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Nonequilibrium Statistical mechanics of Exciton-Polariton systems

Motivation: Equilibrium vs. Non-equilibrium

- Equilibrium paradigm:

low T ——————— x ——————— high T

* correlations

$\langle \psi^*(r)\,\psi(r)\rangle \sim r^{-\alpha}$ $\qquad\qquad$ $e^{-r/\xi}$

* responses: superfluidity

$\rho_s \neq 0$ $\qquad\qquad$ $\rho_s = 0$

* KT transition: unbinding of vortex-antivortex pairs

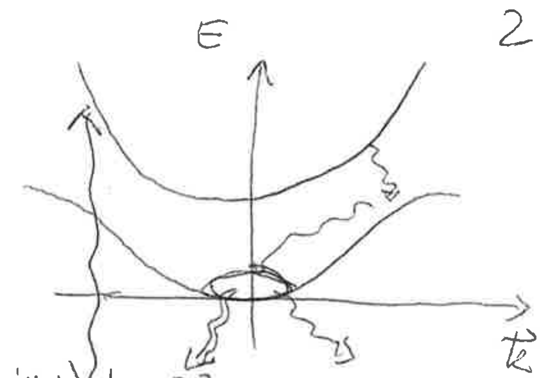
- we will address these questions in ~~an~~ driven out-of-eq. context.

Outline

XP systems
how do they break eq. conditions

KPZ equation
surface roughening

implications for:
* correlations
* superfluidity
* KT transition



XP systems

- stochastic GPE for lower Polariton

$$i\partial_t \psi = \Big[-(K_c - iK_d)\nabla^2 + r_c - i r_d + (u_c - i u_d)|\psi|^2 \Big]\psi + \xi$$

K_c: $\frac{1}{2m_{eff}}$; K_d: diffusion $K_d \ll K_c$; r_c: chem. pot. ; r_d: $\gamma - P$ (loss, pump) ; u_c: elastic coll ; u_d: pump saturation / two body loss

noise $\langle \xi(t,\vec{x}) \rangle = 0 \quad \langle \xi^*(t,\vec{x})\xi(t',\vec{x}') \rangle = 2\sigma\,\delta(t-t')\delta(\vec{x}-\vec{x}')$

$$= -i\frac{\delta H_d}{\delta\psi^*} - \frac{\delta H_c}{\delta\psi^*} + \xi$$

with $H_\alpha = \int_{\vec{x}} \Big[r_\alpha |\psi|^2 + K_\alpha (\vec{\nabla}\psi)^2 + \frac{u_\alpha}{2}|\psi|^4 \Big] \quad \alpha = c, d$

→ coherent and driven-dissip. dynamics on an equal footing

- key ingredients:

* $U(1)$ phase rotation symmetry $\quad \psi(t,\vec{x}) \to e^{i\theta}\psi(t,\vec{x})\,, \ \xi(t,\vec{x}) \to e^{i\theta}\xi(t,\vec{x})$

no coherent forcing

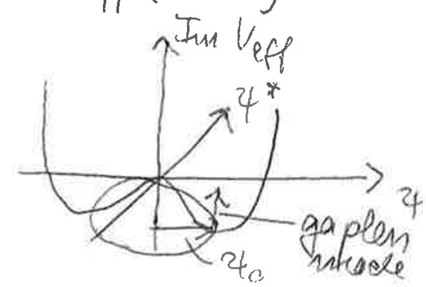
implication: existence of Goldstone mode

→ seen in Mean field theory

$$0 \overset{!}{=} \Big[-(r_d + u_d|\psi_0|^2) + i(r_c + u_c|\psi_0|^2) \Big]\psi_0 \equiv V_{eff}'(|\psi_0|^2)$$

