Multipolar Order and Superconductivity in $Pr(TM)_2(Al,Zn)_{20}$ Kondo Materials

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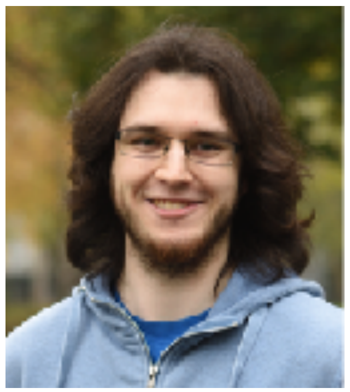
Jan 2018

Collaborators



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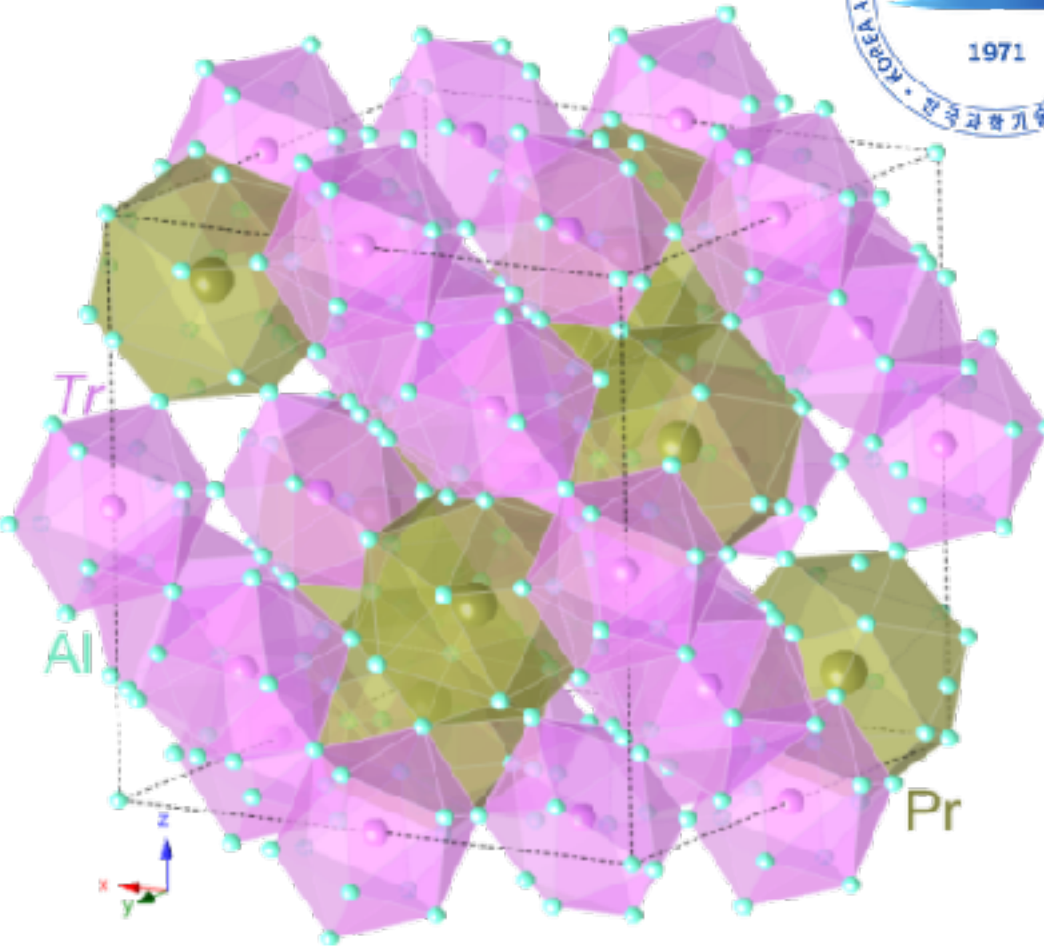
KAIST & KIAS

What is interesting?



$Pr(TM)_2(Al,Zn)_{20}$

Pr based cage compounds
Kondo materials

multipolar order : double transitions
anisotropy in fields

superconductivity : multipolar fluctuation
d-wave pairing

Motivation Strongly Correlated Electronic Systems

Strongly correlated materials

Quantum materials

Looking for new exotic phases ···

unconventional metal,

Mott insulators,

heavy fermions,

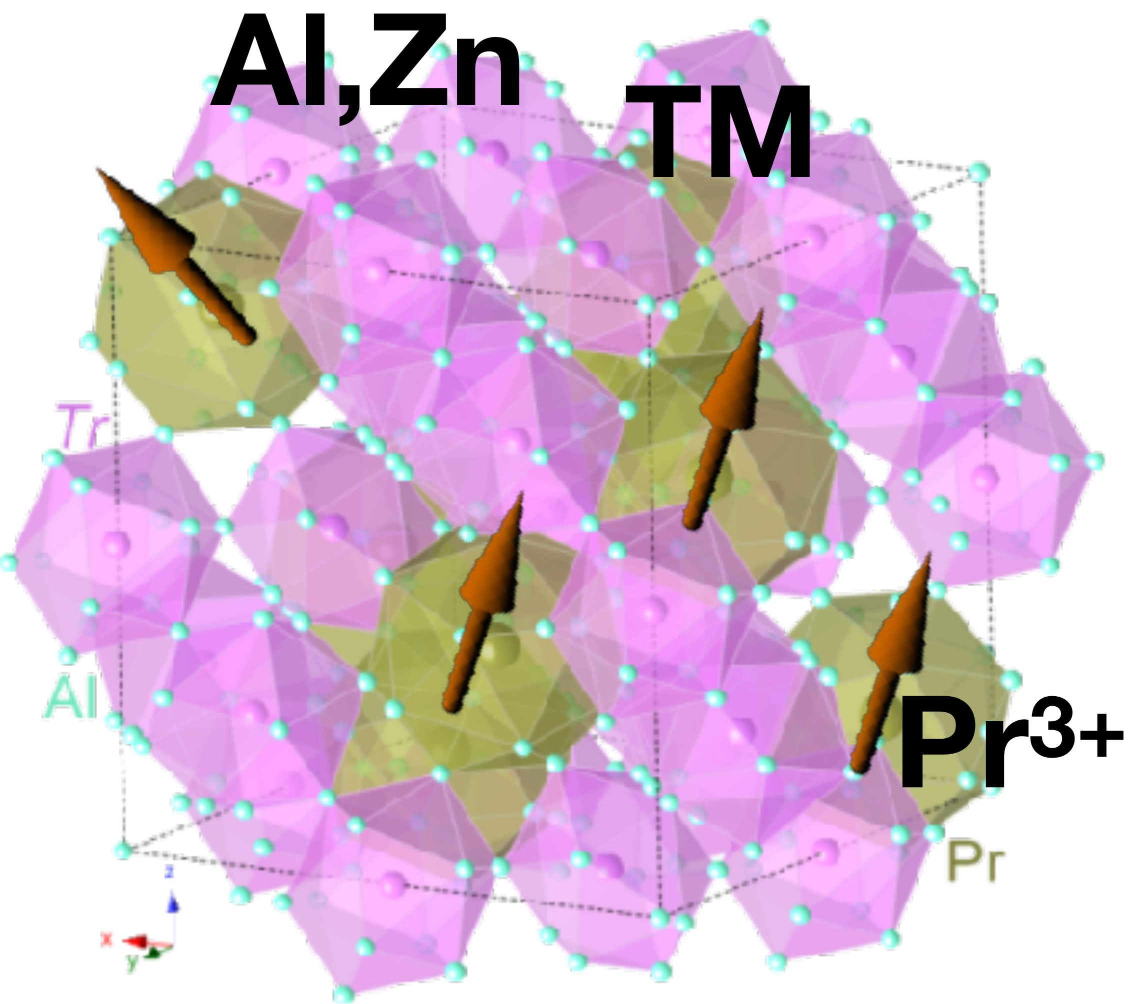
superconductivity,

magnetic order, spin liquids ···

localized moments,
Kondo coupling,
spin orbit coupling,
electronic structure,
electron-electron interactions ...

Motivation Strongly Correlated Electronic Systems

$Pr(TM)_2(Al,Zn)_{20}$



Kondo Materials,
Heavy fermion

Non-Fermi liquid

All interesting phenomena coexist!

Localized moments,
spin orbit coupling + CEF **Pr^{3+}**

Quadrupole-Octupolar ordering

Electronic Structure
spin orbit coupling
Fermi pockets at k=0 **(TM)+ Al,Zn**

e-e interactions
Kondo coupling

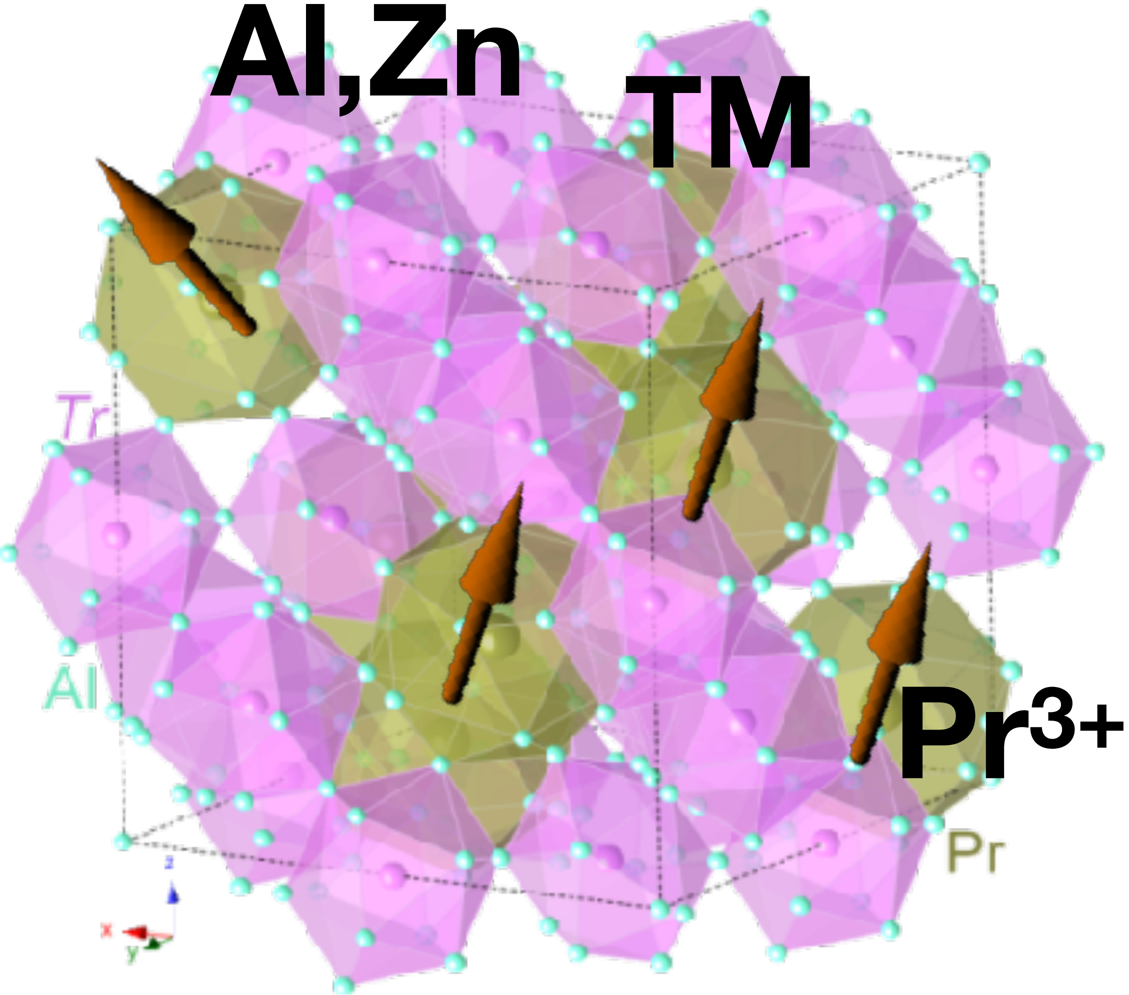
Pr^{3+} + (TM)+ Al,Zn

superconductivity

Motivation Strongly Correlated Electronic Systems

$Pr(TM)_2(Al,Zn)_{20}$

All interesting phenomena coexist!



Pr^{3+} **Quadrupole-Octupolar ordering**

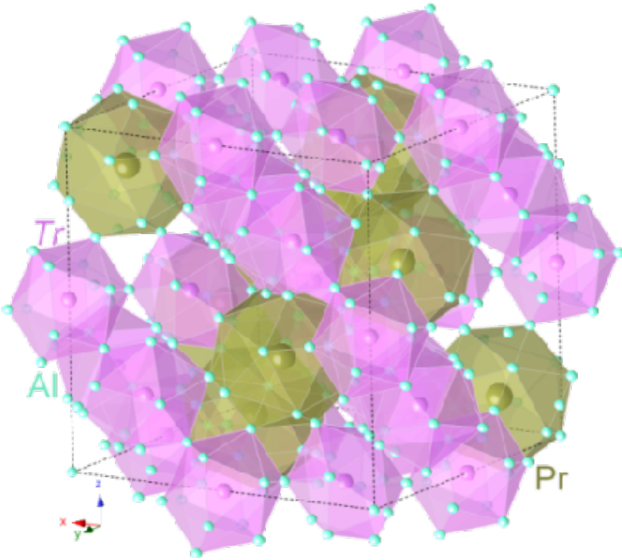
+ (TM)+ Al,Zn

superconductivity

Kondo Materials,
Heavy fermion

Q) How can we understand them?

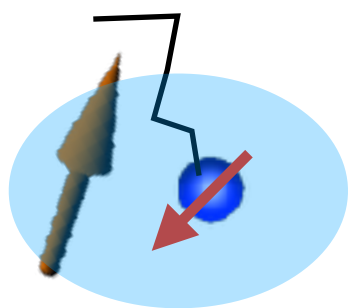
Outline $Pr(TM)_2(Al,Zn)_{20}$



- Introduction
  - Experiments on $Pr(TM)_2(Al,Zn)_{20}$
- Pr^{3+} localized moments
  - Modeling of pseudospin-1/2
  - Magnetic field effect
- Superconductivity
  - Molecular orbital picture and Luttinger semimetal
  - Kondo coupling : Quadrupolar moment

Introduction — Kondo effect

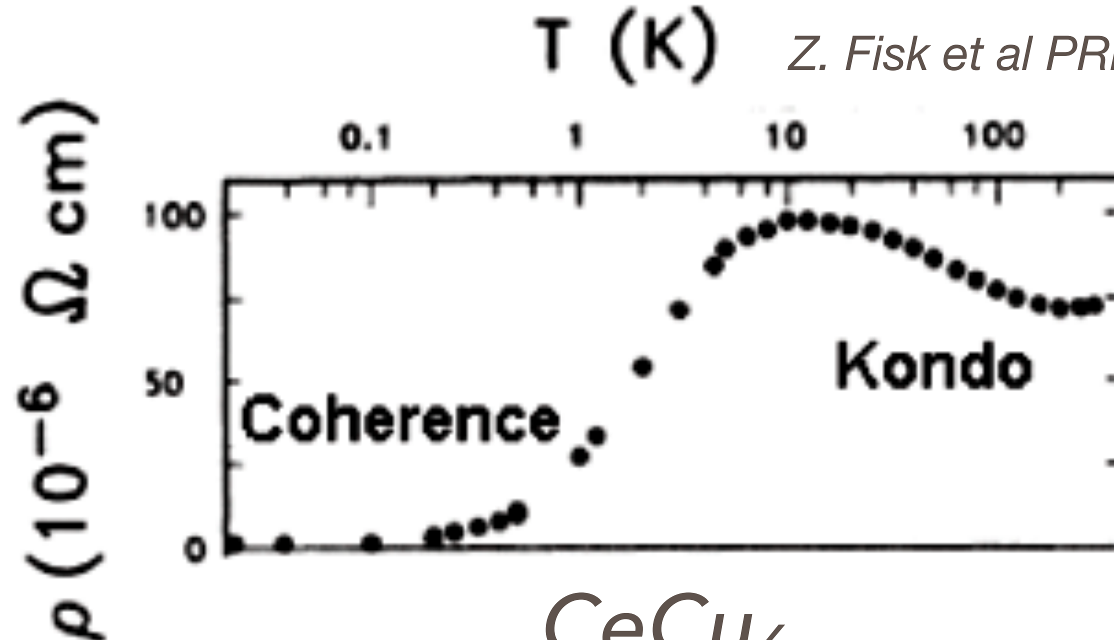
J. Kondo (1964)



