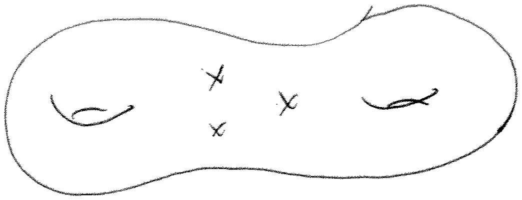
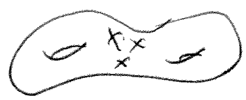
12 August 2010
Y. Tachikawa

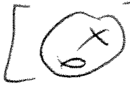
The work of Argyres and Seiberg, con.

6d theory of type $\mathfrak{g}$



$\phi_i : SU(2) \to \mathfrak{g}$. (punctures)

$SCFT_{\mathfrak{g}}$ [] s.t.

$SCFT_{\mathfrak{g}}$ [ ] = $SCFT_{\mathfrak{g}}$ []  +
$SCFT_{\mathfrak{g}}$ []

$SCFT_{\mathfrak{g}}$ [ , ] / G with coupling τ

special puncture

= $SCFT_{\mathfrak{g}}$ []

length $q = e^{2\pi i \tau}$

- n_v, n_h : known, but complicated
- Flavor symmetry $\phi_i : SU(2) \to \mathfrak{g}$ from punctures

Flavor symmetry $\supseteq \bigtimes_i$ { commutant of $\phi_i(SU(2))$ in $\mathfrak{g}$ }

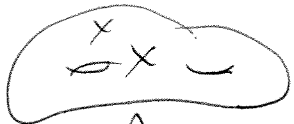
special puncture for gluing: ϕ_i is trivial

$g = SU(N)$

$\varphi_i : SU(2) \to SU(N)$

partition of N $\leftarrow$ N

Coulomb branch

Hitchin eq. on 

A_M

$$\Phi(z) \sim \frac{{}^t\varphi_i(\sigma_1) + \sqrt{-1}\,{}^t\varphi_i(\sigma_2)}{z - z_i}\, dz_i$$

$\sigma_1, \sigma_2, \sigma_3$ generate $SU(2)$ Lie algebra

($SU(N)$ Hitchin eq.)

There is a moduli space $\mathcal{M}$ of sols with that boundary condition.

Coulomb branch = base of the Hitchin fibration. $\mathcal{M} \downarrow B$

(This is the associated algebraic integrable system)

$\tilde{\rho}/G$

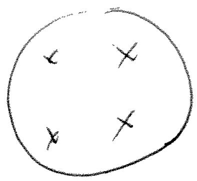
$\mathfrak{g} = SU(2)$.

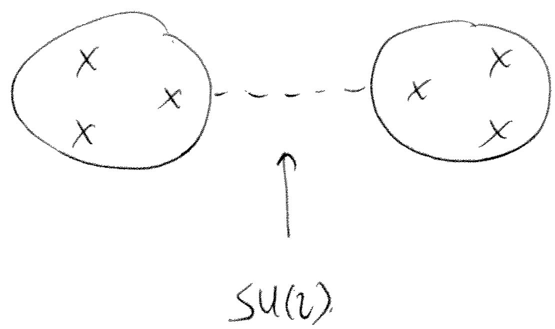
$\varphi : SU(2) \to SU(2)$ — transpose

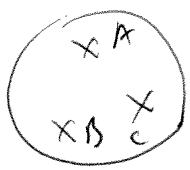
 $\mathbf{1} + \mathbf{1} \leftarrow \mathbf{2}$ 

 ($\mathbf{2} \leftarrow \mathbf{2}$ no singularity) 

So only one type of actual puncture.







