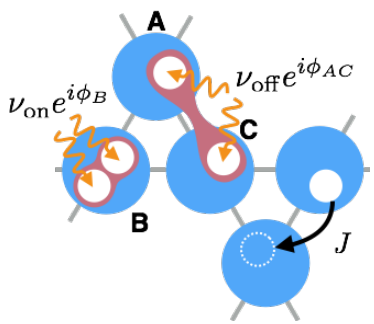
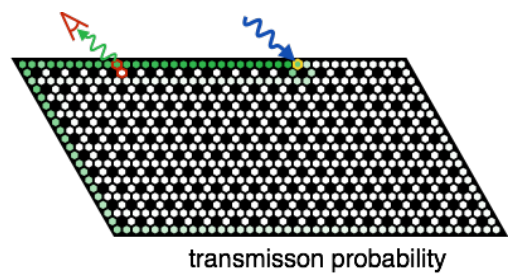
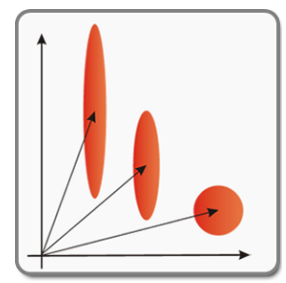
New approaches to topological and non-reciprocal photonic states

Aashish Clerk, Dept. of Physics, McGill University





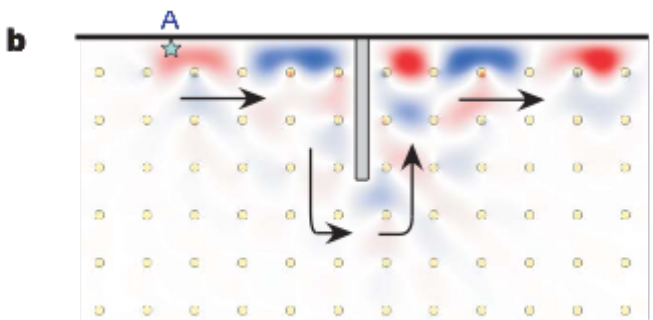


- Squeezing to producing new kinds of topological states (bosonic analogue of a topolgical superconductor?)
  - With **V. Peano**, **M. Houde,** C. Brendel and F. Marquardt; arXiv:1508.01383
- Reservoir engineering to make arbitrary interactions non-reciprocal
  - With A. Metelmann, PRX **5**, 021025 (2015)

Motivation

- Can we realize "simple" topological states of photons (or phonons)?
  - e.g. Chern insulators, QHE-like states (quadratic Hamiltonians!)

- Why?
  - Robust generation of protected edge modes, "one-way" waveguides (recent review: Lu et al, Nat. Photon. 2014)



Possible Realization of Directional Optical Waveguides in Photonic Crystals with Broken Time-Reversal Symmetry

F. D. M. Haldane and S. Raghu*

Department of Physics, Princeton University, Princeton, New Jersey 08544-0708, USA

(Received 23 March 2005; revised manuscript received 30 May 2007; published 10 January 2008)

We show how, in principle, to construct analogs of quantum Hall edge states in "photonic crystals" made with nonreciprocal (Faraday-effect) media. These form "one-way waveguides" that allow electromagnetic energy to flow in one direction only.

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- Basic physics…

$$\hat{H} = -J \sum_{<i,j>} \left(e^{i\phi_{ij}} \hat{c}_i^\dagger \hat{c}_j + h.c. \right) \longrightarrow \left\{ \epsilon_{\mathbf{k}n}, u_{\mathbf{k}n}[j] \right\}$$

  - Topology encoded in single-particle wavefunctions
  - No essential difference for bosons or fermions

Particle non-conseving terms

- What if we consider a more general quadratic Hamiltonian?
  - Allow particle-number non-conserving terms…

$$\hat{H} = \sum_{\mathbf{k},n} \varepsilon_n[\mathbf{k}] b^\dagger_{\mathbf{k},n} b_{\mathbf{k},n} + \sum_{\mathbf{k},n,n'} \left(\lambda_{nn'}[\mathbf{k}] b^\dagger_{\mathbf{k},n} b^\dagger_{-\mathbf{k},n'} + h.c. \right)$$

- Fermions?
  - Generic Hamiltonian of a superconductor
  - "Anomalous" terms can themselves lead to topological structures
    - e.g.. spinless p-wave superconductor

$$H_{\mathrm{BdG}} = \frac{1}{2} \sum_{\mathbf{p}} \Psi^\dagger_{\mathbf{p}} \begin{pmatrix} \epsilon(\mathbf{p}) & 2i\Delta(\sin p_x + i \sin p_y) \\ -2i\Delta^*(\sin p_x - i \sin p_y) & -\epsilon(\mathbf{p}) \end{pmatrix} \Psi_{\mathbf{p}},$$

- Majorana modes at boundaries….
- Treat BdG wavefunctions like ordinary single particle wavefunctions…

$$\sum_j |u_j[\vec{k}]|^2 + |v_j[\vec{k}]|^2 = 1 \qquad \mathcal{A}_l = i\langle \mathbf{k}_l | \nabla_{\mathbf{k}} | \mathbf{k}_l \rangle$$

Particle non-conserving terms

- What if we consider a more general quadratic Hamiltonian?
  - Allow particle-number non-conserving terms…

$$\hat{H} = \sum_{\mathbf{k},n} \varepsilon_n[\mathbf{k}] b^\dagger_{\mathbf{k},n} b_{\mathbf{k},n} + \sum_{\mathbf{k},n,n'} \left(\lambda_{nn'}[\mathbf{k}] b^\dagger_{\mathbf{k},n} b^\dagger_{-\mathbf{k},n'} + h.c. \right)$$

- Fermions?
  - Generic Hamiltonian of a topological superconductor
  - "Anomalous" terms can directly give rise to topological structures
    - Non-trivial winding of order parameter

	d												
Cartan	0	1	2	3	4	5	6	7	8	9	10	11	…
Complex case:													
A	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	…
AIII	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	…
Real case:													
AI	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	…
BDI	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	…
D	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	
DIII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	[illegible]					
[illegible]	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	[illegible]								

(Ryu, Schneider, Furusaki and Ludwig, NJP 2010)

Topological photonic supercondcutors?

- What if we consider a more general quadratic Hamiltonian?
  - Allow particle-number non-conserving terms…

$$\hat{H} = \sum_{\mathbf{k},n} \varepsilon_n[\mathbf{k}] b^\dagger_{\mathbf{k},n} b_{\mathbf{k},n} + \sum_{\mathbf{k},n,n'} \left(\lambda_{nn'}[\mathbf{k}] b^\dagger_{\mathbf{k},n} b^\dagger_{-\mathbf{k},n'} + h.c. \right)$$

- Bosons?
  - No longer identical to the fermionic problem…
    - Pairing terms not constrained by Pauli principle
    - Possibility of instabilities ($\Rightarrow$ usual classification fails?)
    - Definition of a Chern number?
      - No trivial mapping to single particle wavefunctions

$$\sum_s |u_{\mathbf{k}}[s]|^2 - |v_{\mathbf{k}}[s]|^2 = 1$$

- Topological phases?
- Protected edge modes?
  - Transport properties?

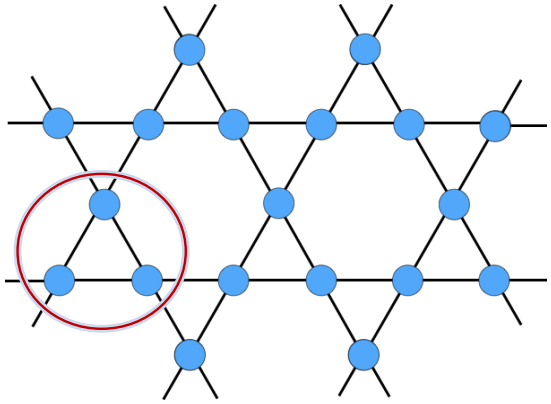
Related work:

- Shindou et al, PRB 2013 (magnons)
- Brandes et al, arXiv.1503.02503, Bardyn et al, arXiv.1503.08824 (weakly interacting condensates)

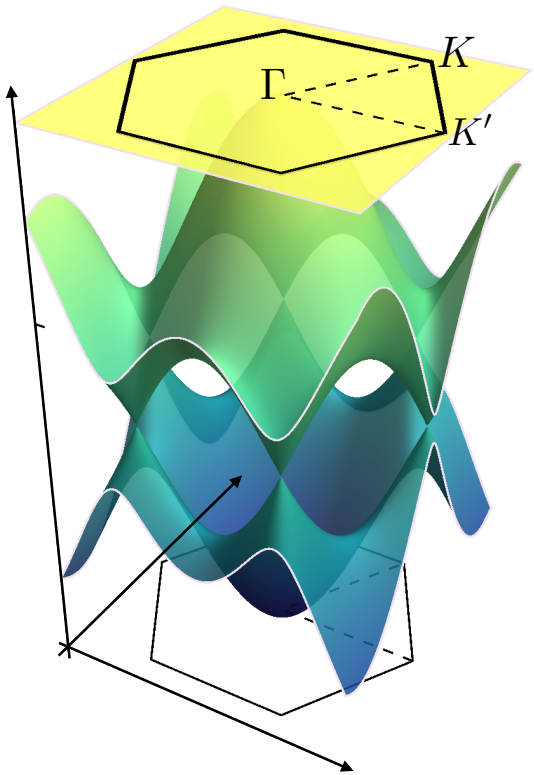
Simple model

- Bosons hopping on Kagome lattice:

$$\hat{H} = -J \sum_{<i,j>} \left(\hat{c}_i^\dagger \hat{c}_j + h.c. \right)$$



- No flux, no anomalous terms
  - Three bands, no gaps
- Realizations?
  - Arrays of superconducting microwave resonators (Koch et al PRA 2010)
  - Arrays of defect cavities in a photonic crystal (e.g. Painter group)



