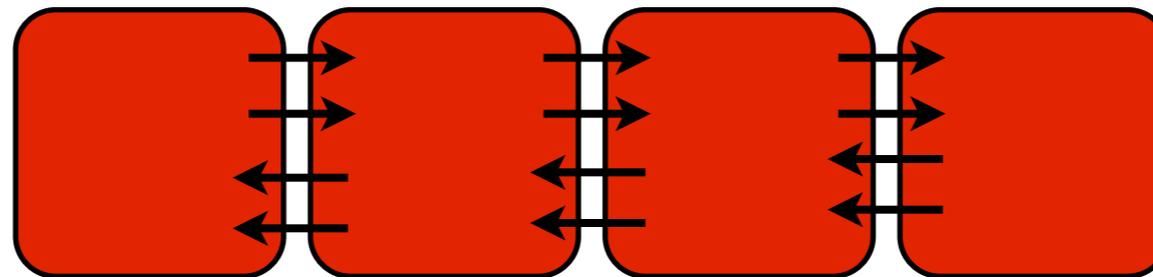
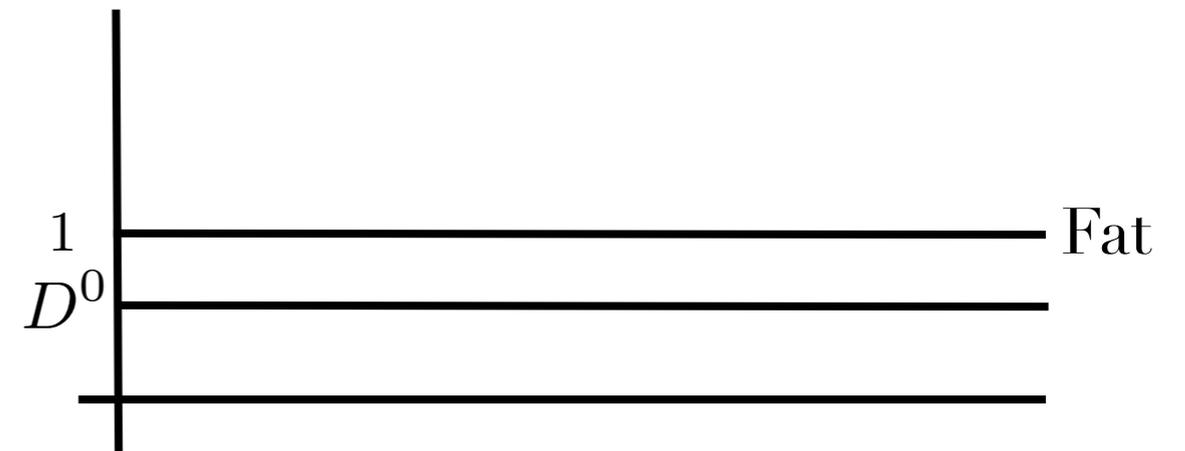


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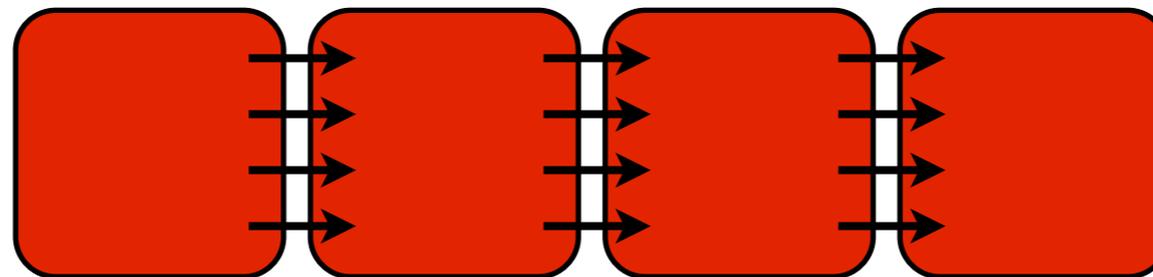
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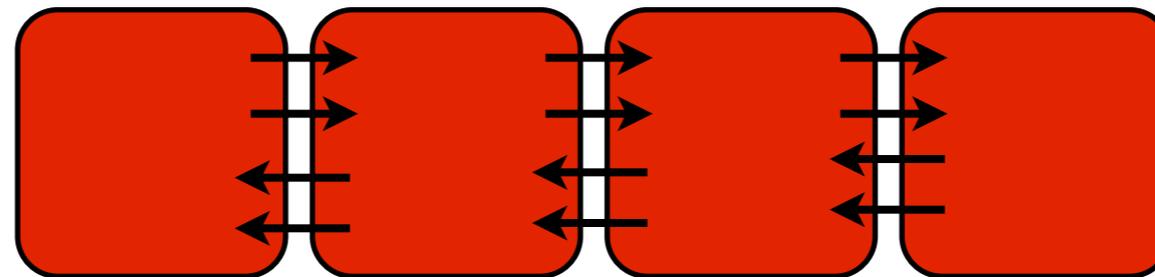
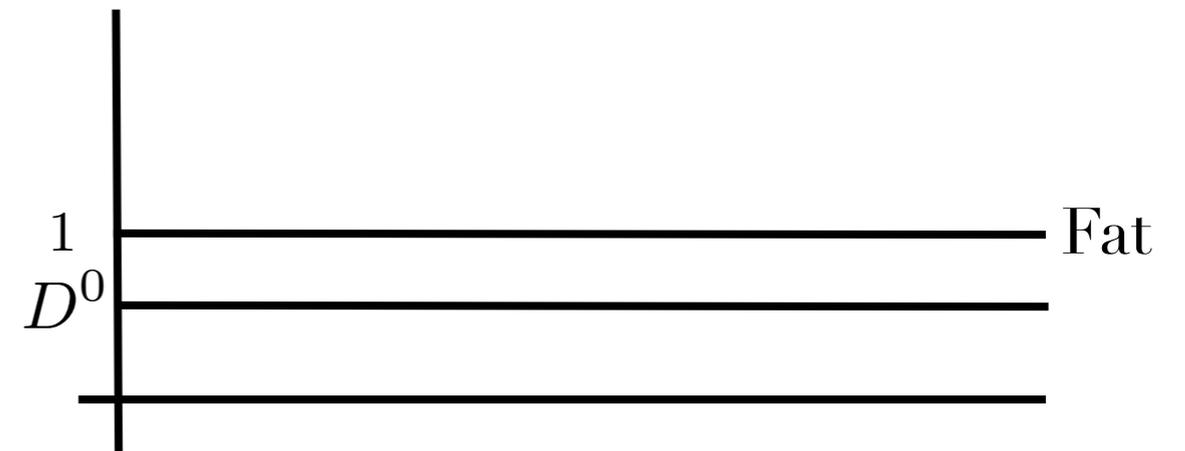
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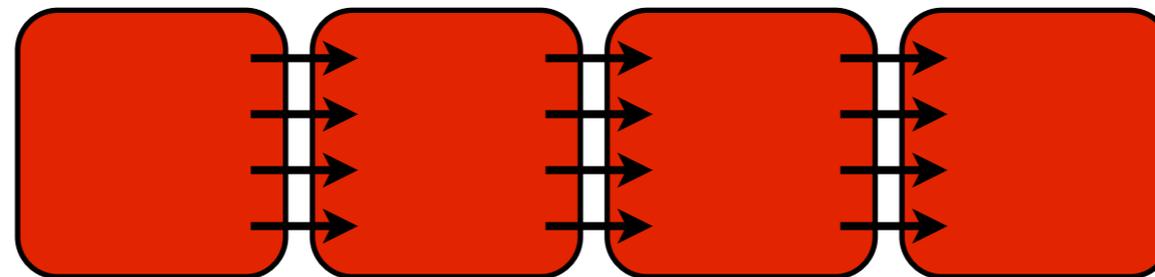
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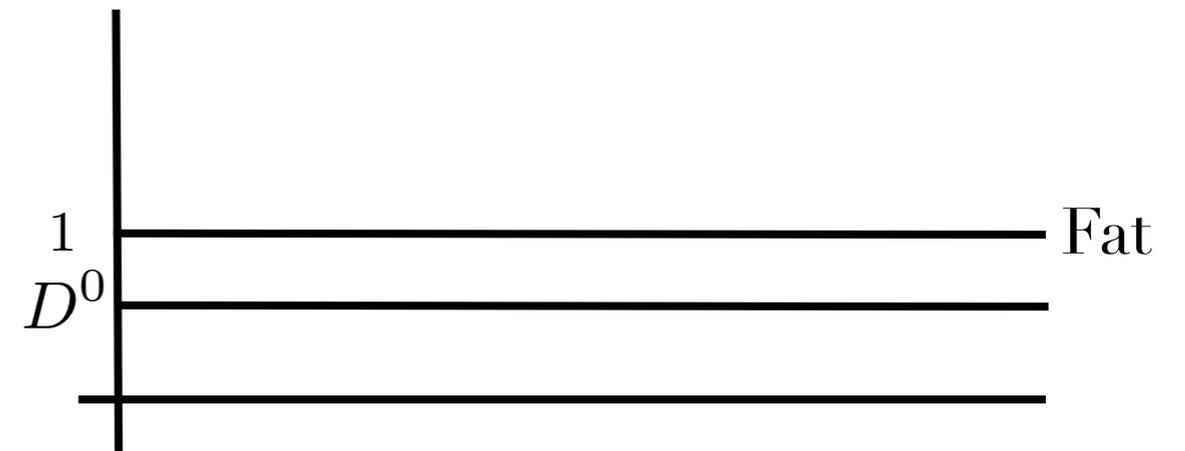
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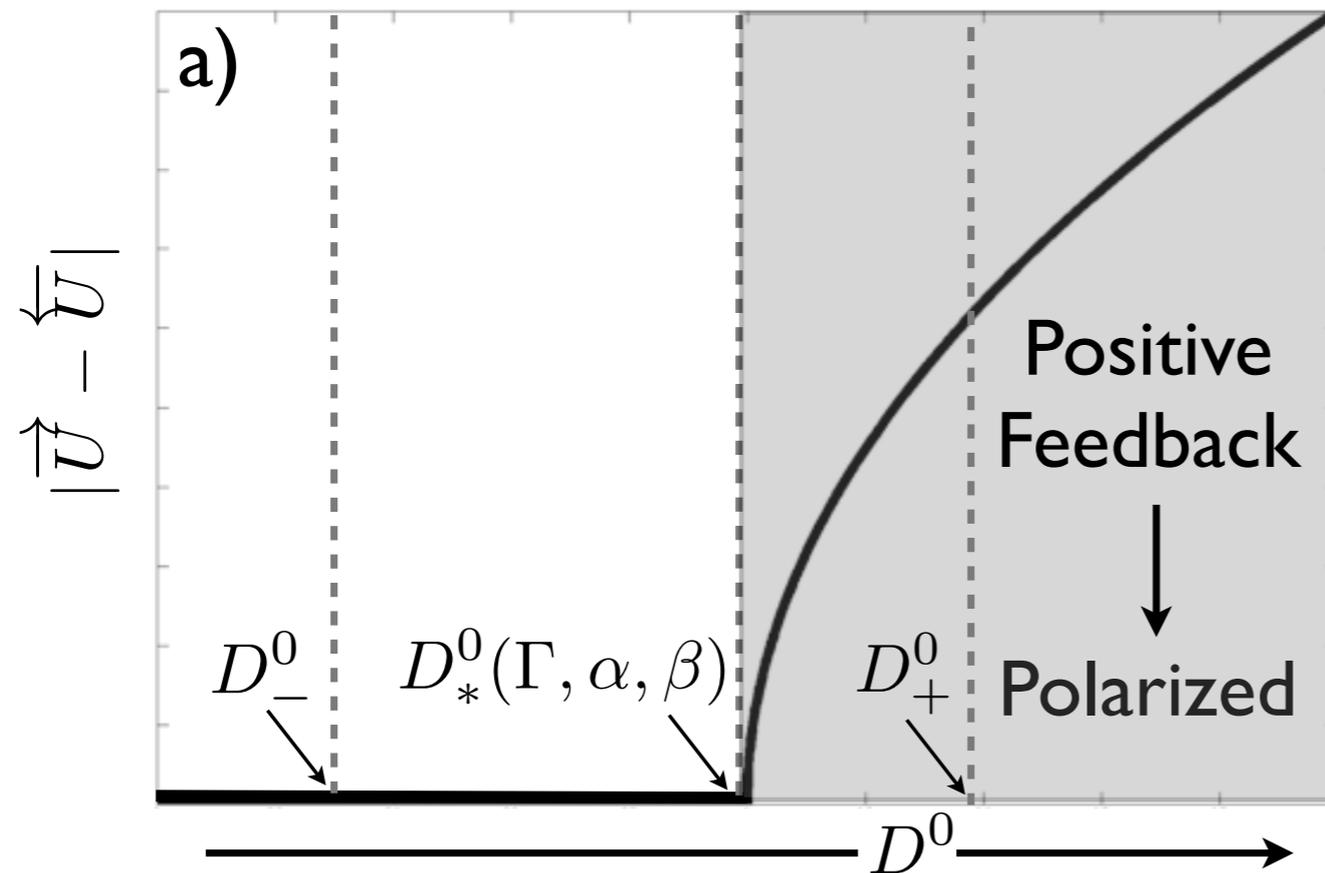
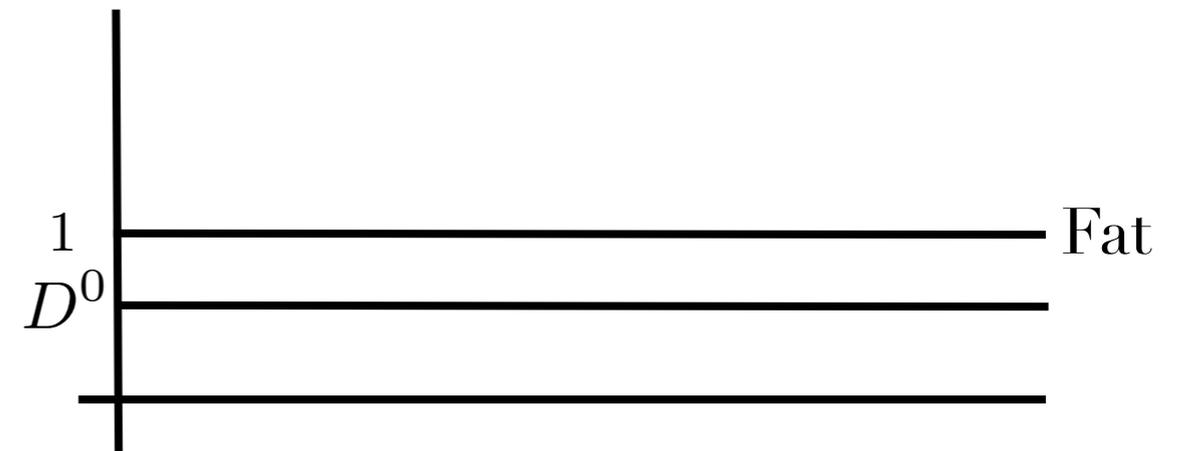


# Analytic solution - Absolute level response

$$FD(1 + \alpha \vec{U}) - \Gamma \vec{U} (1 + \beta \overleftarrow{U}) = 0$$

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$$F = 1 - \vec{U} - \overleftarrow{U} \quad \& \quad D = D^0 - \vec{U} - \overleftarrow{U}$$

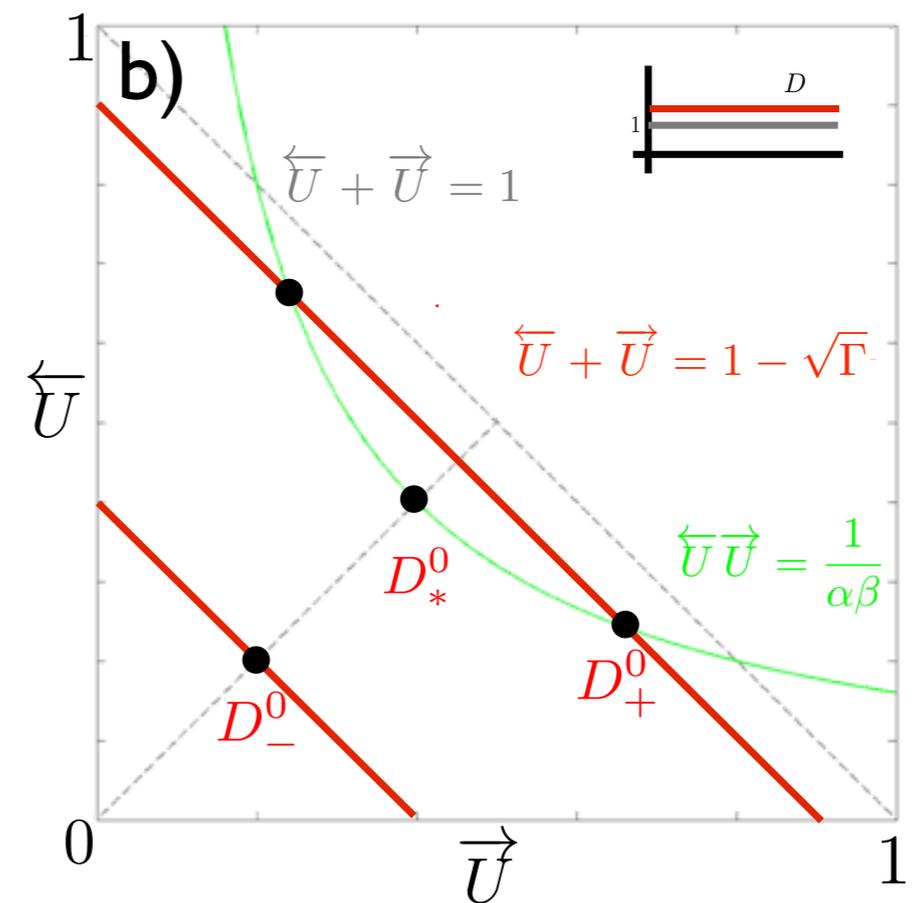
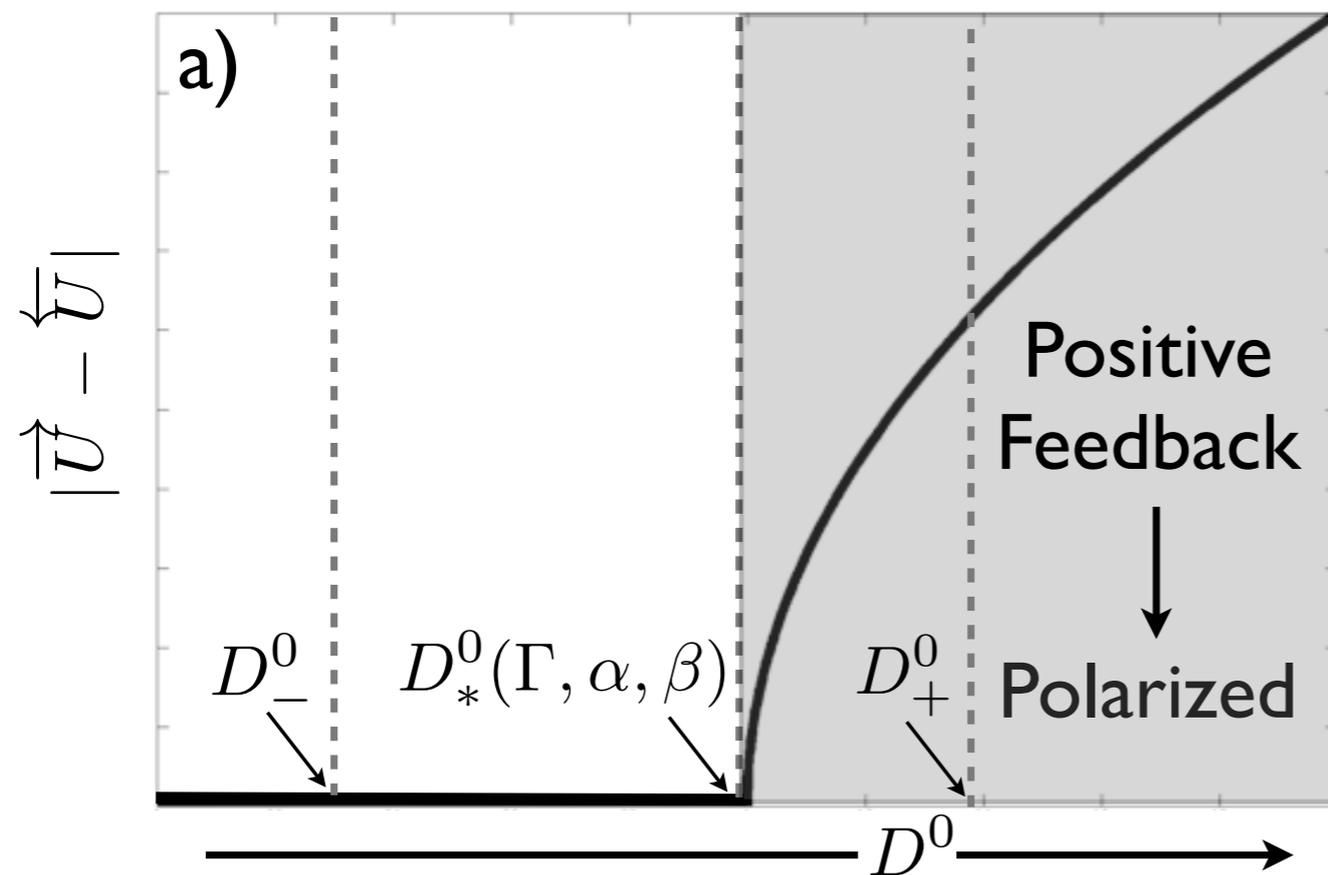
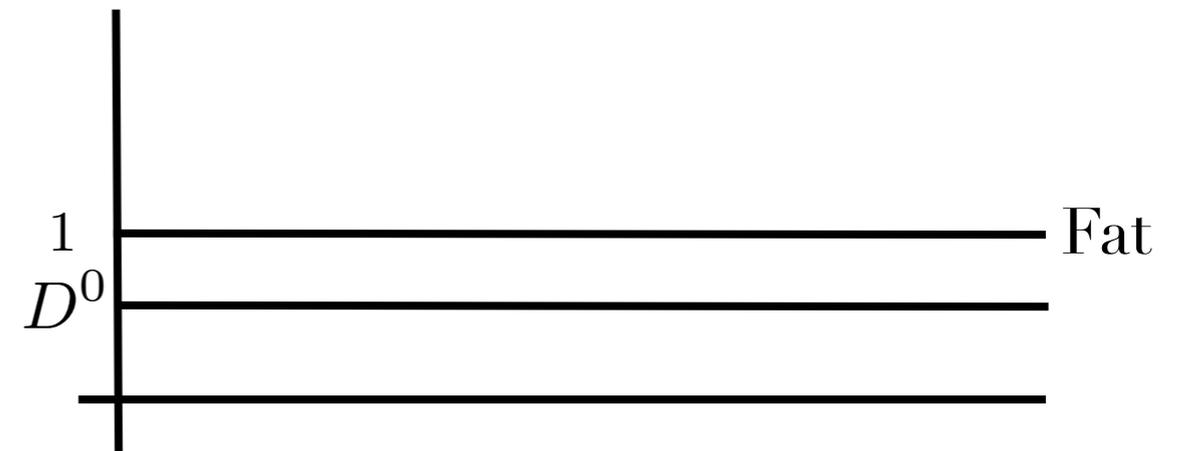


