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Omega Deformation vs. Quantization

Applications

① quantum integrable systems [Nekrasov-Shatashvili]
② Liouville theory, AGT
③ Chern-Simons

[Nekrasov-Witten]

[DGH]
Witten's talk

Get theory on:

2-manifold

① $C \times \mathbb{R}^2_\varepsilon \times \mathbb{R} \times S^1$

② $C \times \mathbb{R} \times S^3_{\varepsilon_1, \varepsilon_2}$

③ $S^1 \times \mathbb{R}^2_\varepsilon \times M_3$

3-manifold

Remark:

① ② $\Rightarrow$ 4d gauge theory with $\mathcal{N}=2$ SUSY

Pick 2. for now.

Liouville theory

$\mathcal{H}$ = Hilbert space

= quantization of Teichmüller space $\mathcal{T}$, ω

[GW]: to quantize (M, ω)

study A-model of (Y, Ω) = complexification of (M, ω)

w.r.t. symplectic form $\omega_Y = \operatorname{Im} \Omega$. $\mathcal{H} = \operatorname{Hom}(\mathcal{B}_{cc}, \mathcal{B}')$

$\mathcal{B}'$ = Lagrangian A-brane (for ω_Y)

Supported on $M \subset Y$

$\mathcal{B}_{cc}$ = coisotropic brane on all of Y $\qquad \omega = \operatorname{Re} \Omega|_M$.

w/ line bundle of curvature

$$F = \operatorname{Re} \Omega$$

$$(\omega_Y^{-1} F)^2 = -1$$

<u>Remark</u>

$F|_M = 0 \rightsquigarrow$ D-modules [KW]

$F|_M \neq 0 \rightsquigarrow$ quantization of M [GW]

$\mathrm{Hom}(\mathcal{B}_{cc}, \mathcal{B}')$

take $M = T$
$\hookrightarrow$ $\underline{Y} = M_H(G_{\cancel{c}} = SU(2), C) = \{(A,\phi) | \ldots\}/\sim$

cplx. str. $I, J, K = IJ$
$\omega_I, \omega_J, \omega_K$

$$\Omega_I = \int_C \mathrm{Tr}\, \delta A \wedge \delta\phi$$

[Hitchin]
* involution: $\phi \to -\phi$
  holo in cplx. str. I
  anti-holo in J, K.

* $T =$ ~~one~~ a component in the fixed point set of this involution

an example of (B, A, A) brane

$\omega_J |_{\text{f.p. set}} = 0$ $\quad$ $\omega_K |_{\text{f.p. set}} = 0$

$$\Omega_I = \omega_J + i\omega_K$$

Example $Y = \mathbb{C}_z \times \mathbb{C}_w = \mathbb{R}^4$

$$\Omega = dz \wedge dw = \omega_J + i\,\omega_K$$

$$\omega_I \sim dz \wedge d\bar{z} + dw \wedge d\bar{w}$$

$M = \{w = 0\}$

(B, A, A)

$\mathcal{H} = \mathrm{Hom}(\mathcal{B}_{cc}, \mathcal{B}')$

$\mathcal{B}' = (B, A, A)$ supported on T

$\mathcal{B}_{cc}$ = coisotropic (B, A, A) brane

w/ curvature $F = \omega_I$

$\mathcal{N} = 2$ gauge theory

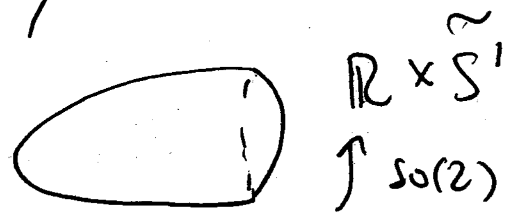
on $\mathbb{R}_{time} \times S^1 \times \mathbb{R}^2_\varepsilon$

