27 August 2010

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Gauge theory loop operators and Liouville theory

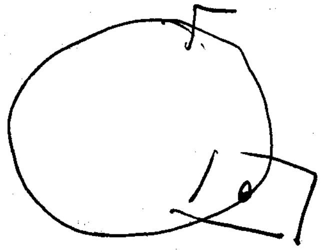
- Introduction
- Gauge theory loop operators — with Drukker, Morrison
- Liouville loop operators — with Drukker, Gomis, Teschner; also: AGGTV
- Localization for the 't Hooft loop — with Gomis, Pestun

4D G, $A = A_\mu dx^\mu$ (matter fields)

$$\int \mathcal{D}A \; e^{-S[A]} \qquad S \sim F^2$$

$$W \sim \mathrm{Tr}\, P \exp \oint_{\substack{\text{circle} \\ \subset \text{spacetime}}} A$$

$$\langle T \rangle \equiv \int_{\substack{\text{boundary} \\ \text{condition}}} \mathcal{D}A \; e^{-S[A]}$$


S^1

$\| \times \mathbb{R}^3$

$\mathbb{R}^3$

$\bullet \quad \vec{B} = \frac{\mu}{r^2}\hat{r}$

$B_i = \frac{1}{2}\varepsilon_{ijk}F_{jk}$

$F = dA + A \wedge A$

$\mu \in Lie(G)$

"order parameters"

$\langle W \rangle \sim e^{-\#\text{Area}}$

$\langle T \rangle \sim e^{-\#\text{Area}}$

Supersymmetric loops in $N=2$ gauge theories

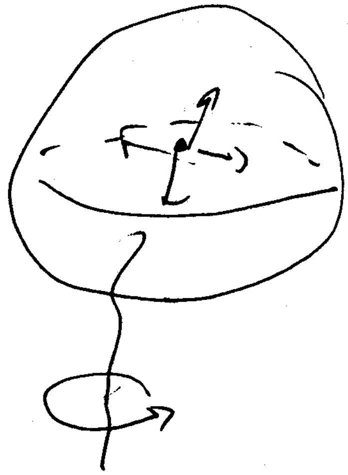
$W_{rep} = Tr_{rep} P e^{\oint A + i\phi ds}$

G

rep $\longleftrightarrow q \in \Lambda_{weight}$

't Hooft loop, $\mu \in$ Cartan.

Dirac quantization



$S^2 \subset \mathbb{R}^3$

$$A = \frac{\mu}{2}(1-\cos\theta)\, d\varphi$$

$$(\text{field})_R \to \underbrace{e^{2\pi i R(\mu)}}_{\underline{1}} (\text{field})_R$$

$\langle \mu, \text{weight of } R \rangle \in \mathbb{Z}.$

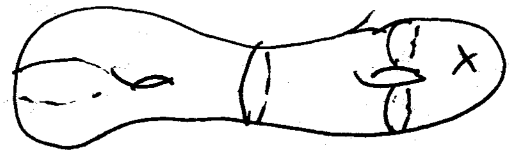
$\Lambda_{\text{cochar}} \subset \text{Cartan}$

$$\frac{\Lambda_{\text{weight}}(G) \times \Lambda_{\text{cochar}}(G)}{\text{Weyl}}$$

$\Updownarrow$ 1-1 corr.

$\{$loop operators of the gauge theory$\}$

2 M5 branes on $C_{g,n}$



Pants decomp $\leftrightarrow$ S-duality frame

$3g-3+n$ pants curves $\leftrightarrow$ $G = SU(2)_1 \times \cdots \times SU(2)_{3g-3+n}$

$2g-2+n$ pairs of pants $\leftrightarrow$ hypermultiplets, scalars ϕ_{IJK}

n punctures $\leftrightarrow$ $SU(2)$ flavor

$$\Lambda_{weight} = \mathbb{Z}^{3g-3+n}$$

$$\Lambda_{cochar} = \{ \vec{p} \in \mathbb{Z}^{3g-3+n} \mid p_i + p_j + p_k \in 2\mathbb{Z} \}$$

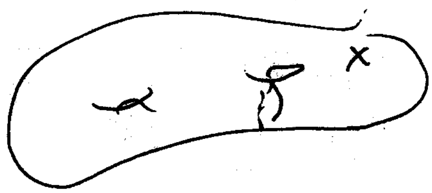
if i, j, k corresp to a pair of pants.