Localized magnetic moment
+ itinerant electrons

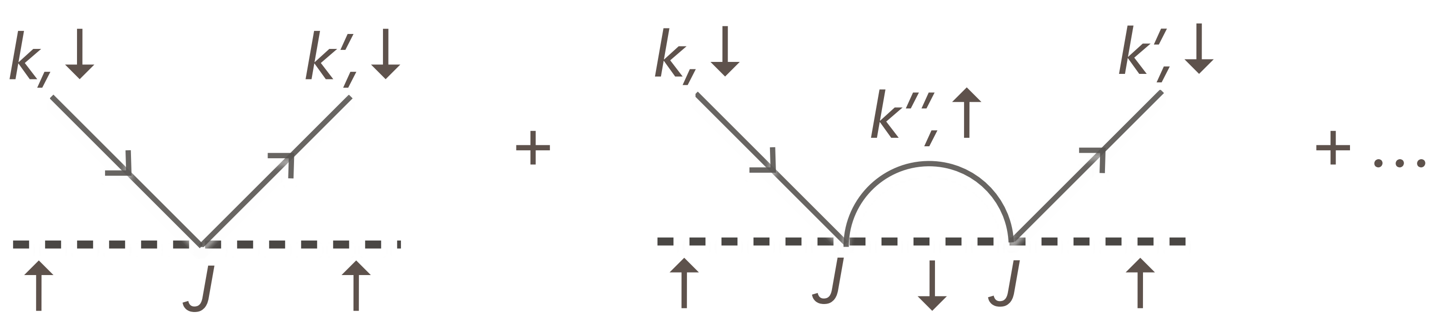
resistivity upturn in metal

Z. Fisk et al PRB 34, 5959 (1986)



$CeCu_6$

$H_K = J\, S_i \cdot c^\dagger_{i\,\alpha} \sigma_{\alpha\beta}\, c_{i\beta}$

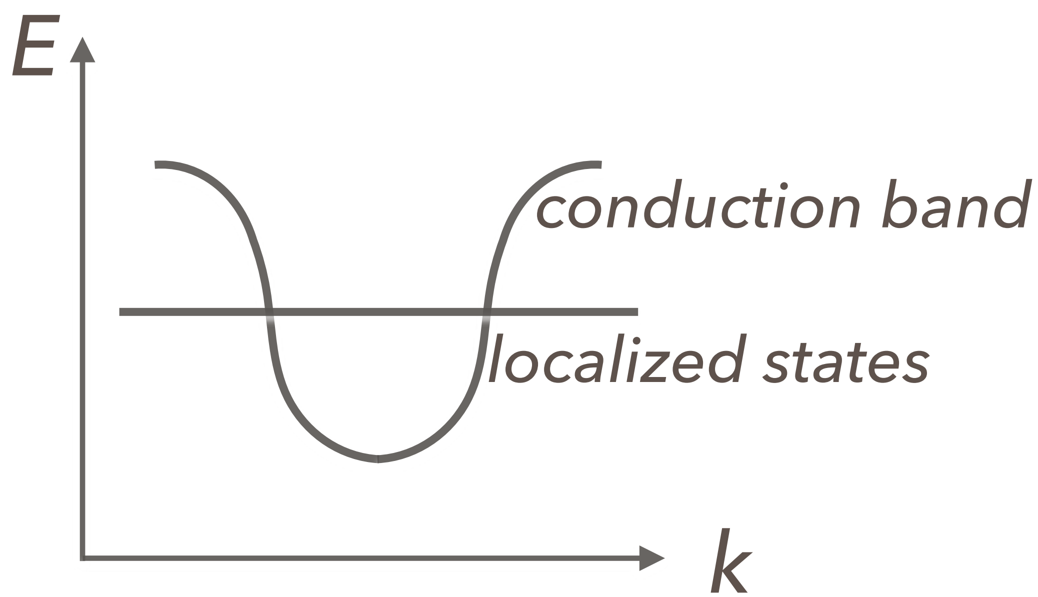


Logarithmic correction in resistivity due to spin flip scattering

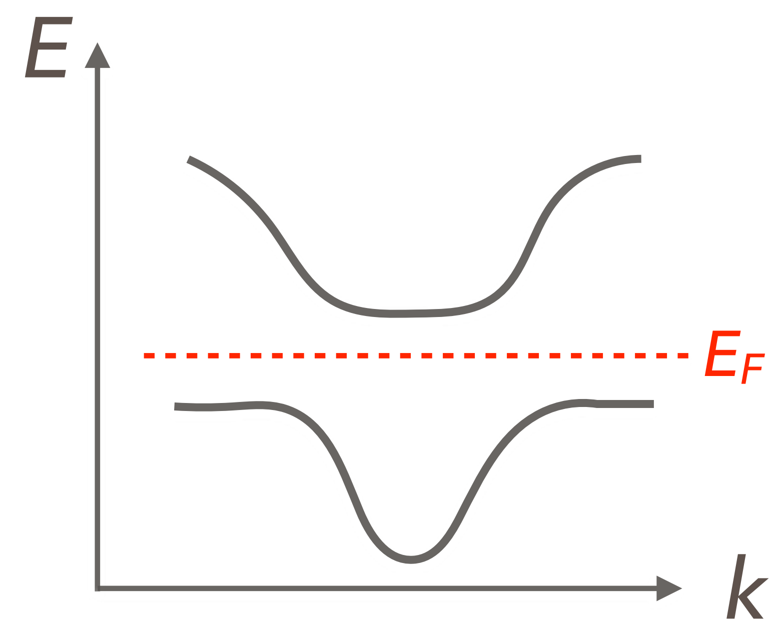
J

$J^2\rho \int dE_{k''}\, (1-f_{Ek''})/(E_k-E_{k''}) \sim J^2\rho \log (E_F/T)$

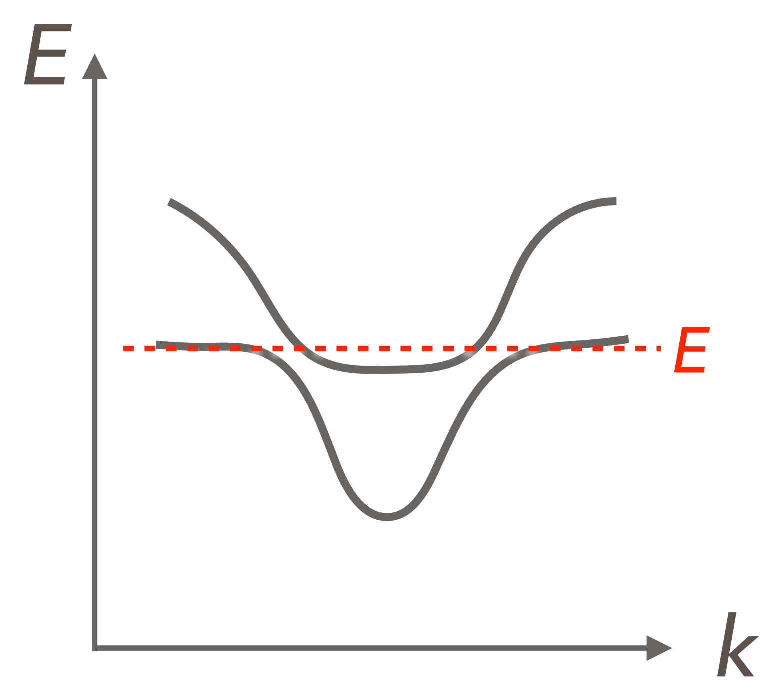
Introduction — Heavy Fermions

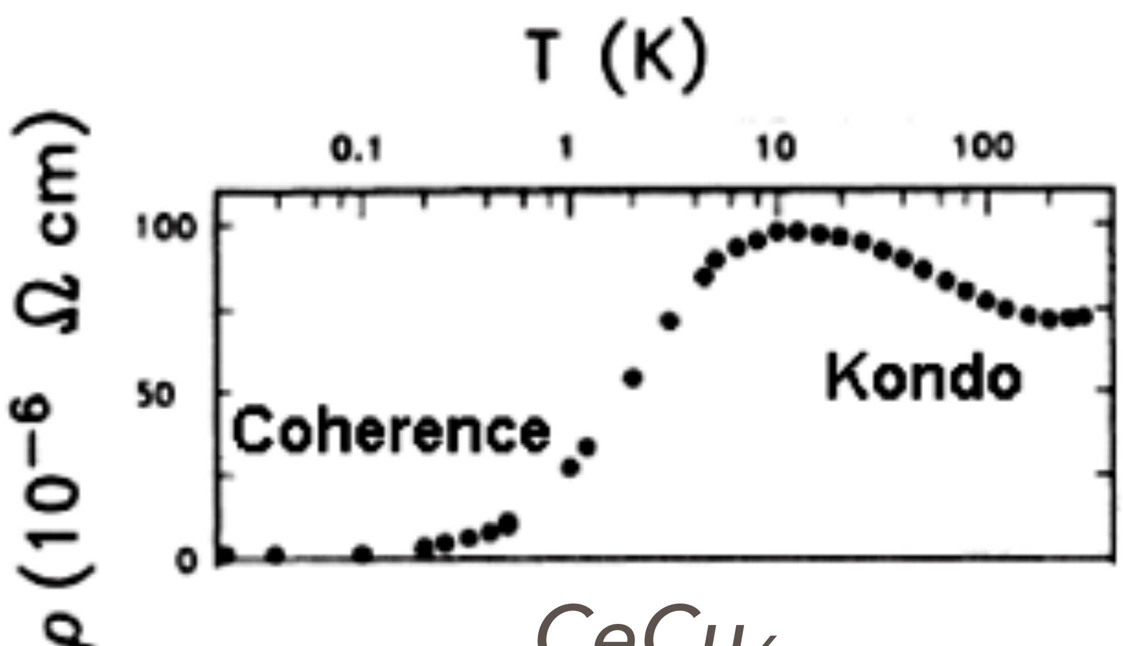


Kondo Insulator



Heavy Fermions



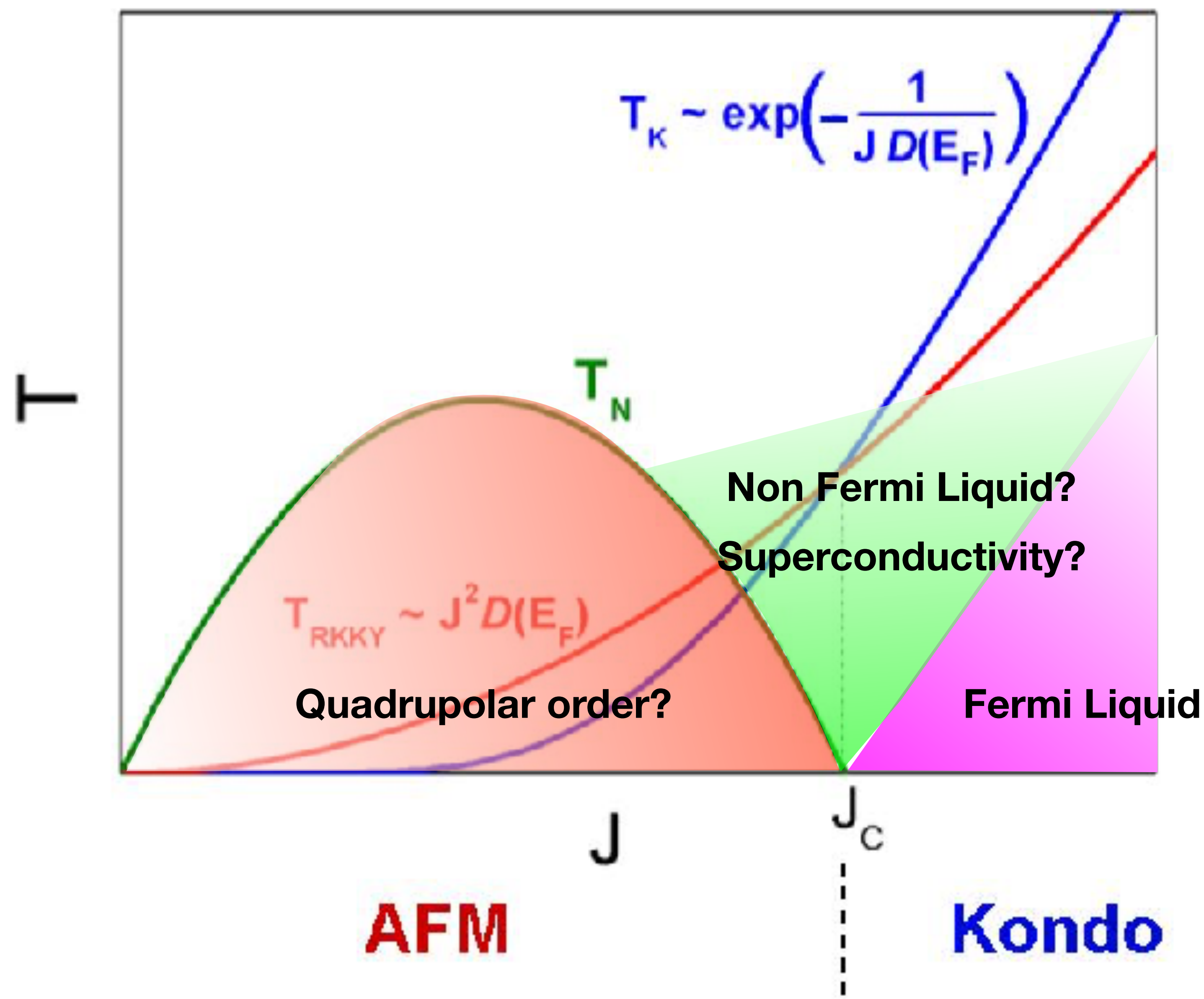


$CeCu_6$

Introduction — Doniach Phase Diagram

Doniach phase diagram

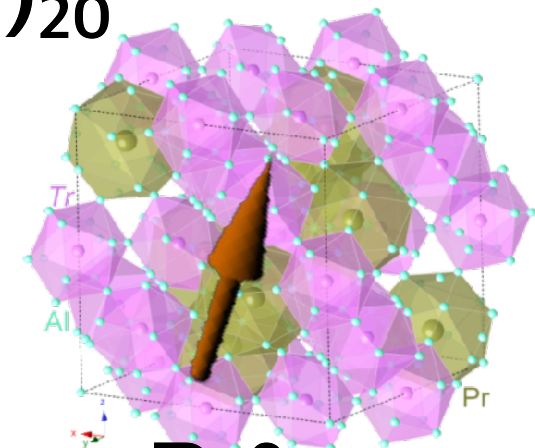
S. Doniach, Physica B **91**, 231 (1977)



Q) Search for new Doniach phase diagram with multipolar order ?

Multipolar order in $Pr(TM)_2(Al,Zn)_{20}$

Multipolar order with Γ_3 doublets in $Pr(TM)_2(Al,Zn)_{20}$



Pr^{3+} $4f^2$ non Kramers Γ_3 doublets

Spin orbit coupling
+ Crystalline Electric Field

$$|\Gamma_1\rangle = \frac{1}{2}\sqrt{\frac{5}{6}}|+4\rangle + \frac{1}{2}\sqrt{\frac{7}{3}}|0\rangle + \frac{1}{2}\sqrt{\frac{5}{6}}|-4\rangle$$

$$|\Gamma_{5\pm}^{(2)}\rangle = \frac{1}{2}\sqrt{\frac{7}{2}}|\pm 3\rangle - \frac{1}{2}\sqrt{\frac{1}{2}}|\mp 1\rangle$$

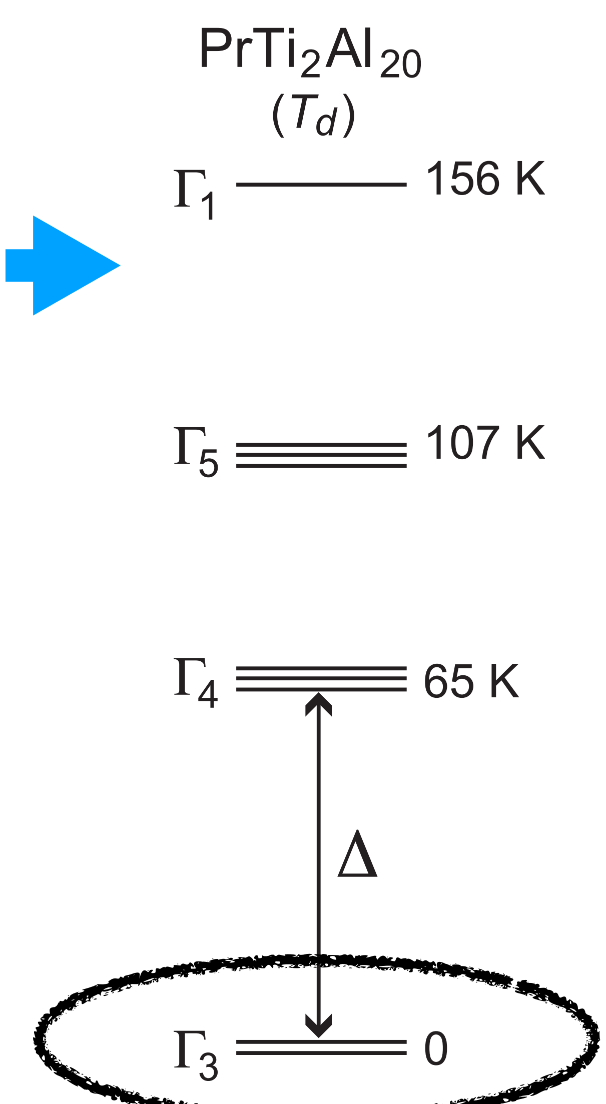
$$|\Gamma_5^{(2)}\rangle = \sqrt{\frac{1}{2}}|+2\rangle - \sqrt{\frac{1}{2}}|-2\rangle$$

$$|\Gamma_{4\pm}^{(1)}\rangle = \frac{1}{2}\sqrt{\frac{1}{2}}|\mp 3\rangle + \frac{1}{2}\sqrt{\frac{7}{2}}|\pm 1\rangle$$

$$|\Gamma_4^{(2)}\rangle = \sqrt{\frac{1}{2}}|+4\rangle - \sqrt{\frac{1}{2}}|-4\rangle$$

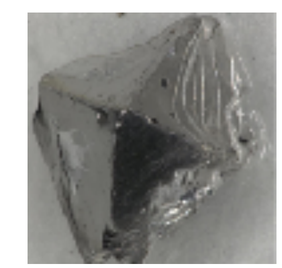
$$|\Gamma_3^{(1)}\rangle = \frac{1}{2}\sqrt{\frac{7}{6}}|+4\rangle - \frac{1}{2}\sqrt{\frac{5}{3}}|0\rangle + \frac{1}{2}\sqrt{\frac{7}{6}}|-4\rangle$$

$$|\Gamma_3^{(2)}\rangle = \sqrt{\frac{1}{2}}|+2\rangle + \sqrt{\frac{1}{2}}|-2\rangle$$

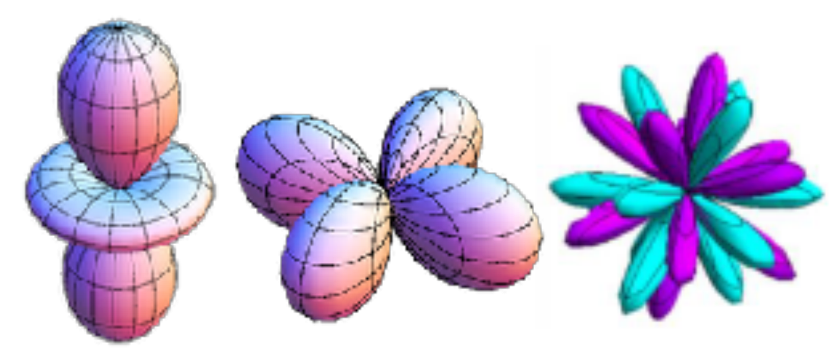


TJ Sato et al PRB 86, 184419 (2012)