Simple model

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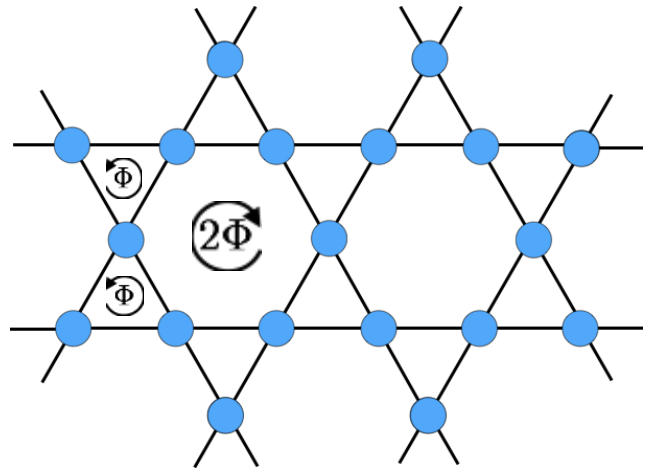
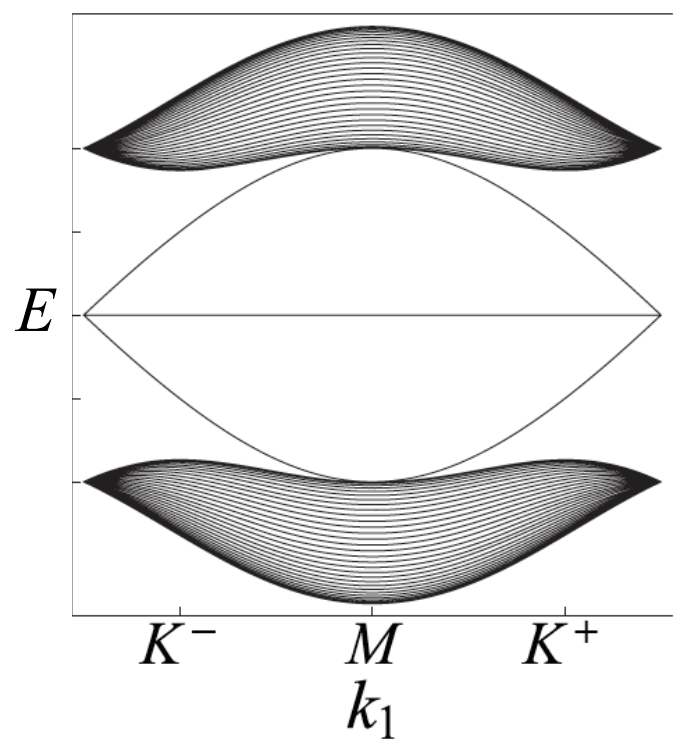
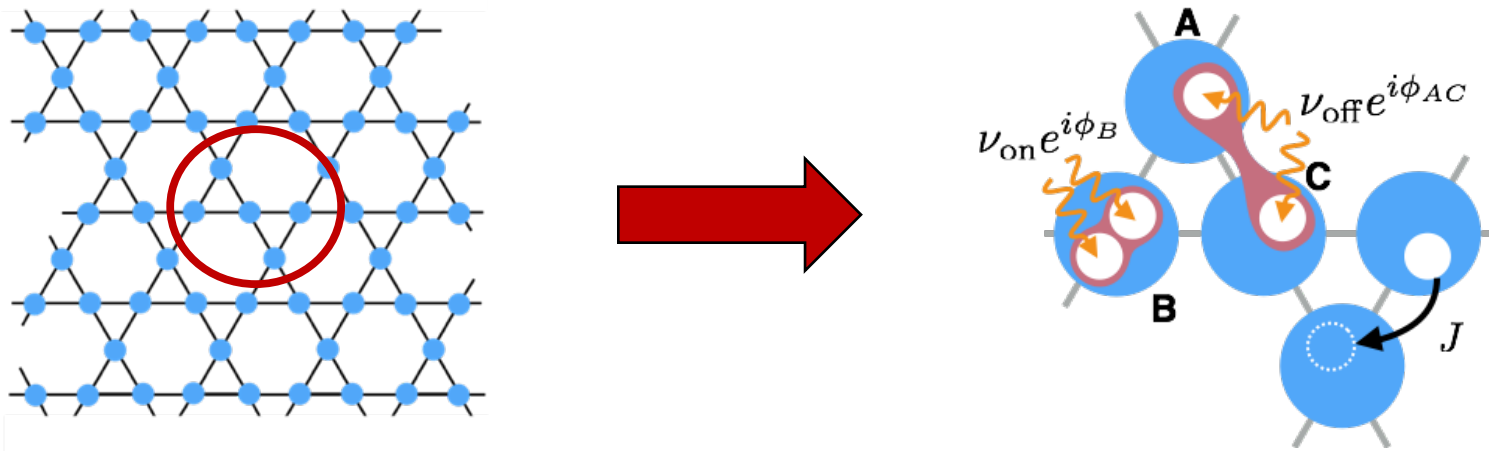


- No flux, no anomalous terms
  - Three bands, no gaps
- Add hopping phases
  - Break TRS, anomalous QHE **(Ohgushi, Murakami, Nagaosa, PRB 2000)**
  - Gaps, non-zero Chern numbers
  - Protected edge states



(Petrescu & LeHur, PRA 2012)

Break TRS via parametric driving



$$\hat{H} = \hat{H}_0 + \hat{H}_{\rm par} \qquad \hat{H}_0 = \sum_j \omega_0 \hat{a}_j^\dagger \hat{a}_j - J \sum_{\langle ij \rangle} \hat{a}_i^\dagger \hat{a}_j$$

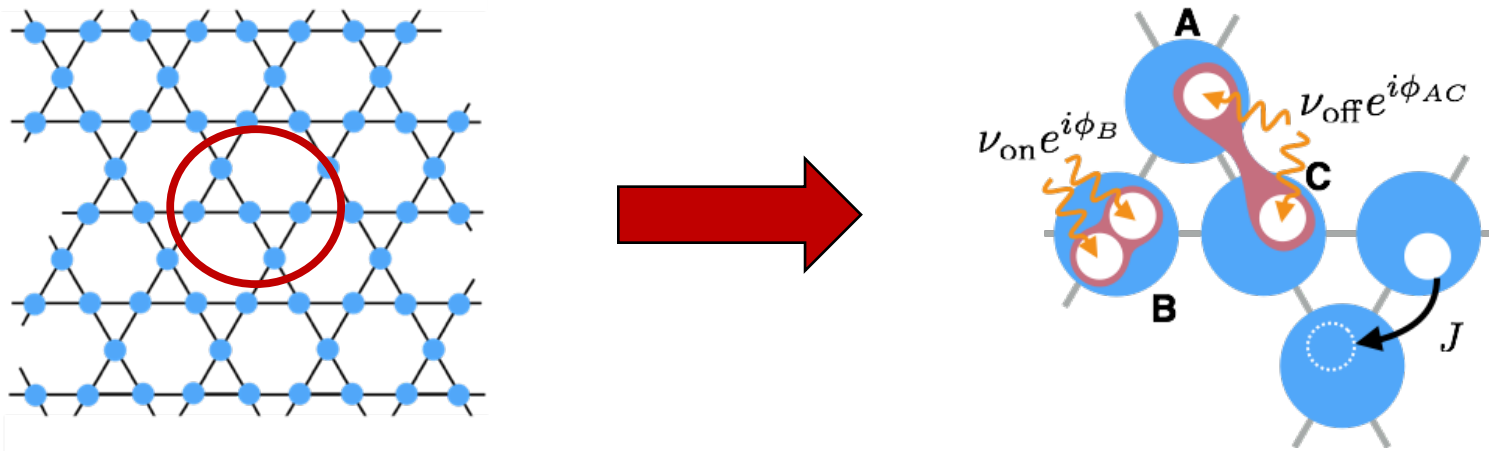
no fluxes!

$$\hat{H}_{\rm par} = -\frac{\nu_{\rm on}}{2} \sum_j e^{i\phi_j} \hat{a}_j^\dagger \hat{a}_j^\dagger - \frac{\nu_{\rm off}}{2} \sum_{\langle ij \rangle} e^{i\phi_{ij}} \hat{a}_i^\dagger \hat{a}_j^\dagger + h.c.$$