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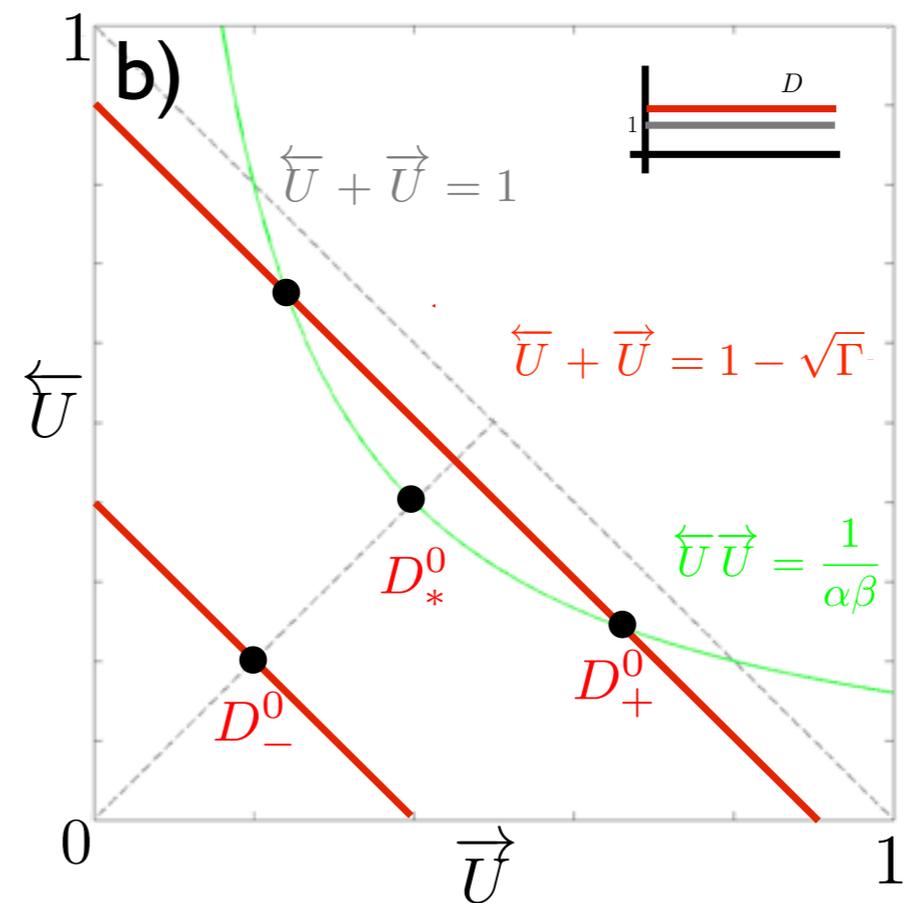
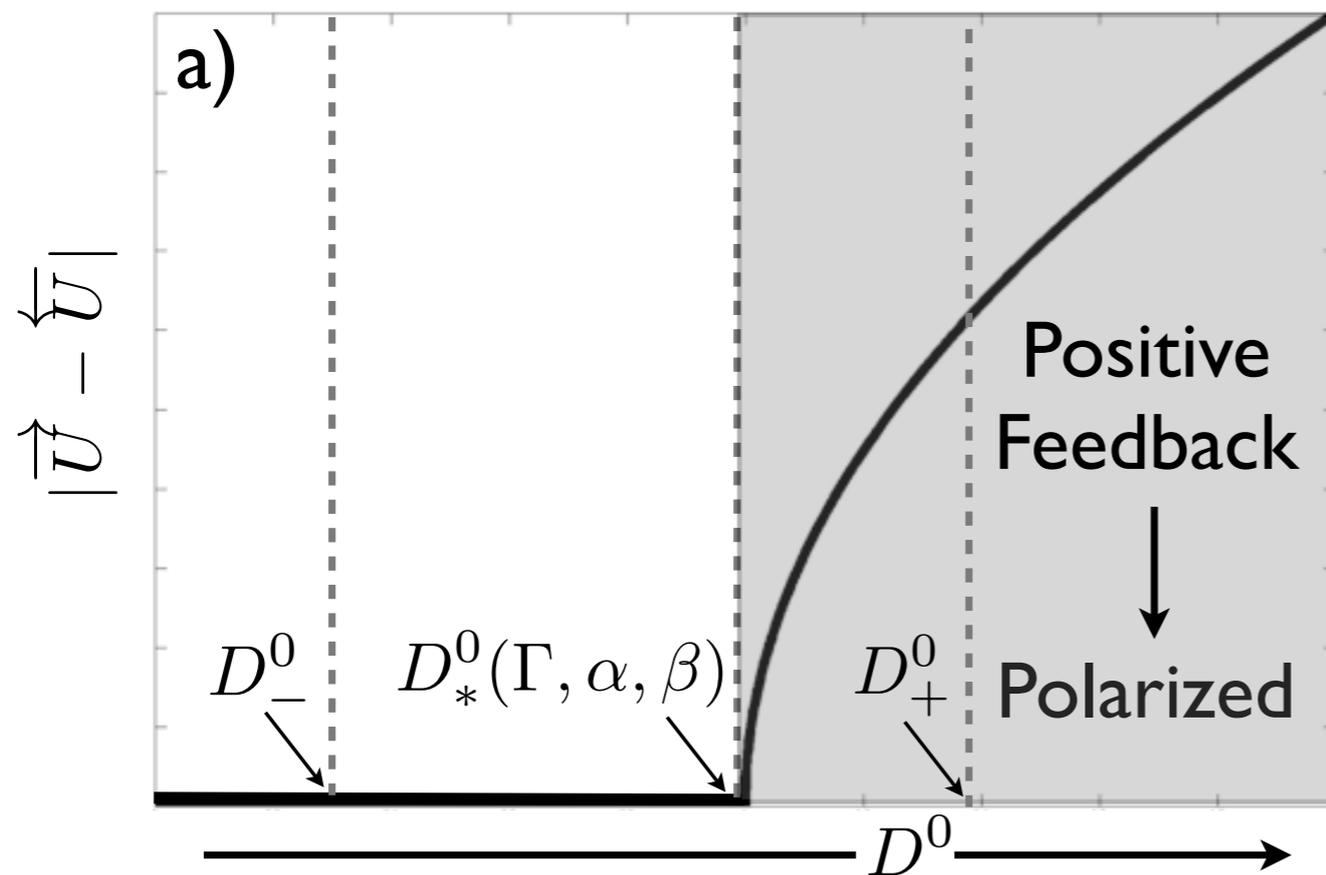
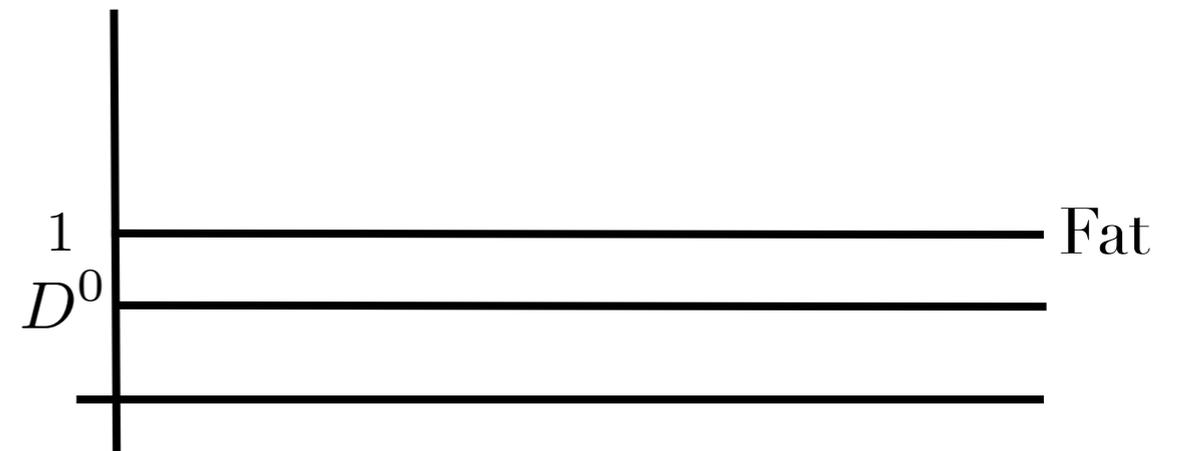


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$\alpha \& \beta \longrightarrow$  Polarity

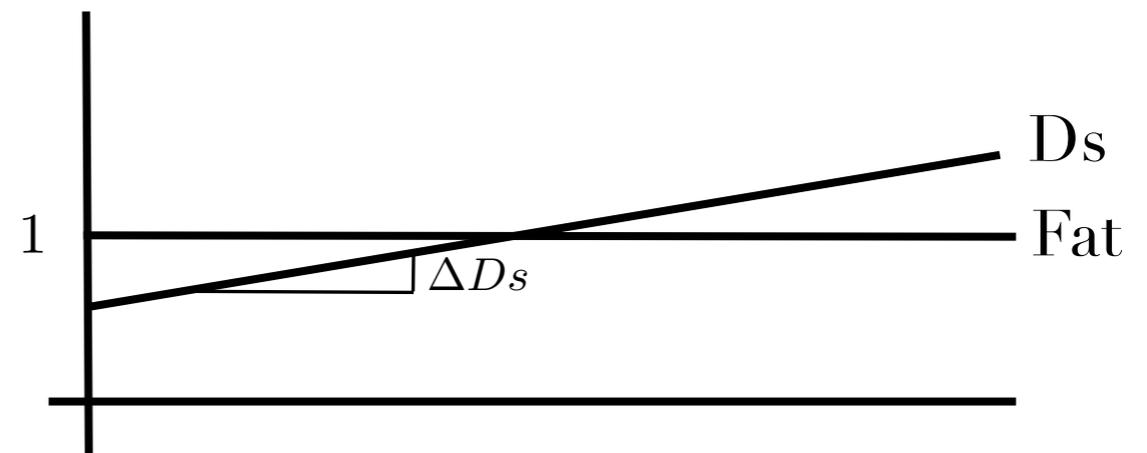
Critical relative level of Dachsaus

# Analytic solution - Gradient response (main slide)

$$F_i D_{i+1} (1 + \alpha \vec{U}_i) - \Gamma \vec{U}_i (1 + \beta \overleftarrow{U}_{i+1}) = 0$$

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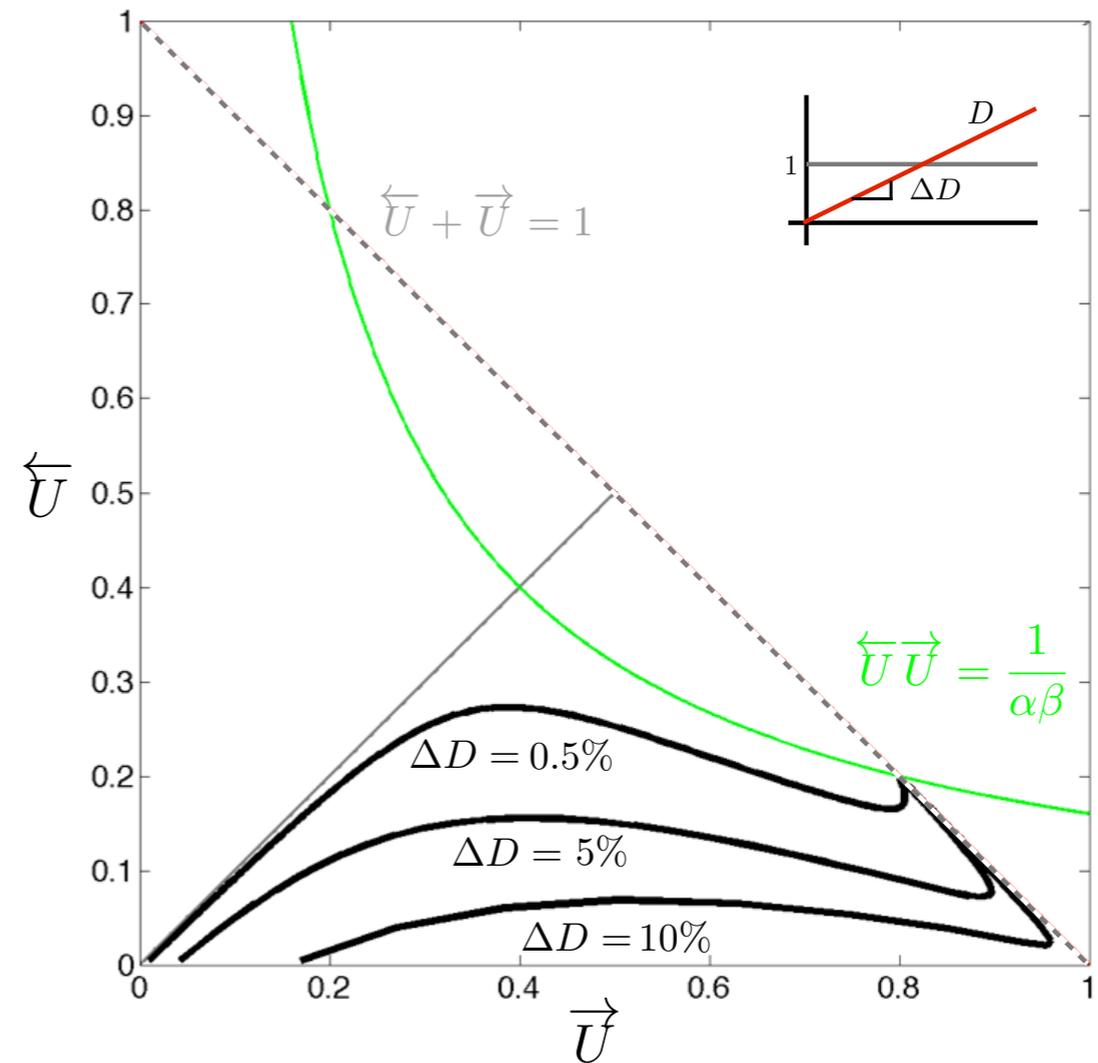
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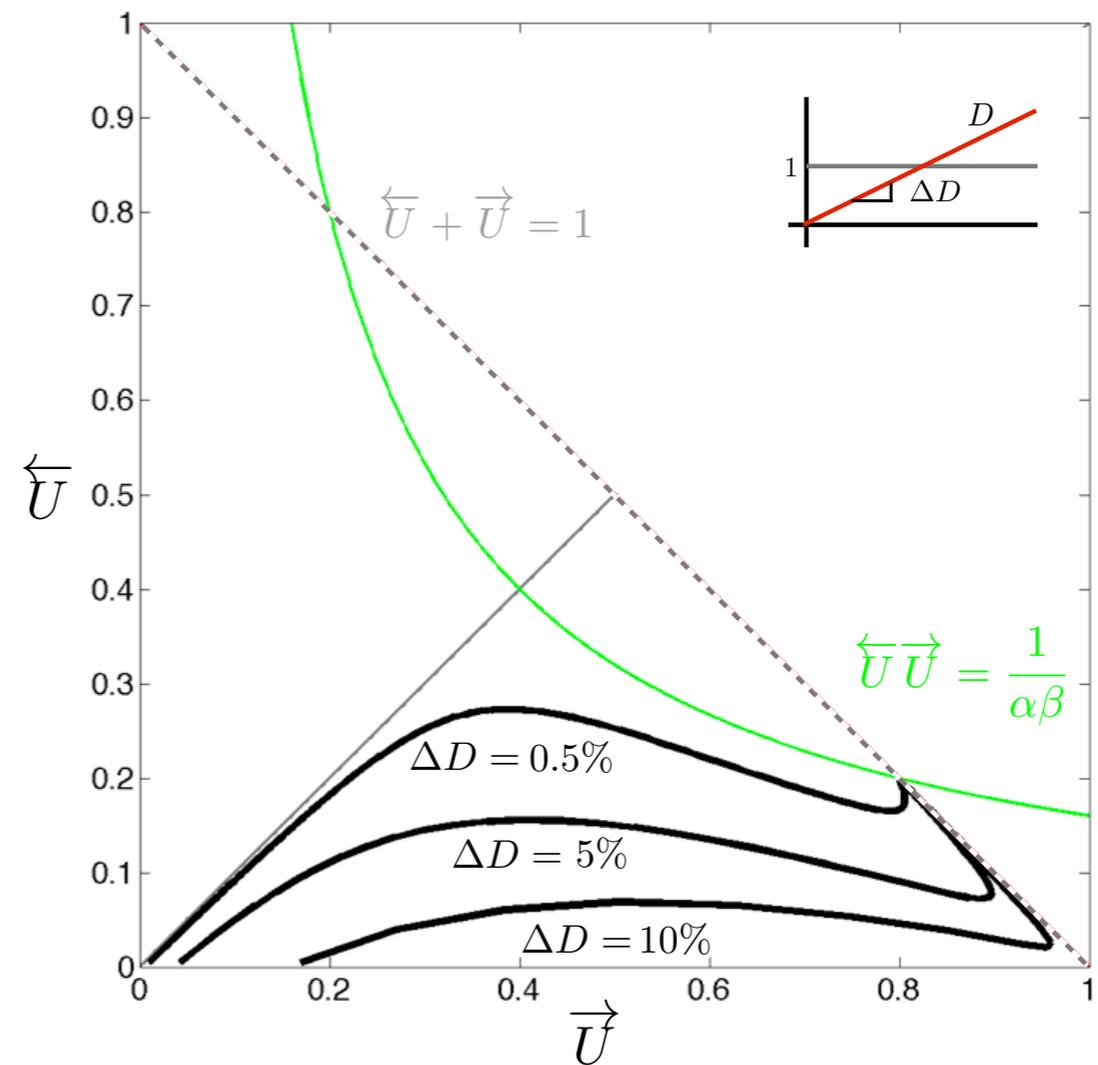


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Questions about the curve:

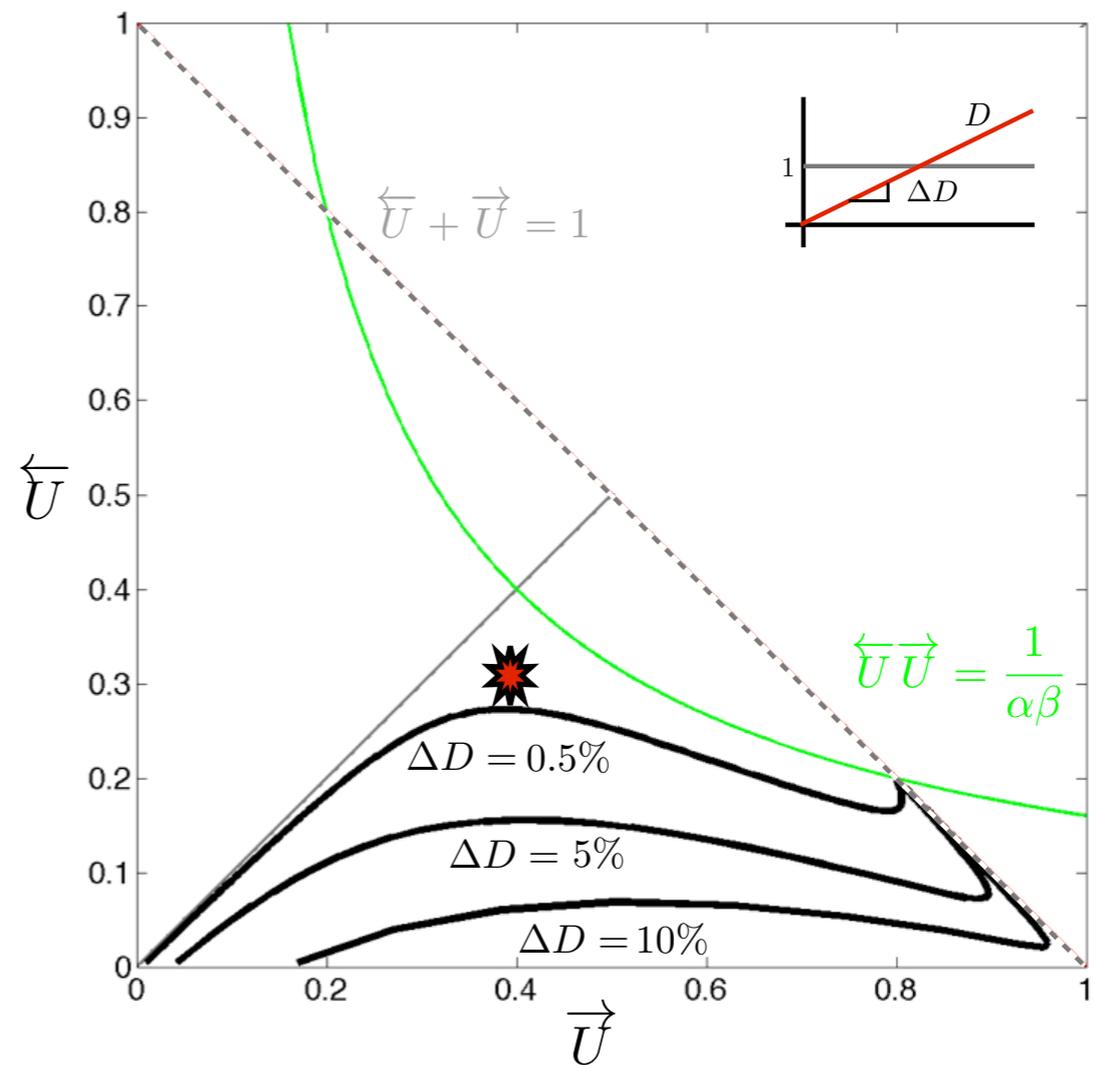
- 1) Response at the critical point
- 2) Why does the curve settle at intersection?
- 3) When does it turn around?

