Interaction-induced Edge effects in 1D open-boundary lattices

Masud Haque

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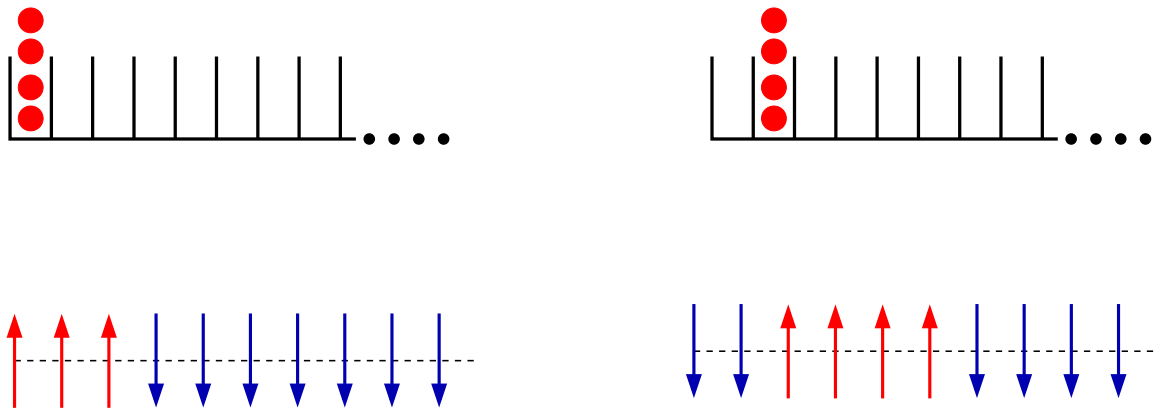
Edge-localization in 1D lattice models

Bose-Hubbard chain

spinless fermion model

XXZ chain



PHYSICS:

Far-from-equilibrium dynamics

Eigenstates far from ground state

Intricate structures in spectrum (FRACTAL)

QUANTUM CONTROL:

Locking and release of magnetization/state

Designing a quantum switch

For details ....

R. A. Pinto, M. Haque, and S. Flach; Phys. Rev. A **79**, 052118 (2009).

Edge-localized states in quantum one-dimensional lattices.

M. Haque, Phys. Rev. A **82**, 012108 (2010).

Self-similar spectral structures and edge-locking hierarchy in open-boundary spin chains.

Nice people thank their collaborators

Sergej Flach

MPI-PKS Dresden

Ricardo Pinto

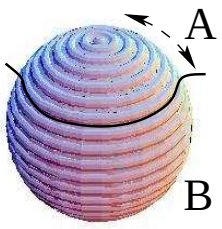
MPI-PKS $\longrightarrow$ Riverside

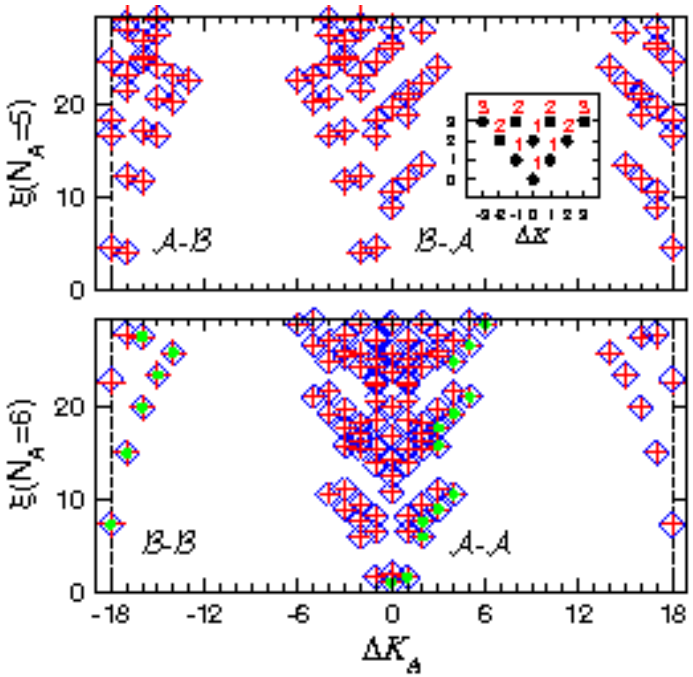
Motivated by related phenomena in the

Discrete nonlinear Schrodinger equation (DNLS)

A.K.A. Discrete Gross-Pitaevskii equation

Entanglement & topological order in FQH states

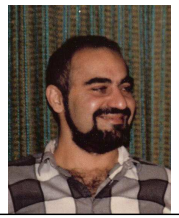




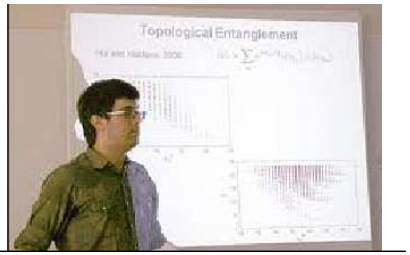
Haque, Zozulya, Schoutens; P.R.L. 2007
Zozulya, Haque, Schoutens, Rezayi; P.R.B 2007
Zozulya, Haque, Regnault; P.R.B 2009
Läuchli, Bergholtz, Suorsa, Haque; P.R.L. 2010
Läuchli, Bergholtz, Haque; N.J.P. 2010






Related series of work:
Bernevig,
Regnault, Haldane, ...

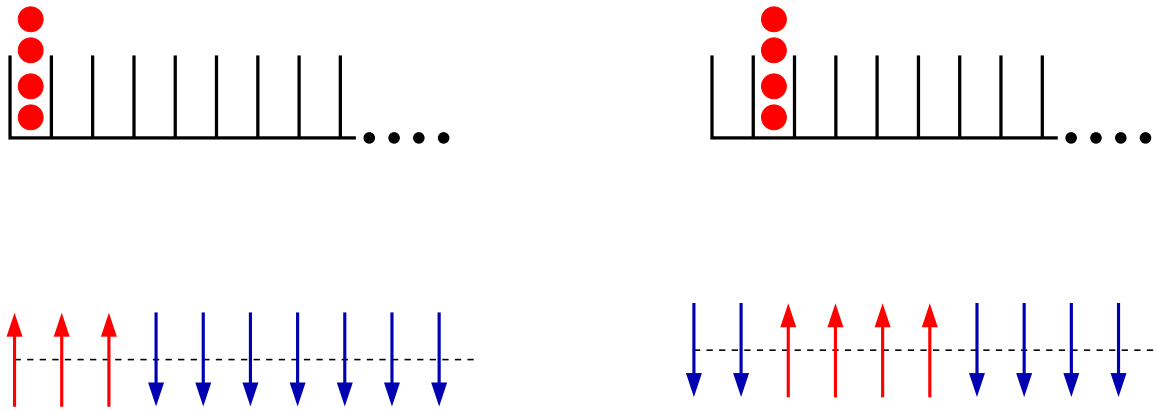


Edge-localization in 1D lattice models

Bose-Hubbard chain

spinless fermion model

XXZ chain



PHYSICS:

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Intricate structures in spectrum (FRACTAL)

QUANTUM CONTROL:

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Designing a quantum switch

Hamiltonians & small parameters

Hamiltonians:	Small Parameters:
$H_{\text{Bose.Hubbard}} = -t\sum\left(a_j^\dagger a_{j+1} + a_{j+1}^\dagger a_j\right) + \frac{U}{2}\sum a_j^\dagger a_j^\dagger a_j a_j$	t/U
$H_{\text{sp.ferm.}} = -t\sum\left(c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j\right) + V\sum c_j^\dagger c_{j+1}^\dagger c_{j+1} c_j$	t/V
$H_{XXZ} = J_x\sum\left[S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z\right]$	$1/\Delta$

I take these Hamiltonians seriously!

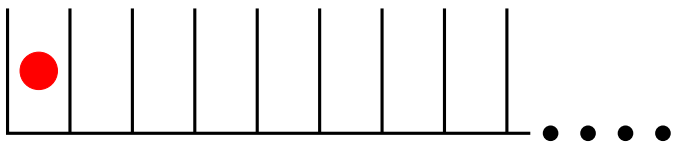
- not only low-energy sector
- no dissipation mechanism

Simple evolutions; Bose-Hubbard model

1D Bose-Hubbard model in an OPEN chain (has edges)

$$\hat{H} = -t \sum_{j=1}^{L-1} \left(a_j^\dagger a_{j+1} + a_{j+1}^\dagger a_j \right) + \frac{U}{2} \sum_{j=1}^{L} a_j^\dagger a_j^\dagger a_j a_j$$

I'm interested in large U/t. $U = 10$ or $U = -10$

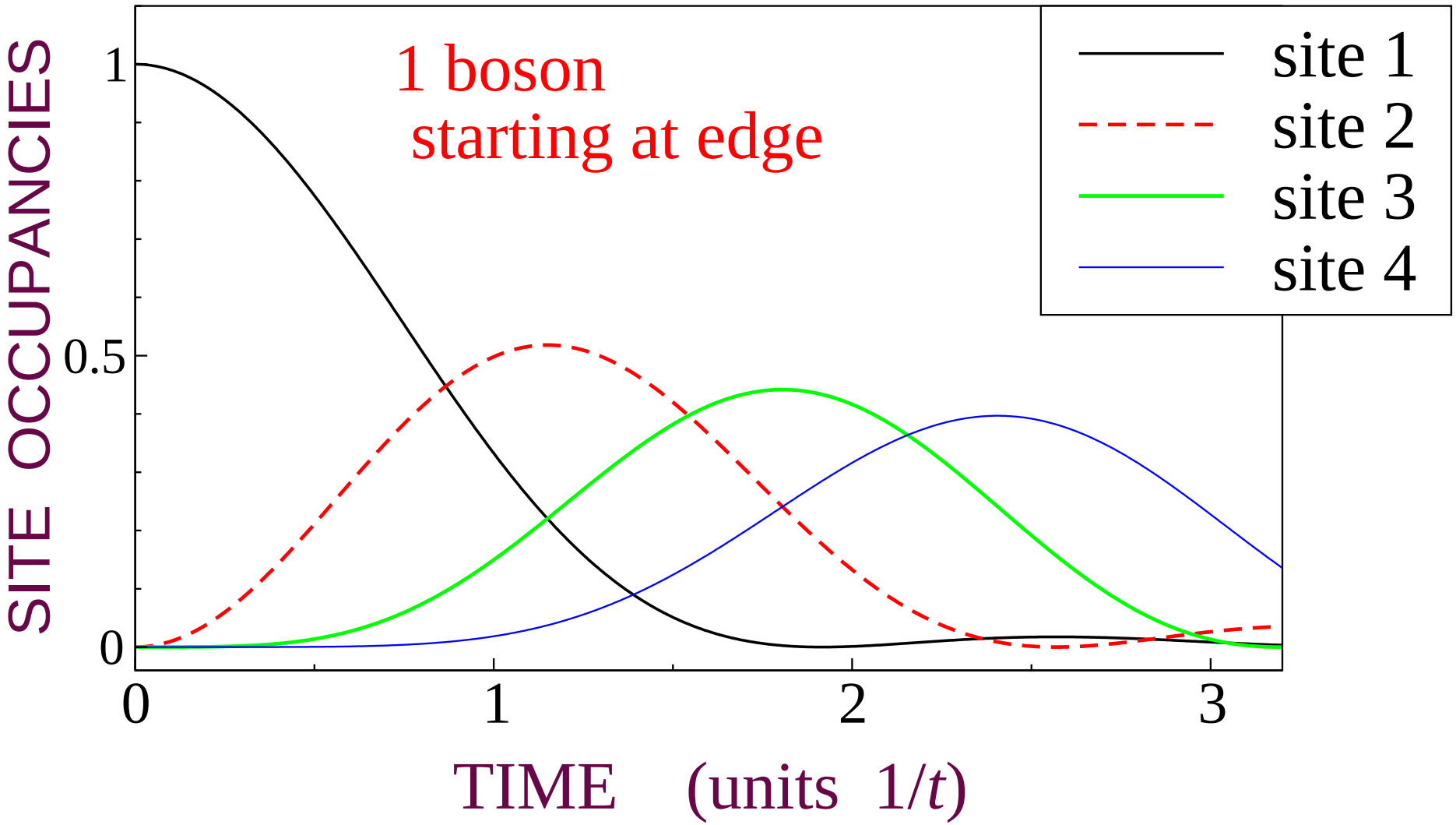


1 0 0 0 0 0 0 0 .....

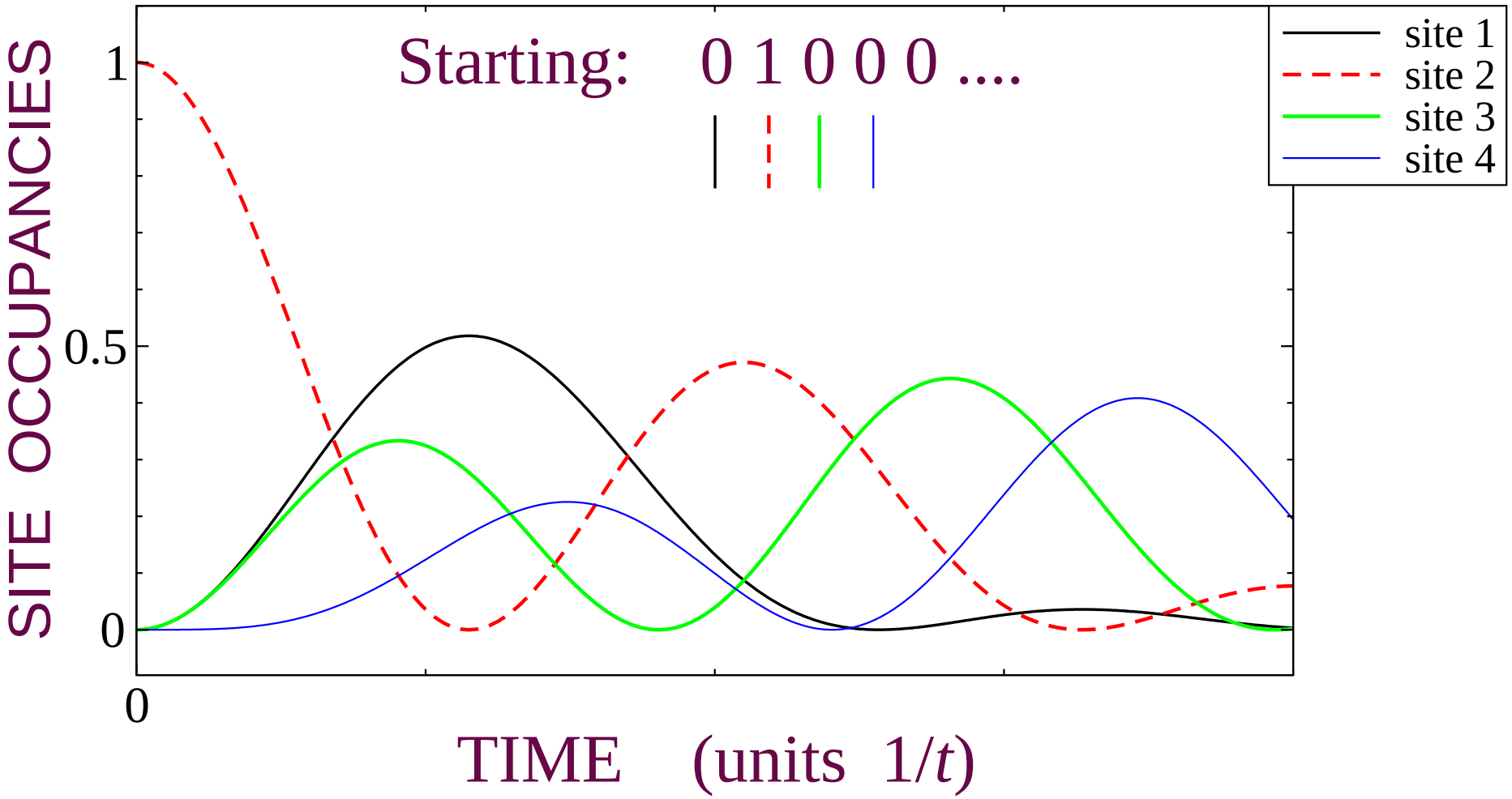
How does this evolve?

