24 August 2010
S. Cherkis

Bow diagrams and Yang-Mills instantons on Multi-Taub-NUT spaces

Motivation

- (0,2) super conf. theory
- geometric Langlands for surfaces
    - Nakajima
    - Braverman, Finkelberg
    - Witten
    - Tan
    - Dijkgraaf, Hollands, Sulkowski, Vafa

L^2 cohom. of mod.-sp. of instantons on k-degenerate mTN

= weight space of reps of $\widetilde{LG}$ at level k

= Gravitational instantons

ALE, ALF, ALG, ALH, compact

More natural to study instantons and geometry of these spaces

— R. Frost

G (0,2) theory on $\mathbb{R}\times S^1\times TN$

Stack of M5-branes

M theory $\mathbb{R}_t\times TN\times S^1\times\mathbb{R}^5$

M5's $S^1_M=S^1$ $\mathbb{R}_t\times TN\times S^1$

$S^1_M=S^1_{TN}$.

IIA on $\mathbb{R}_t\times TN\times\mathbb{R}^5$

D4's $\mathbb{R}_t\times TN$

IIA on $\mathbb{R}_t\times\mathbb{R}^3\times S^1\times\mathbb{R}^5$

D6 $\mathbb{R}_t\times S^1\times\mathbb{R}^5$

D4's $\mathbb{R}_t\times\mathbb{R}^3\times S^1$

Config. space of the QM problem

= space of BPS config

= $\mathcal{M}_{inst}(TN)$

Holom. WZW model on D4 & D6 intersection, and gauge theory on $\mathbb{R}_t\times\mathbb{R}^3\times S^1$

BPS states

L^2 cohomology of $\mathcal{M}_{inst}(TN_k)$

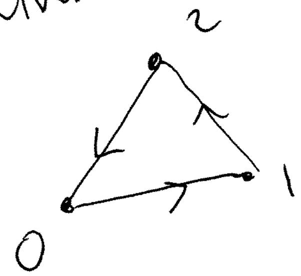
TN_k k degenerate = k D6-branes

Level k Kac-Moody module W_j from WZW on $\mathbb{R}_t\times S^1$

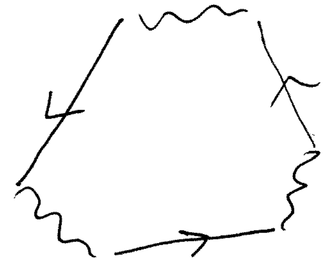
$$V=\bigoplus_{\substack{TN\\ \text{centers}\\ j}} W_j$$

Bows

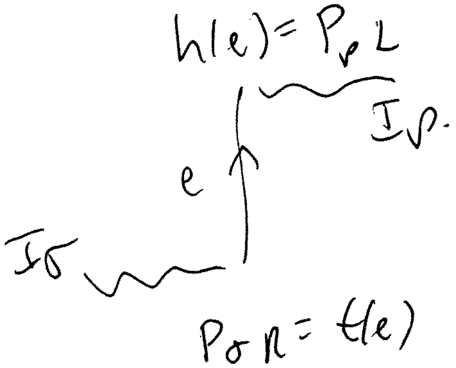
Quiver

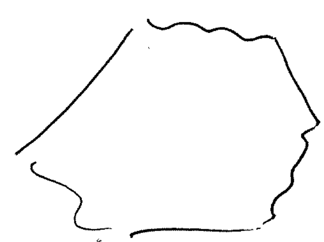


Bow



A **bow** is a collection $\mathcal{I} = \{ I_\sigma \}$ of oriented intervals $I_\sigma = [p_{\sigma L}, p_{\sigma R}]$ and edges $\mathcal{E} = \{ e \}$ with $t(e) = p_{\sigma R}$, $h(e) = p_{\rho L}$.