⇒ Im part: $\quad |\psi_0|^2 = -\frac{r_d}{u_d} > 0 \quad$ fixes state

Re part: $\quad r_c = -u_c|\psi_0|^2 \quad$ choice of rot frame



* open system / no particle number conservation

<u>implication</u> : Goldstone mode is diffusive at leading order

$$\omega = i D \vec{q}^{\,2}$$

vs. closed system :

$$\omega = c|\vec{q}| \quad \text{sound mode}$$

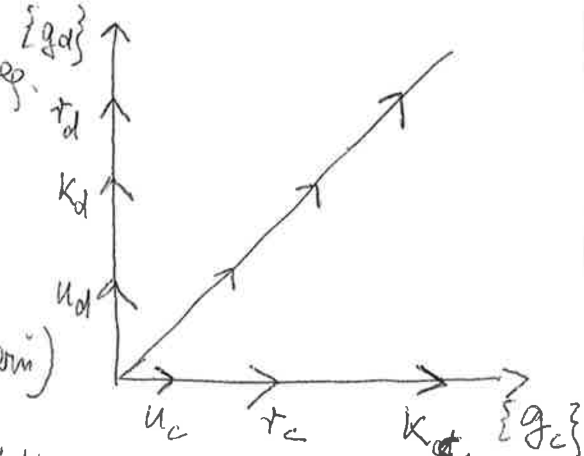
* open system / no energy conservation

→ Hamiltonian is not the only resource of dynamics

→ consider two extreme limits :

1) $H_d \equiv 0$; $\xi \equiv 0$ → Gross-Pitaevskii eq.

2) $H_c \equiv 0$ → "Model A" for classical equilibrium dynamical criticality (Hohenberg-Halperin)



→ both correspond to equilibrium conditions

→ couplings aligned on one ray in complex plane

→ can be shown :

equilibrium dynamics obtains for

$$\boxed{H_c = R\, H_d}\ , \quad R \in \mathbb{R}$$

* intuition : rescale EoM with $Z = 1 - iR$

$$\Rightarrow i\partial_t \longrightarrow \frac{i}{Z}\partial_t = \frac{i\partial_t}{1+R^2} + \frac{R}{1+R^2}\partial_t$$

imag. time damping into Gibbs ensemble stat. state.

* more formally:

→ use equivalence of stochastic PDE w/ MSR functional integral $Z = \int \mathcal{D}\psi\, \mathcal{D}\tilde{\psi}\, e^{iS[\psi,\tilde{\psi}]}$, where $\tilde{\psi} \Longleftrightarrow \xi$ (noise field)

→ eq. conditions $\Longleftrightarrow$ existence of symmetry

$$\psi(t,\vec{x}) \longrightarrow \psi(-t,\vec{x})$$
$$\mathrm{T}: \quad \tilde{\psi}(t,\vec{x}) \longrightarrow \tilde{\psi}(-t,\vec{x}) - \frac{1}{T}\partial_t \psi(-t,\vec{x})$$
$$i \longrightarrow -i$$

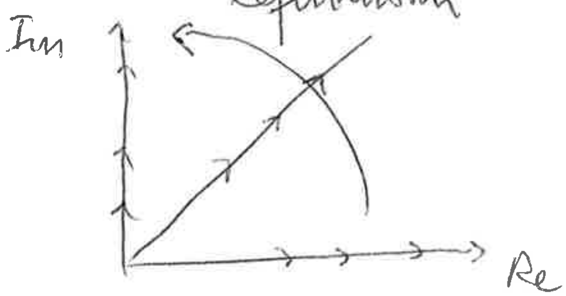
* this is a concatenation of time reversal and time translation

* the conserved charge is energy

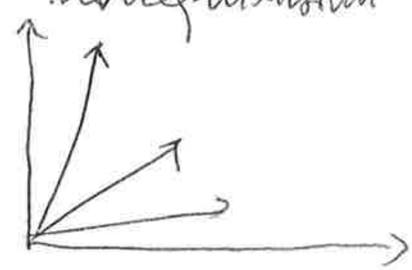
* if this symmetry is present ($\mathrm{T}\, S[\psi,\tilde{\psi}] = S[\psi,\tilde{\psi}]$), thermal fluctuation-dissipation relations of arbitrary order follow

* If this is a symmetry, all couplings lie on a single ray

equilibrium



nonequilibrium



- diss. and coherent dyn. not independent
- RG perspective: couplings "rotate" from Re to Im axis under RG (decoherence)

- d & c dyn. independent
- but also decoherence (& thermalization!) takes place.