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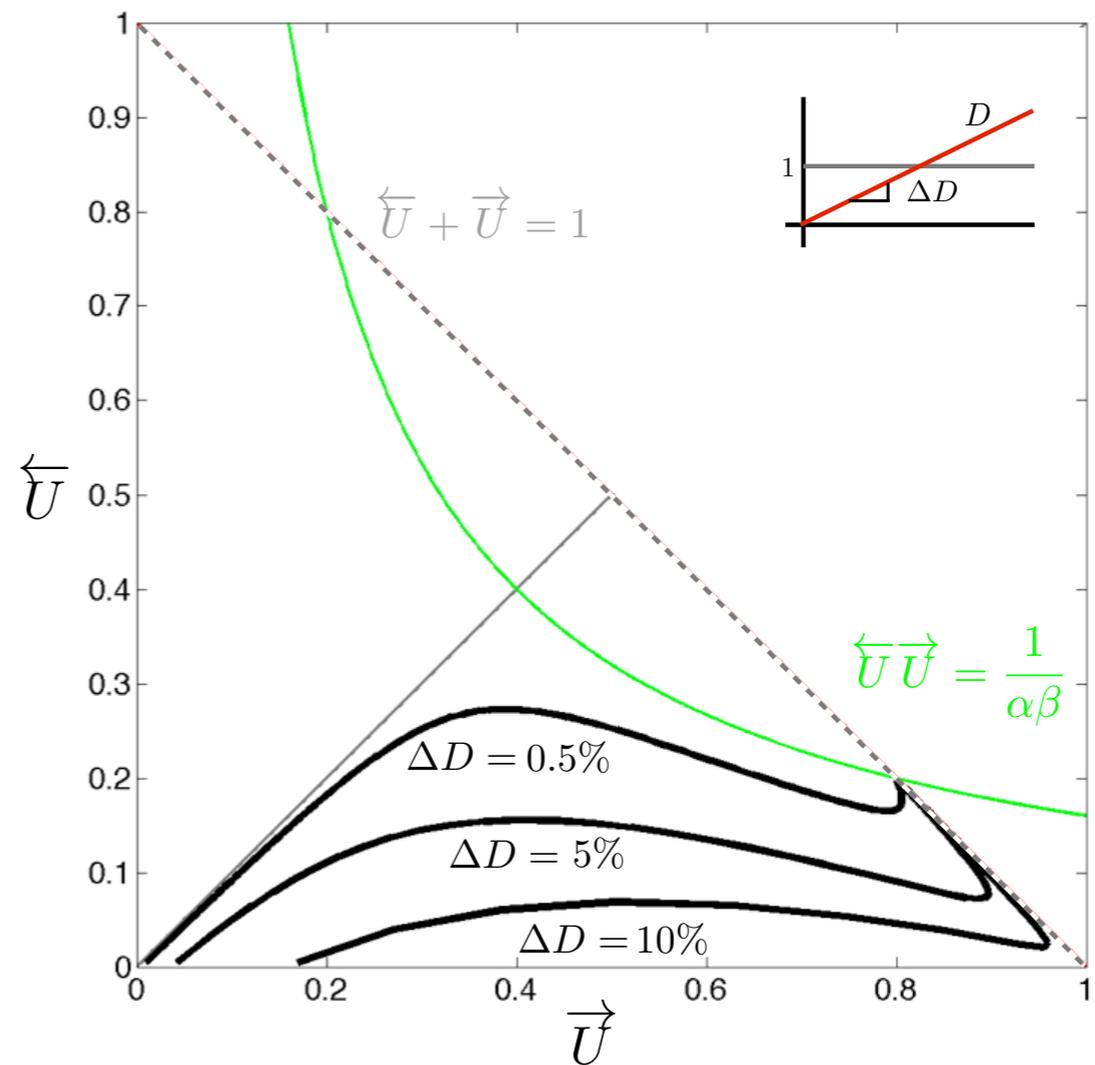
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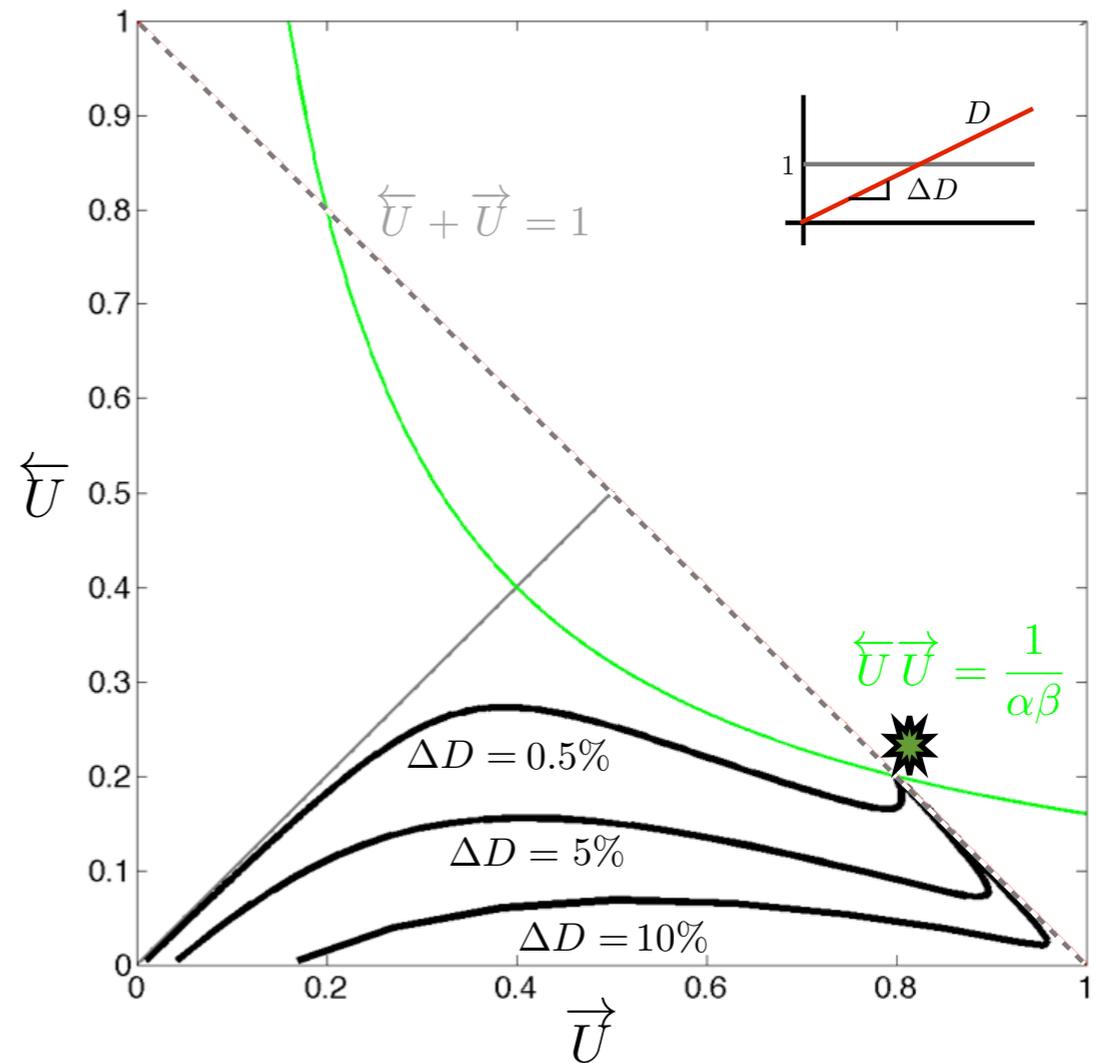
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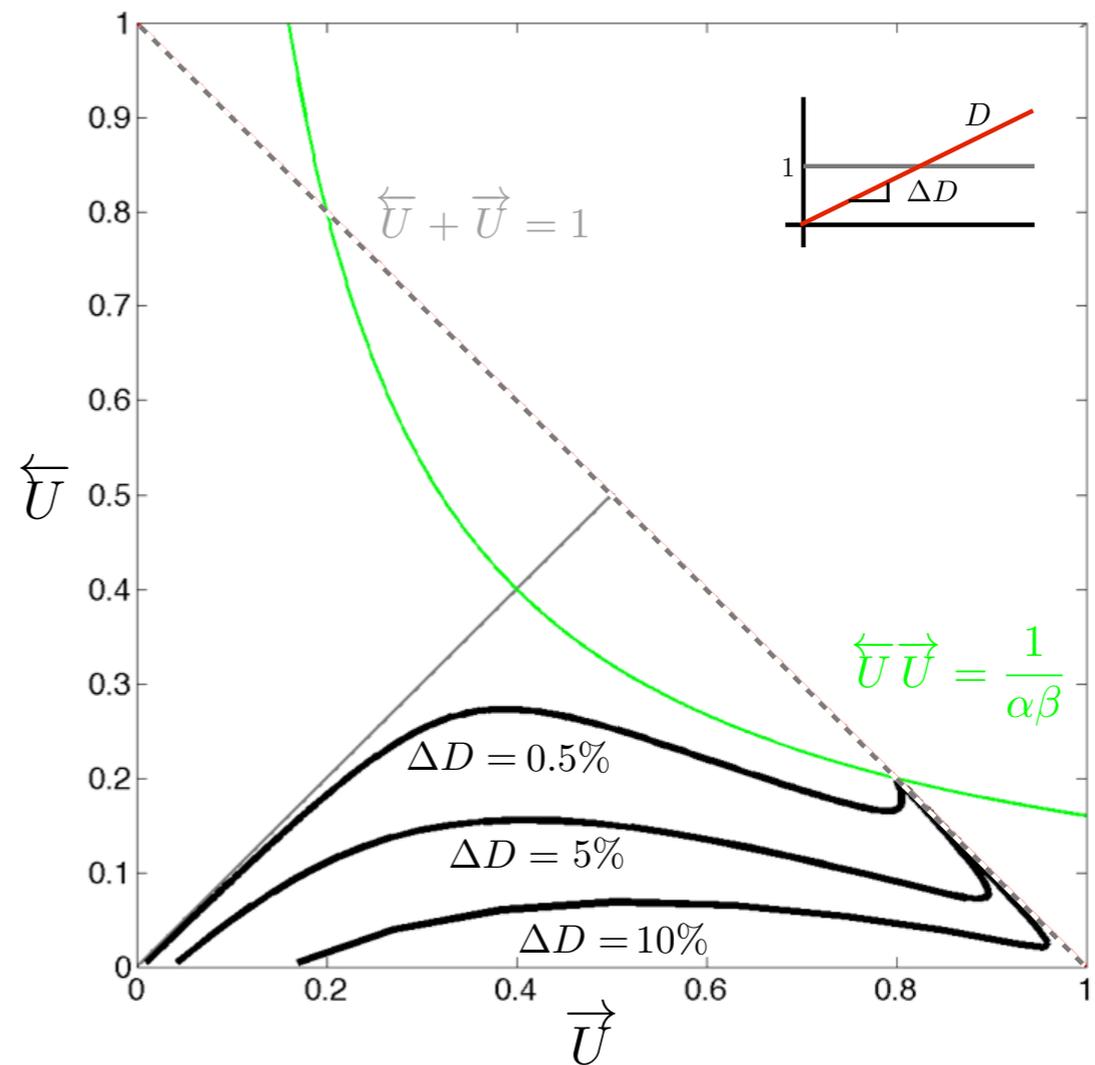
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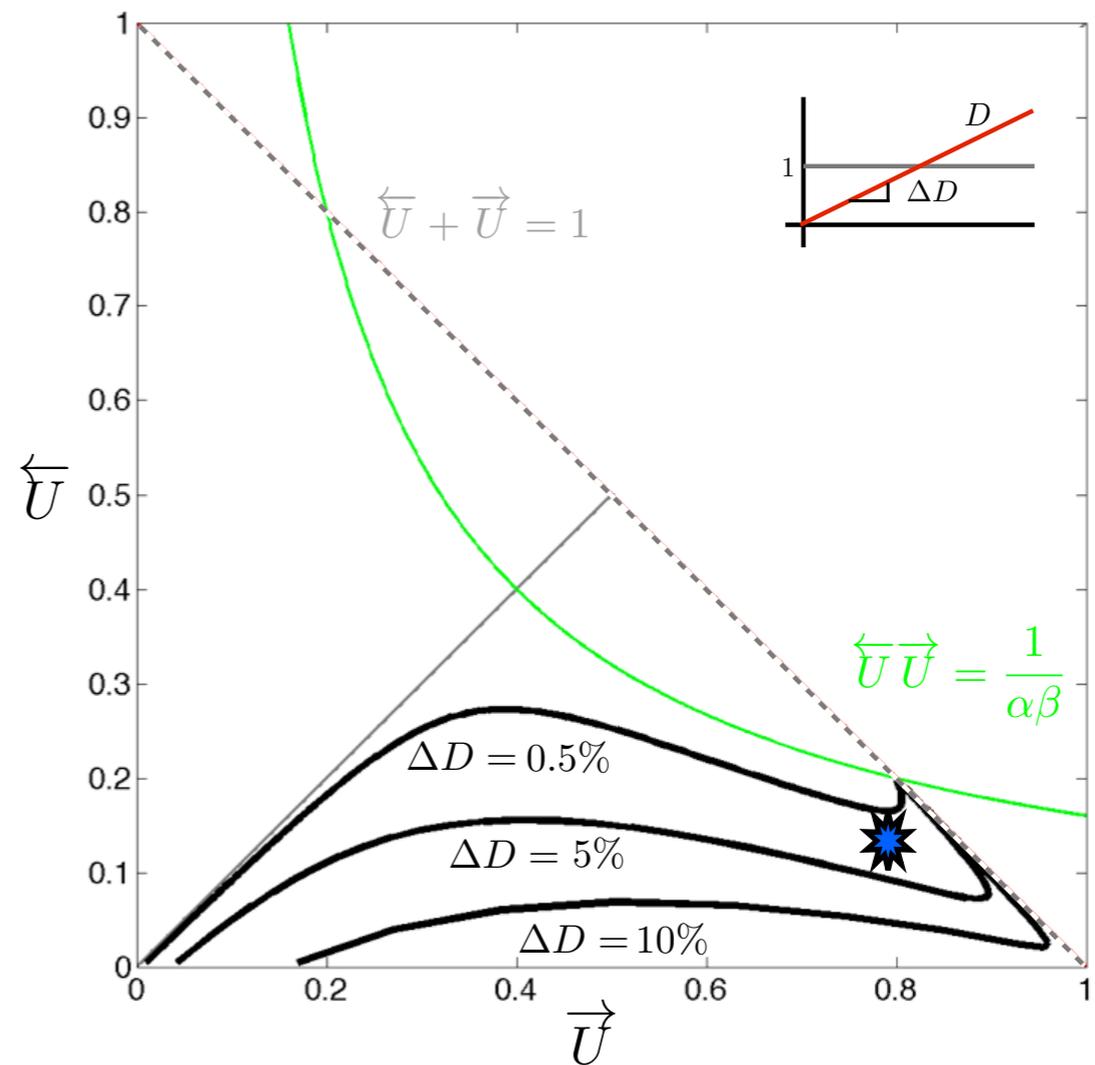
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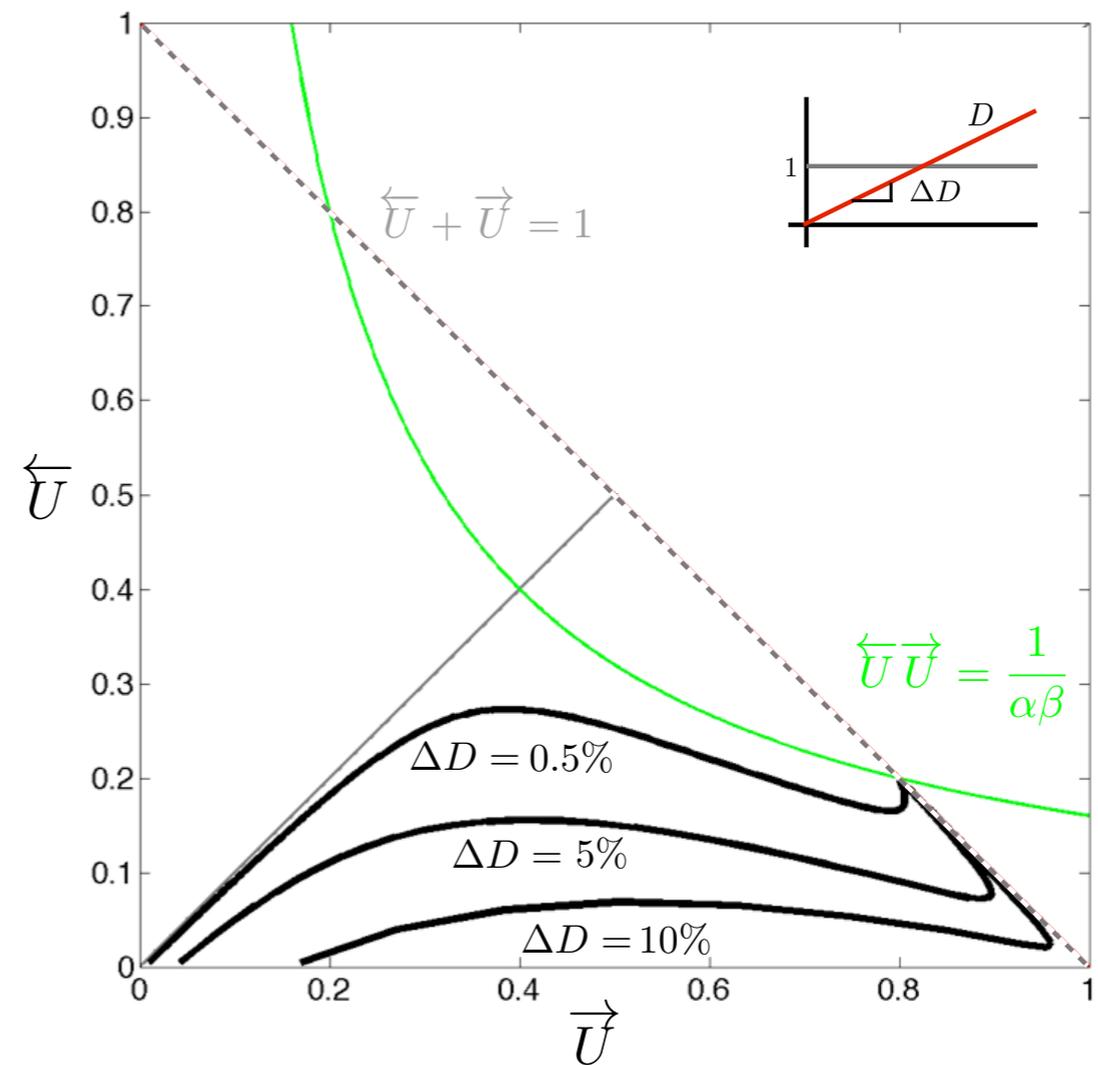
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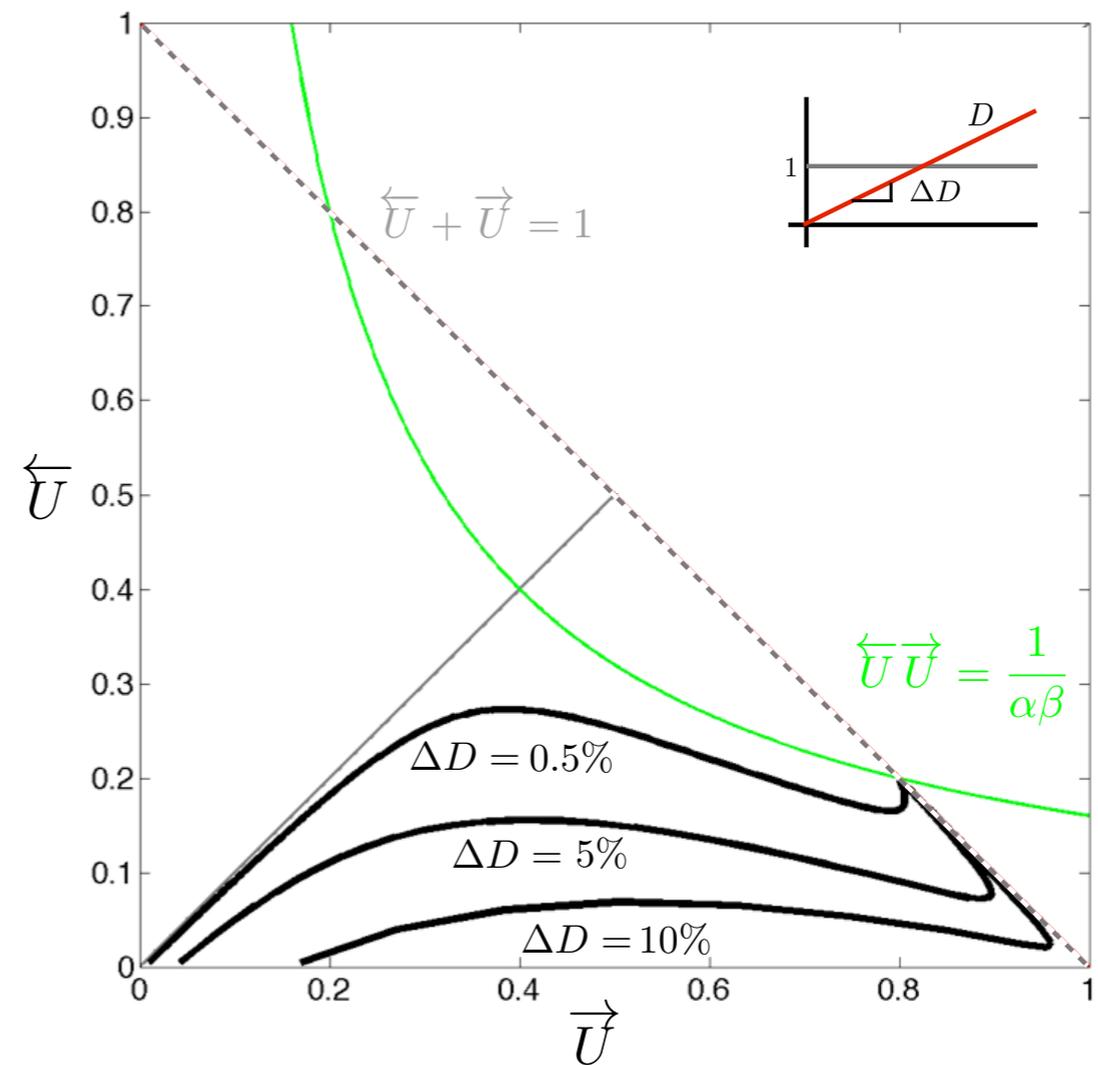
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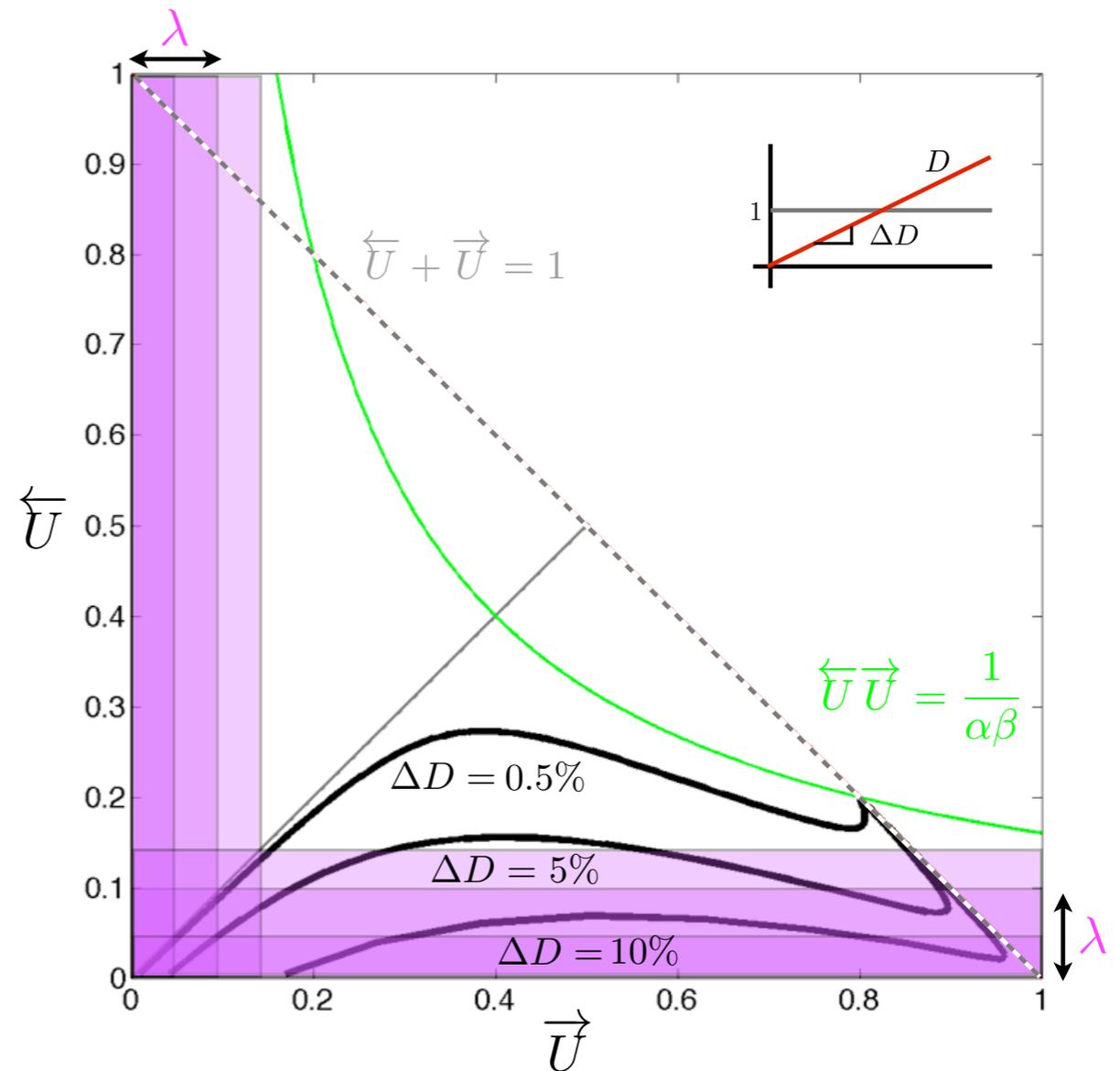
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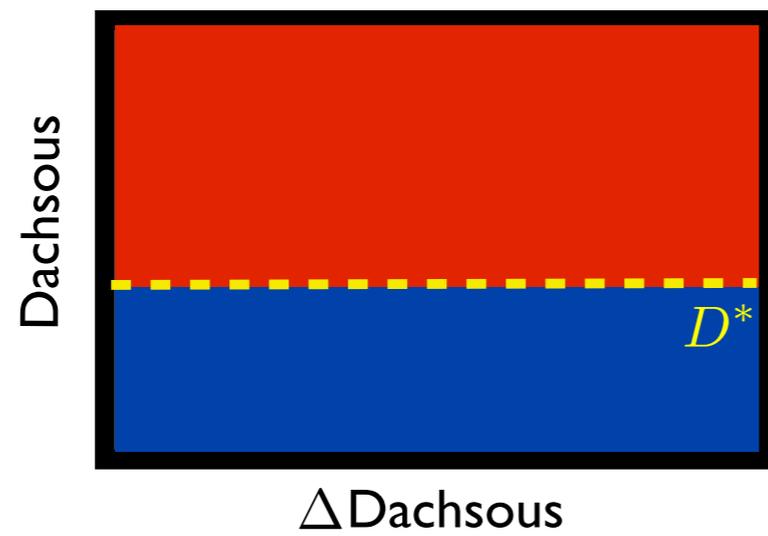
Questions about the curve:

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All 3 levels of model  
in one figure

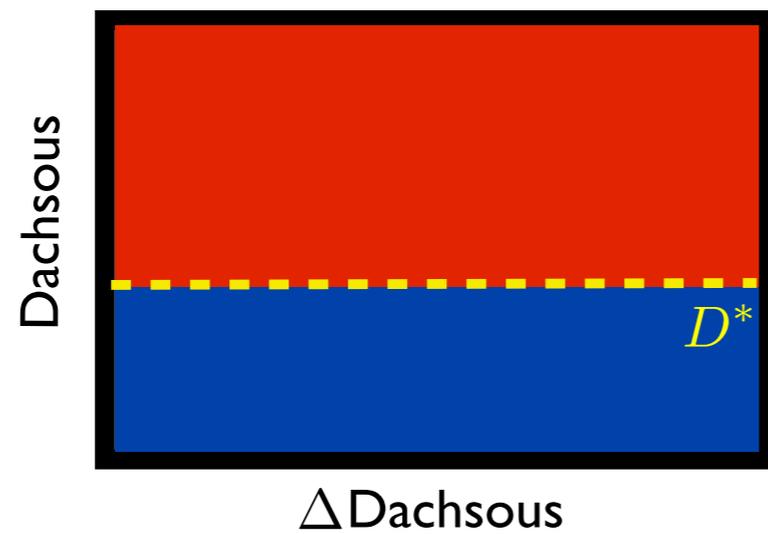
# Framework - Map from expression levels of core components to growth/polarity response

a) Level Detector

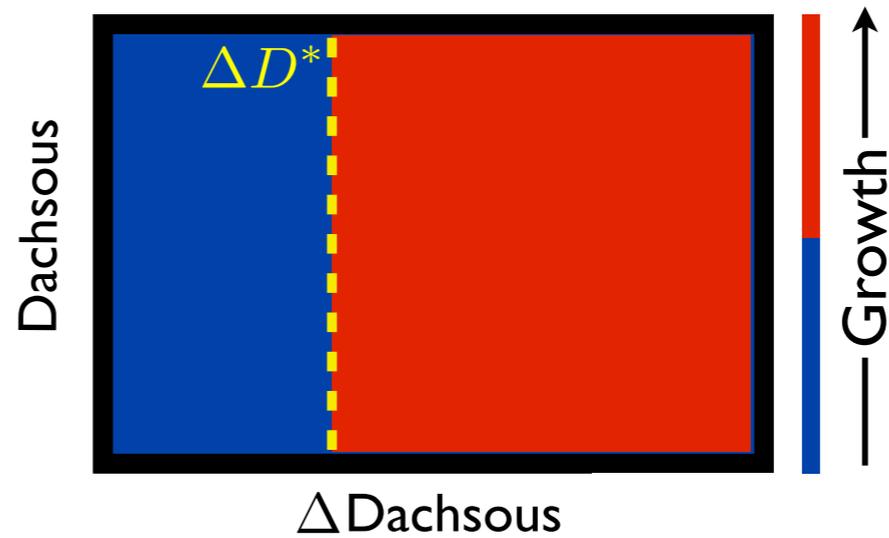


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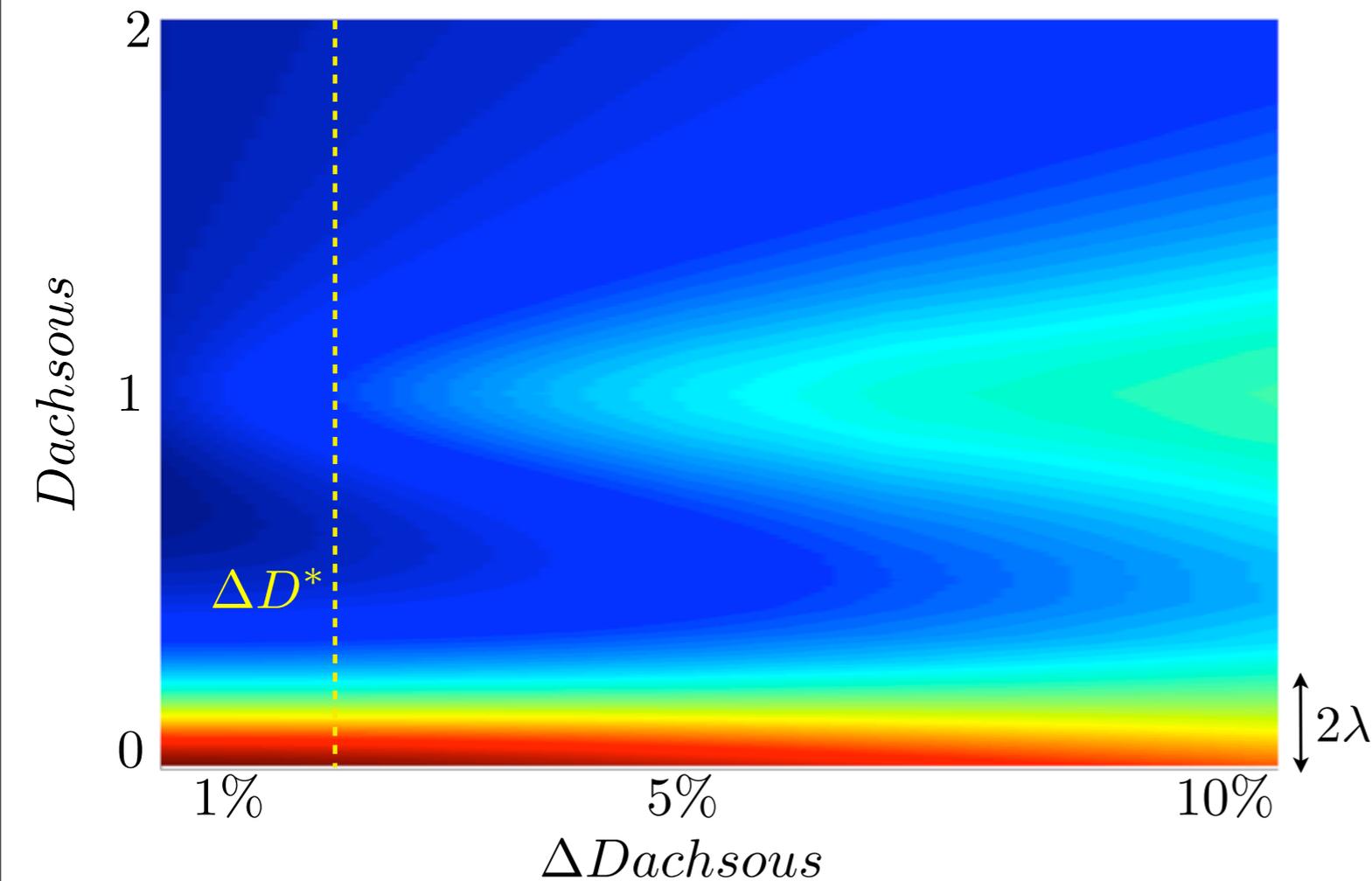
a) Level Detector



b) Gradient Detector



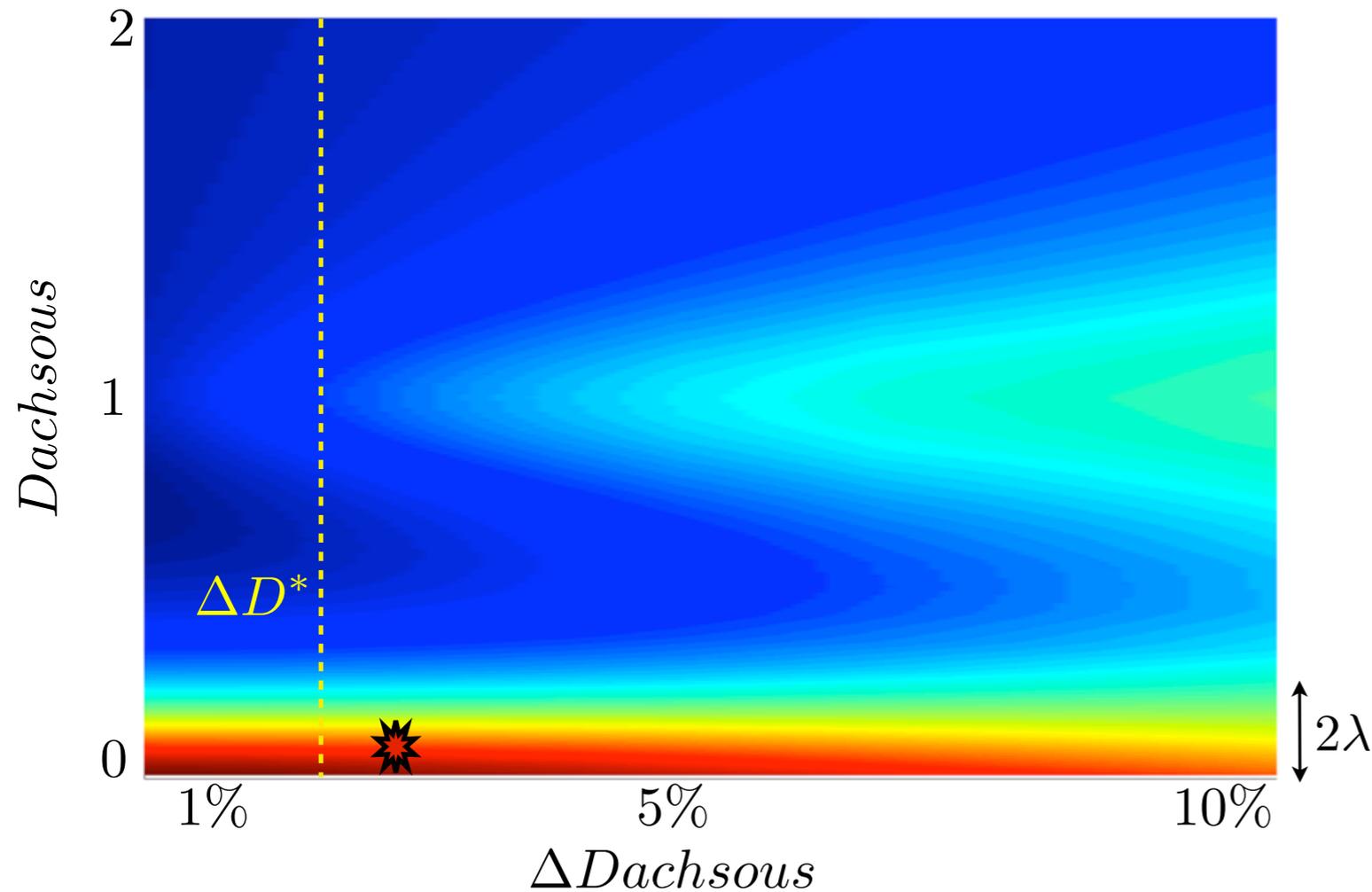
# Phase diagram



## Features

- 1) Knockout
- 2) Signaling peak
- 3) “Critical” gradient
- 4) More gradient --> More growth
- 5) Low (a weak knockout) levels give less growth than the signaling peak
- 6) Over-expression of Ds over Fat leads to no growth regardless of gradient

