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Omega Backgrounds, con.

$\mathcal{N}=2$ vector multiplet

		$SU(2)_L \times SU(2)_R \times SU(2)_I$ ← R-symmetry
Ω^1	A_μ	$(\frac{1}{2}, \frac{1}{2}, 0)$
	φ	$(0, 0, 0)$
S^-	ψ_L	$(\frac{1}{2}, 0, \frac{1}{2})$
S^+	ψ_R	$(0, \frac{1}{2}, \frac{1}{2})$

after twist

$$SU(2)_{R'} = \mathrm{diag}\,(SU(2)_R \times SU(2)_I)$$

	$SU(2)_L \times SU(2)_{R'}$		
A_μ	$(\frac{1}{2}, \frac{1}{2})$		
φ	$(0, 0)$		
ψ_L	$(\frac{1}{2}, \frac{1}{2})$	ψ_μ	$Q A_\mu = \psi_\mu$
ψ_R	$(0, 1)$	$\chi^+_{\mu\nu}$	
	$(0, 0)$	λ	

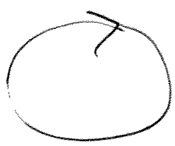
$$\Omega^{2+} \otimes \mathfrak{g}$$
$$\downarrow$$
$$\mathcal{A}$$

$\mathcal{N}=2$ untwisted on S^4

conformal

$S^4 \to \mathbb{R}^4_x \cup \{\infty\}$

$Q_\alpha,\ Q_{\dot\alpha}$	$\varepsilon(x) = \varepsilon_s + \Gamma^\mu x^\mu \varepsilon_c$
$S_\alpha,\ S_{\dot\alpha}$	$\nabla_\mu \varepsilon = \Gamma_\mu \tilde{\varepsilon}$
	$Q_\varepsilon = \xi_{s\alpha\dot\alpha} Q^{\alpha\dot\alpha} + \varepsilon_{c\alpha\dot\alpha} S^{\alpha\dot\alpha}$



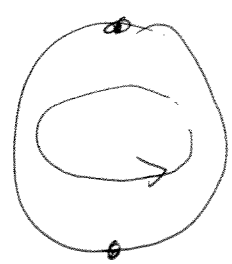
C

$$Q_\varepsilon\left(\left\langle \mathrm{tr}_R\, P\exp\left(\int A_\mu dx^\mu + i\varphi\, ds\right)\right\rangle\right) = 0$$

$$Q^2 = \mathcal{L}_v + R + \text{gauge}$$

$$v = \frac{\partial}{\partial\varphi_1}\,\frac{\partial}{\partial\varphi_2}$$

N



S

(The magnitude of the $\frac{\partial}{\partial\varphi_j}$ generators is the same)

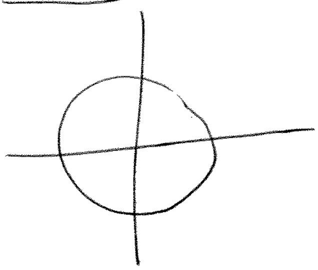
$\varepsilon_1 = \varepsilon_2 = 1/R$, R = radius of S^4.

Claim $\left(S_{phys}(S^4), Q^{conf}\right)$ near $x = 0$ (x coord on $\mathbb{R}^4$) (stereographic projection)

as $x \to 0$

$\cong S_{twisted}\left(\mathbb{R}^4_{\varepsilon} \text{ near } x=0, Q_{\varepsilon_1 = \varepsilon_2 = 1/R}\right)$

level of approximation:



$\sqrt{1-x^2} \approx 1 - \frac{x^2}{2}$

$x \to \infty$, or South pole: $y = \frac{x}{|x|^2}$ coord

$y \to 0$, get the conjugate of the twisted theory.

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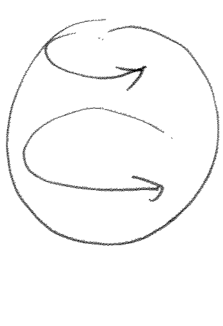
top. twisted $W=2$ $F^+ = 0$

anti-top twisted $W=2$ $F^- = 0$

physical on S^4: interpolates between



$U(1)$ action on S^4, tangent to $V = \frac{\partial}{\partial \varphi_1} \frac{\partial}{\partial \varphi_2}$

(the vector field to which Q squares)

$Q^2 = \mathcal{L}_V$

0 denotes direction of V

$\mathbb{R}^3$ 1, 2, 3

$$F_{0i} = E_i$$

$$\frac{1}{2} F_{ij} \epsilon_{ijk} = B_k$$

$F^+ = 0$ means: $E_i = -B_i$

$F^- = 0$ means: $E_i = +B_i$

eqn depends on θ, angle from $\theta = \frac{\pi}{2}$ at N to $\theta = -\frac{\pi}{2}$ at S.

