α_s and QCD tests at hadron colliders

- world summary of α_s
- newest results (selection)
- α_s from hadron colliders
- remarks

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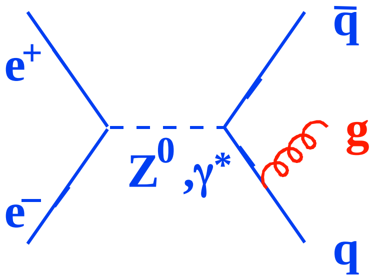
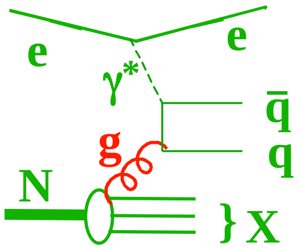
Experimental Determination of α_s

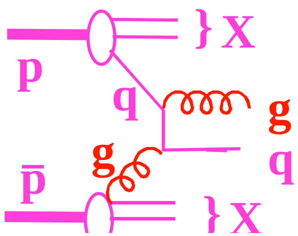
in all types of reactions which contain gluons:



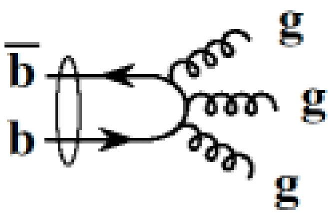
- e^+e^- annihilation
  - total hadronic cross section
  - hadronic decay width of Z bosons and τ leptons
  - jet rates and event shapes observables



- deep inelastic lepton-nucleon-scattering
  - scaling violations of structure functions
  - sum rules of structure functions
  - jet rates and event shapes observables



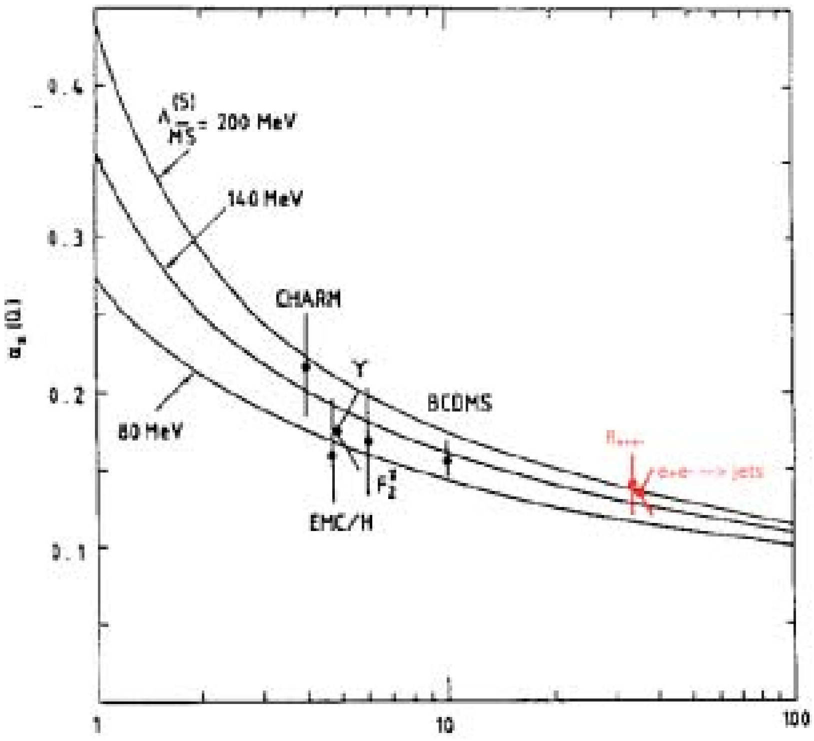
- proton-(anti-)proton collisions
  - jet rates
  - photoproduction
  - inklusive production of b-quarks



- heavy quarkonia decays

World summary of α_s

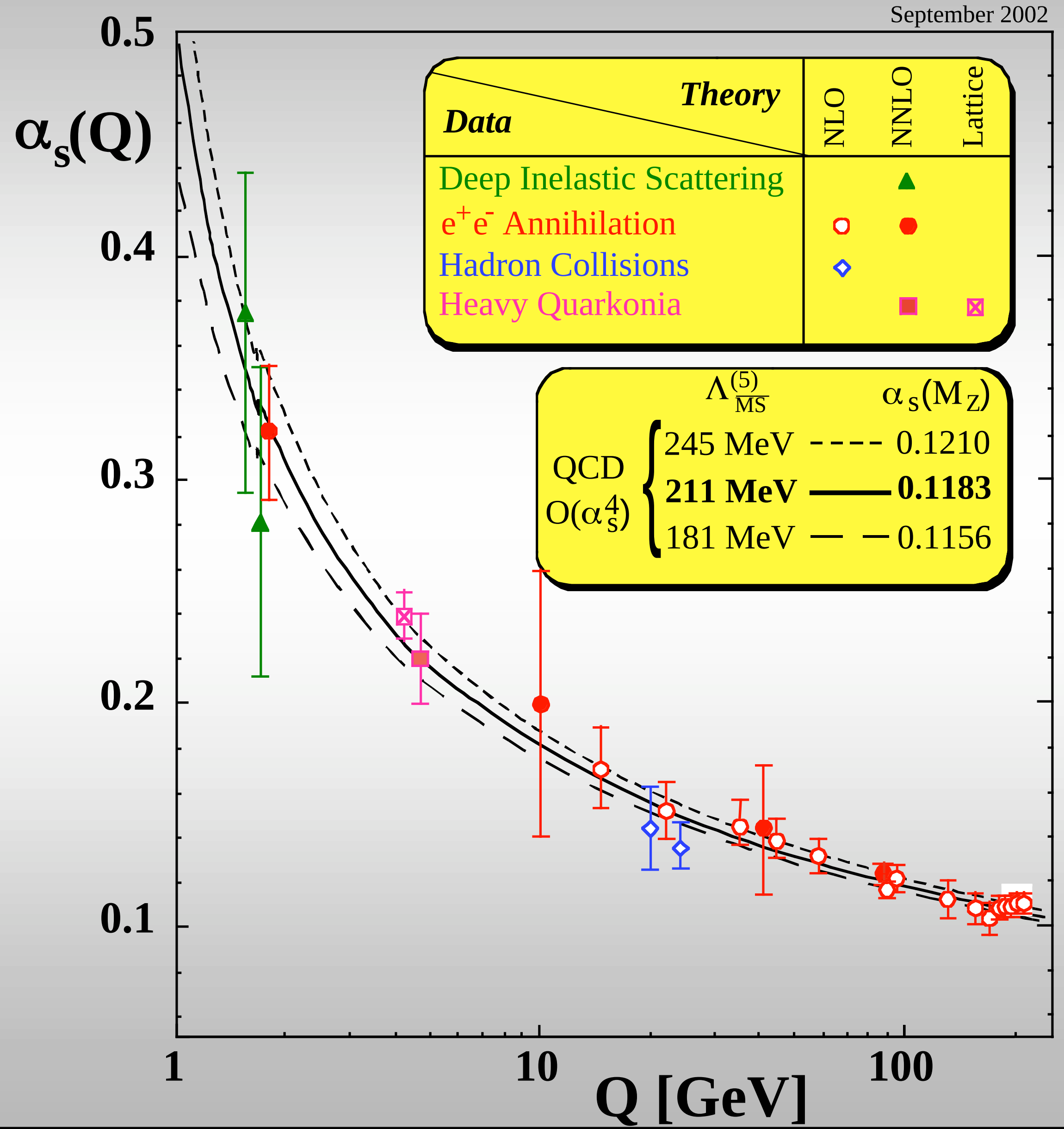
1989



$$\alpha_s(M_Z) = 0.110^{+0.006}_{-0.008} \text{ (NLO)}$$

G. Altarelli, Ann. Rev. Nucl. Part. Sci. 39, 1989

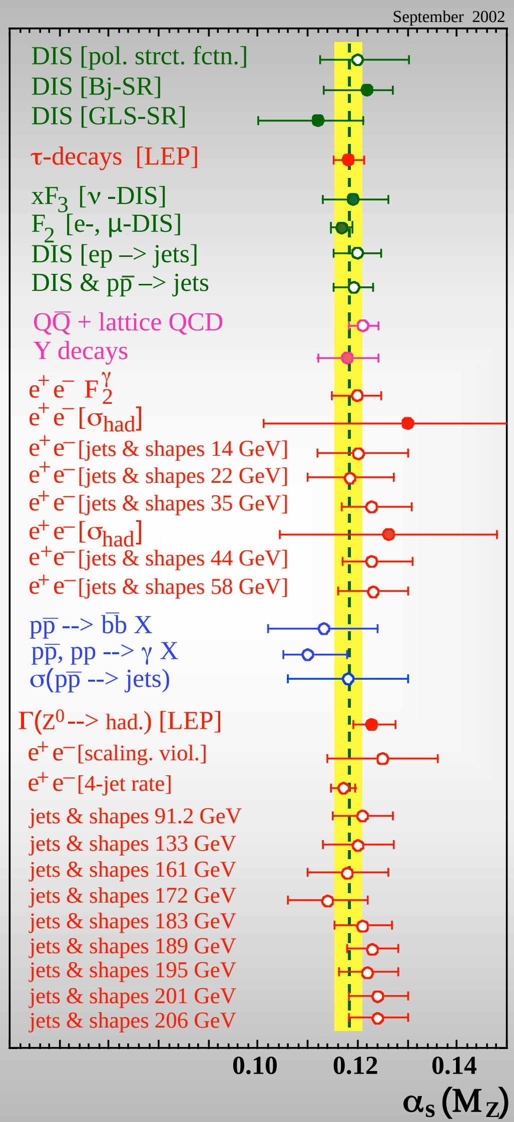
World Summary of $\alpha_s(Q)$ (Sep. 2002)