At timescales $\sim \hbar/t$

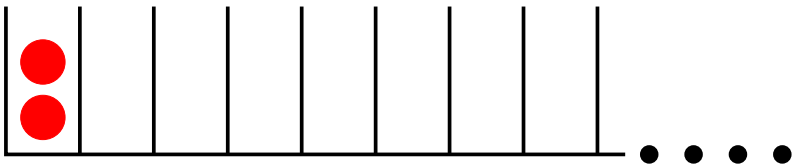
ONE BOSON STARTING AT SITE 1



1 BOSON STARTING AT SITE 2 (NEXT-TO-EDGE)



Next: two bosons



2 0 0 0 0 0 0 .....

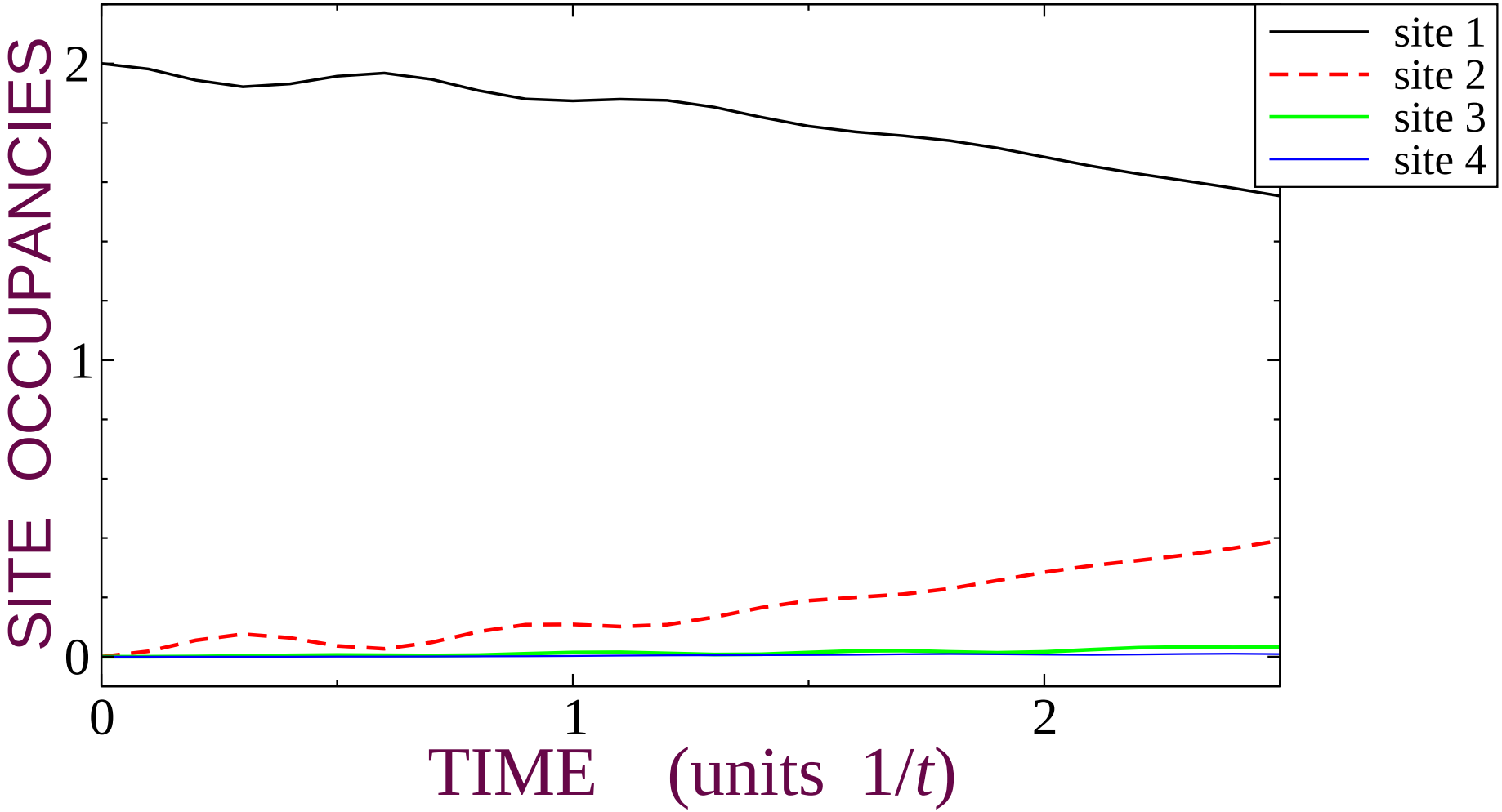
How does this evolve?

At timescales $\sim 1/t \sim 1$

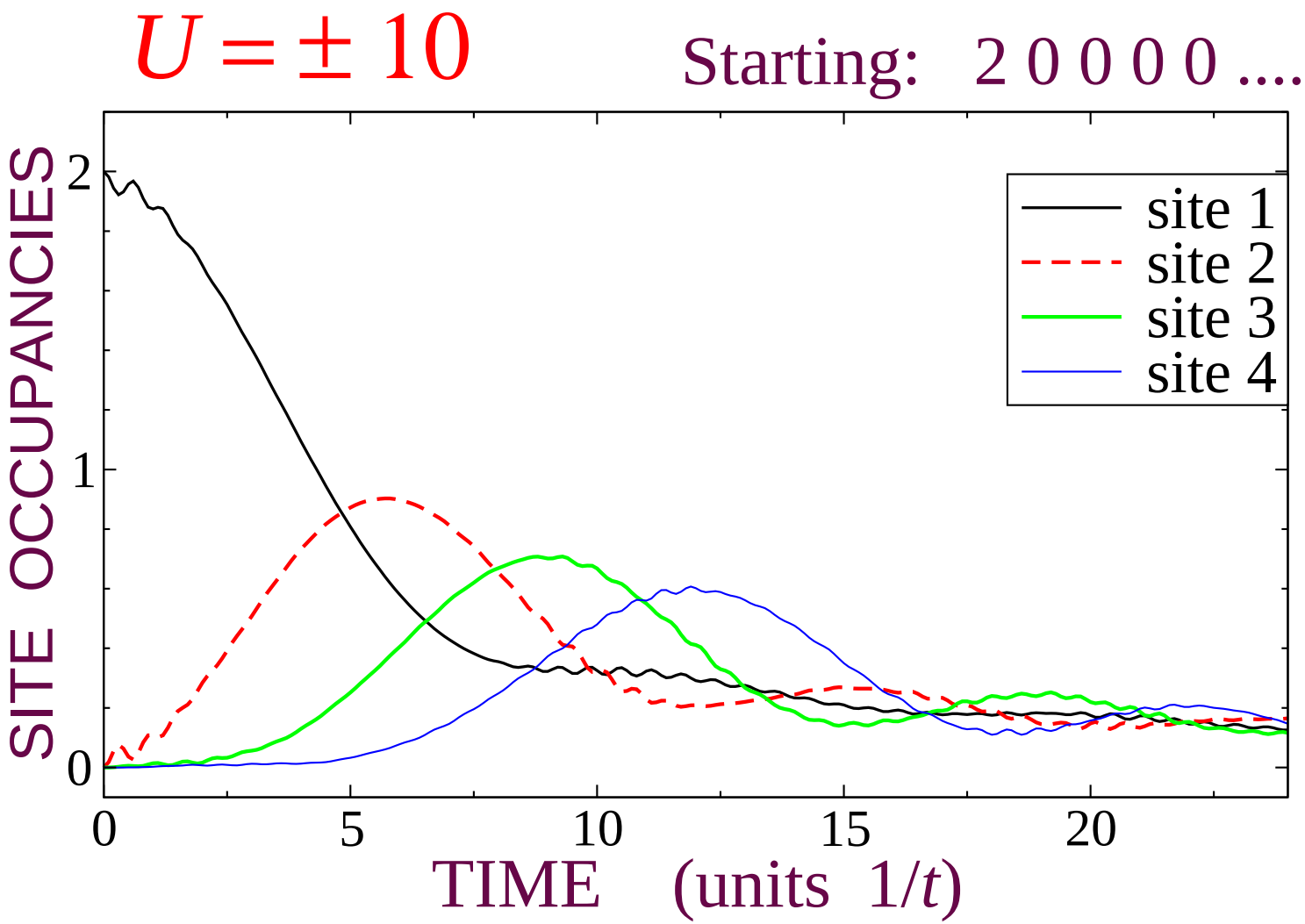
At timescales $\sim 1/(t^2/U) \sim U$

Two bosons at edge: timescales $\sim 1/t \sim 1$

$U = 10$ Starting: 2 0 0 0 0 ....



2 BOSONS AT EDGE: TIMESCALES $\sim 1/(t^2/U) \sim U$



Large U encourages correlated pair motion

Single particle hopping timescale $\sim 1/t \sim 1$

Pair hopping time scale $\sim 1/\left(\frac{t^2}{U}\right) \sim U$

"Repulsively bound pairs"

Triplet hopping time scale $\sim 1/\left(\frac{t^3}{U^2}\right) \sim U^2$

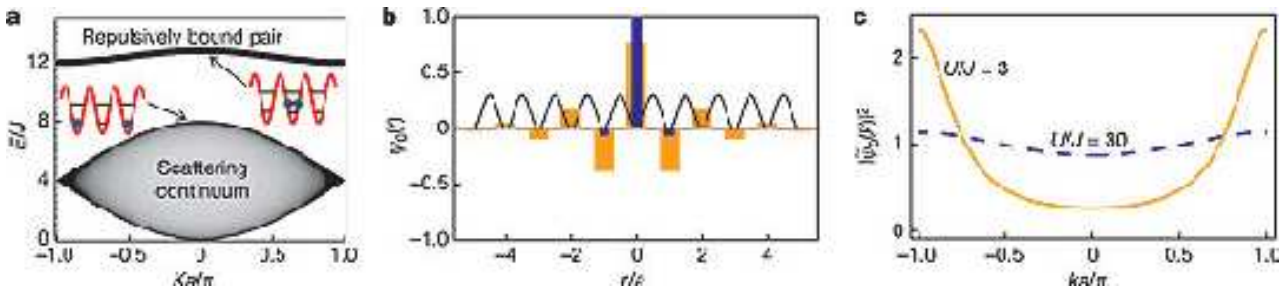
Repulsively bound pairs

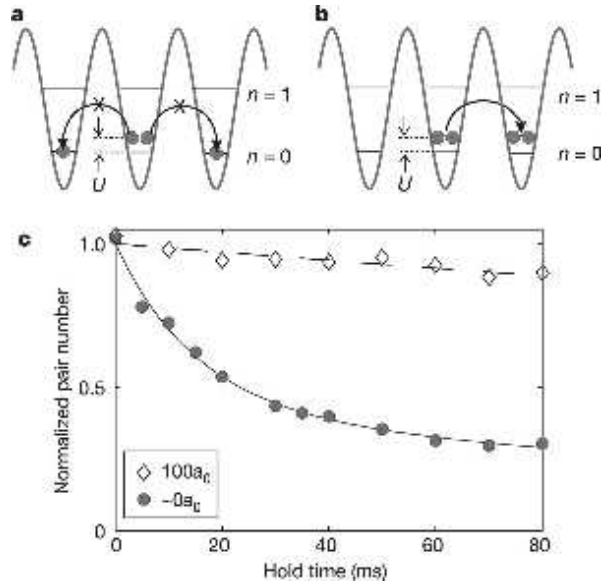
Vol 441|15 June 2006|doi:10.1038/nature04918 nature

LETTERS

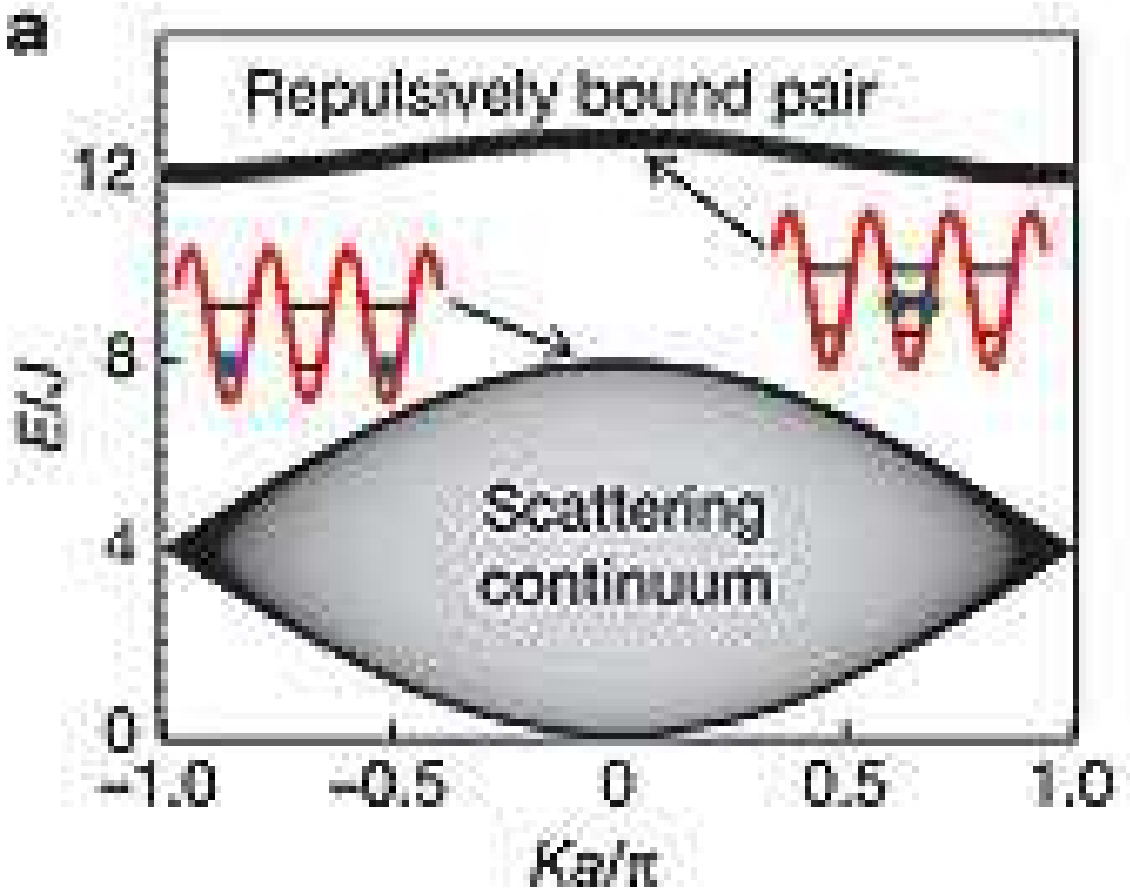
Repulsively bound atom pairs in an optical lattice

K. Winkler[1], G. Thalhammer[1], F. Lang[1], R. Grimm[1,3], J. Hecker Denschlag[1], A. J. Daley[2,3], A. Kantian[2,3], H. P. Büchler[2,3] & P. Zoller[2,3]





