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Can the graviton have mass?

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Theories with massless particles
describe long-range forces.

It seems natural that the theory with
massless particles can be viewed
as a smooth limit of a certain
massive theory with a finite range
forces.

Feynman '63

We know that this works for
spin 0 and $\frac{1}{2}$ particles. It also works
for QED which can be viewed as
a limit of massive vector ($s=1$) theory.

Subtlety starts with nonabelian
Yang-Mills theory, continuity there is
achieved with Higgs mechanism of
mass generation.

In the most dramatical way
the problem shows up in theory of
gravity.

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Compare the long-range interaction in Einstein theory (exchange by massless graviton) with the finite-range interaction due to exchange by massive spin 2 particles in the limit when the mass goes to zero.

This comparison was done in three independent works in 1970.

Ixrasaki
van Dam, Veltman
Zakharov

It was shown that in the linearized approximation the limit does exist but does not coincide with the Einstein theory.

The discontinuity leads to a different values for the bending of light by the sun: the limit from the massive theory is $3/4$ of the Einstein theory.

Generalities

Poincare group representations
Different for $m \neq 0, m=0$
 $\underline{m \neq 0}$, $p_\mu, \mu=0, 1, 2, 3$ $p_\mu p^\mu = m^2$
 spin S , $2S+1$ polarization states

$m=0$ $p_\mu, p_\mu p^\mu = 0$
 helicity h , one-dimensional rep.
 CPT relates h and $-h$

In case of $S=0, Y_2$ there is no discontinuity in the number of polarization states when $m \rightarrow 0$ (accounting for discrete symmetries).

When $S=1$ we deal with three states at $m \neq 0$, and with only two states, $h=\pm 1$, for $m=0$.

Let us consider this case in more detail.

Neutral vector field

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For massless photon the QED lagrangian has the form

$$\mathcal{L} = -\frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} \gamma^\mu (i\gamma_5 + \gamma_\mu) \psi - m \bar{\psi} \psi$$

It contains four fields A_μ while there are only two helicity states for photon. The lagrangian does not depend on all four fields due to gauge invariance

$$A_\mu \rightarrow A_\mu + \partial_\mu \lambda, \quad \psi \rightarrow e^{i\lambda} \psi$$

Fixing gauge (e.g. Coulomb one, $\vec{\nabla}\vec{A}=0$) and excluding nondynamical A_0 (Lagrange multiplier) we come to two degrees of freedom.

The massive vector field is described by adding (Proca)

$$\mathcal{L}_m = \frac{m^2}{2e^2} A_\mu A^\mu$$

Excluding the nondynamical A_0 we get three degrees of freedom.

Often is said that the introduction of mass breaks gauge invariance. It is not correct, # of d.o.f., it is what matters. Gauge invariance can be restored by (Stückelberg's method)

$$\mathcal{L}_m = \frac{m^2}{2e^2} (A_\mu + \frac{1}{m} \partial_\mu \varphi)^2$$

with the extra field φ , transforming as

$$\varphi \rightarrow \varphi - m$$

under the gauge transformation.

Three polarization states of massive vector particle with the momentum k_μ are described by ϵ_μ

$$k^\mu \epsilon_\mu = 0$$

In the rest frame $k^\mu = (m, 0, 0, 0)$

$$\epsilon^{(1)} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \epsilon^{(2)} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \epsilon^{(3)} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

After boosting along x-axis, $k^\mu = (E, K, 0, 0)$

$$\epsilon^{(1)} = \begin{pmatrix} \frac{K}{m} \\ \frac{E}{m} \\ 0 \\ 0 \end{pmatrix}, \quad \epsilon^{(2)} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \epsilon^{(3)} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$h=0$ $h=+1$

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The kinematical growth of ϵ^{μ} for the zero helicity state could imply a growth of interaction. However,

$$\epsilon_{(1)}^\mu = \frac{E^\mu}{m} + \frac{m}{E+k} \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad (1)$$

The first term drops out from the interaction in ϵ^μ due to the current conservation, the second vanishes at the $m \rightarrow 0$ limit.