hep-ex/0211012

	Q			$\Delta\alpha_s(M_{Z^0})$		
Process	[GeV]	$\alpha_s(Q)$	$\alpha_s(M_{Z^0})$	exp.	theor.	Theory
DIS [pol. strct. fctn.]	0.7 - 8		$0.120^{+0.010}_{-0.008}$	$^{+0.004}_{-0.005}$	$^{+0.009}_{-0.006}$	NLO
DIS [Bj-SR]	1.58	$0.375^{+0.062}_{-0.081}$	$0.121^{+0.005}_{-0.009}$	–	–	NNLO
DIS [GLS-SR]	1.73	$0.280^{+0.070}_{-0.068}$	$0.112^{+0.009}_{-0.012}$	$^{+0.008}_{-0.010}$	0.005	NNLO
τ-decays	1.78	0.323 ± 0.030	0.1181 ± 0.0031	0.0007	0.0030	NNLO
DIS [ν; xF_3]	2.8 - 11		$0.119^{+0.007}_{-0.006}$	0.005	$^{+0.005}_{-0.003}$	NNLO
DIS [e/μ; F_2]	1.9 - 15.2		0.1166 ± 0.0022	0.0009	0.0020	NNLO
DIS [e-p $\rightarrow$ jets]	6 - 100		0.120 ± 0.005	0.002	0.004	NLO
DIS & p$\bar{\text{p}}$ $\rightarrow$jets	1 - 400		0.119 ± 0.004	0.002	0.003	NLO
$Q\overline{Q}$ states	4.1	$0.239^{+0.012}_{-0.010}$	0.121 ± 0.003	0.000	0.003	LGT
Υ decays	4.75	0.217 ± 0.021	0.118 ± 0.006	–	–	NNLO
e^+e^- [F_2^γ]	1.4 - 28		$0.1198^{+0.0044}_{-0.0054}$	0.0028	$^{+0.0034}_{-0.0046}$	NLO
e^+e^- [σ_{had}]	10.52	0.20 ± 0.06	$0.130^{+0.021}_{-0.029}$	$^{+0.021}_{-0.029}$	0.002	NNLO
e^+e^- [jets & shapes]	14.0	$0.170^{+0.021}_{-0.017}$	$0.120^{+0.010}_{-0.008}$	0.002	$^{+0.009}_{-0.008}$	resum
e^+e^- [jets & shapes]	22.0	$0.151^{+0.015}_{-0.013}$	$0.118^{+0.009}_{-0.008}$	0.003	$^{+0.009}_{-0.007}$	resum
e^+e^- [jets & shapes]	35.0	$0.145^{+0.012}_{-0.007}$	$0.123^{+0.008}_{-0.006}$	0.002	$^{+0.008}_{-0.005}$	resum
e^+e^- [σ_{had}]	42.4	0.144 ± 0.029	0.126 ± 0.022	0.022	0.002	NNLO
e^+e^- [jets & shapes]	44.0	$0.139^{+0.011}_{-0.008}$	$0.123^{+0.008}_{-0.006}$	0.003	$^{+0.007}_{-0.005}$	resum
e^+e^- [jets & shapes]	58.0	0.132 ± 0.008	0.123 ± 0.007	0.003	0.007	resum
p$\bar{\text{p}}$ $\rightarrow$ b$\bar{\text{b}}$X	20.0	$0.145^{+0.018}_{-0.019}$	0.113 ± 0.011	$^{+0.007}_{-0.006}$	$^{+0.008}_{-0.009}$	NLO
p$\bar{\text{p}}$, pp $\rightarrow$ γX	24.3	$0.135^{+0.012}_{-0.008}$	$0.110^{+0.008}_{-0.005}$	0.004	$^{+0.007}_{-0.003}$	NLO
σ(p$\bar{\text{p}}$ $\rightarrow$ jets)	40 - 250		0.118 ± 0.012	$^{+0.008}_{-0.010}$	$^{+0.009}_{-0.008}$	NLO
e^+e^- [$\Gamma(Z^0 \rightarrow$ had.)]	91.2	$0.1227^{+0.0048}_{-0.0038}$	$0.1227^{+0.0048}_{-0.0038}$	0.0038	$^{+0.0029}_{-0.0005}$	NNLO
e^+e^- scal. viol.	14 - 91.2		0.125 ± 0.011	$^{+0.006}_{-0.007}$	0.009	NLO
e^+e^- 4-jet rate	91.2	0.1170 ± 0.0026	0.1170 ± 0.0026	0.0001	0.0026	NLO
e^+e^- [jets & shapes]	91.2	0.121 ± 0.006	0.121 ± 0.006	0.001	0.006	resum
e^+e^- [jets & shapes]	133	0.113 ± 0.008	0.120 ± 0.007	0.003	0.006	resum
e^+e^- [jets & shapes]	161	0.109 ± 0.007	0.118 ± 0.008	0.005	0.006	resum
e^+e^- [jets & shapes]	172	0.104 ± 0.007	0.114 ± 0.008	0.005	0.006	resum
e^+e^- [jets & shapes]	183	0.109 ± 0.005	0.121 ± 0.006	0.002	0.005	resum
e^+e^- [jets & shapes]	189	0.109 ± 0.004	0.121 ± 0.005	0.001	0.005	resum
e^+e^- [jets & shapes]	195	0.109 ± 0.005	0.122 ± 0.006	0.001	0.006	resum
e^+e^- [jets & shapes]	201	0.110 ± 0.005	0.124 ± 0.006	0.002	0.006	resum
e^+e^- [jets & shapes]	206	0.110 ± 0.005	0.124 ± 0.006	0.001	0.006	resum

World Summary of $\alpha_s(Q)$ (Sep. 2002)



$$\alpha_s(M_z) = 0.1183 \pm 0.0027$$

running α_s up to 4^{th} order:

$$Q^2 \frac{\partial \alpha_s(Q^2)}{\partial Q^2} = \beta\left(\alpha_s(Q^2)\right)$$

$$\beta(\alpha_s(Q^2)) = -\beta_0 \alpha_s^2(Q^2) - \beta_1 \alpha_s^3(Q^2) - \beta_2 \alpha_s^4(Q^2) - \beta_3 \alpha_s^5(Q^2) + \mathcal{O}(\alpha_s^6)$$

$$\beta_0 = \frac{33 - 2N_f}{12\pi} ,$$

$$\beta_1 = \frac{153 - 19N_f}{24\pi^2} ,$$

$$\beta_2 = \frac{77139 - 15099N_f + 325N_f^2}{3456\pi^3} ,$$

$$\beta_3 \approx \frac{29243 - 6946.3N_f + 405.089N_f^2 + 1.49931N_f^3}{256\pi^4}$$

$$\alpha_s(Q^2) = \frac{1}{\beta_0 L} - \frac{1}{\beta_0^3 L^2}\beta_1 \ln L + \frac{1}{\beta_0^3 L^3}\left(\frac{\beta_1^2}{\beta_0^2}(\ln^2 L - \ln L - 1) + \frac{\beta_2}{\beta_0}\right) + \frac{1}{\beta_0^4 L^4}\left(\frac{\beta_1^3}{\beta_0^3}\left(-\ln^3 L + \frac{5}{2}\ln^2 L + 2\ln L - \frac{1}{2}\right) - 3\frac{\beta_1\beta_2}{\beta_0^2}\ln L + \frac{\beta_3}{2\beta_0}\right)$$

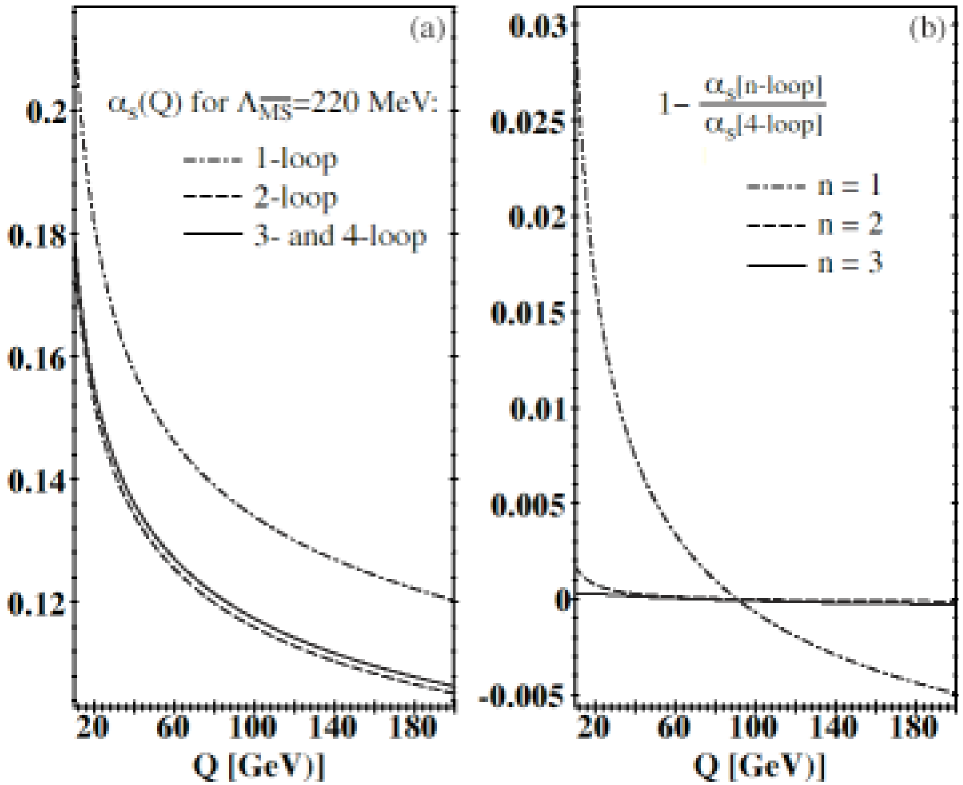
Ritbergen, Vermaseren, Larin

$$L = \ln \frac{Q^2}{\Lambda^2_{\overline{MS}}}$$

β_0 and β_1 do not depend on renormalisation scheme; β_2 and β_3 … do !

choose $\overline{\mathrm{MS}}$ scheme for all of the following discussion.