# Phase diagram

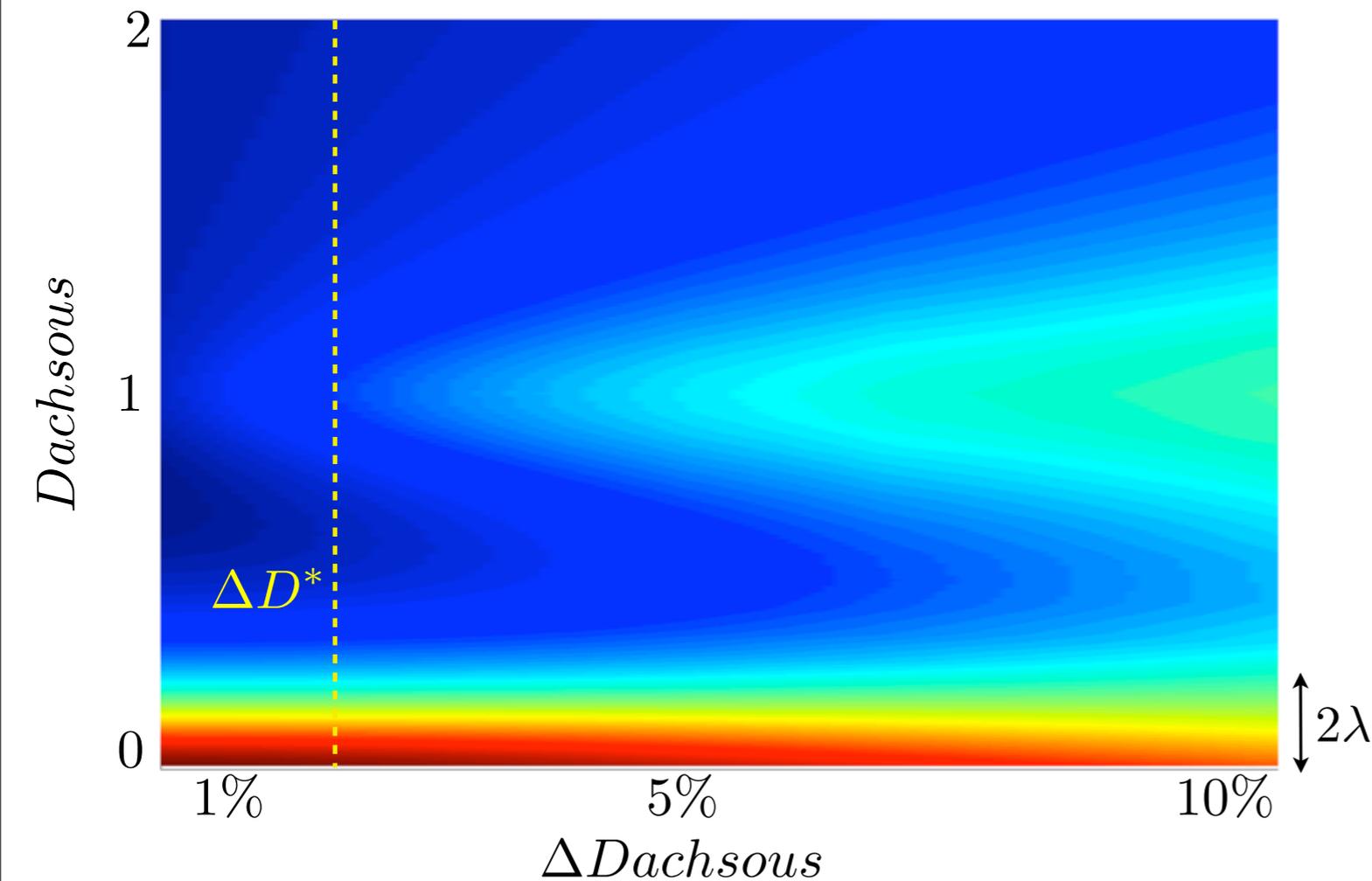


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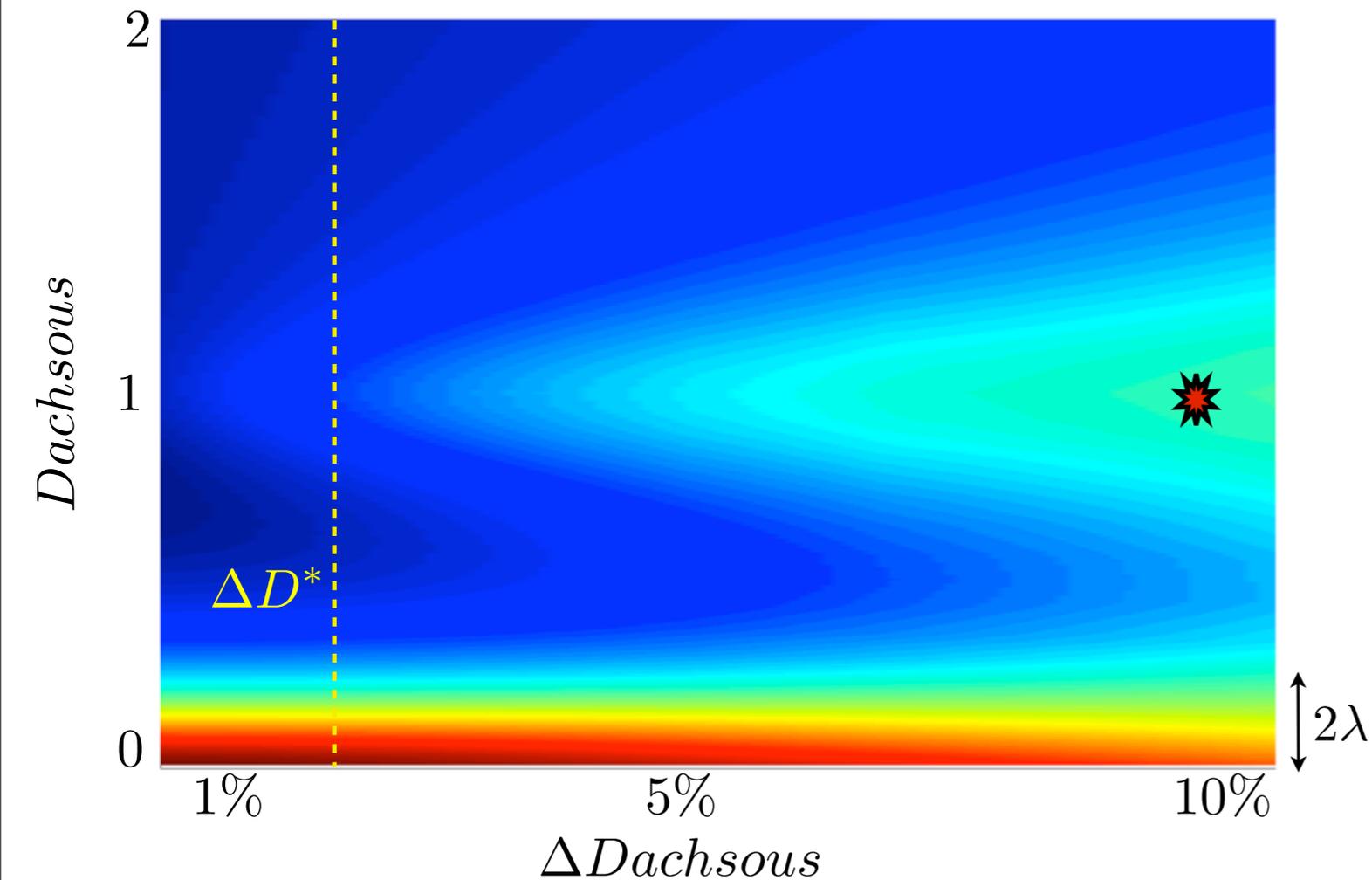
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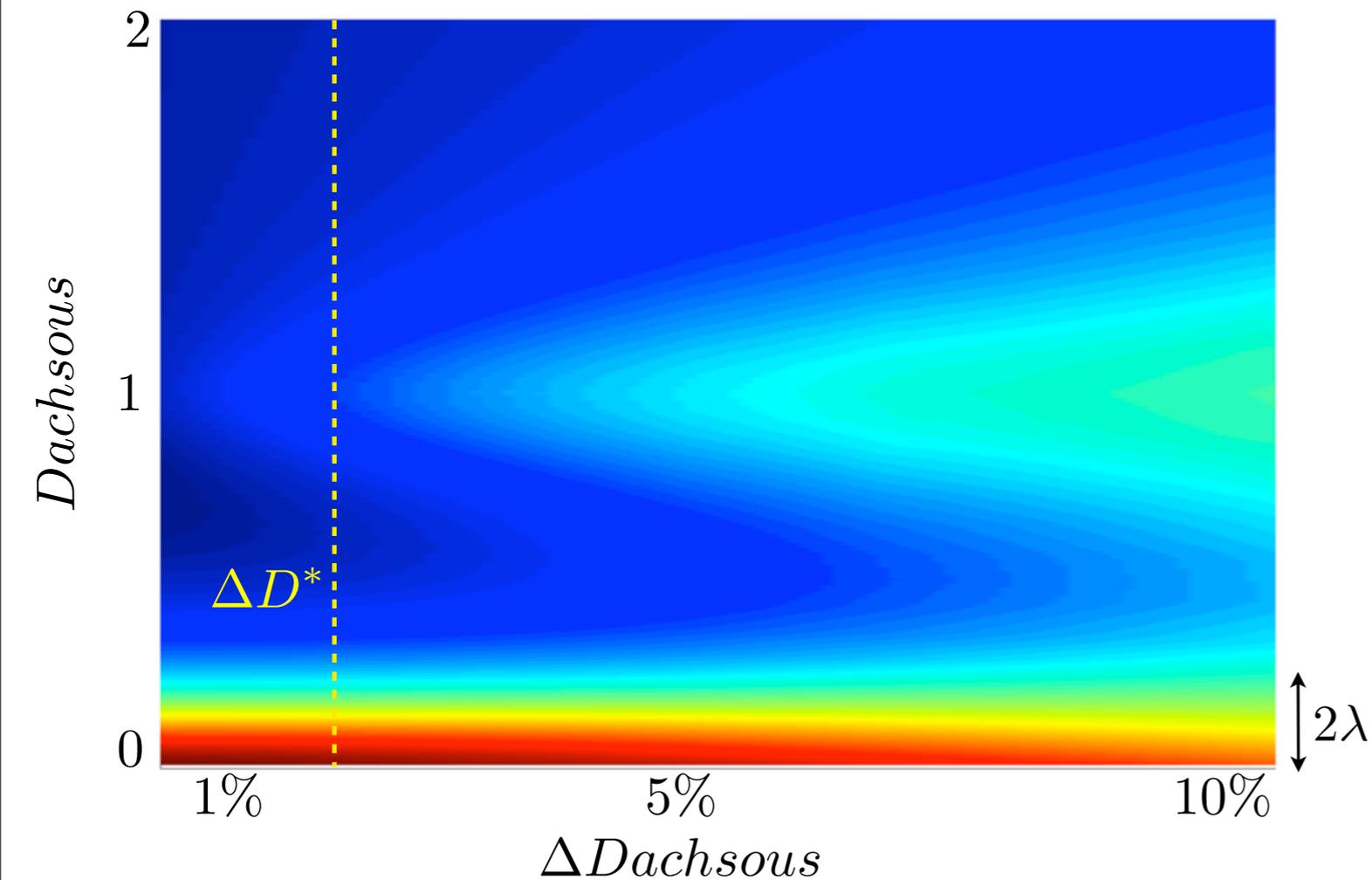
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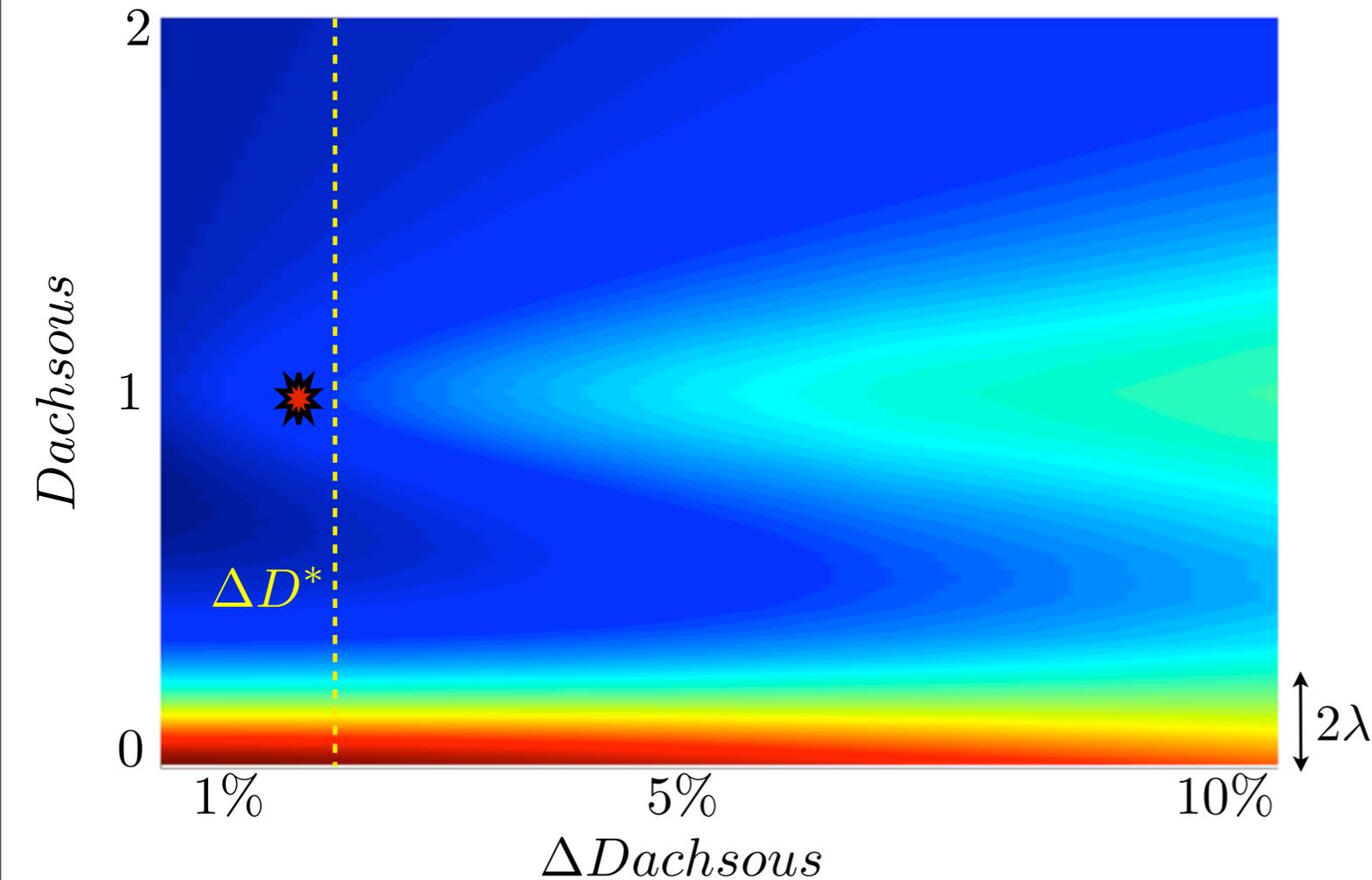
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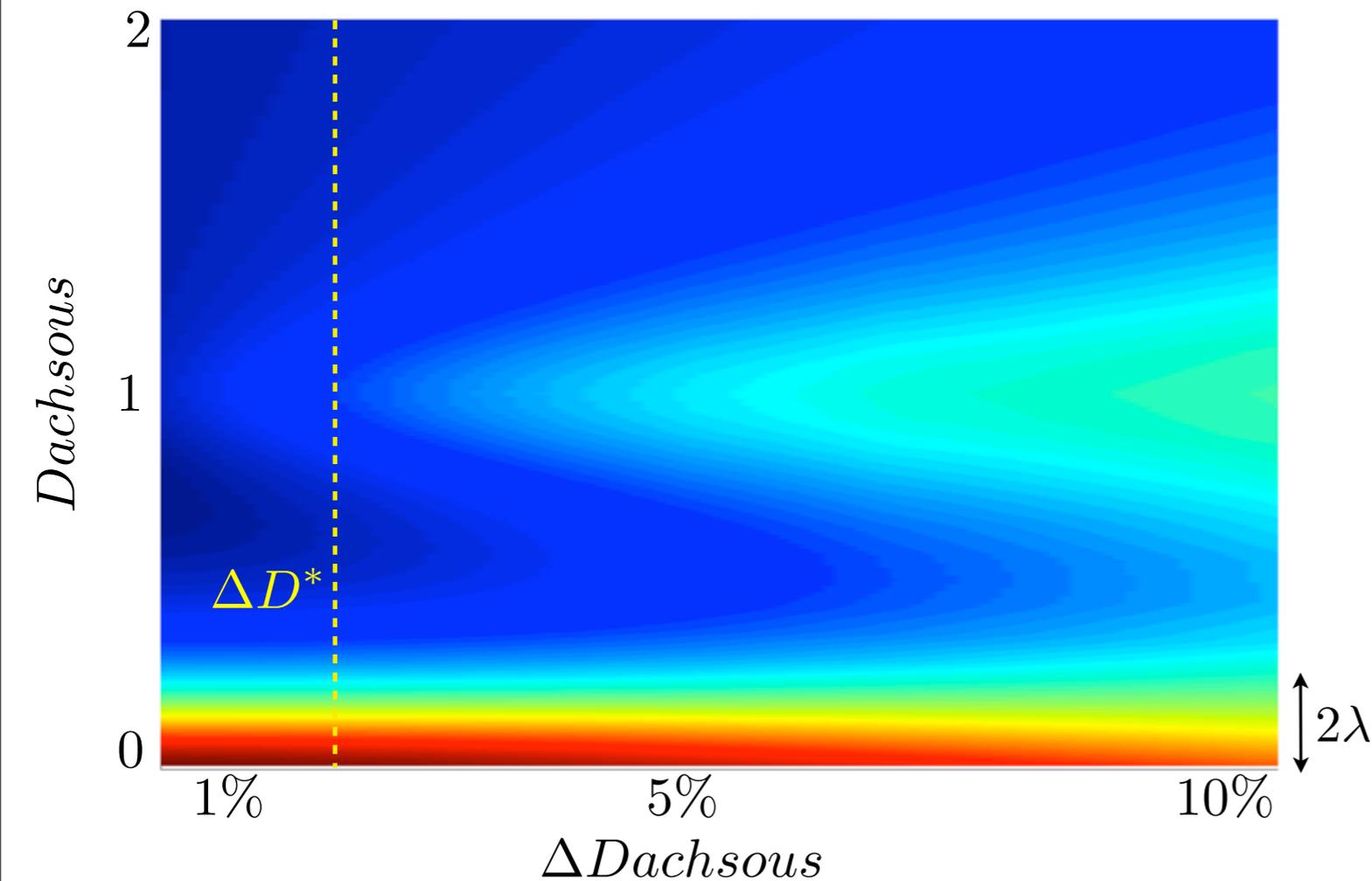
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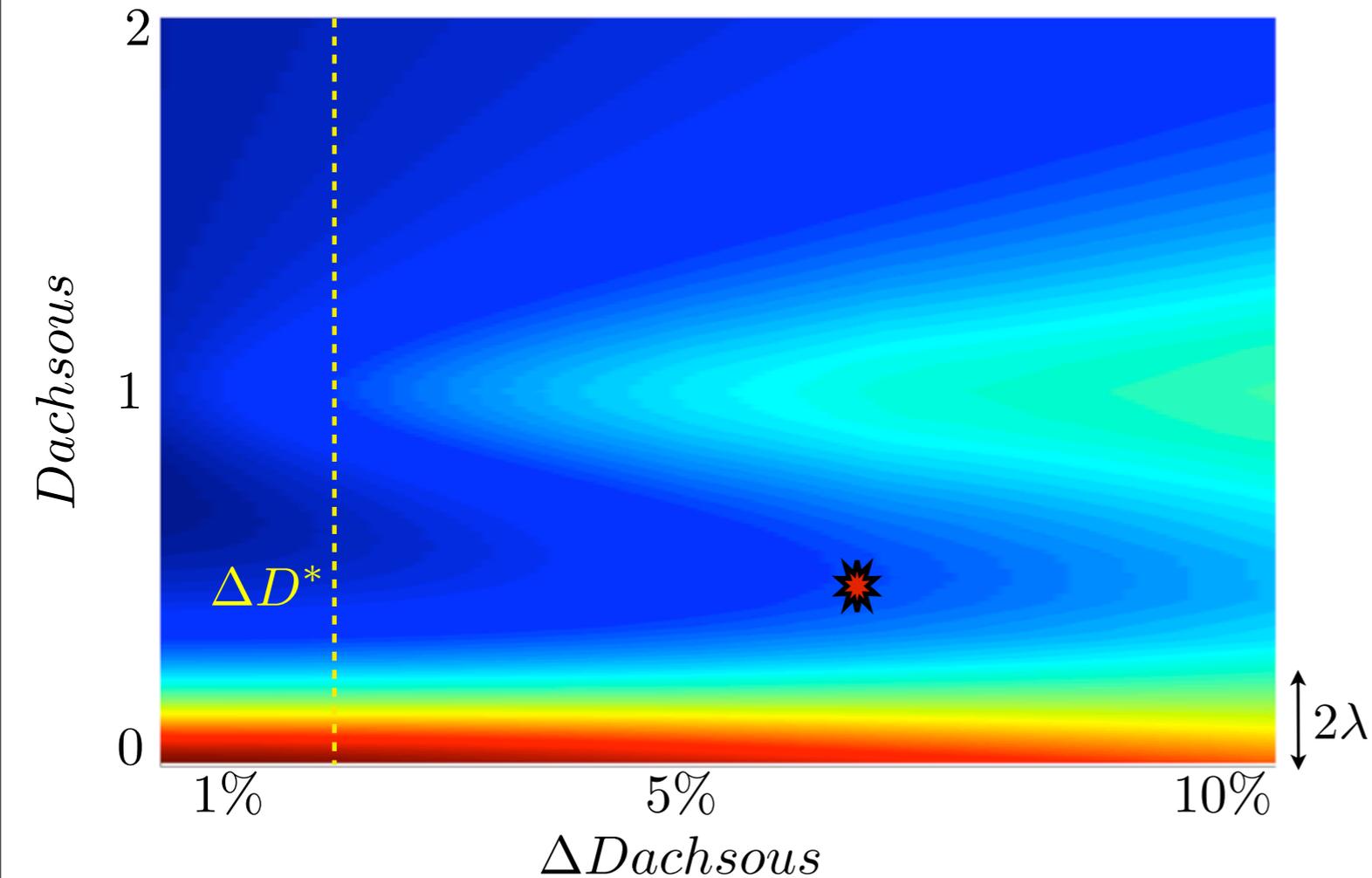
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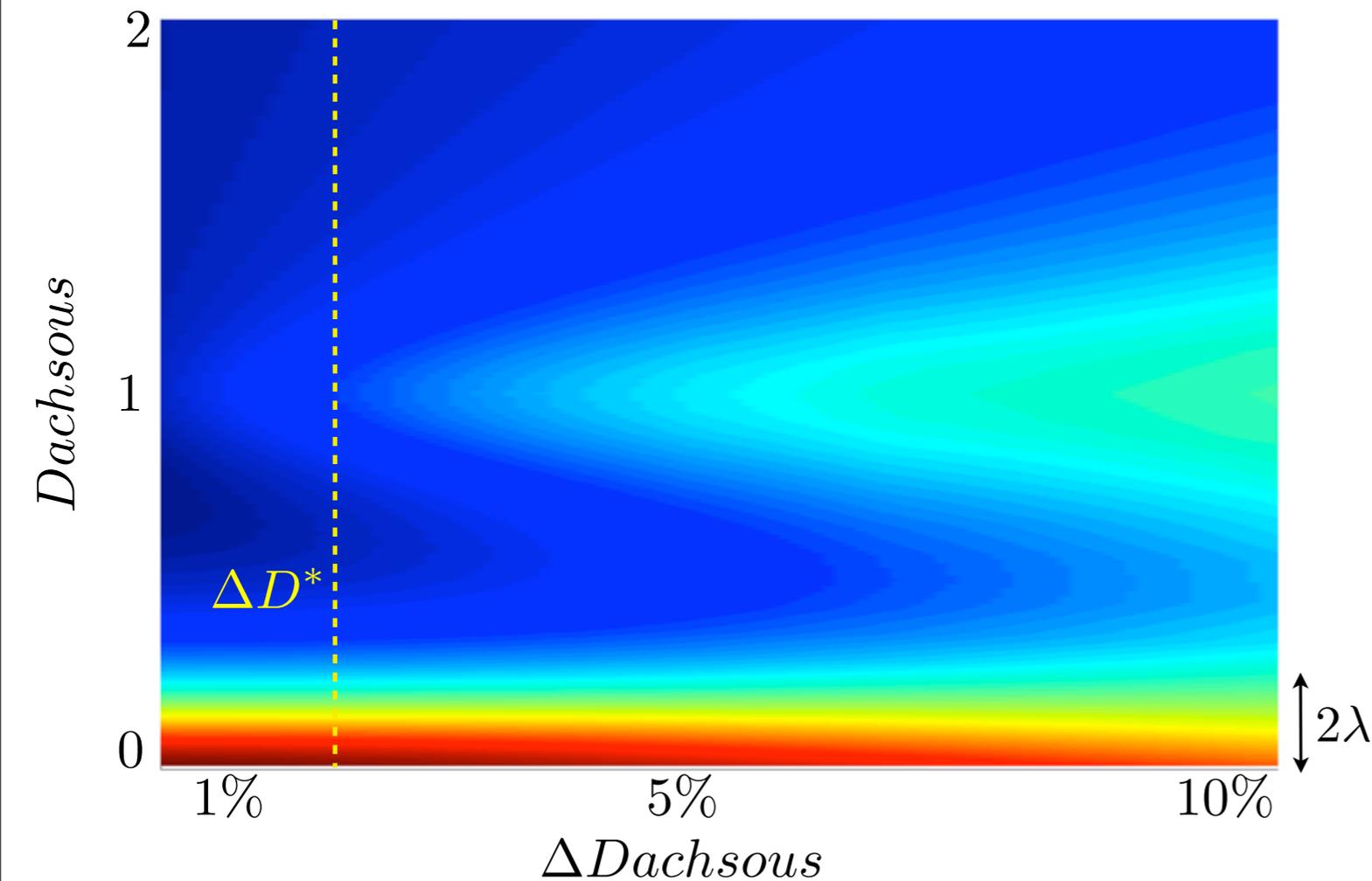
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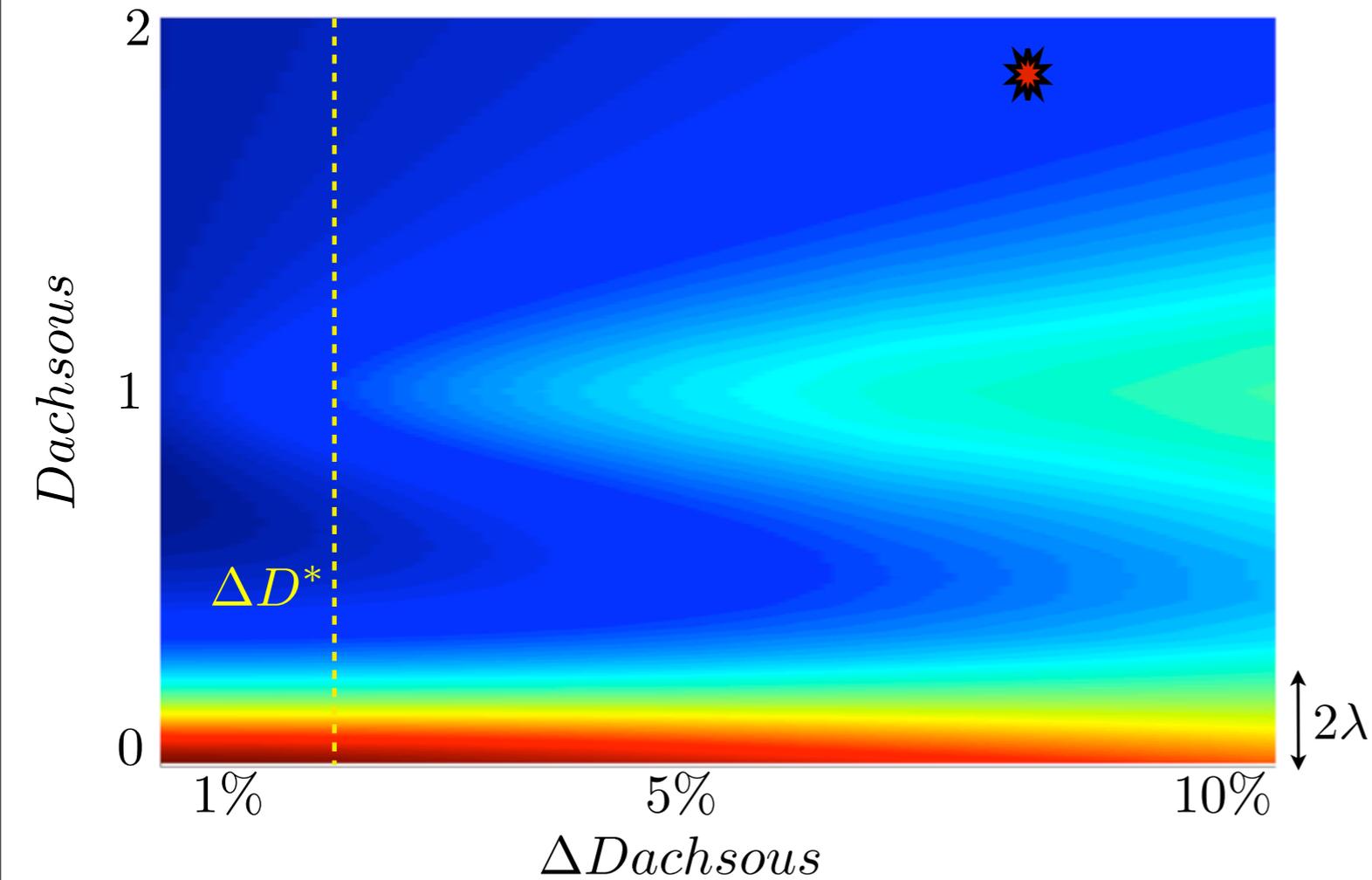
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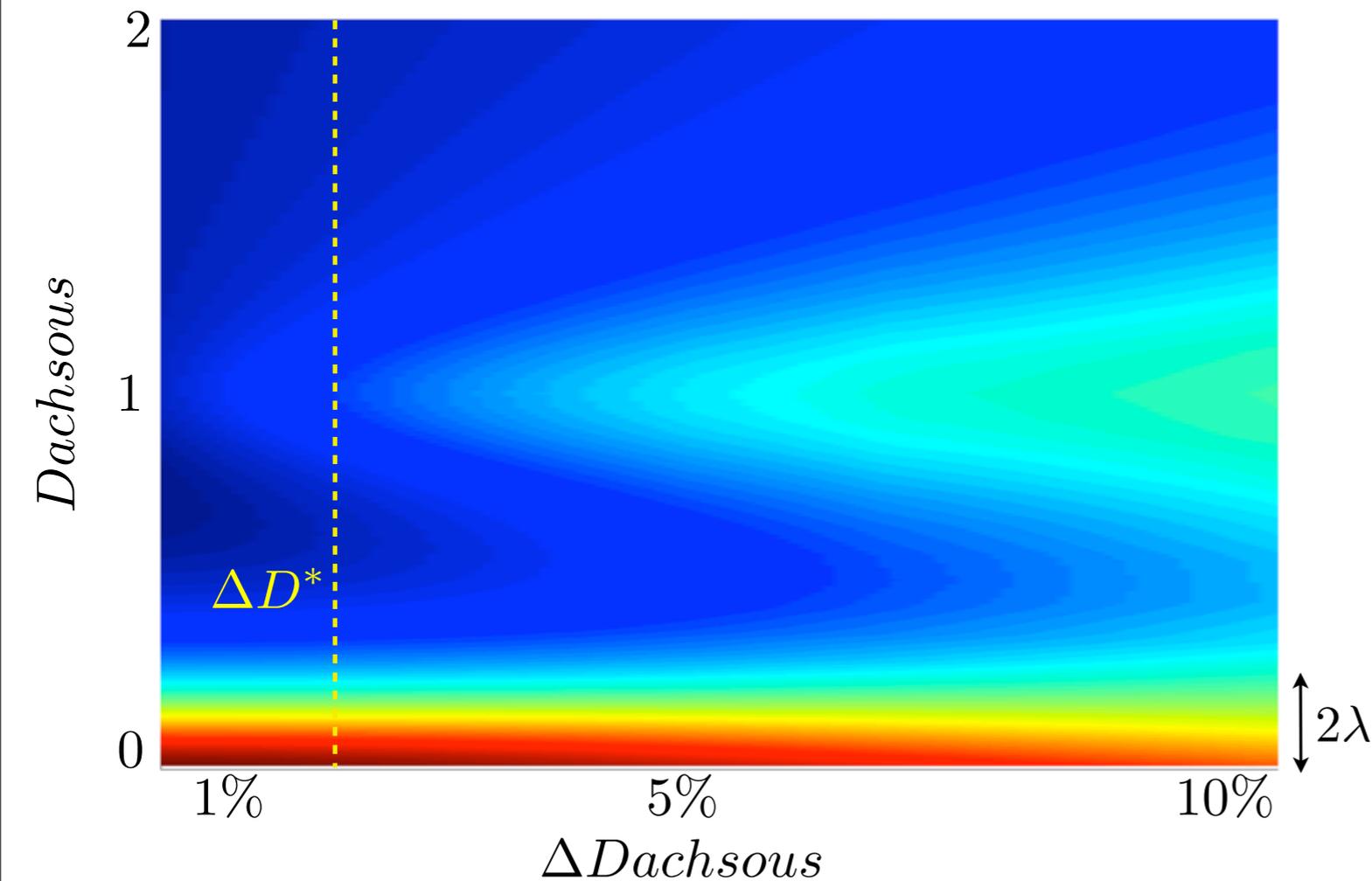
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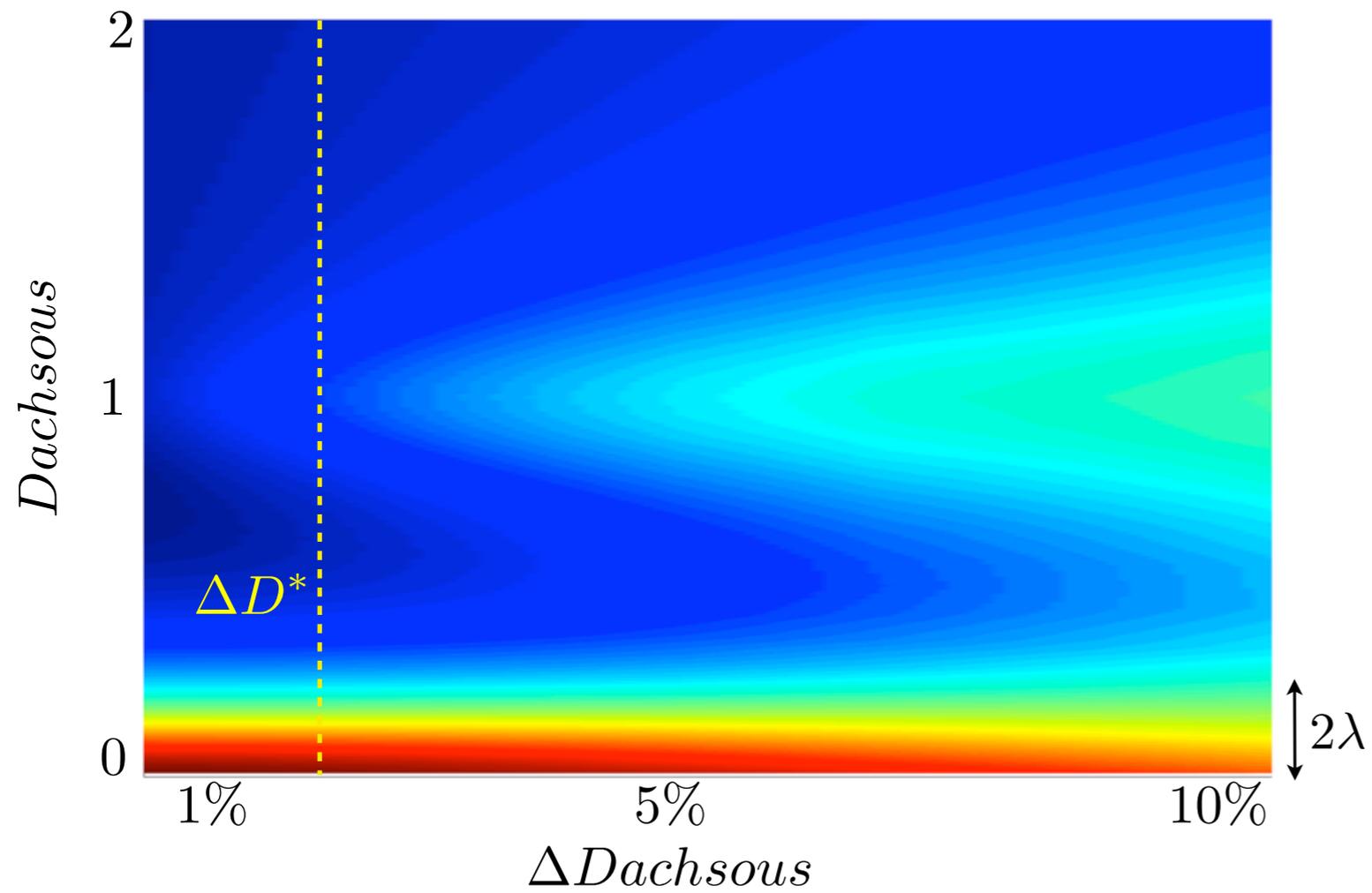
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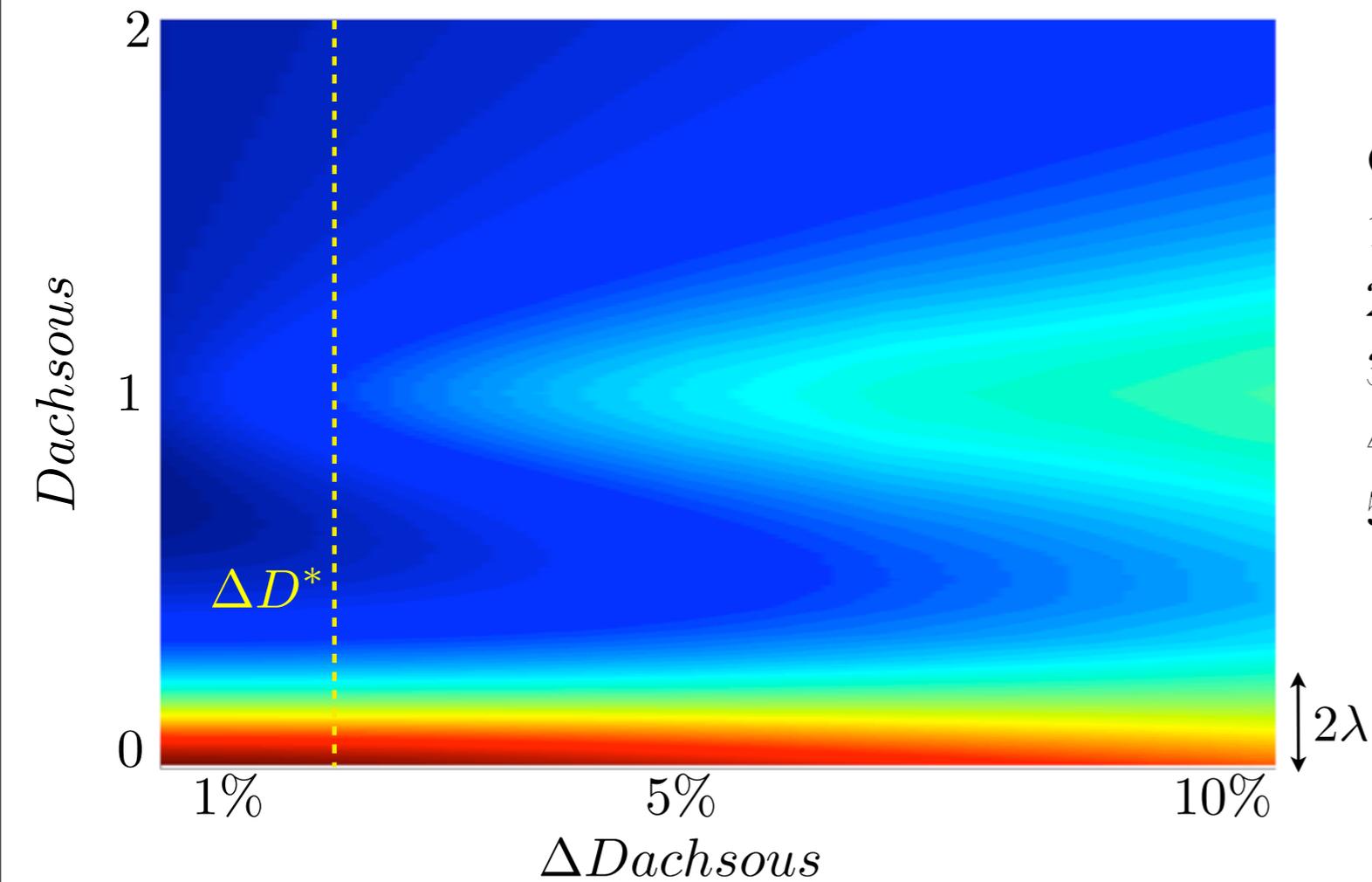
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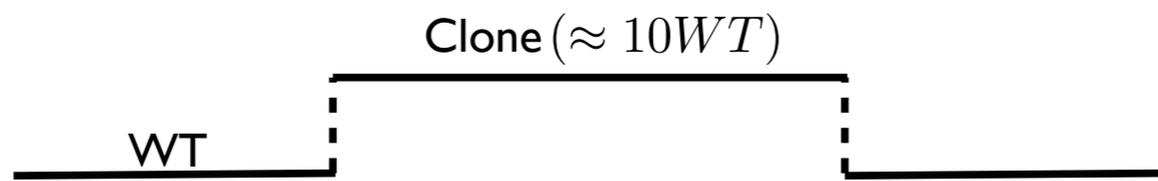
# Phase diagram