"Bands" in energy spectrum, 2 bosons



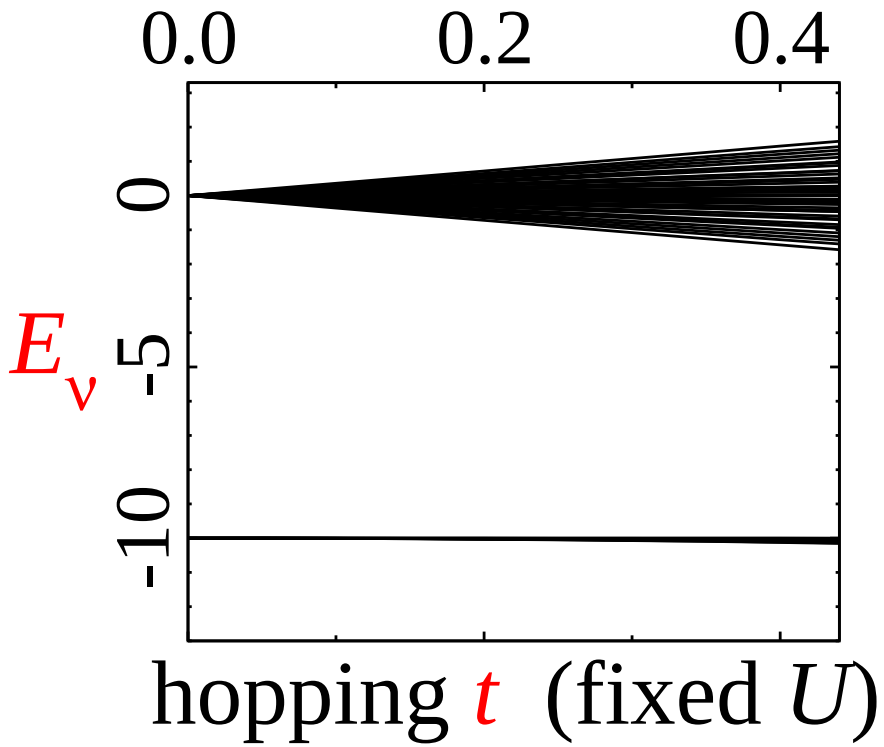
(translation-invariant picture)

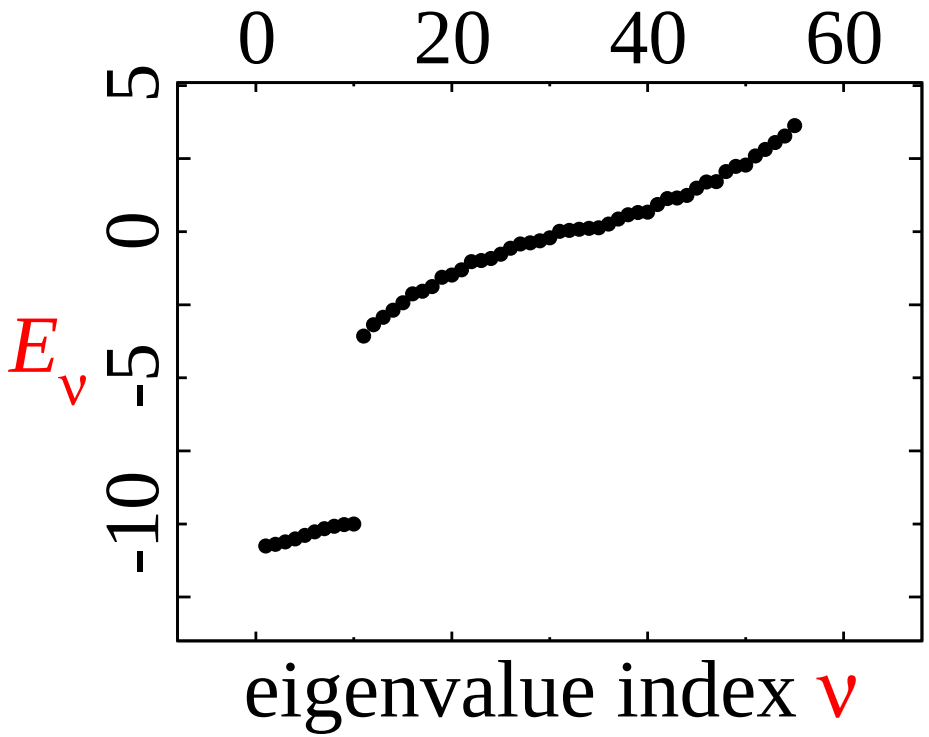
Pairs cannot break without losing energy,

$\Longrightarrow$ without energy relaxation mechanism.

2-BOSON SPECTRUM, BANDS

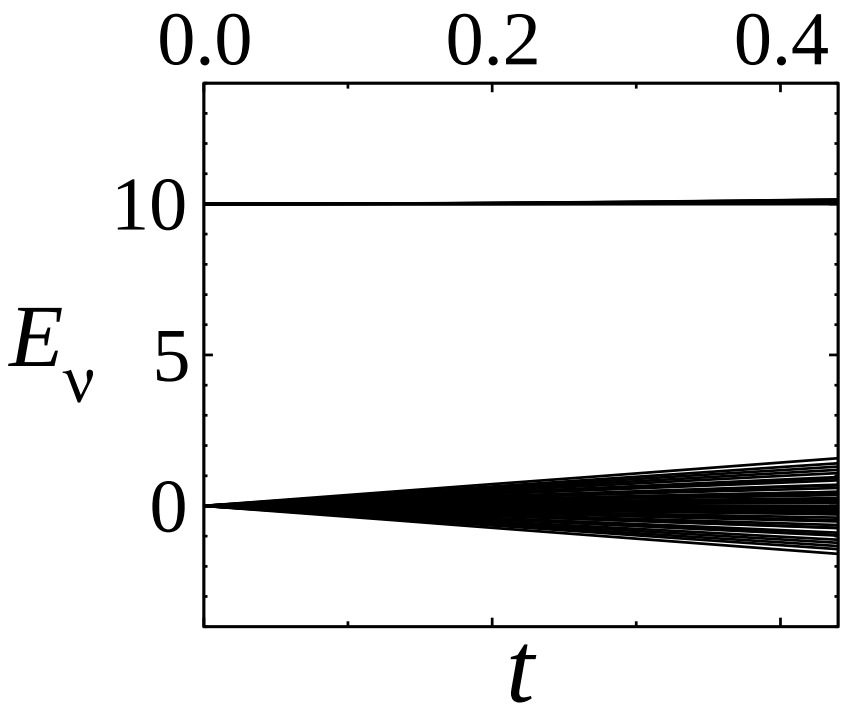
2 Bosons in 10-site open chain. Negative U !! $U = -10$

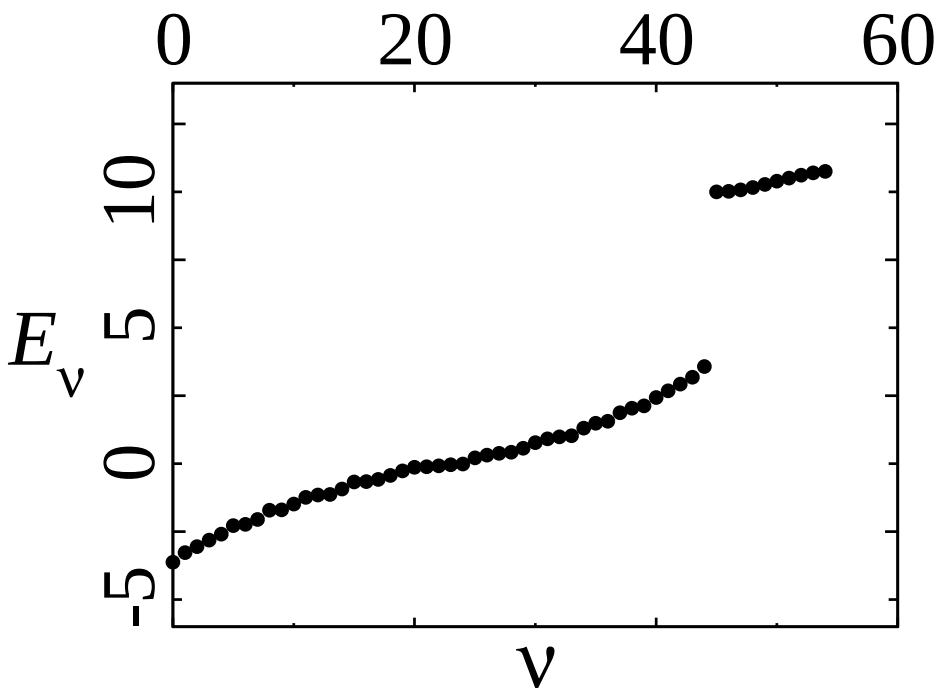




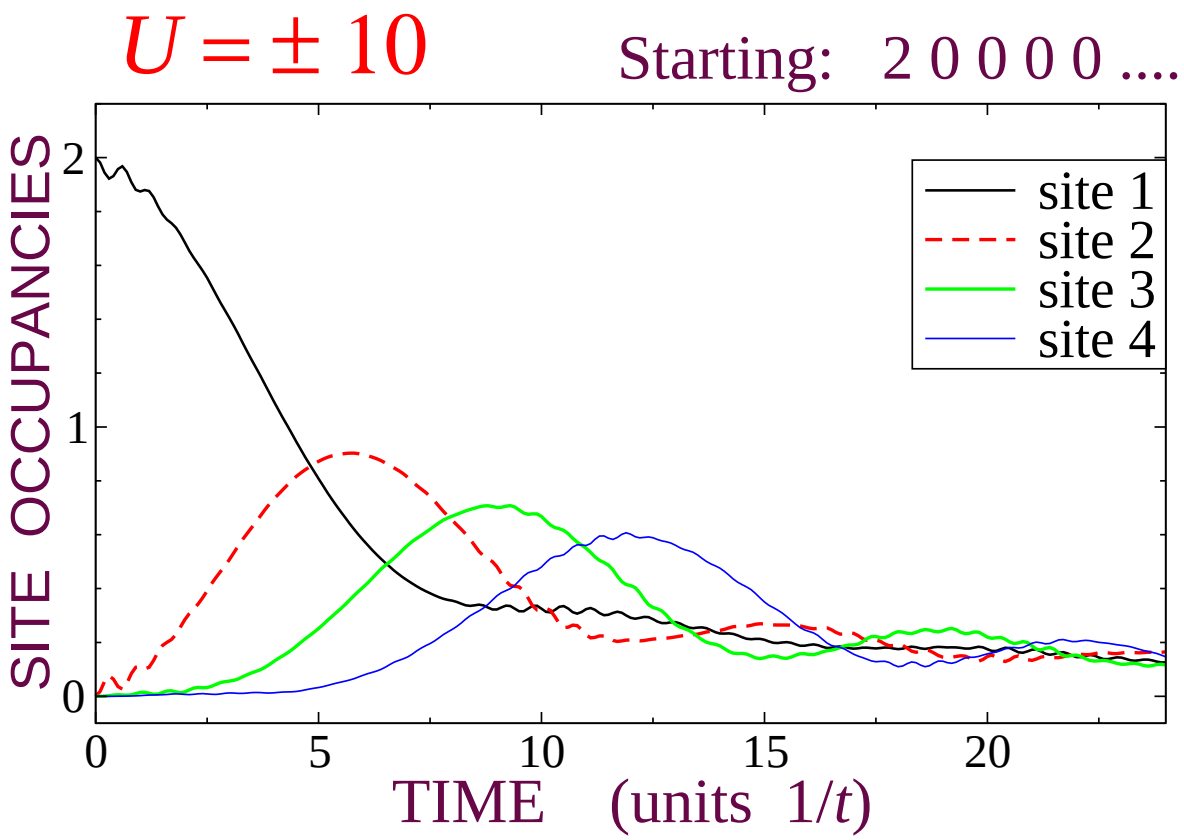
2-BOSON SPECTRUM, BANDS, POSITIVE U

2 Bosons in 10-site open chain. $U = +10$





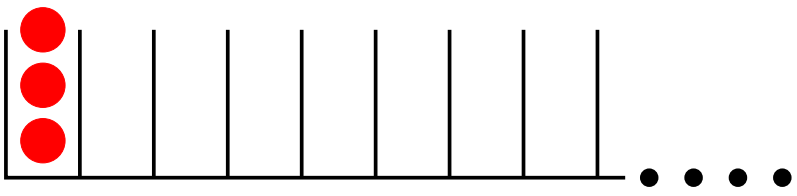
Two bosons



Long time-scale $\rightarrow$ hopping mostly within bound-pair band.

High-frequency oscillations $\rightarrow$ inter-band processes.

Let's move on: THREE bosons



3 0 0 0 0 0 0 .....

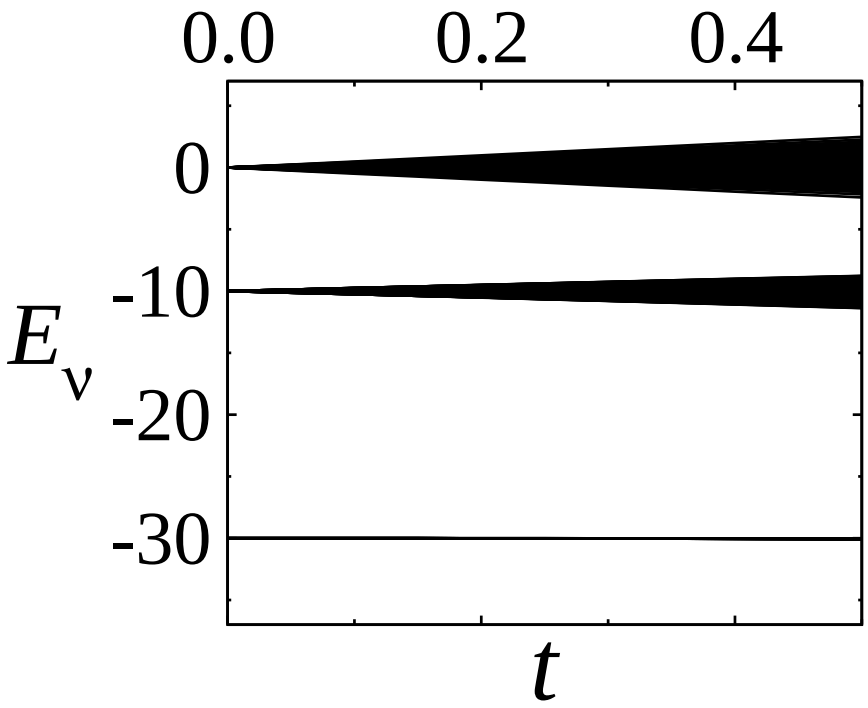
How does this evolve?