relative size of higher order corrections



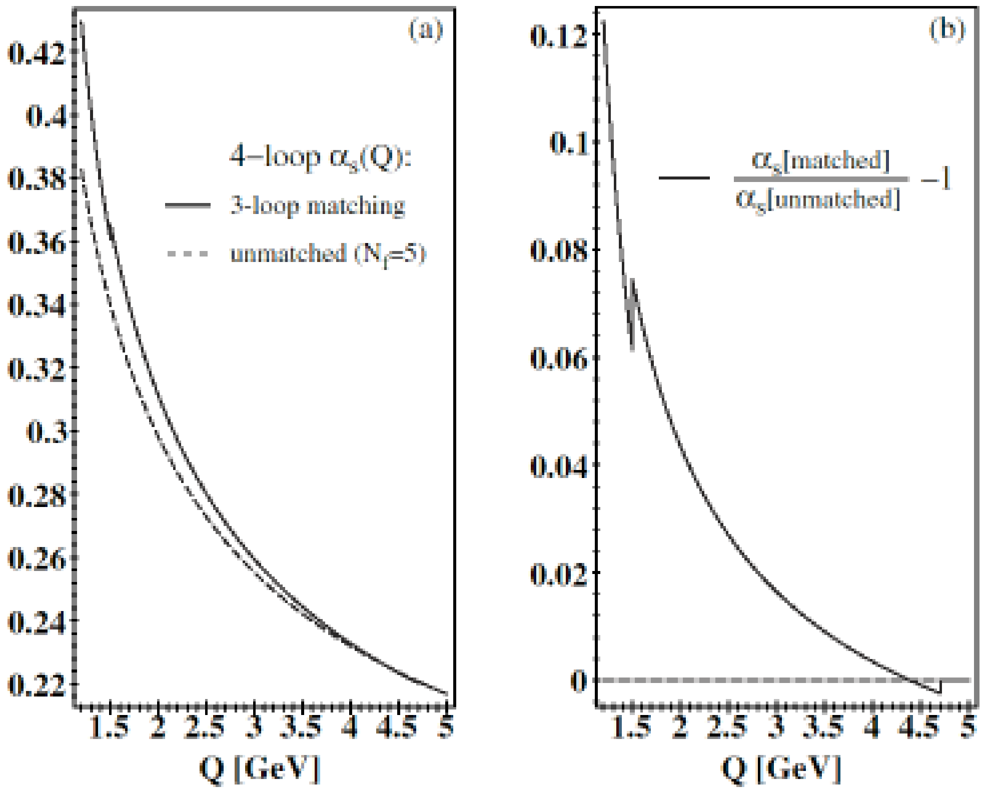
heavy quark threshold matching

Matching conditions for the choice $\mu^{(Nf)} = M_q$ (pole mass definition):

$$\frac{a'}{a} = 1 + C_2\, a^2 + C_3\, a^3 \qquad (\text{with } a' = \alpha_s^{(Nf-1)}/\pi\,; \quad a = \alpha_s^{(Nf)}/\pi\,)$$

$$C_2 = -0.291667 \text{ and } C_3 = -5.32389 + (N_f - 1) \cdot 0.26247$$

(3-loop condition; Chetyrkin, Kniehl, Steinhauser)



perturbative predictions for physical quantities

$$\mathcal{R}(Q^2) = P_l \sum_n R_n \alpha_s^n$$
$$= P_l \left(R_0 + R_1 \alpha_s(\mu^2) + R_2(Q^2/\mu^2)\alpha_s^2(\mu^2) + ...\right)$$

in n^{th} order perturbation theory

R_1 : “leading order coefficient” (lo)
R_2 : “next to leading coefficient” (nlo)
R_3 : “next-next-to leading” (nnlo)

Resummation of logs arising from soft and collinear singularities:

$$\Sigma(\mathcal{R}) \equiv \int_0^{\mathcal{R}} \frac{1}{\sigma}\frac{d\sigma}{d\mathcal{R}} d\mathcal{R} = C(\alpha_s)\exp\left[G(\alpha_s, L)\right] + D(\alpha_s, \mathcal{R}) \qquad L = \ln(1/\mathcal{R}) \qquad C(\alpha_s) = 1 + \sum_{n=1}^{\infty} C_n \hat{\alpha}_s^n$$

$$G(\alpha_s, L) = \sum_{n=1}^{\infty}\sum_{m=1}^{n+1} G_{nm}\hat{\alpha}_s^n L^m$$
$$\equiv L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \alpha_s^2 g_4(\alpha_s L)\cdots$$

	Leading logs	Next-to-Leading logs	Subleading logs	Non-log. terms	
$\ln\Sigma(\mathcal{R}) =$	$G_{12}\hat{\alpha}_s L^2$	$+G_{11}\hat{\alpha}_s L$		$+\alpha_s\mathcal{O}(1)$	$\mathcal{O}(\alpha_s)$
	$+G_{23}\hat{\alpha}_s^2 L^3$	$+G_{22}\hat{\alpha}_s^2 L^2$	$+G_{21}\hat{\alpha}_s^2 L$	$+\alpha_s^2\mathcal{O}(1)$	$\mathcal{O}(\alpha_s^2)$
	$+G_{34}\hat{\alpha}_s^3 L^4$	$+G_{33}\hat{\alpha}_s^3 L^3$	$+G_{32}\hat{\alpha}_s^3 L^2 + \cdots$	$+\cdots$	$\mathcal{O}(\alpha_s^3)$
	$+\cdots$	$+\cdots$	$+\cdots$	$+\cdots$	$\vdots$
$=$	$L g_1(\alpha_s L)$	$+g_2(\alpha_s L)$	$+\cdots$	$+\cdots$	

renormalisation scale dependence

$$\mathcal{R} \equiv \mathcal{R}(Q^2/\mu^2, \alpha_s); \;\; \alpha_s \equiv \alpha_s(\mu^2)$$

since choice of μ is arbitrary, physical observables $\mathcal{R}$ should not depend on μ

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \mathcal{R}(Q^2/\mu^2, \alpha_s) = \left(\mu^2 \frac{\partial}{\partial \mu^2} + \mu^2 \frac{\partial \alpha_s}{\partial \mu^2} \frac{\partial}{\partial \alpha_s} \right) \mathcal{R} \stackrel{!}{=} 0$$

$$\begin{aligned} 0 = \mu^2 \frac{\partial R_0}{\partial \mu^2} + \alpha_s(\mu^2) \mu^2 \frac{\partial R_1}{\partial \mu^2} &+ \alpha_s^2(\mu^2) \left[\mu^2 \frac{\partial R_2}{\partial \mu^2} - R_1 \beta_0 \right] \\ &+ \alpha_s^3(\mu^2) \left[\mu^2 \frac{\partial R_3}{\partial \mu^2} - [R_1 \beta_1 + 2 R_2 \beta_0] \right] \\ &+ \mathcal{O}(\alpha_s^4) \; . \end{aligned}$$

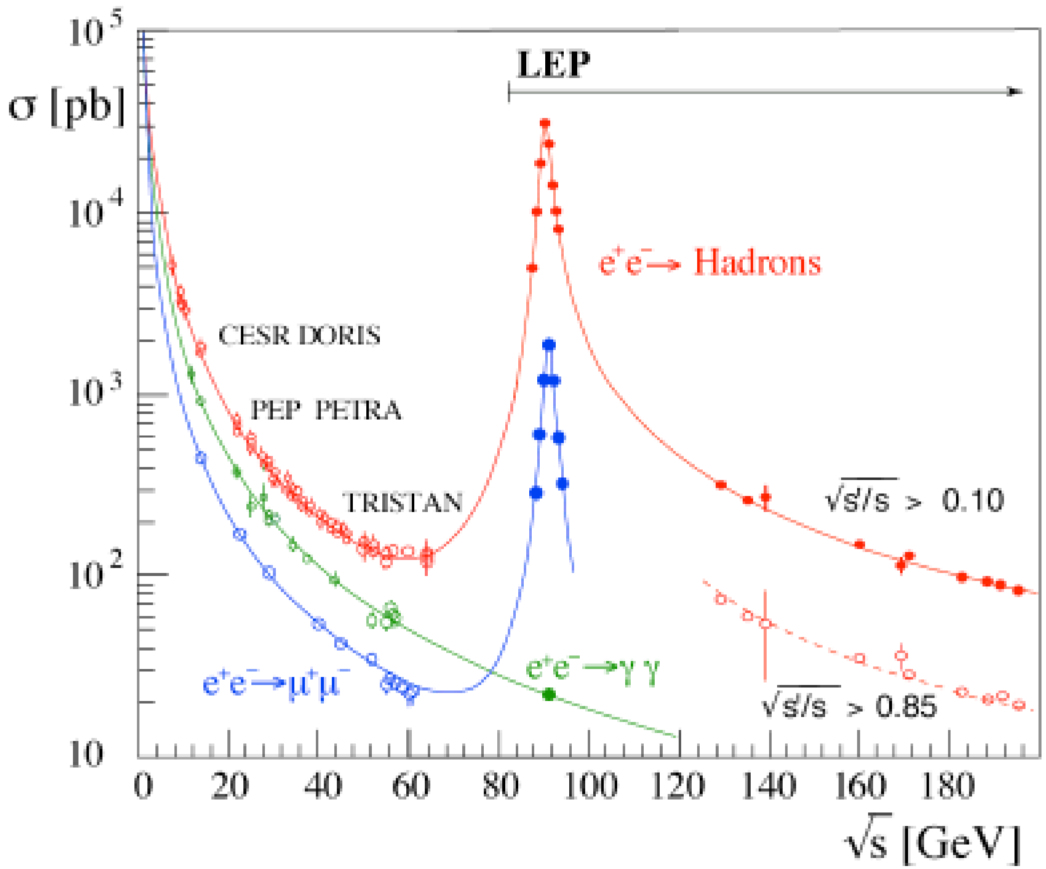
–>

$$\begin{aligned} R_0 &= \text{const.} \; , \\ R_1 &= \text{const.} \; , \\ R_2 \left(\frac{Q^2}{\mu^2} \right) &= R_2(1) - \beta_0 R_1 \ln \frac{Q^2}{\mu^2} \; , \\ R_3 \left(\frac{Q^2}{\mu^2} \right) &= R_3(1) - [2 R_2(1) \beta_0 + R_1 \beta_1] \ln \frac{Q^2}{\mu^2} + R_1 \beta_0^2 \ln^2 \frac{Q^2}{\mu^2} \end{aligned}$$

Perturbative QCD coefficients beyond leading order become renormalisation scale dependend !

This dependence is used to quantify theoretical uncertainties due to unknown higher orders.