- "Pairing terms" break both time-reversal symmetry and charge conservation
  - Keep C3 rotation symmetry for simplicity
  - Pairing terms have a phase-winding in real space

$$\phi_\alpha = 0, \pm 2\pi/3$$

Break TRS via parametric driving



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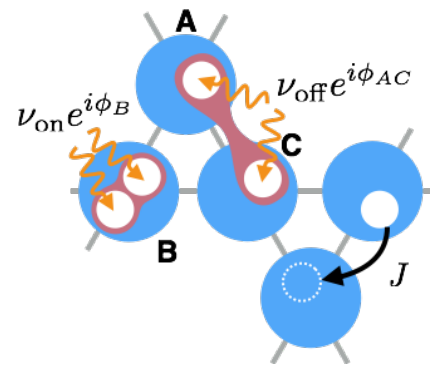
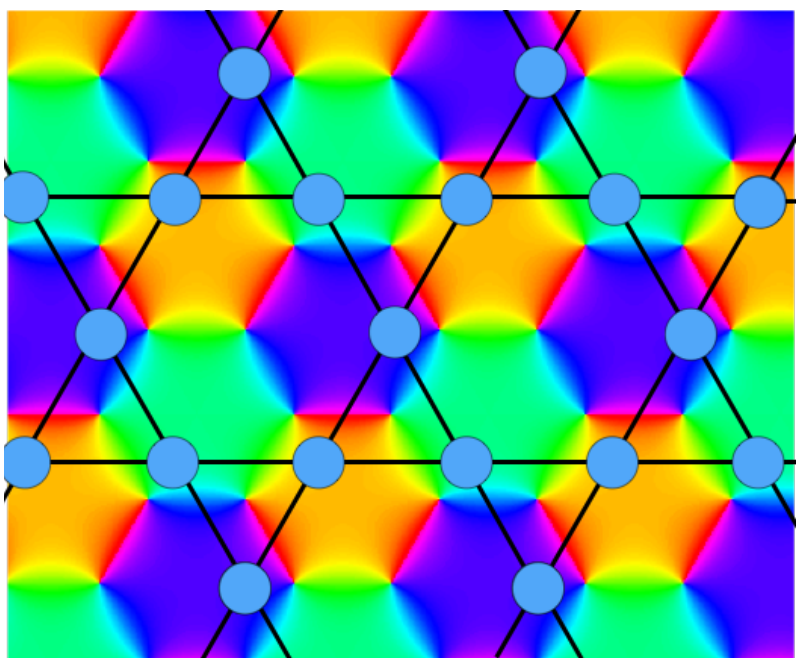
- "Pairing terms" break both time-reversal symmetry and charge conservation (keep C3 rotation symmetry)
- **Focus on regime where system remains stable without dissipation ($\nu < \omega_0 - 4J$)**

Physical realization?

$$\hat{H}_{\rm par} = -\frac{\nu_{\rm on}}{2}\sum_j e^{i\phi_j}\hat{a}_j^\dagger\hat{a}_j^\dagger - \frac{\nu_{\rm off}}{2}\sum_{\langle ij\rangle} e^{i\phi_{ij}}\hat{a}_i^\dagger\hat{a}_j^\dagger + h.c.$$



- Array of defect cavities in a nonlinear photonic crystal (χ_2 material)
- Phase control: pump e.g. with three lasers….

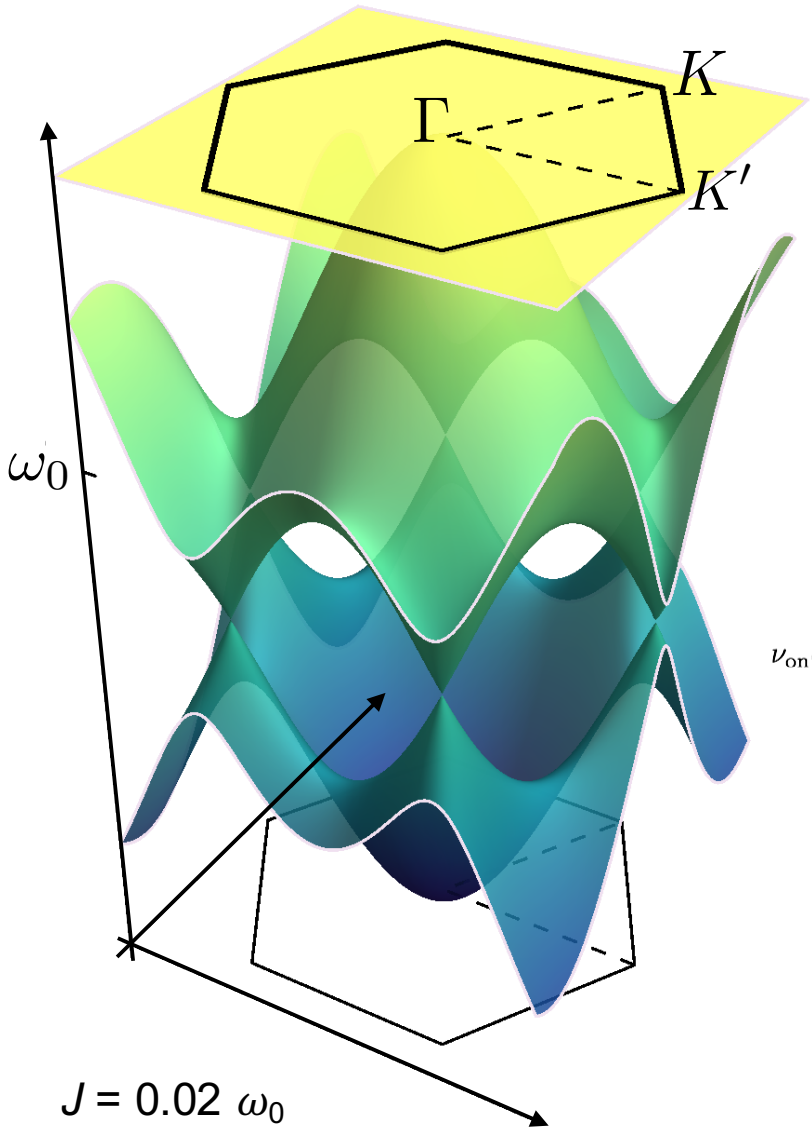


$$\hat{H}_{\rm NL} = \Lambda\hat{a}^\dagger\hat{a}^\dagger\hat{b} + h.c.$$
$$\simeq \Lambda\hat{a}^\dagger\hat{a}^\dagger\langle\hat{b}\rangle + h.c.$$

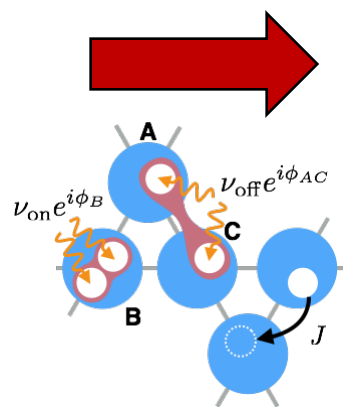
- Other possibilities?
  - Optomechanical arrays?
  - Arrays of nonlinear superconducting cavities…