It works for any number of the $h=0$ quanta and shows the decoupling of the helicity zero when $m \rightarrow 0$.

Note that decoupling refers to EM interaction. The $h=0$ particles do not decouple from gravity.

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Non-Abelian vector field

The Lagrangian of Yang-Mills theory

$$\mathcal{L} = -\frac{1}{4g^2} (G_{\mu\nu}^a)^2$$

$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f^{abc} A_\mu^b A_\nu^c$ describes the multiplet of fields A_μ^a , an adjoint rep of the group G .

Again, one can introduce the "hard" mass m , adding

$$\mathcal{L}_m = \frac{m^2}{2g^2} (A_\mu^a)^2$$

The amplitude of the process with some number of the massive quanta can be written as

$$M = \bar{q}_1 q_1 \dots \bar{q}_n q_n \cdot \epsilon^{a_1 a_1} \dots \epsilon^{a_n a_n}$$

Let us choose polarizations ϵ_i to be zero helicity, i.e.

$$\epsilon^{aa} = \left(\frac{E^\mu}{m} + \frac{m}{E+k} n^\mu \right) \cdot \chi^a, \quad n^a = \left(-1, \frac{\vec{E}}{|E|} \right)$$

If only one polarization is longitudinal we see the same decoupling as in the abelian case.

The term $\epsilon^{\mu_1 \mu_2 \dots \mu_n} T^{a_1 a_2 \dots a_n} \epsilon^{\nu_1 \nu_2 \dots \nu_n} t^{a_1 a_2 \dots a_n} = 0$ ⁽⁹⁾
due to the conservation of current.

If two quanta are longitudinal we get

$$\frac{m}{E_1 + E_2} n^{m_1} f_1^{a_1} \frac{k_2^m}{m} f_2^{a_2} g^{a_1 a_2 \dots a_n} \epsilon^{a_3 a_4 \dots a_n}$$

which does not vanish when the commutator $[t^{a_1}, t^{a_2}] \neq 0$ (Ward identities)

Thus, for two longitudinal quanta we see a finite discontinuity at $m \rightarrow 0$. For more than two $b=0$ quanta we get a singular behaviour

$$M_{m=0} \propto \left(\frac{E}{m}\right)^{n_{b=0}-2} g^{n_{b=0}} \quad \text{Khriplovich, A.V. '71}$$

It shows both: absence of the zero mass limit and nonrenormalizability of the theory with the "hard" mass in perturbation theory.

To isolate the singular behavior it's convenient to use Stückelberg's method,

$$A_\mu = U^{-1} B_\mu U + i U^{-1} \partial_\mu U, \quad A_\mu = A_\mu^{ab} t^a \\ G = SU(N) \quad U^\dagger U = 1 \quad d\det U = 1$$

The substitution brings in a gauge redundancy⁽¹⁰⁾

$$B_\mu \rightarrow S B_\mu S^{-1} + i S \partial_\mu S^{-1}$$

$$U \rightarrow SU$$

Let us fix the gauge by putting B_μ to be related to the transversal quanta, the matrix U then describe the longitudinal quanta.

Then

$$L_m = \frac{m^2}{4g^2} \text{Tr} (\partial_\mu U^{-1} \partial^\mu U)$$

gives amplitudes of processes with longitudinal quanta. It is nonrenormalizable chiral model with the coupling

$$\frac{g^2}{m^2}$$

singular at $m \rightarrow 0$.

On the other hand it gives a hint how to overcome the strong coupling: nonlinear δ -model in weak coupling will stop the growth at energies larger than masses of extra δ particles.

It is just what used for the Higgs mechanism of generation of mass for the nonabelian vector field.

