Parton Distribution Functions — Hadron Collider Phenomenology

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- overview
- current status of pdf 'global analyses'
- some outstanding issues
  - pdf uncertainties: understanding the differences between various pdf sets
  - is there a problem with the NLO DGLAP DIS fit at small x?
  - $\sin^2\theta_W$ from νN scattering
  - QED effects in pdfs
- conclusions

factorisation in QCD → precision predictions

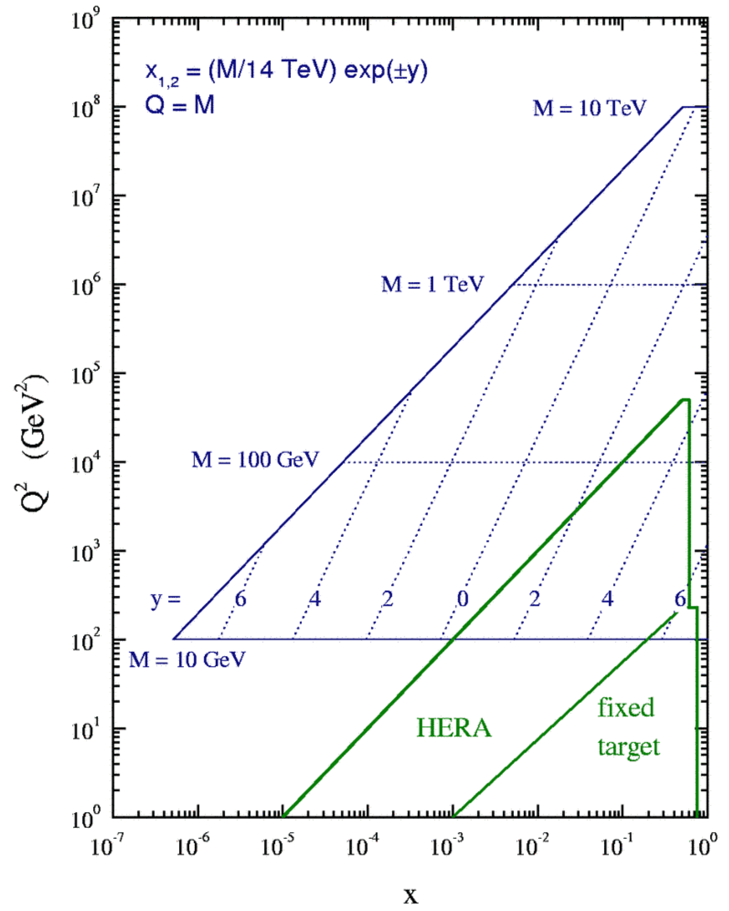
For short-distance ('hard–scattering') inclusive processes, the QCD factorisation theorem applies

- perturbative collinear singularities are universal

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2\, f_a(x_1, \mu_F^2)\, f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab\to X}\left(x_1, x_2, \{p_i^\mu\}; \alpha_S(\mu_R^2), \alpha(\mu_R^2), \frac{Q^2}{\mu_R^2}, \frac{Q^2}{\mu_F^2}\right)$$

where $X = W,\ Z,\ H,\ Q\bar{Q}$, high-E_T jets, $\tilde{q}\tilde{\bar{q}}, \ldots$ etc.

- $\hat{\sigma}$ known either
  - completely to some fixed order (e.g. NLO or NNLO) in pQCD and EW
  - in some leading logarithm approximation (LL, NLL, ...) to all orders via resummation
  - or combination of both (matching)
- x_1 and x_2 fixed by the mass and rapidity of X
- Note: 'higher-twist' power-suppressed contributions (e.g. double parton scattering) and more exclusive event selection will break this factorisation



strategy

- make theoretical predictions as precisely as possible for all relevant (SM and BSM) processes through
  - calculation of HO corrections
  - precision determination of input pdfs

  to give $\sigma_{\text{th}} \pm \delta\sigma_{\text{th}}$
- compare with measurements of 'standard candle' processes, e.g. $\sigma(Z)$, $\sigma(\text{jet})$, $\sigma(t\bar{t})$, ...
- ...and perhaps refine predictions as a result (e.g. improved determination of the gluon pdf)
- incorporate as many HO corrections as possible into parton shower models, for improved event simulation (e.g. MC@NLO, Frixione & Webber, hep-ph/0309186)

This was the topic of the recent Binn Workshop on "Precision Cross Section Measurements at the LHC" – see http://wwweth.cern.ch/WorkShopBinn/

parton distributions from global fits

- DIS structure functions (e.g. F_2^{ep})

$$\mathcal{F}_i(x,Q^2) = \int_x^1 \frac{dy}{y}\Big\{\sum_j C_{ij}(y,\alpha_S)\, q_j(\frac{x}{y},Q^2) + C_{ig}(y,\alpha_S)\, g(\frac{x}{y},Q^2)\Big\}$$

- hadronic cross sections (e.g. $\sigma(p\bar{p}\to WX)$)

$$d\sigma_X = \sum_{\text{partons } a,b} \int dx_a dx_b\, f_a(x_a,Q^2)\, f_b(x_b,Q^2)\, d\hat{\sigma}_{ab\to X}$$

- DGLAP evolution

$$\frac{\partial q_i(x,Q^2)}{\partial \log Q^2} = \frac{\alpha_S}{2\pi}\int_x^1 \frac{dy}{y}\Big\{P_{q_iq_j}(y,\alpha_S)\, q_j(\frac{x}{y},Q^2) + P_{q_ig}(y,\alpha_S)\, g(\frac{x}{y},Q^2)\Big\}$$

$$\frac{\partial g(x,Q^2)}{\partial \log Q^2} = \frac{\alpha_S}{2\pi}\int_x^1 \frac{dy}{y}\Big\{P_{gq_j}(y,\alpha_S)\, q_j(\frac{x}{y},Q^2) + P_{gg}(y,\alpha_S)\, g(\frac{x}{y},Q^2)\Big\}$$

- NLO: coefficient functions C, $\hat{\sigma}$ and splitting functions P evaluated to next-to-leading order in α_S at present

recent work

H1, ZEUS: ongoing fits for pdfs + uncertainties from HERA and other DIS data

Martin, Roberts, WJS, Thorne (MRST): updated 'MRST2001' global fit (hep-ph/0110215); LO/NLO/'NNLO' comparison (hep-ph/0201127); parton distribution uncertainties: from experiment (hep-ph/0211080) and theory (hep-ph/0308087)

Pumplin et al. (CTEQ): updated 'CTEQ6' global fit (hep-ph/0201195), including uncertainties on pdfs; dedicated study of high E_T jet cross sections for the Tevatron (hep-ph/0303013); strangeness asymmetry from neutrino dimuon production (hep-ph/0312323)

Giele, Keller, Kosower (GKK): restricted global fit, focusing on data-driven pdf uncertainties (hep-ph/0104052)

Alekhin: restricted global fit (DIS data only), focusing on effect of both theoretical and experimental uncertainties on pdfs and higher-twist contributions (hep-ph/0011002); updated and including 'NNLO' fit (hep-ph/0211096)

Comprehensive repository of past and present polarised and unpolarised pdf codes (with online plotting facility) can be found at the HEPDATA pdf server web site: `http://durpdg.dur.ac.uk/hepdata/pdf.html` — this is also the home of the LHAPDF project

ingredients: data typically used in current global fits

H1, ZEUS $F_2^{e^+p}(x,Q^2)$, $F_2^{e^-p}(x,Q^2)$

BCDMS $F_2^{\mu p}(x,Q^2), F_2^{\mu d}(x,Q^2)$

NMC $F_2^{\mu p}(x,Q^2), F_2^{\mu d}(x,Q^2), (F_2^{\mu n}(x,Q^2)/F_2^{\mu p}(x,Q^2))$

SLAC $F_2^{\mu p}(x,Q^2), F_2^{\mu d}(x,Q^2)$

E665 $F_2^{\mu p}(x,Q^2), F_2^{\mu d}(x,Q^2)$

CCFR $F_2^{\nu(\bar\nu)p}(x,Q^2), F_3^{\nu(\bar\nu)p}(x,Q^2)$

$\rightarrow$ q, $\bar q$ at all x and g at medium, small x

H1, ZEUS $F_{2,c}^{e^+p}(x,Q^2) \rightarrow c$

E605, E772, E866 Drell-Yan $pN \rightarrow \mu\bar\mu + X \rightarrow \bar q\ (g)$

E866 Drell-Yan p,n asymmetry $\rightarrow \bar u, \bar d$

CDF W rapidity asymmetry $\rightarrow u/d$ ratio at high x

CDF, D0 Inclusive jet data $\rightarrow g$ at high x

CCFR, NuTeV Dimuon data constrains strange sea s, $\bar s$

Note: nowadays, no prompt photon data included in fits

method and assumptions

Choose theoretical framework (e.g. $\overline{\mathrm{MS}}, \mathrm{NLO}$) and data set

Perform fit by minimizing χ^2 to all data, including both statistical and (correlated, where available) systematic errors $\longrightarrow$ 'best fit' set of pdfs + pdf errors (see later)

Start evolution at some Q_0^2 (e.g. $= 1\ \mathrm{GeV}^2$), where pdfs parametrised with functional form, e.g.
$xf(x,Q_0^2) = (1-x)^\eta(1+\epsilon x^{0.5}+\gamma x)x^\delta$

Cut data at $Q^2 > Q^2_{\mathrm{min}}$ and at $W^2 > W^2_{\mathrm{min}}$ to avoid higher twist contamination