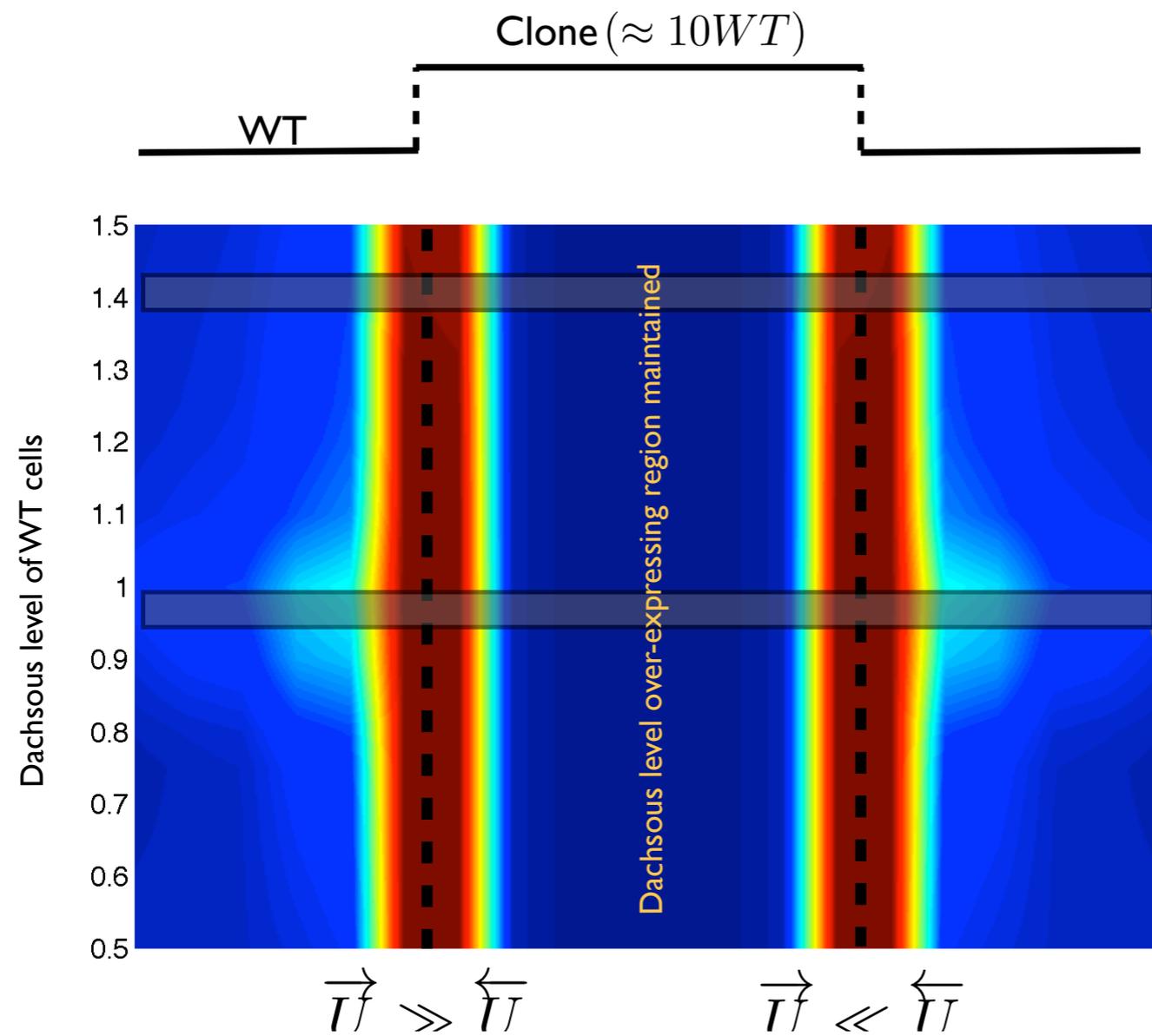
## Questions

- 1) Signaling peak width?
- 2) Predictions? Quantitative vs Qualitative
- 3) Multiple gradients?
- 4) Dynamic picture?
- 5) Clones?

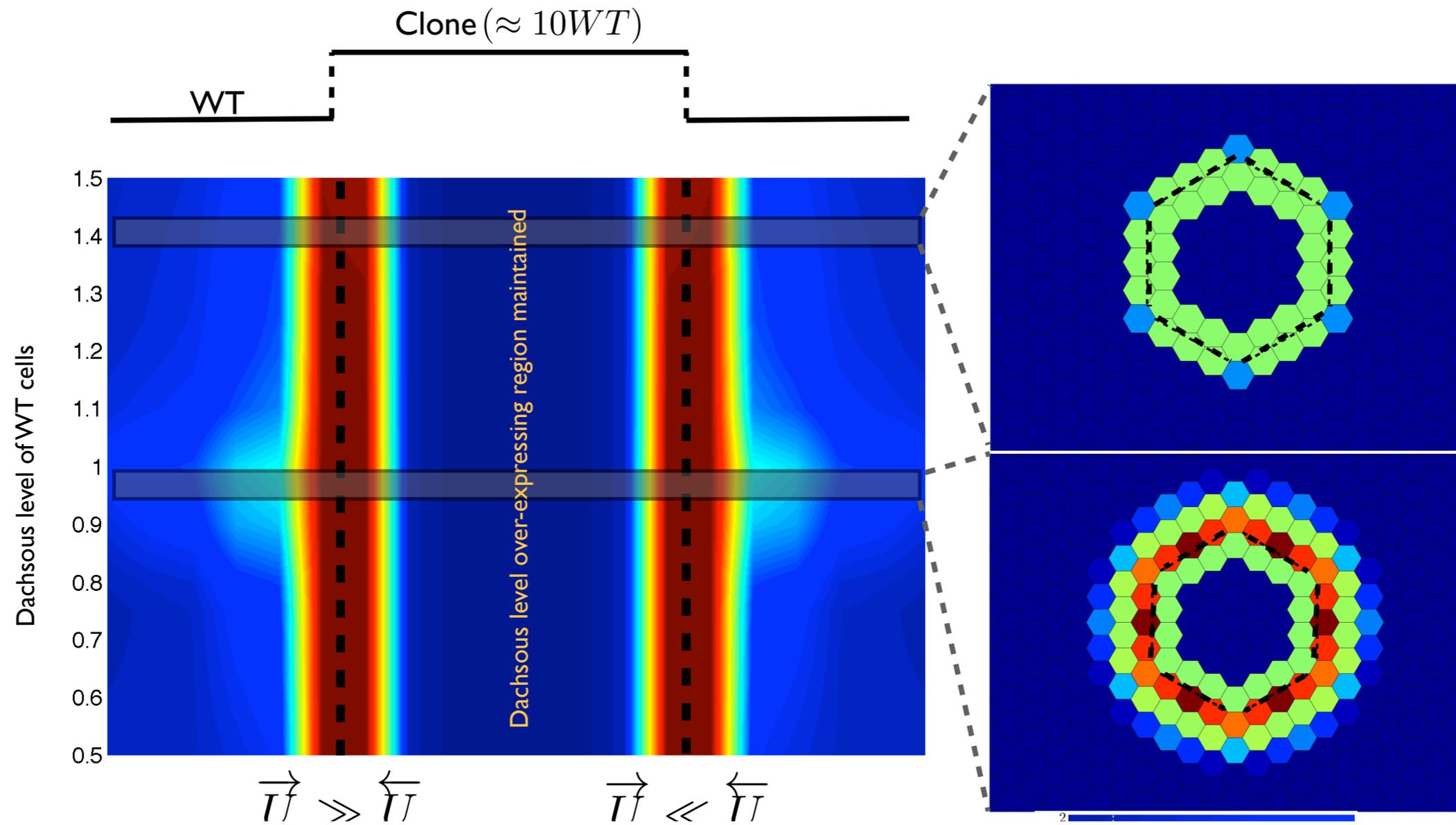
# Response to over-expression (clones)