Massive gravity

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The Einstein theory of gravity is described by the Lagrangian

$$\mathcal{L} = -M_{Pl}^2 \sqrt{-g} R \quad M_{Pl}^2 = \frac{1}{16\pi G_N}$$

where R is the scalar curvature for the metric $g_{\mu\nu}$ and $g = \det[g_{\mu\nu}]$. In quadratic approximation near the flat vacuum, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, the field $h_{\mu\nu}$ describes the graviton with two polarization states, $h=\pm 2$.

One can pass to the massive $S=2$ tensor field adding a Pauli-Fierz term

$$\mathcal{L}_m = -M_{Pl}^2 \cdot \frac{m^2}{4} \left[(h_{\mu\nu})^2 - (h_r^{\mu\nu})^2 \right] \quad \text{Fierz, Pauli '39}$$

Let us write down $h_{\mu\nu}$ for the zero helicity state. In the rest frame

$$h^{\mu\nu} = \frac{1}{\sqrt{6}} \begin{pmatrix} 0 & 2 & 0 \\ 0 & -1 & -1 \\ 0 & -1 & -1 \end{pmatrix}$$

After the boost along x -axis

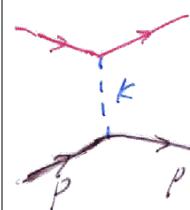
(12)

$$h^{\mu\nu} = \frac{2}{\sqrt{6}} \begin{pmatrix} \frac{K^2}{m^2} & \frac{KE}{m^2} & 0 & 0 \\ \frac{KE}{m^2} & \frac{E^2}{m^2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} \end{pmatrix} =$$

$$= \frac{2}{\sqrt{6}} \left[\frac{KK^r}{m^2} + \begin{pmatrix} -1 & 0 \\ 0 & +1 \\ 0 & -\frac{1}{2} \\ -\frac{1}{2} & 0 \end{pmatrix} \right]$$

Consider emission of the $h=0$ quanta. The term KK^r/m^2 drops out because interaction $T_{\mu\nu} h^{\mu\nu}$ contains the conserved energy-momentum tensor $T_{\mu\nu}$. The second term gives the finite amplitude of emission. This is an origin of Iwasaki-Van Dam-Veltman-Zakharov discontinuity.

More specifically, for the bending of light we need to know the amplitude of a graviton exchange between two sources (the sun, and the light).



$$A = -\frac{2}{M_{Pl}^2} T_{\mu\nu} D^{\mu\nu; \alpha\beta} T_{\alpha\beta}$$



$T_{\mu\nu}, T^{\alpha\beta}$ are m.e. of the energy-momentum tensor, $T_{\mu\nu} = q^\nu T_{\mu\nu}(q)$, normalized as $2p_\mu p_\nu$ at zero momentum transfer.

In the massless case the propagator

$$D_0^{\mu\nu; \alpha\beta} = \frac{1}{2k_\mu^2} (\eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha} - 2\eta^{\mu\nu}\eta^{\alpha\beta})$$

is fixed by the condition of unitarity in the exchange by the $h=\pm 2$ states.

These states are described by

$$h_{mn} \quad (m, n = 2, 3) \quad h_{mm} = 0$$

$$h^{(1)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$h^{(2)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\frac{1}{2} (\delta^{mn}\delta^{nl} + \delta^{ml}\delta^{nk} - \frac{2}{3} \delta^{mn}\delta^{lk})$$

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The potential for interaction of two massive sources is then

$$V_0(q) = -\int \frac{d^3 k}{(2\pi)^3} e^{ik\cdot q} \frac{A}{4M_1 M_2} = -\frac{G_N M_1 M_2}{\epsilon}$$

In the massive case there are 5 states which in the rest frame are given by h_{mn} living in $d=3$ now

$$h_{mn} \quad (m, n = 1, 2, 3) \quad h_{mm} = 0$$

$$\frac{1}{2} [\delta^{mk}\delta^{nl} + \delta^{ml}\delta^{nk} - \frac{2}{3} \delta^{mn}\delta^{lk}]$$

After the boost $\delta^{mk} \rightarrow -g^{\mu\nu} + \frac{k^\mu k^\nu}{m^2}$ and

$$D_m^{\mu\nu; \alpha\beta} = \frac{1}{2(k_m^2 - m^2)} (\eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha} - \frac{2}{3} \eta^{\mu\nu}\eta^{\alpha\beta})$$

up to noncontributing terms with k^A .