Allow $\bar u \neq \bar d$ as implied by e.g. E866 Drell-Yan asymmetry data

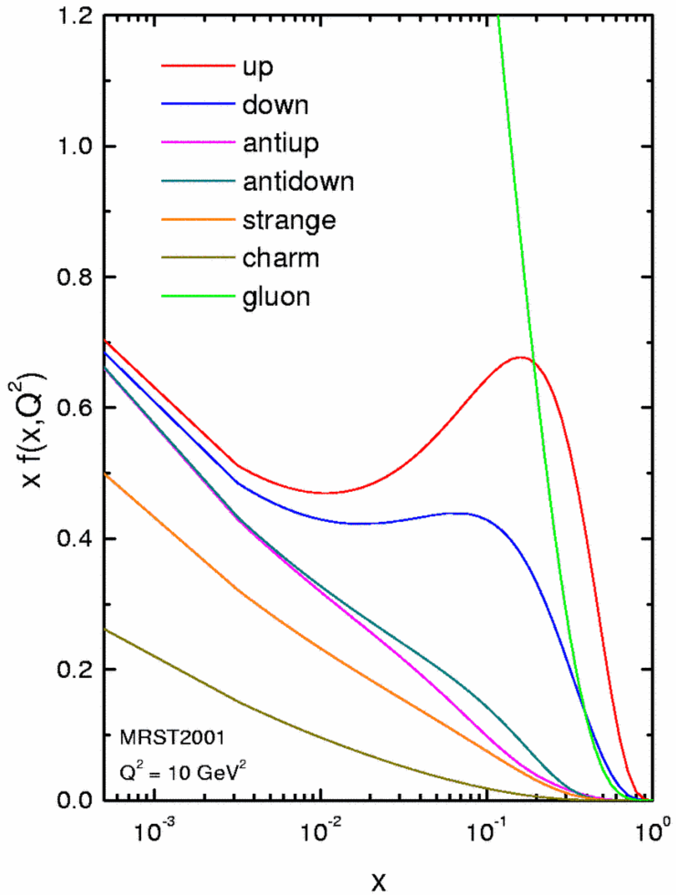
$s(=\bar s?)$ is constrained by CCFR & NuTeV Dimuon data $\Rightarrow$
$s \approx 0.45(\bar u + \bar d)/2$ at $Q_0^2 = 1$ GeV^2

For heavy (c,b) quarks use a VFNS (improves global fit compared to ZM-VFNS and FFNS)

By-product of fit is

$$\alpha_S^{\overline{MS},NLO}(M_Z^2) = 0.116 - 0.120$$

MRST 2001 parton distributions



pdf uncertainties

- direct effect on Tevatron, LHC cross section predictions, i.e. $\pm\delta\sigma_{\rm pdf}$
- currently receiving a lot of attention; various approaches being used. For review see Thorne et al. hep-ph/0205233 (Working Group at IPPP Statistics Workshop, March 2002)

  1. Hessian (error matrix) approach (H1, ZEUS, CTEQ, Alekhin,...)

  2. Offset method (H1, ZEUS, Zomer, Pascaud, Botje,...)

  3. Statistical method (Giele, Keller, Kosower)

  4. Lagrange Multiplier method (CTEQ, MRST, ...)

  contrast **1.** ($\rightarrow$ generic pdfs sets which form uncertainty 'envelope') with **4.** ($\rightarrow$ predicted error on particular observable due to pdfs)

  ... in both cases, the main problem is **normalising** the overall uncertainty, i.e. $\Delta\chi^2 = ??$
- in examples below, will use the MRST2001E package of 31 pdf sets as illustration (MRST, hep-ph/0211080)

Hessian Matrix approach

$$\chi^2 - \chi^2_{min} \equiv \Delta\chi^2 = \sum_{i,j} H_{ij}(a_i - a_i^{(0)})(a_j - a_j^{(0)})$$

The Hessian matrix H is related to the covariance matrix of the parameters by

$$C_{ij}(a) = \Delta\chi^2 (H^{-1})_{ij}$$

Then using the standard formula for linear error propagation:

$$(\Delta F)^2 = \Delta\chi^2 \sum_{i,j} \frac{\partial F}{\partial a_i}(H)^{-1}_{ij}\frac{\partial F}{\partial a_j}$$

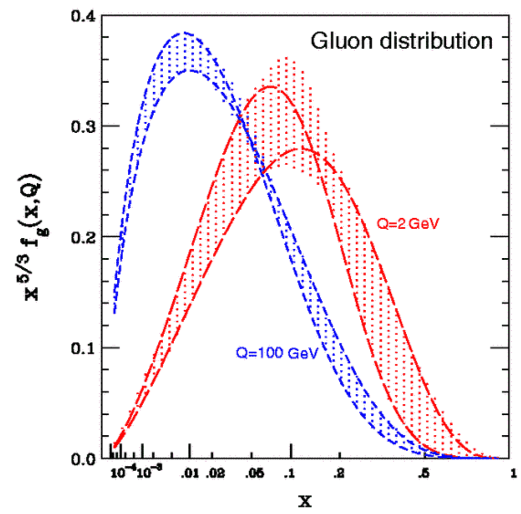
Problem due to extreme variations in $\Delta\chi^2$ in different directions in parameter space solved by finding and rescaling eigenvectors of H leading to diagonal form $\Delta\chi^2 = \sum_i z_i^2$

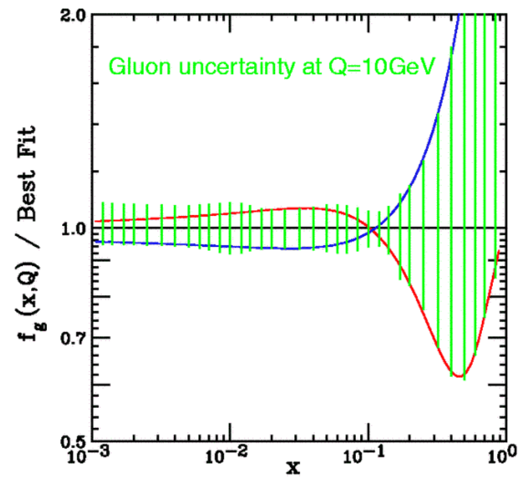
Uncertainty on physical quantity then given by

$$(\Delta F)^2 = \sum_i \left(F(S_i^{(+)}) - F(S_i^{(-)})\right)^2$$

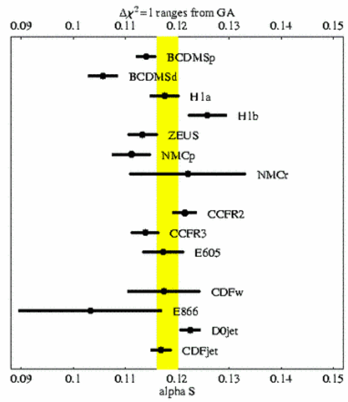
where $S_i^{(+)}$ and $S_i^{(-)}$ are pdf sets displaced along eigenvector direction by given $\Delta\chi^2$. Art in choosing 'correct' $\Delta\chi^2$ given complication of errors in full fit: CTEQ choose $\Delta\chi^2 \sim 100$

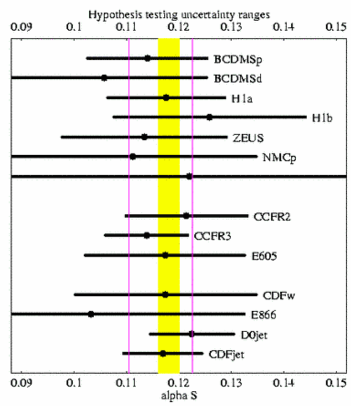
Results of CTEQ Hessian approach for gluon uncertainty





CTEQ α_S values, with $\Delta\chi^2 = 1,\ 100$





Statistical approach (GKK)

Construct an ensemble of distributions labeled by $\mathcal{F}$ each with probability $P(\{\mathcal{F}\})$. Mean μ_O and deviation σ_O of observable O then given by

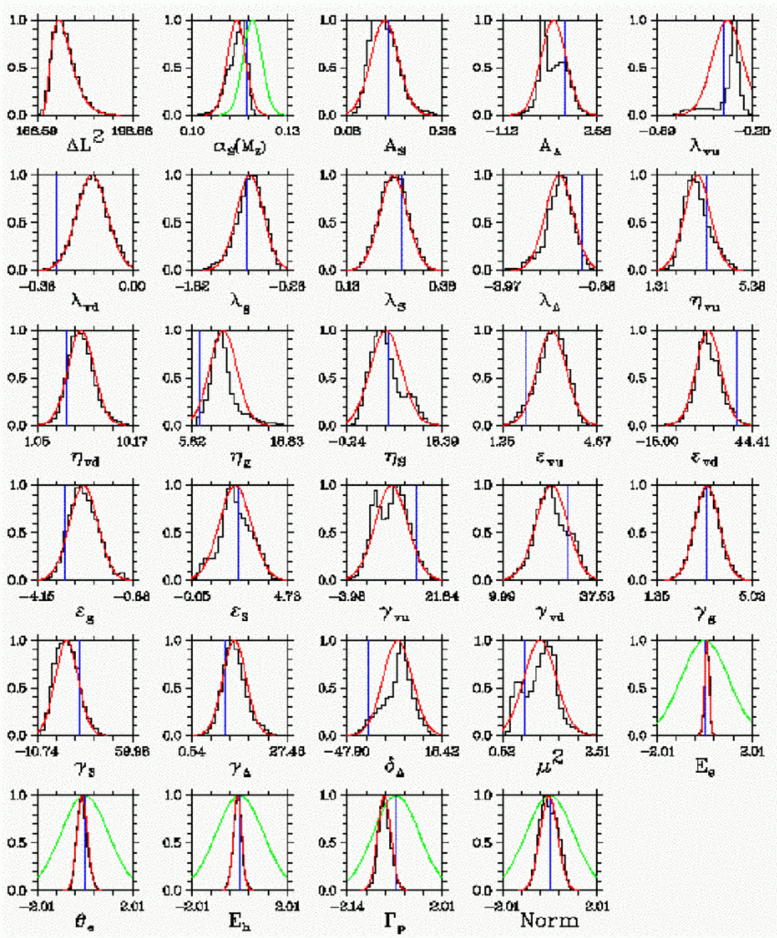
$$\mu_O = \sum_{\{\mathcal{F}\}} O(\{\mathcal{F}\})P(\{\mathcal{F}\}), \quad \sigma_O^2 = \sum_{\{\mathcal{F}\}} (O(\{\mathcal{F}\}) - \mu_O)^2 P(\{\mathcal{F}\}).$$

Note that this is statistically correct, and does not rely on the approximation of linear propagation errors in calculating observables. However it is somewhat inefficient – in practice generate $N_{pdf} \sim 100$ different sets of pdfs with unit weight but distributed according to $P(\{\mathcal{F}\})$

$$\mu_O = \frac{1}{N_{pdf}} \sum_{1}^{N_{pdf}} O(\{\mathcal{F}\}), \quad \sigma_O^2 = \frac{1}{N_{pdf}} \sum_{1}^{N_{pdf}} (O(\{\mathcal{F}\}) - \mu_O)^2.$$

Can incorporate full information about measurements and their error correlations in the calculation of $P(\{\mathcal{F}\})$

Currently uses only proton DIS data sets in order to avoid complicated uncertainty issues such as shadowing effects for nuclear targets etc



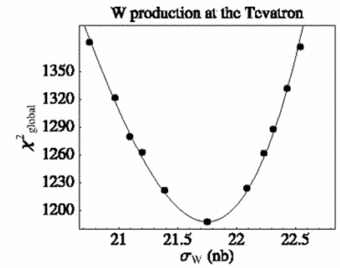
'H1' set of parton parameters from GKK. Red curve Gaussian approx. and blue line MRST value. Green curve for α_S is LEP result.