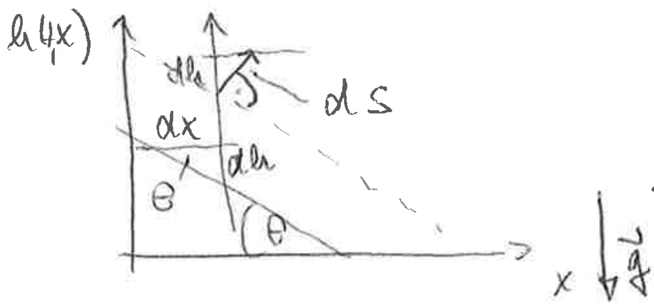
• KPZ equation

* point particles : Brownian motion

$$\partial_t n(t,x) = D \nabla^2 n(t,x) + \xi(t,x)$$

*Q : analogue of Brownian motion of surfaces?

$\longrightarrow$ Qualitatively distinct in the presence of drive (geom. effect)
(Kardar, Parisi, Zhang 1986)



• growth : $ds = \lambda dt$ ← deposition rate

• geometry :

$$dh = \frac{ds}{\sqrt{1+\left(\frac{dh}{dx}\right)^2}} \approx \lambda dt\left(1-\frac{1}{2}\left(\frac{dh}{dx}\right)^2\right)$$

$\Rightarrow$ Brownian motion corrected by terms $\sim \lambda$:

$$\frac{\partial h}{\partial t} = D\nabla^2 h \underline{+\lambda} - \frac{\lambda}{2}(\vec{\nabla} h)^2 + \xi$$

* Properties from a phase analogy ; consider behavior of complex field $\psi(t,\vec{x}) = \rho(t,\vec{x})\, e^{i\theta(t,\vec{x})}$; $h \hat{=} \theta$

1) comoving / rotating frame trafo $\cong$ time-local gauge trafo

$$\psi(t,\vec{x}) \longrightarrow e^{i\lambda t}\psi(t,\vec{x})$$

i.e. $\theta(t,\vec{x}) \longrightarrow \theta(t,\vec{x}) \longrightarrow \theta(t,\vec{x}) + \lambda t$

$\Rightarrow$ we can absorb free λ (describes average growth of interface)

$$\partial_t h = D \vec{\nabla}^2 h - \frac{1}{2}(\vec{\nabla} h) + \xi \qquad \text{KPZ eq.}$$

$\Rightarrow$ λ has a nontrivial effect only under noneq. conditions!

indeed $\theta = 0 \Rightarrow$ ~~$\frac{dh}{dx}$~~ $dh = ds = \lambda dt \to$ linear eq.

balance of forces

2) Scale invariance = global gauge invariance

$$\psi(t,\vec{x}) \to e^{i\alpha}\psi(t,\vec{x})$$

i.e. $\theta(t,\vec{x}) \to \theta(t,\vec{x}) + \alpha$

~~also~~ EOM remains gapless : "self-organized criticality", scaling of correlation functions. Eg.

$$H(t,\vec{x}) := \langle [h(t,x) - h(0,0)]^2 \rangle = r^{2\chi} f_{KPZ}\left(\frac{t}{r^z}\right)$$

w/ $f_{KPZ}(y \to 0) = \text{const}$

$f_{KPZ}(y \to \infty) \sim y^{2\chi/z} \qquad \Rightarrow \quad H(t,\vec{x}) \sim t^{2\chi/z}$

χ - "roughness exponent"

$\chi > 0$: height variance grows w/ y : "rough phase"

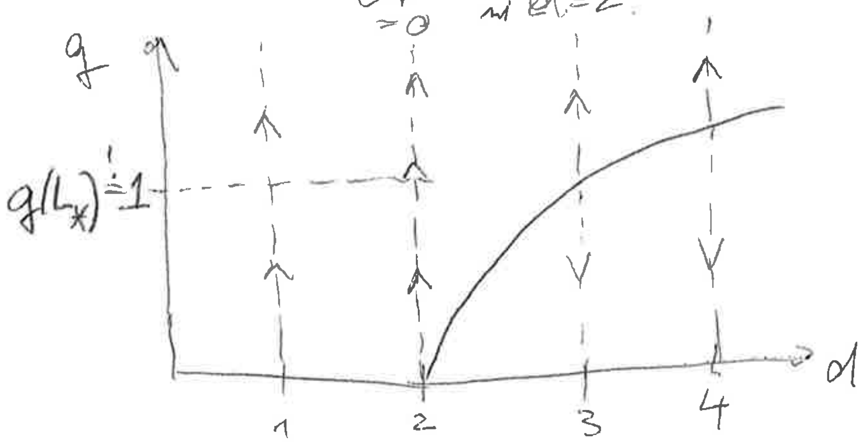
$\chi < 0$ — " — shrinks, "smooth phase"