Example: hadronic width of Z^0 boson

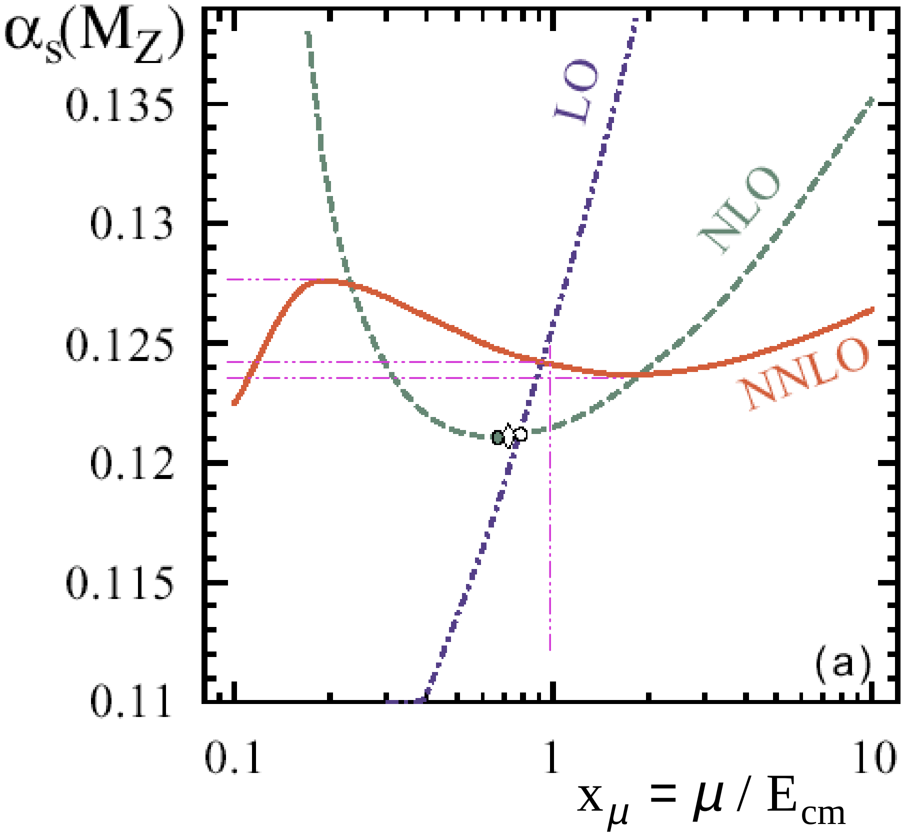


Renormalisation scale dependence

$$R_Z = \frac{\Gamma(Z^0 \to \text{hadrons})}{\Gamma(Z^0 \to \text{leptons})} = 20.767 \pm 0.0024$$

$$R_Z = 19.934\left[1 + 1.045\frac{\alpha_s(\mu)}{\pi} + 0.94\left[\frac{\alpha_s(\mu)}{\pi}\right]^2 - 15\left[\frac{\alpha_s(\mu)}{\pi}\right]^3\right]$$

Larin, van Ritbergen,, Vermaseren, Chetyrkin,Tarasov, Kühn, Steinhauser, Hoang,…...



$$\Rightarrow \alpha_s(M_Z) = 0.124 \pm 0.004 \ (\text{exp.}) \pm 0.002 \ (M_H, M_{top}) \ {}^{+0.003}_{-0.001} \ (\text{QCD})$$

error source	$\Delta\alpha_s(M_{Z^0})$
$\Delta M_{Z^0} = \pm 0.0021$ GeV	± 0.00003
$\Delta M_t = \pm 5$ GeV	± 0.0002
$M_H = 100 \ldots 1000$ GeV	± 0.0017
$\mu = (\frac{1}{4} \ldots 4)\ M_{Z^0}$	${}^{+0.0028}_{-0.0004}$
renormalization schemes	± 0.0002
total	${}^{+0.003}_{-0.002}$

Improved Calculation of F_2 and xF_3

hep-ph/0102247 (Santiago / Yndurain):

- complete NNLO calculations for moments of structure functions
- use all available DIS data (3.5 $GeV^2 \leq Q^2 \leq 230$ GeV^2)
- extract α_s in NNLO for ep-scattering (F_2) and for νN-scattering (xF_3).
- error calculation:

Source of error	$\Lambda(n_f = 4; 3\text{ loop})$	$\Delta\Lambda(n_f = 4; 3\text{ loop})$	$\Delta\alpha_s(M_Z^2)$
No TMC	279	5	0.0004
Interpolation	279	5	0.0004
HT	268	6	0.0004
Quark mass effect	269	5	0.0004
Q_0^2 to 12 GeV^2	279	5	0.0004
NNNLO	265	10	0.0006

- results:

$$\alpha_s(M_Z) = 0.1166 \pm 0.0009\ (\text{stat}) \pm 0.0010\ (\text{sys}) \qquad (F_2)$$

$$\alpha_s(M_Z) = 0.1153 \pm 0.0040\ (\text{stat}) \pm 0.0061\ (\text{sys}) \qquad (xF_3)$$

Global analysis of DIS and hadron collider data

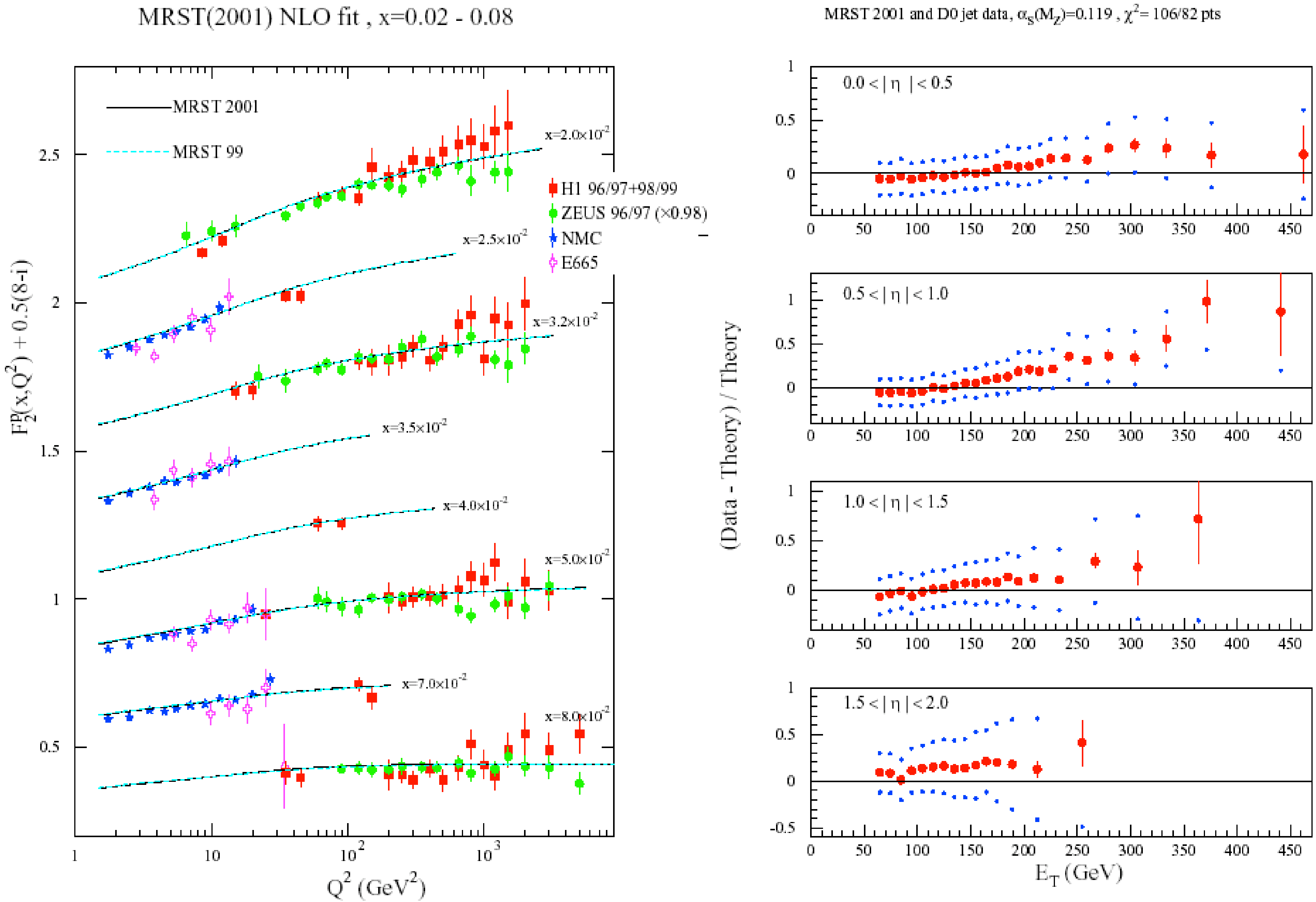
hep-ph/0307262 (Martin, Roberts, Stirling, Thorne; MRST03):

NLO: $\alpha_s(M_z) = 0.1165 \pm 0.002 \pm 0.003$

NNLO: $\alpha_s(M_z) = 0.1153 \pm 0.002 \pm 0.003$

Note: “NNLO” QCD not complete for DIS, absent for hadron collider jet production –> not a complete NNLO analysis

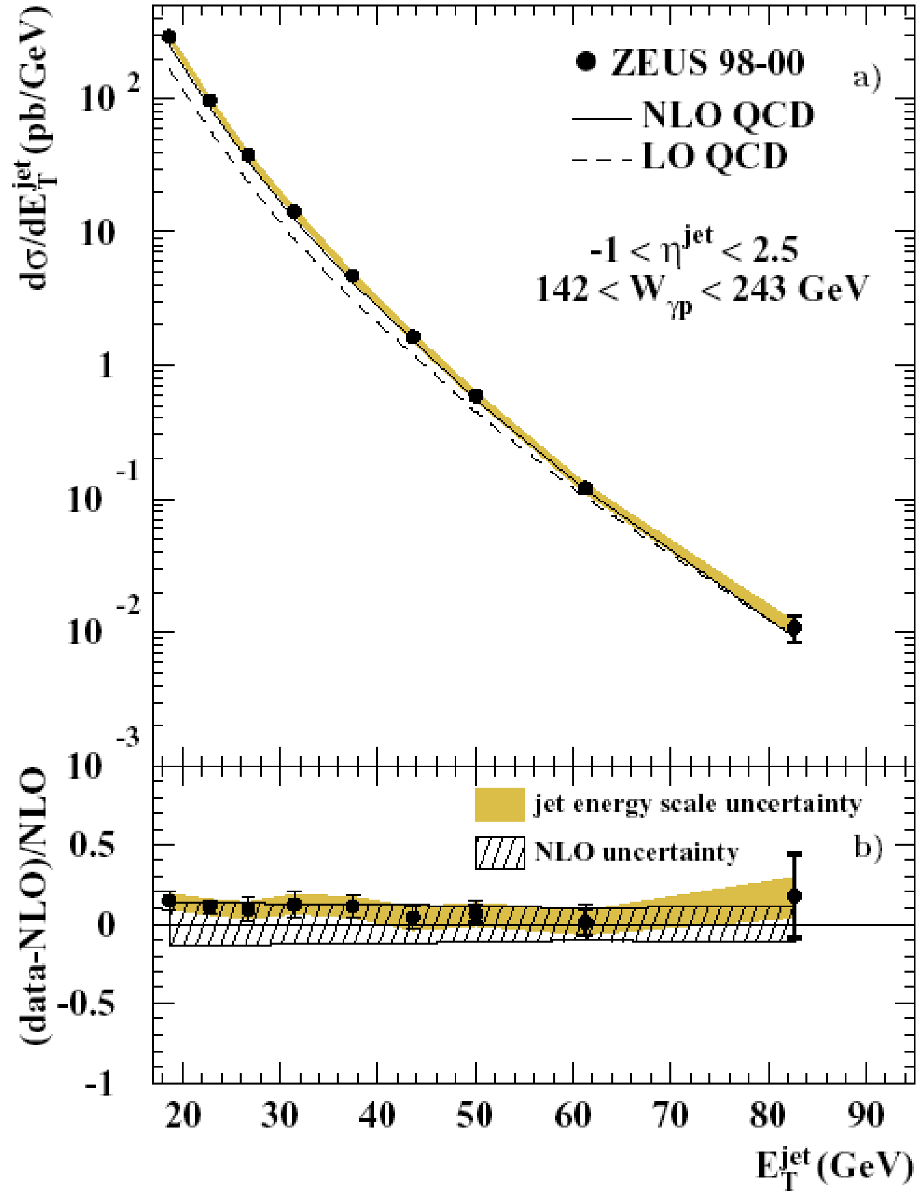
α_s from DIS and pp–>jets hep-ph/0110215 A. Martin et al.