Pr^{3+}



pseudospin-1/2 with Γ_3 doublets describes



$3J_z^2 - J^2$ **$J_x^2 - J_y^2$** **$J_xJ_yJ_z$**

Quadrupole **Octupole**

Multipolar order in $Pr(TM)_2(Al,Zn)_{20}$

Multipolar order with Γ_3 doublets in $Pr(TM)_2(Al,Zn)_{20}$

Pr^{3+} $4f^2$ non Kramers Γ_3 doublets

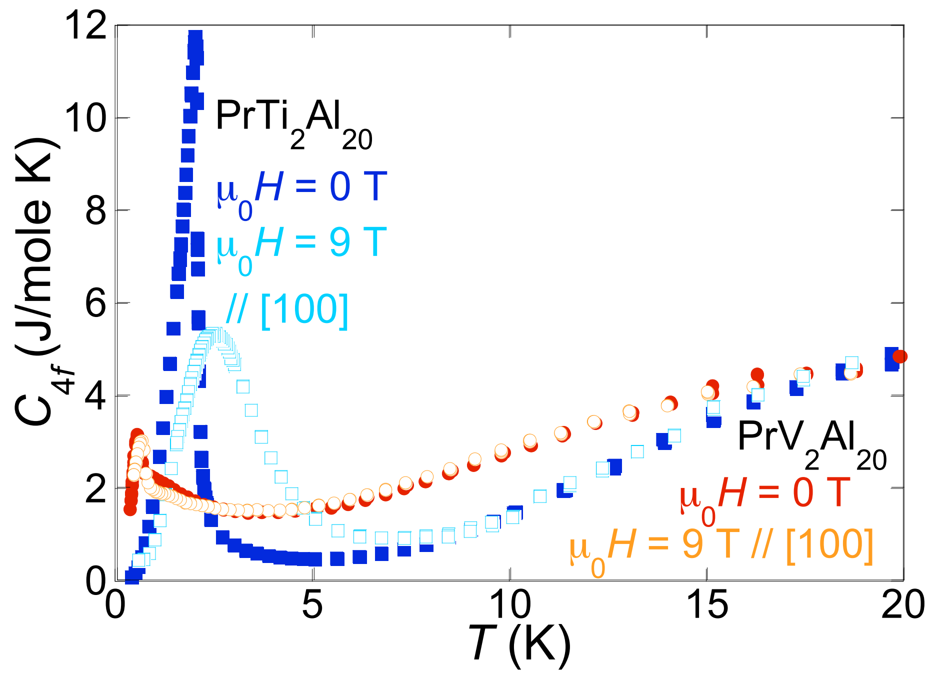
Quadrupolar order at T_Q

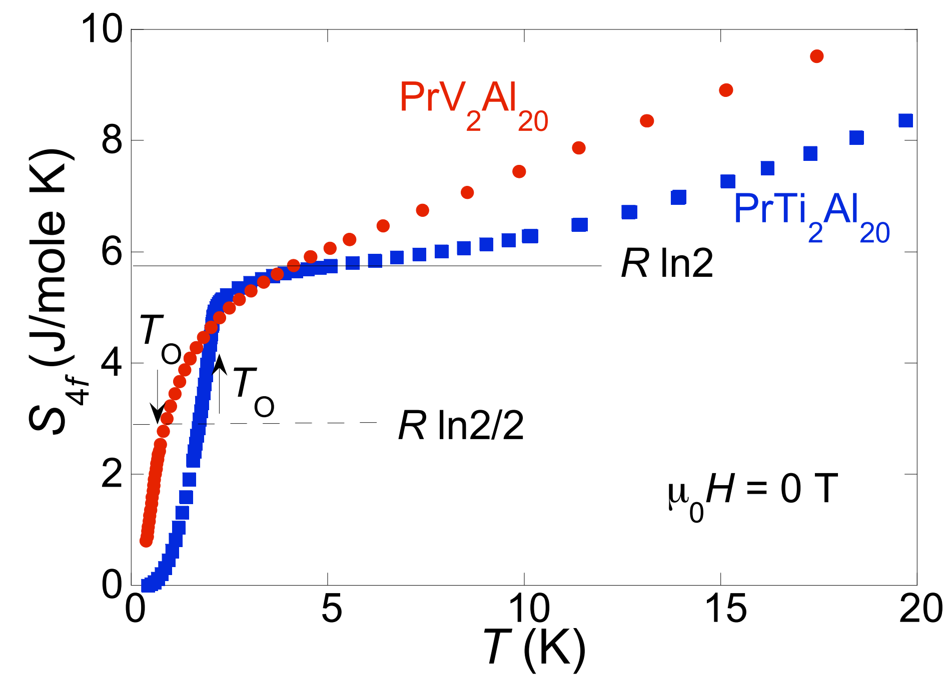
	$T_s(K)$	$T_{N/Q}(K)$	$T_c(K)$	SC type	structure ($T < T_s$)	$\rho(\mu\Omega cm)$ (at RT)	pressure
Al	-	-	1.2	I	fcc	2.82	$T_c \downarrow$
Zn	-	-	0.85	I	hexagonal	5.9	$T_c \uparrow$
$PrTi_2Al_{20}$	-	2	0.2	II	$Fd\bar{3}m$	?	$T_c \uparrow$
PrV_2Al_{20}	-	0.9	-	-	$Fd\bar{3}m$	?	?
LaV_2Al_{20}	-	-	-	-	$Fd\bar{3}m$	?	?
$Al_{0.3}V_2Al_{20}$	-	-	1.49	?	$Fd\bar{3}m$	80	?
$Ga_{0.2}V_2Al_{20}$	-	-	1.66	?	$Fd\bar{3}m$	100	?
YV_2Al_{20}	-	-	0.69	?	$Fd\bar{3}m$	60	?
$PrRh_2Zn_{20}$	140	0.06	0.06	?	?	80	?
$PrIr_2Zn_{20}$	-	0.2	0.05	?	$Fd\bar{3}m$	90	$T_Q \uparrow$
$LaIr_2Zn_{20}$	200	-	0.6	?	?	100	$T_s \uparrow, T_c \downarrow$
$PrRu_2Zn_{20}$	138	none	-	-	?	90	$T_s \downarrow$
$LaRu_2Zn_{20}$	150	-	0.2	?	?	100	$T_s \downarrow, T_c \downarrow$

Multipolar order in $Pr(TM)_2(Al,Zn)_{20}$

Multipolar order with Γ_3 doublets in $Pr(Ti, V)_2Al_{20}$

A. Sakai and S. Nakatsuji, J. Phys. Soc. Jpn . **80** (2011) 063701



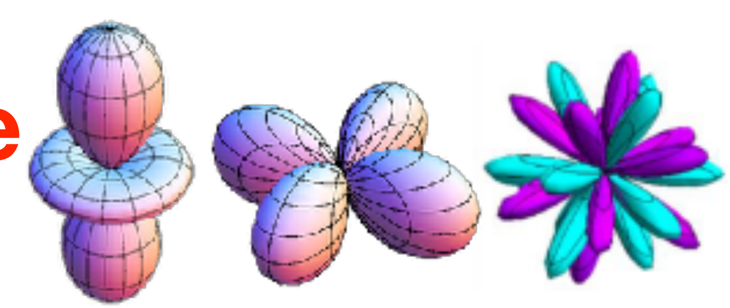


- Phase transitions at T = 2.0K (Ti), T=0.9K, 0.6K (V)
- No Zeeman gap even at H=9T

Nonmagnetic ground state

- Crystalline Electric Field gap Δ ~ 60K (Ti), 40K (V)
- Entropy S ~ R ln 2

ground doublet Γ_3



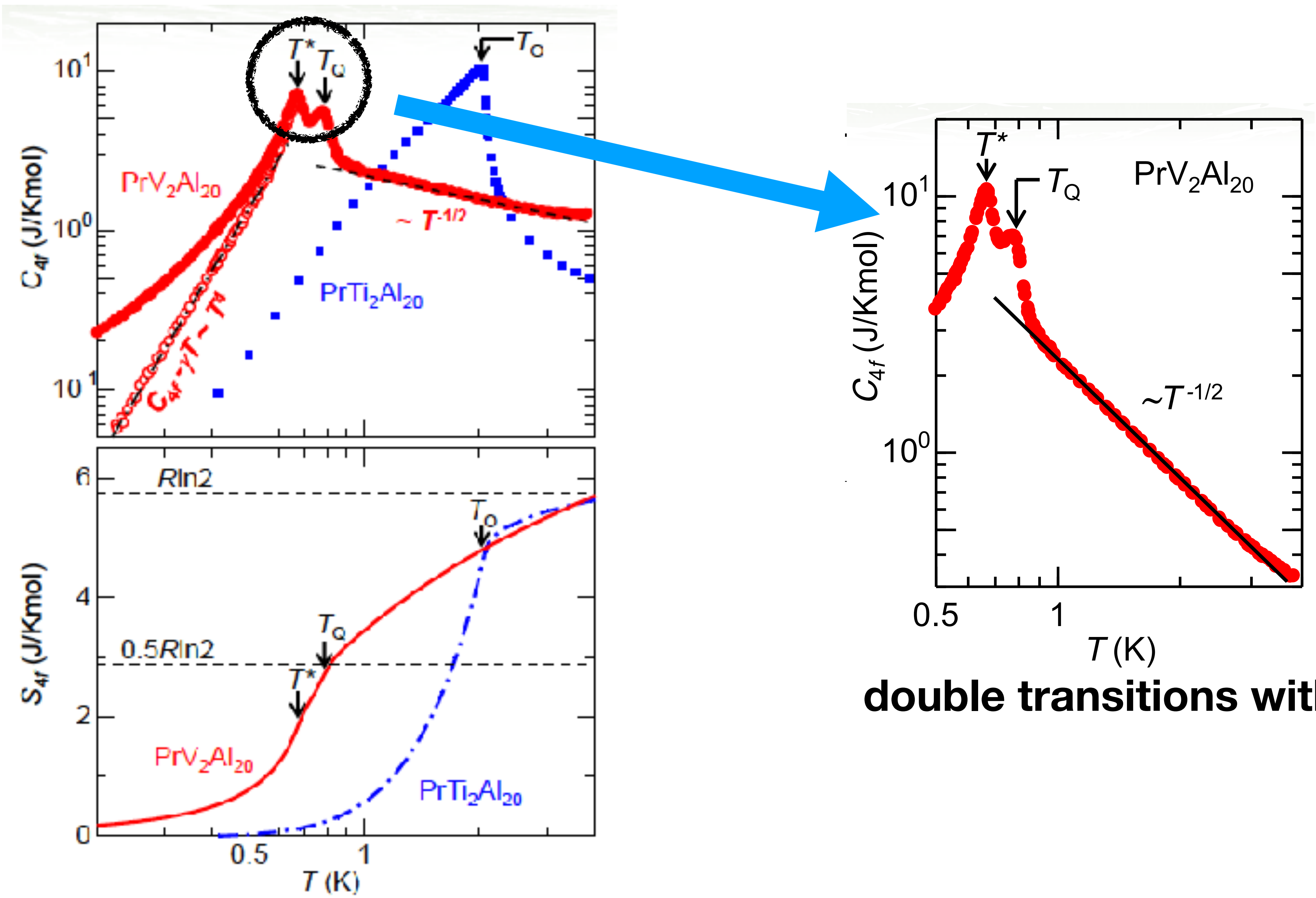
$3J_z^2 - J^2$ **$J_x^2 - J_y^2$** **$J_xJ_yJ_z$**

Quadrupole **Octupole**

Multipolar order in $Pr(TM)_2(Al,Zn)_{20}$

Multipolar order with Γ_3 doublets in $Pr(Ti, V)_2Al_{20}$

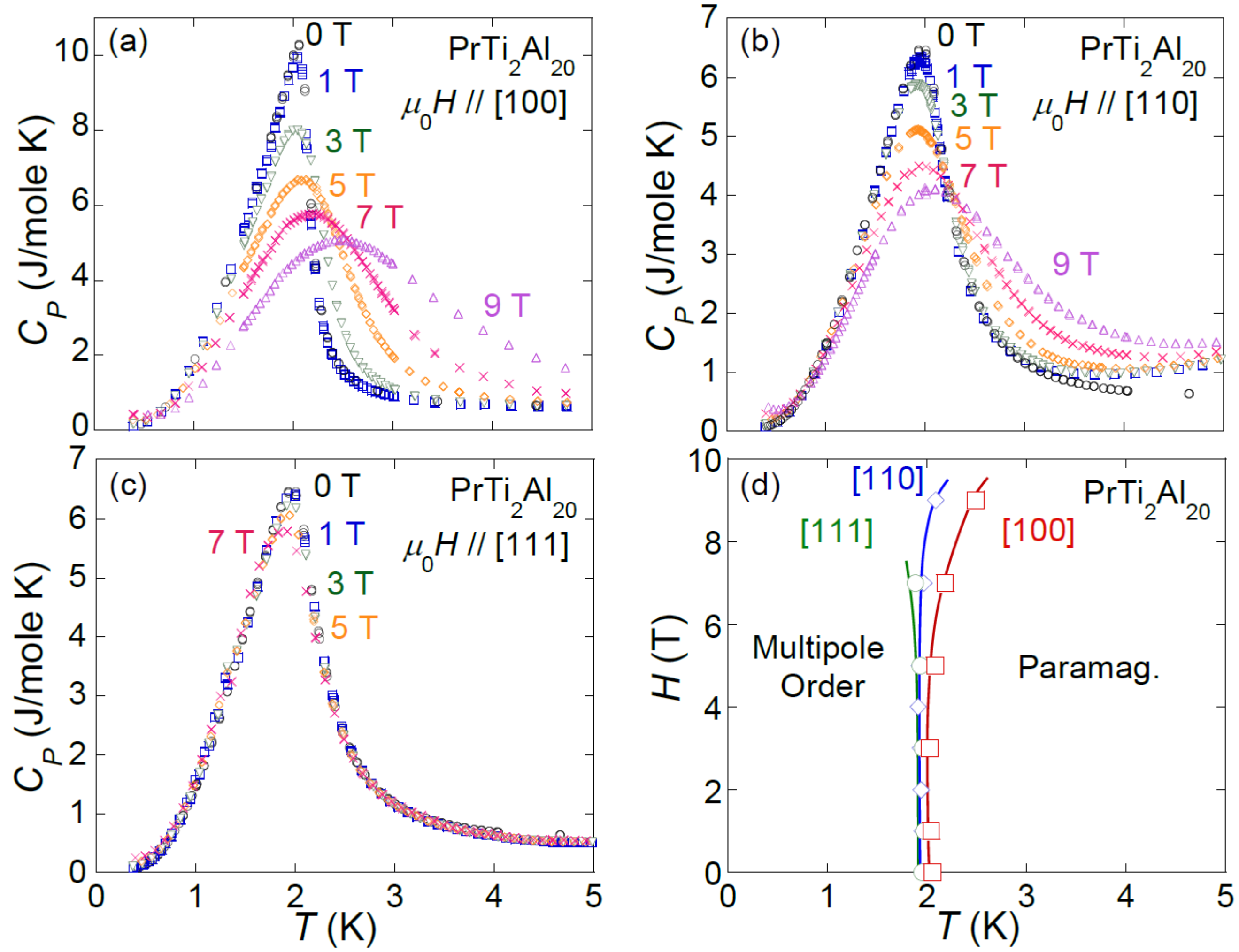
Single transition (Ti) vs double transition (V)



double transitions with V ?

Multipolar order in fields

Multipolar order with Γ_3 doublets in $PrTi_2Al_{20}$ in fields

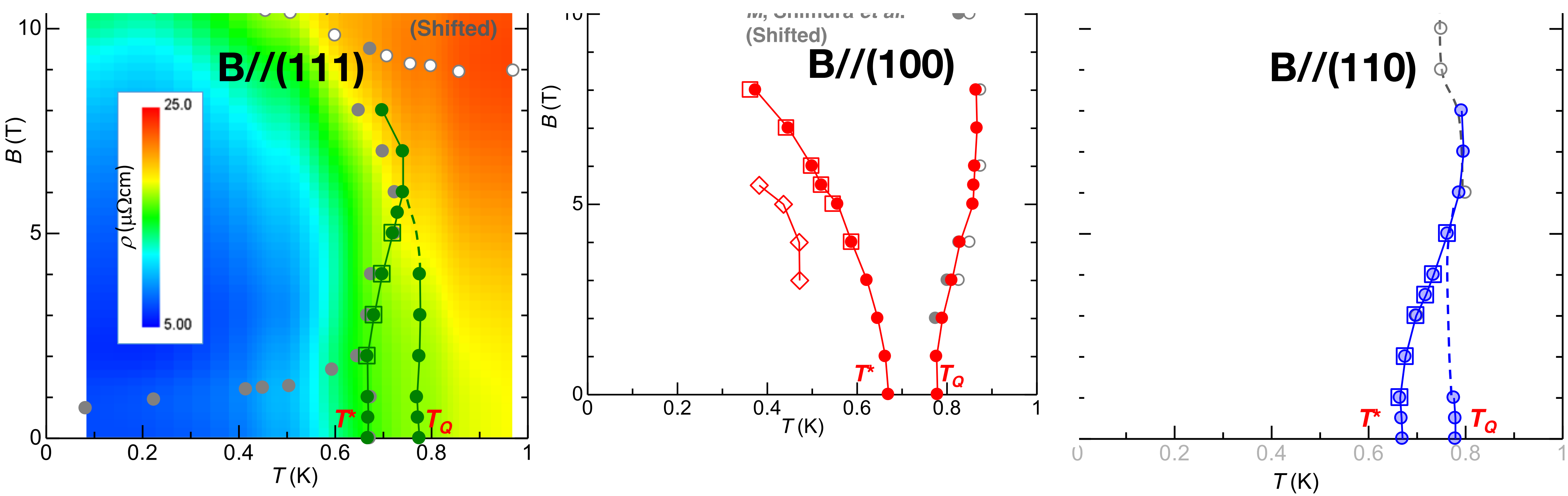


$PrTi_2Al_{20}$ **Ferro-quadrupolar order does not couple to B//(111)**

- **specific heat barely changes with field**
- **enhanced Tc for B//(100) or (110) - expected for ferro**
- **insensitive up to 6T - large crystal field splitting Δ (huff ~ B^2/Δ)**

Multipolar order in fields

Multipolar order with Γ_3 doublets in PrV_2Al_{20} in fields



Y. Shimura *et al.*, JPSJ **82**, 043705(2013).

Y. Shimura *et al.*, Physical Review B **91**, 241102(R) (2015).

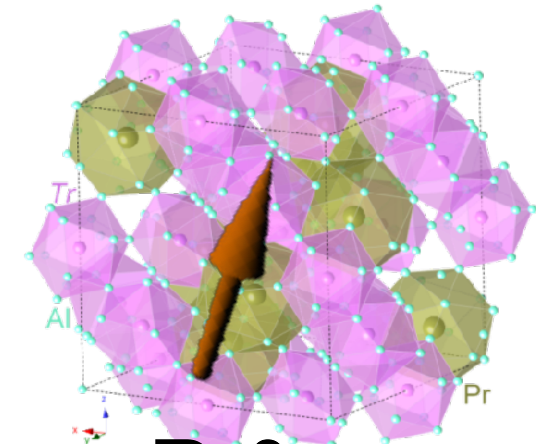
PrV_2Al_{20} **Antiferro-quadrupolar order - insensitive with small fields**

- **double transition - 0.9K (high-T transition) and 0.65K (low-T transition)**

Q) How can we understand these phenomena?

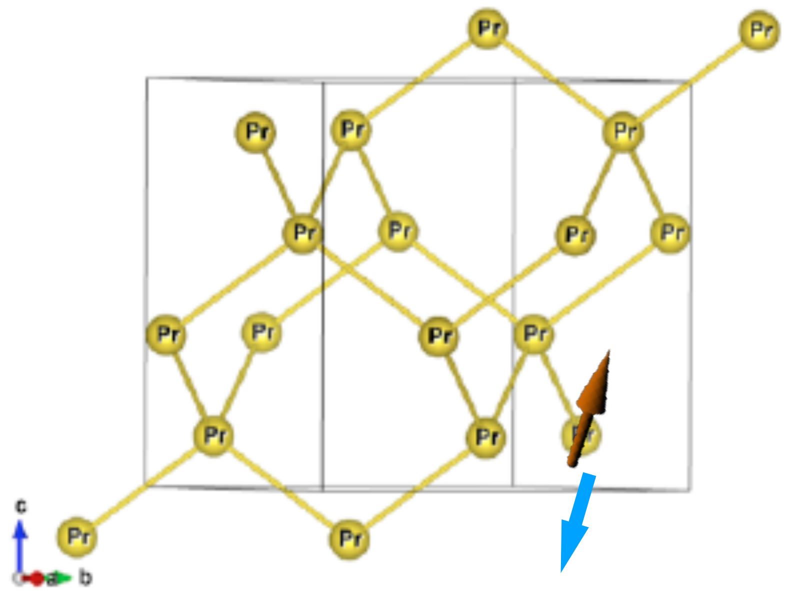
Multipolar order and finite T transitions

Multipolar order with Γ_3 doublets in Pr^{3+}



Pr^{3+}

Pr^{3+} ions form a diamond lattice



Modeling of pseudospin-1/2 — τ

Kondo coupling with itinerant electrons
—> multiple spin interactions

Γ_3 doublets describes

$J_x^2 - J_y^2$ $3J_z^2 - J^2$ $J_xJ_yJ_z$

Quadrupole Octupole

τ_x τ_y τ_z

in pseudospin-1/2 basis

Quadrupolar moments - TR even

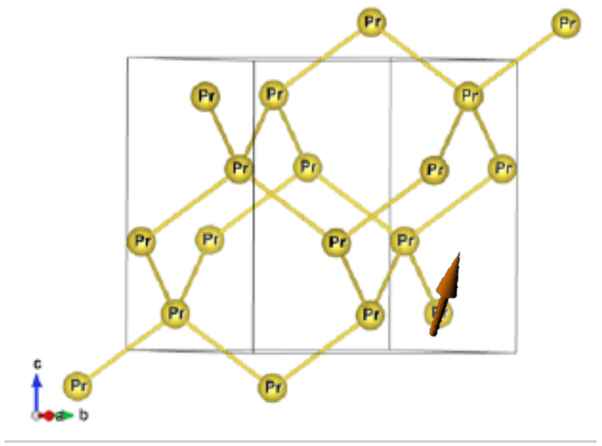
$$H = \frac{1}{2}\sum_{i,j} J_{ij}(\vec{\tau}_i^{\perp}\cdot\vec{\tau}_j^{\perp} + \lambda\tau_i^z\tau_j^z) - K\sum_{\langle\langle ij\rangle\langle km\rangle\rangle}\vec{\tau}_i^{\perp}\cdot\vec{\tau}_j^{\perp}\tau_k^z\tau_m^z .$$

Octupolar moments -TR odd

Multipolar order and finite T transitions

Multipolar order with Γ_3 doublets in Pr^{3+}

Modeling of pseudospin-1/2 — τ



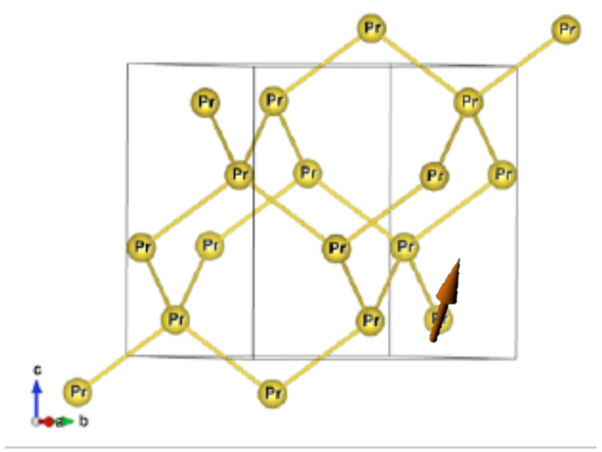
$$H = \frac{1}{2}\sum_{i,j} J_{ij}(\vec{\tau}_i^{\perp}\cdot\vec{\tau}_j^{\perp} + \lambda\tau_i^z\tau_j^z) - K\sum_{\langle\langle ij\rangle\langle km\rangle\rangle}\vec{\tau}_i^{\perp}\cdot\vec{\tau}_j^{\perp}\tau_k^z\tau_m^z\,.$$

Q) Possible phases and finite T transitions ?

