10 August 2010
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1002.0385 (with K. Setter & K. Vyas)

1002 4241 (with K. Vyas)

1004.~~4~~2307

$\mathcal{N}=4$ $d=4$ SYM

$\downarrow$

4d TFT

$\downarrow$ compactify on C (Riemann surface)

2d TFT

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1. What if $g(C)=0$ or 1, or what do we get if we replace C by S^1?

2. How do we classify & describe properties of nonlocal operators? (line ops, surface ops)

Atiyah-Segal axioms for TFT in D dimensions

$M_D \rightsquigarrow Z(M) \in \mathbb{C}$

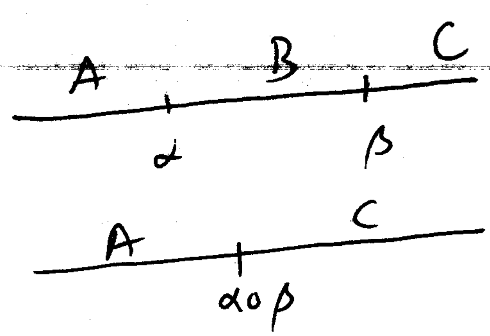
(compact oriented)

$M_{D-1} \rightsquigarrow \mathcal{H}(M_{D-1})$ ("Hilbert" space)

$M_{D-2} \rightsquigarrow ?$ category of branes in the effective 2d TFT on $\mathbb{R}^2 \times M_{D-2}$

Why do branes form a "category"?

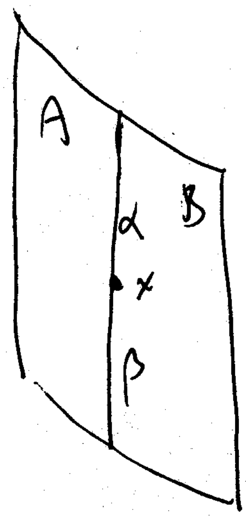
$$\begin{tikzcd} A \arrow[r, "\alpha"] \arrow[rr, bend right, "\alpha \circ \beta"'] & B \arrow[r, "\beta"] & C \end{tikzcd}$$



α, β

boundary-changing operators

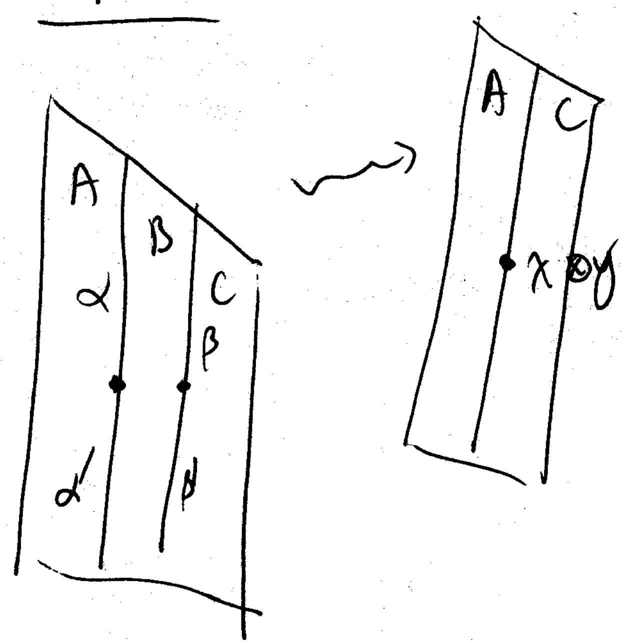
$M_{D-3} \rightsquigarrow$ "2-category" of boundary conditions
(get a 3d TFT)



$x \in V_{\alpha,\beta}$

A, B objects
α, β 1-morphisms
x 2-morphism

composition:



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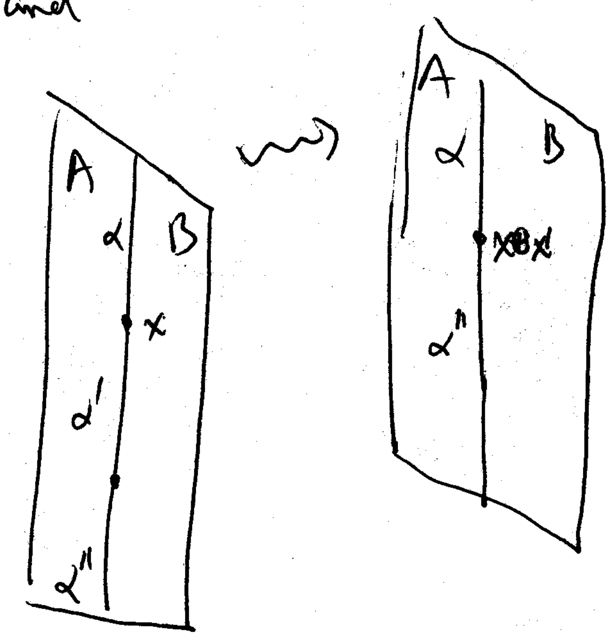
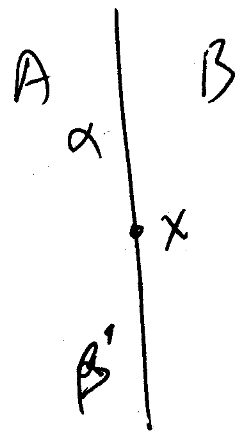
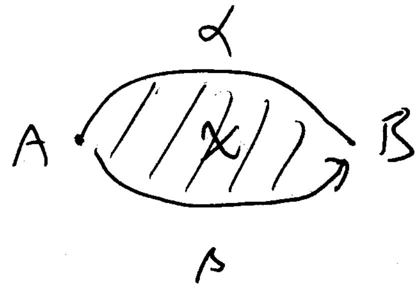
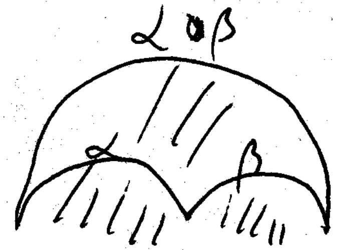






Composition:



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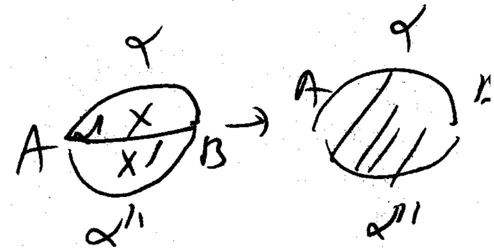


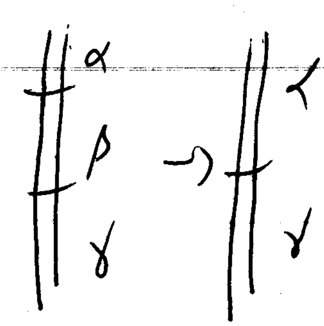
3d TFT

$S^1 \rightsquigarrow$ category of boundary conditions in the effective 2d TFT

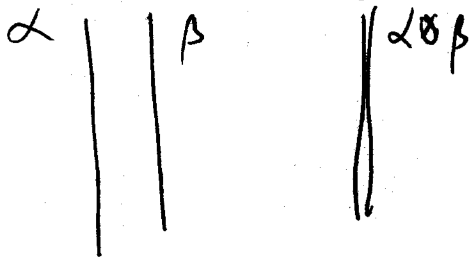
Another ~~[illegible]~~ definition: line operators in 3d TFT

$\mathbb{R}_t \times \mathbb{R}_+ \times S^1 \rightsquigarrow$ get a 2d TFT on $\mathbb{R}_+ \times \mathbb{R}$

(category of line operators)



Two defects can come together:



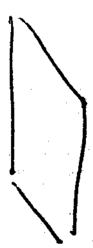
1) monoidal category
2) not quite commutative, but "braided" (Aharonov-Bohm effect)

3d TFT

- $\rightsquigarrow$ 2-category of boundary conditions.