A representation of a Bow

- a collection of λ-points $\Lambda = \{ \lambda_\sigma^\alpha \}$ s.t. $\lambda_\sigma^\alpha \in I_\sigma$, $\alpha = 1, \dots, r_\sigma$. Splitting I_σ into I_σ^β, $\beta = 0, \dots, r_\sigma$
- a collection of Hermitian vector bundle $E \to \mathcal{I}$

$$E_\sigma^\beta \to I_\sigma^\beta$$

- $R_\sigma^\beta = \mathrm{rk}\, E_\sigma^\beta$

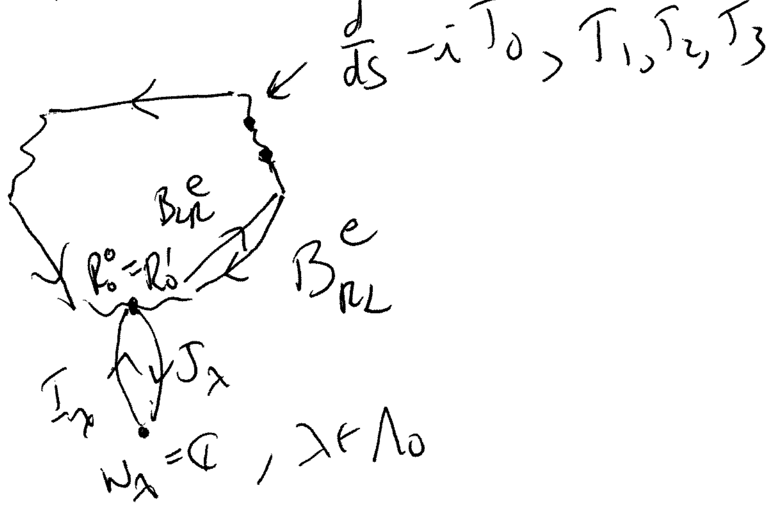
Let $\Lambda_0 \subset \Lambda$

λ_σ^α for which $R_\sigma^{\alpha-1} = R_\sigma^\alpha$

- Herm. vector spaces $W_\lambda = \mathbb{C}$.
  for each $\lambda \in \Lambda_0$

Bow data for a bow rep.

$\mathcal{R} = (\Lambda, \{R_\sigma^\beta\})$



$$\mathrm{Data}(\mathcal{R}) = \mathrm{Hom}(W, E_{\Lambda_0}) \oplus \mathrm{Hom}(E_{\Lambda_0}, W) \oplus \mathrm{Hom}^{\mathcal{E}}(E_R, E_L)$$
$$\oplus \mathrm{Hom}^{\mathcal{E}}(E_L, E_R)$$
$$\bigoplus_{\sigma \in I} \bigoplus_{\beta=0}^{r_\infty} \mathrm{Con}(E_\sigma^\beta) \oplus \mathrm{End}(E_\sigma^\beta) \oplus \mathrm{End}(E_\sigma^\beta) \oplus \mathrm{End}(E_\sigma^\beta)$$

Affine Hyperkähler space

Pick some 2-dim rep of quaternionic units.

$$e_j = -i\sigma_j \,, \quad e_0 = \mathbb{1} \quad \text{w/ rep space } S.$$

$$Q_\lambda = \begin{pmatrix} J_\lambda^\dagger \\ I_\lambda \end{pmatrix} \text{ for } \lambda \in \Lambda_0$$

$$B_e = \begin{pmatrix} (B_e^{RL})^\dagger \\ B_e^{LR} \end{pmatrix} : E_{h(e)} \to S \otimes E_{t(e)}$$

$$\mathcal{B}_e = \begin{pmatrix} (B_e^{LR})^\dagger \\ -B_e^{RL} \end{pmatrix} : E_{t(e)} \to S \otimes E_{h(e)}$$

$$\mathbb{T} = e_\mu \otimes T_\mu = \mathbb{1} \otimes T_0 + e_1 \otimes T_1 + e_2 \otimes T_2 + e_3 \otimes T_3$$

Triholomorphic action of the gauge ~~group~~ transf. on $\mathrm{Dat}(R)$

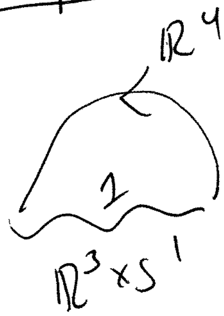
$$\mu(Q,B,T) = \mathrm{Im}\,(-i)\Big(i\frac{d}{ds}\mathbb{T}^* + \mathbb{T}\mathbb{T}^* + \sum_{\lambda\in\Lambda_0}\delta(s-\lambda)Q_\lambda Q_\lambda^\dagger$$
$$+ \sum_{e\in E}\delta(s-t_e)\,\mathcal{B}_e\mathcal{B}_e^\dagger + \delta(s-h_e)\,B_e B_e^\dagger\Big)$$