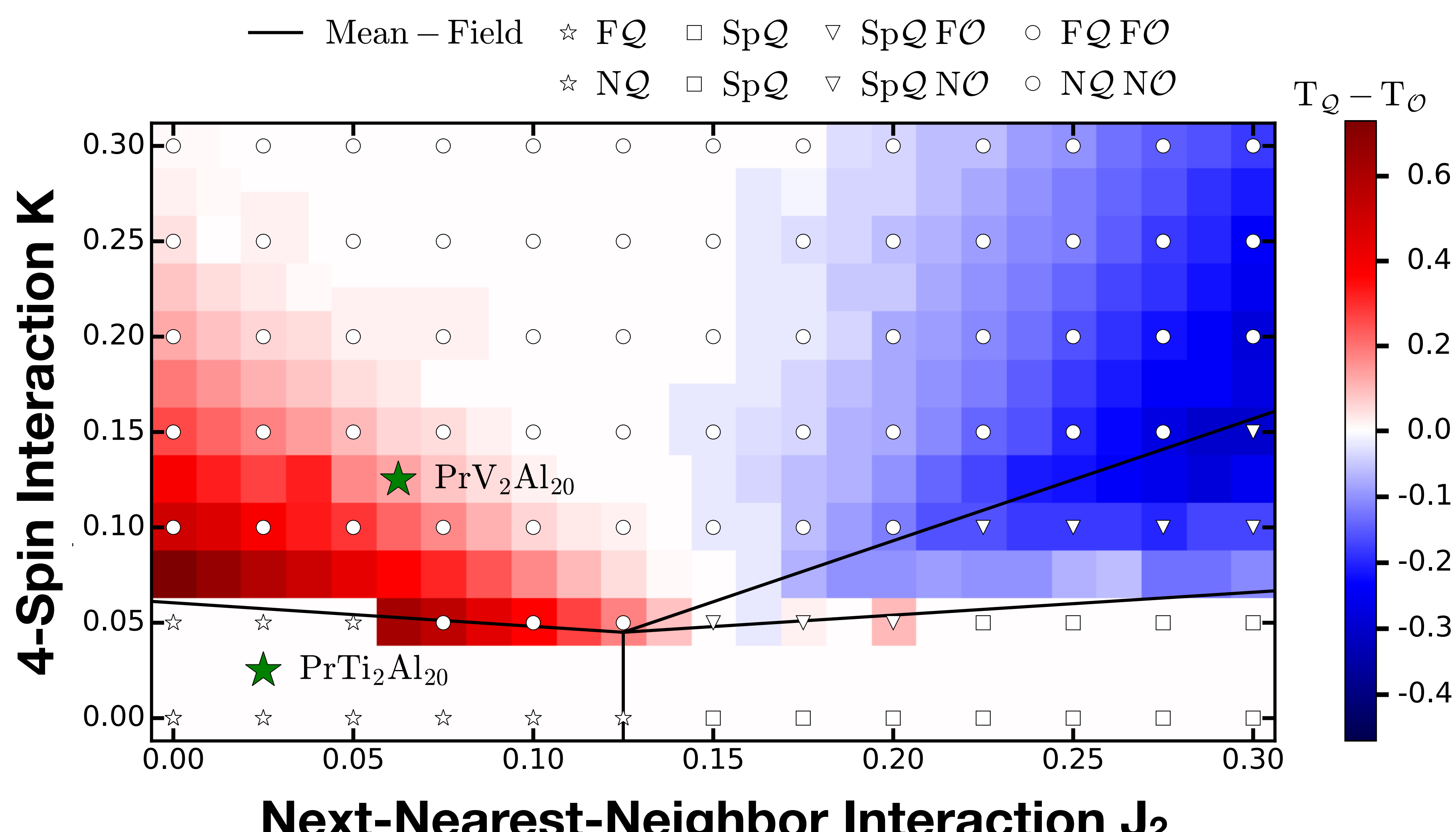
Multipolar order and finite T transitions

Quadrupolar, Octupolar orderings

$$H = \frac{1}{2}\sum_{i,j} J_{ij}\left(\vec{\tau}_i^{\perp}\cdot\vec{\tau}_j^{\perp} + \lambda \tau_i^z \tau_j^z\right) - K \sum_{\langle\langle ij\rangle\langle km\rangle\rangle} \vec{\tau}_i^{\perp}\cdot\vec{\tau}_j^{\perp}\tau_k^z\tau_m^z \,.$$



Monte-Carlo Results



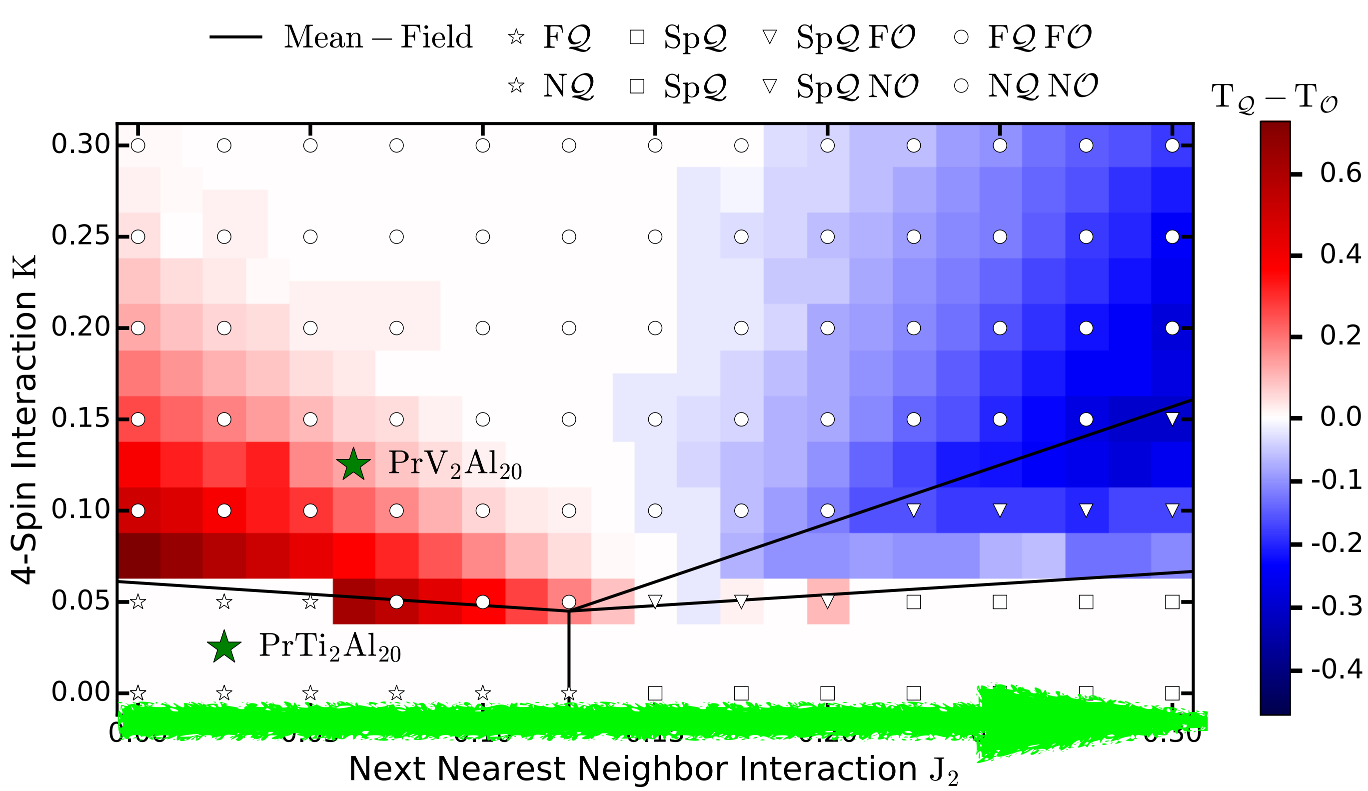
multispin interactions + frustration

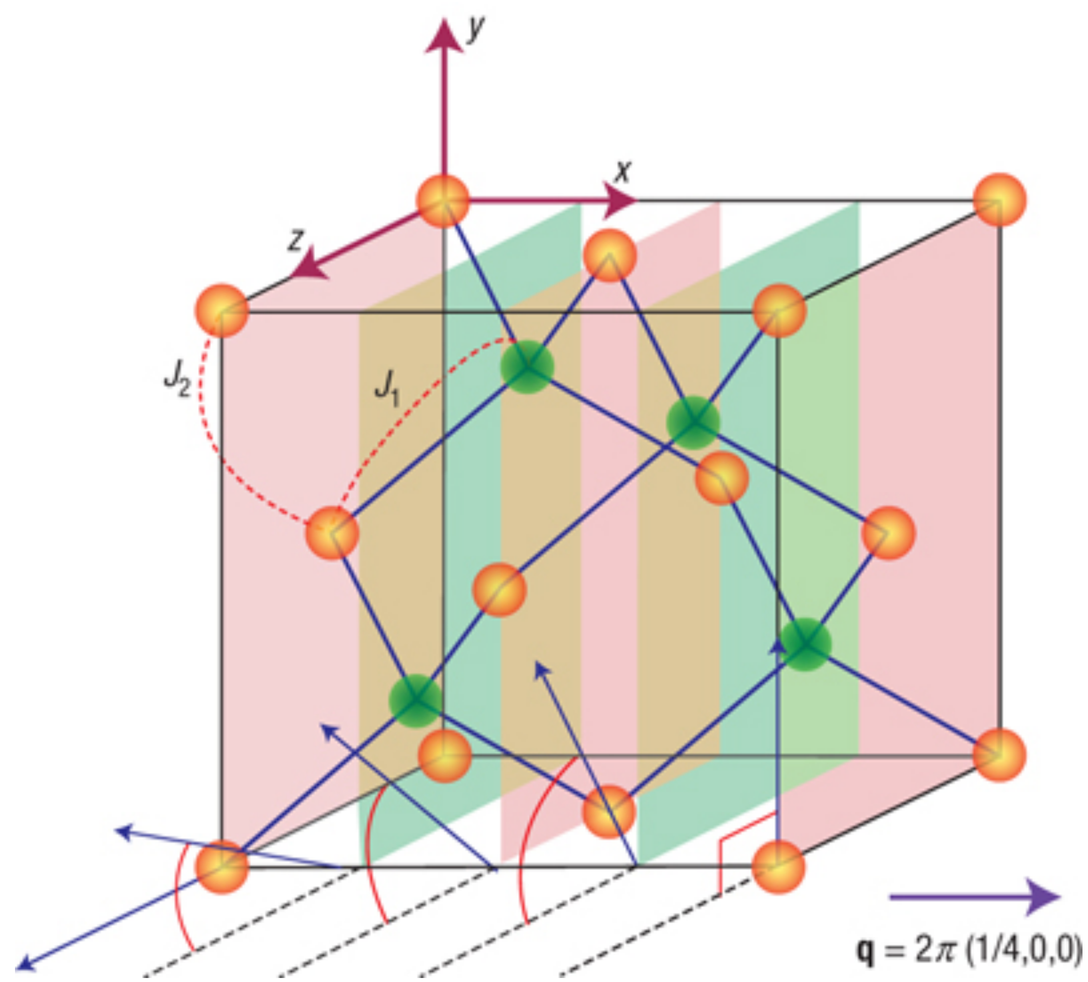
F. Freyer, J. Attig, **SBL**, A. Paramekanti, S. Trebst and Y.B Kim **ArXiv 1709.06094 (2017)**

Multipolar order and finite T transitions

Quadrupolar, Octupolar orderings

Phase diagram based on MC

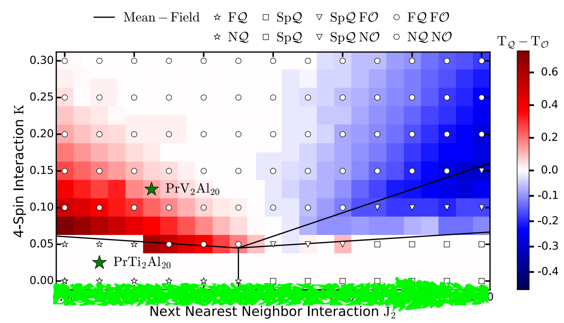




Quadrupolar order : (Anti-) Ferro → spiral order with finite-Q

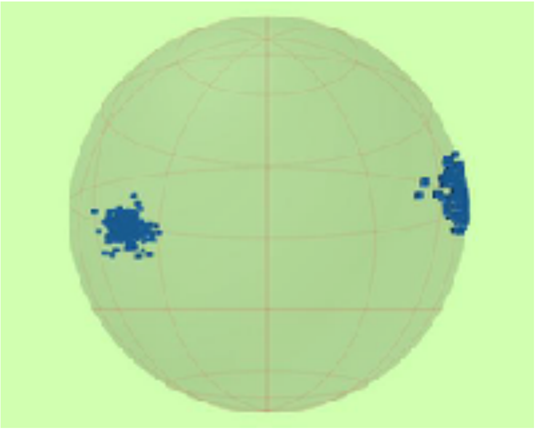
Multipolar order and finite T transitions