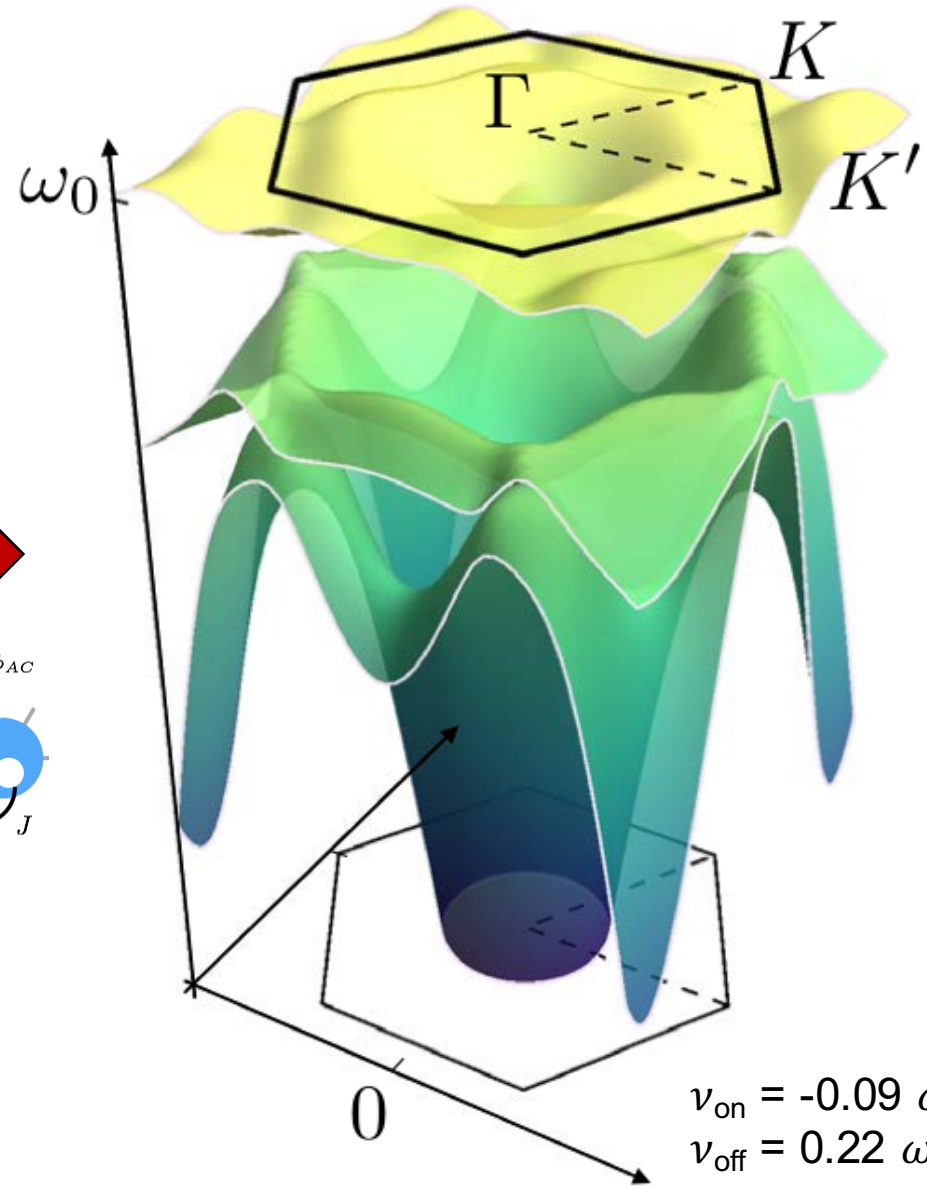
Band structure: bulk

- Parametric driving opens gaps…



J = 0.02 ω_0

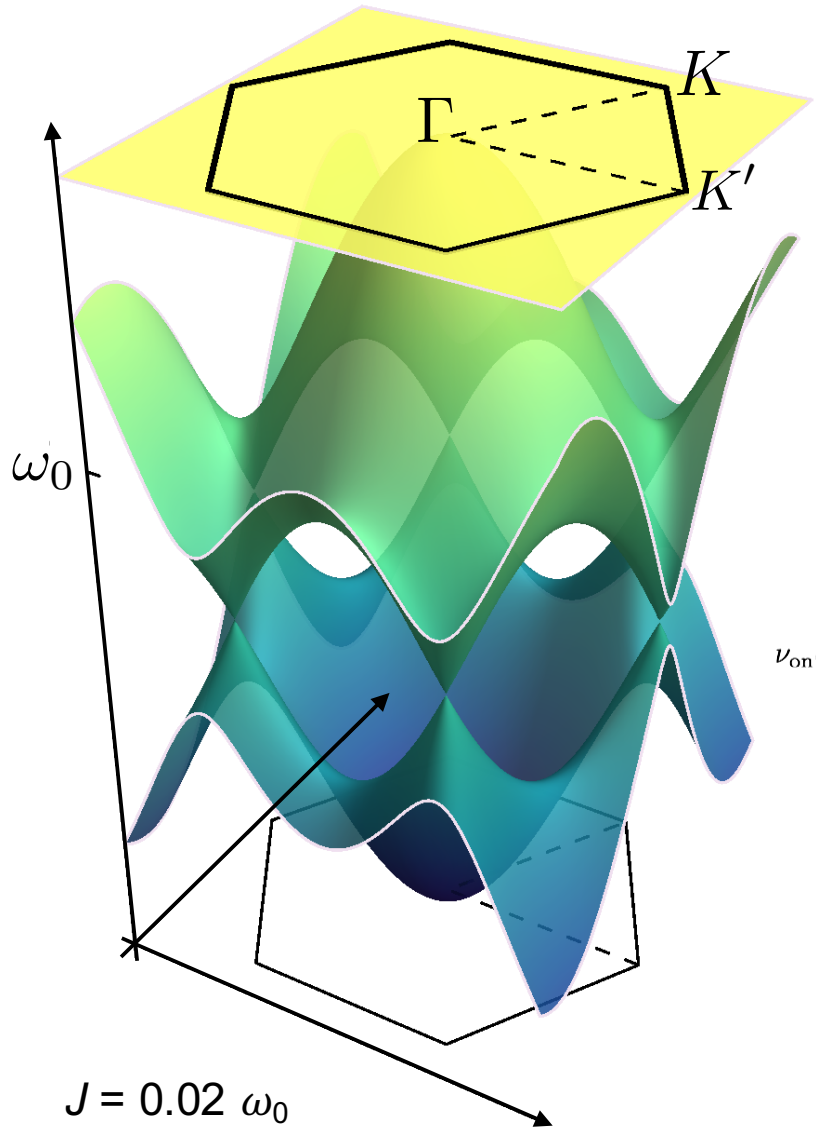




$\nu_{\rm on}$ = -0.09 ω_0
$\nu_{\rm off}$ = 0.22 ω_0

Band structure: bulk

- Parametric driving opens gaps…



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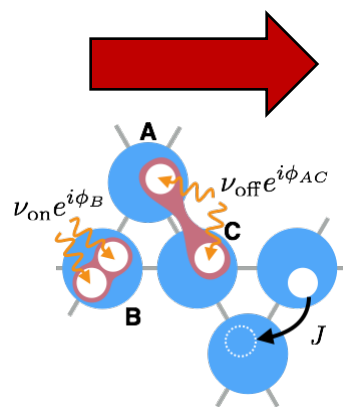
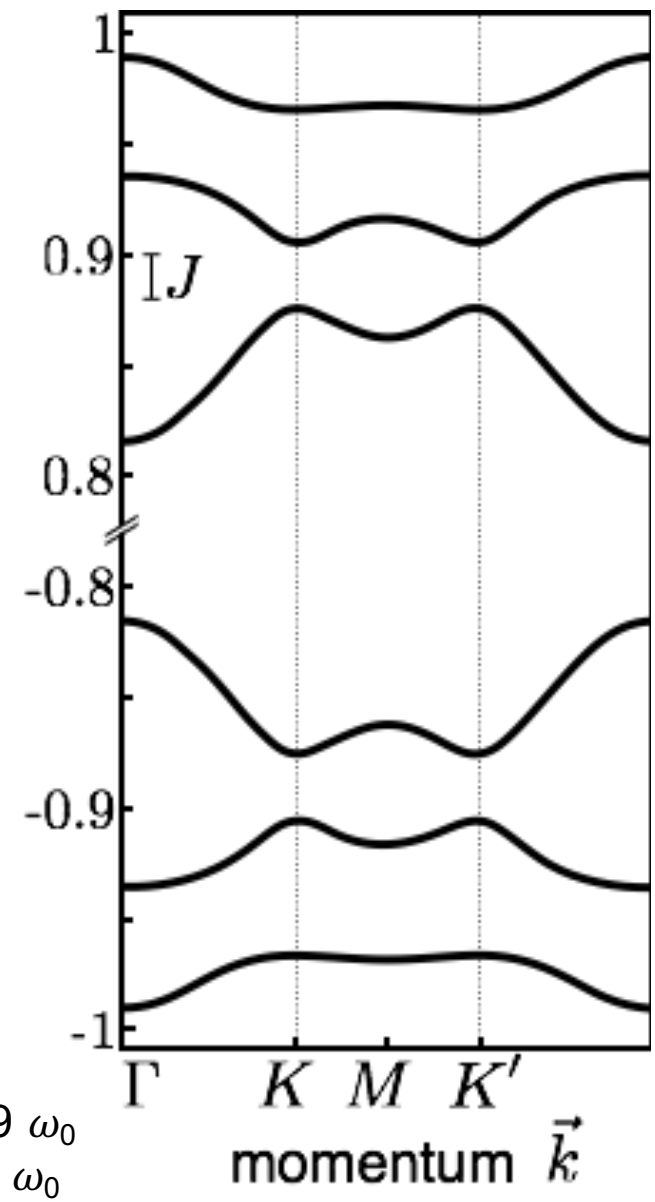
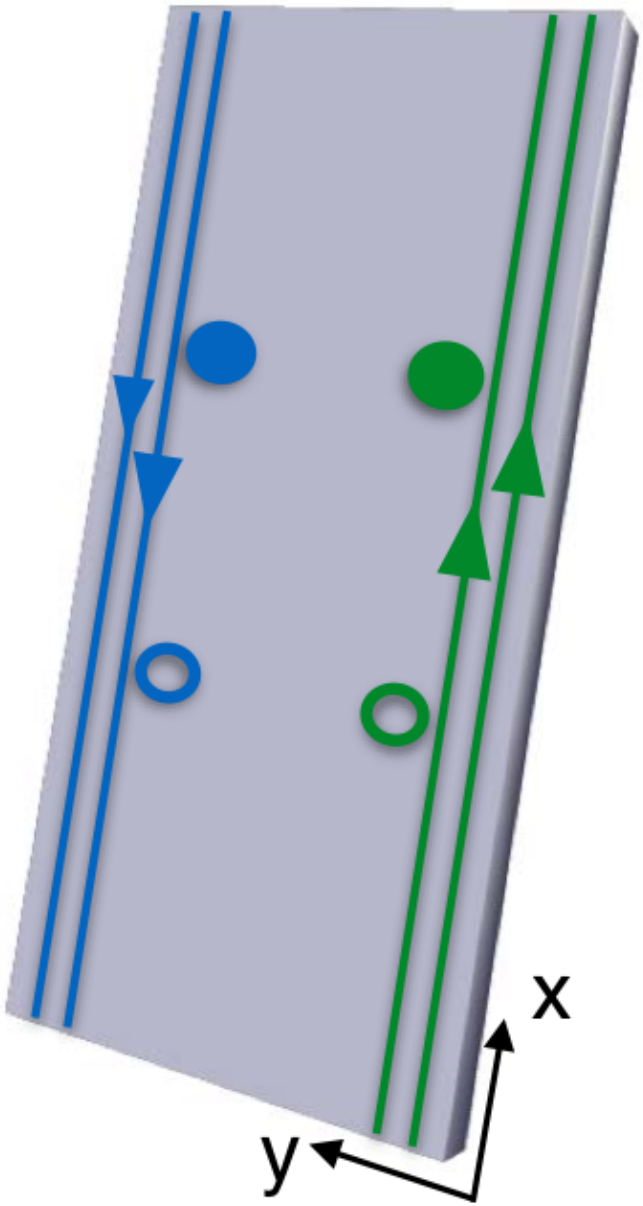
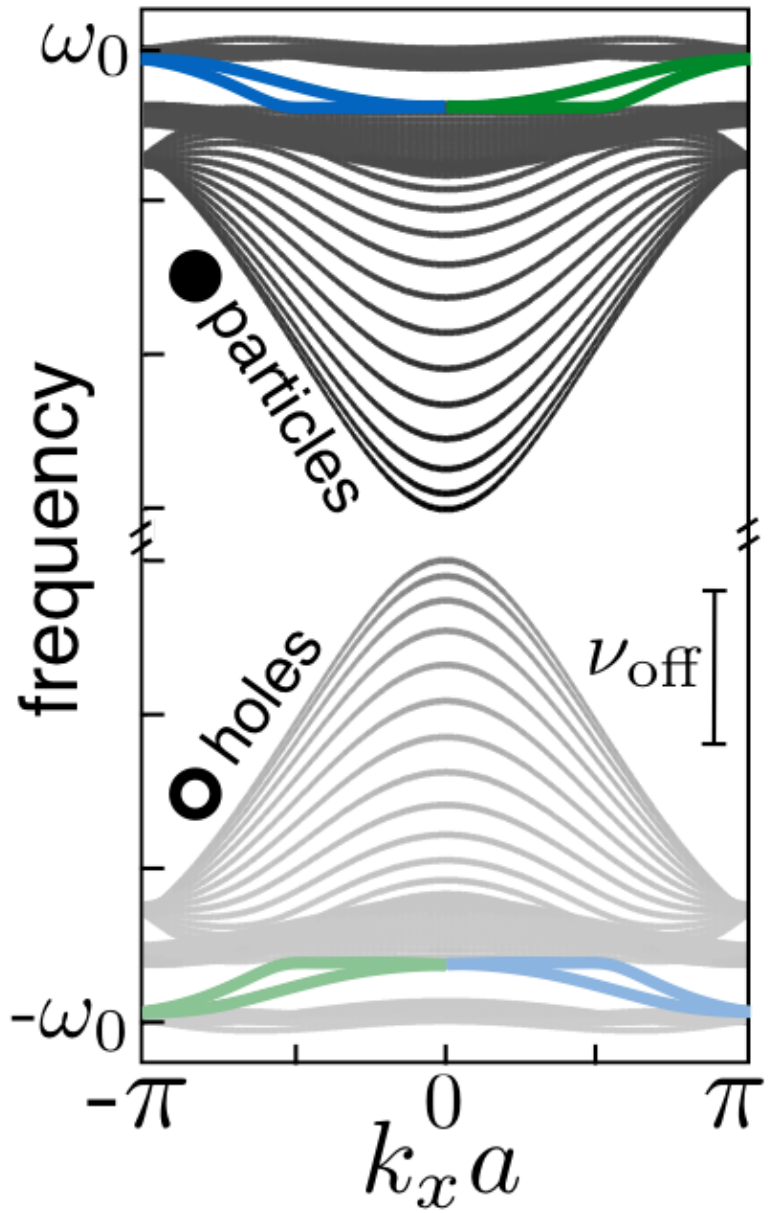