Lagrange Multiplier method

First suggested by CTEQ. Perform fit while constraining value of some physical quantity F. Minimize

$$\Psi(\lambda, a) = \chi^2_{\mathrm{global}}(a) + \lambda F(a)$$

for various values of λ and parton parameters $\{a\}$. Gives set of best fits for particular values of parameter $F(a)$ without relying on Gaussian approximation for χ^2, e.g. W cross section at Tevatron:

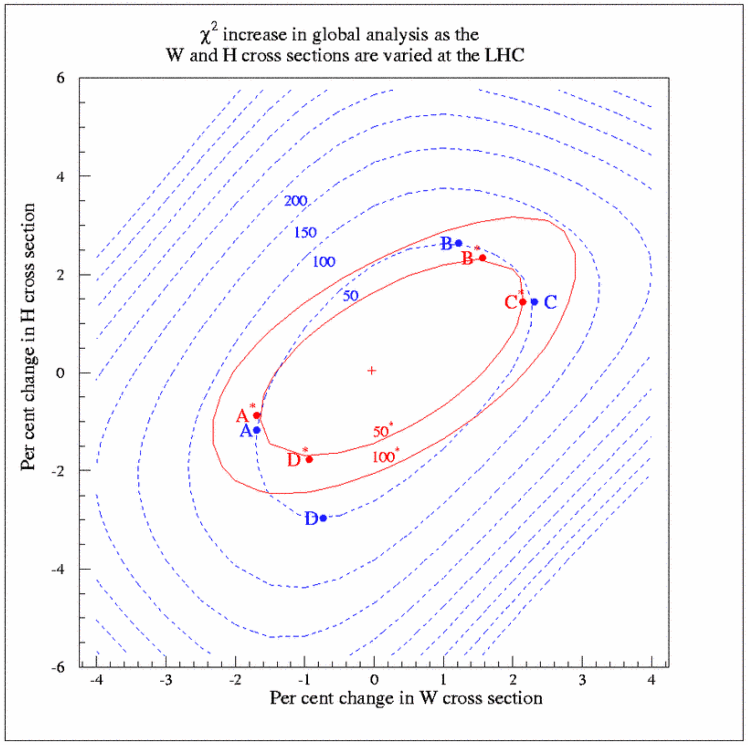


Uncertainty then determined by deciding allowed range of $\Delta\chi^2$. Can also see which data sets in global fit most directly influenced by variation in $F(a)$. Typically deterioration in χ^2 comes from 2 or 3 data sets. MRST impose (rough) criterion that for all data well fit by central best fit no data set has worse than 1% confidence level $\Rightarrow \Delta\chi^2 \approx 50$

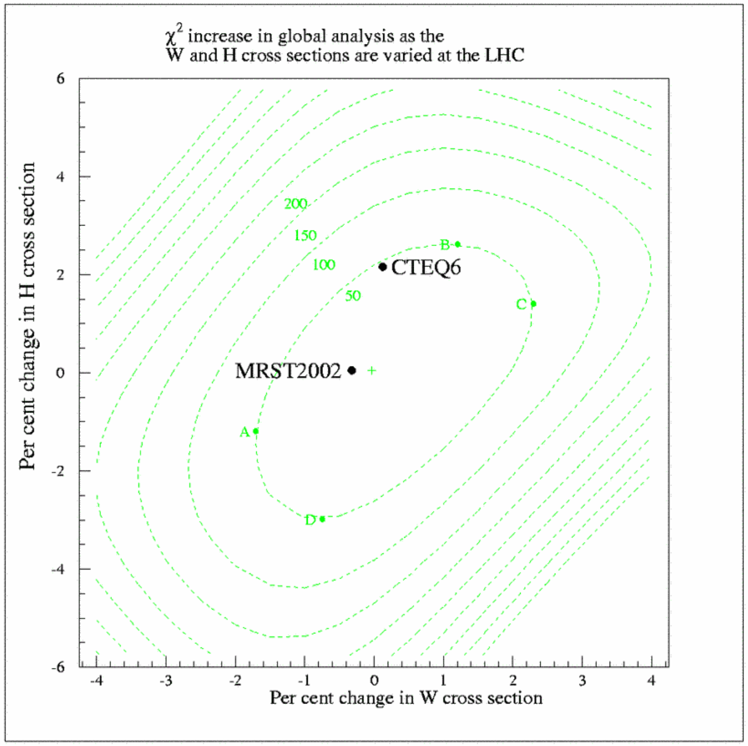
As a specific example, we can consider W and Higgs cross sections at Tevatron and LHC

Results of MRST for Higgs, W cross section pdf uncertainty at LHC

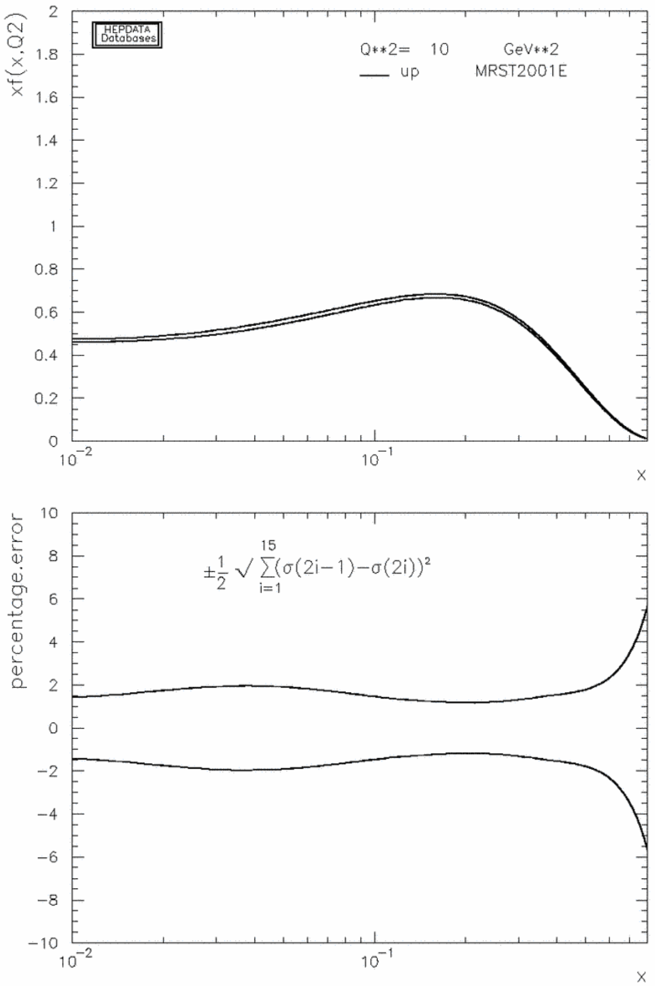
blue contours: α_S fixed in fits
red contours: α_S varied in fits