$$\text{Weyl} = \mathbb{Z}_2^{3g-3+n}$$

$$(p_i, q_x) \to (-p_i, -q_x).$$

Liouville theory

$$S=\int (\partial\phi)^2+\mu e^{2b\phi}$$



$V_m \sim e^{2m\phi}$

$C_{g,n}$

Verlinde,

Classification: non-self-intersecting closed closed curves

<u>Dehn's thm</u>

Closed curves are labeled by $(p_i, q_i)_{i=1}^{3g-3+n}$ with same

p_i = # intersection with the i^{th} pants curve

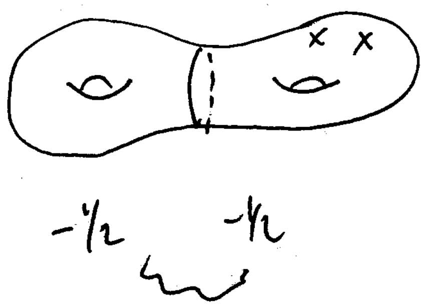
q_i = winding #

$\{$ gauge loops $\} \longleftrightarrow \{$ Liouville loop operators $\}$

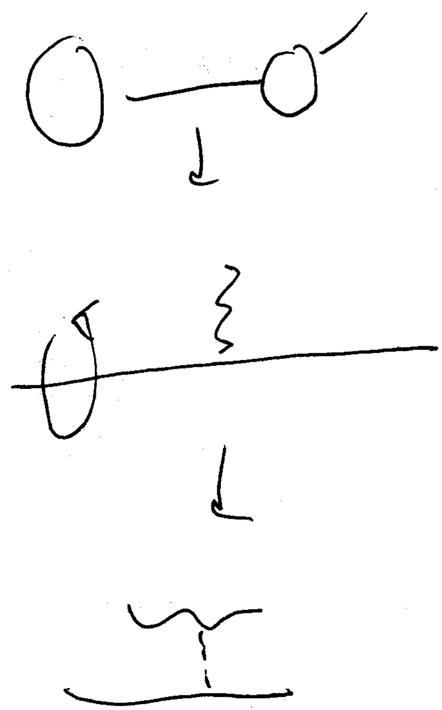
Quantitative

Def by <u>Verlinde</u>

Can compute!



$$V_{-3/2} = e^{-3\phi}$$
$$C_{g,n}$$

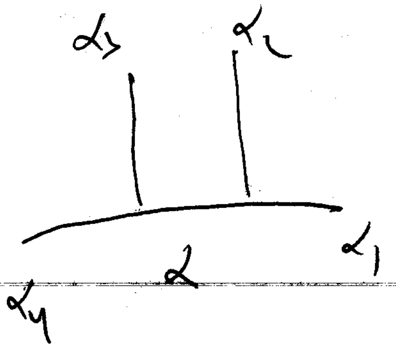


$$V_\phi = e^{2b\phi}$$

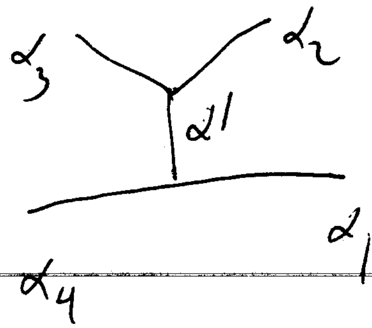
conf $\rightsquigarrow$ $\cos 2\pi a$ - conf

$$\alpha = \frac{Q}{2} + a, \quad Q = b + \frac{1}{b}$$

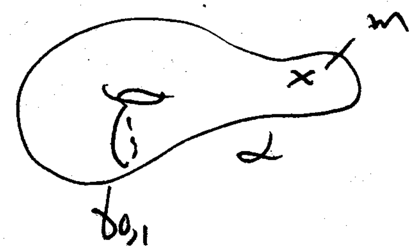
conf. block $\rightsquigarrow$ Nar conf. block