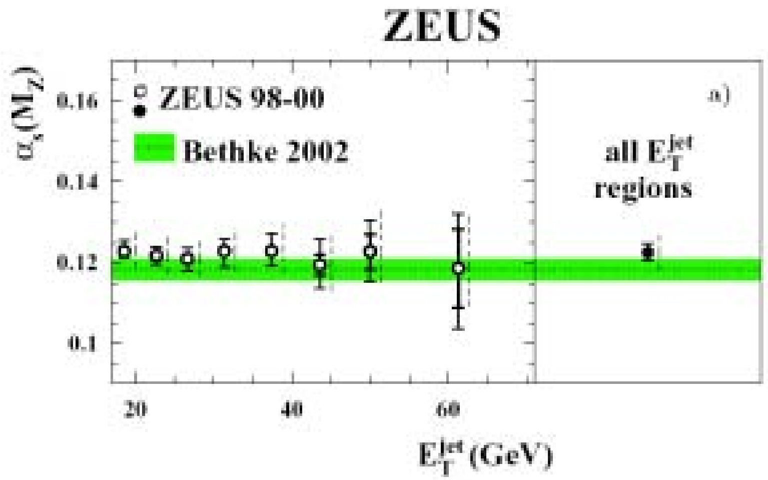


$$\alpha_s(M_z) = 0.119 \pm 0.002(\text{exp.}) \pm 0.003(\text{theor.})$$

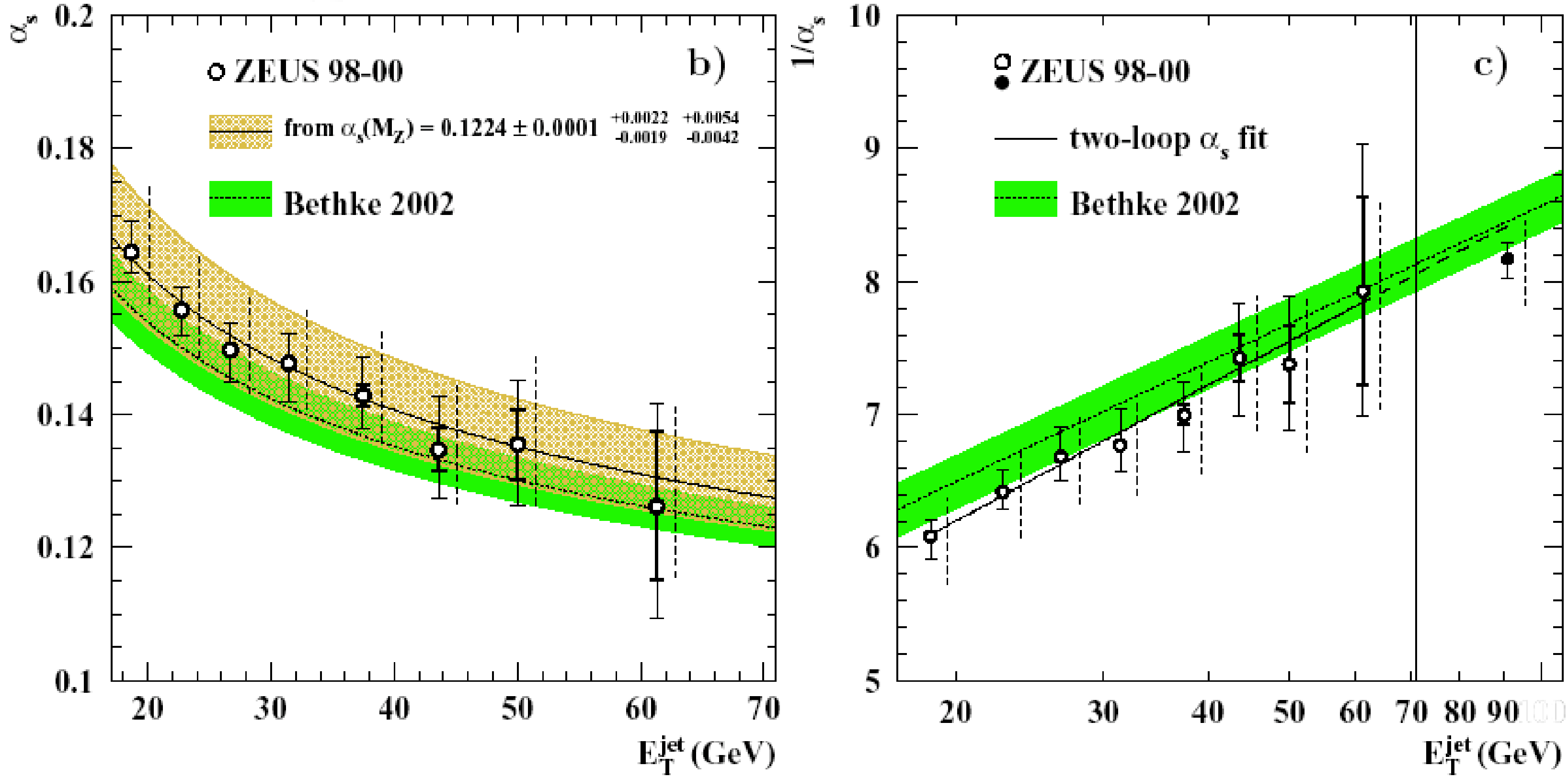
ZEUS: scaling violation from jet production in γp interactions at HERA

Phys. Lett. B560 (2003) 7



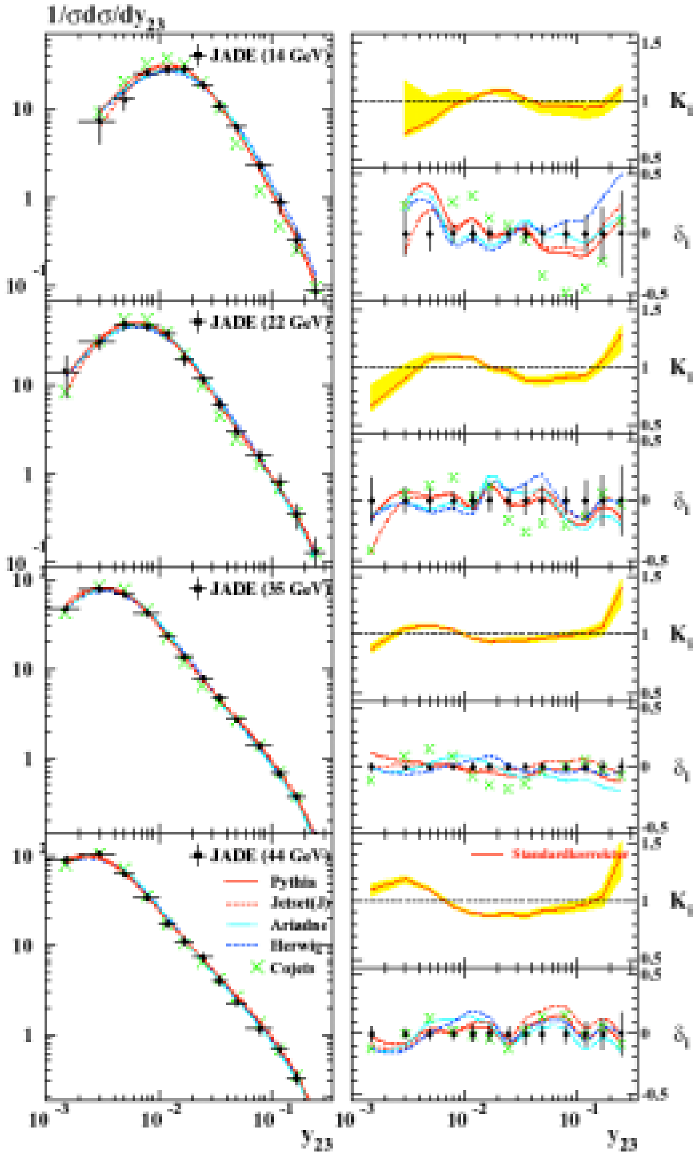


$$\alpha_s(M_Z) = 0.1224 \pm 0.0001(\text{stat})^{+0.0022}_{-0.0019}(\text{exp})^{+0.0054}_{-0.0042}(\text{th})$$