The static potential becomes

$$V_m(q) = -\frac{4}{3} \frac{G_N M_1 M_2}{\epsilon} e^{-mq}$$

The additional attraction is due the helicity zero (graviscalar) exchange.

The discrepancy increases if an extra scalar is added.

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Of course, one can introduce :

$$\tilde{G}_N = \frac{4}{3} G_N$$

to normalize the static interaction to the same form. However, when it comes to the light for which $\eta_{\mu}^{\alpha=0}$ we get the factor $\frac{3}{4}$

$$A_0 = -\frac{8\pi G_N}{k_1^2} \left(\eta_{\mu\nu} \eta^{\alpha\beta} - \frac{1}{2} \eta_{\mu}^{\alpha} \eta^{\beta}_{\alpha} \right)$$

$$A_m = -\frac{3}{4} \frac{8\pi \tilde{G}_N}{k_1^2 m^2} \left(\eta_{\mu\nu} \eta^{\alpha\beta} - \frac{1}{3} \eta_{\mu}^{\alpha} \eta^{\beta}_{\alpha} \right)$$

Non linear corrections

What are corrections due to nonlinear terms in graviton interaction? In case of massive Yang-Mills field every extra longitudinal particle gives a factor

in the amplitude. Similarly for $h=0$: particle in the massive gravity

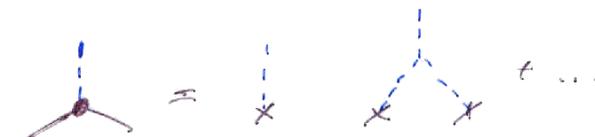
$$\frac{E}{M_p} \cdot \frac{E^2}{m^2}$$

$$E_{\text{eff}} \sim (M_p m^2)^{1/3}$$

Arkani-Hamed, Georgi, Schwarz '02

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First calculation was 30 years earlier. It was in application to the field of the static source (Schwarzschild problem)



where the expansion parameter is

$$\frac{M_1}{M_p^2 m^2} \left(\frac{1}{r^2 m^2} \right)^2$$

A. V. '72

It implies that corrections are small at

$$r \gg r_c \quad r_c = \left(\frac{M_1}{M_p^2 m^4} \right)^{1/5} = \left(\frac{4 M_1}{m^4} \right)^{1/5}$$

For the largest $m = \frac{1}{10^{26} \text{ cm}}$ PDG

and $r_{\text{sun}} \sim 3 \cdot 10^{15} \text{ cm}$ we get $r_c \sim 10^{21} \text{ cm}$

What is much larger than the solar system size 10^{15} cm . At these distances the next-to-leading corrections are about 10^{32} times bigger than the leading term — one cannot rely on weak coupling.

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Nonperturbative screening

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In case of massive YM fields the continuous transition to zero mass and weak coupling was possible to achieve by introduction of extra fields and Higgs mechanism.

Having no analogue for the massive gravity we have to consider what happens with the bending of light by the Sun within the theory with ultra-strong coupling.

Studying the classical nonlinear equations is one possible route to go beyond perturbation theory. The equations are

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \frac{1}{2} m^2 (h_{\mu\nu} - g_{\mu\nu} h) = \frac{1}{M_P^2} T_{\mu\nu}$$

Although the perturbative solution generates strongly coupled zero helicity modes nonperturbatively they can screen themselves providing a continuous limit to $m=0$ coinciding with the massless Einstein theory.

Such nonlinear solution was constructed for the Schwarzschild problem. A.V. '72

The interval

$$-dx^\mu dx^\nu g_{\mu\nu} = -e^\nu dt^2 + e^\theta d\theta^2 + e^\phi (d\phi^2 + \sin\theta d\psi^2)$$

in the spherical coordinates t, θ, ϕ .