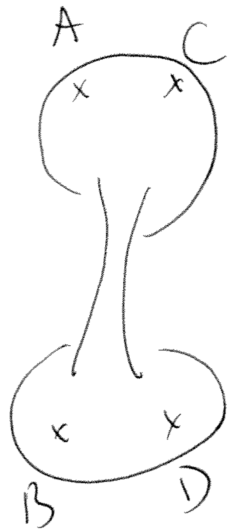
flavor sym. $SU(2)_A \times SU(2)_B \times SU(2)_C$

trivial thing from $\tilde{\rho} = \mathbf{2}_A \otimes \mathbf{2}_B \otimes \mathbf{2}_C$

$G = SU(2)$

$$\tilde{\rho} = \mathbf{2} \otimes \mathbf{2}_A \otimes \mathbf{2}_B \oplus \mathbf{2} \otimes \mathbf{2}_C \otimes \mathbf{2}_D = \mathbf{2} \otimes_{\mathbb{R}} \mathbb{R}^8_V$$

$SU(2) \times SO(8)$ sym. of which only $SO(4) \times SO(4)$ is manifest



$G = SU(2)$

$\tilde{\rho} = \mathcal{D} \otimes \mathcal{D}_A \otimes \mathcal{D}_C$

$\oplus \mathcal{D} \otimes \mathcal{D}_B \otimes \mathcal{D}_D$

$= \mathcal{D} \otimes_{\mathbb{R}} \mathbb{R}^8_s$ (triality)

$g = SU(3)$

$\varphi : SU(2) \to SU(3)$

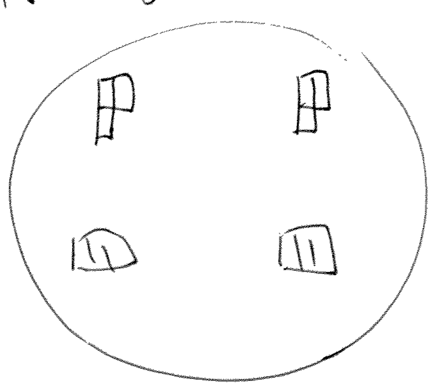
Flavor

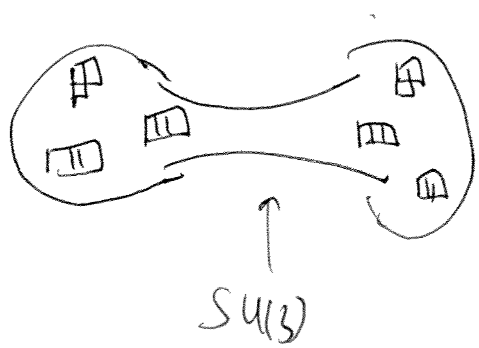
$SU(3) \leftarrow$ $\quad 1+1+1 \leftarrow 3$

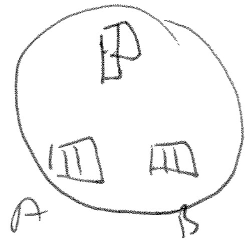
$U(1) \leftarrow$ $\quad 2+1$

$(\; 3 \quad)$ trivial

Two degenerations:



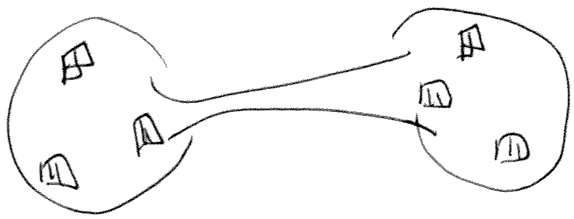




$\tilde{\rho} = 3_A \otimes \bar{3}_B \oplus \bar{3}_A \otimes 3_B$

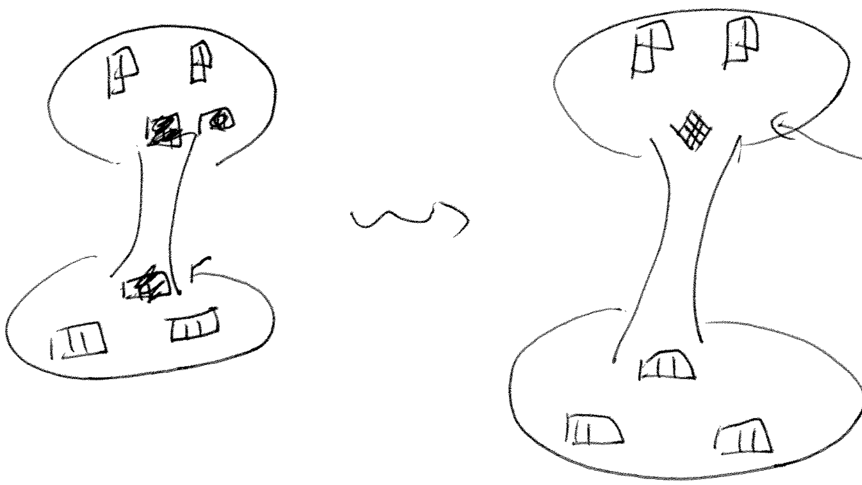
$+1 \qquad -1 \qquad$ U(1) charge

$SU(3)_A \times SU(3)_B \times U(1)$



gets $SU(3)$ with $\tilde{\rho} = \bar{3}\otimes\bar{6} \oplus \bar{3}\otimes 6$

$SU(3)\times U(6)$



$\tilde{\rho} = \mathbf{2}\otimes_{\mathbb{R}}\mathbb{R}^2$

$SU(2)\times SO(2)$

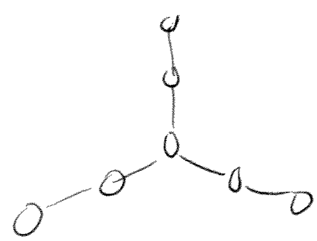
special puncture

$\Downarrow$

$SU(2)$ gauge group

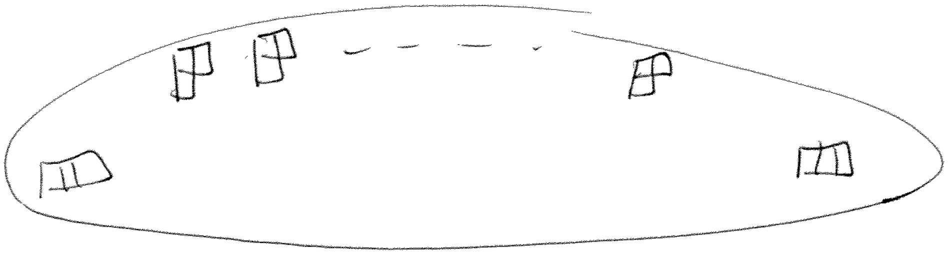
$\uparrow$ $SU(3)^3$ symmetry

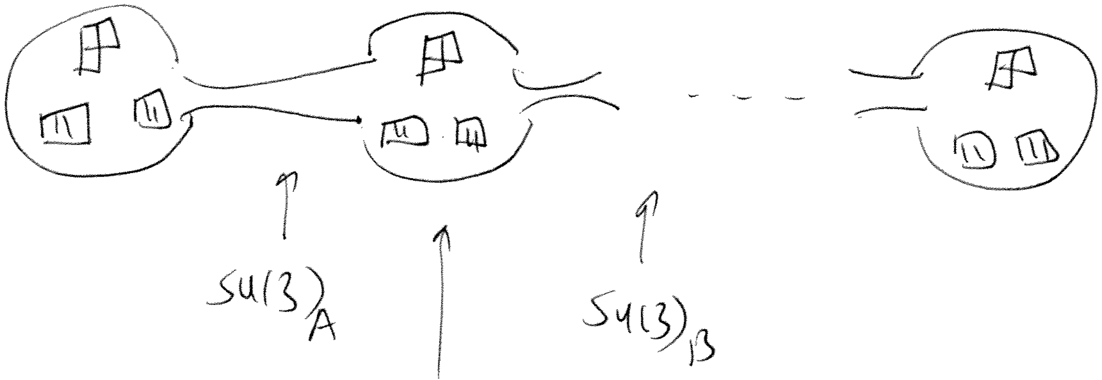
$MN(E_6)$



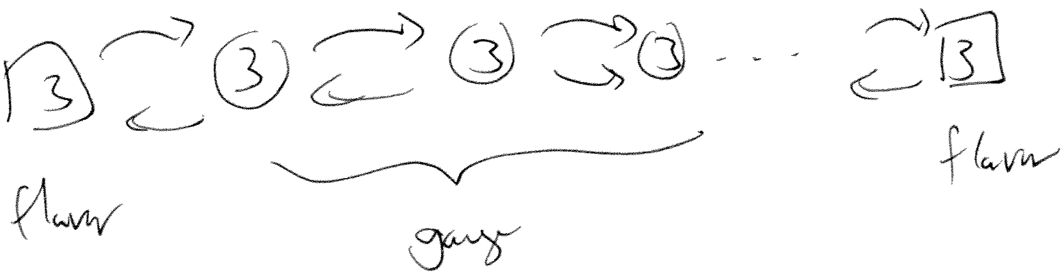
$SU(3)^3 \subseteq E_6$

Consider

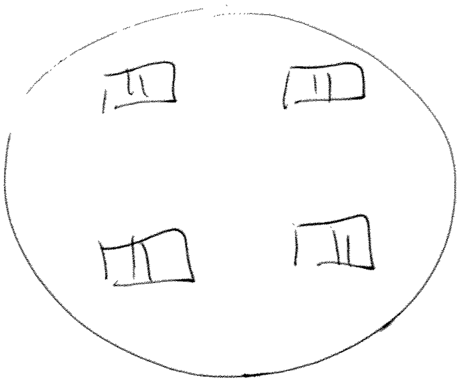


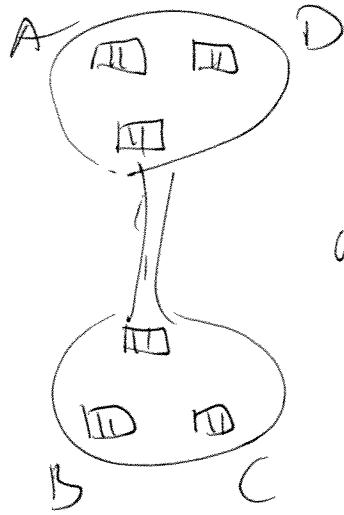