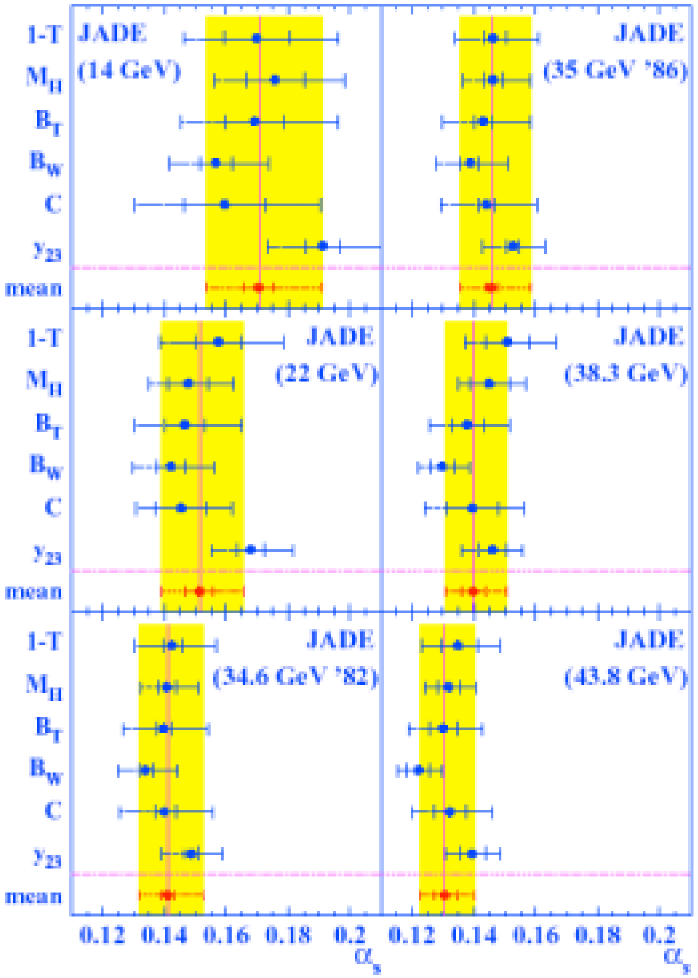


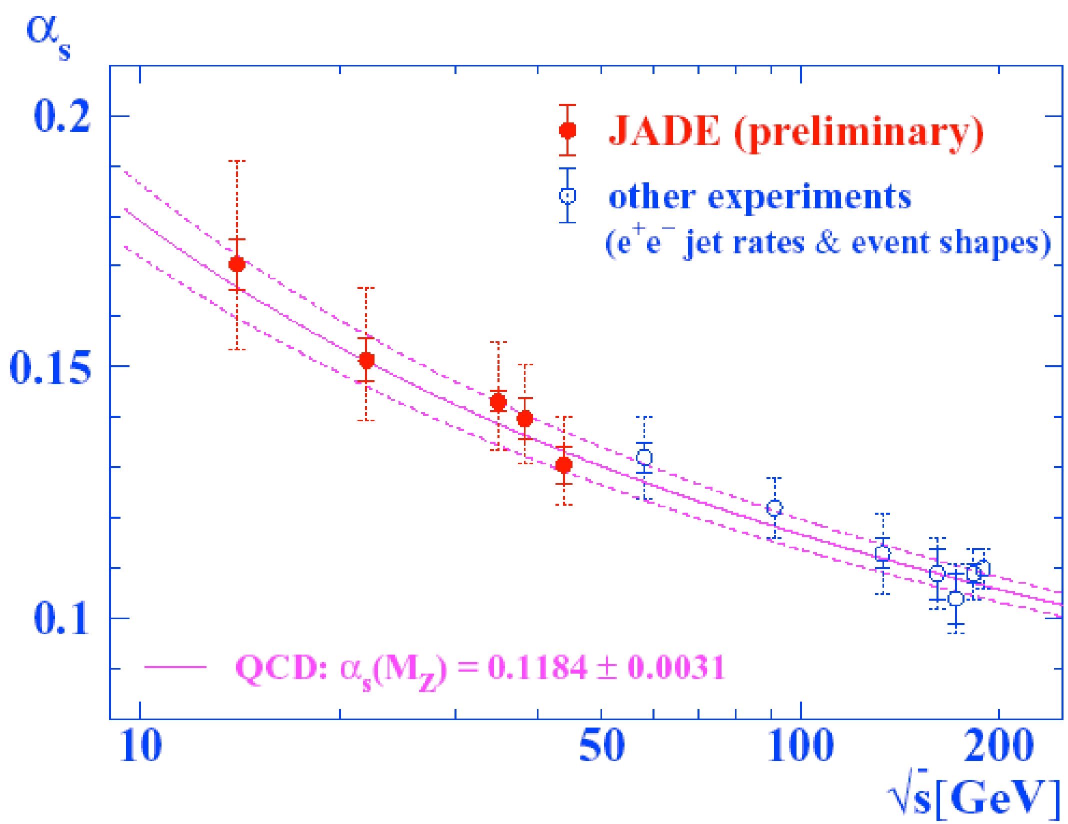
H1 and ZEUS combined (jet prod.): $\alpha_s(M_z)$ = 0.120 ± 0.002 (exp) ± 0.004 (th)

Reanalysis of data from Jade at PETRA: α_s from hadronic event shapes hep-ex/0205014



in resummed $O(\alpha_s^2)$ (P. Fernandez)

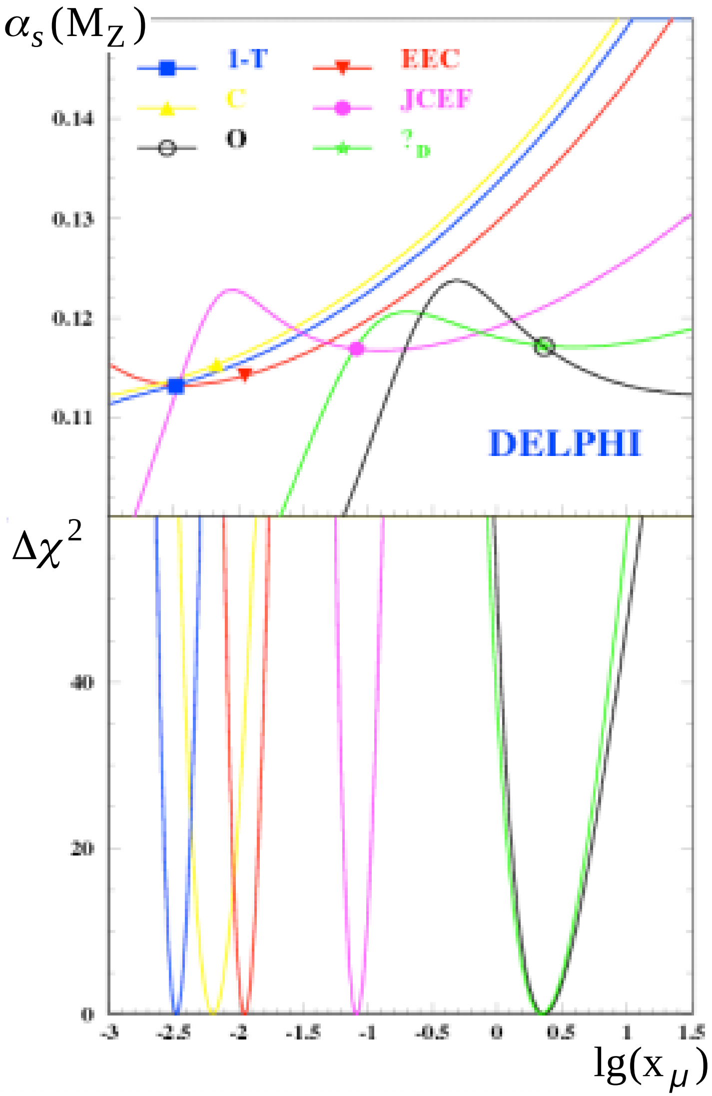


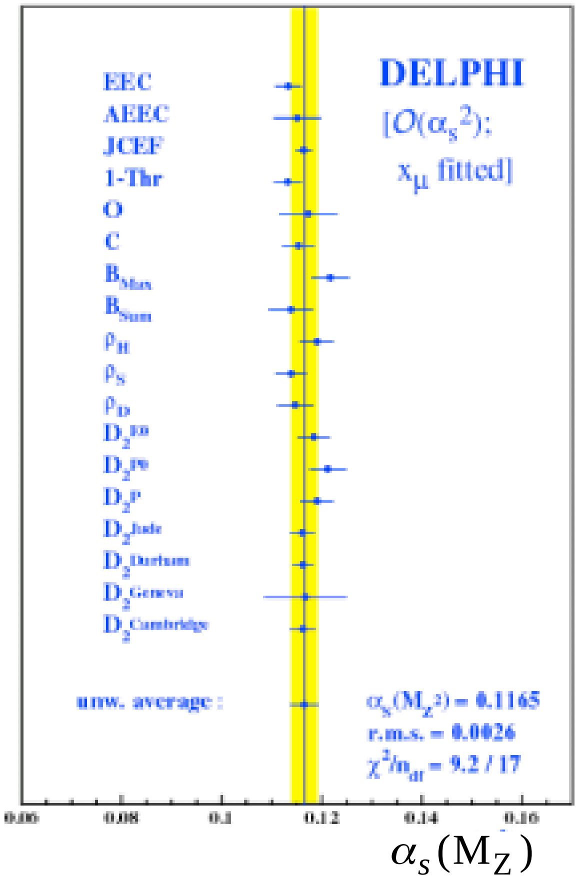


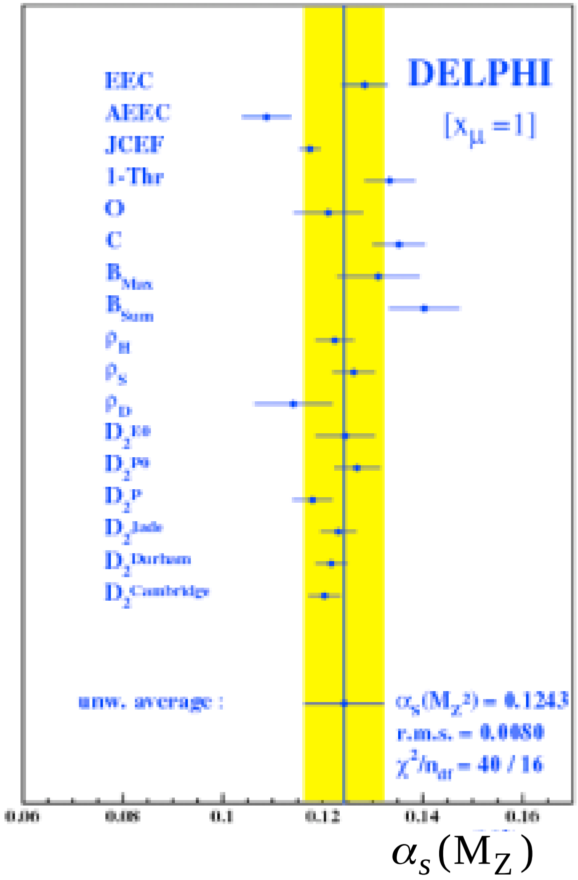
$\sqrt{s}$ [GeV]	$\alpha_S(\sqrt{s})$	fit	exp.	hadr.	higher ord.	total
14.0	0.1704	±0.0051		+0.0141 −0.0136	+0.0143 −0.0091	+0.0206 −0.0171
22.0	0.1513	±0.0043		±0.0101	+0.0101 −0.0065	+0.0144 −0.0121
34.6 ('82)	0.1409	±0.0012	±0.0017	±0.0071	+0.0086 −0.0057	+0.0114 −0.0093
35.0 ('86)	0.1457	±0.0011	±0.0020	±0.0076	+0.0096 −0.0064	+0.0125 −0.0101
38.3	0.1397	±0.0031	±0.0026	±0.0054	+0.0084 −0.0056	+0.0108 −0.0087
43.8	0.1306	±0.0019	±0.0032	±0.0056	+0.0068 −0.0044	+0.0096 −0.0080