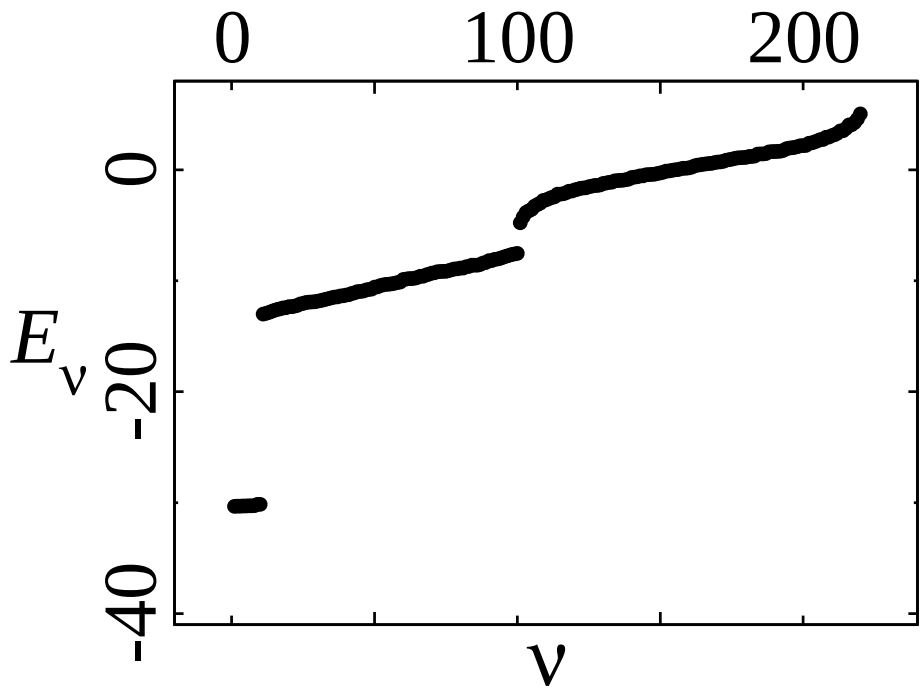
At timescales $\sim 1/t$

At timescales
$\sim 1/(t^3/U^2) \sim U^2/t^3$

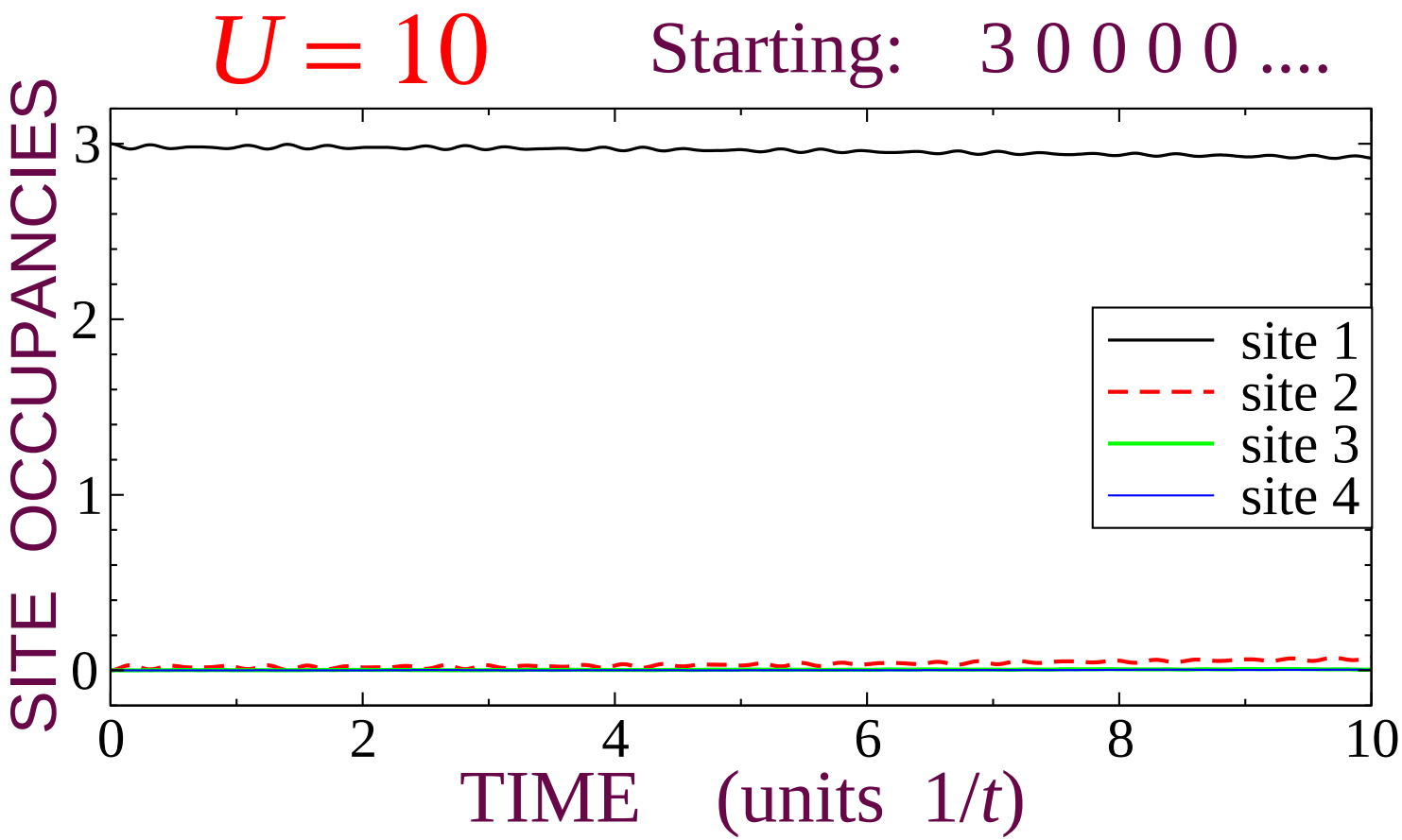
3-BOSON SPECTRUM, BANDS

3 Bosons in 10-site open chain. Negative U; $U = -10$



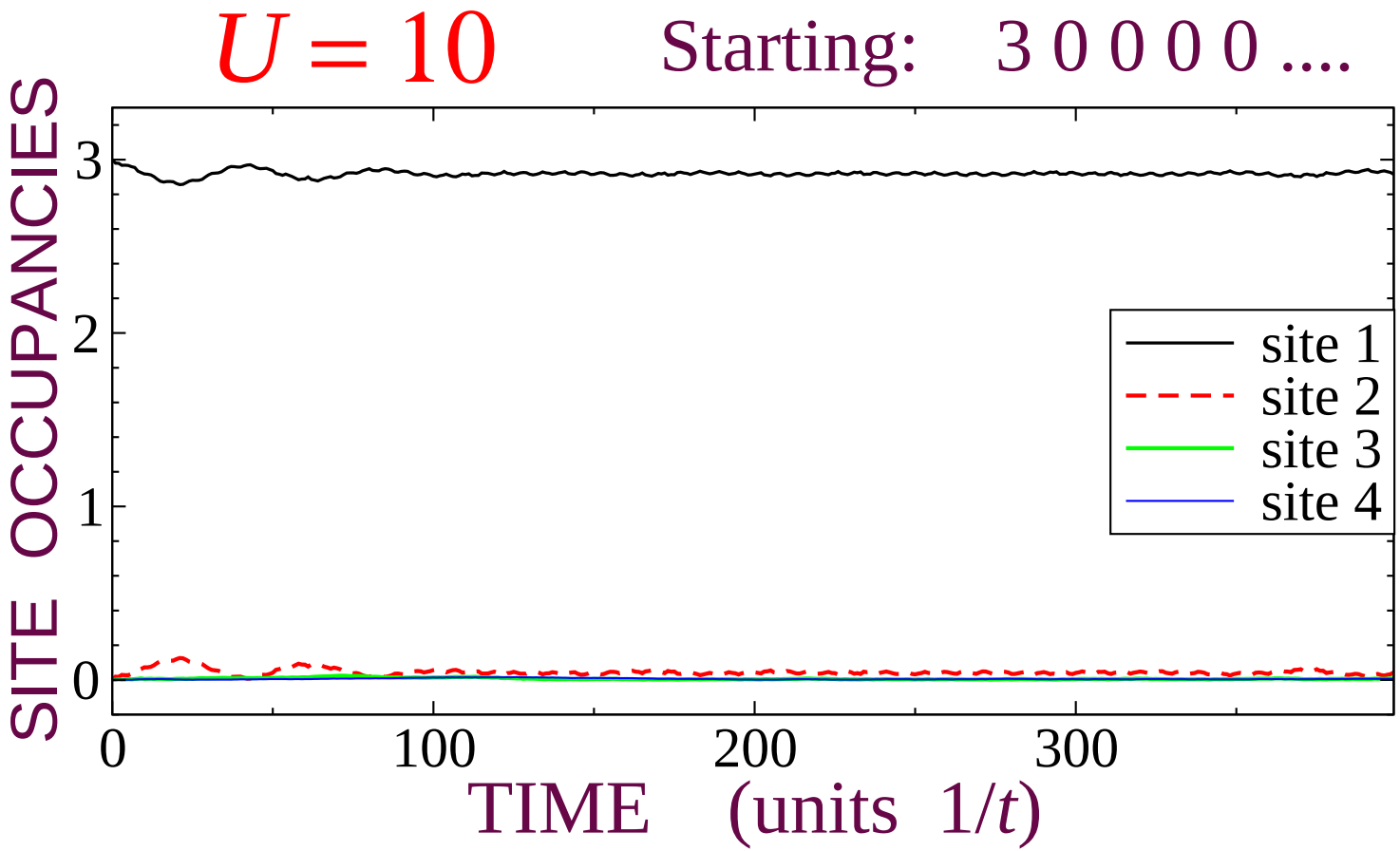


THREE BOSONS AT EDGE: TIMESCALES $\sim 1/t$



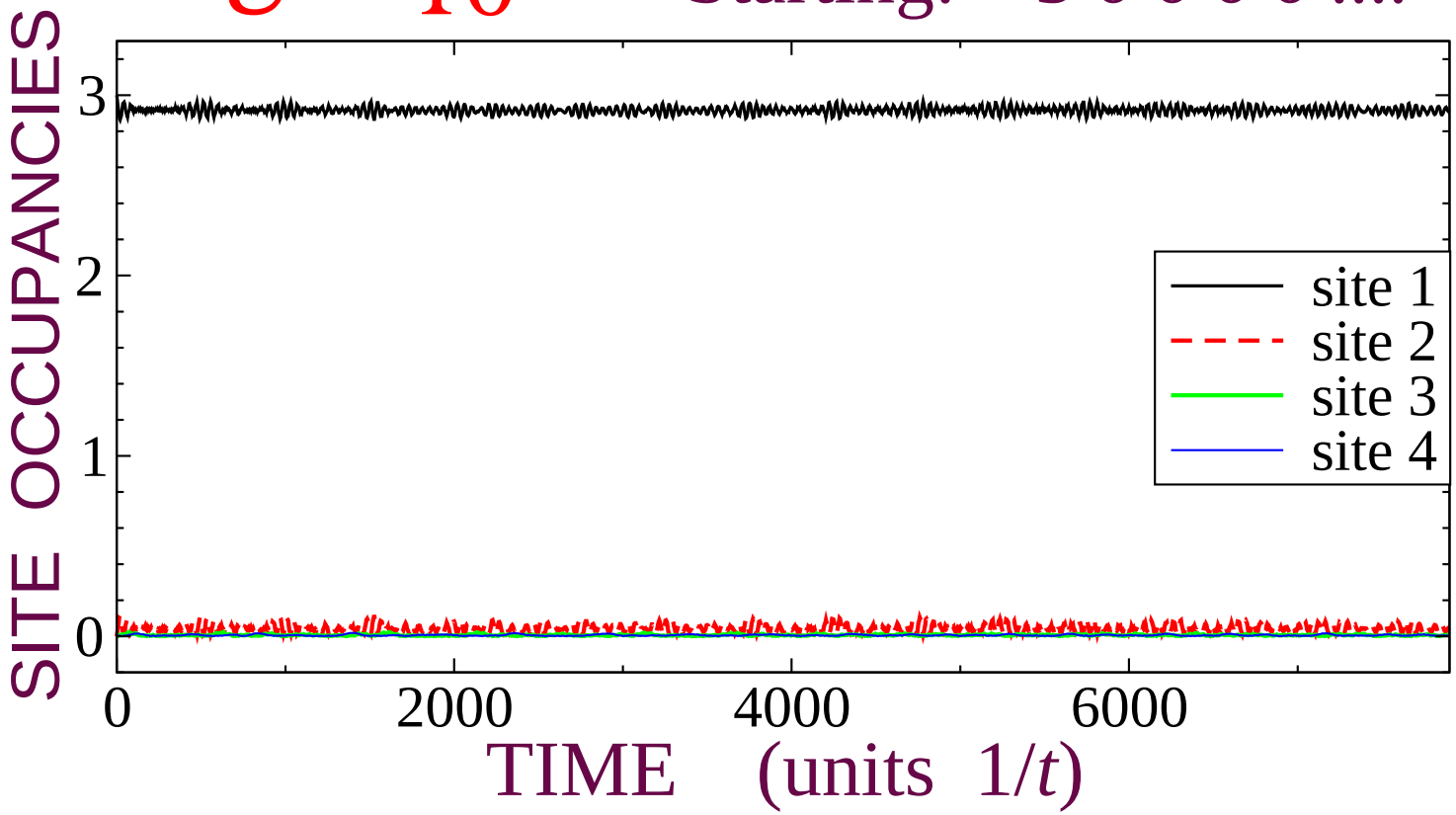
No big surprise.

THREE BOSONS AT EDGE: TIMESCALES $\sim U^2$



TRYING TIMESCALES $\gg \sim U^2$

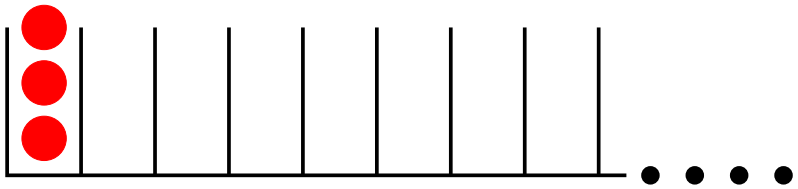
$U = 10$ Starting: 3 0 0 0 0 ....



? ? ? ? ? ? ? ? ?

You should be surprised

We've found a STABLE state

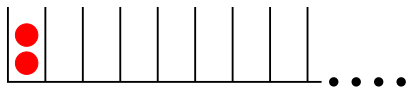


3 0 0 0 0 0 0 .....

For $n \geq 3$ bosons, edge states are stable.

Stable should mean "close" to an eigenstate?