$\nu_{\rm on}$ = -0.09 ω_0
$\nu_{\rm off}$ = 0.22 ω_0

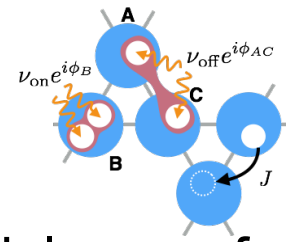
Band structure: strip geometry



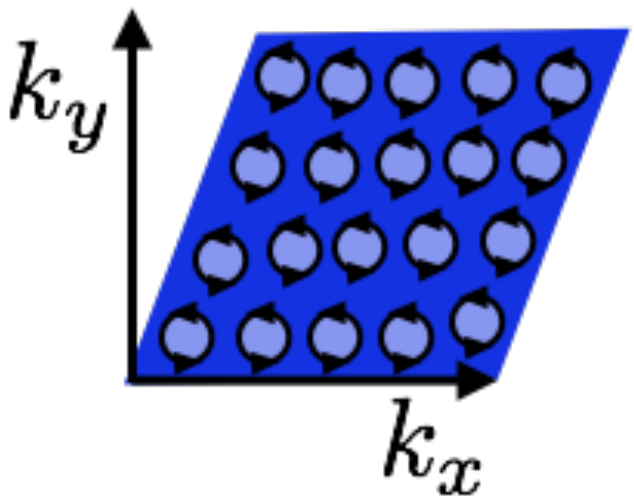


Defining a topological invariant

$$\hat{H}_{\rm par} = -\frac{\nu_{\rm on}}{2}\sum_j e^{i\phi_j}\hat{a}_j^\dagger\hat{a}_j^\dagger - \frac{\nu_{\rm off}}{2}\sum_{\langle ij\rangle} e^{i\phi_{ij}}\hat{a}_i^\dagger\hat{a}_j^\dagger + h.c.$$



- Can't directly take BdG wavefunctions ~ single-particle wavefunction
- Back to basics: consider Berry phase of QP states



$$C_l = -\frac{1}{2\pi}\int_{BZ} d^2k(\nabla_{\mathbf{k}}\times\mathcal{A}_{\mathbf{l}}(\mathbf{k}))\cdot\mathbf{e}_z$$

1. *H*[***k***]: effective 3-mode parametric amplifier problem
   1. Ground state: $|\Omega[\vec{k}]\rangle$
   2. QP state: $|n,\vec{k}]\rangle = \hat{\beta}^\dagger[\vec{k},n]|\Omega[\vec{k}]\rangle$

Defining a topological invariant

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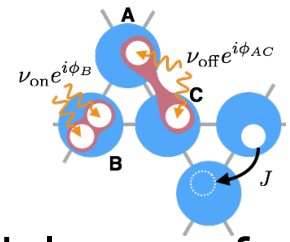

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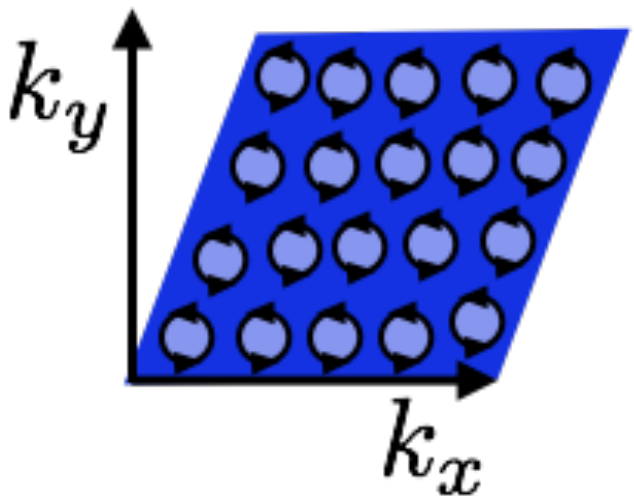


$$C_l = -\frac{1}{2\pi}\int_{BZ} d^2k(\nabla_{\mathbf{k}}\times\mathcal{A}_{\mathbf{l}}(\mathbf{k}))\cdot\mathbf{e}_z$$

(see also Shindou et al, PRB 2013)

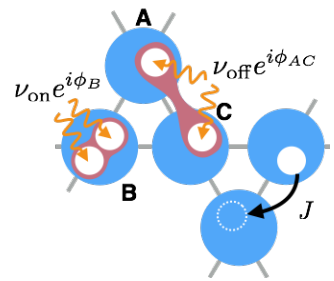
1. *H*[***k***]: effective 3-mode parametric amplifier problem
   1. Ground state: $|\Omega[\vec{k}]\rangle$
   2. QP state: $|n,\vec{k}]\rangle = \hat{\beta}^\dagger[\vec{k},n]|\Omega[\vec{k}]\rangle$
2. Berry's phse for a single quasiparticle:

$$\vec{\mathcal{A}}[\vec{k}] = i\langle n,\vec{k}|\vec{\nabla}|n,\vec{k}\rangle$$
$$= \vec{\mathcal{A}}[\vec{k}]_{\rm gnd} + \vec{\mathcal{A}}[\vec{k}]_{\rm qp}$$

$$\vec{\mathcal{A}}[\vec{k}]_{\rm qp} = i\left(\sum_j u_j^*\vec{\nabla}u_j - \sum_j v_j^*\vec{\nabla}v_j\right)$$

Chern numbers

$$\hat{H}_{\rm par} = -\frac{\nu_{\rm on}}{2}\sum_j e^{i\phi_j}\hat{a}_j^\dagger\hat{a}_j^\dagger - \frac{\nu_{\rm off}}{2}\sum_{\langle ij\rangle} e^{i\phi_{ij}}\hat{a}_i^\dagger\hat{a}_j^\dagger + h.c.$$