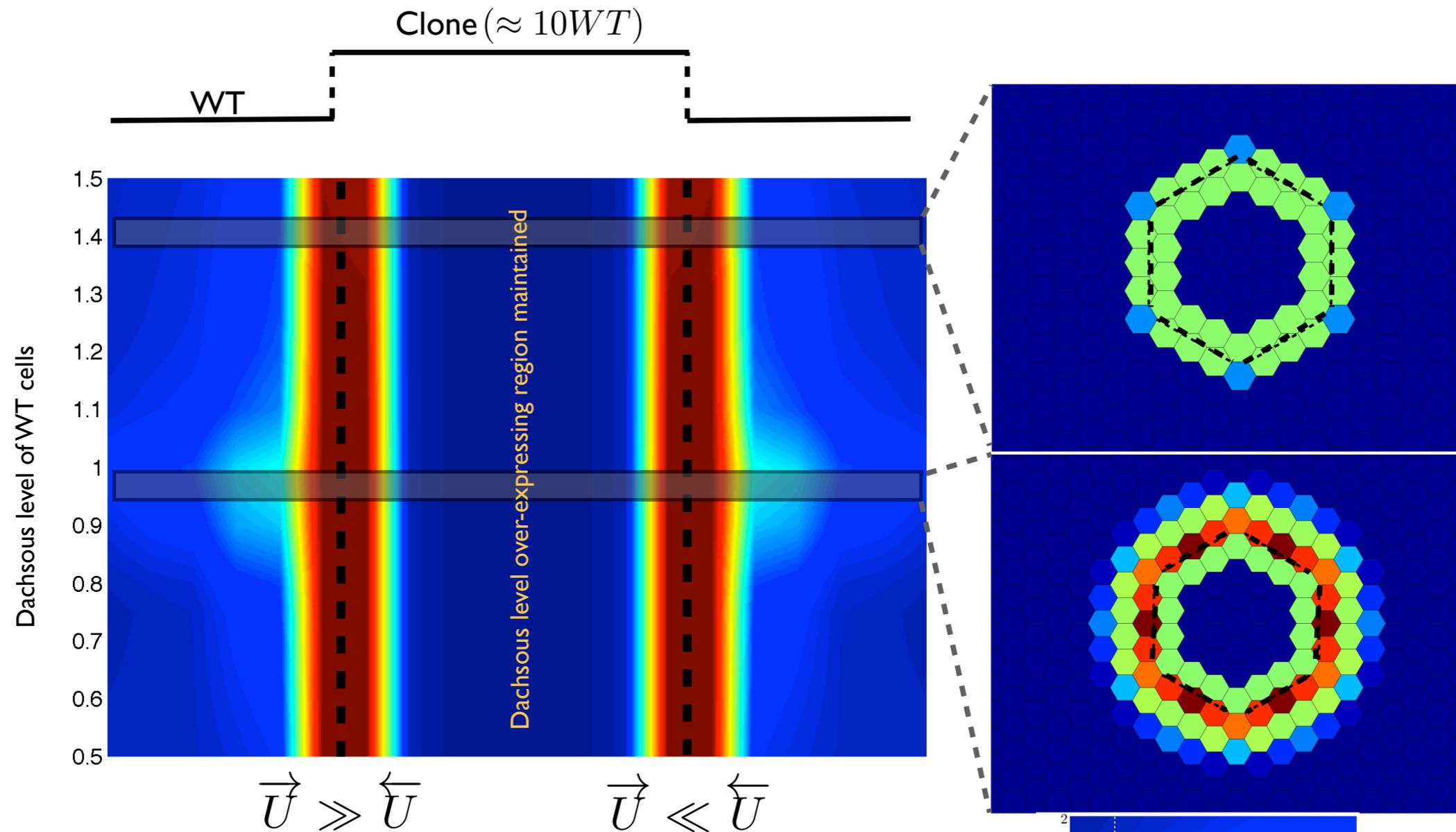
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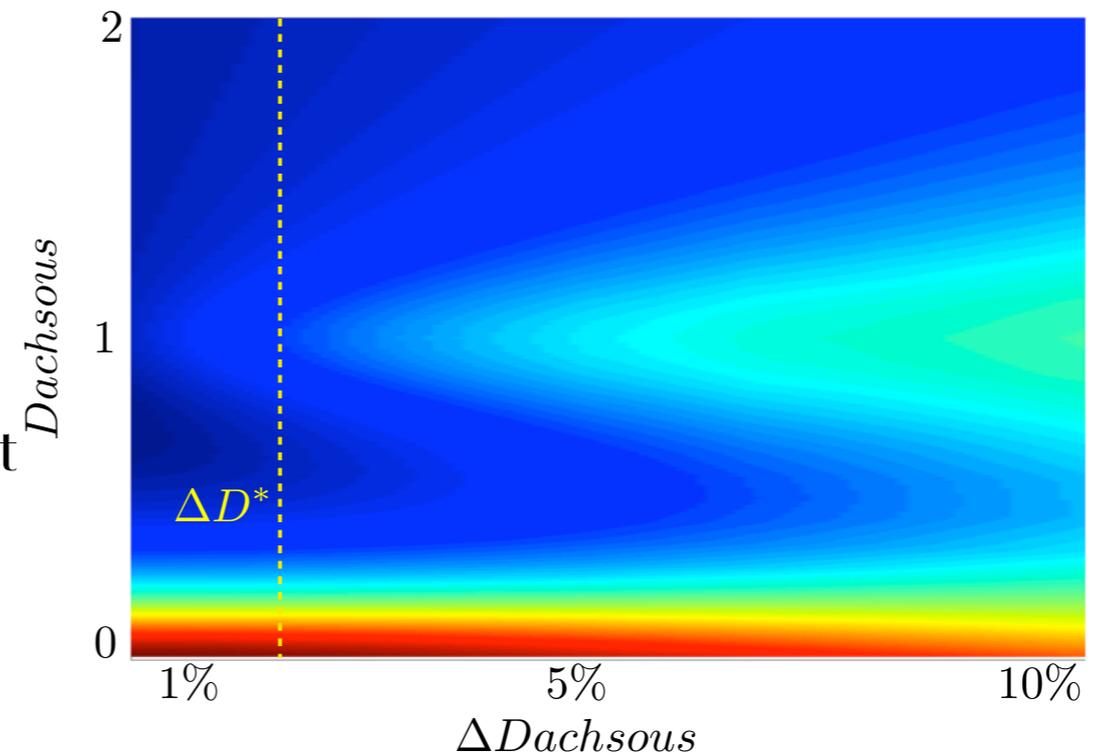
Features:

- 1) Edge response (with different polarity)
- 2) Low Fat-pathway activity within OE-region
- 3) Non-autonomous signaling that is Dachsoous dependent (positional dependence of clone)
- 4) Interpolating polarity within OE-region

# Hypotheses

Qualitative hypotheses (in vivo)

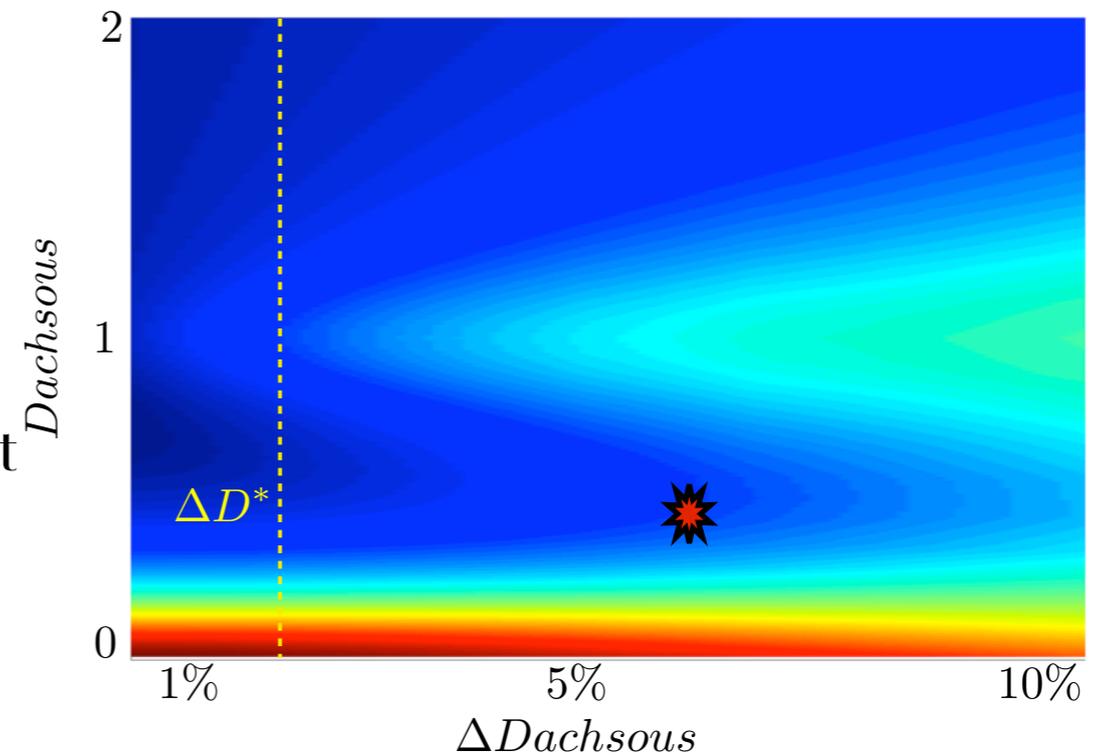
- 1) Weak knockout --> undergrowth
- 2) Clones with gradient will not instigate growth
- 3) Position of Dachsaus clone
- 4) Fat and Dachsaus in approximately equal amount
- 5) Polarity in the interior of the clone ought to interpolate over a correlation length



# Hypotheses

Qualitative hypotheses (in vivo)

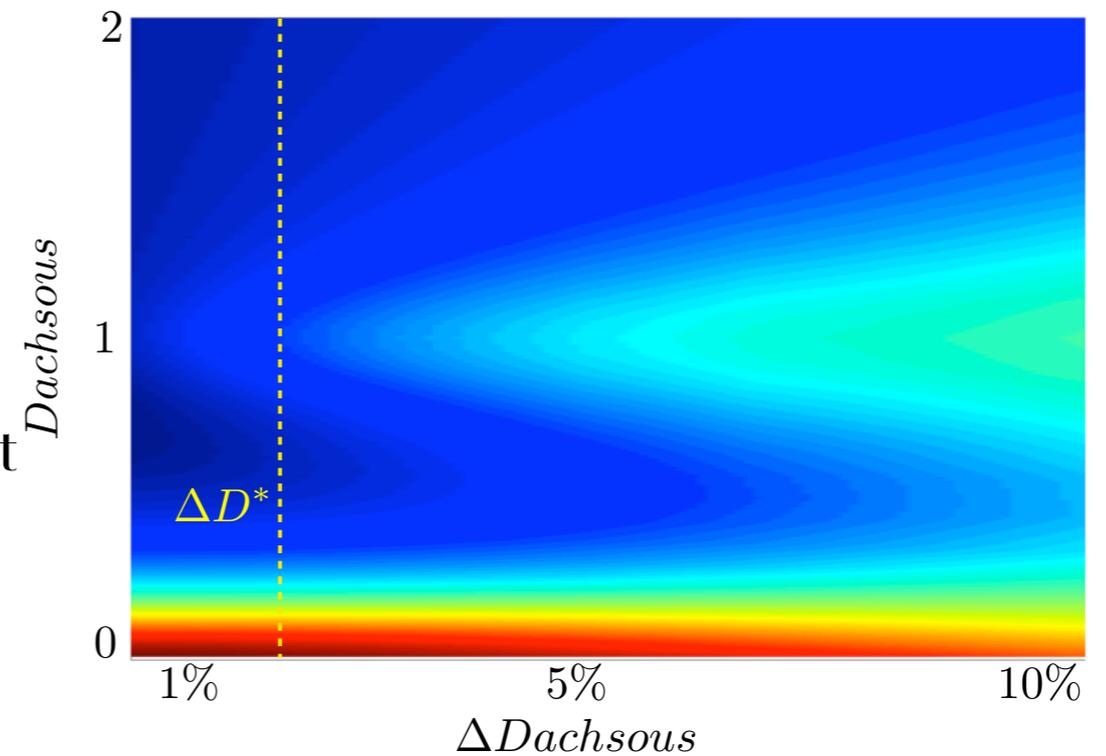
- 1) Weak knockout --> undergrowth ✱
- 2) Clones with gradient will not instigate growth
- 3) Position of Dachsaus clone
- 4) Fat and Dachsaus in approximately equal amount
- 5) Polarity in the interior of the clone ought to interpolate over a correlation length