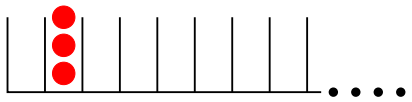
Hierarchy of edge-locked states



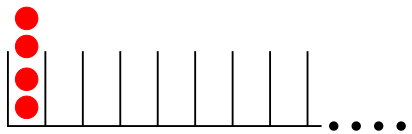
NOT STABLE

NOT STABLE

STABLE

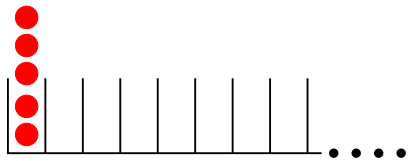


NOT STABLE

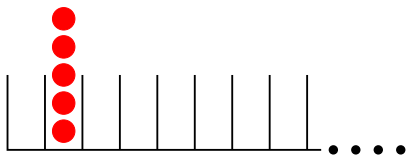


STABLE

NOT STABLE

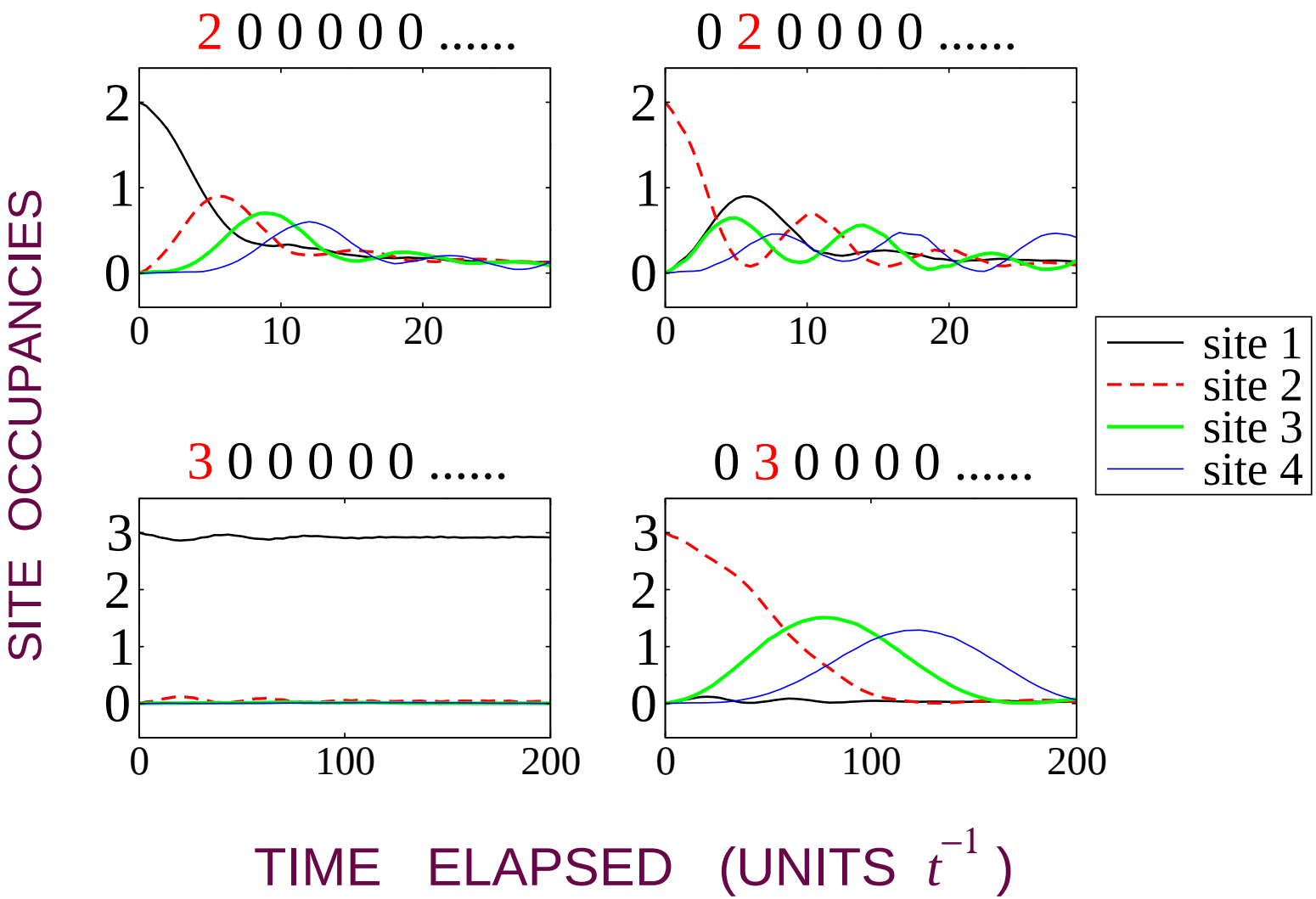


STABLE

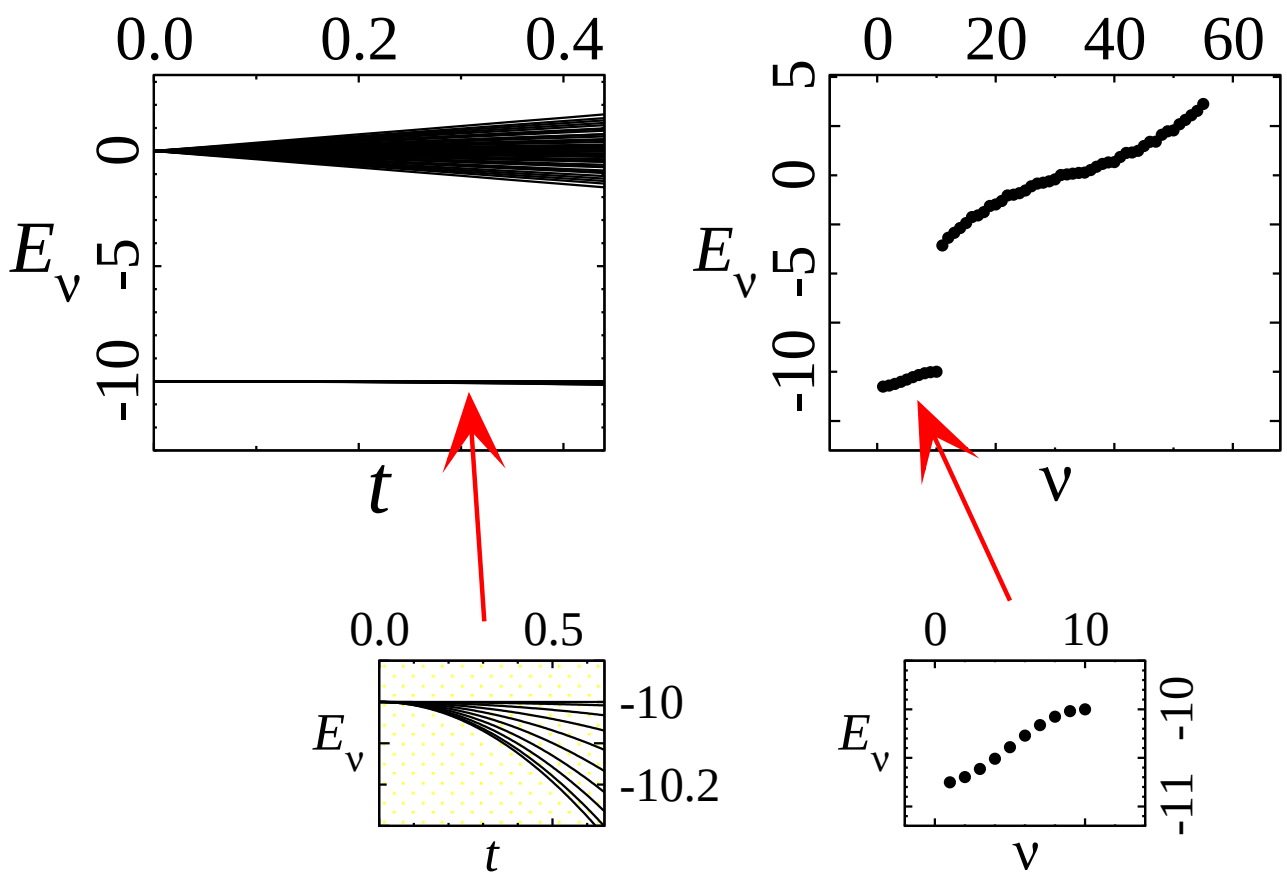


STABLE

TIME EVOLUTION SUMMARY



Structure of 'bound' band: two bosons



Linear combinations of

$|20000.....000\rangle$

$|02000.....000\rangle$

$|00200.....000\rangle$

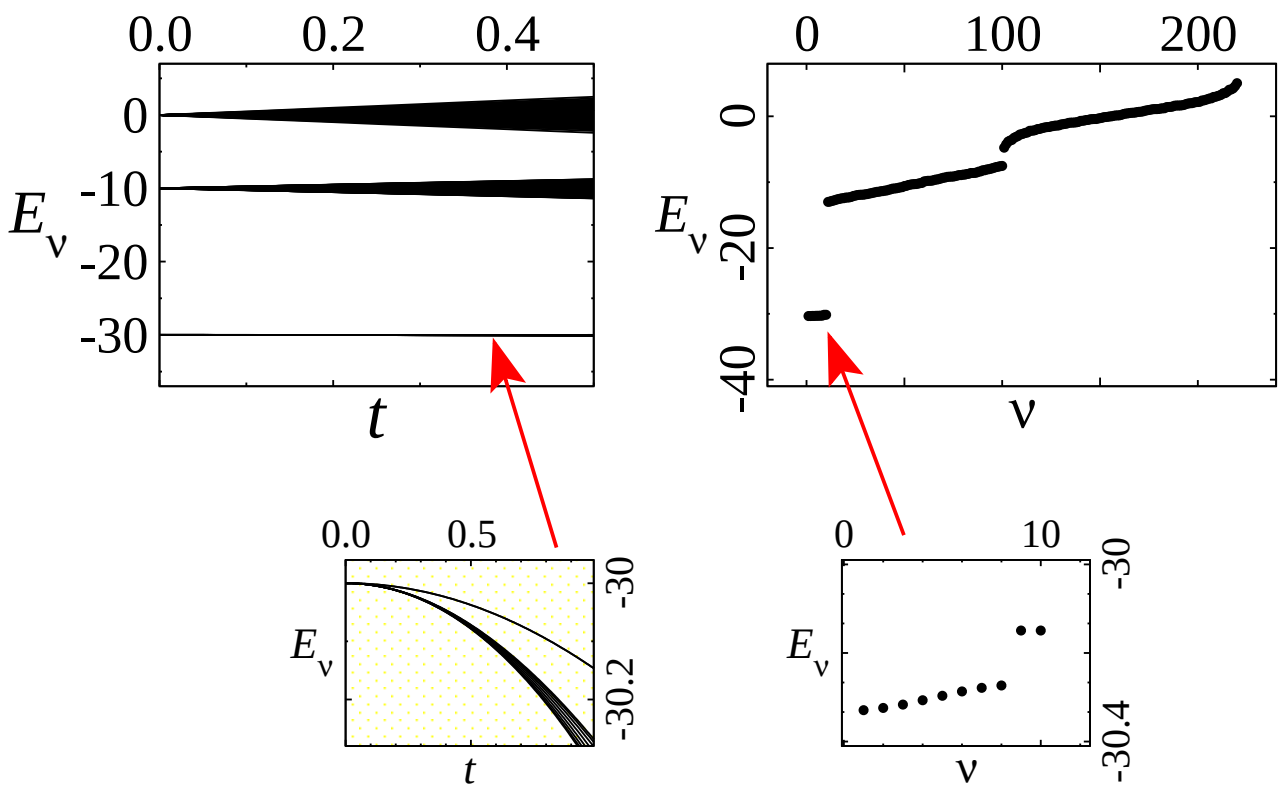
$|00020.....000\rangle$

..

..

$|0000.....002\rangle$

‘Bound’ band: three bosons



Linear combinations of

$|03000.....000\rangle$

$|00300.....000\rangle$

$|00030.....000\rangle$

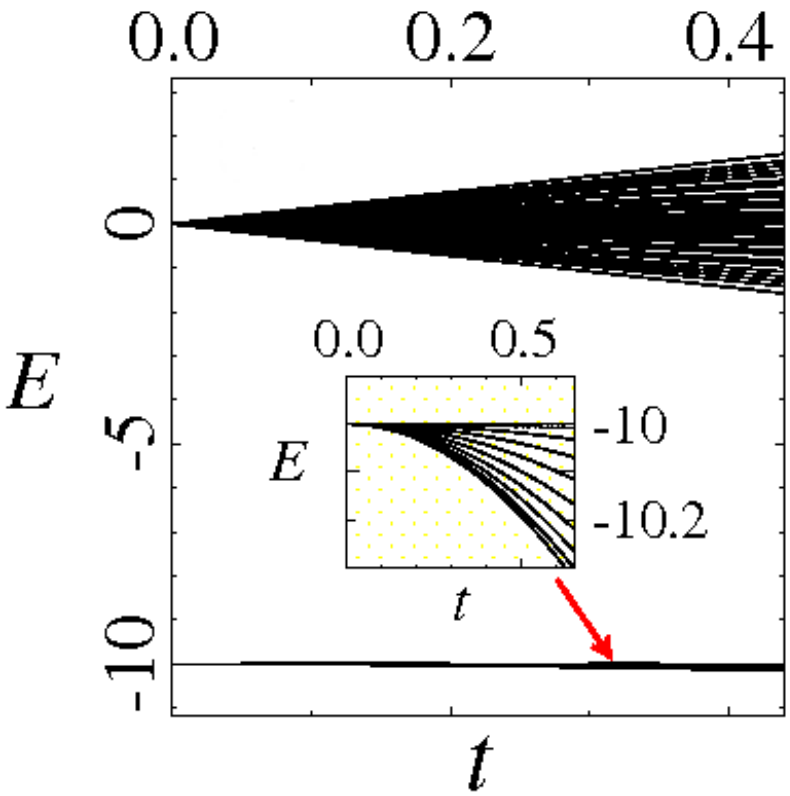
..

..

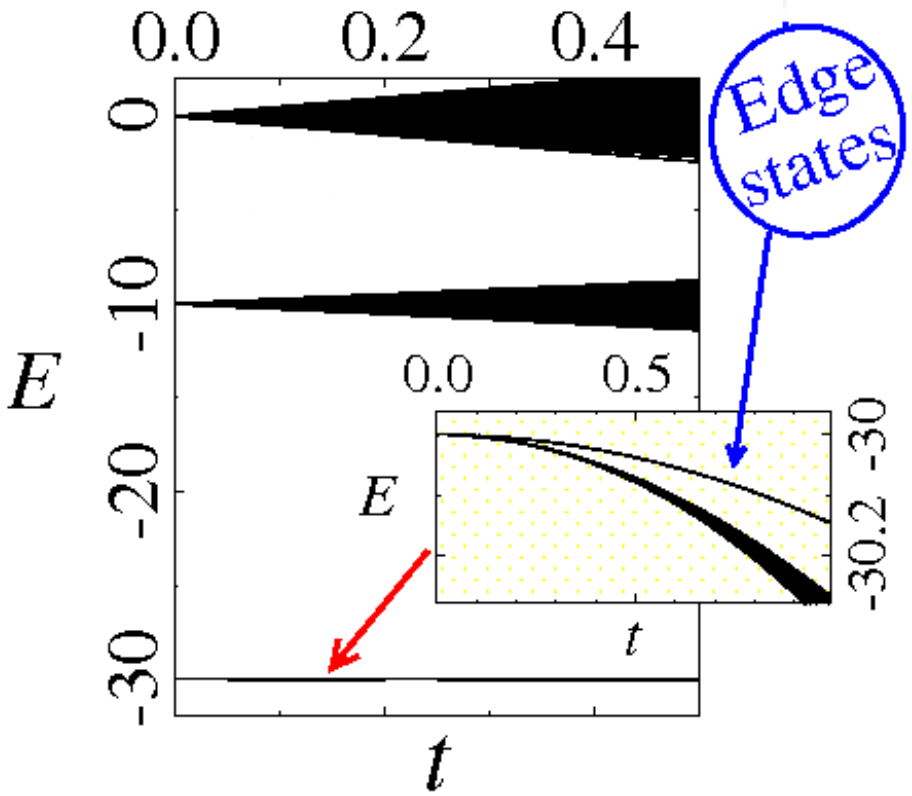
$|0000.....030\rangle$

Separated out from the rest: $|30000.....000\rangle$ and $|0000.....003\rangle$.

2 bosons



3 bosons



| L > =  | R > = 

Edge eigenstates: | L > + | R > and | L > − | R >

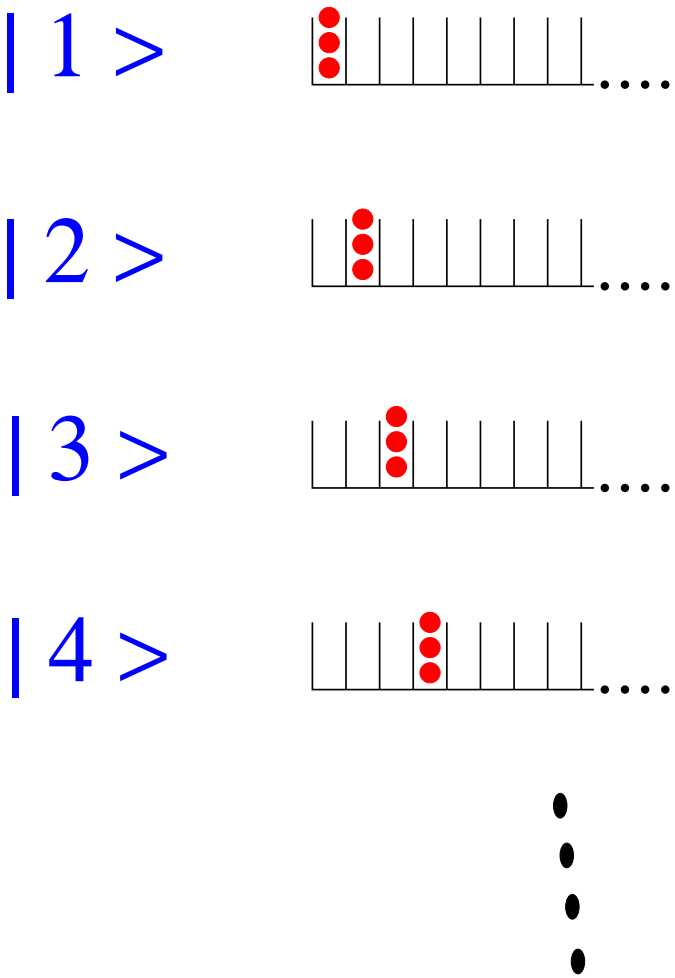
Spectral separation explains stability of edge states

Who ordered the spectral separations?

Degenerate perturbation theory.

Competition between energy shifts at $\mathcal{O}(t^2)$ and manifold mixing at $\mathcal{O}(t^n)$.

Degenerate perturbation theory



Degenerate manifold at $t/U = 0$.

States $|j\rangle$ and $|j+1\rangle$ connect at $\mathcal{O}(t^n)$.

$\Longrightarrow$ mixing / dispersion at $\mathcal{O}(t^n)$.