- Phases with "Standard" Chern numbers

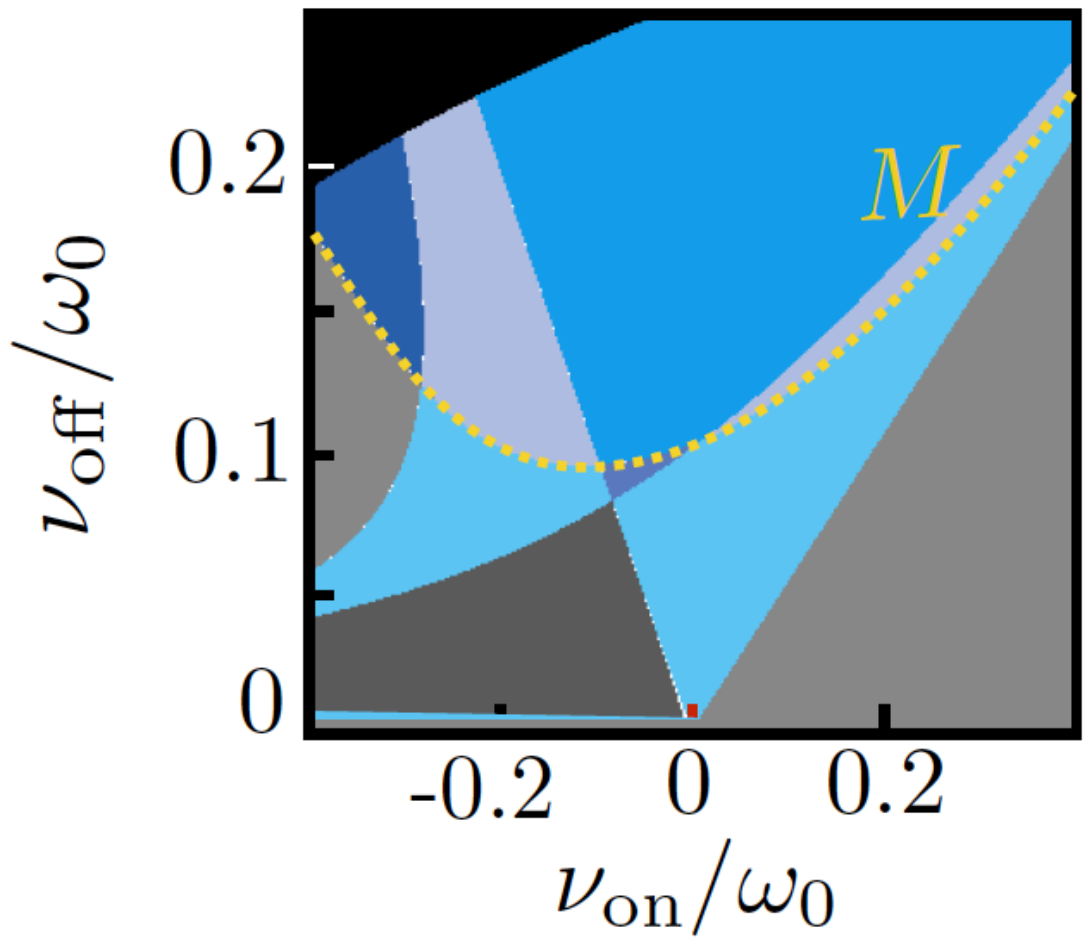
[-1, 0, 1] [1, 0, -1]

- Phases not found in particle-conserving model:

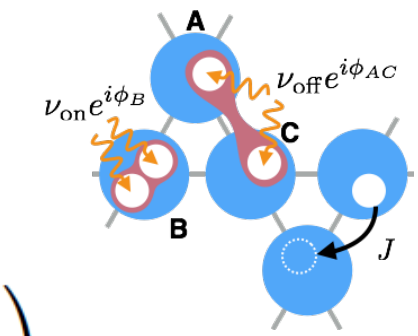
[-2, 1, 1] [1, -4, 3] [1, -2, 1]

[-2, -1, 3] [-4, 3, 1]

- NB: Chern numbers of "particle" bands must sum to zero (no zero modes)



Effective model

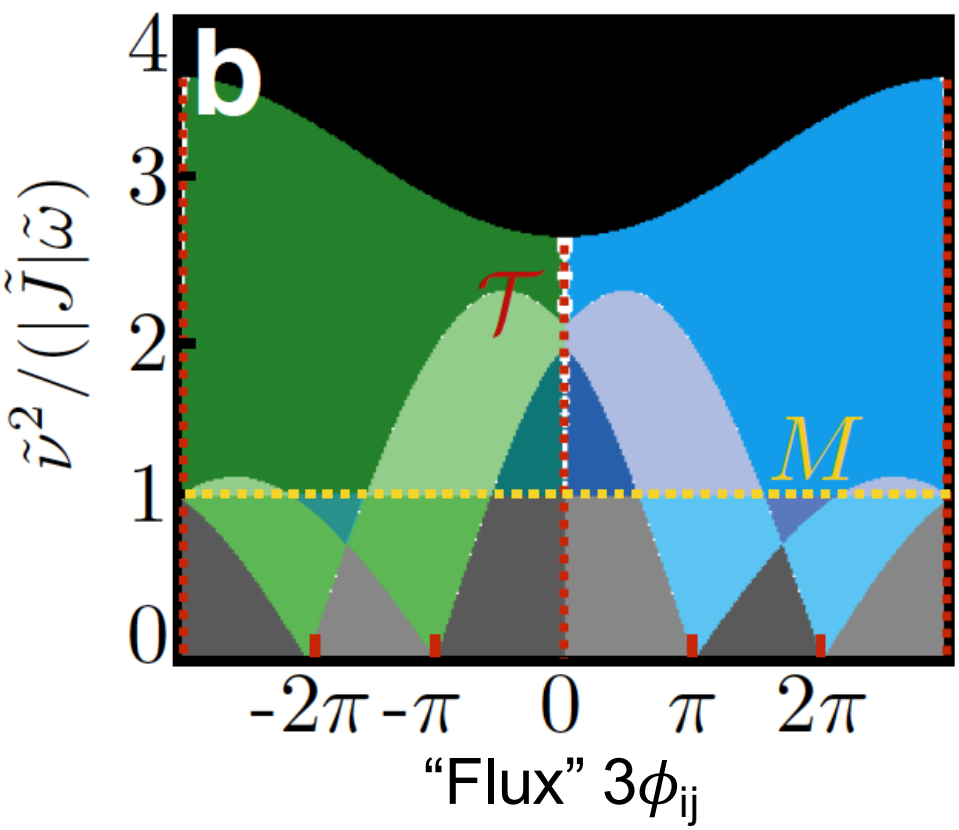


- Simpler structure: diagonalize on-site Hamiltonian….

$$\hat{H} = \sum_{\mathbf{j}} \tilde{\omega} \hat{\alpha}_{\mathbf{j}}^{\dagger} \hat{\alpha}_{\mathbf{j}} - \sum_{\langle \mathbf{j},\mathbf{l} \rangle} \tilde{J}_{\mathbf{j}\mathbf{l}} \hat{\alpha}_{\mathbf{j}}^{\dagger} \hat{\alpha}_{\mathbf{l}} - \left(\frac{\tilde{\nu}}{2} \sum_{\langle \mathbf{j},\mathbf{l} \rangle} \hat{\alpha}_{\mathbf{j}}^{\dagger} \hat{\alpha}_{\mathbf{l}}^{\dagger} + h.c. \right)$$

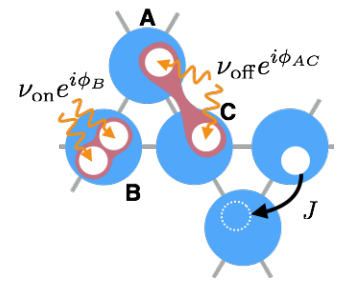
- Berry connection invariant under local squeezing transformation
- Reduced number of dimensionless parameters:

$$\tilde{\nu}, \ \tilde{\omega}, \ \tilde{\phi}_{AB}$$

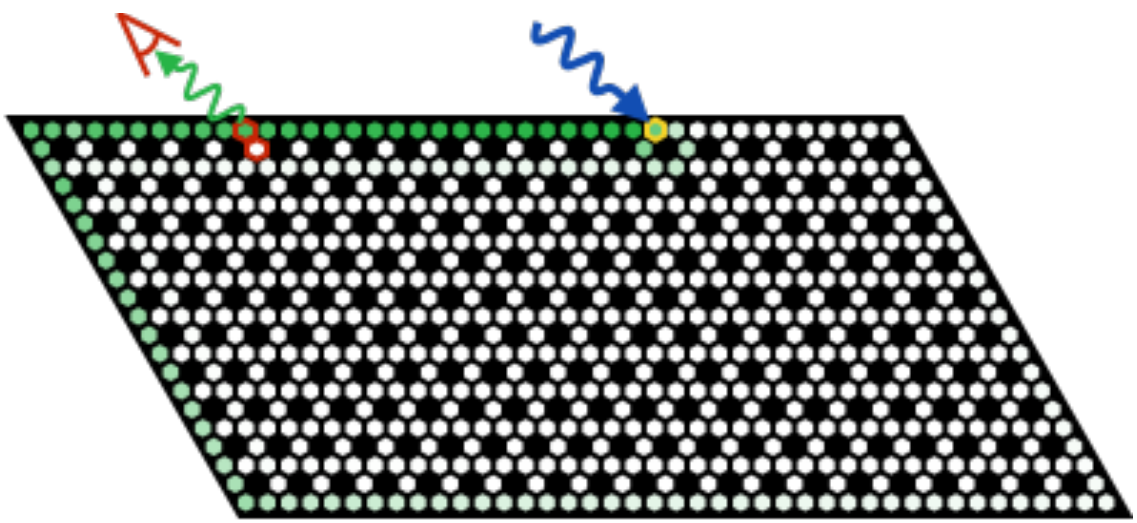


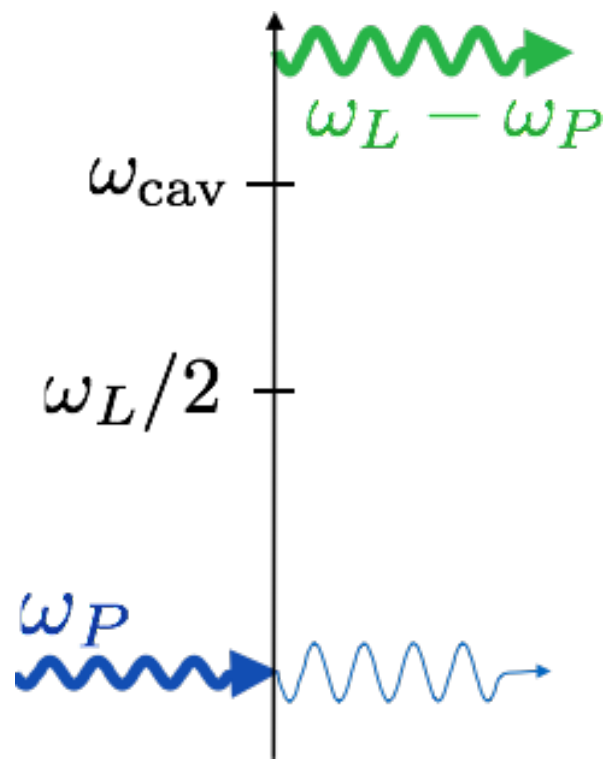
Transport

$$\hat{H}_{\rm par} = -\frac{\nu_{\rm on}}{2}\sum_j e^{i\phi_j}\hat{a}_j^\dagger\hat{a}_j^\dagger - \frac{\nu_{\rm off}}{2}\sum_{\langle ij\rangle} e^{i\phi_{ij}}\hat{a}_i^\dagger\hat{a}_j^\dagger + h.c.$$



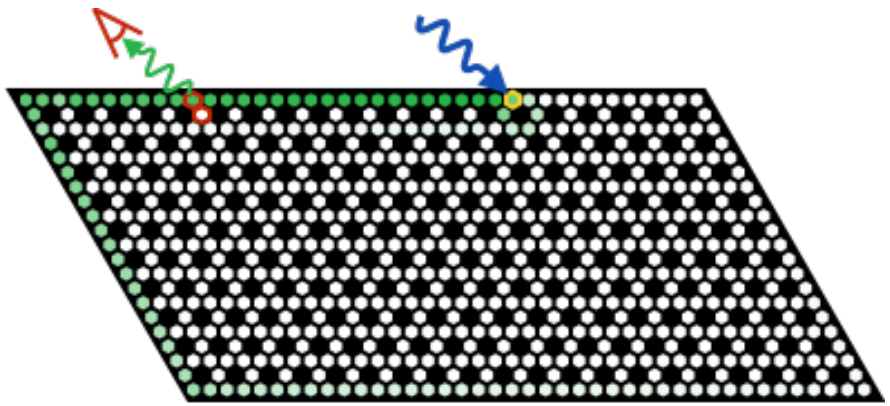
- As usual, edge states mediate chiral, protected transport
  - BUT: elastic and inelastic transport channels!



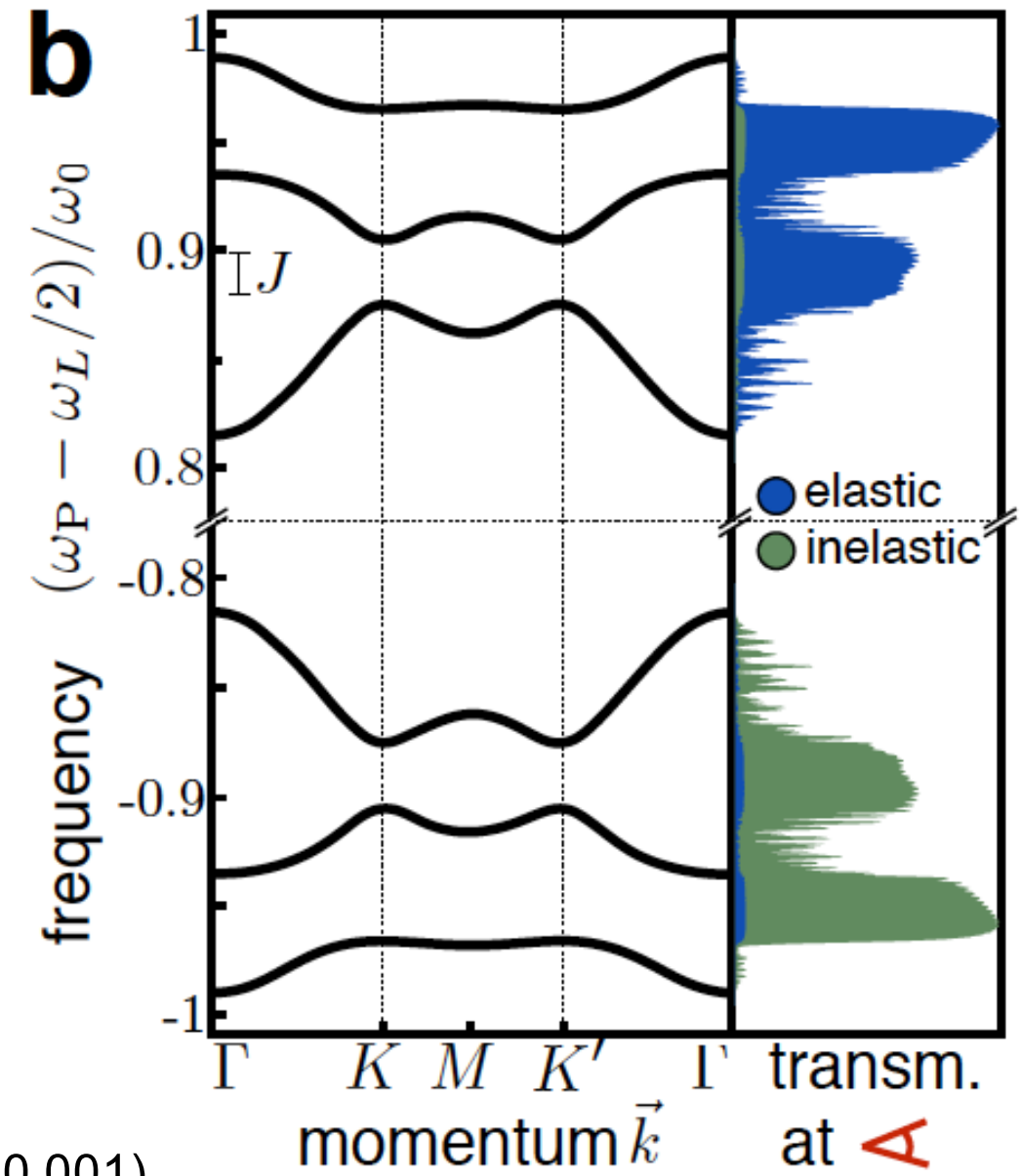


- Recall: parametric driving at freq. ω_L
  - This was shifted to zero in our rotating frame

Transport



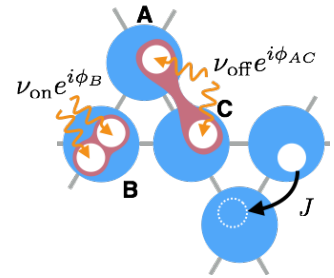
- Edge states mediate chiral, protected “inelastic” transport
  - Inelastic transport dominates for driving of “hole” bands
- Amplifying behaviour possible (even without entering unstable regime)



(ω_0 =1, J = 0.02, ν_{on} = 0.4, ν_{off} = 0.02, κ= 0.001)

Classification

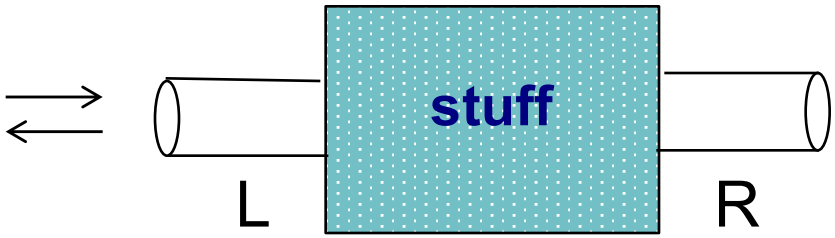
$$\hat{H}_{\rm par} = -\frac{\nu_{\rm on}}{2}\sum_j e^{i\phi_j}\hat{a}_j^\dagger\hat{a}_j^\dagger - \frac{\nu_{\rm off}}{2}\sum_{\langle ij\rangle} e^{i\phi_{ij}}\hat{a}_i^\dagger\hat{a}_j^\dagger + h.c.$$



- Can we use standard classification of fermionic non-interacting Hamiltonians?
  - Standard route (Ryu et al, NJP 2010):
    - Consider disorder problem (ensemble of random matrices with given symmetry)
    - Look at corresponding NLSM, possibility of adding topological terms
- Issue here:
  - Cannot formulate disorder problem the same way
    - Condition of stability → matrix elements of H correlated
    - Standard symmetry classes of limited used (Guararie and Chalker, PRL 2002, PRB 2003)

Nonreciprocal transport?

