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Overview of the new links between 4D and 2D theories

2D conformal field theory

- vertex operator algebras
- (spaces of) conformal blocks
- integrable systems

$\vdots$

4D gauge theory

- G = (compact) gauge group
- M_4 = 4-manifold
- PDE's for gauge connection A

  e.g. $F_A^+ = 0$

  $\mathcal{M}_{inst}^{c_2 = k} = \{ A \mid F_A^+ = 0 \} / \text{gauge}$

character of a 2D CFT $\leftarrow$ $Z = \sum_k q^k \chi(\mathcal{M}_{inst}^k (G, M_4))$

(mysterious) six-dimensional superconformal theory $\mathcal{T}$
(labeled by $G = A_N, D_N, E_N$)

on $M_4 \times C$

$\xrightarrow{Vol(C) \to 0}$ "effective" four-dimensional superconformal theory on M_4
$\hat{}$
gauge

gauge/matter determined by C

$\xrightarrow{Vol(M_4) \to 0}$ 2D CFT on C

CFT determined by M_4

Examples

① C.F. $M_4 = K3$ ⇝ "effective" 2D CFT
(single 5brane) on the worldsheet of the heterotic string

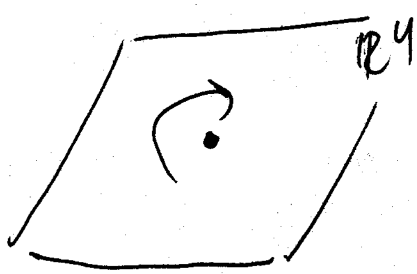
$$\Gamma = H^2_{\mathrm{free}}(M_4, \mathbb{Z}) = \Gamma_+ \oplus \Gamma_-$$

② $M_4 = S^4$ ⇝ (Alday Gaiotto Tachikawa)

$G = A_1$: Liouville theory ← correlation function

$G = A_n$: Toda CFTs conformal blocks

②' $M_4 = \mathbb{R}^4_{\epsilon_1, \epsilon_2}$ ($= \mathbb{R}^4$ equipped with $\underset{\subset SO(4)}{SO(2)_1 \times SO(2)_2}$ acting on $\mathbb{R}^4$)



"Ω-backgrounds"
(see Pestun's talk later this week)

Next week: we will have an intro to Liouville theory (Teschner)
and AGT (Tachikawa)

Vol(C) → 0 limit.

One finds $\mathcal{N}=2$ superconformal theory in 4d. (determined by C)
Maldacena-Nuñez + others (Gaiotto...)
(lecture tomorrow by Morrison)

Also this week: an intro to the duality group of the 4d theory (determined by C) Argyres–Seiberg, Gaiotto, ---

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- relation to Geometric Langlands / $\mathcal{N}=4$ Super-Yang-Mills on $\Sigma \times C$

- start in 6d; the mysterious theory can be put on

$\mathbb{R}^2_\varepsilon \times C \times T^2$

Omega-background

reduce on T^2: get $\mathcal{N}=4$ SYM on $\mathbb{R}^2_\varepsilon \times C$ with $\tau_{YM} = \tau(T^2)$

reduce on C: get the 4d theories above, on $\mathbb{R}^2_\varepsilon \times T^2$

Nekrasov–Witten

or better: $\mathbb{R}^4_{\varepsilon_1 \varepsilon_2} \times C$