3) Galilean invariance

- Large scale physics of KPZ eq : RG approach

  * gradually integrate out short scale gapless fluctuations
  * RG flow equation (perturbative)

$$\partial_\ell g = \underbrace{(2-d)}_{=0 \text{ in } d=2} g + k_d g^2 \qquad g = \frac{\lambda^2 \Delta}{D^3} \quad \leftarrow \text{noise level}$$



Interpretation : "rough phase"

$g \to \infty$: strong noneq. KPZ fixed PT

$g \to 0$: effective emergent equil. behavior / thermaliz. "smooth"

- <u>Connection</u>

* go back to driven GPE, decompose $\psi(t,x) = (\underbrace{M_0}_{\text{from mean field}} + \underbrace{\chi(t,x)}_{\text{density}})\, e^{i\underbrace{\theta(t,\vec{x})}_{\text{phase}}}$

$$\Rightarrow \partial_t \chi = -2u_d M_0^2 \chi - k_d M_0 (\vec{\nabla}\theta)^2 - k_c M_0 \vec{\nabla}^2\theta + \mathrm{Re}\,\xi \quad (1)$$

$$M_0 \partial_t \theta = -2u_c M_0 \chi - k_d M_0 \vec{\nabla}^2\theta - k_c M_0 (\vec{\nabla}\theta)^2 + \mathrm{Im}\,\xi \quad (2)$$

(1) $\to$ gapped $\to$ adiab. elimination, linearization justified
$\partial_t \chi \overset{!}{=} 0$ fast on scale of θ

$$\Rightarrow \partial_t \theta = \underbrace{D \vec{\nabla}^2 \theta}_{\text{phase diffusion}} + \underbrace{\lambda (\vec{\nabla}\theta)^2}_{\text{KPZ nonlin.}} + \xi$$

with $D = K_d [1 + R_K R_u)$ $\quad \langle \xi(t',x') \xi(t,x) \rangle = 2\Delta\, \delta(t-t')\, \delta(x-x')$

$\lambda = 2K_c [\frac{R_u}{R_K} - 1]$ $\quad \Delta = \frac{u_d^2 + u_c^2}{2 u_d m_d}$

$R_K = \frac{K_d}{K_c}$; $R_u = \frac{u_d}{u_c}$

equil: $R = R_k = R_u = \ldots \Rightarrow \lambda = 0$ protected by symm.

$\Rightarrow \lambda \neq 0$ signals noneq.

*Implications

→ generation of crossover length scale

from sol. of RG flow in 2D (~~eg~~ marginally relevant)

$$L_* = \xi_0 e^{\frac{2\pi}{g_0}}$$

g_0: g @ micro. scale

ξ_0: microscopic scale (healing length)

→ interpretation: study spatial corr. fn.

$$\langle \psi^*(0,\vec{x}) \psi(0,0) \rangle \underset{\text{ampl. fluct}}{\overset{\text{neglect gapped}}{\cong}} m_0^2 \langle e^{i(\theta(0,\vec{x}) - \theta(0,0))} \rangle$$

$$\underset{\text{cumulant exp}}{\cong} M_0^2\, e^{-\frac{1}{2} \langle [\theta(0,\vec{x}) - \theta(0,0)]^2 \rangle}$$

cf. the correlator above!

$\Rightarrow$ two regimes:

① $L \ll L_* \Rightarrow \lambda(L) \ll 1$

$\Rightarrow$ equilibrium / Bogol. FP is appropriate:

$$\langle [\theta \ldots]^2 \rangle \sim K \log |\vec{x}|/x_0$$

$$\Rightarrow \langle \phi^*(0,\vec{x}) \phi(0,0) \rangle \cong M_0^2 |\vec{x}|^{-K} \quad \text{algebraic}$$

o $L \gg L_*$ $\Rightarrow$ $\lambda(L) \gg 1$

$\Rightarrow$ KPZ FP appropriate (known numerically !)

$$\langle [\quad \cdot \quad]^2 \rangle \sim |x|^{2\chi} \qquad \chi = 0.4 \text{ in 2D}$$

$$\rightarrow \langle \phi^*(0,\vec{x})\,\phi(0,0) \rangle \sim e^{-k'|\vec{x}|^{2\chi}} \quad \text{stretched exp.}$$