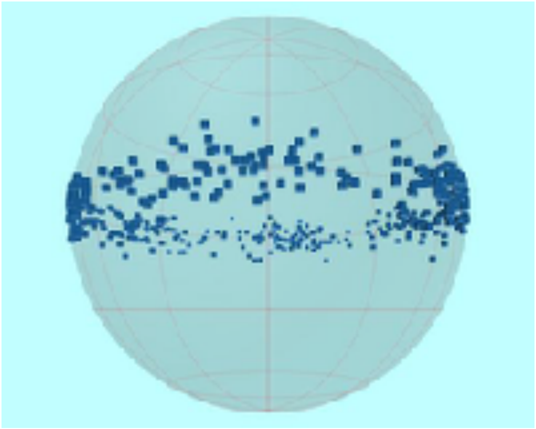
Quadrupolar, Octupolar orderings

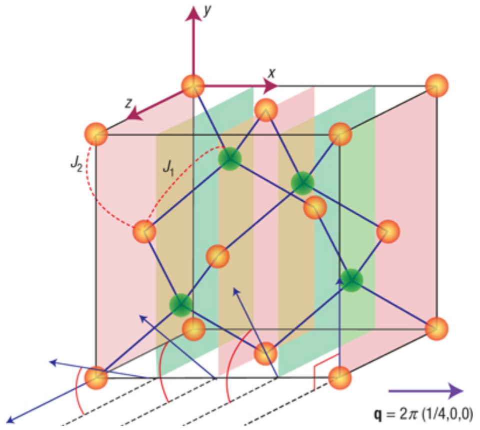


Quadrupolar order : (Anti-) Ferro **spiral order with finite-Q**

Common origin plots of τ







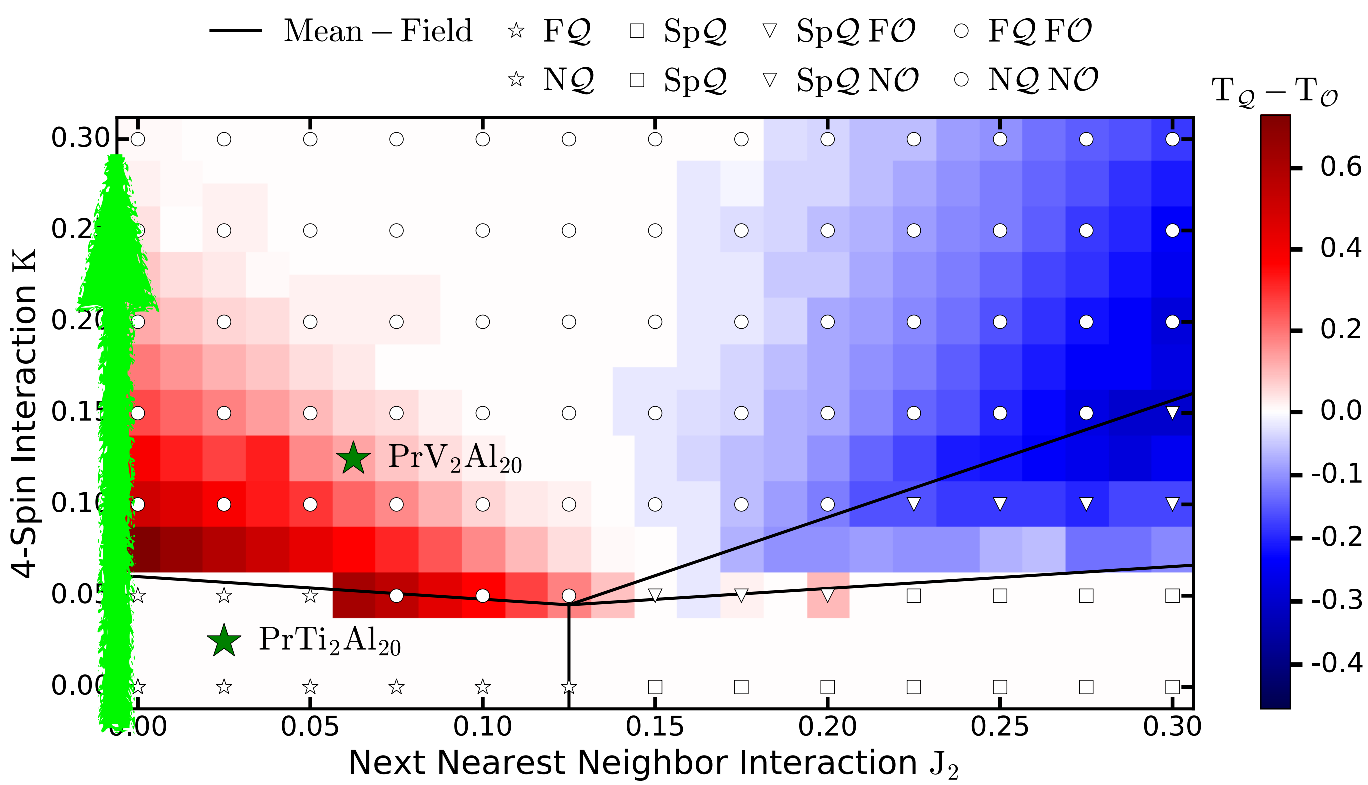
J_2

with increasing next-nearest neighbor J_2

Multipolar order and finite T transitions

Quadrupolar, Octupolar orderings

Phase diagram based on MC



$T_Q > T_O$

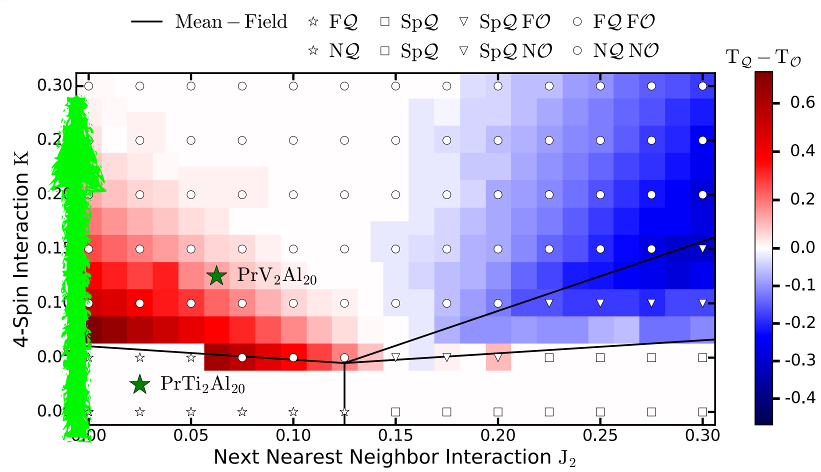
$T_Q < T_O$

Pure (anti-) ferro quadrupolar order

→ **quadrupolar + octupolar coexisting order with** $T_Q > T_O$

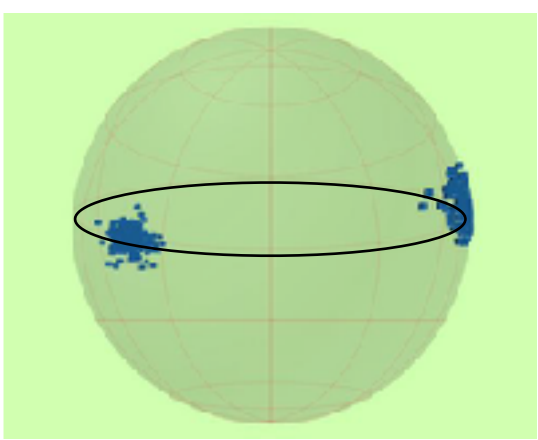
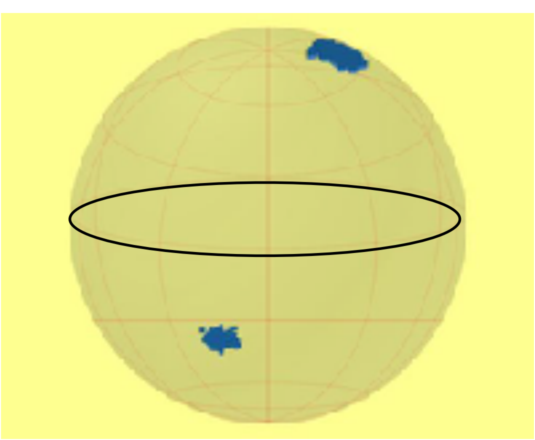
Multipolar order and finite T transitions

Quadrupolar, Octupolar orderings



pure quadrupolar order → **quadrupolar + octupolar coexisting order**

Common origin plots of τ

 → 

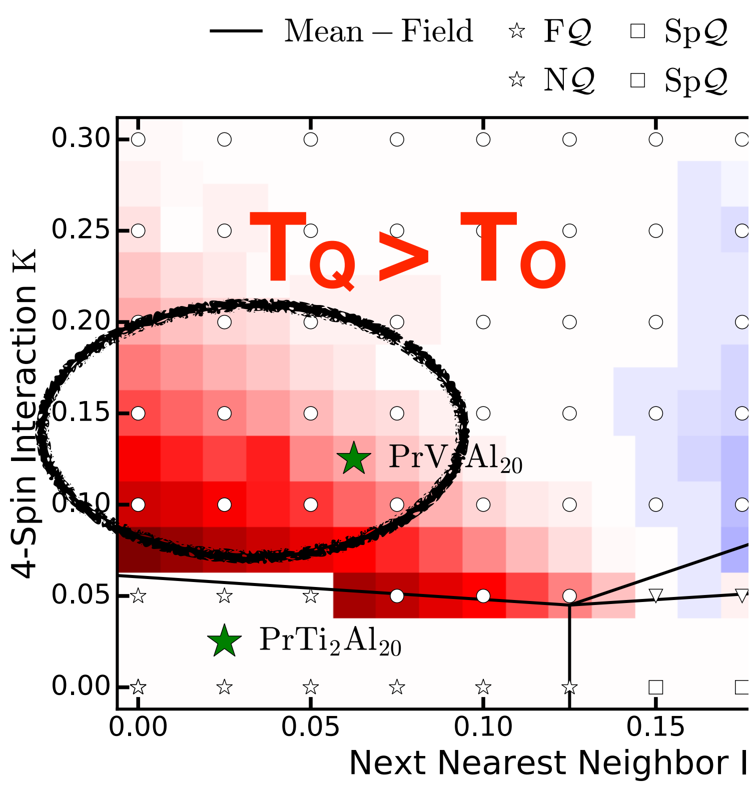
K

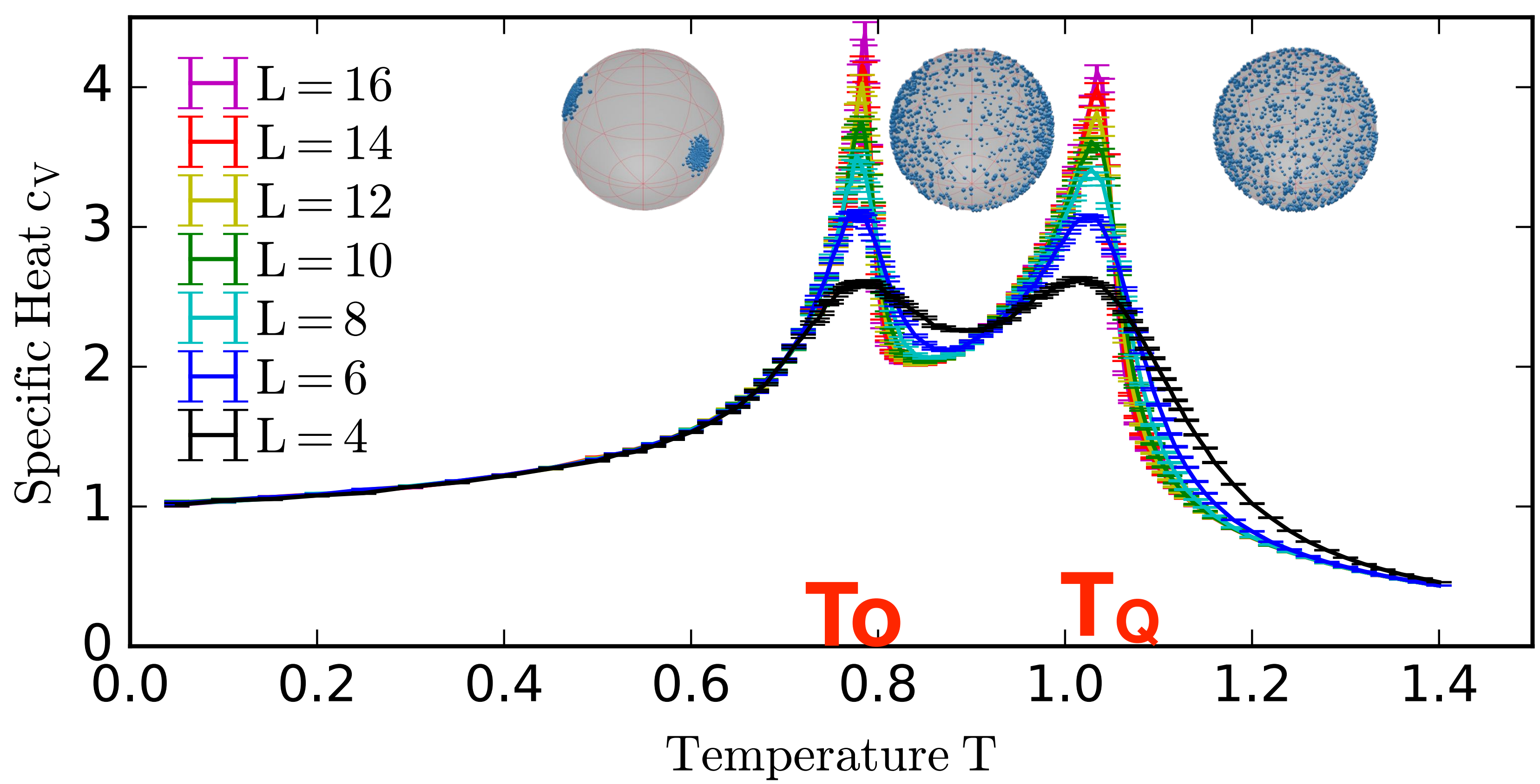
with increasing four spin interaction K

Multipolar order and finite T transitions

Quadrupolar, Octupolar orderings

quadrupolar + octupolar coexisting order at finite T ?

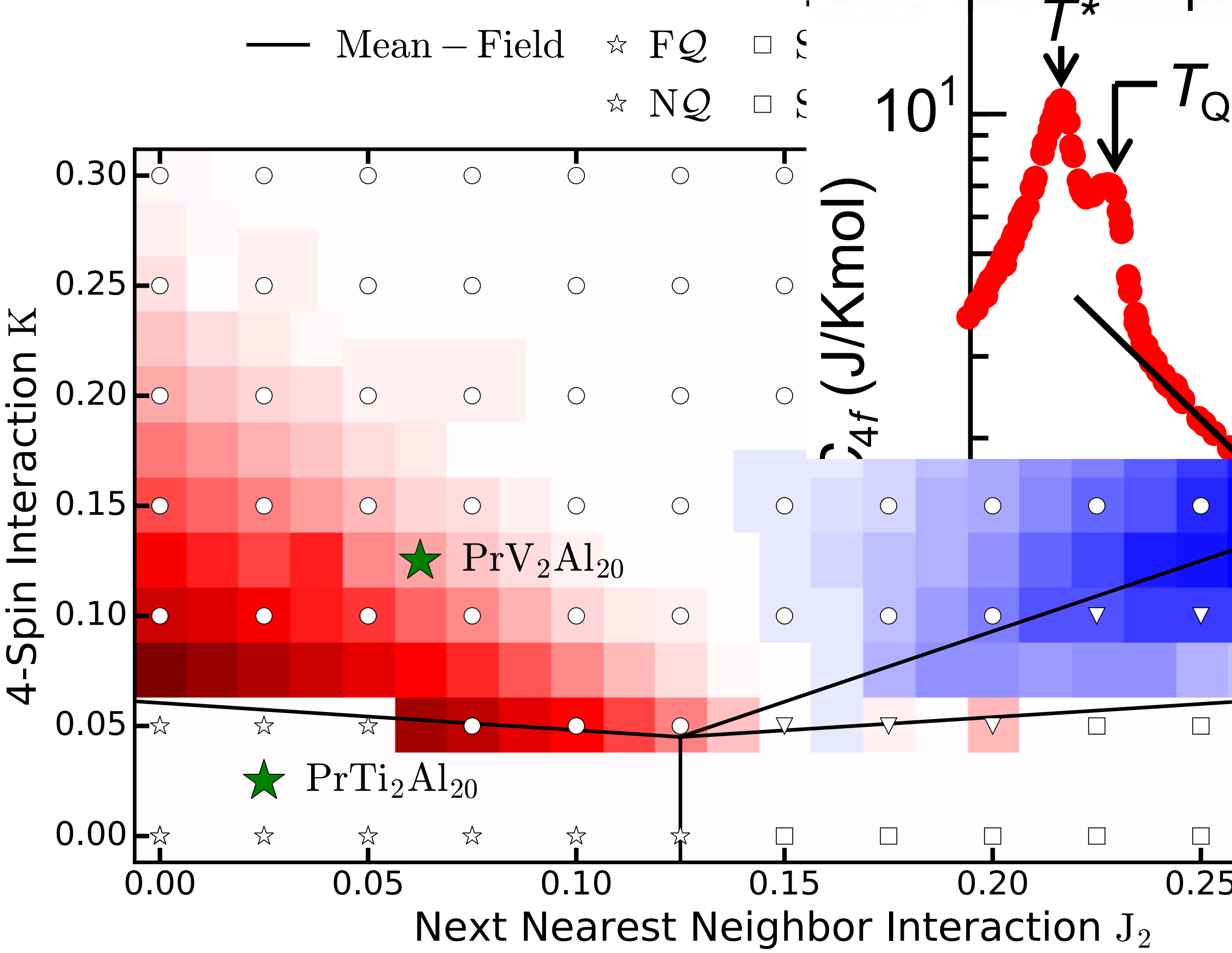


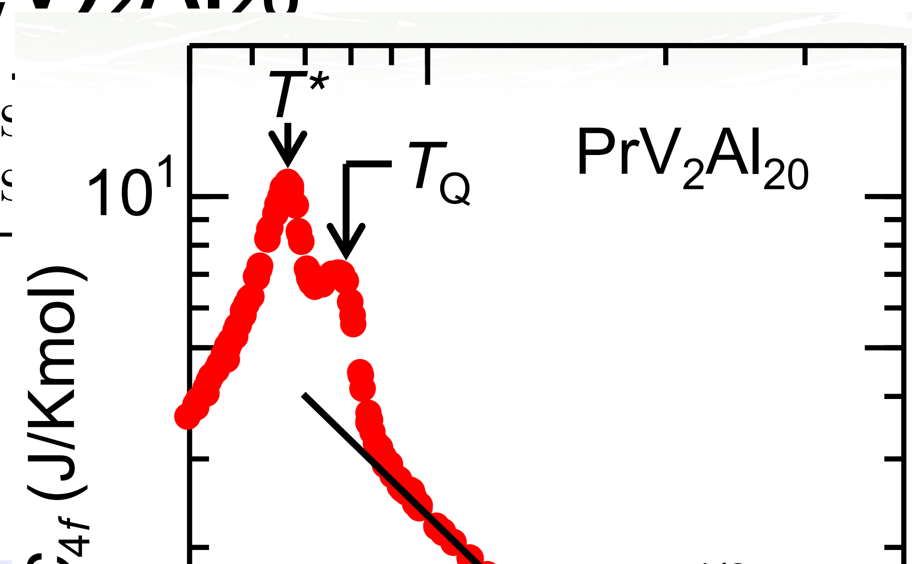


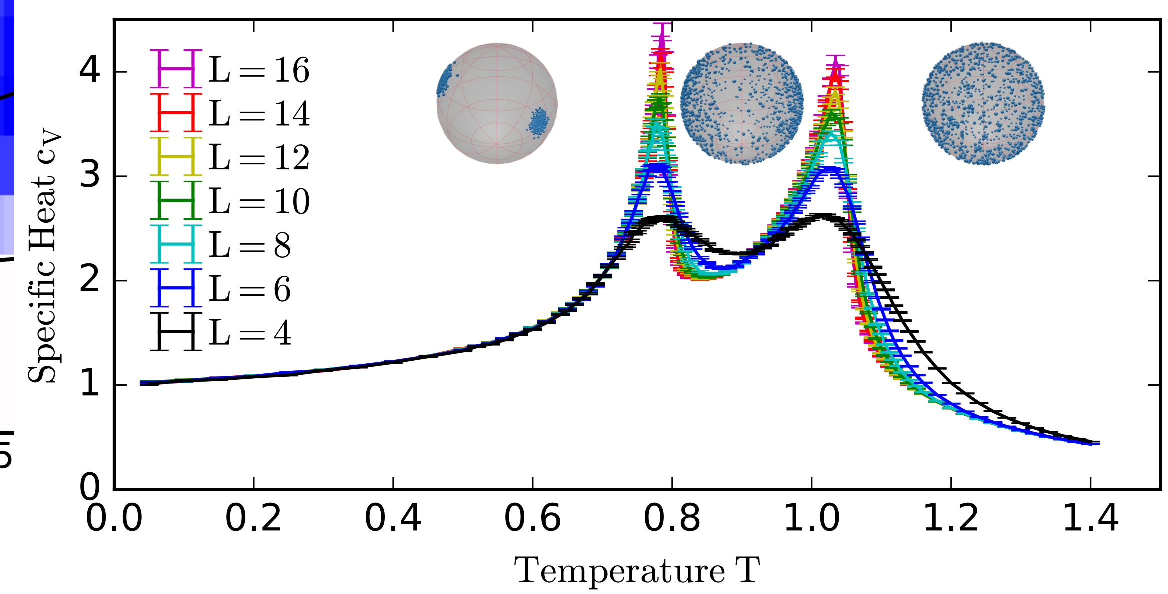
double transitions

Multipolar order and finite T transitions

Comparison with Pr(Ti,V)$_2$Al$_{20}$







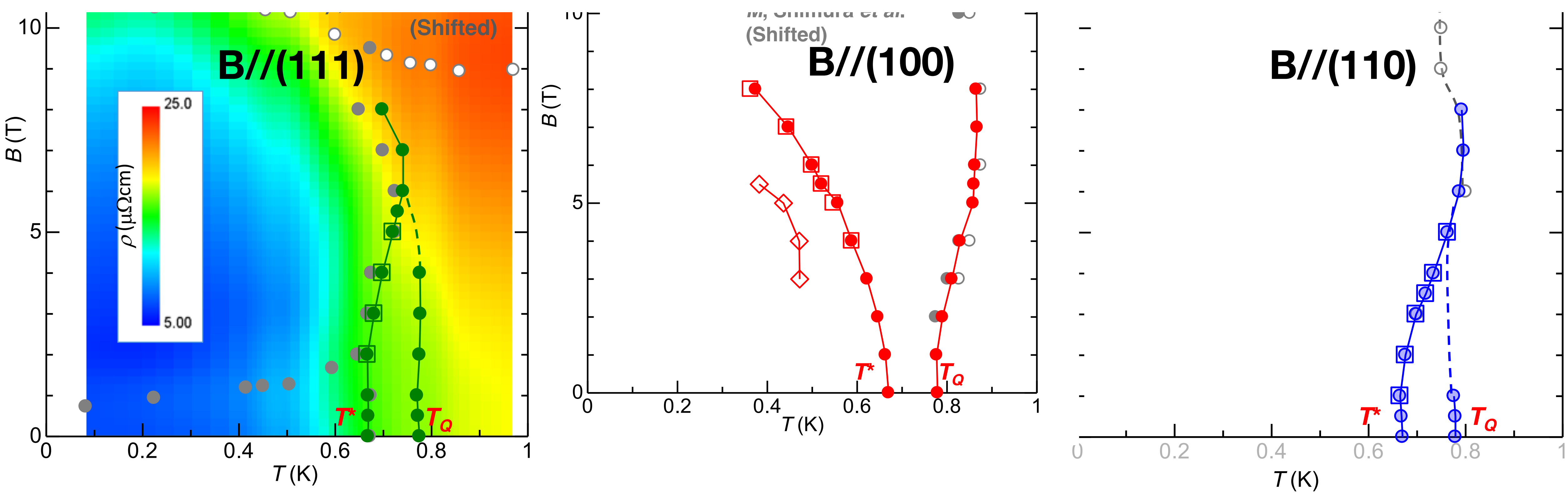
PrTi$_2$Al$_{20}$ → **PrV$_2$Al$_{20}$**