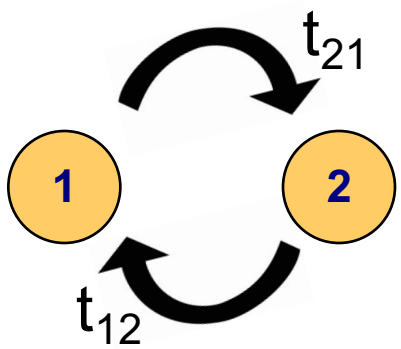
- Scattering description:



$$\begin{bmatrix} a_{L,out} \\ a_{R,out} \end{bmatrix} = \begin{pmatrix} 0 & 0 \\ s_{21} & 0 \end{pmatrix} \begin{bmatrix} a_{L,in} \\ a_{R,in} \end{bmatrix}$$

- Tunneling interaction:

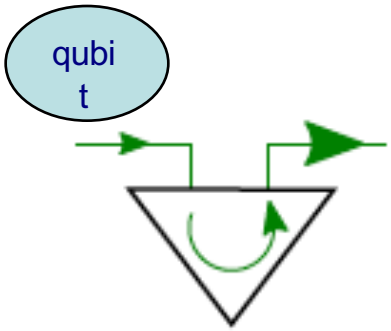
$$\hat{H} = t_{21}\hat{d}_2^\dagger \hat{d}_1 + t_{12}\hat{d}_1^\dagger \hat{d}_2$$



Want: $t_{12} \neq t_{21}$

- Asymmetric phases ~ TRS breaking
- Asymmetric magnitudes = ????

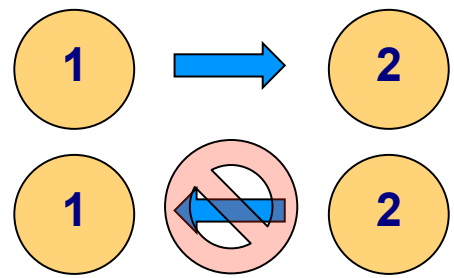
- Motivation for reciprocity-breaking interactions?
  - Devices: isolators, circulators, **directional amplifiers**, …
    - Without magnetic materials / magneto-optic effects?
  - A route to new kinds of quantum phases?



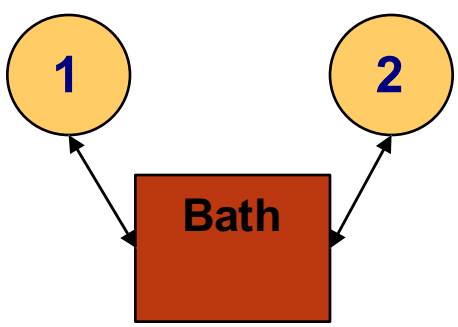
Dissipative quantum interactions?

- Goal: make a given interaction "one-way" (non-reciprocal)
  (A. Metelmann and AC, PRX 2015)

$$\hat{H}_{coh} = \lambda \left(\hat{O}_1 \hat{O}_2 + h.c. \right)$$



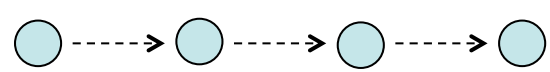
- General solution:
  - use additional interaction mediated by a dissipative reservoir



$$\frac{d}{dt}\hat{\rho} = -i[\hat{H}_{\rm coh}, \hat{\rho}] + \Gamma \left(\hat{z}\hat{\rho}\hat{z}^\dagger - \frac{1}{2}\{\hat{z}^\dagger \hat{z}, \hat{\rho}\} \right)$$

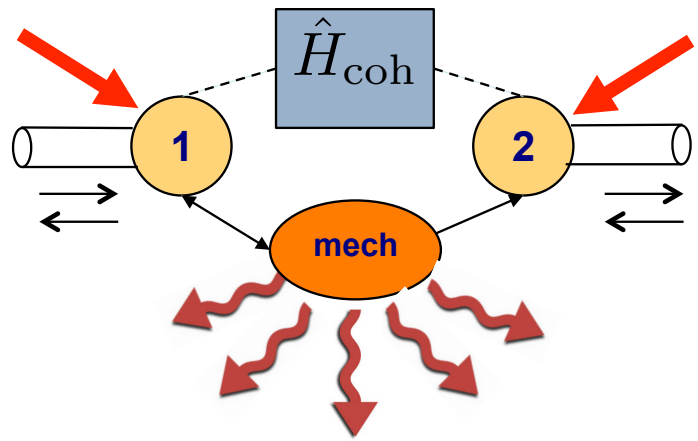
$$\hat{z} = \hat{O}_1 + e^{i\phi}\hat{O}_2^\dagger$$

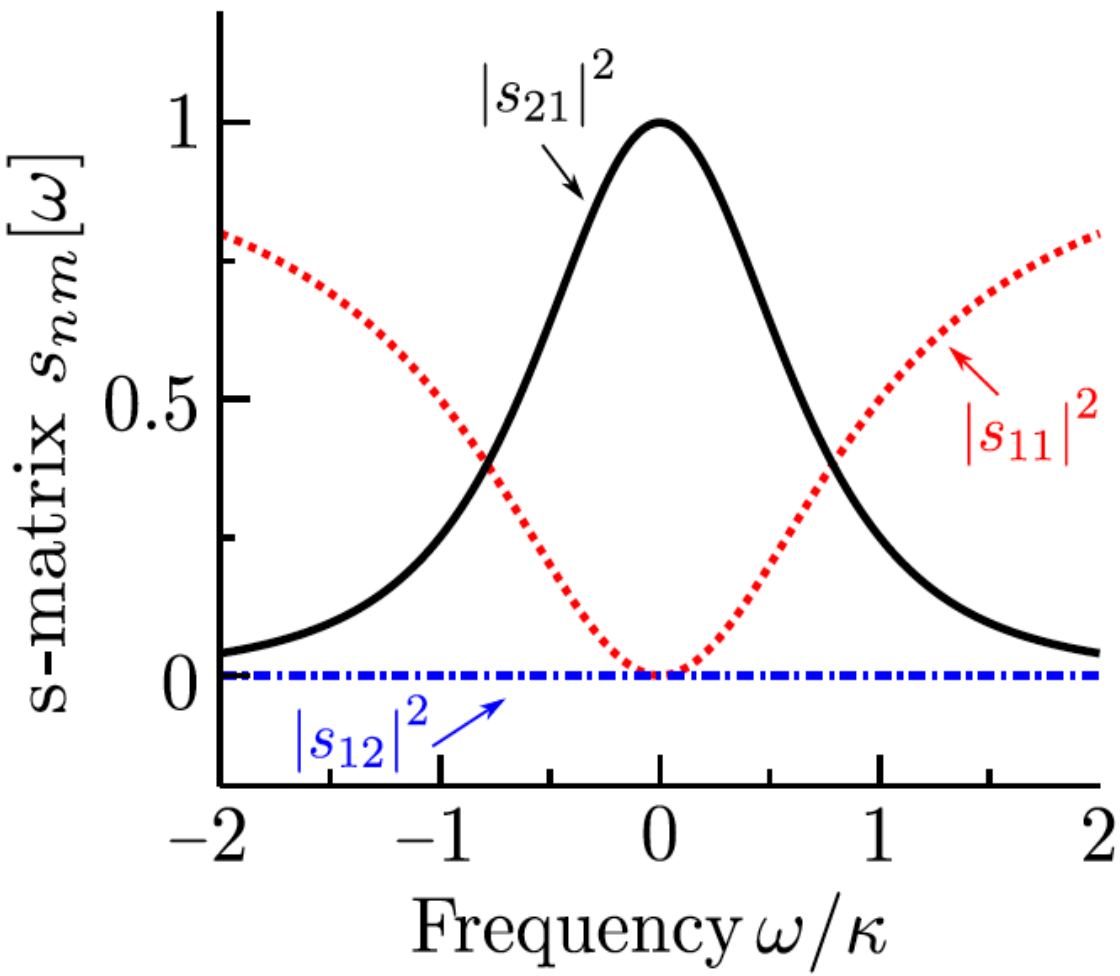
- Makes any interaction unidirectional…
  - Applications: non-reciprocal photonic devices
  - Non-reciprocal many-body interactions?



Dissipative quantum interactions?

- E.g.: Reservoir-engineered isolators & amplifiers (A. Metelmann and AC, PRX 2015)



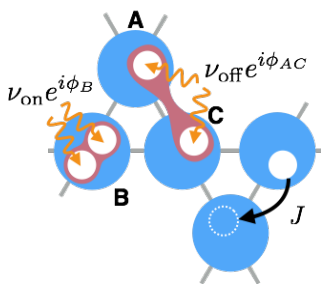


- Bandwidth of non-reciprocity set by response time of bath
- Generalization of “cascaded quantum systems” theory

Conclusions

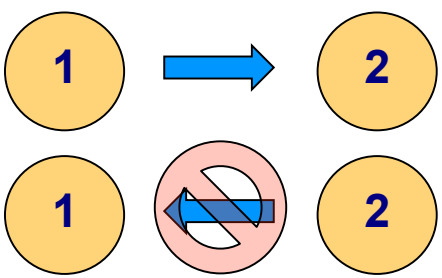
- Parametric driving as a route to new kinds of topological states
  (Peano, Houde, Brendel, Marquardt & AC, arXiv:1508.01383)

$$\hat{H}_{\rm par} = -\frac{\nu_{\rm on}}{2}\sum_j e^{i\phi_j}\,\hat{a}_j^\dagger\hat{a}_j^\dagger - \frac{\nu_{\rm off}}{2}\sum_{\langle ij\rangle} e^{i\phi_{ij}}\,\hat{a}_i^\dagger\hat{a}_j^\dagger + h.c.$$



  - Edge states which faciliate protected & directional inelastic transport and even amplification

- Reservoir engineering as a route to non-reciprocal interactions
  (A. Metelann and AC, PRX 2015)



  - A route to new kinds of non-recirpocal devices
  - A route to new kinds of interacting photonic phases?