Choose $\mu(Q,B,T) = \sum_{e\in E}\big(\delta(s-t_e) - \delta(s-h_e)\big)\, \nu_e \otimes 1_E$

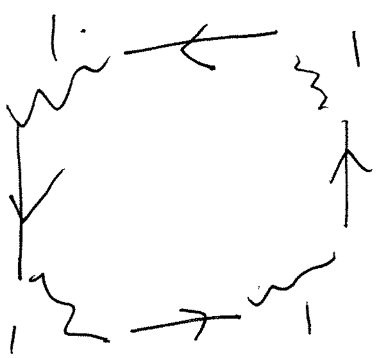
$$\sum_{e\in E} \nu_e = 0.$$

$$M_{\mathfrak{R}} = \mu^{-1}(\nu_e)/\mathcal{G} = \mathrm{Dat}(\mathfrak{R})/\!/\!/\mathcal{G}.$$

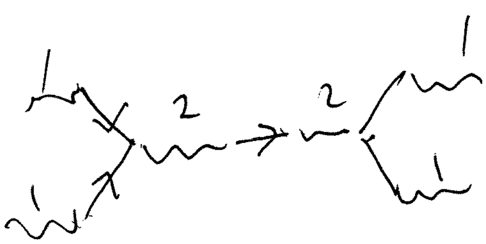
<u>Examples</u>



Taub-NUT
$\shortparallel$
A_0 ALF



A_{k-1} ALF
$\shortparallel$
TN_k

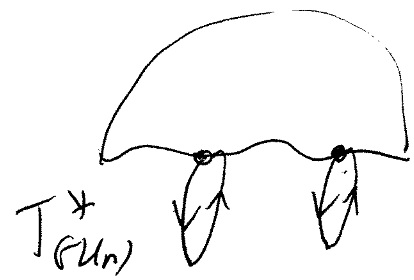


D_k ALF

What are Bows good for?

- Moduli spaces of instantons on ALF spaces are <u>isometric</u> to moduli space of a bow.

a) as <u>finite</u> HK quotients '08



b) explicit metrics

- Explicit solutions
  - 1 inst. on TN, no monopole charge '09.
  - 0 inst #, monopole number 1 '07

<u>Next</u>

- Instantons for other simple gauge groups (SO, Sp)

  (open problem for ALE)

- Instantons on ALG (Sliring)

4-dim Riemannian manifold W, $R = *R$.

Pick some point $x \in W$.

B_x^R be a ball of radius R in W.

Vol B_x^R as $R \to \infty$

ALE	R^4
ALF	R^3
ALG	R^2
ALH	$R^{h<2}$

Dirac operator (Weyl)

$\cancel{D_2} =$

$f \in \Gamma(S \otimes E)$

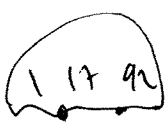
$$Df = \begin{pmatrix} \left(i\frac{d}{ds} + i T^*\right) f \\ Q_\lambda^\dagger f(\lambda) \\ B^\dagger f(p_{\sigma_1}) \\ B^\dagger f(p_{\sigma_2}) \end{pmatrix}$$

$$\operatorname{Im} D^\dagger D = M(Q, B, T)$$

Bow

large rep. $\mathcal{L}$

Small representation $\mathcal{S}$



E

e

Pick a solution (T, Q, B)

$(t, b) \in TN$

$D^{T,Q,B}$

$D^{t,b}$

$$D_t = D^{T,Q,B} \otimes 1_e + 1_E \otimes D^{t,b}$$

$$\operatorname{Ker} D_t^{+}$$

$$\downarrow$$

$\mathcal{M}(\delta)$ (= Taub-NUT)

Bow Solutions $\xrightarrow[\text{transform}]{\text{Nahm}}$ Inst.

inverse Nahm.