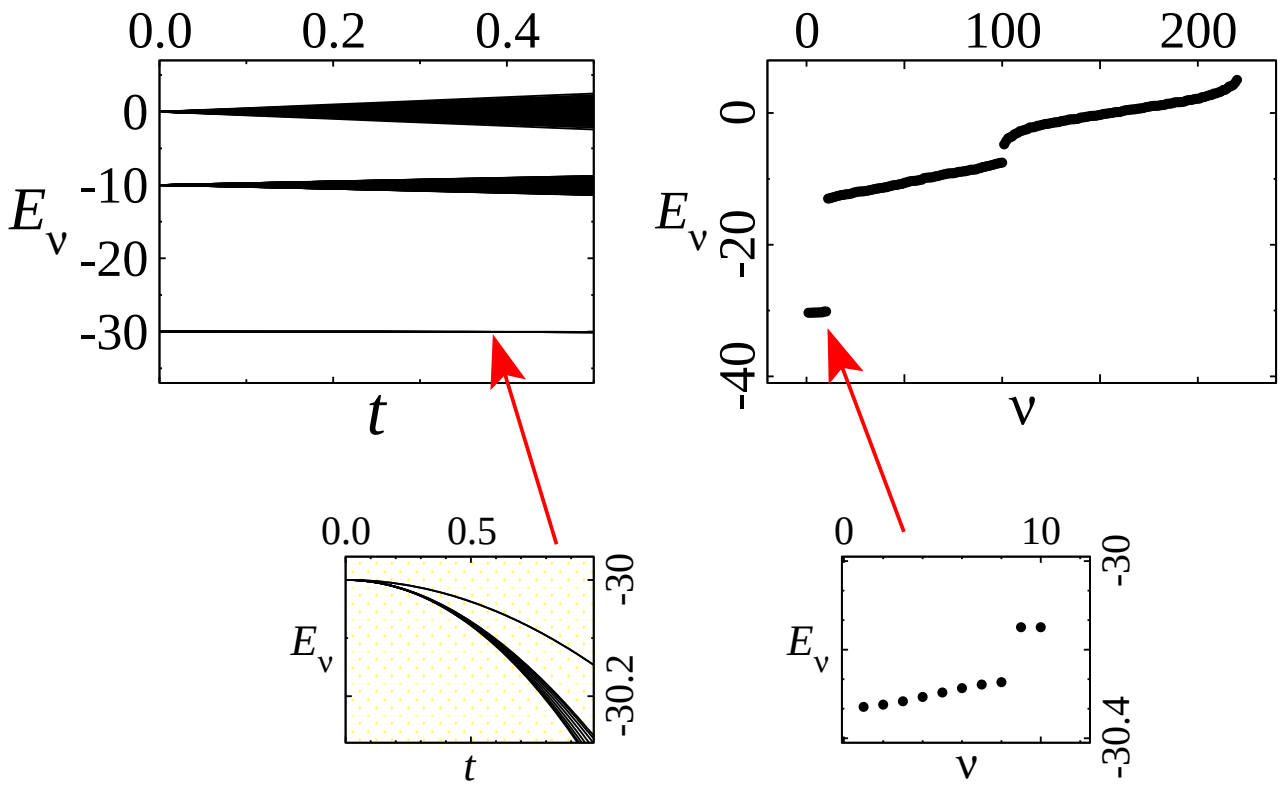
State $|1\rangle$ acquires different shift at $\mathcal{O}(t^2)$.

$\Downarrow$

Spectral separation if $\mathcal{O}(t^2)$ beats $\mathcal{O}(t^n)$.
(1st level of hierarchy)

State $|2\rangle$ acquires different shift at $\mathcal{O}(t^4)$.
(2nd level) ....

THREE BOSONS: $\mathcal{O}(t^2)$ VERSUS $\mathcal{O}(t^3)$



Linear combinations of

$|03000.....000\rangle$

$|00300.....000\rangle$

$|00030.....000\rangle$

..

..

$|0000.....030\rangle$

Separated out from the rest: $|30000.....000\rangle$ and $|0000.....003\rangle$.

What I'm missing....

There should be a

sum over histories

interpretation

3	0	0	0	0	.....
0	3	0	0	0	.....
0	0	3	0	0	.....
2	1	0	0	0	.....
2	0	1	0	0	.....
1	2	0	0	0	.....
1	0	2	0	0	.....
1	1	1	0	0	.....
1	1	0	1	0	.....

Spinless fermion model: similar hierarchy

$$\hat{H} = -t\sum_{j=1}^{L-1}\left(c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j\right) + V\sum_{j=1}^{L-1} c_j^\dagger c_{j+1}^\dagger c_{j+1} c_j$$

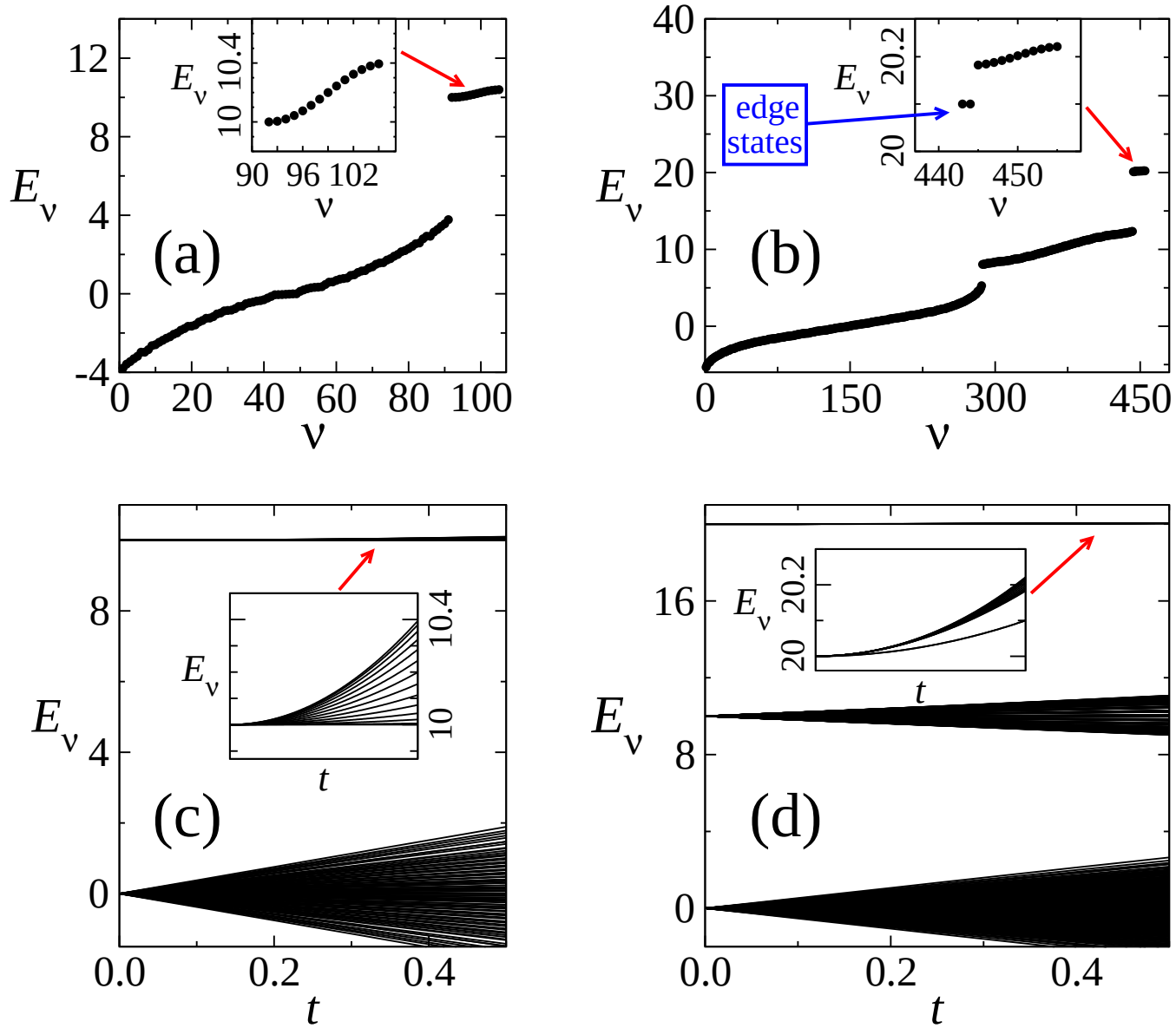
Sometimes called t-V model or Heisenberg-Ising model.

1 1 1 0 0 0 0 0 0 0 0	1 0 0 0 0 0 0 0 0 0 0 .....
1 1 1 1 0 0 0 0 0 0 0	1 1 0 0 0 0 0 0 0 0 0 .....
1 1 1 1 1 0 0 0 0 0 0	0 1 0 0 0 0 0 0 0 0 0 .....
1 1 1 1 1 1 0 0 0 0 0	0 1 1 0 0 0 0 0 0 0 0 .....
0 1 1 1 1 1 0 0 0 0 0	0 1 1 1 0 0 0 0 0 0 0 .....
0 1 1 1 1 1 1 0 0 0 0	0 1 1 1 1 0 0 0 0 0 0 .....

SPINLESS FERMIONS: SPECTRUM

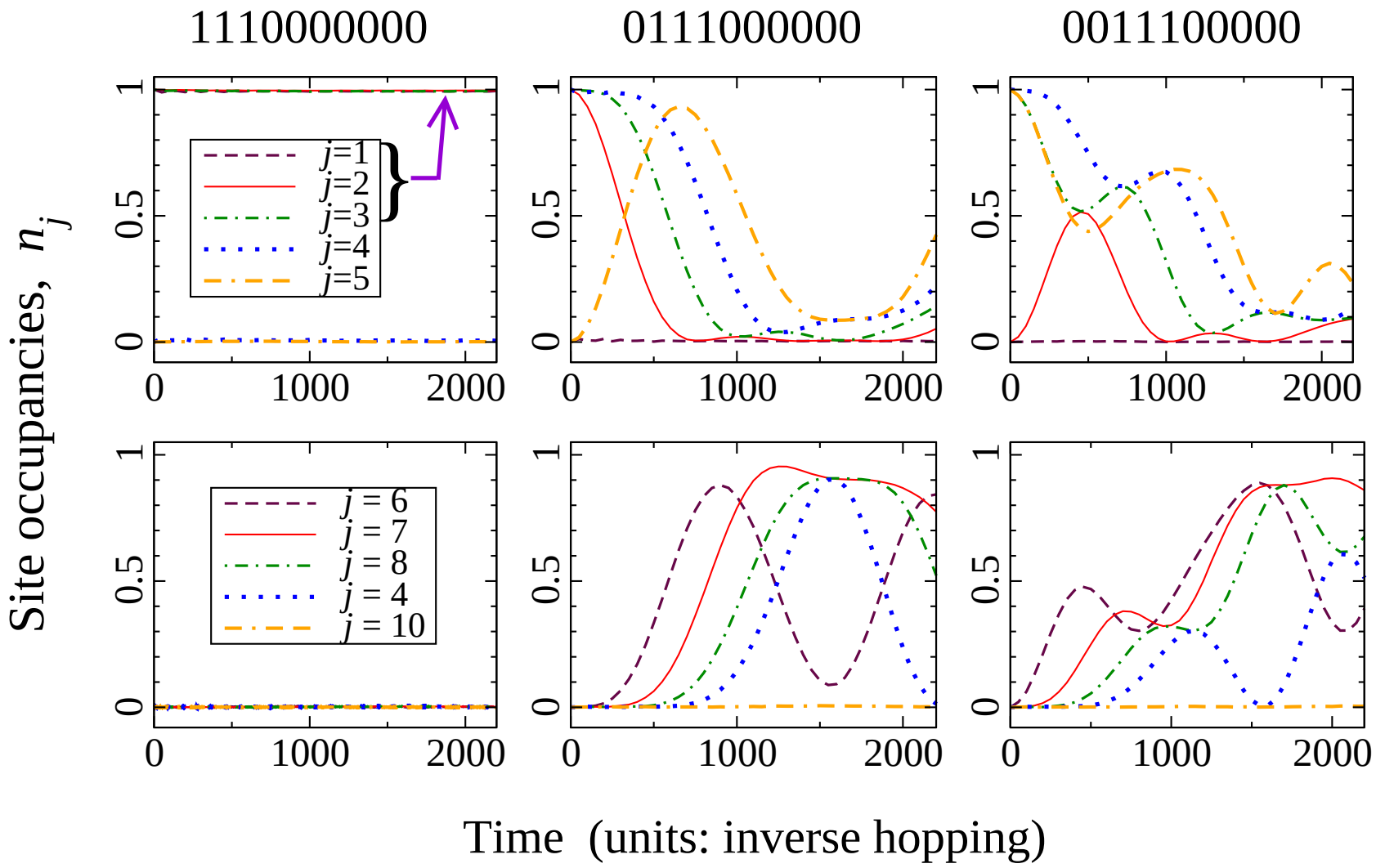
2 fermions

3 fermions



Two, three fermions in 15 sites

Spinless fermions: dynamics



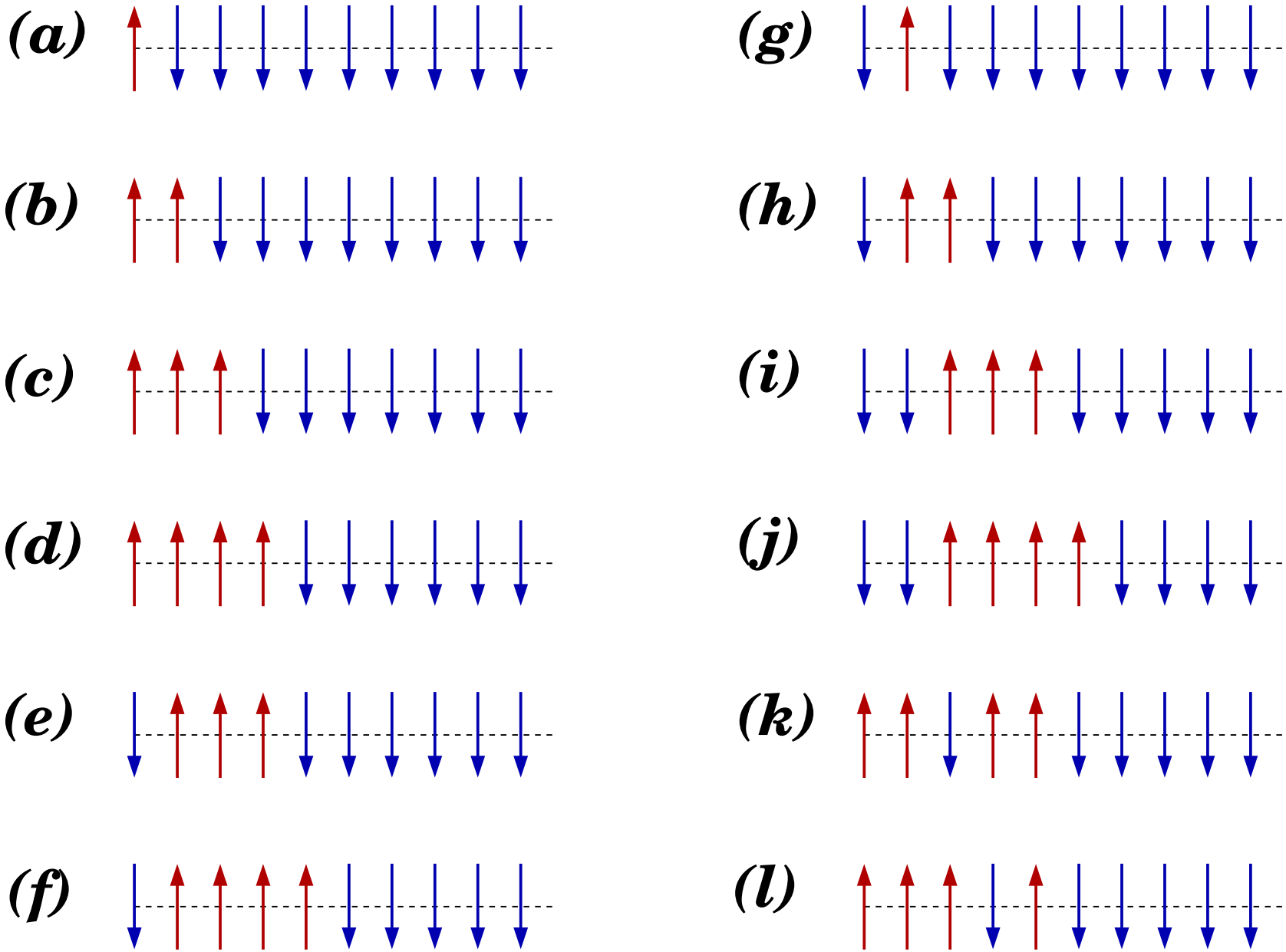
Anisotropic Heisenberg (XXZ) chain

$$H = J_x \sum_{j=1}^{L-1} \left[S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z \right]$$