Results of MRST for Higgs, W cross section pdf uncertainty at LHC

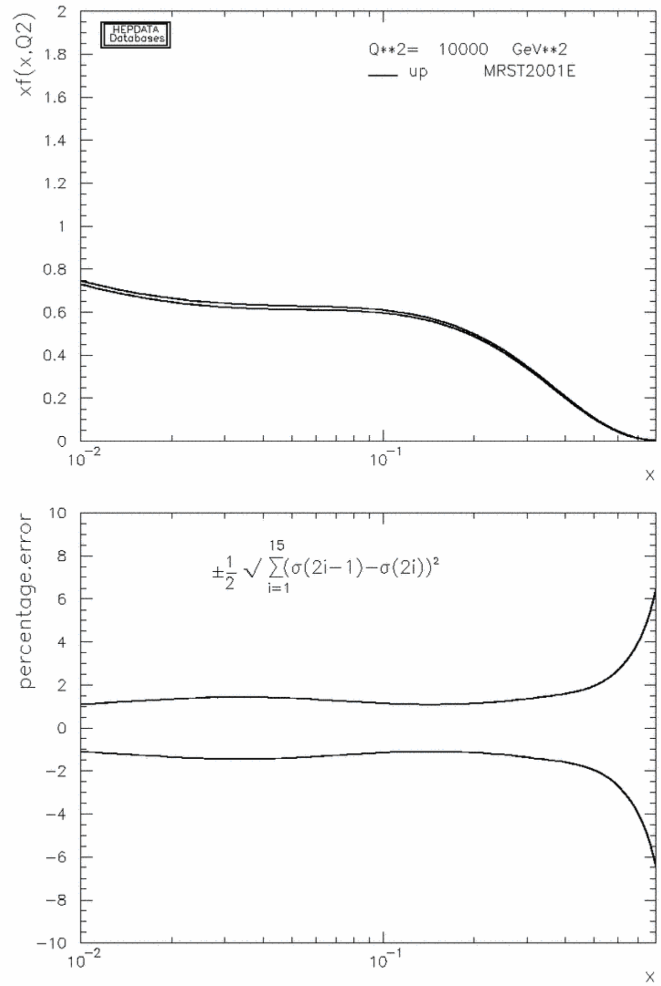


error on up distribution at 10 GeV^2



IP^3 18

error on up distribution at 10^4 GeV^2



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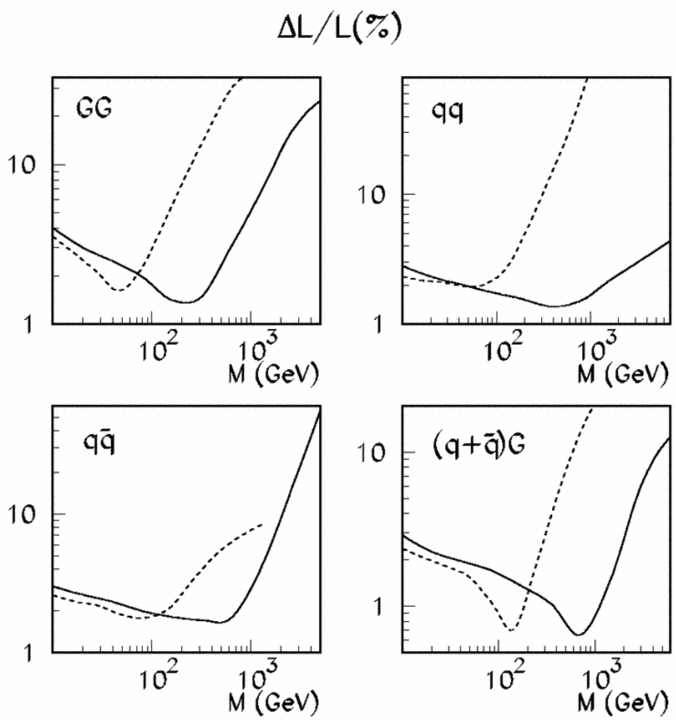
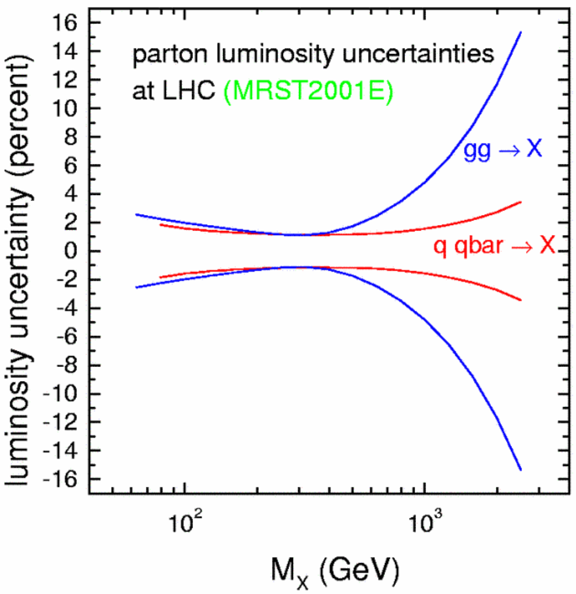


The dependence of parton-parton luminosities for the LHC (full curves) and the Tevatron (dashes) on the produced mass M

Alekhin, hep-ph/0211096

qq, gg luminosity uncertainties at LHC as estimated by MRST2001E

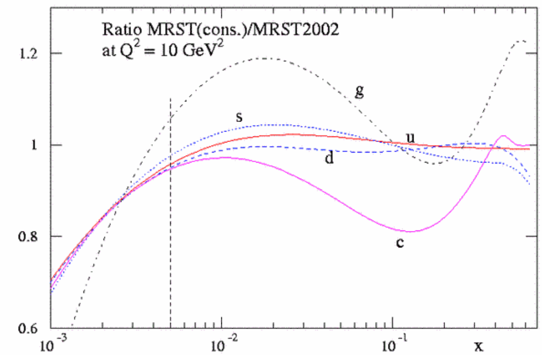


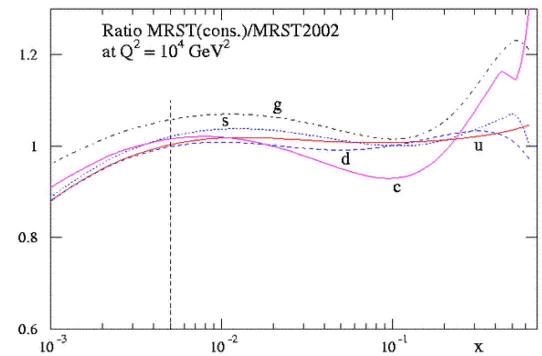
pdf uncertainties contd.

- the above represent experimental pdf uncertainties, i.e. due to errors on experimental data used in global analysis
- much more difficult to quantify are theoretical systematic pdf uncertainties, which are the reason why MRST2002 $\neq$ CTEQ6 for example
- sources of these include (see for example MRST, hep-ph/0211080)
  - selection of data fitted
  - presence of $\ln(1/x)$, $\ln(1-x)$, HT contributions
  - input assumptions: choice of parameterisation, heavy target corrections, isospin and strange-antistrange asymmetry violation, etc.

  ... in the MRST study, the effect on the extracted pdfs (and on some reference cross sections, e.g. σ_W) of varying the 'standard assumptions' was investigated
- particularly interesting was the effect of systematically removing small x and small Q^2 DIS data from the NLO and 'NNLO' global fits

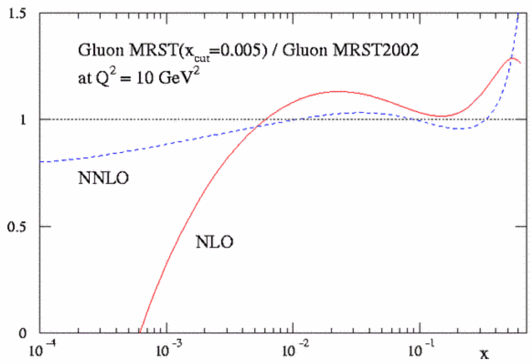
- effect of removing small x, Q^2 data from the fit
  - start with $x_{\rm cut} = 0$, $Q^2_{\rm cut} = 2$ GeV^2
  - systematically increase these to remove DIS data
  - notice that the quality of the fit to the remaining data signifcantly improves, until stability is reached for $x_{\rm cut} \simeq 0.005$, $Q^2_{\rm cut} \simeq 10$ GeV^2
  - call the resulting partons 'the conservative set'
  - repeat at 'NNLO'

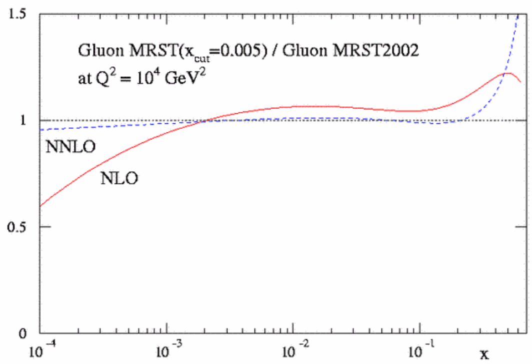




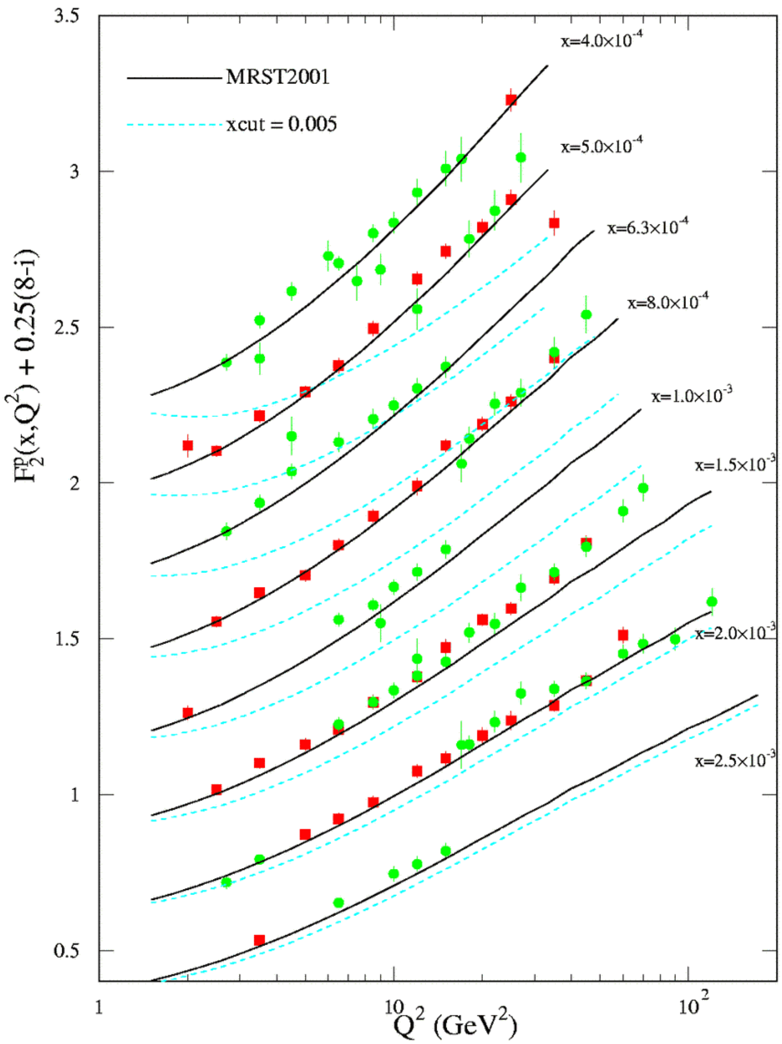
- the gluon adjusts to be larger at medium x (better fit to DIS data in this region) and at high x (better fit to Tevatron jet data)

- the adjustment of the pdfs is much less at 'NNLO', suggesting that higher-order perturbative corrections ($\alpha_S^n \ln^m(1/x)$) could be responsible for the problems with the NLO fit at small x