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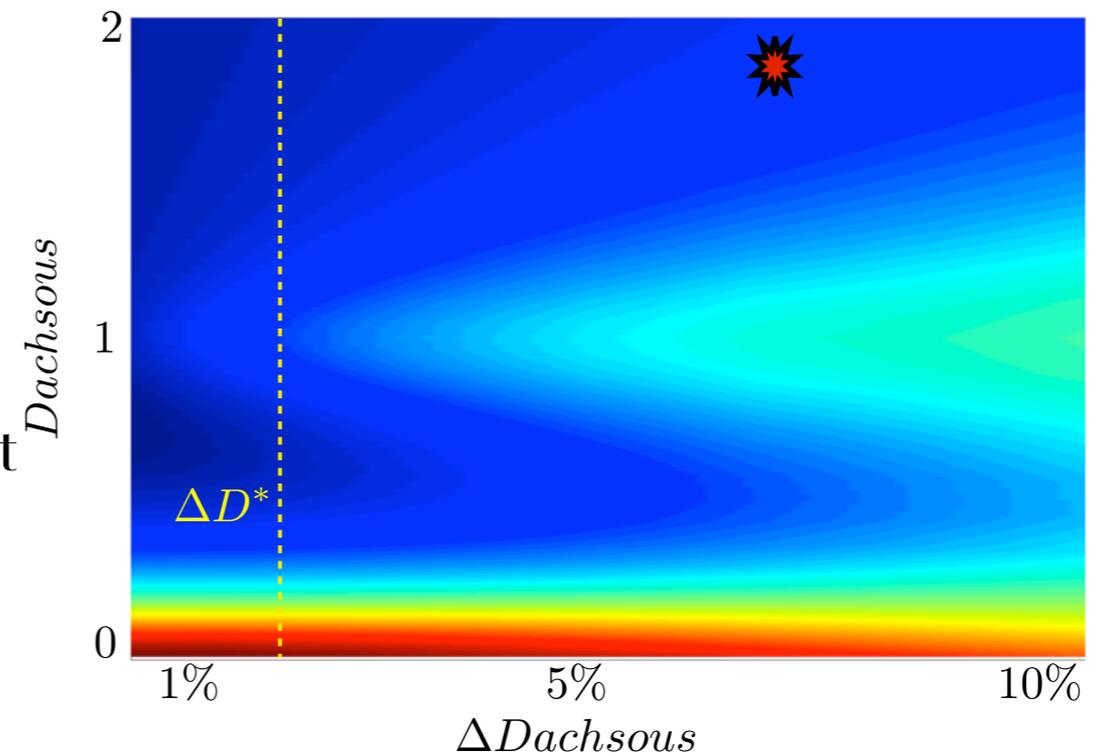
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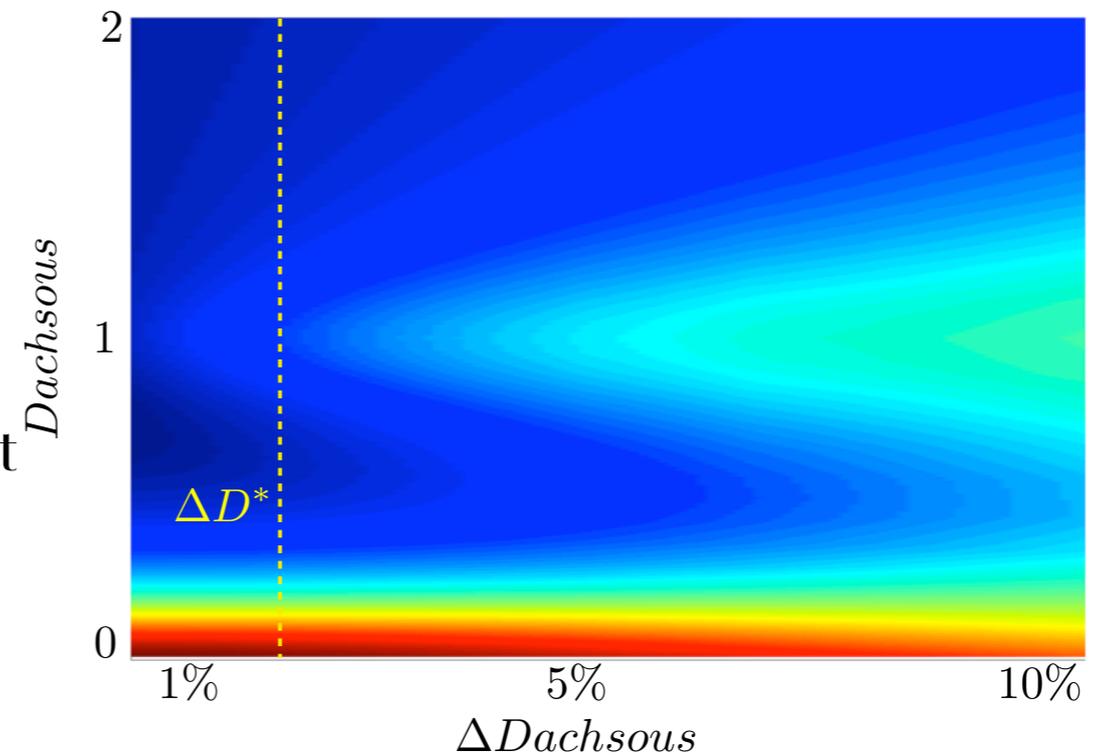
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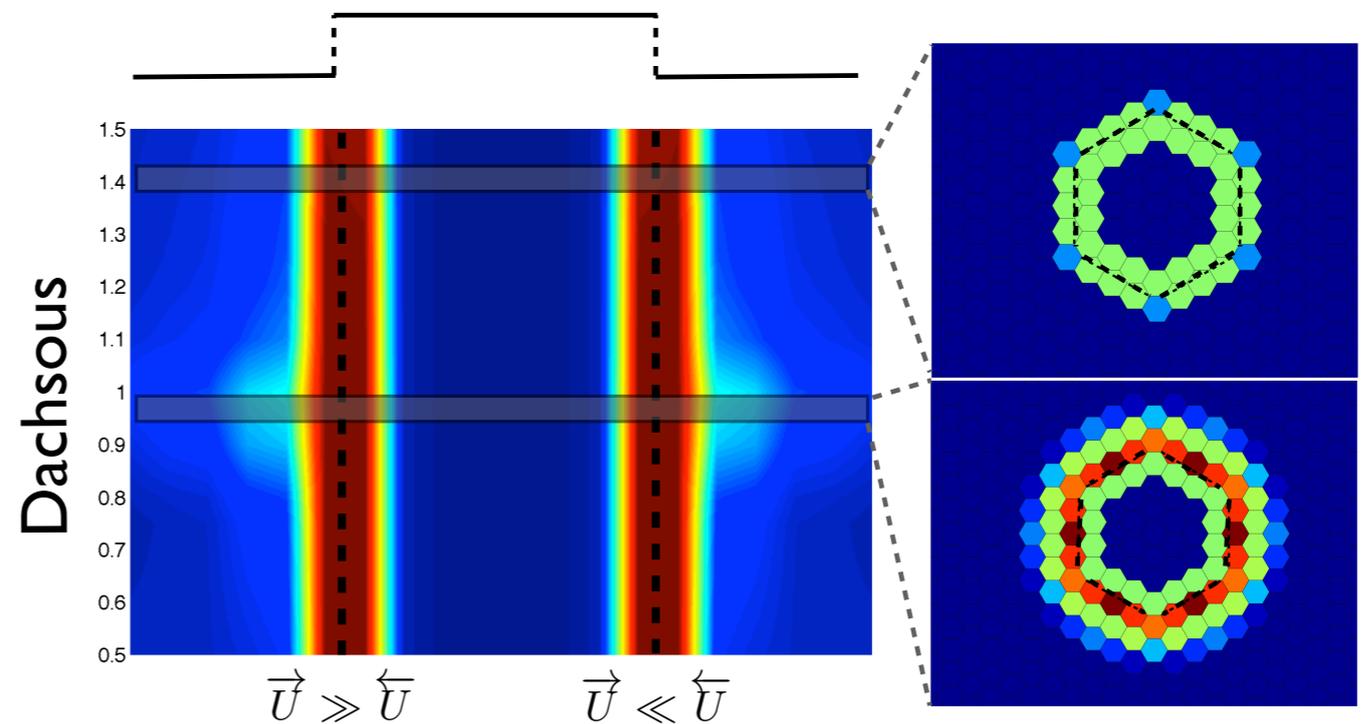
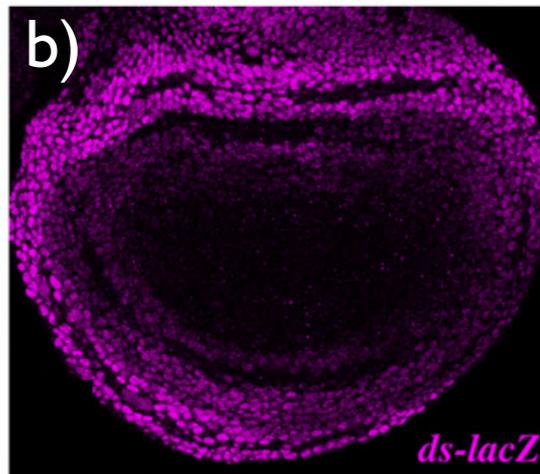
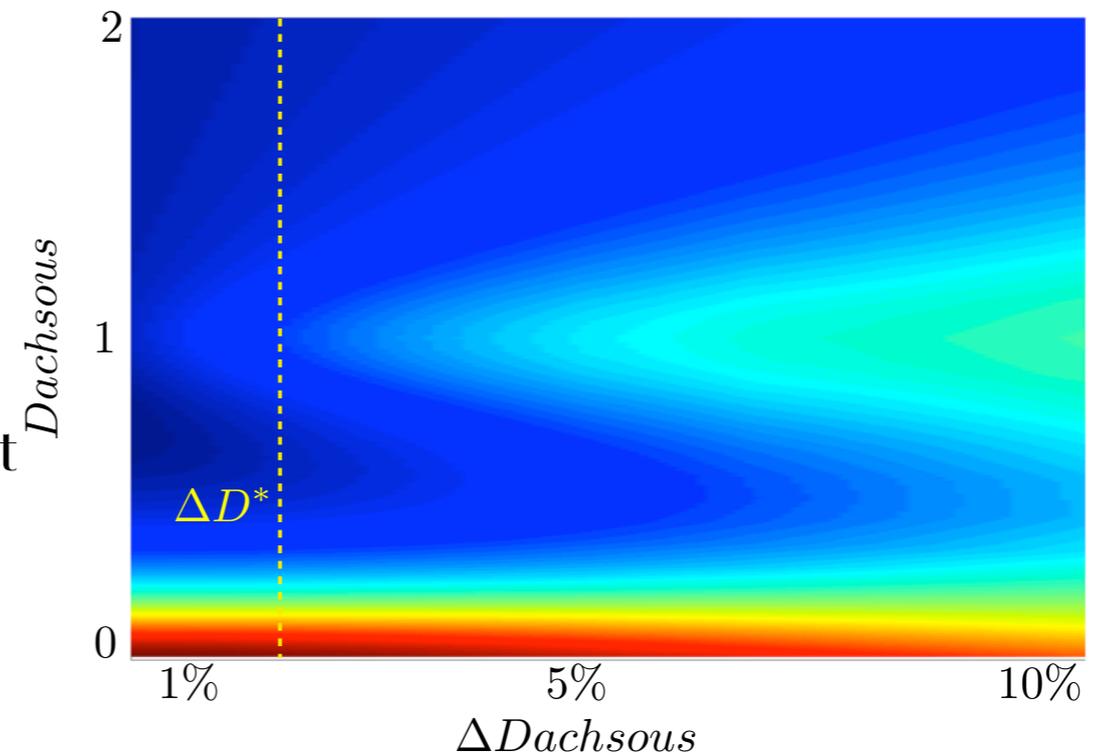
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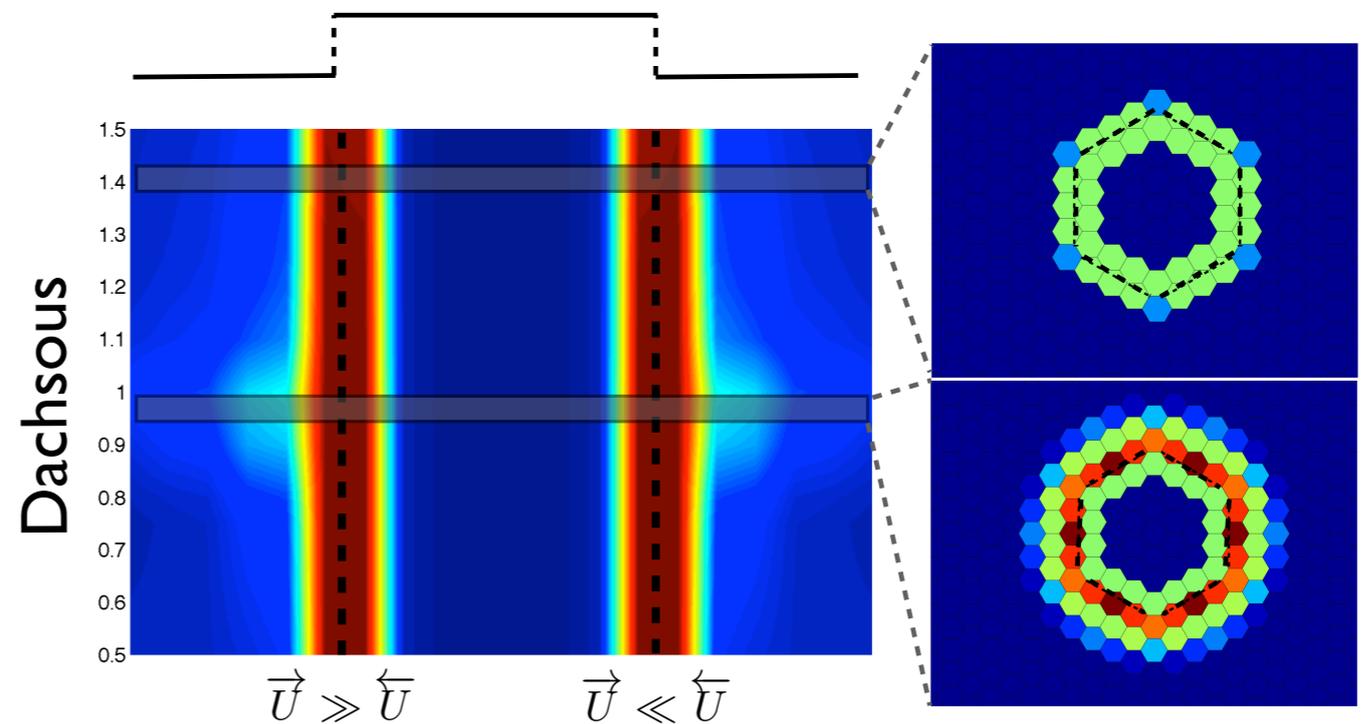
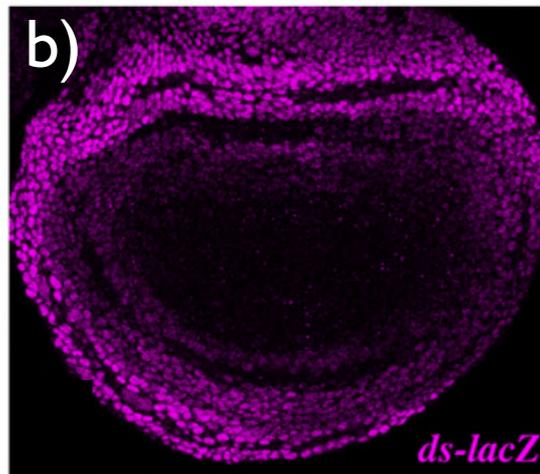
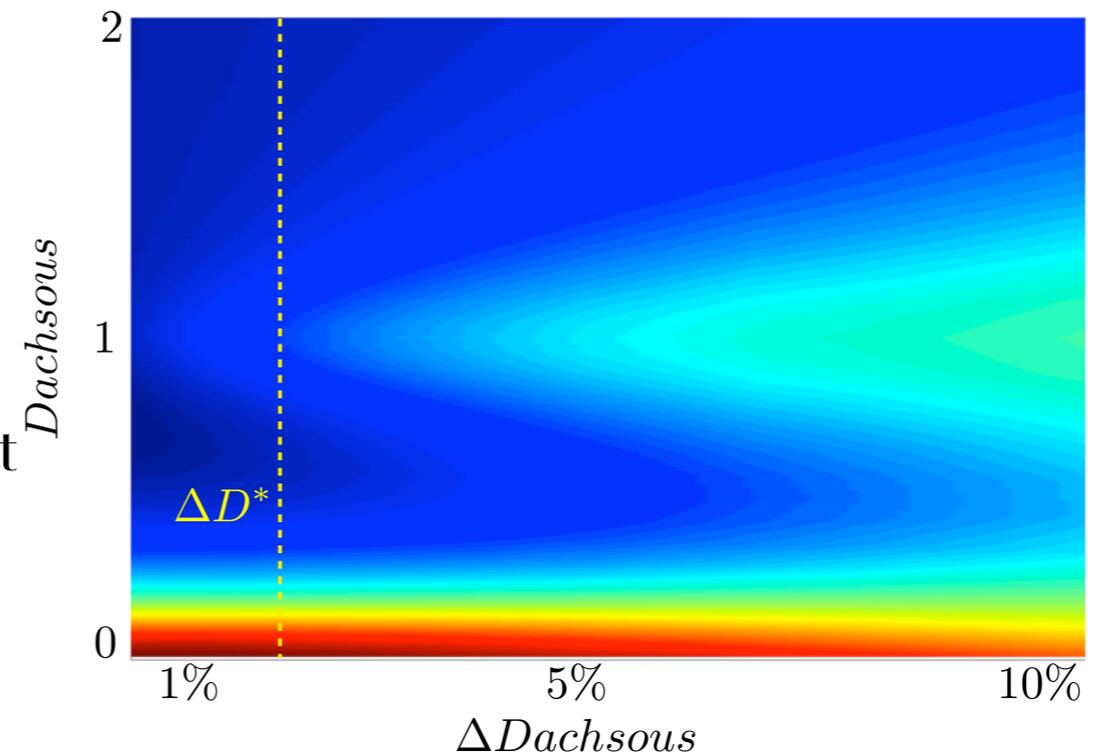
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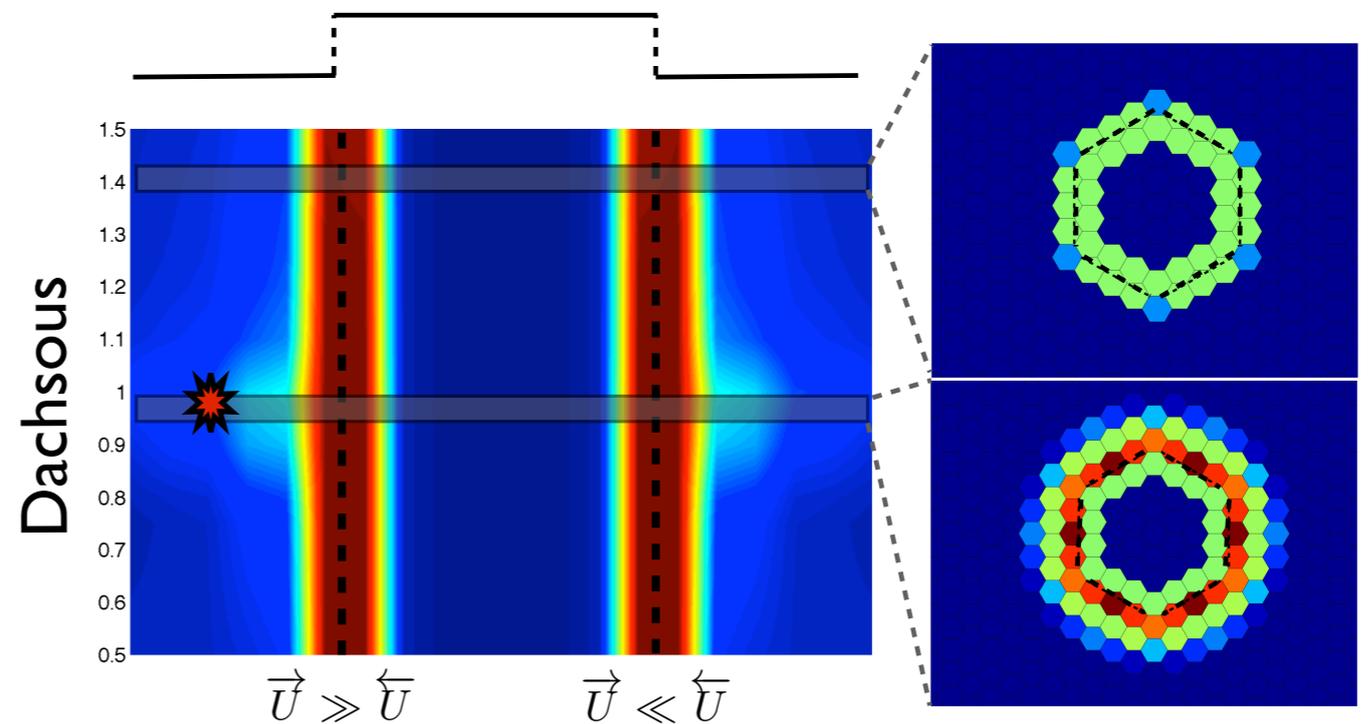
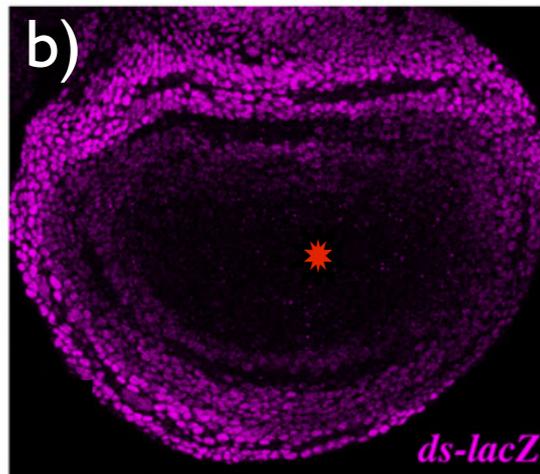
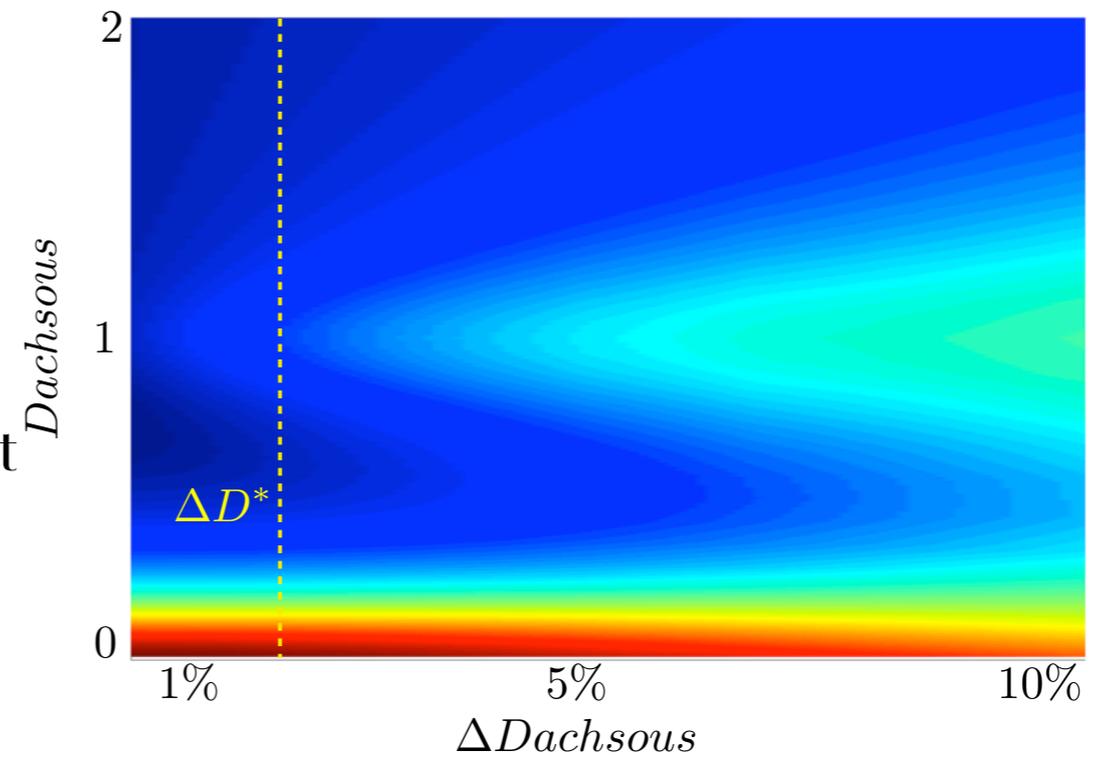
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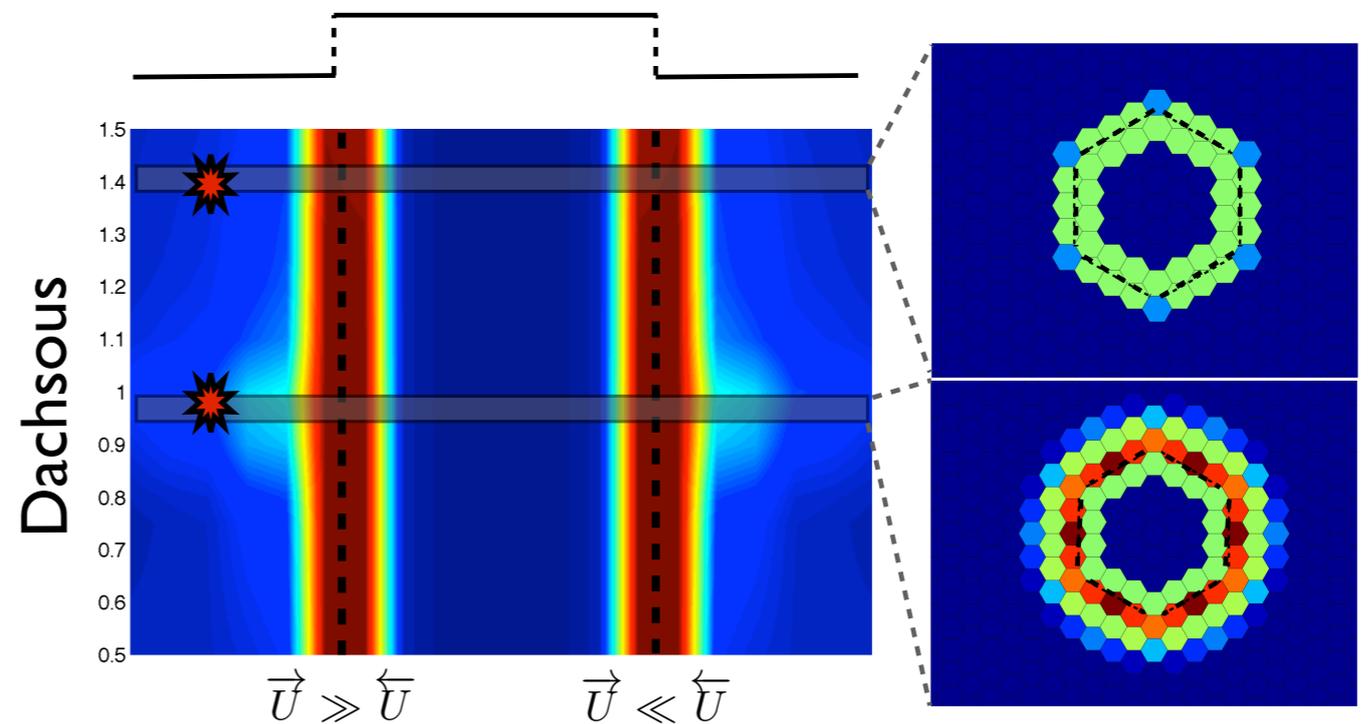
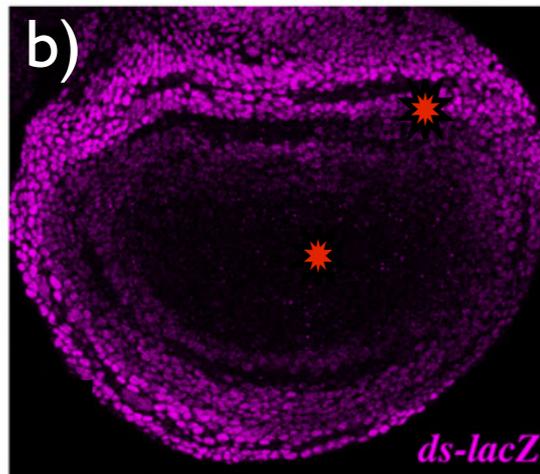
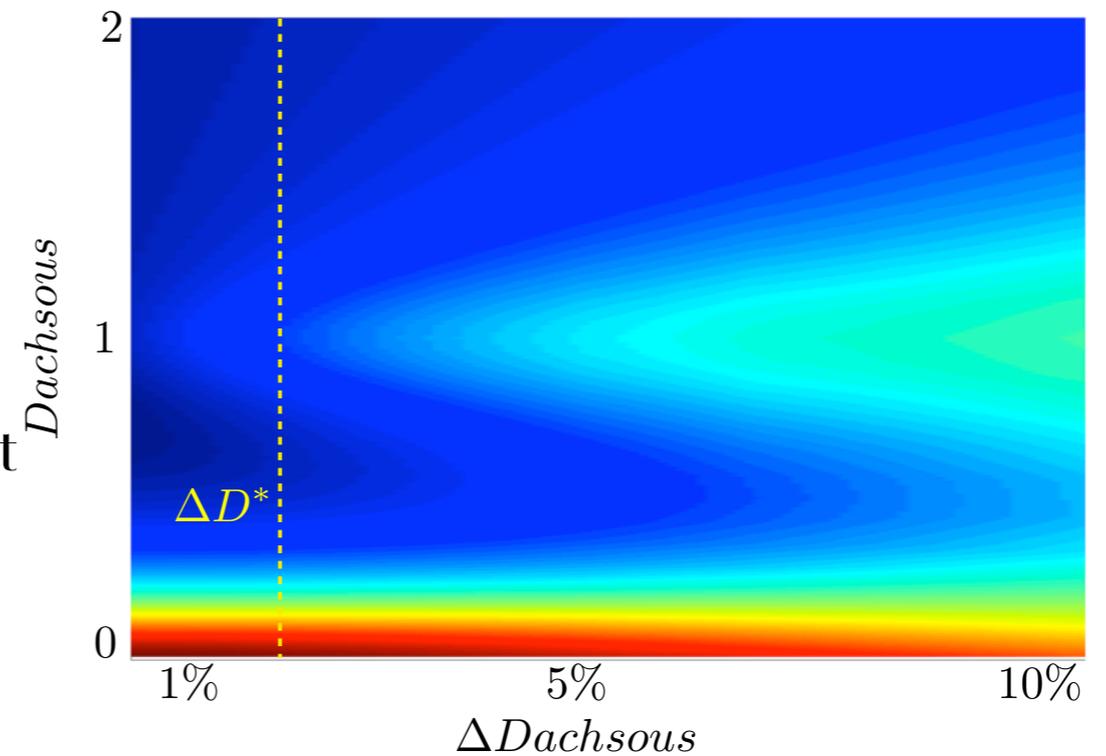
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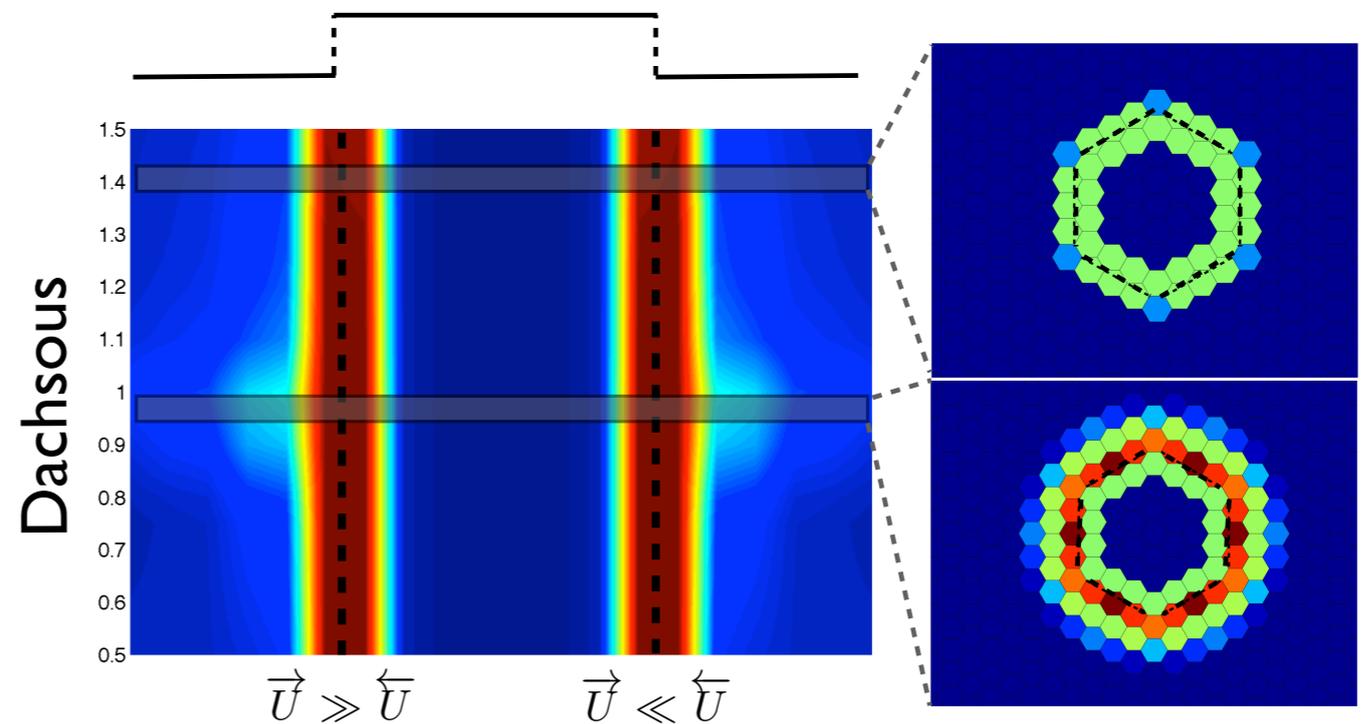
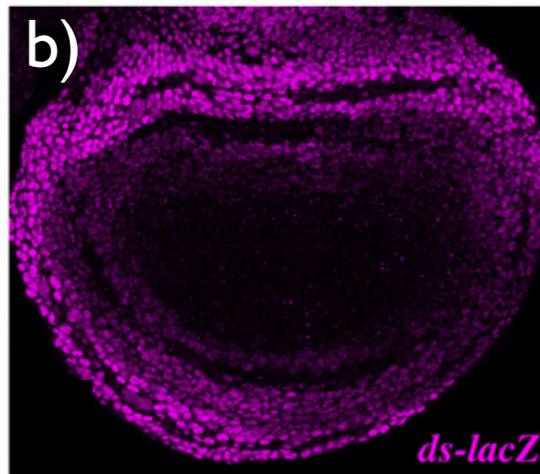
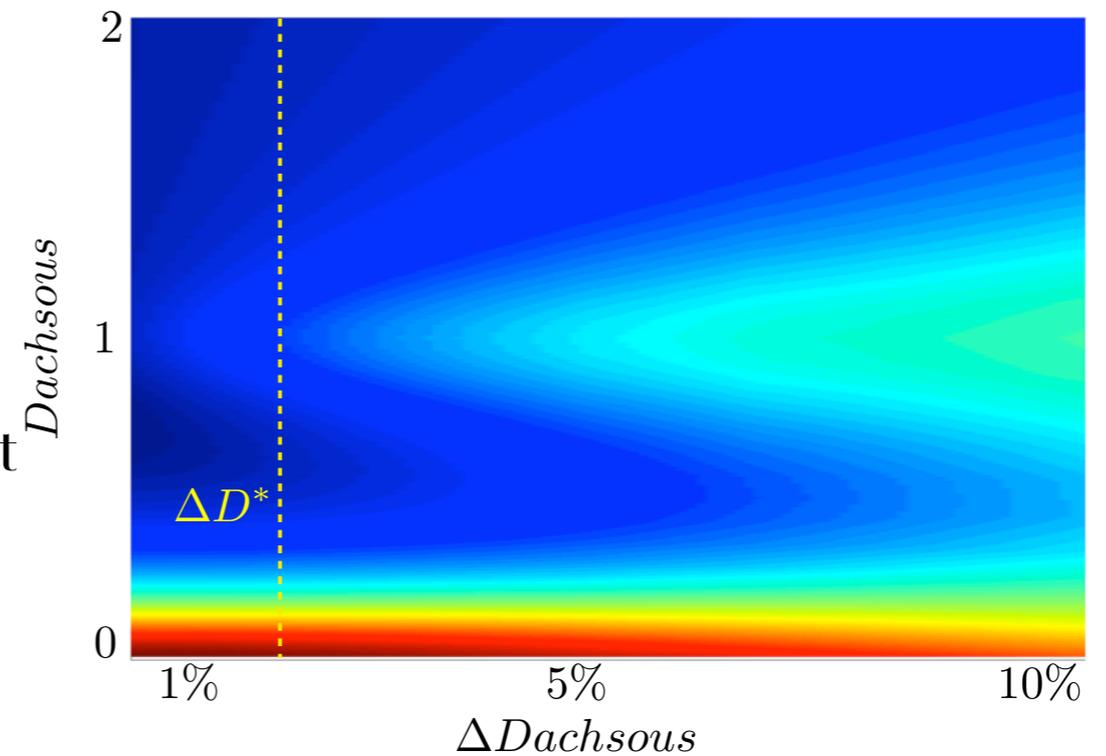
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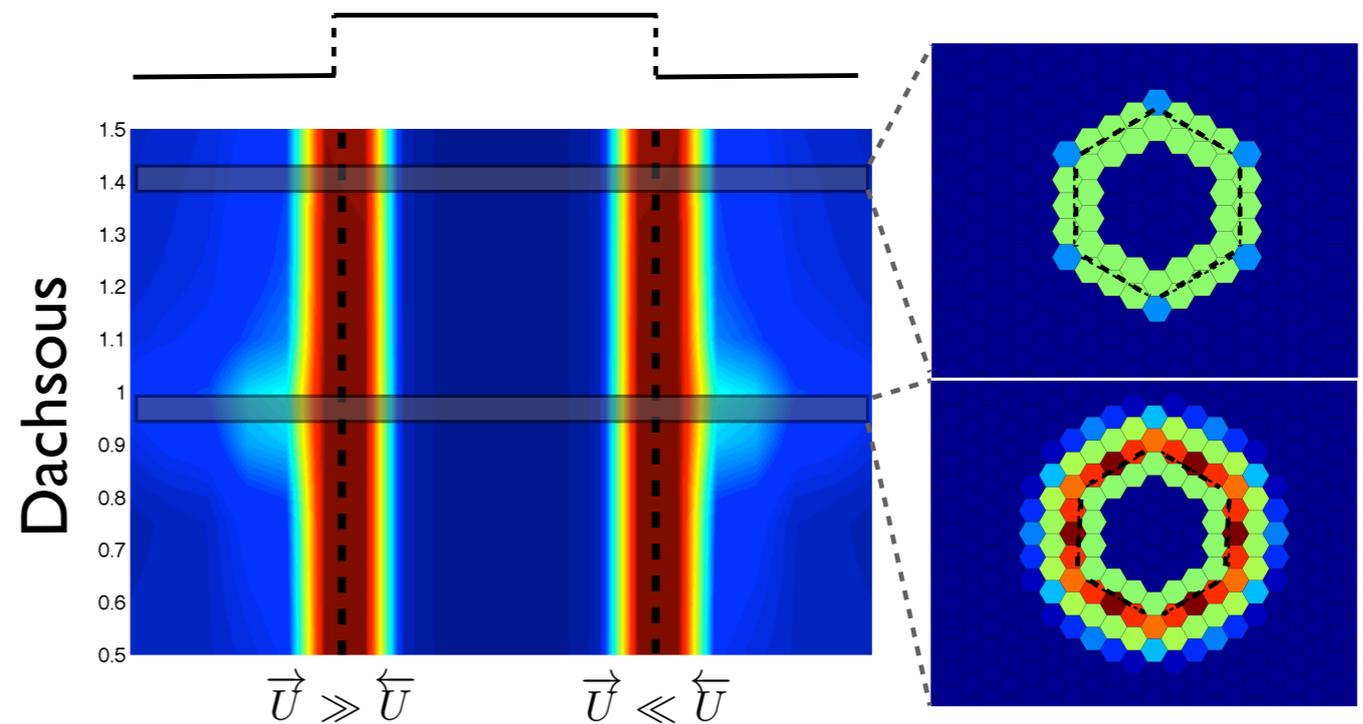
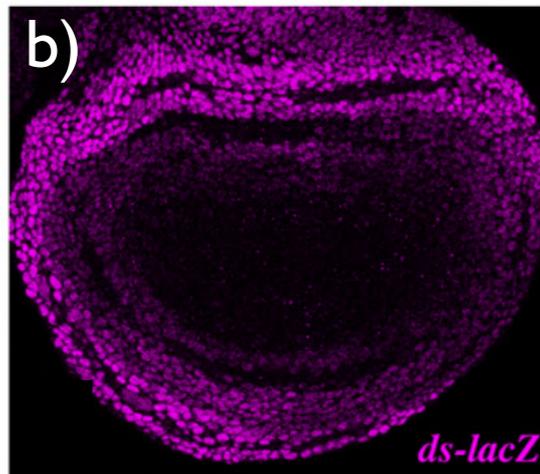
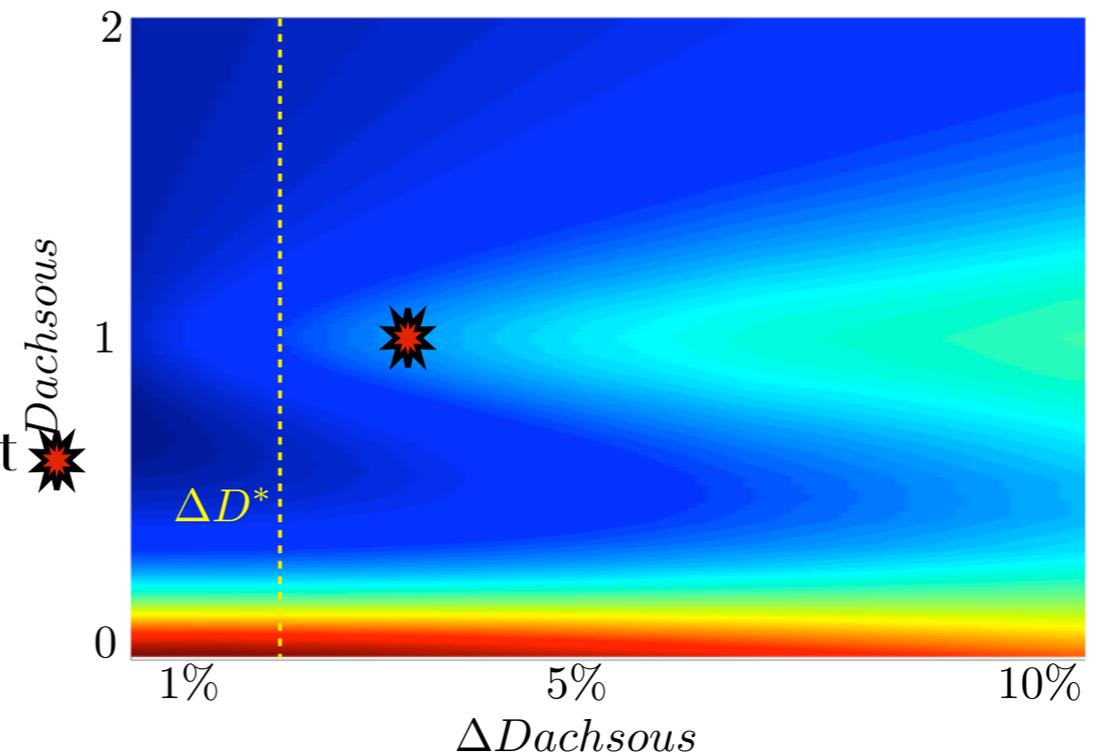
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# Hypotheses