$$\tilde{\rho} = 3_A \otimes \bar{3}_B \oplus \bar{3}_A \oplus 3_B$$

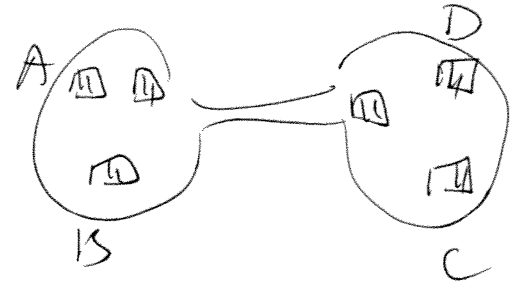


Higgs branch is almost the standard quiver variety





or



These theories are equivalent, so these should agree.

$$\mathcal{N}_{min}(E_6)\times\mathcal{N}_{min}(E_6)///SU(3)$$

using specific $SU(3)$'s on right/left

$$SU(3)\times SU(3)\times \underline{SU(3)} \qquad \underline{SU(3)}\times SU(3)\times SU(3)$$

diagonal

$SU(3)^4$.

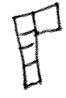
$\Rightarrow$ all $SU(3)$ trinion actions are the same.

$SO(N)$

$1+1+\cdots+1 \leftarrow N$

$(N-1)+1 \leftarrow N$

$(N \leftarrow N)$ trivial

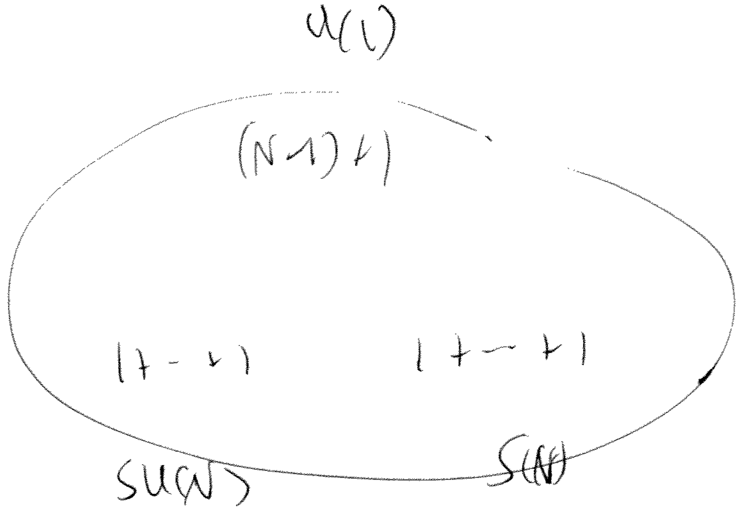


Toda

general monodromy

semi degenerate,

fund. wt.



$N\times\bar{N} + \bar{N}\times N$