$\rightsquigarrow$



$\mathcal{N}=2^*$ theory : mass deformation of $\mathcal{N}=4$



$C_{1,1}$

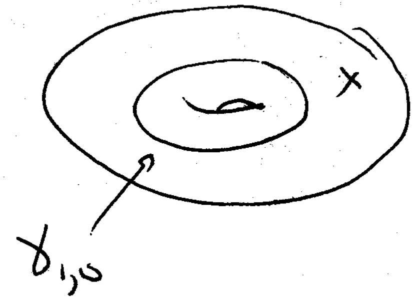
$\mathcal{F}_\alpha^m$

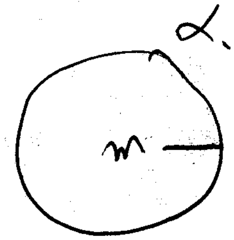
$$\left[\mathcal{L}(\gamma_{0,1}) \cdot \mathcal{F} \right]_\alpha^m$$

$$= \cos(2\pi a)\, \mathcal{F}_a^m$$

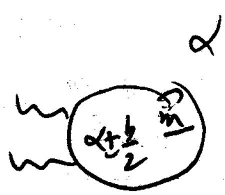
Wilson ?

'tHooft $\gamma_{1,0}$ $\hat{p}$ $\ddot{q}$

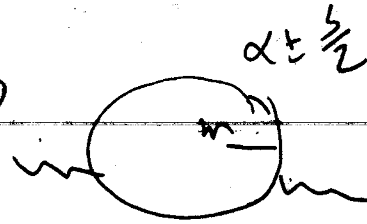




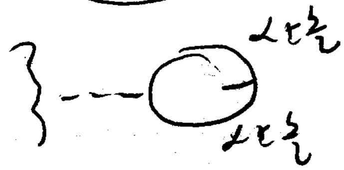
$\Longrightarrow$



$\Longrightarrow$



$\Longrightarrow$



$$[\mathcal{L}_{1,0}\cdot \mathcal{F}]^m_\alpha$$

$$= H_+(\alpha)\mathcal{F}^m_{\alpha+\frac{b}{2}} + H_-(\alpha)\mathcal{F}^m_{\alpha-\frac{b}{2}}$$

$$H_\pm(\alpha) =$$

<u>Remark</u> $b=1 \to S^4$

<u>$\mathcal{N}=4$ limit</u>

$b=1,\ b^2=\frac{\varepsilon_1}{\varepsilon_2},\qquad \varepsilon_1=\varepsilon_2.$

$$m = \frac{1}{\text{radius of } S^4}$$

$\Rightarrow$ enhanced SUSY $\mathcal{N}=4$.

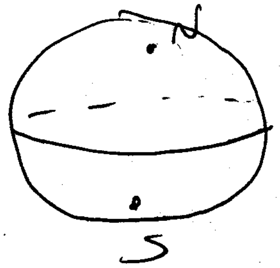
$$\langle W\rangle_2^{\mathcal{N}=4} = \langle T\rangle_{-1/2}^{\mathcal{N}=4}$$

$$[\mathcal{L}_{1,0}\cdot \mathcal{F}]^m_\alpha.$$

$$[(\mathcal{L}_{1,0})^p \mathcal{F}]^m_\alpha$$

Localization

$\mathcal{N}=2$ on S^4



S^4

$S^1 \subset S^3$

$$e^{-S_{\mathcal{N}=2} - tQV}$$

$Q\cdot V = 0 \iff Q$-invariance

$\Rightarrow$ Path integral $\rightsquigarrow$ finite dim'l integral.

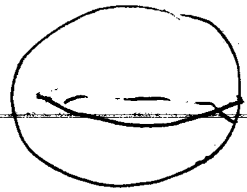
$$\int da\, e^{\text{gaussian}}\, Z_{1\text{-loop}}\, |Z_{\text{Nek}}|^2 \times \mathrm{Tr}\, e^{2\pi a}$$

Q-inv. $\Rightarrow F=0$ except at N-, S- Poles

$QV \Rightarrow$ Donaldson-twisted action at N-Pole.

$\Rightarrow$ Anti-Donaldson twist at S-Pole.

Ingredient: Equivariant cohomology, $\mathcal{M}_{\text{inst}}$.



S^1 't Hooft loop.

$$F = \frac{M}{2}\, \mathrm{vol}_{S^2}$$

Q-inv $\Rightarrow$ Bogomolny eqn.

Ingredient: $M_{\text{monopoles}}$ > equivariant cohomology

Index $\rightsquigarrow$ $Z_{1\text{-loop}}$

Sum over solns