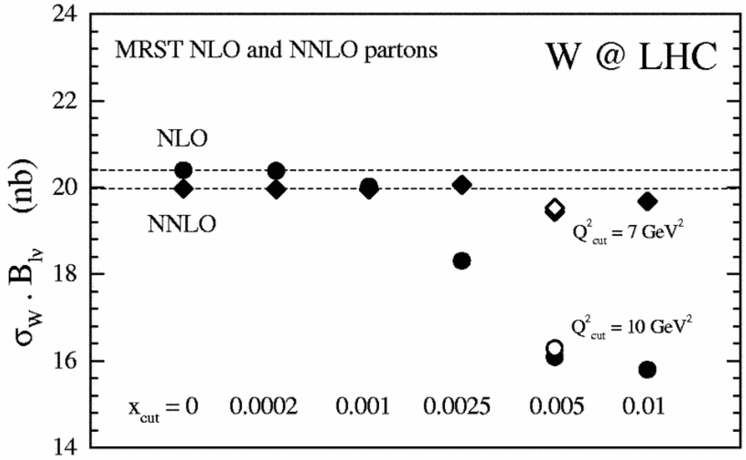


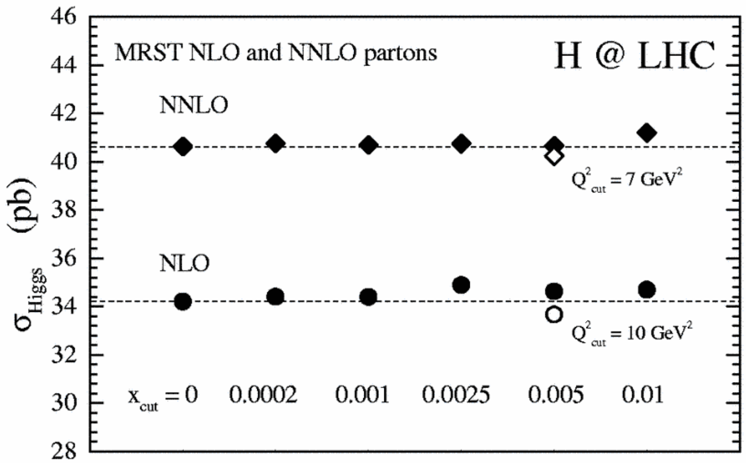


MRST(2001) NLO fit , x=0.0004 - 0.0025

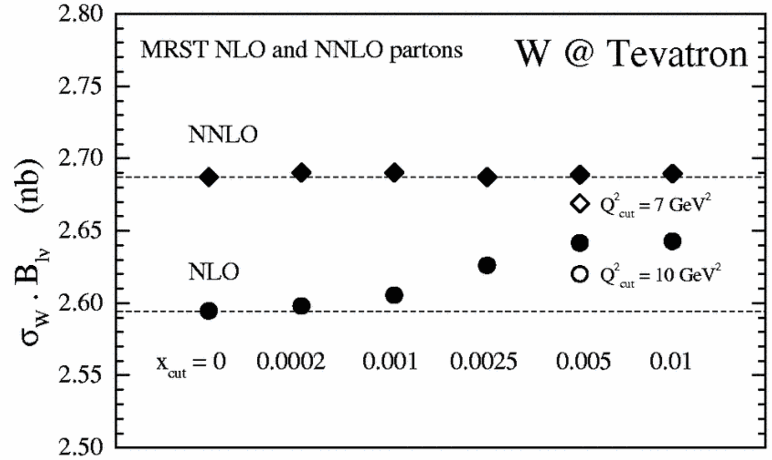


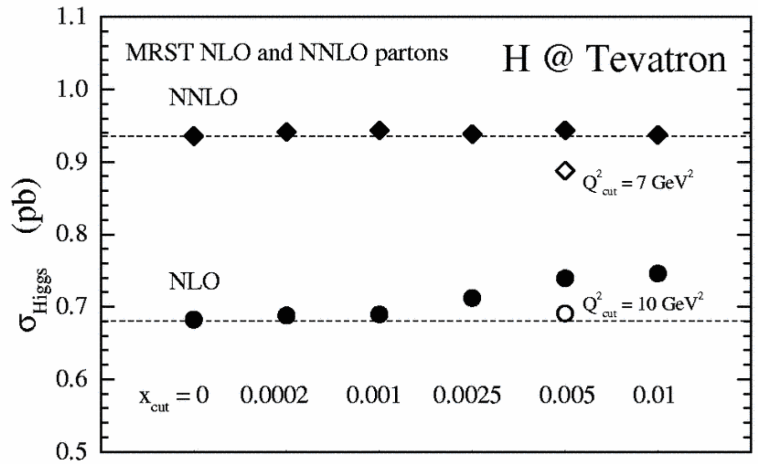
effect on W, H cross sections at LHC



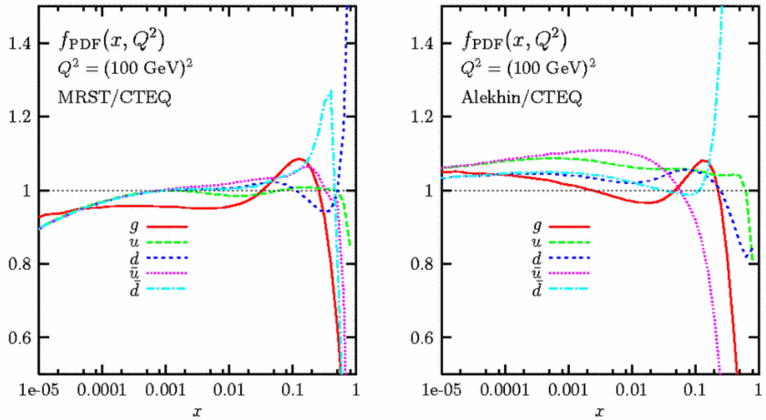


effect on W, H cross sections at Tevatron



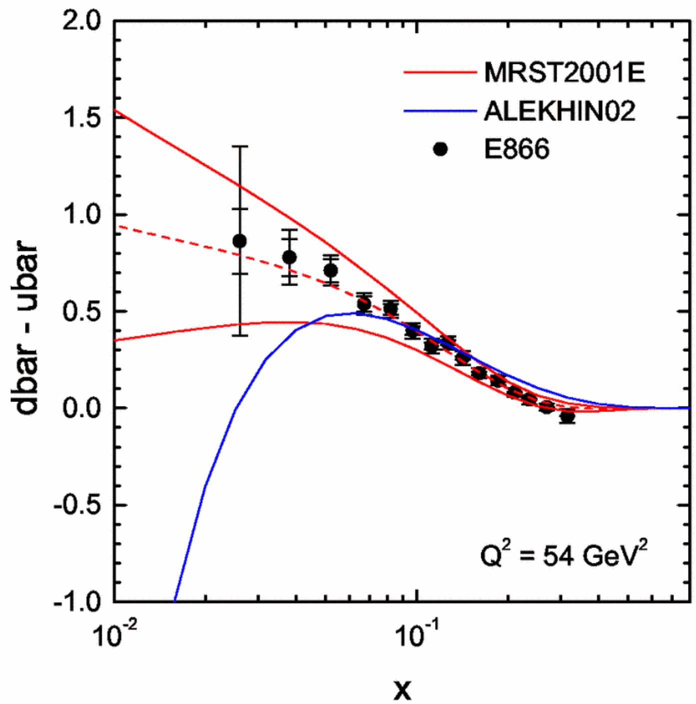


comparison between CTEQ6, MRST2001, Alekhin02 NLO pdfs



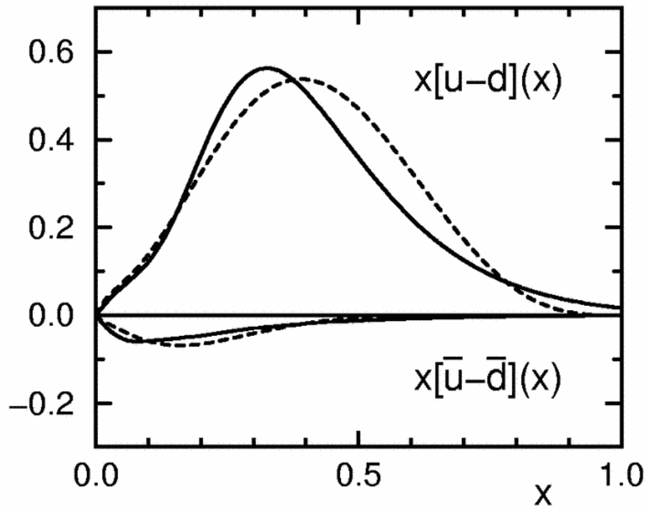
Djouadi & Ferrag hep-ph/0310209

- CTEQ – MRST differences understood, see hep-ph/0211080 (mainly, CTEQ gluon at Q_0^2 required to be positive at small x means $g_{\rm CTEQ} > g_{\rm MRST}$ there)
- Alekhin gluon smaller at high x (no Tevatron jet data in fit) and different flavour content of sea at small x (different assumption about (i) $\bar{u}-\bar{d}$ as $x \to 0$ and (ii) $s/\langle\bar{u}+\bar{d}\rangle$)



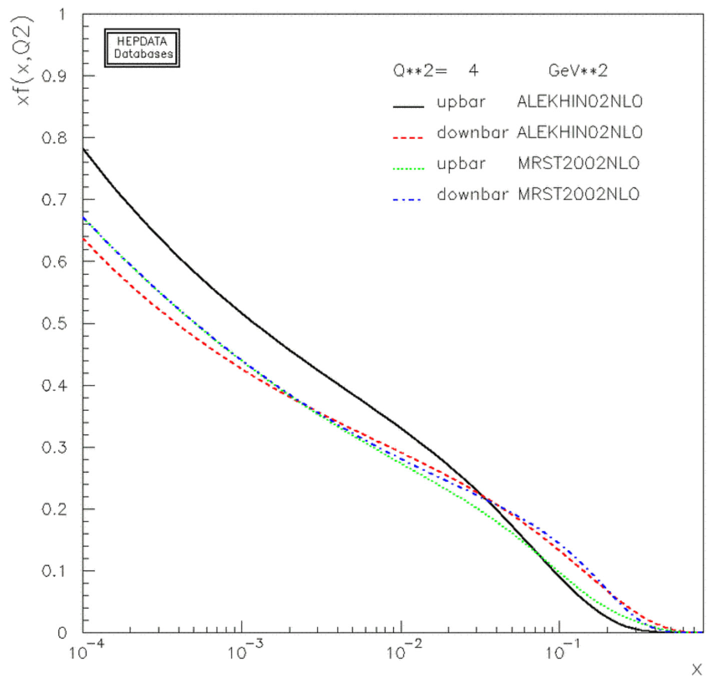
- evidence for
  - $\bar{d} > \bar{u}$
  - $x(\bar{d}-\bar{u}) \to 0$