stronger hybridization leads to larger K, J

Quadrupole (T$_Q$)-octupole (T$_O$) double transitions

Multipolar order in magnetic fields

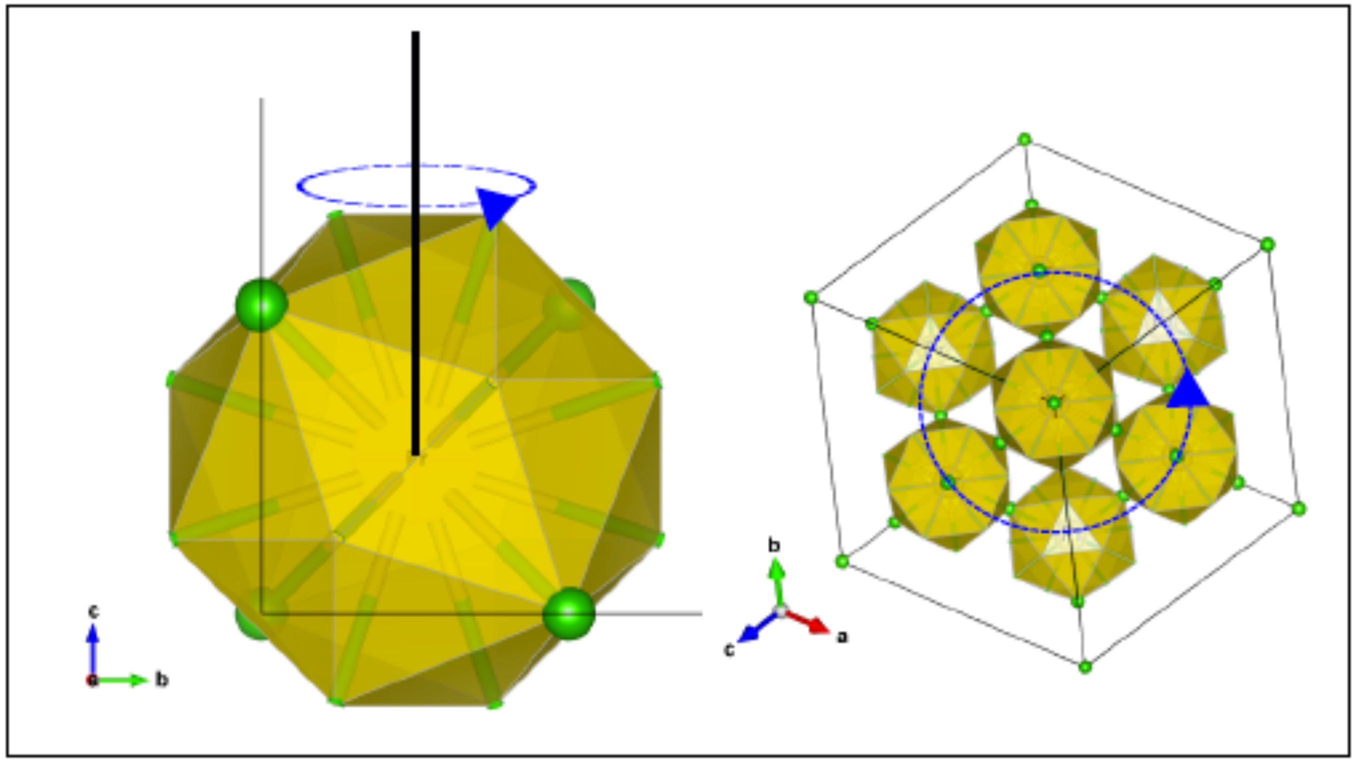
PrV_2Al_{20}



Anisotropy in fields with double transitions

Multipolar order in magnetic fields

Local symmetry



$$\Theta : \tau_\mu^z \to -\tau_\mu^z,$$
$$\mathcal{I} : \tau_\mu^A \leftrightarrow \tau_\mu^B,$$
$$\mathcal{S}_{4z} : \tau_\mu^\pm \to -\tau_\mu^\mp$$
$$\mathcal{C}_{31} : \tau_\mu^\pm \to e^{i\theta}\tau_\mu^\pm$$

symmetry analysis with antiferroquadrupolar order parameter ϕ and octupolar order parameter m

ferro- case **antiferro- case**

$$\phi \equiv \langle \tau_\mu^+ \rangle \qquad \tilde{\phi} \equiv \langle \tau_A^+ \rangle - \langle \tau_B^+ \rangle$$
$$\leftrightarrow$$
$$m \equiv \langle \tau_z^\mu \rangle \qquad \tilde{m} \equiv \langle \tau_z^A \rangle - \langle \tau_z^B \rangle$$

Multipolar order in magnetic fields

Landau Theory Analysis — No field

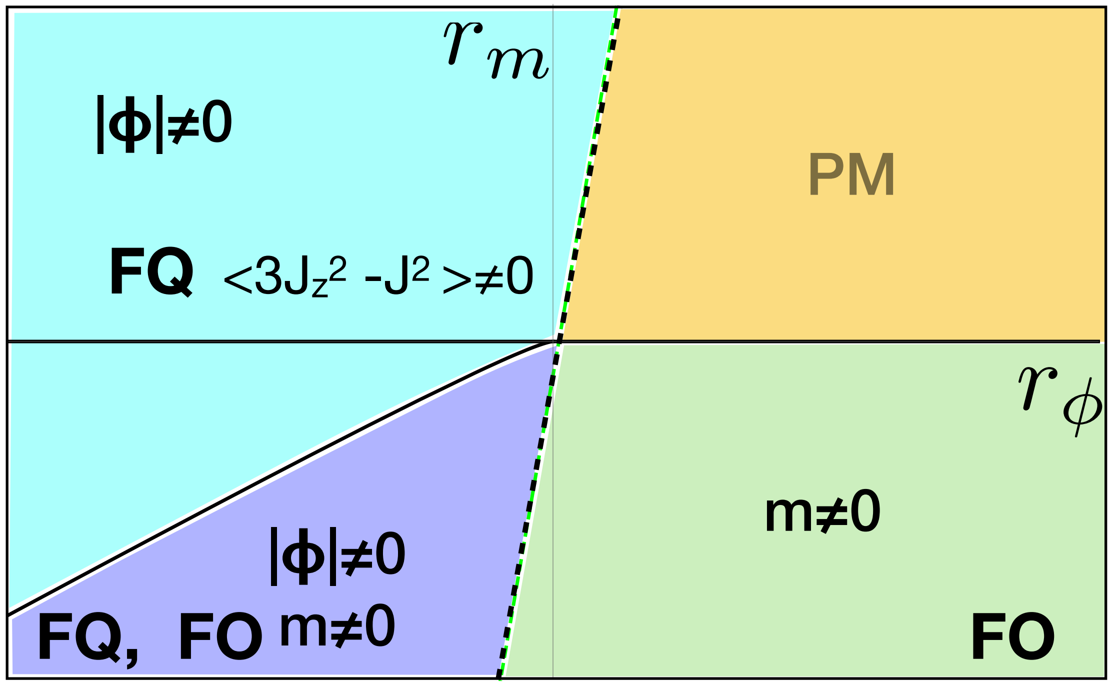
$$\Theta: \; m \to -m, \; \dot{m} \to -\dot{m}$$
$$\mathcal{I}: \; \tilde{\phi} \to -\tilde{\phi}, \; \tilde{m} \to -\tilde{m},$$
$$\mathcal{S}_{4z}: \; \phi \to -\phi^*, \; m \to -m, \; \tilde{\phi} \to -\tilde{\phi}^*, \; \tilde{m} \to -\tilde{m}$$
$$\mathcal{C}_{31}: \; \phi \to e^{i\theta}\phi, \; \tilde{\phi} \to e^{i\theta}\tilde{\phi}$$

$$F^{(2)}_{\phi,m} = r_\phi|\phi|^2 + r_m m^2,$$
$$F^{(4)}_{\phi,m} = u_\phi|\phi|^4 + u_m m^4 + u_{\phi m}|\phi^2|m^2.$$

ferroquadrupole $2v_\phi|\phi|^3 \sin 3\theta_\phi.$

anti-ferroquadrupole $-w_{\tilde{\phi}}|\tilde{\phi}|^6 \cos 6\theta_{\tilde{\phi}}$

Ferro- case



$$u_{\phi m} > 0, v_\phi \neq 0$$

SBL, A. Paramekanti, F. Freyer, J. Attig, S. Trebst and Y.B Kim **To appear in ArXiv soon**

Multipolar order in magnetic fields

Landau Theory Analysis with fields

Focus on double transitions

$$F^{(2)}_{\phi,m} = r_\phi|\phi|^2 + r_m m^2,$$

$$F^{(4)}_{\phi,m} = u_\phi|\phi|^4 + u_m m^4 + u_{\phi m}|\phi^2|m^2.$$

couple quadrupolar-octuplar moments

Phase locking of quadrupolar order $\phi = \tau_x + i\,\tau_y$

anti-ferroquadrupole $-w_{\tilde\phi}|\tilde\phi|^6 \cos 6\theta_{\tilde\phi}$

competition!

with magnetic field B $|B^2||\phi^2| \left(\tilde r_h + r_h \sin(\theta_h + 2\theta_\phi)\right)$

$$H_{\rm field} = \gamma B^2\left(\frac{\sqrt{3}}{2}(\hat B_x^2 - \hat B_y^2)\tau^x + \frac{1}{2}(3\hat B_z^2 - 1)\tau^y\right)$$

Quadrupole moments coupled to field B^2

Multipolar order in magnetic fields

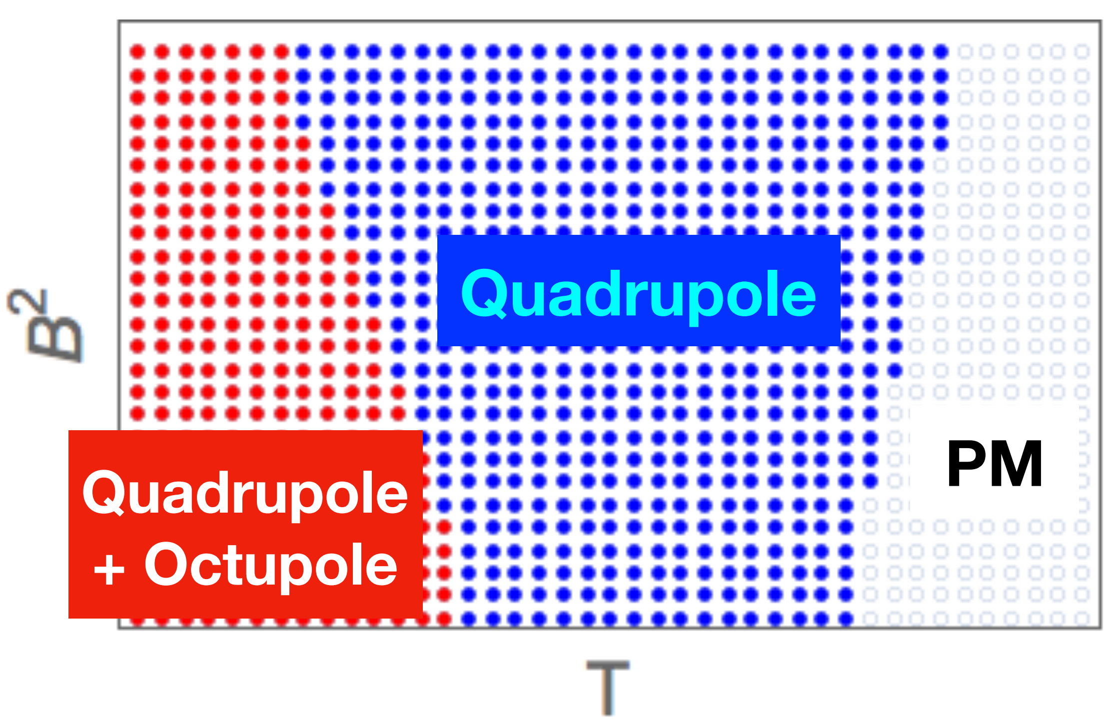
Landau Theory Analysis with fields

Focus on double transitions

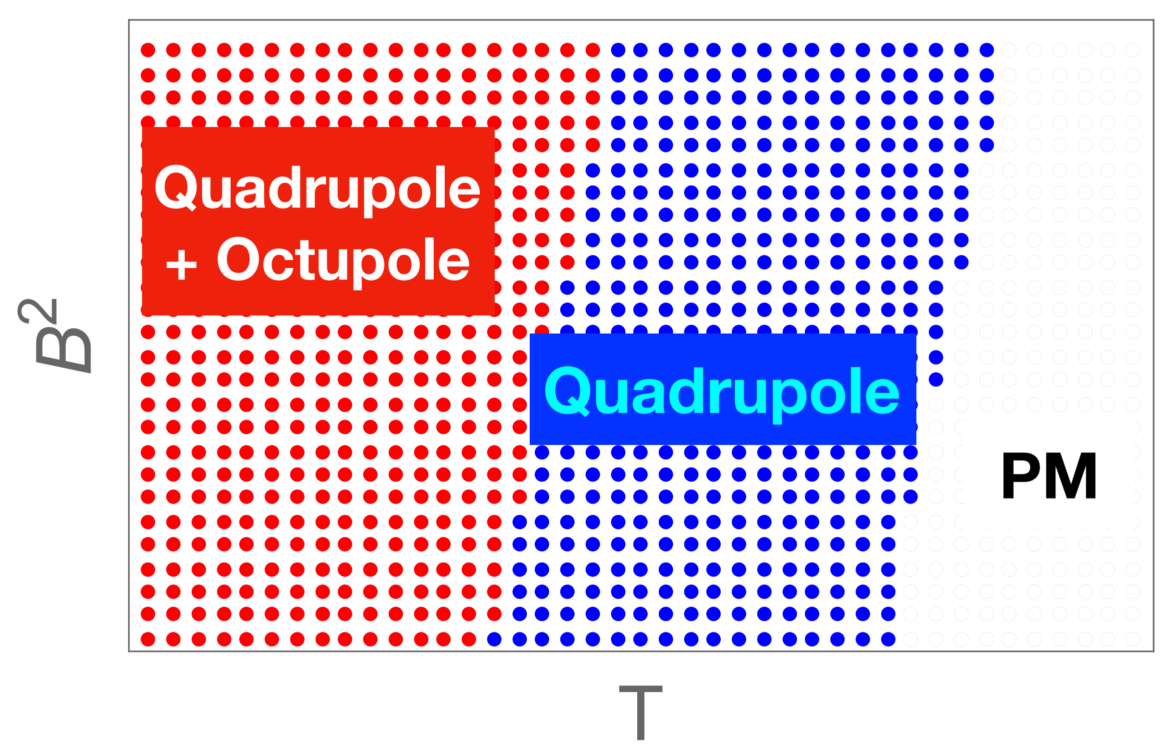
$$+u_{\phi m}|\phi^2|m^2. \quad -w_{\tilde{\phi}}|\tilde{\phi}|^6\cos 6\theta_{\tilde{\phi}} \quad |B^2||\Phi^2|\left(\tilde{r}_h + r_h\sin(\theta_h + 2\theta_\phi)\right)$$

Field effect based on Landau theory

B//(100)



B//(110) or // (111)

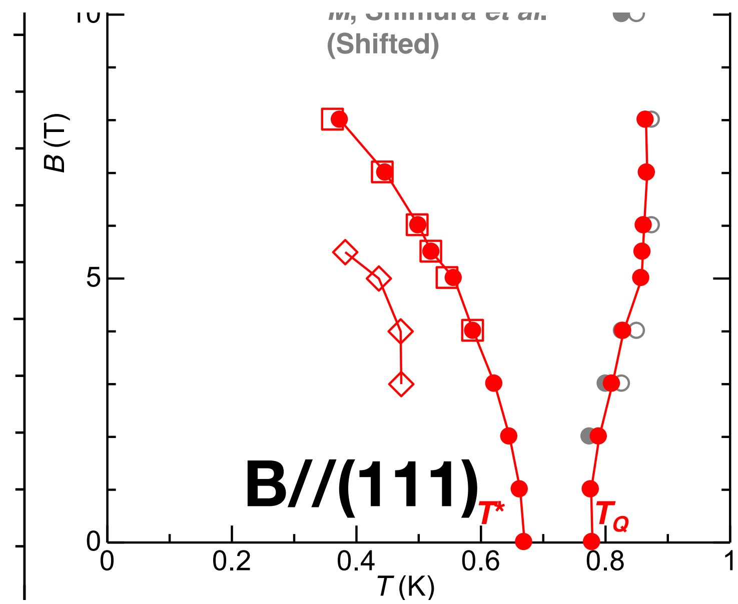


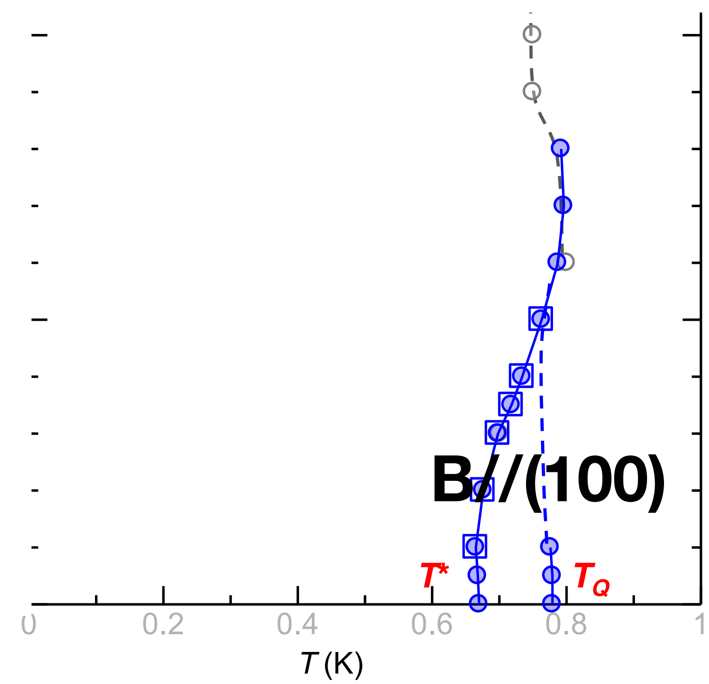
octupolar order transition temperature T_O is very sensitive to B direction