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Qualitative hypotheses (synthetic - David Sprinzak)

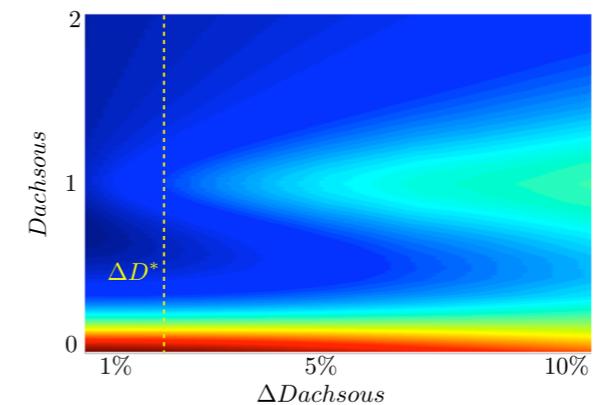
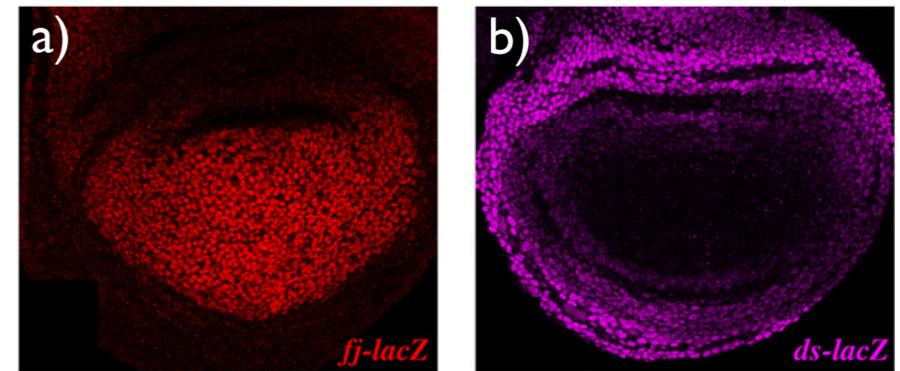
- 1) Testing the existence of cooperative interactions (strength of alpha, beta etc)
- 2) Possibly see transition as a function of absolute level
- 2) Non-autonomy in a multi-cell assay

# Shortcomings, Cynicism and things I don't understand

1) Why are the profiles of Fj and Ds what they are? Fail

Potentially helpful observations would be:

- a) Dynamics of initiation of these profiles
- b) Fat profile

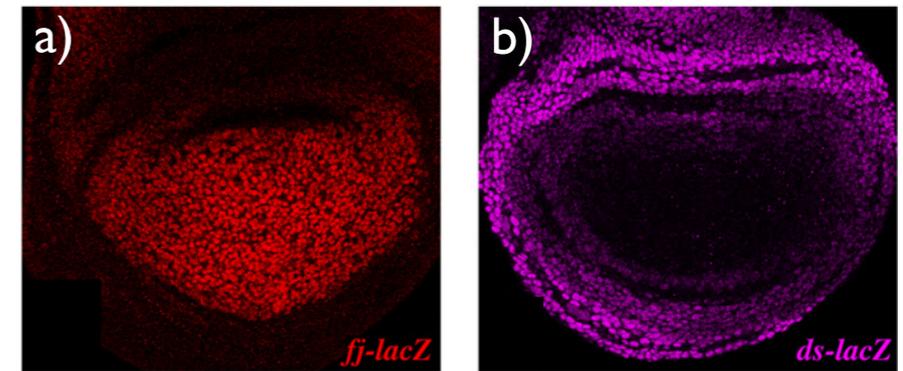


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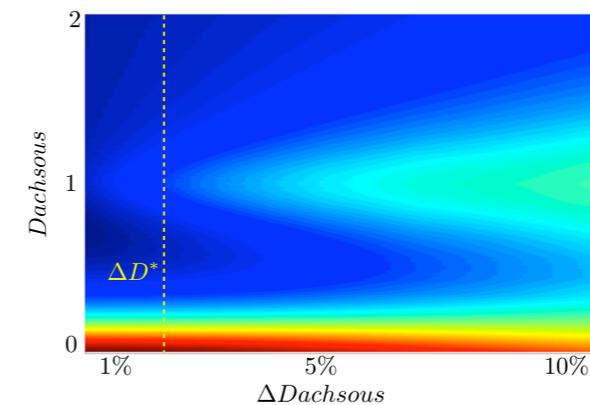
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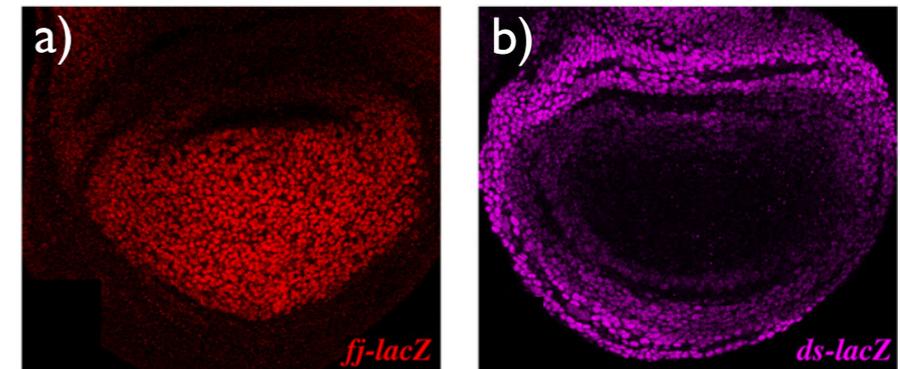


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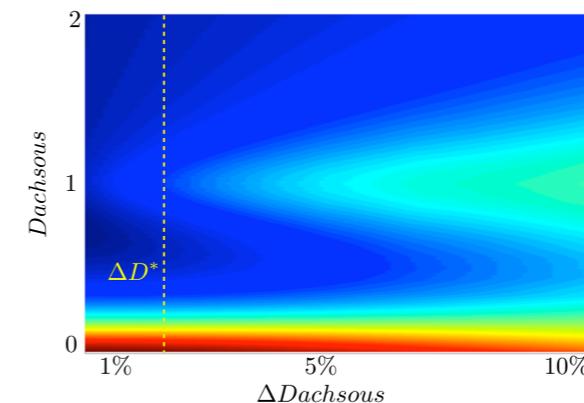
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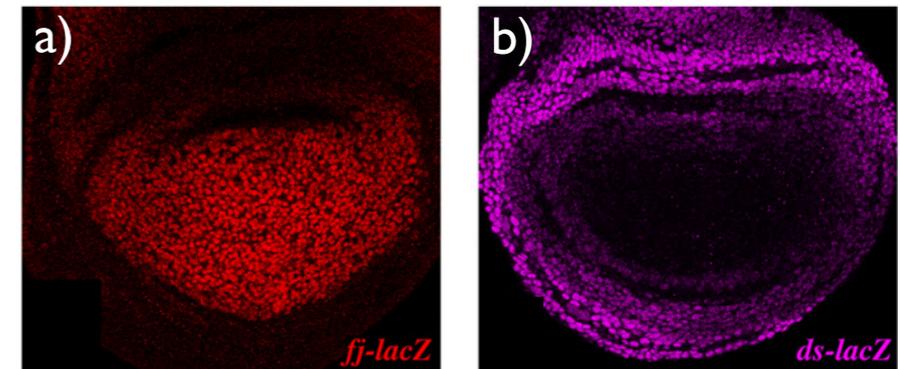
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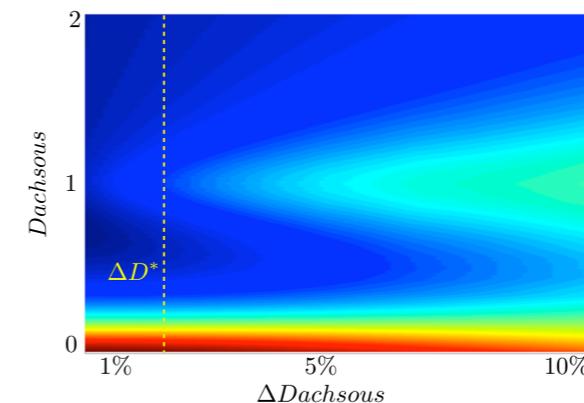
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Could the Fat pathway allow communication across the wing-disc?

# A hypothesis

*Growth*

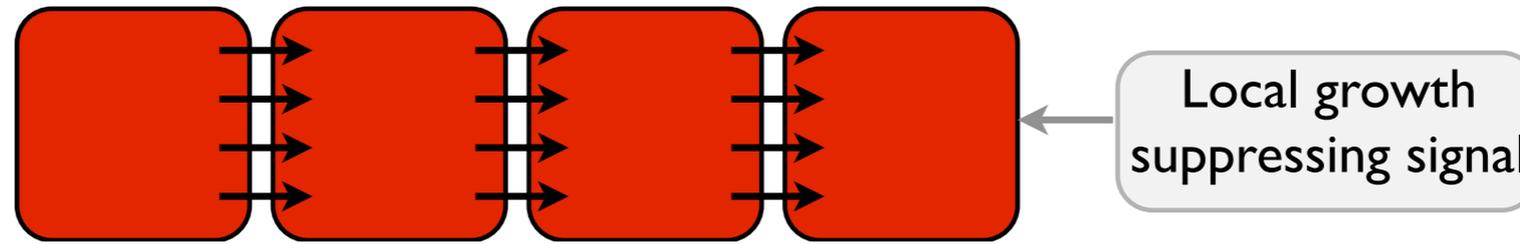


*Four—  
jointed*



$\alpha$  &  $\beta$

# A hypothesis



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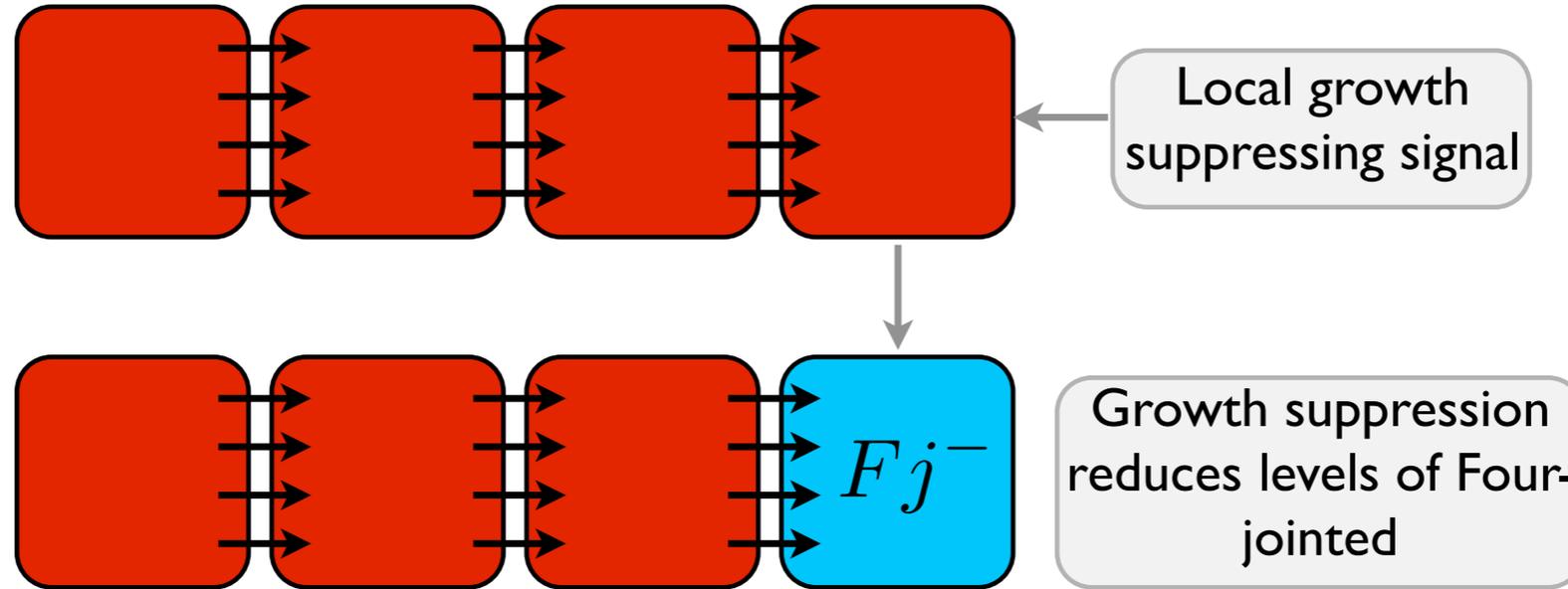
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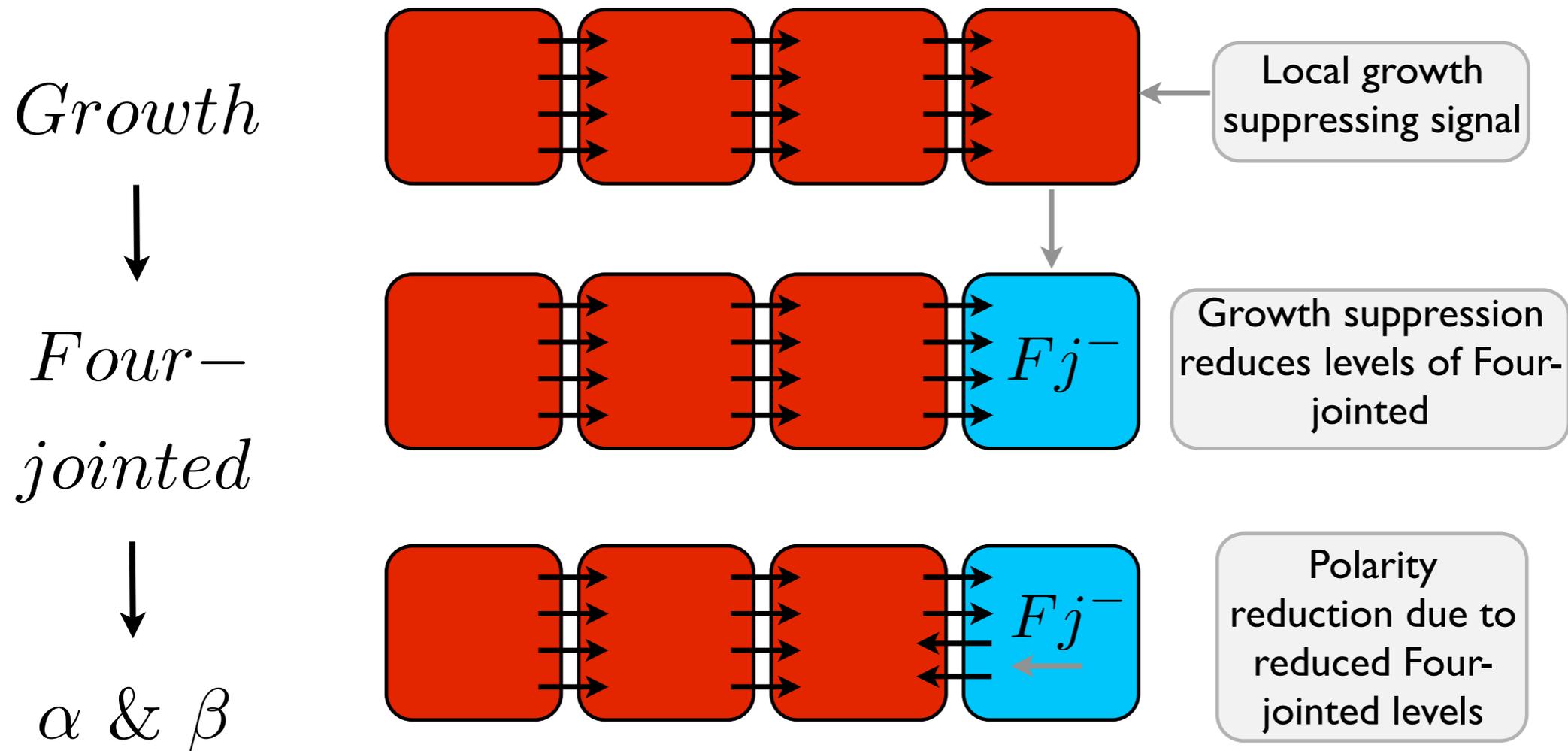


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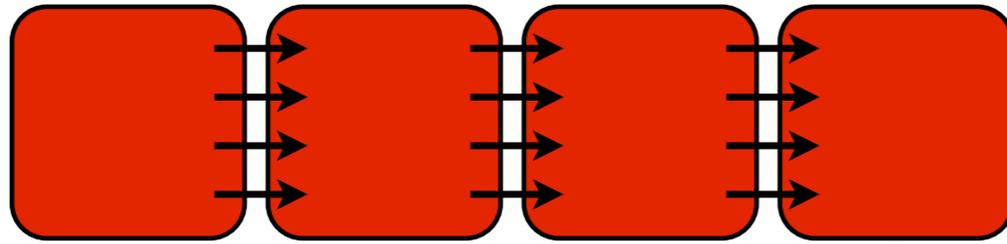
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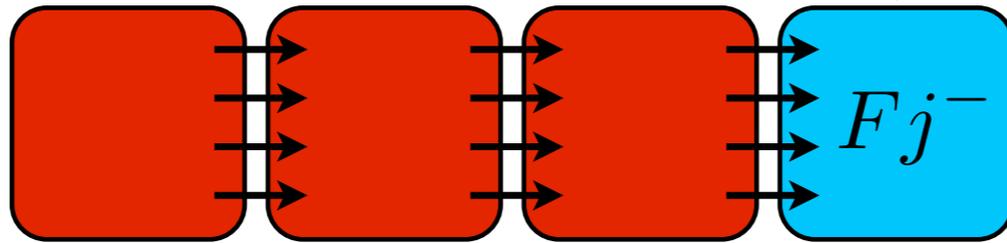
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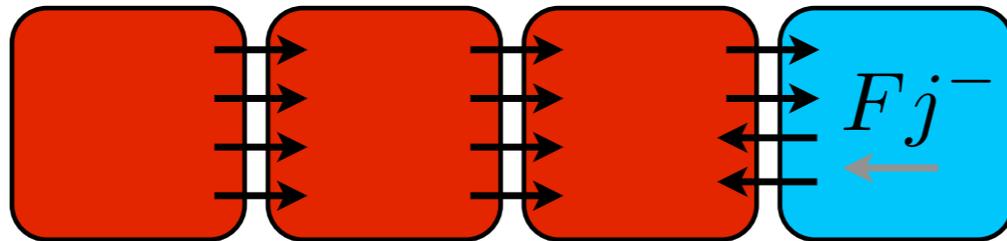


Local growth  
suppressing signal

*Four-  
jointed*

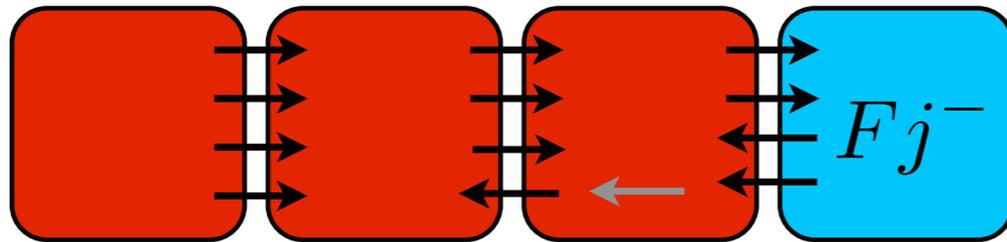


Growth suppression  
reduces levels of Four-  
jointed



Polarity  
reduction due to  
reduced Four-  
jointed levels

$\alpha$  &  $\beta$



Depolarization of  
neighboring interface

# A hypothesis

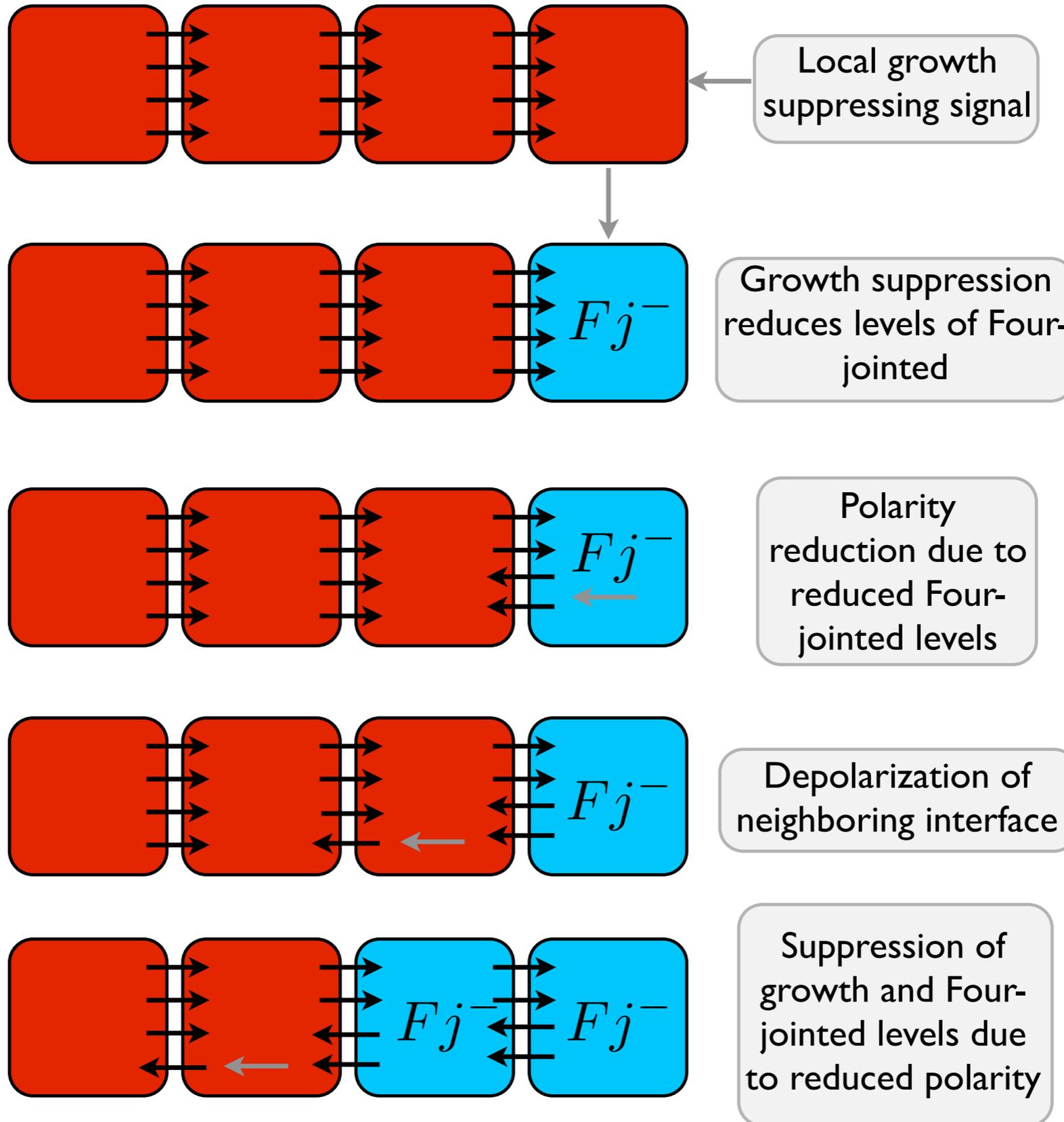
*Growth*



*Four-jointed*



$\alpha$  &  $\beta$



Thank you for listening

Thank you for listening

Thank you to Sid, Ken and Boris

# Thank you for listening

## Thank you to Sid, Ken and Boris

This fruit fly  
It reveals to us  
How nature designs  
Amidst all the fuss

We relentlessly study  
With all our might  
The order and beauty  
Of your bristles and stripes

When will you ever reveal  
Your singular principles  
Or will we forever feel  
Ensnared with something mystical