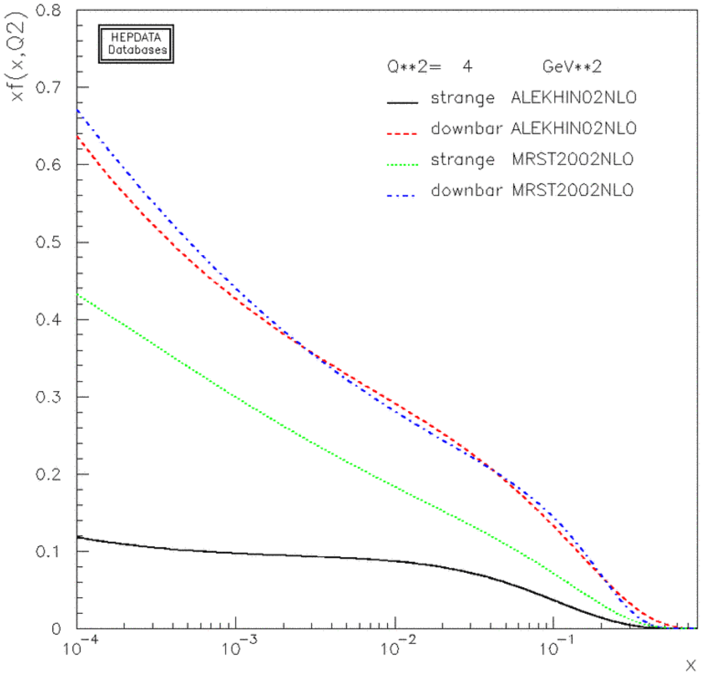
  as $x \to 0$?
- HERA3 (ep and ed DIS at small x) could provide an interesting measurement!



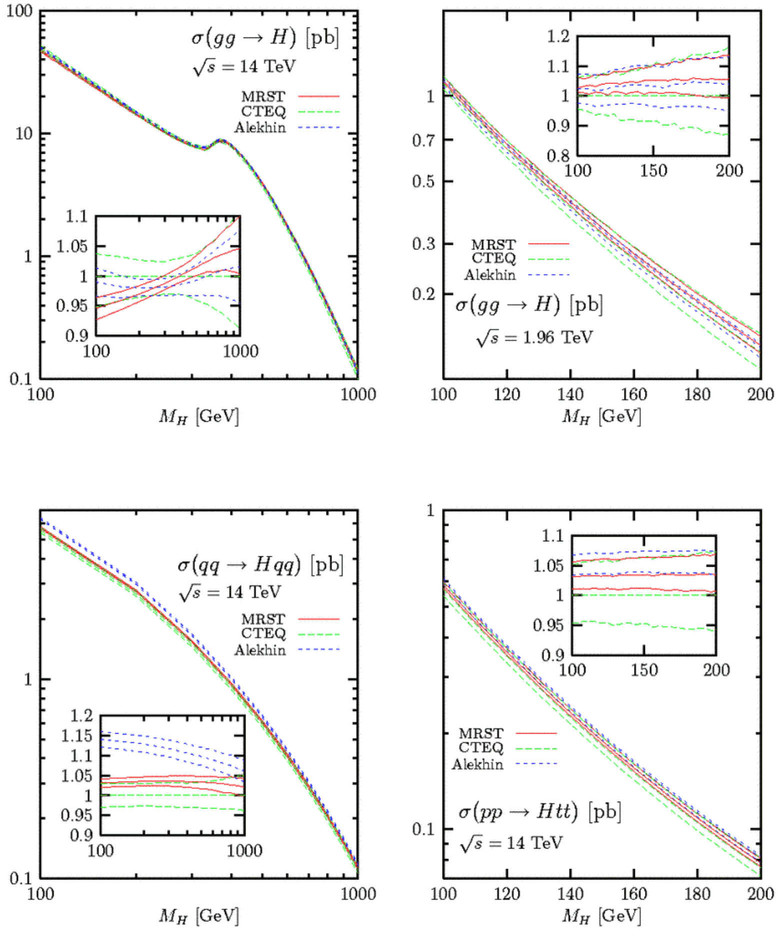
solid lines = large N_c non-perturbative (chiral soliton) model of Pobylitsa et al., hep-ph/9804436

dashed lines = GRV dynamical parton model



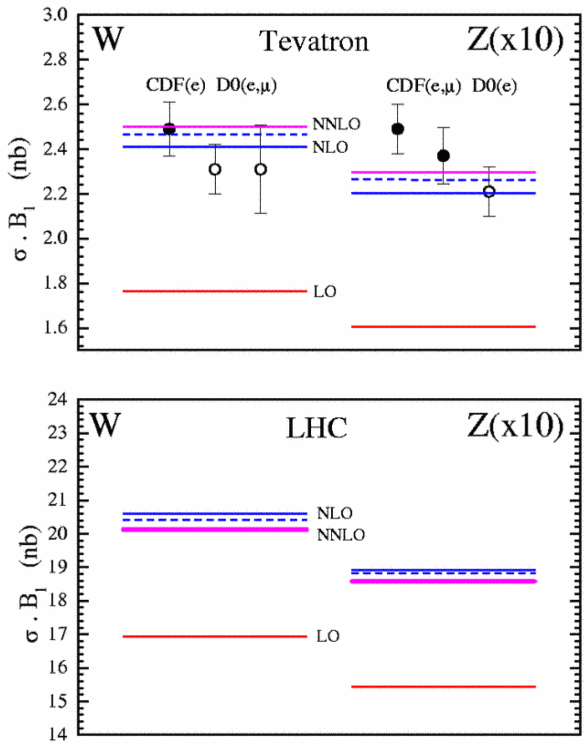


Higgs cross sections at the LHC



cross section predictions at LHC

1. W,Z total cross sections

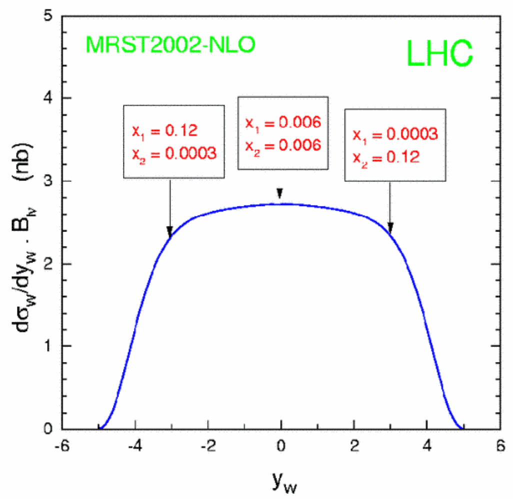


partons: MRST2001
NNLO evolution: van Neerven, Vogt approximation to Vermaseren et al. moments
NNLO W,Z corrections: van Neerven et al. with Harlander, Kilgore corrections

- current best (MRST) estimate

$$\delta\sigma_{W,Z}^{\rm NNLO}(\text{total pdf}) = \pm 4\%$$

- *cf.* $\pm 2\%$ for 'expt. pdf' errors only
- but note that there is a much greater uncertainty in the NLO prediction, due to problems at small x in the global fit to DIS data (see previous)
- this is because the large rapidity W and Z total cross sections sample very small x



2. cross section ratios

- $\sigma(W^+)/\sigma(W^-)$ is gold-plated (MRST, hep-ph/9907231)

$$R_\pm = \frac{\sigma(W^+)}{\sigma(W^-)} \simeq \frac{u(x_1)\bar{d}(x_2)}{d(x_1)\bar{u}(x_2)} \simeq \frac{u(x_1)}{d(x_1)}$$

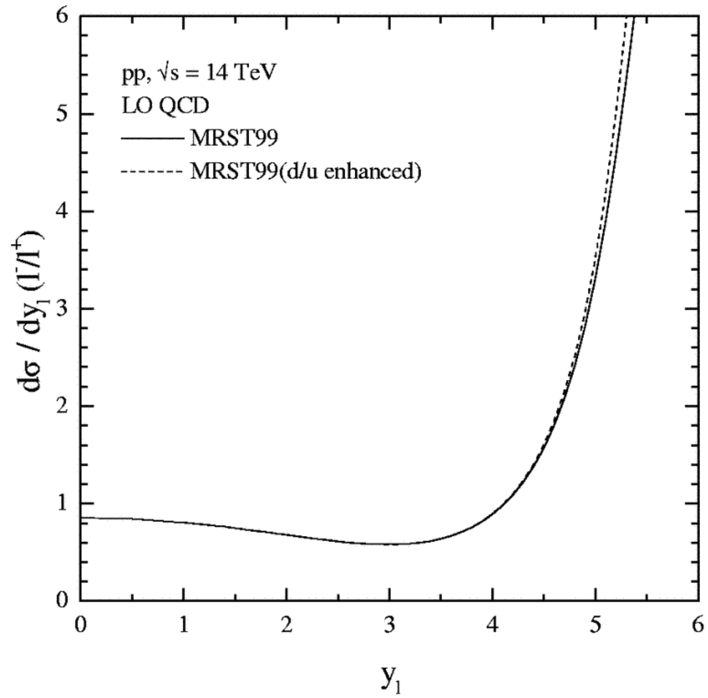
iff we assume that the sea is u, d symmetric at small x (see previous discussion on Alekhin02 $vs.$CTEQ, MRST) and using MRST2001E