$\circlearrowleft$

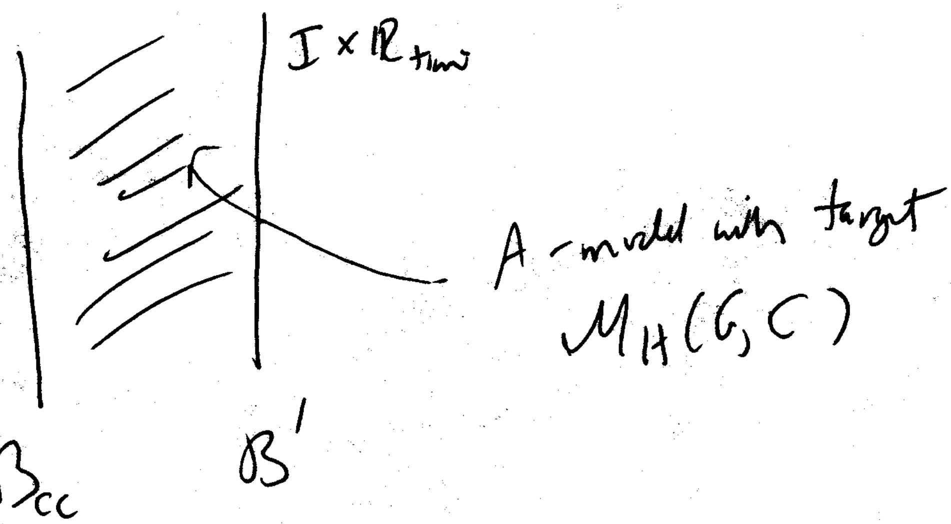
$SO(2)$

$\mathcal{H}$ = Hilbert space of states

i) Replace $\mathbb{R}^2_\varepsilon$ by

$D^2_\varepsilon =$ 

ii) compactify on $S^1 \times \tilde{S}^1$



$\mathcal{B}'$ = Lagrangian brane

$\mathcal{B}_{cc}$ = coisotropic brane supported on all of $\mathcal{M}_H(G,C)$.

(because condition on A or ϕ at the tip of $D_{\varepsilon=0}$)

$\uparrow$

brane of type (B,B,B)

iii) turning on ε gives $F = \varepsilon\,\omega_J$

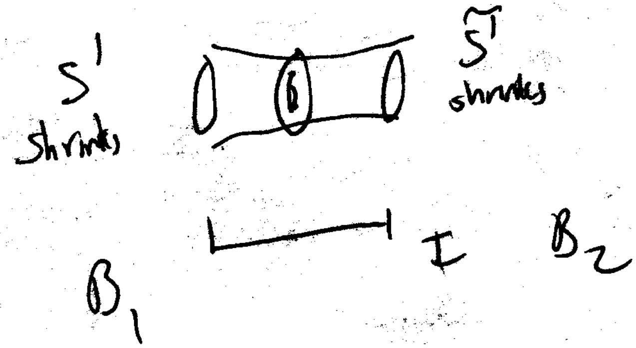
i.e. $\mathcal{B}_{cc}$ coisotropic of type (A,B,A)

Going back to 2

$$6d \text{ theory on } C \times \underbrace{\mathbb{R}_{time} \times S^3_{\varepsilon_1,\varepsilon_2}}_{\text{space time manifold}} \circlearrowleft SO(2)_1 \times SU(2)_2$$

space time manifold, $\mathcal{N}=2$ 4d theory

$$S^3 = \begin{array}{c} S^1 \times \tilde{S}^1 \\ \downarrow \\ I \end{array}$$



$$\left.\begin{array}{l} \mathcal{B}_2 = \text{Coisotropic } (A,B,A) \text{ brane} \\ \mathcal{B}_1 = \text{Lagrangian } (A,A,B) \text{ brane} \end{array}\right\} \text{ on } \mathcal{M}_H(G,C)$$

S-duality

$\tilde{\mathcal{B}}_2 = \text{Lagrangian } (A,B,A)$ brane

$\tilde{\mathcal{B}}_1 = \text{Coisotropic } (A,A,B)$ brane

$$\mathcal{H} = \mathrm{Hom}(\mathcal{B}_1, \mathcal{B}_2) = \mathrm{Hom}(\tilde{\mathcal{B}}_1, \tilde{\mathcal{B}}_2)$$