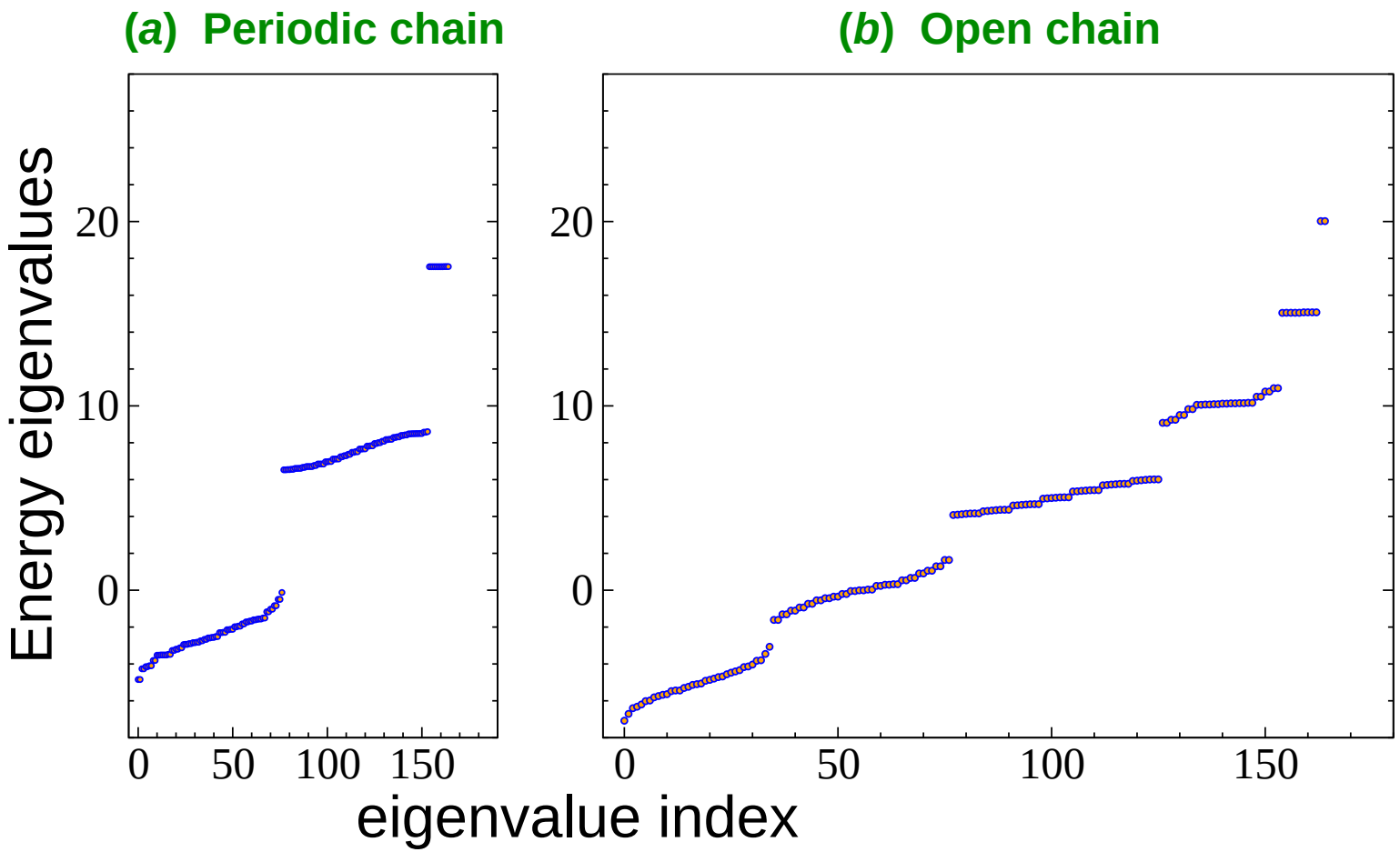
Edge-locking hierarchy $\rightarrow$ surprisingly different from t-V model.

Physical t-V model has $V n_i n_{i+1}$, not $V(n_i - \frac{1}{2})(n_{i+1} - \frac{1}{2})$.

Physical t-V model does not have
empty-empty or empty-occupied energy.
(Only occupied-occupied energy.)


(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
(i)
(j)
(k)
(l)

XXZ CHAIN: PERIODIC VERSUS OPEN SPECTRA

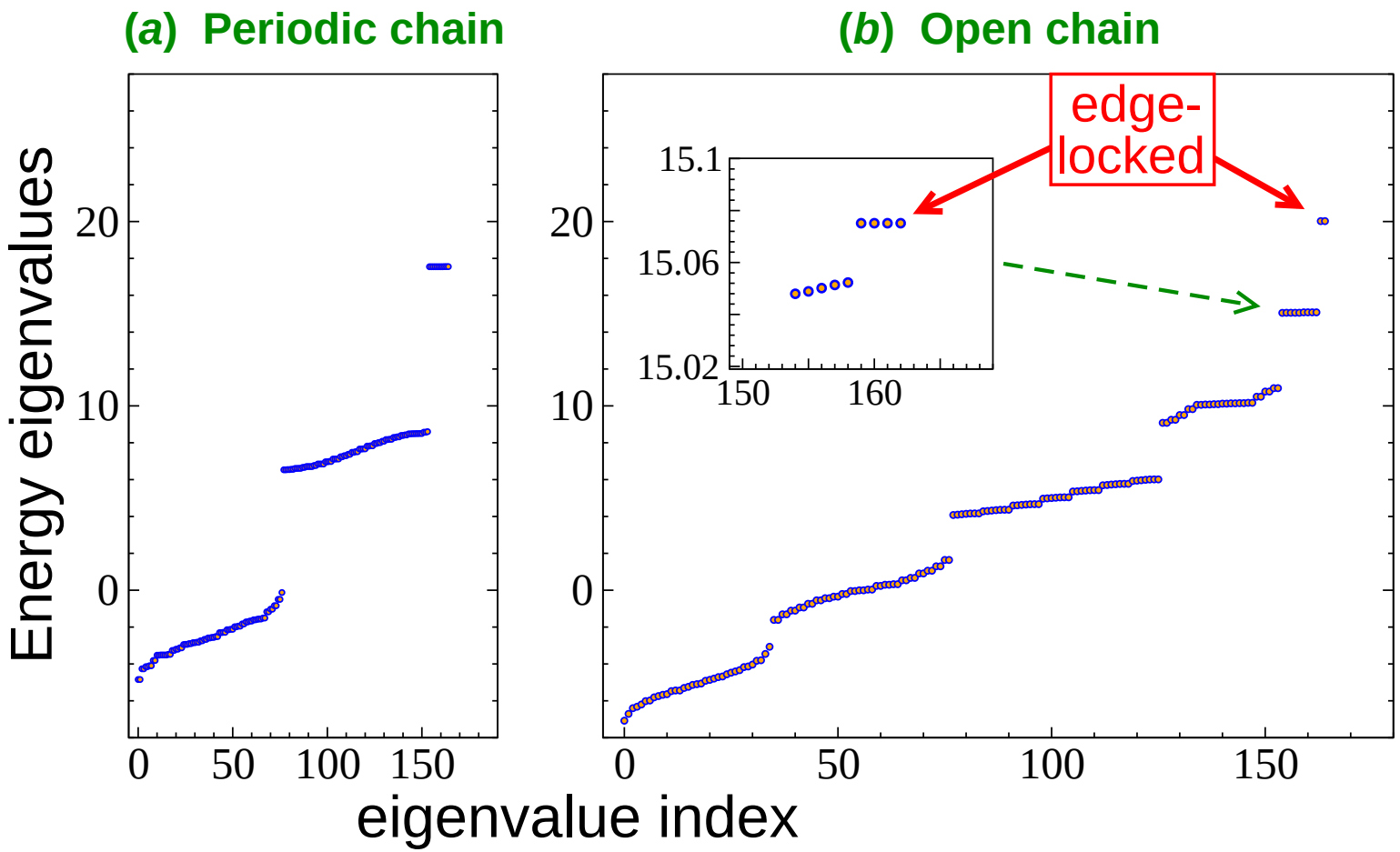


$N_\uparrow = 3$

11 sites

Many extra spectral features in open chain

XXZ CHAIN: PERIODIC VERSUS OPEN SPECTRA

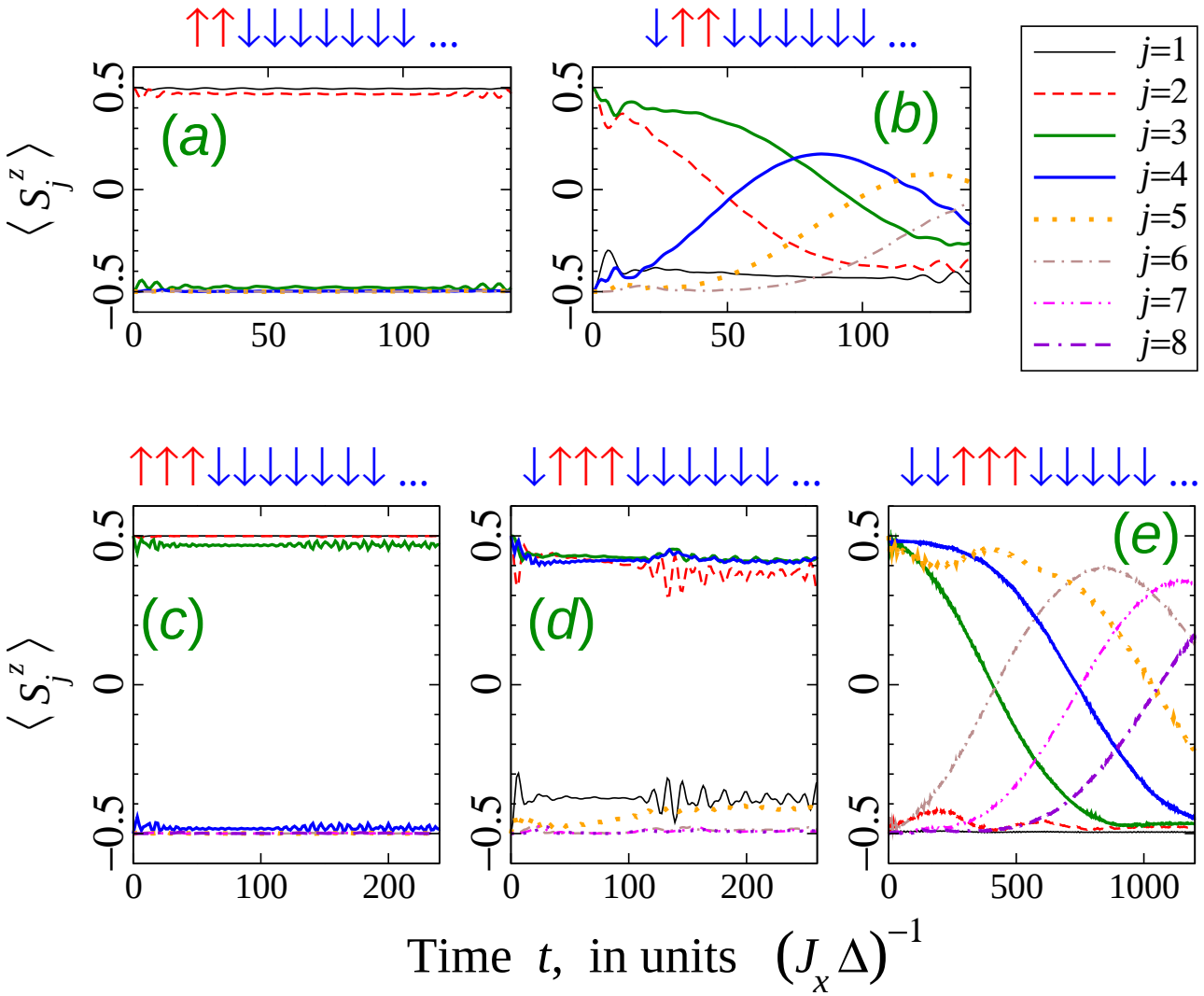


$N_\uparrow = 3$

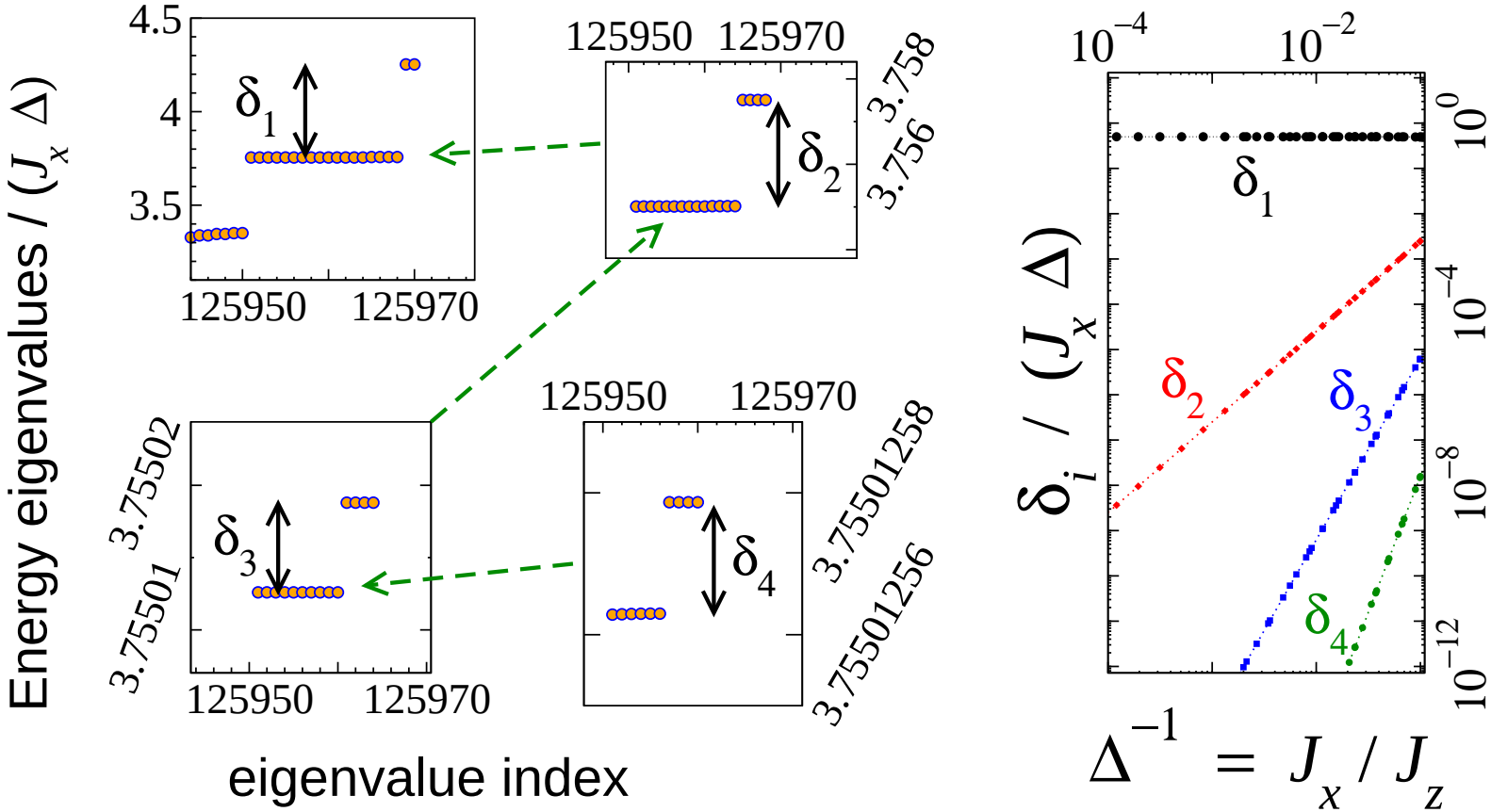
11 sites

Many extra spectral features in open chain

XXZ CHAIN: DYNAMICS



XXZ CHAIN: HIERARCHY



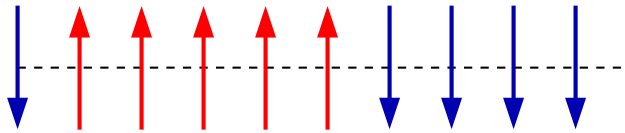
$N_\uparrow = 8$; 20 sites. $\delta_1 \sim \Delta^0$ $\delta_2 \sim \Delta^{-2}$ $\delta_3 \sim \Delta^{-4}$

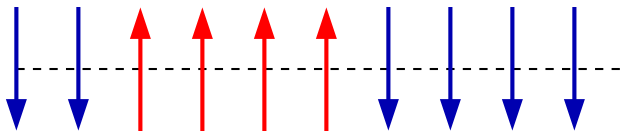
Hierarchy of Spectral Separations

Energy spectrum contains structures at
many different scales.