Multipolar order in magnetic fields

Landau Theory Analysis with fields

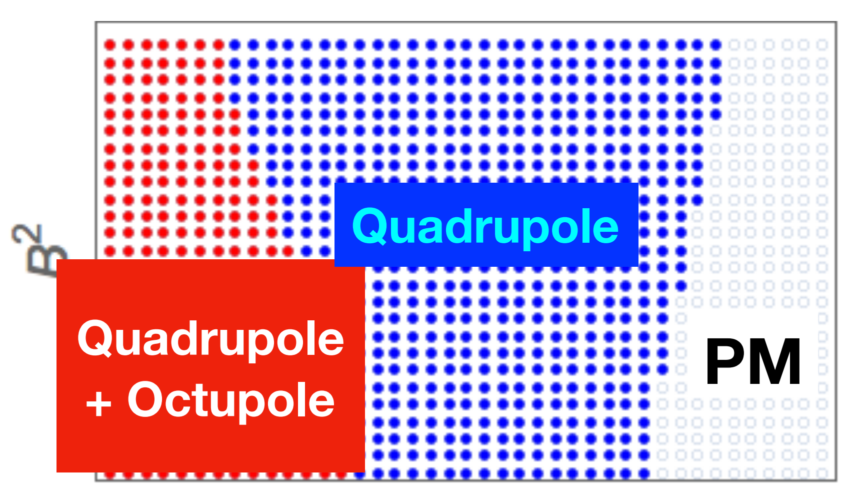
Comparison with PrV_2Al_{20}



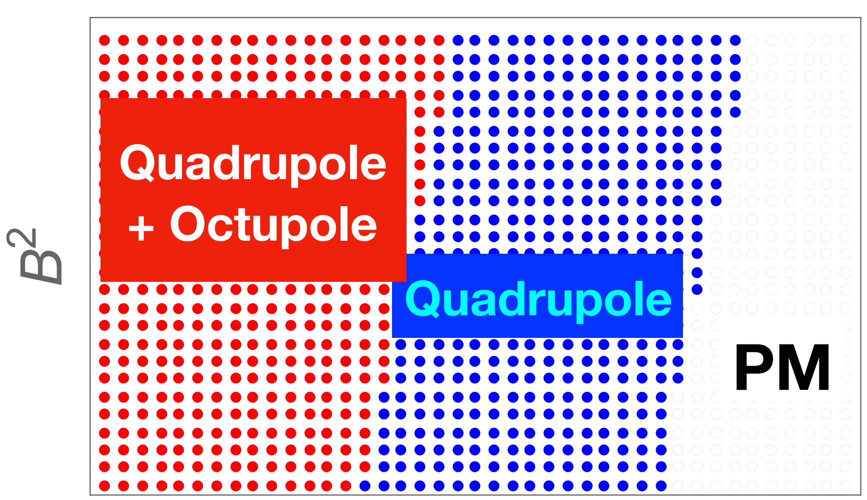


Field effect based on Landau theory

B//(100)



B//(110) or // (111)



Quadrupolar and octupolar double transition in fields

—> Field direction matters! anisotropic (100) vs (110) or (111)

Multipolar order and Superconductivity

Superconductivity in $Pr(TM)_2(Al,Zn)_{20}$

	$T_s(K)$	$T_{N/Q}(K)$	$T_c(K)$	SC type	structure ($T < T_s$)	$\rho(\mu\Omega cm)$ (at RT)	pressure
Al	-	-	1.2	I	fcc	2.82	$T_c \downarrow$
Zn	-	-	0.85	I	hexagonal	5.9	$T_c \uparrow$
$PrTi_2Al_{20}$	-	2	0.2	II	$Fd\bar{3}m$	?	$T_c \uparrow$
PrV_2Al_{20}	-	0.9	-	-	$Fd\bar{3}m$	?	?
LaV_2Al_{20}	-	-	-	-	$Fd\bar{3}m$	?	?
$Al_{0.3}V_2Al_{20}$	-	-	1.49	?	$Fd\bar{3}m$	80	?
$Ga_{0.2}V_2Al_{20}$	-	-	1.66	?	$Fd\bar{3}m$	100	?
YV_2Al_{20}	-	-	0.69	?	$Fd\bar{3}m$	60	?
$PrRh_2Zn_{20}$	140	0.06	0.06	?	?	80	?
$PrIr_2Zn_{20}$	-	0.2	0.05	?	$Fd\bar{3}m$	90	$T_Q \uparrow$
$LaIr_2Zn_{20}$	200	-	0.6	?	?	100	$T_s \uparrow, T_c \downarrow$
$PrRu_2Zn_{20}$	138	none	-	-	?	90	$T_s \downarrow$
$LaRu_2Zn_{20}$	150	-	0.2	?	?	100	$T_s \downarrow, T_c \downarrow$

Multipolar order and Superconductivity

$Pr(Rh,Ir)_2(Al,Zn)_{20}$

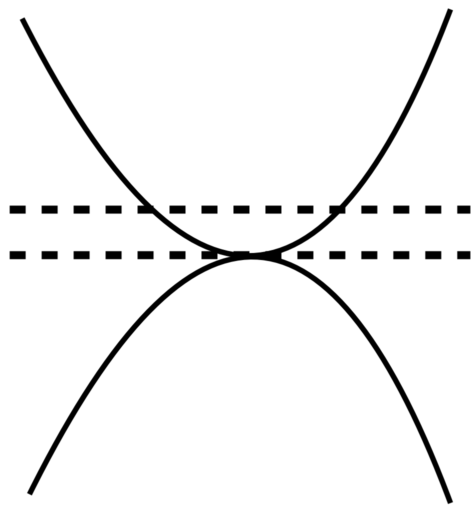
strong SOC + cubic symmetry

Quadratic band touching at k=0 point : Near Fermi level or not?

→ **Luttinger Hamiltonian with J=3/2**

$$d_1 = \sqrt{3}k_y k_z,$$
$$d_2 = \sqrt{3}k_x k_z,$$
$$d_3 = \sqrt{3}k_x k_y,$$
$$d_4 = \frac{\sqrt{3}}{2}(k_x^2 - k_y^2),$$
$$d_5 = \frac{1}{2}(3k_z^2 - \mathbf{k}^2)$$

Luttinger Hamiltonian with J=3/2



Quadratic band touching with cubic symmetry

Gamma matrices for J=3/2

$$\mathcal{H}_0(\mathbf{k}) = \psi^\dagger_{\mathbf{k}}\left(c_0\mathbf{k}^2 - \mu + \sum_{i=1}^{5} c_i\ d_i(\mathbf{k})\gamma_i\right)\psi_{\mathbf{k}}$$

t_{2g} & e_g in k

GB Sim, A. Mishra, G-Y Cho and **SBL** **In preparation**

Multipolar order and Superconductivity

$Pr(Rh,Ir)_2(Al,Zn)_{20}$

strong SOC + cubic symmetry

Quadratic band touching at k=0 point **+ interaction**

$$\mathcal{H}_0(\mathbf{k}) = \psi^\dagger_{\mathbf{k}}\Big(c_0\mathbf{k}^2 - \mu + \sum_{i=1}^{5} c_i\ d_i(\mathbf{k})\gamma_i\Big)\psi_{\mathbf{k}}$$

t_{2g} & e_g in k

+

$$\mathcal{H}_{int}(\mathbf{k}) = u(\psi^\dagger\psi) + \sum_{i=1}^{5} v_i\ (\psi^\dagger\gamma_i\psi)^2$$

Fierz Identity

$$\Delta_i \equiv \langle \sum_{\mathbf{k}} \psi^T_{-\mathbf{k}}\gamma_{13}\gamma_i\psi_{\mathbf{k}}\rangle$$

exactly decoupled into s and d wave pairing channels

$$(\frac{u}{4}+\frac{3v}{4}+\frac{w_1}{4}+\frac{w_2}{4})(\psi^\dagger(\gamma_{13})^*\psi^*\Delta_0+\psi^T(\gamma_{13})^T\psi\Delta_0^*)$$
$$+(\frac{u}{4}-\frac{v}{4}-\frac{w_1}{4}-\frac{w_2}{4})(\psi^\dagger i\gamma_{13}\gamma_1\psi^*)(\psi^T i\gamma_{13}\gamma_1\psi)$$
$$+(\frac{u}{4}-\frac{v}{4}-\frac{w_1}{4}-\frac{w_2}{4})(\psi^\dagger\gamma_{13}\gamma_2\psi^*)(\psi^T\gamma_{13}\gamma_2\psi)$$

Multipolar order and Superconductivity

$Pr(Rh,Ir)_2(Al,Zn)_{20}$

strong SOC + cubic symmetry

$$\Delta_i \equiv \langle \sum_{\mathbf{k}} \psi^T_{-\mathbf{k}} \gamma_{13} \gamma_i \psi_{\mathbf{k}} \rangle$$

$$\mathcal{H}_{int}(\mathbf{k}) = u(\psi^\dagger\psi) + \sum_{i=1}^{5} v_i \ (\psi^\dagger \gamma_i \psi)^2$$

Fierz Identity

$$
\begin{aligned}
&= (\frac{u}{4} + \frac{3v}{4} + \frac{w_1}{4} + \frac{w_2}{4})(\psi^\dagger \gamma_{13} \psi^*)(\psi^T \gamma_{13} \psi) \\
&+(\frac{u}{4} - \frac{v}{4} - \frac{w_1}{4} - \frac{w_2}{4})(\psi^\dagger i\gamma_{13}\gamma_1 \psi^*)(\psi^T i\gamma_{13}\gamma_1 \psi) \\
&+(\frac{u}{4} - \frac{v}{4} - \frac{w_1}{4} - \frac{w_2}{4})(\psi^\dagger \gamma_{13}\gamma_2 \psi^*)(\psi^T \gamma_{13}\gamma_2 \psi) \\
&+(\frac{u}{4} - \frac{v}{4} - \frac{w_1}{4} - \frac{w_2}{4})(\psi^\dagger i\gamma_{13}\gamma_3 \psi^*)(\psi^T i\gamma_{13}\gamma_3 \psi) \\
&+(\frac{u}{4} - \frac{3v}{4} + \frac{w_1}{4} - \frac{w_2}{4})(\psi^\dagger \gamma_{13}\gamma_4 \psi^*)(\psi^T \gamma_{13}\gamma_4 \psi) \\
&+(\frac{u}{4} - \frac{3v}{4} - \frac{w_1}{4} + \frac{w_2}{4})(\psi^\dagger \gamma_{13}\gamma_5 \psi^*)(\psi^T \gamma_{13}\gamma_5 \psi)
\end{aligned}
$$

exactly decoupled into s and d wave pairing channels

$$
\begin{aligned}
&= (\frac{u}{4} + \frac{3v}{4} + \frac{w_1}{4} + \frac{w_2}{4})(\psi^\dagger (\gamma_{13})^\dagger \psi^*)(\psi^T \gamma_{13} \psi) \\
&+(\frac{u}{4} - \frac{v}{4} - \frac{w_1}{4} - \frac{w_2}{4})(\psi^\dagger (\gamma_{13}\gamma_1)^\dagger \psi^*)(\psi^T \gamma_{13}\gamma_1 \psi) \\
&+(\frac{u}{4} - \frac{v}{4} - \frac{w_1}{4} - \frac{w_2}{4})(\psi^\dagger (\gamma_{13}\gamma_2)^\dagger \psi^*)(\psi^T \gamma_{13}\gamma_2 \psi) \\
&+(\frac{u}{4} - \frac{v}{4} - \frac{w_1}{4} - \frac{w_2}{4})(\psi^\dagger (\gamma_{13}\gamma_3)^\dagger \psi^*)(\psi^T \gamma_{13}\gamma_3 \psi) \\
&+(\frac{u}{4} - \frac{3v}{4} + \frac{w_1}{4} - \frac{w_2}{4})(\psi^\dagger_{\mathbf{-k}} (\gamma_{13}\gamma_4)^\dagger \psi^*_{\mathbf{k}})(\psi^T_{\mathbf{k}'} \gamma_{13}\gamma_4 \psi_{\mathbf{-k}'}) \\
&+(\frac{u}{4} - \frac{3v}{4} - \frac{w_1}{4} + \frac{w_2}{4})(\psi^\dagger (\gamma_{13}\gamma_5)^\dagger \psi^*)(\psi^T \gamma_{13}\gamma_5 \psi)
\end{aligned}
$$

Multipolar order and Superconductivity

$Pr(Rh,Ir)_2(Al,Zn)_{20}$

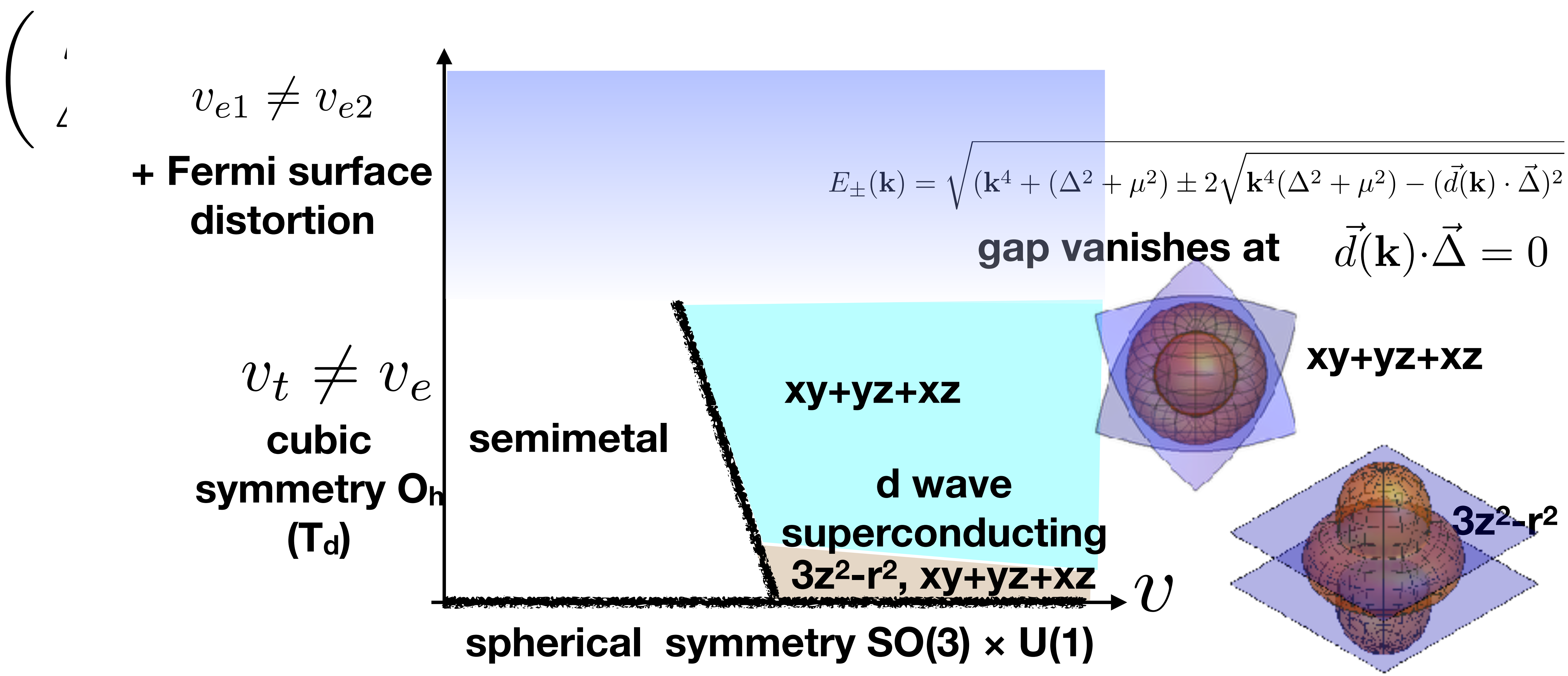
strong SOC + cubic symmetry

$$\Delta_i \equiv \langle \sum_{\mathbf{k}} \psi^T_{-\mathbf{k}} \gamma_{13} \gamma_i \psi_{\mathbf{k}} \rangle$$

Quadrupolar fluctuation drives d-wave superconductivity

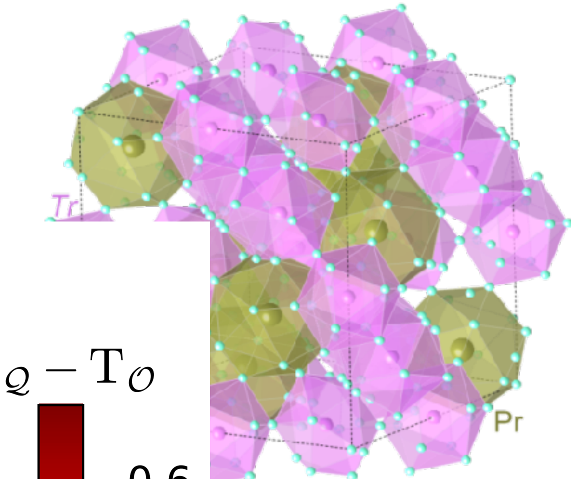
$$E_{\pm}(\mathbf{k}) = \sqrt{(\mathbf{k}^4 + (\Delta^2 + \mu^2) \pm 2\sqrt{\mathbf{k}^4(\Delta^2+\mu^2) - (\vec{d}(\mathbf{k}) \cdot \vec{\Delta})^2}}$$

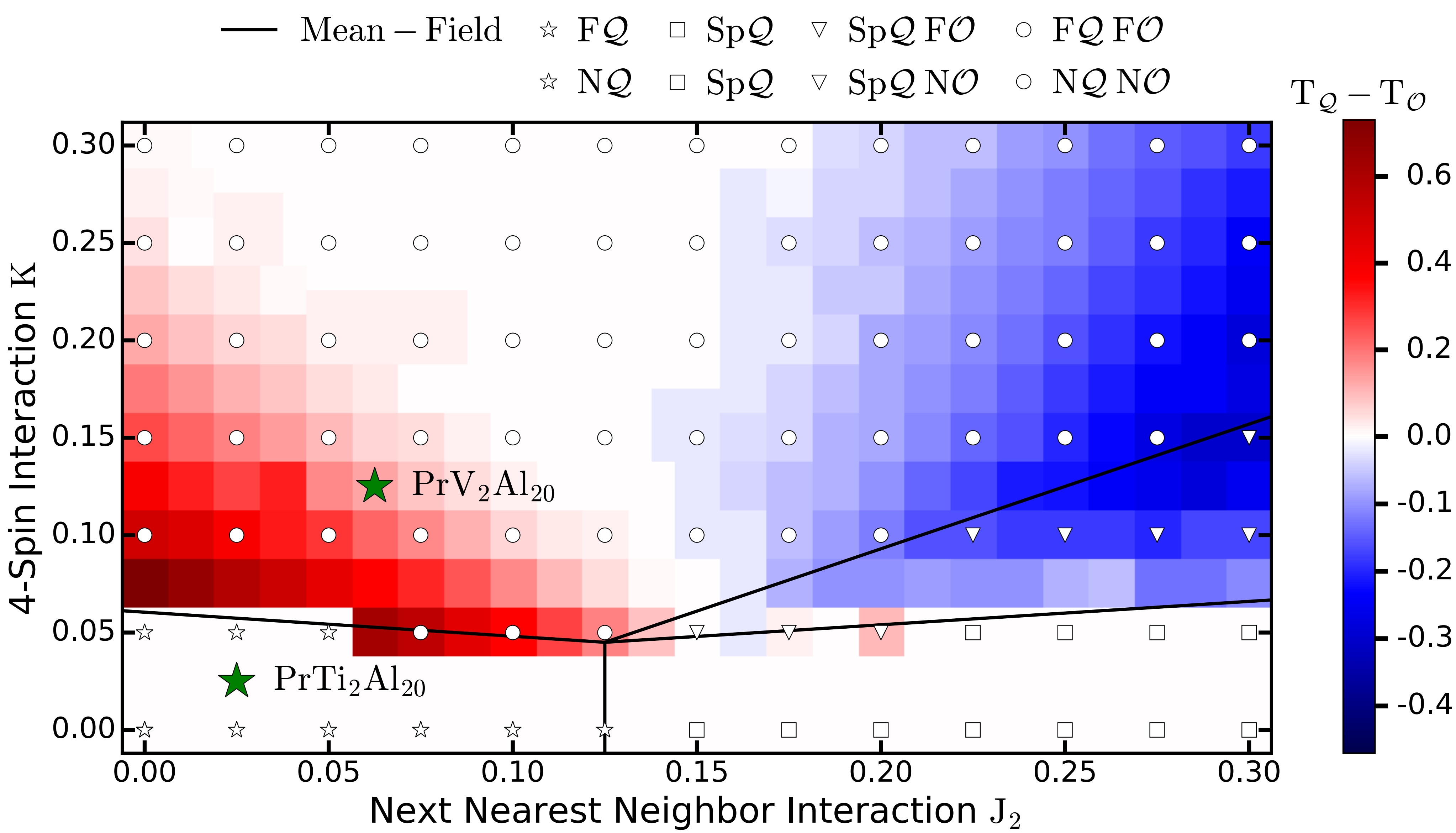
gap vanishes at $\vec{d}(\mathbf{k}) \cdot \vec{\Delta} = 0$