4d TFT

$\mathbb{R}^2 \times \mathbb{R}_t \times S^1$



Surface ops $\longleftrightarrow$ boundary conditions on 3d TFT obtained by compactifying on S^1.

4d TFT on S^2

$\Downarrow$

2d TFT

$\mathbb{R} \times \mathbb{R}_t \times S^2$

4d topologically twisted $\mathcal{N}=4$ SYM

Extra parameter $t \in \mathbb{C} \cup \{\infty\}$

$$Q_t = Q_\ell + t\, Q_r$$

$t = \pm i$ is particularly simple $W_R = R(P\exp(\oint A + i\varphi))$

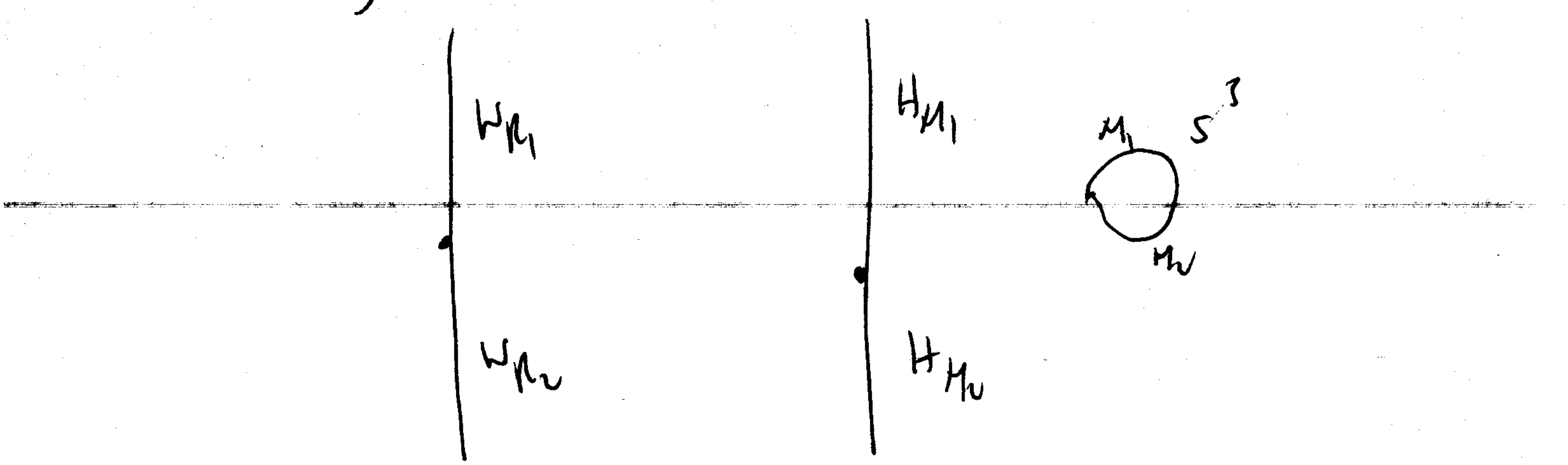
$$\delta A_\mu = \psi_\mu \quad \rightarrow \quad \delta(A_\mu + i\varphi_\mu) = 0$$
$$\delta \varphi_\mu = i\psi_\mu$$
$$\delta \sigma = 0 \qquad (\sigma \text{ is Lie algebra valued})$$

Reduce on S^2:

get a σ-model with target $\mathfrak{g}[2]$ — gauged — coordinate has R-charge 2

(topological B-model)

Boundary conditions: equivariant complexes of vector bundles on $\mathfrak{g}[2]$.



Surface ops at $t=i$

Reduce on S^1

Get a gauged topological σ-model in 3d (Rozansky-Witten model)

target: $T^*G_{\mathbb{C}}$. (Rozansky-Witten model)

(action by conjugation)

Take $G_{\mathbb{C}}$-equivariant holomorphic vector bundles on $G_{\mathbb{C}}$