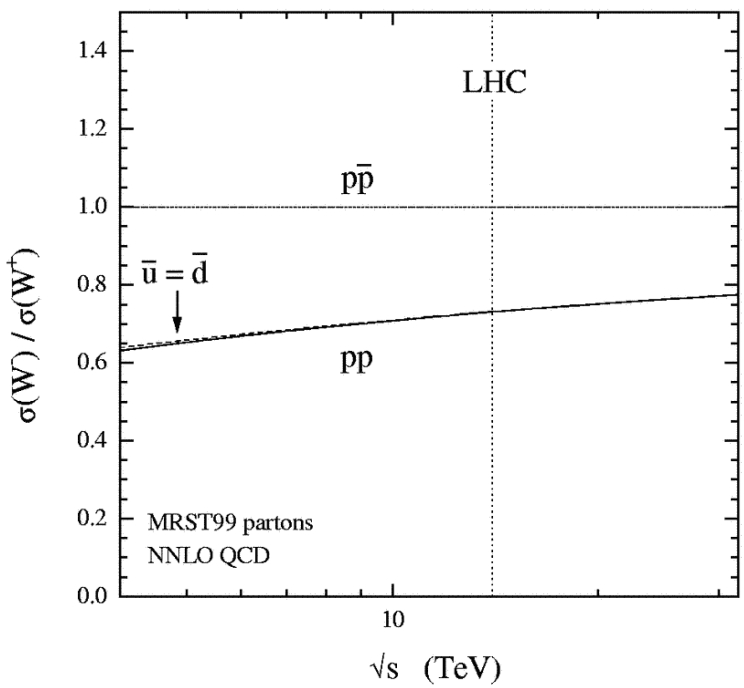
$\delta\sigma_{W^\pm}$(expt. pdf) $= \pm 2\%$, $\delta R_\pm$(expt. pdf) $= \pm 1.4\%$

Assuming all other uncertainties cancel, this is probably the most accurate SM cross section test at LHC

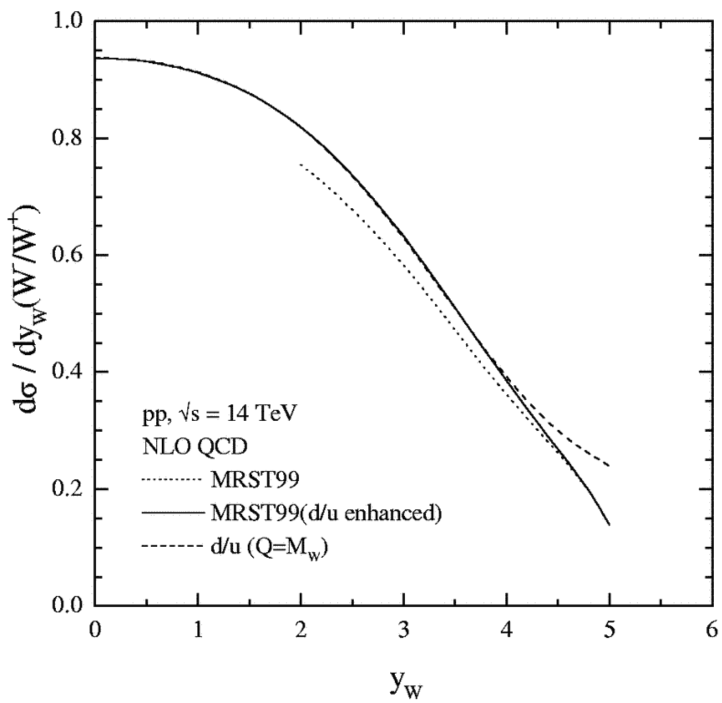
Note: attempt to pin down d/u ratio at large x using forward $W^\pm$ production appears hopeless

ratio of l⁻ and l⁺ rapidity distributions

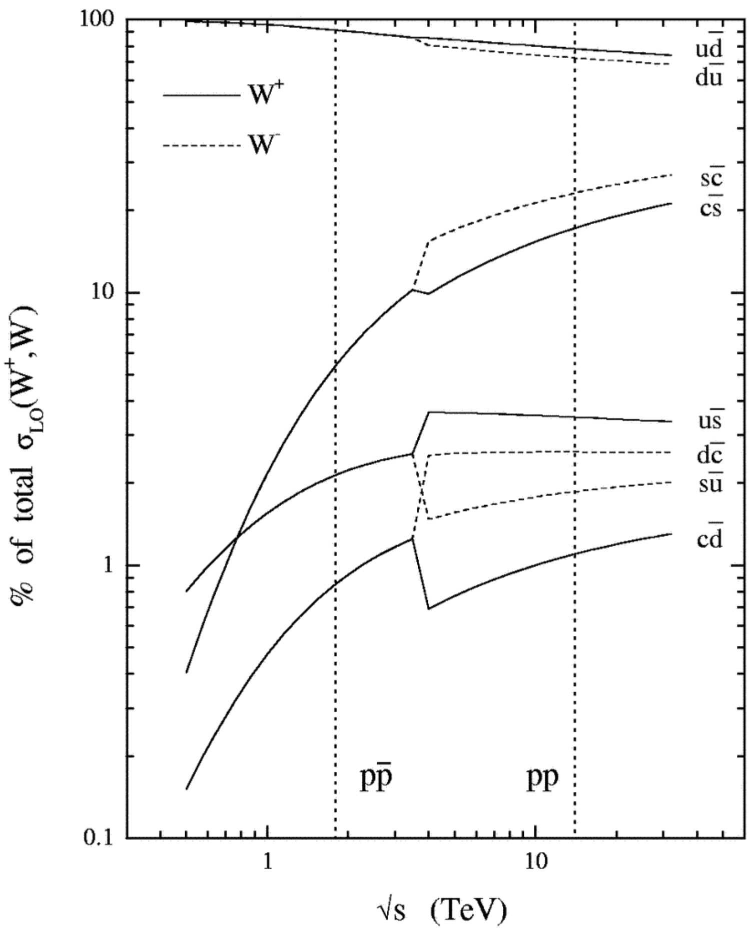




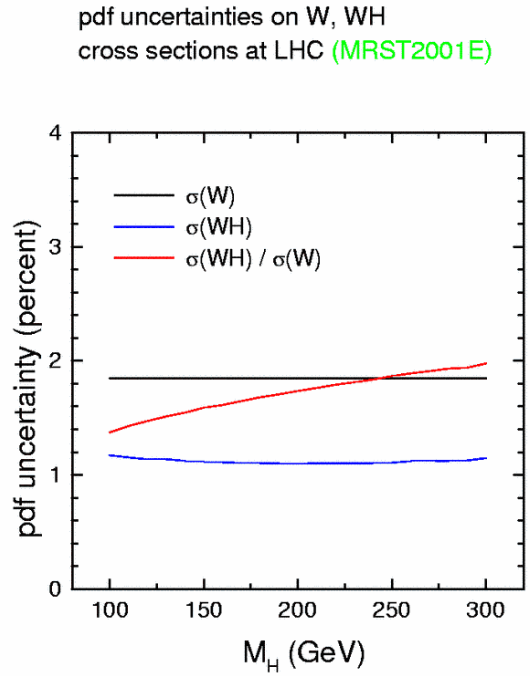
ratio of W^- and W^+ rapidity distributions



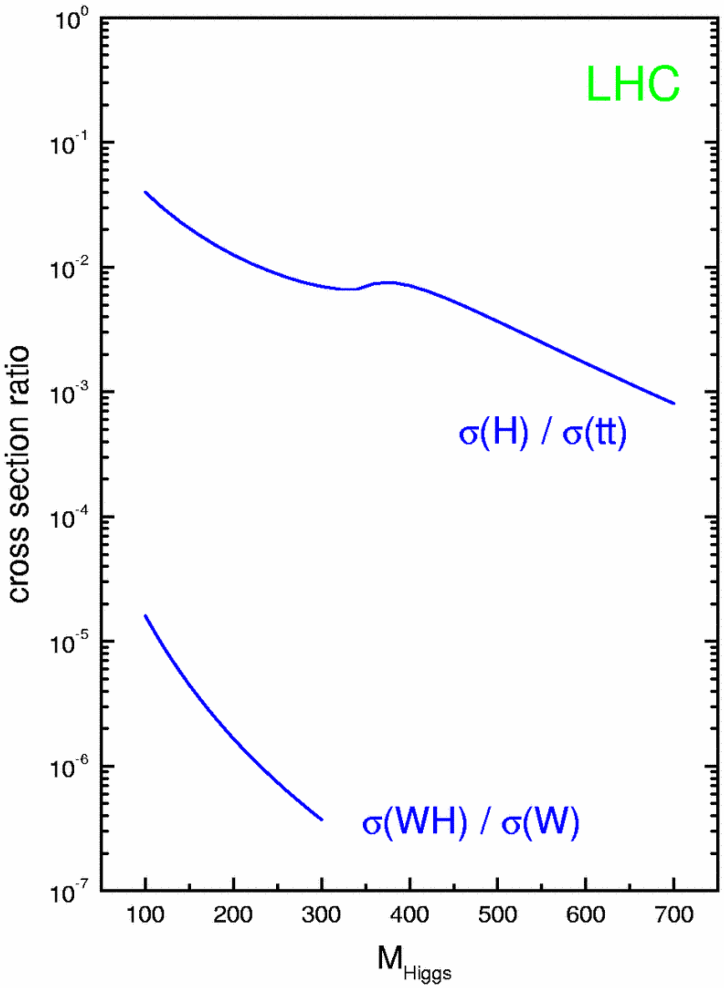
flavour decomposition of W cross sections



- using $\sigma(W)$ or $\sigma(Z)$ to calibrate other cross sections, e.g. $\sigma(WH)$, $\sigma(Z')$



- $\sigma(WH)$ more precisely predicted because it samples quark pdfs at higher x than $\sigma(W)$!