The functions ν, θ, ϕ depends only on r .

New coordinate z instead of r

$$z = r e^{\frac{\mu(\rho)}{2}}, \quad e^\lambda = \frac{e^{\theta-\mu}}{1 - \frac{r_0}{r} \frac{dp}{dr}}$$

It's a gauge transformation for the massless theory, but not for the massive one. For the massless theory

$$\nu^{Schr}(z) = -\lambda^{Schr}(z) = \ln\left(1 - \frac{m}{z}\right) = -\frac{m}{z} - \frac{1}{2}\left(\frac{m}{z}\right)^2$$

$$\mu^{Schr}(z) = 0$$

The perturbative solution for the massive theory

$$\nu = -\frac{m}{z} \left(1 + \frac{7}{32} \frac{m}{m^4 z^5}\right), \quad \lambda = \frac{1}{2} \frac{m}{z} \left(1 - \frac{21}{8} \frac{m}{m^4 z^5}\right)$$

$$\mu = \frac{1}{2} \frac{m}{m^2 z^3} \left(1 + \frac{21}{4} \frac{m}{m^4 z^5}\right)$$

$$\underline{m \gg m}$$

Doing expansion in powers of m instead one finds ⁽⁴⁹⁾

$$\mathcal{V}(r) = -\frac{\gamma_M}{r} + \mathcal{O}(m^2 \sqrt{\gamma_M r^3})$$

$$\lambda(r) = \frac{\gamma_M}{r} + \mathcal{O}(m^2 \sqrt{\gamma_M r^3})$$

$$\mu(r) = \sqrt{\frac{8\gamma_M}{13r}} + \mathcal{O}(m^2 r^2)$$

This is valid in the interval

$$\gamma_M < r < r_c \quad r_c = \left(\frac{\gamma_M}{m^4}\right)^{1/5}$$

$$\gamma_M = 2MGN$$

The solution is nonanalytical in G_N .

Its asymptotics at $r \gg r_c$ is not known
- could be not decreasing.

It seems that a continuous in m
^{solution} local \mathcal{V} is always possible but an existence of global solution with exponentially decaying asymptotics is strongly doubtful.
Moreover, the very definition of a stable nonlinear theory of the massive spin 2 field is in doubt too.

Unresolved problems

The detailed study of the massive gravity was in the early paper

Boulware, Deser '72

While at linear level the theory of massive $S=2$ field is well defined, at the nonlinear level it stops to be a theory of 5 degrees of freedom.

From the equations of motions follows (at the linear level):

$$\partial^\mu (\eta_{\mu\nu} - \eta_{\mu\nu} h) = 0 \quad h = h_\mu \delta^\mu$$

This condition leads to

$$R^h = 2\partial^\mu \partial^\nu h_{\mu\nu} - \partial^\mu h = 0$$

Then, from L.O.m.

$$h = \frac{1}{3m^2 M_{pl}^2} T_\mu^\mu \delta^\mu$$

As a result we get $\partial^\mu h_{\mu\nu} = 0$, $h = 0$, i.e. 5 degrees of freedom.

At nonlinear level $R \neq 0$ and h becomes the sixth degree of freedom.

Moreover, energy can be made with
help of \hbar an arbitrarily large negative-
the theory is unstable. The instability
of Minkowski metric was studied by
Zun, Kang '86 and recently by Babadadeh¹⁰³
Gruzinov

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A different class of nonperturbative
solutions was found by

Salam, Strathdee '77

They used two "gravitational fields approach"
called it "strong gravity". Recently, it
reappeared in the paper by Arkani-Hamed, Georgi,
Schwarz, mentioned above.

Their solution coincides with the Schwarzschild-
de Sitter solution, the graviton mass
generates the cosmological term.
The continuity is present but the asymptotic
is very different from Minkowski metric.

Recently, Damour, Kogan, Pappasoglou '03
tried to find a numerical solution for
the Schwarzschild problem in massive gravity.
They conclude that there is no global solution
with asymptotically Minkowski metric.