FRACTAL structure in spectrum

"Quantum control" of magnetization transport





Single-site π-pulse

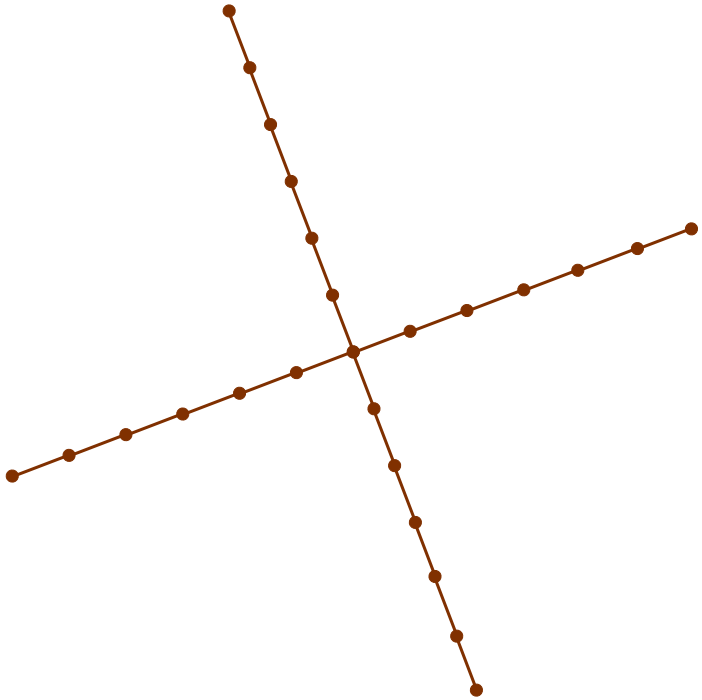
$\Downarrow$

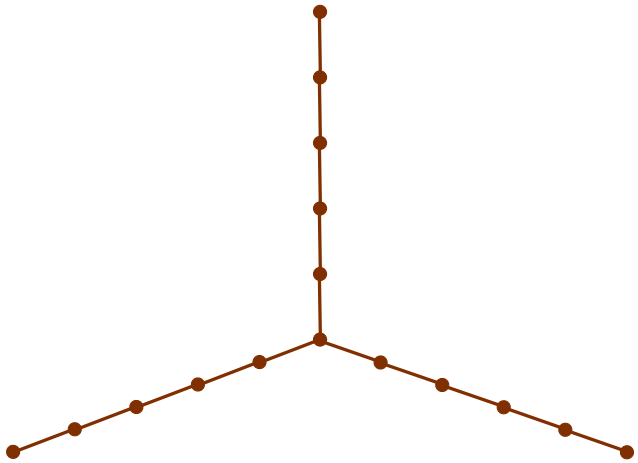
quantum switch

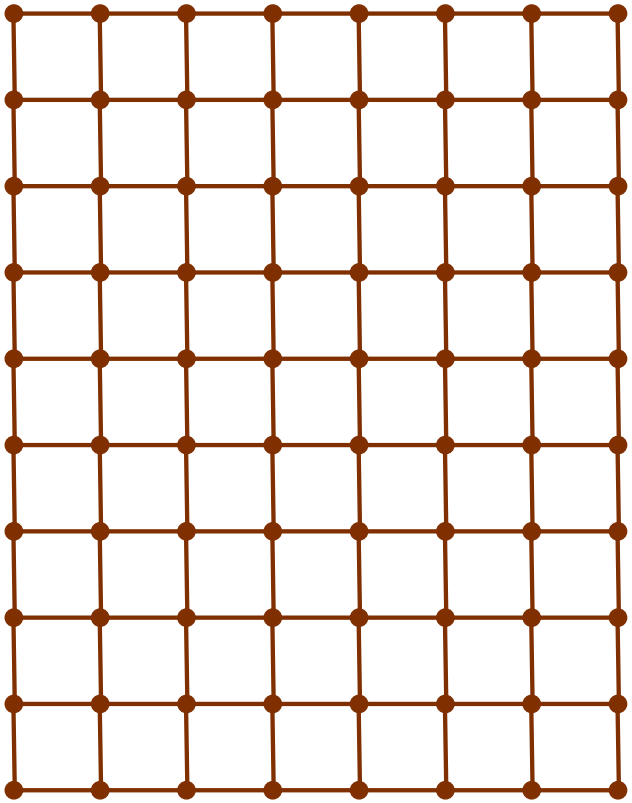
for unlocking magnetization.

Many other control protocols....

Other geometries: junctions, 2D





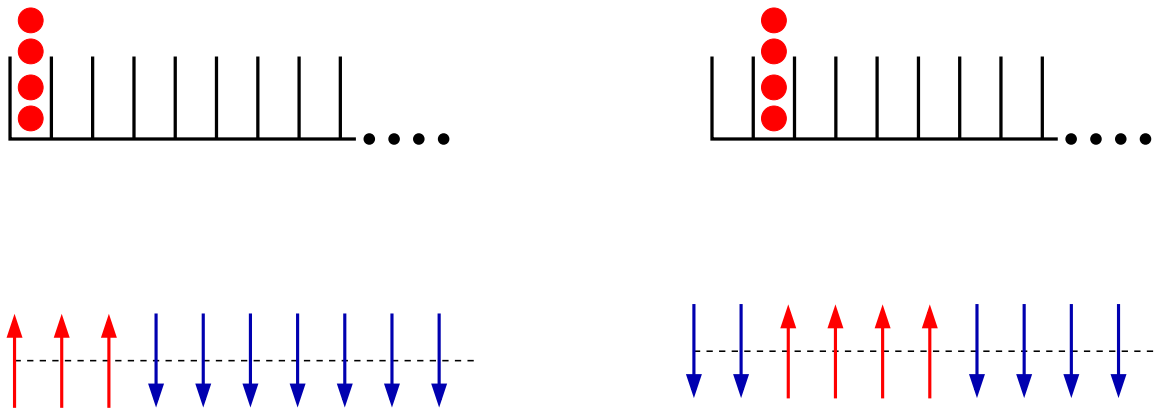


Edge-localization in 1D lattice models

Bose-Hubbard chain

spinless fermion model

XXZ chain



PHYSICS:

Far-from-equilibrium dynamics

Eigenstates far from ground state

Intricate structures in spectrum (FRACTAL)

QUANTUM CONTROL:

Locking and release of magnetization/state

Designing a quantum switch

Edge localization: more bosons

For more bosons, a hierarchy of localization patterns.

$n \geq 5$ bosons $\longrightarrow$ can also be bound in site 2

$n \geq 7$ bosons $\longrightarrow$ can also be bound in site 3

... ...etc

Actually, several hierarchies, with other localization patterns:

2 2 0 0 0 0 .....

Isn't this just self-trapping?

Question from
nonlinear dynamics
and/or BEC
community

Isn't this just self-trapping?

NO

Aren't edge states ubiquitous?

yes, but

No single-particle edge-localization in our tight-binding model

Our edge-locking is an interaction effect

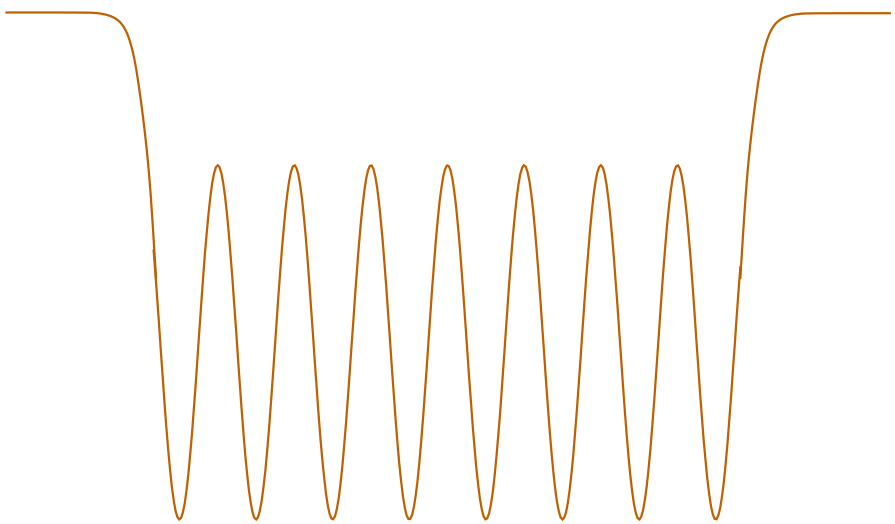
Aren't edge states ubiquitous?

yes, but

No single-particle edge-localization in our tight-binding model

Typical solid-state situation:

$$\hat{H} = -t \sum_{j=1}^{L-1} \left(c_j^\dagger c_{j+1} + c_{j+1}^\dagger c \right) - \lambda c_1^\dagger c_1 - \lambda c_L^\dagger c_L$$



Experimental realizations

We would like

Clean and sharp edge, Single-site addressibility

Possible experimental settings:

Cavity polariton arrays	1D optical lattices
Josephson junction arrays	Solid-state magnets with chain structures.
(ideal, but not yet realized)	(not so ideal)

Tunnel to other edge?

$$|L\rangle = |3000.....00\rangle \qquad \text{and} \qquad |R\rangle = |00.....0003\rangle$$

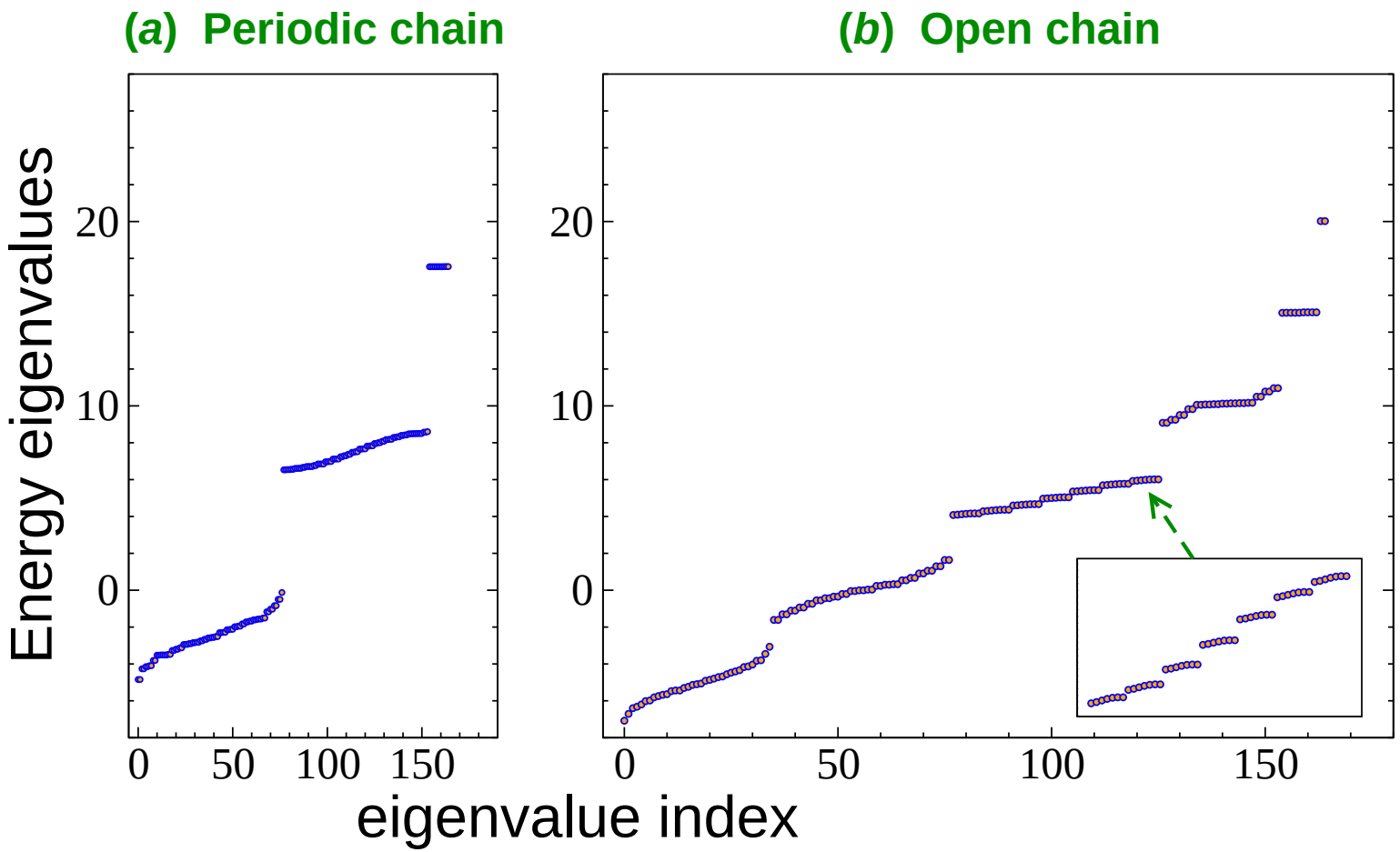
Question: Why doesn't $|L\rangle$ tunnel to $|R\rangle$?

Answer: It will. After some astronomically long time.

$|L\rangle \leftrightarrow |R\rangle$ tunneling exponentially suppressed.

Splitting between $|L\rangle + |R\rangle$ and $|L\rangle - |R\rangle$ exponentially small.

XXZ CHAIN: PERIODIC VERSUS OPEN SPECTRA



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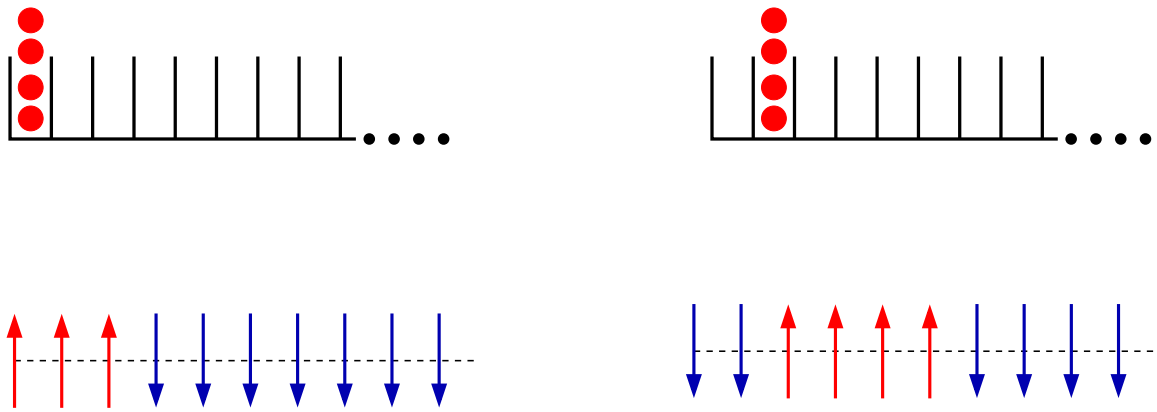

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