Summary 1. $Pr(TM)_2(Al,Zn)_{20}$

- Pr^{3+} localized moments





$H =$

ts

eractions

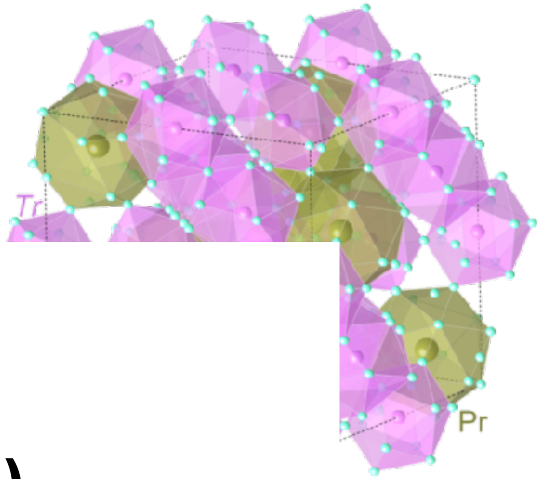
Frustration & Multiple spin interactions

—> double transitions of quadrupole - octuple orderings

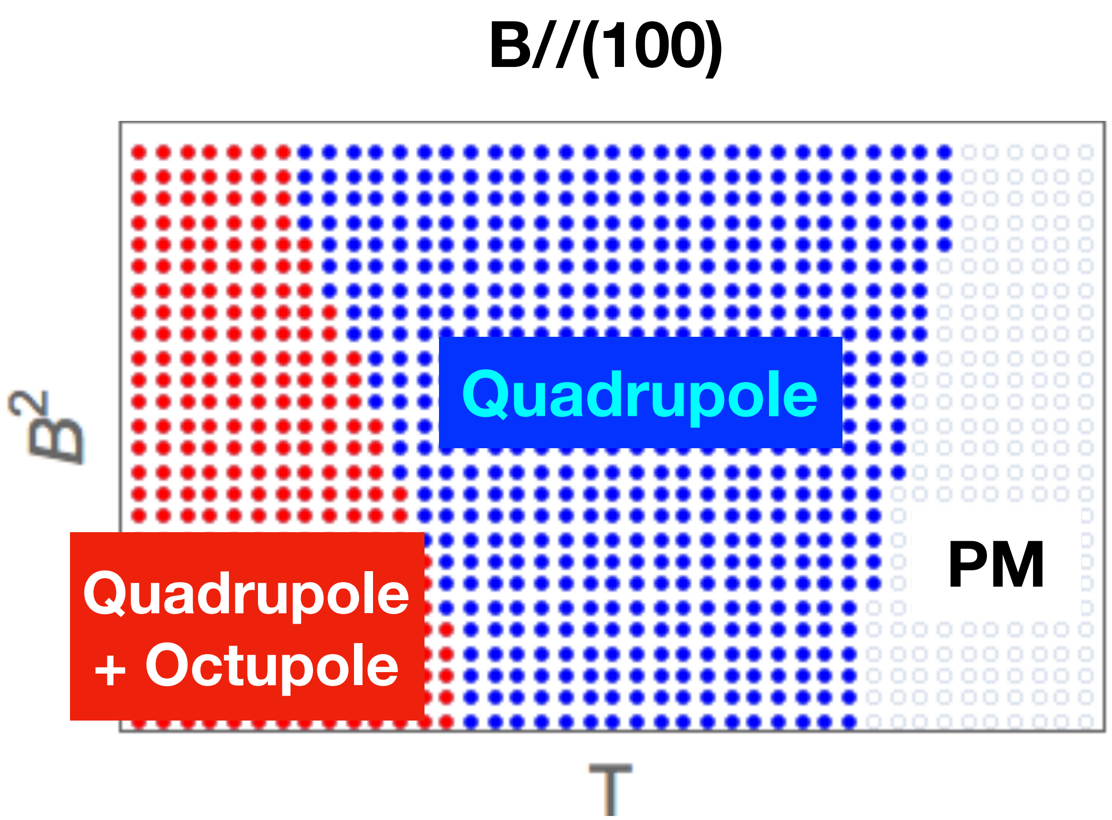
F. Freyer, J. Attig, **SBL**, A. Paramekanti, S. Trebst and Y.B Kim **ArXiv 1709.06094 (2017)**

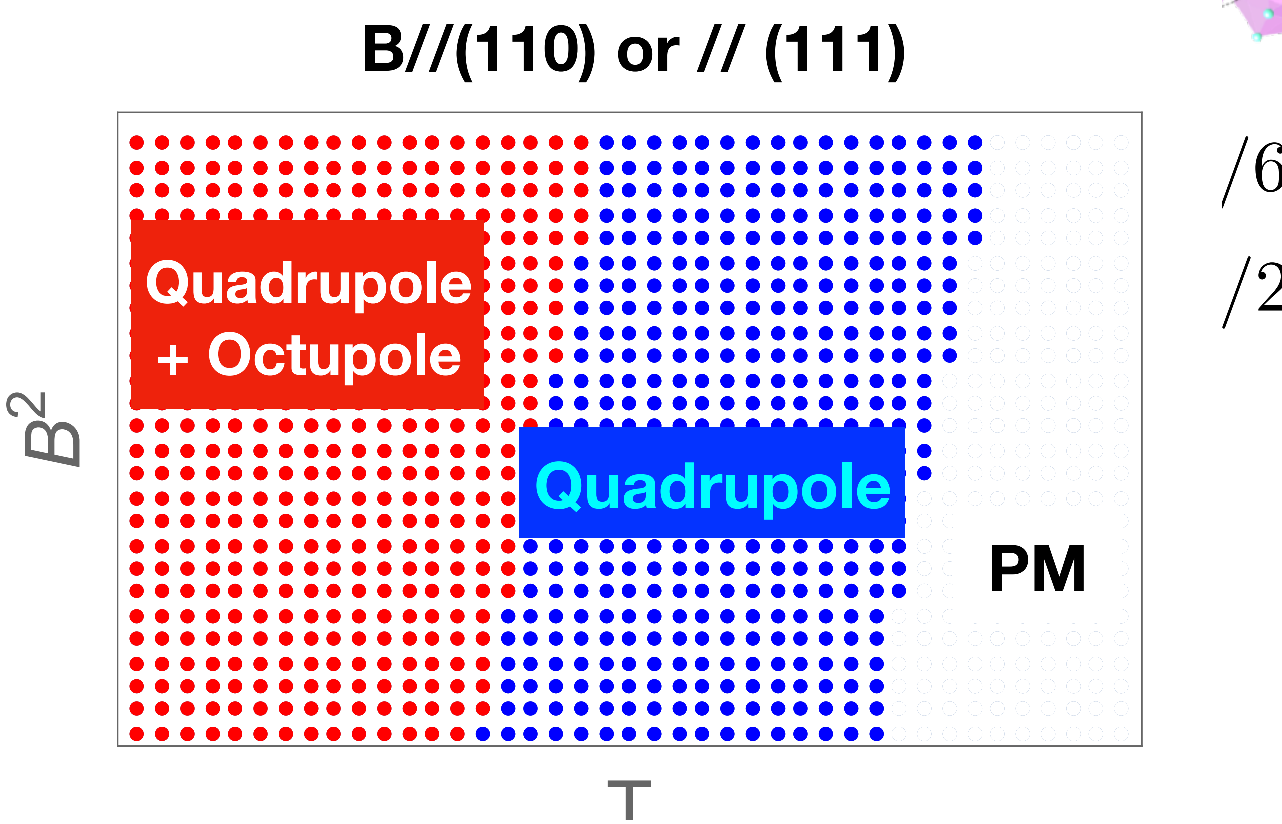
Summary 2. $Pr(TM)_2(Al,Zn)_{20}$

- Pr^{3+} localized moments



Field effect based on Landau theory





Quadrupolar and octupolar double transition in fields

—> Field direction matters! anisotropic (100) vs (110) or (111)

SBL, A. Paramekanti, F. Freyer, J. Attig, S. Trebst and Y.B Kim **To appear in ArXiv soon**

Summary 3. $Pr(TM)_2(Al,Zn)_{20}$

- Superconductivity

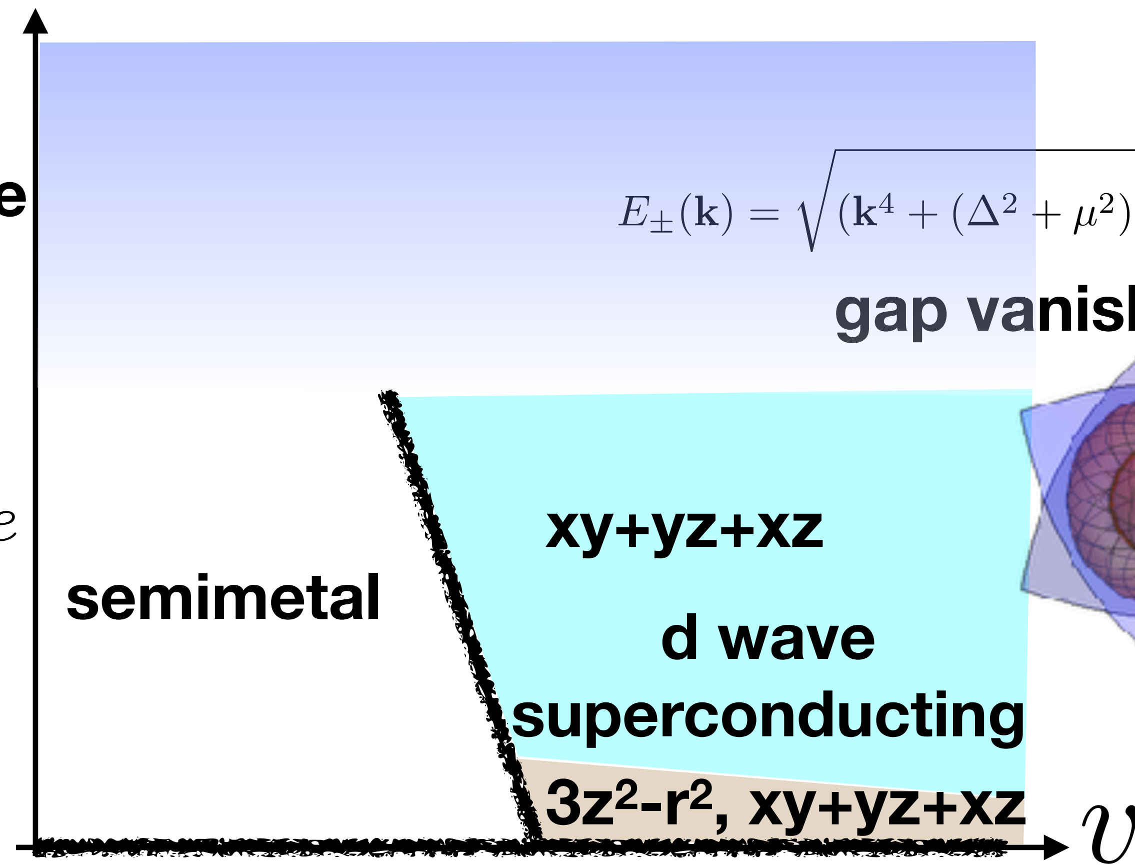
$v_{e1} \neq v_{e2}$

+ Fermi surface distortion

L

$v_t \neq v_e$

cubic symmetry O_h (T_d)

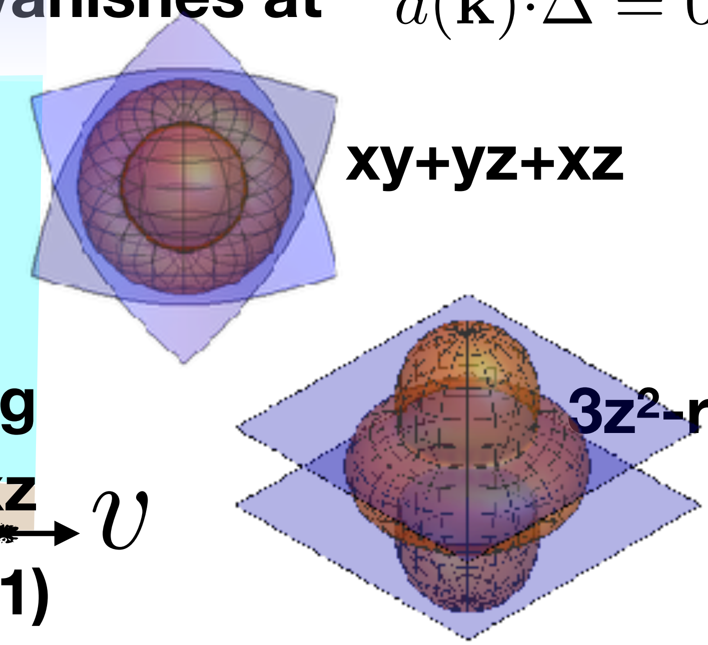


spherical symmetry SO(3) × U(1)

$$E_\pm(\mathbf{k}) = \sqrt{(\mathbf{k}^4 + (\Delta^2 + \mu^2) \pm 2\sqrt{\mathbf{k}^4(\Delta^2 + \mu^2) - (\vec{d}(\mathbf{k}) \cdot \vec{\Delta})^2}}$$

=3/2

gap vanishes at $\vec{d}(\mathbf{k}) \cdot \vec{\Delta} = 0$



irings

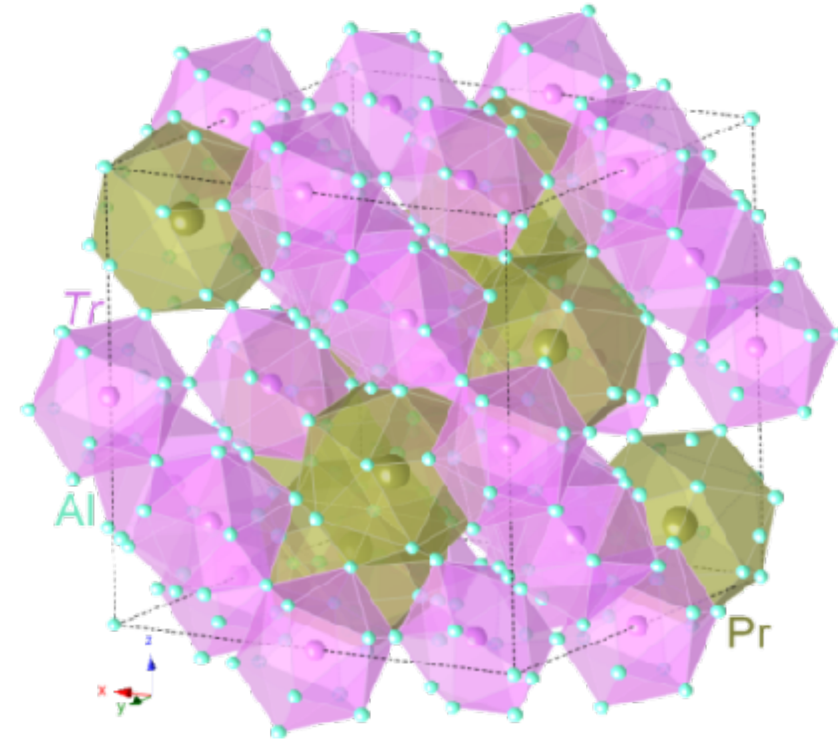
Quadrupolar fluctuation drives d-wave superconductivity

$$(u - 3v_t + v_{e1} - v_{e2})\left(\Delta_4 \psi^\dagger \gamma_{13}^* \psi^* + h.c\right)$$

GB Sim, A. Mishra, G-Y Cho and **SBL** **In preparation**

Summary

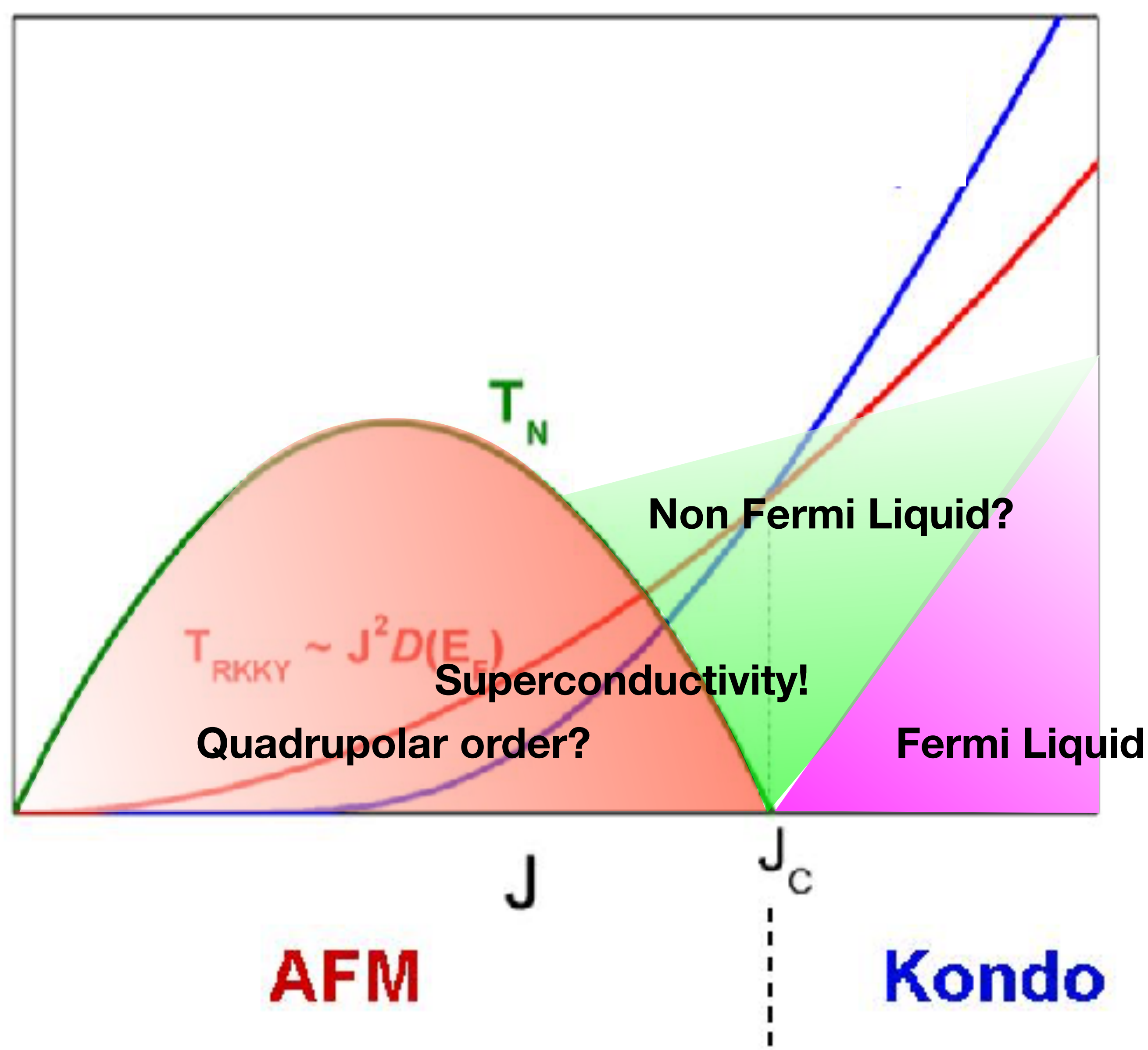
$Pr(TM)_2(Al,Zn)_{20}$



multipo

superc

S. Doniach, Physica B **91**, 231 (1977)



Thank you!