Renormalisation scale dependence in NLO

(event shapes and jet rates)



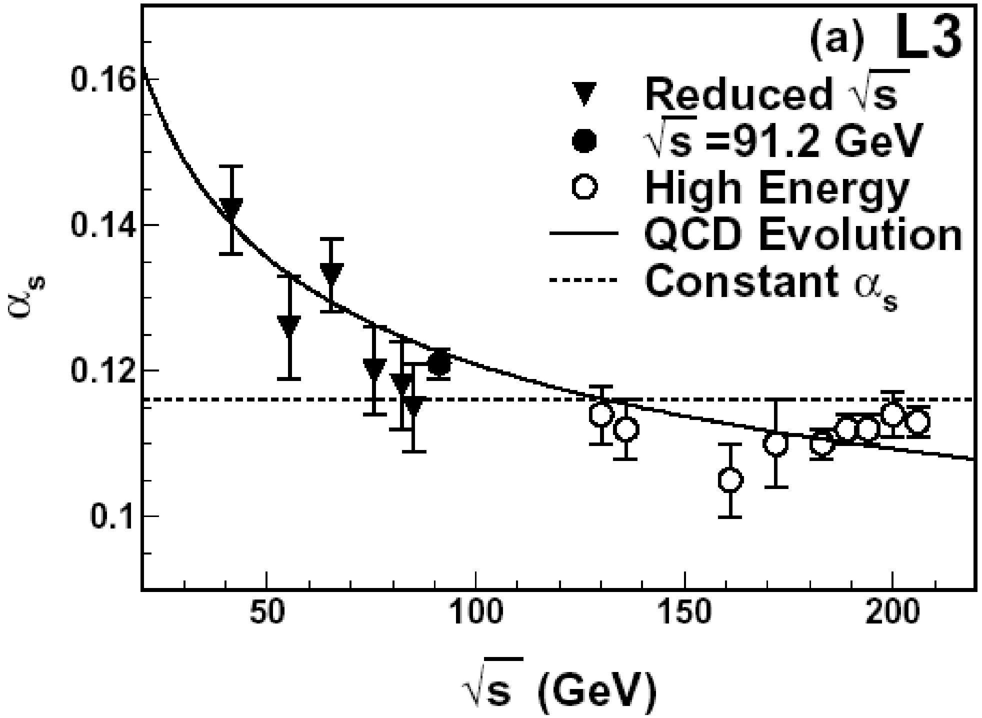




- exp. scale optimisation gives consistent results in NLO
- how to define the corresponding scale uncertainty?

e^+e^- annihilation:
α_s from hadronic event shapes

in resummed NLO QCD, i.e. resummed $O(\alpha_s^2)$



CERN-EP/2002-029

Combination of LEP results at major energy points:
S.B., J. Phys. G26 (2000) R27; hep-ex/0211012

Soon to come: new and standardised error definition and treatment,
LEP QCD WG, R. Jones et al., see JHEP 0312:007,2003

LEP-II: α_s from hadronic event shapes

(most are preliminary)

in resummed NLO QCD, i.e. resummed $O(\alpha_s^2)$

LEP:
====

alphas(q) ± (stat) ± (sys)

Exp	189 GeV	195 GeV	201 GeV	206 GeV
A	.1119(15)(32)	.1065(27)(39)	.1133(30)(47)	.1051(28)(41)
D	.1102(23)(30)	<198:> .1094(19)(45)		.109 (2) (5)
L	.1105(18)(58)	.1123(14)(53)	.1138(18)(54)	.1132(14)(53)
O	.107 (1) (4)	.103 (2) (5)	.104 (2) (4)	.107 (2) (4)
tot	.1090(10)(40)	.1088(11)(47)	.1101(12)(47)	.1100(11)(46)
(Mz)	.1213(13)(49)	.1217(14)(60)	.1239(16)(60)	.1242(14)(59)

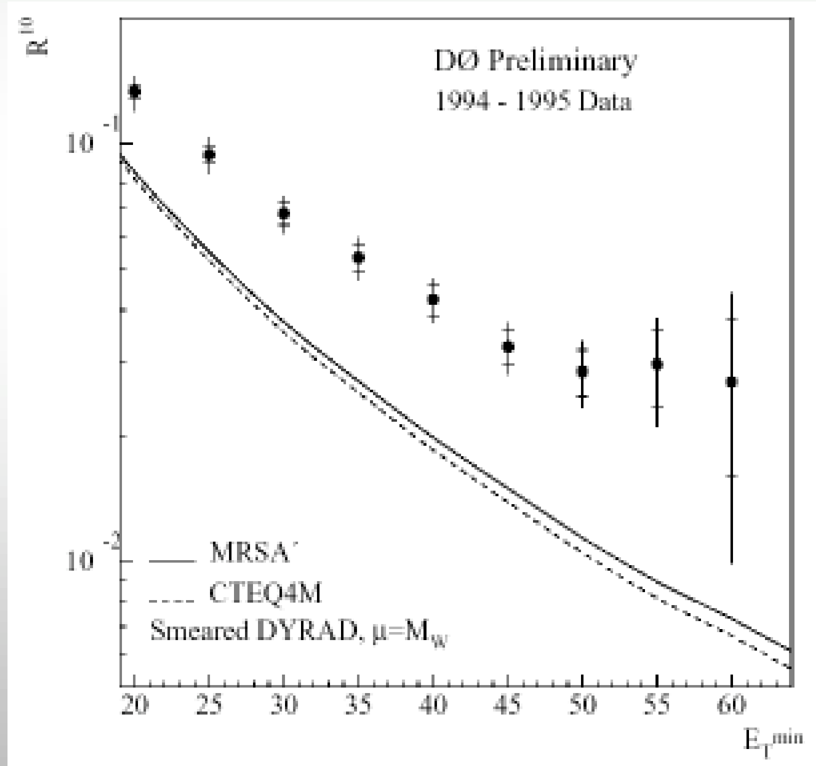
all as(Mz): .1228(7)(57)

α_s determinations from hadron colliders (1)

- α_s from W plus jet production

UA1, UA2 (1991/92): $\alpha_s(M_z) = 0.121 \pm 0.017 \pm 0.016$

D0 (1997):



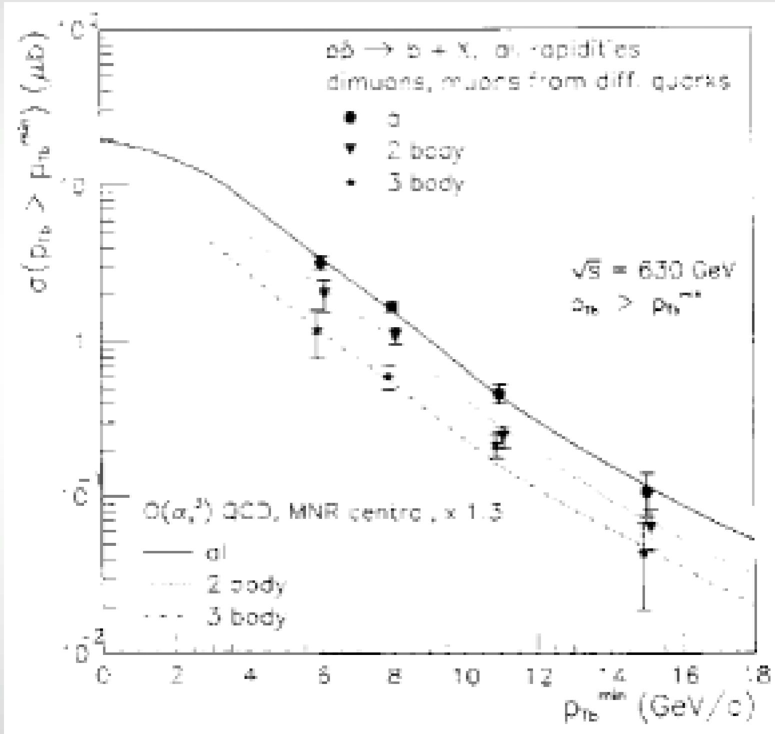
no match with
NLO QCD
predictions !

α_s determinations from hadron colliders(2)

- α_s from b cross sections

UA1 (1996):

$$\alpha_s(M_Z) = 0.113 \,^{+0.007}_{-0.006}\,^{+0.008}_{-0.009}$$



NLO QCD:
Mangano, Nason, Ridolfi
(1992)

α_s determinations from hadron colliders (3)

- α_s from prompt photon production $\left[\sigma\left(p\bar{p} \to \gamma X\right) - \sigma\left(pp \to \gamma X\right)\right]$

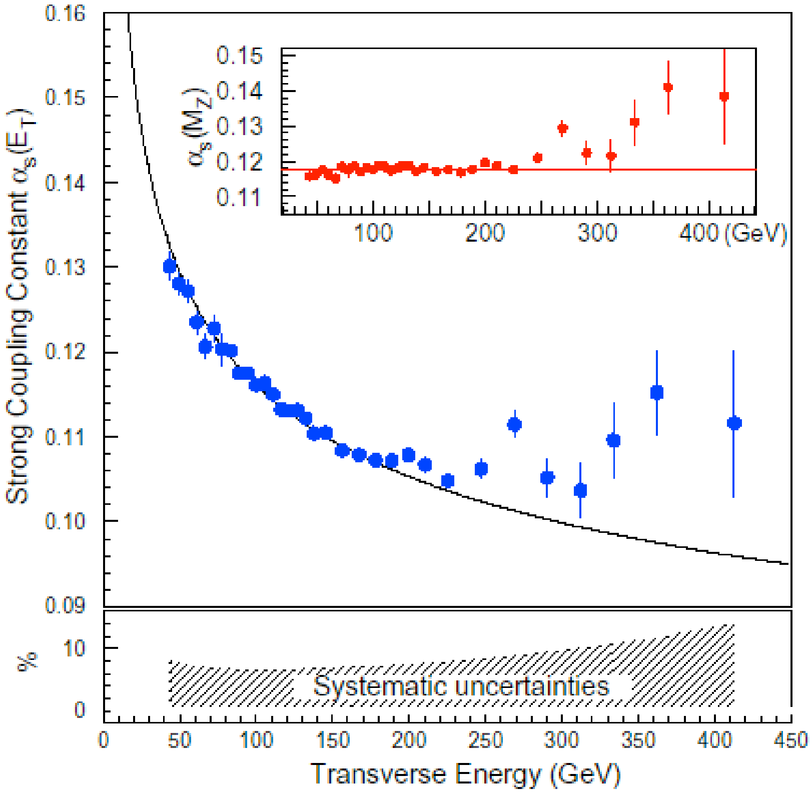
UA6 (1996): $\alpha_s(M_z) = 0.110 \pm 0.004 \, {}^{+0.007}_{-0.003}$