eqn: $\boxed{B_i = -E_i \cos\theta}$

$$F^{+2} = F^+_{\mu\nu} H^+_{\mu\nu} + H^{+2}_{\mu\nu}$$

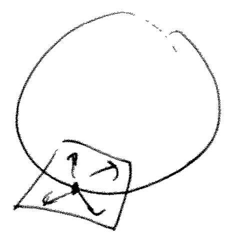
$$Q\,\psi_\mu = D_\mu \phi$$

$$Q(\psi_\mu, \overline{D_\mu \phi})$$

$\boxed{F^+}$

$$\Omega^0 \xrightarrow{\nabla_A^+} \Omega^1 \otimes \mathrm{ad}\,g \xrightarrow{\nabla_A^+} \Omega^{2+} \otimes \mathrm{ad}\,g$$

$G \circlearrowright M$



$g \in G. \quad g(x)$

transversal elliptic if the symbol is elliptic in normal direction

M/G gets an elliptic operator.

$$\text{"} \; Z_{pert}(a) = \left| Z_{pert,\, twisted\ \varepsilon_1 = \varepsilon_2 = 1/R}(a) \right|^2 \; \text{"}$$

$$Z_{twisted} = \int_{\substack{\varphi(x) = a \\ \text{as } x \to \infty}} D\varphi \, e^{S}$$

In fact,

$$Z_{pert} = \int \left| Z_{pert,\, twisted,\, \varepsilon_1 = \varepsilon_2 = 1/R}(a) \right|^2 \Delta(a)^2 e^{-a^2}$$

$$\int_M \alpha = \int \frac{i_F \alpha}{e(N_F)}$$

$$\langle W_R(C)\rangle = \int \left|Z_{\varepsilon_1=\varepsilon_2=1/r}(a)\right|^2 \Delta(a)^2 e^{-a^2}\, \mathrm{tr}_R\, e^{a} \Big/ Z_{pert}$$

Wilson loop on the equator.

$$\partial\varphi\,\partial\varphi + \frac{R}{6}\varphi^2 \qquad (R = \text{scalar curvature} > 0)$$

Recall

$$Z^{vector}_{pert} = \prod_{\alpha\in \text{roots}} \Gamma^{-1}_{\varepsilon_1=\varepsilon_2=1/r}(\alpha(a))$$

Z^{matter}_{pert} = slightly shifted version

$$Z^{hyper}_{pert} = \prod_{w\in R} \Gamma_{\varepsilon_1=\varepsilon_2=1/r}(w(a)+m)$$

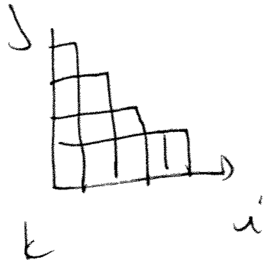
mass of hyper.

Twisted in Ω background

$$\sum_{k=0}^{\infty} q^k \int_{\mathcal{A}/\mathcal{G}_\infty} eu(\Omega^{2+}\otimes \mathfrak{g}) = \sum_{k=0}^{\infty}\Big(Z_{pert}\int_{\substack{\{F^+=0\}\\ \mathcal{M}_k}} 1\Big) q^k$$

(x,y) $\mathbb{C}^2$

$x^i y^j$



N colors $\quad \alpha = 1, \ldots, N$ partitions

$$k = k_1 + \cdots + k_N$$

$$\sum_k q^k = \sum_{|\vec{Y}|=k} (\qquad)$$

$$\langle W_R(C) \rangle_{S^4} = \int da \, \Delta(a)^2 \left| Z_{\text{twisted}}\left(\epsilon_1 = \tfrac{1}{R} = \epsilon_2, a\right) \right|^2 \operatorname{tr}_R e^a$$

$$Z_{\text{twisted}} = e^{-a^2/2} Z_{\text{pert}} Z_{\text{inst}}$$

classical piece

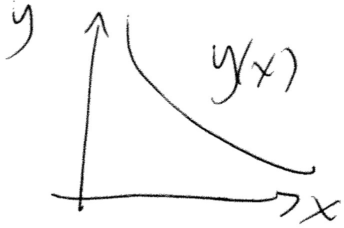
$$\lim_{\epsilon_1, \epsilon_2 \to 0} Z_{\text{twisted}}(\epsilon_1, \epsilon_2, a) = \exp\left(- \frac{F_{SW}(a)}{\epsilon_1 \epsilon_2} \right)$$

i.e. to compute $F_{SW}(a)$, take

$-\epsilon_1 \epsilon_2 \log Z_{\text{twisted}}$, expand in ϵ_1, ϵ_2.

Z_{twisted} is a sum over colored partitions

$k \to \infty$ There is a limit shape ; described by Seiberg-Witten curve



$y(x) = \text{SW's } y(x)$

$$\prod_{x=1}^{\infty} \frac{1}{(1-q^x)}$$

$$Z_{BH} = \int |Z_{\text{top strings}}|^2$$