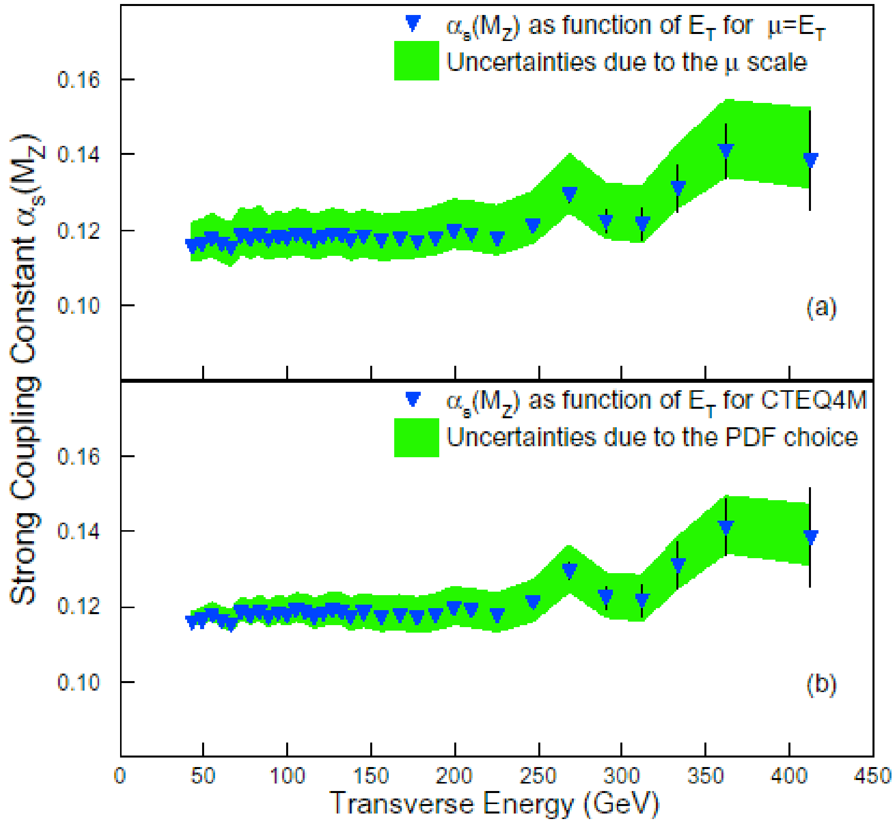
NLO QCD: P. Aurenche et al., 1988

α_s determinations from hadron colliders (4)

- CDF: inclusive jet production at the Tevatron   PRL 88 (2002) 042001

NLO QCD: Giele, Kosover, Yu (1996)





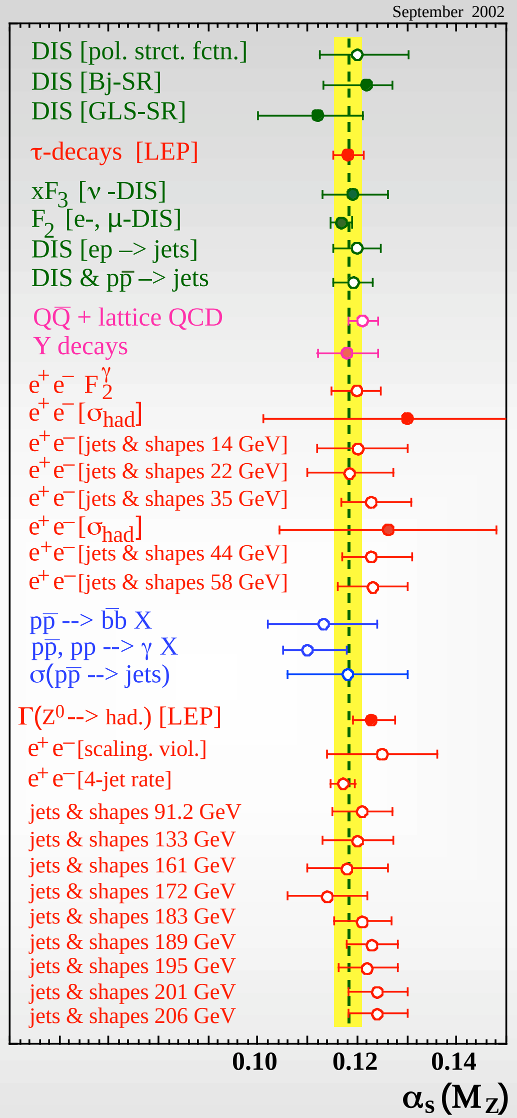
$$\alpha_s(M_Z) = 0.1178 \pm 0.0001\ (\text{stat})\ {}^{+0.008}_{-0.010}\ (\text{exp.sys})\ {}^{+0.007}_{-0.005}\ (\text{ren.scale})\ \pm 0.006\ (\text{pdf})$$

Remarks on α_s determinations from hadron colliders:

- due to large (exp *and* theoretical) uncertainties, do not significantly influence or determine the world average of α_s

- limited to uncertainties of
  – NLO QCD (higher orders; matching with exp. jet algorithm),
  – structure functions,
  – quality of available MC's,
  – underlying events and beam remnants,
  – energy calibration ...

- improvements needed:
  – NNLO QCD calculations, and/or resummation
  – improved parton distributions
  – alternative and improved tools (jet algorithms)
  – improved MC models
  – correction for nonperturbative effects (hadronisation); parton level analyses
  – more data statistics with highest quality
  – more data at different energies



$$\alpha_s(M_Z) = 0.1183 \pm 0.0027$$

World average of $\alpha_s(M_z)$ - and it's overall error

- problem: errors of most results are dominated by theoretical uncertainties!
- therefore these errors are correlated to an unknown degree!
- there is no accepted unique definition of the theoretical uncertainties!

so how to estimate overall uncertainties?

try several definitions and treatment of errors:

- "optimised correlation": assume overall correlation factor such that overall χ^2 equals 1 per degree of freedom
- compare with error assuming all measurements being totally uncorrelated
- "simple rms": unweigthed rms of all results (without their errors)
- "rms box": assume rectengular shaped error (rather than gaussian) of each result, sum up resulting weigths (inverse of total error squared) in histogram, take resulting rms of that distribution

Summary 2002 / 2003

see: hep-ex/0211012

Subsamples; various methods of error calculation:

sample	#entries	$\alpha_s(M_z)$	corr.err.	corr.fact	uncorr.err	simple	box.err.
all	33	.1189	.0037	.65	0.0009	.0042	.0051
NNLO all	9	.1183	.0031	.67	0.0015	.0053	.0049
" " - F2	8	.1196	.0042	.75	0.0020	.0055	.0052
NNLO <.008	6	.1183	.0027	.60	0.0015	.0022	.0035
" " - F2	5	.1197	.0038	.75	0.0020	.0020	.0037
DIS only	7	.1178	.0034	.81	0.0016	.0031	.0042
DIS - F2	6	.1192	.0054	.90	0.0024	.0033	.0044
e+e- only	22	.1195	.0041	.66	0.0022	.0040	.0050

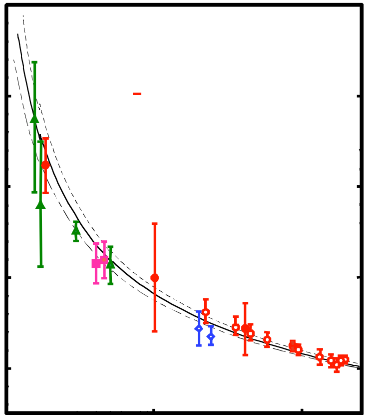
$$\alpha_s(M_Z) = 0.1183 \pm 0.0027$$

- shift & overall error largely dominated by Santiago/Yndurain result
- small error reliable ?

- no significant changes since Sept 2002 —> keep 2002 average for 2003.

quantifying the running of α_s:

determination of β_0, of N_c and of the functional Q-dependence



- Choose 5 maximally uncorrelated α_s measurements:
  - from τ decays — $\alpha_s(1.78\ \mathrm{GeV}) = 0.323 \pm 0.030$
  - from moments of F_2 — $\alpha_s(2.96\ \mathrm{GeV}) = 0.249 \pm 0.010$
  - from scal. viol. of F_3 — $\alpha_s(4.75\ \mathrm{GeV}) = 0.217 \pm 0.021$
  - from $\Gamma(Z^0)$ — $\alpha_s(91.2\ \mathrm{GeV}) = 0.123 \pm 0.004$
  - from event shapes — $\alpha_s(206\ \mathrm{GeV}) = 0.110 \pm 0.004$

- fit simple functional forms:
  - $\alpha_s(Q) = B / \ln(Q^2 / C^2)$ (l.o. QCD; $B = 1/\beta_0$; $C = \Lambda_{\overline{MS}}$)
  - $\alpha_s(Q) = A + C\,Q$ (straight line)
  - $\alpha_s(Q) = A + C / Q$

quantifying the running of α_s: fit results

$\alpha_s(Q)$ =	A	B	C	χ^2 / dof	prob.
$B / \ln(Q/C)^2$	–	1.64±.08	0.115±.027	0.78 / 3	0.85
A + C Q	0.173±.005	–	-0.00037	130 / 3	10^{-62}
A + C / Q	0.115±.002	–	0.40±.03	5.5 / 3	0.14

B = 1.64 ± 0.08 –> N_c = 3.00 ± 0.10 (for N_f = 5 quark flavours)

Conclusions

- huge amounts of data, from different h.e. processes, in large range of energies (1 ... 400 GeV).
- consistent behaviour of data: running α_s convincingly proven
- complete NNLO now for 9 classes of measurements
- 2002/2003 world average (in NNLO): $\alpha_s(M_Z) = 0.1183 \pm 0.0027$
- results from NLO QCD compatible with NNLO world average
- no significant difference between DIS and e+e-

Future:

- error limited by theoretical uncertainties –> need more NNLO calculations (DIS; hadron collider jets; e+e- jets & shapes)
- need consistent definition and treatment of syst. uncertainties
- need optimisation of tools; high statistics collider data at diff. energies ...

The End