$(gg \to)$ Higgs cross section

- a light (SM or MSSM) Higgs dominantly produced via $gg \to H$ and the cross section has small pdf uncertainty because $g(x)$ at small x is well constrained by HERA DIS data
- current best (MRST) estimate, for $M_H = 120$ GeV:

$$\delta\sigma_H^{\rm NNLO}(\text{total pdf}) = \pm 3\%$$

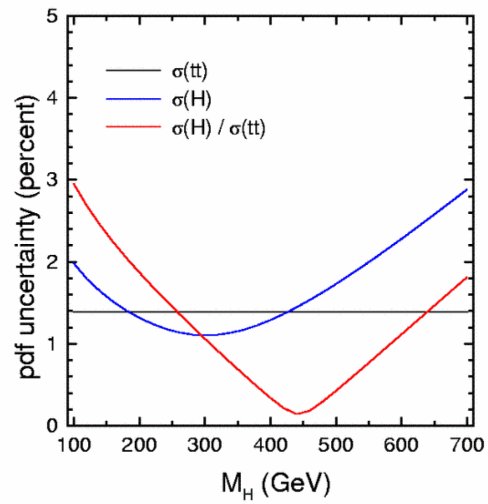
... with less sensitivity to small x than $\sigma(W)$.

- this is *much* smaller than the uncertainty from higher-order corrections, for example (Catani et al, hep-ph/0306211):

$$\delta\sigma_H^{\rm NNLO}(\text{scale variation}) = \pm 10\%,$$
$$\delta\sigma_H^{\rm NNLL}(\text{scale variation}) = \pm 8\%$$

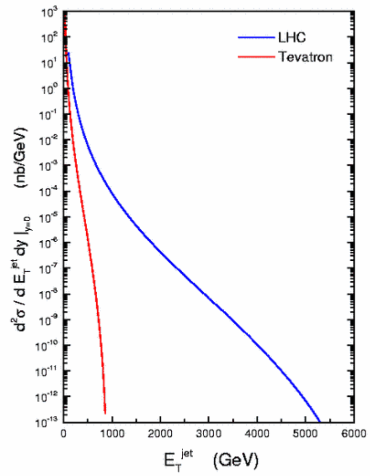
- no obvious advantage in using $\sigma(t\bar{t})$ as a calibration SM cross section, except maybe for large M_H:

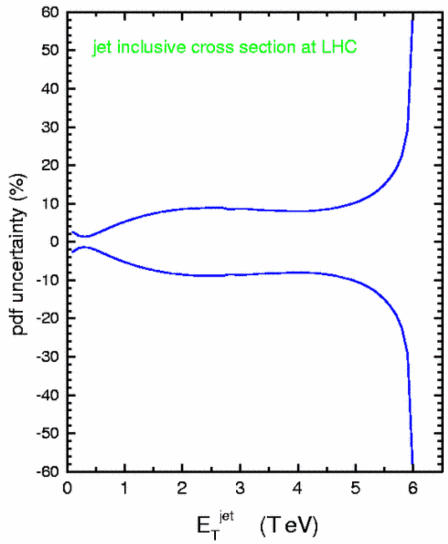
pdf uncertainties on top, (gg→) H cross sections at LHC (MRST2001E)



- Note: the MRST2001E sets have fixed α_S; the variation in e.g. $\sigma(H)$ is slightly larger when the uncertainty in α_S is also taken into account. This is the advantage of the Lagrange Multiplier method.

single jet inclusive

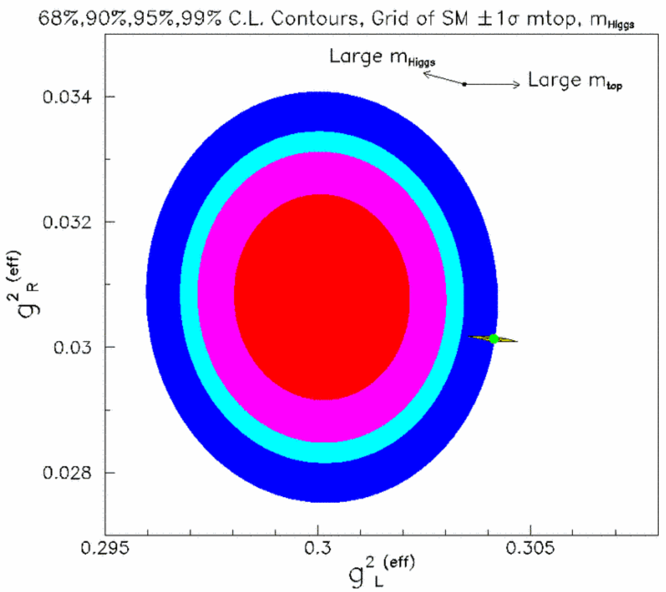




$\sin^2\theta_W$ from νN scattering

NuTeV (2001): $\sigma^{\nu N,\bar{\nu}N} \Rightarrow \sin^2\theta_W = 0.2277 \pm 0.0016$

cf. world average: $\sin^2\theta_W = 0.2227 \pm 0.0004$



new physics?

The NuTeV measurement of $\sin^2\theta_W$ assumes

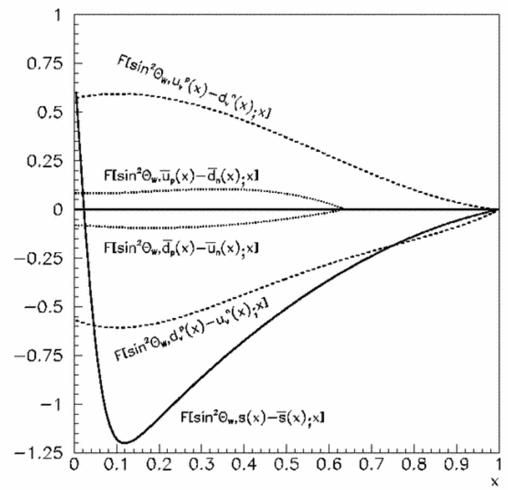
— isospin symmetry: $u_p(x) = d_n(x)$ etc.
— strange–antistrange symmetry: $s(x) = \bar{s}(x)$

otherwise

$$\Delta \sin^2\theta_W = \int_0^1 dx F_s(x)[s(x) - \bar{s}(x)] + \int_0^1 dx F_I(x)[u_p(x) - d_n(x)] + \dots$$

and in particular $s > \bar{s}$ around $x \sim 0.1 \Rightarrow \sin^2\theta_W \downarrow$.

Further constraints on $s(x)$, $\bar{s}(x)$ from CCFR and NuTeV dimuon data, $\nu N, \bar{\nu}N \to \mu^+\mu^- + X$



- Paschos-Wolfenstein ratio

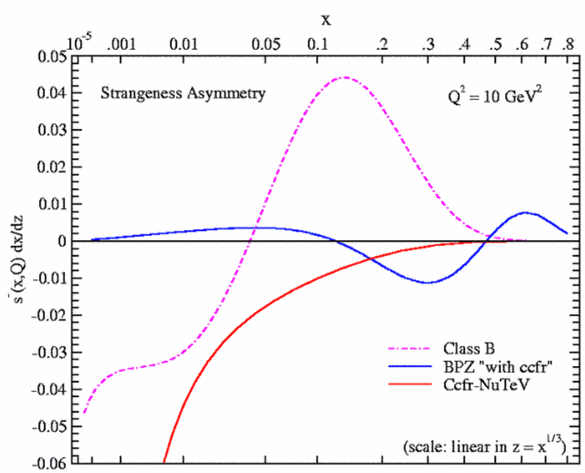
$$R^- \equiv \frac{\sigma^\nu_{NC} - \sigma^{\bar\nu}_{NC}}{\sigma^\nu_{CC} - \sigma^{\bar\nu}_{CC}} \simeq \frac{1}{2} - \sin^2\theta_W + \delta R^-_A + \delta R^-_{EW} + \delta R^-_{NLO} + \delta R^-_s + \delta R^-_{iso}$$

- new analysis by Kretzer et al., hep-ph/0312322, hep-ph/0312323 focuses on

$$\delta R^-_s = -\left(\frac{1}{2} - \frac{7}{6}\sin^2\theta_W\right) \frac{\langle x(s-\bar s)\rangle}{\langle x(u-\bar u + d - \bar d)/2\rangle}$$

via global (CTEQ) fit including νN dimuon data for $s^-(x) \equiv s(x) - \bar s(x)$

$$\Rightarrow \quad -0.005 < \delta R^-_s < +0.001$$



(BPZ = Barone et al., Eur. Phys. J. C12, 243 (2000))

- recent MRST analysis (hep-ph/0308087) investigated possible isospin symmetry breaking, i.e.

$$u^n_V(x) = d^p_V(x) + \kappa f(x)\,, \qquad d^n_V(x) = u^p_V(x) - \kappa f(x)$$

where $f(x) = x^{-0.5}(1-x)^4(x-0.0909)$ and $\int_0^1 f dx = 0$.

- global fit slightly *prefers* $\kappa \neq 0$; best fit is

$$\kappa = -0.2 \quad \Rightarrow \quad \delta(\sin^2\theta_W) = 0.002$$

with $-0.8 < \kappa < +0.65$ at 90%cl

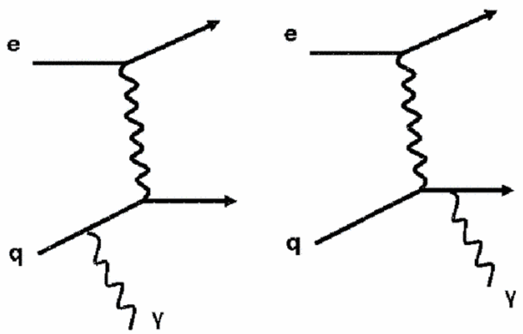
$$\Rightarrow \quad -0.007 < \delta R^-_{iso} < +0.007$$

conclusion

Uncertainties in detailed parton structure needed to relate R^- to $\sin^2\theta_W$ are substantial on the scale of the precision of the NuTeV data – consistency with the Standard Model does not appear to be ruled out at present.

QED effects in pdfs

QED corrections to DIS include



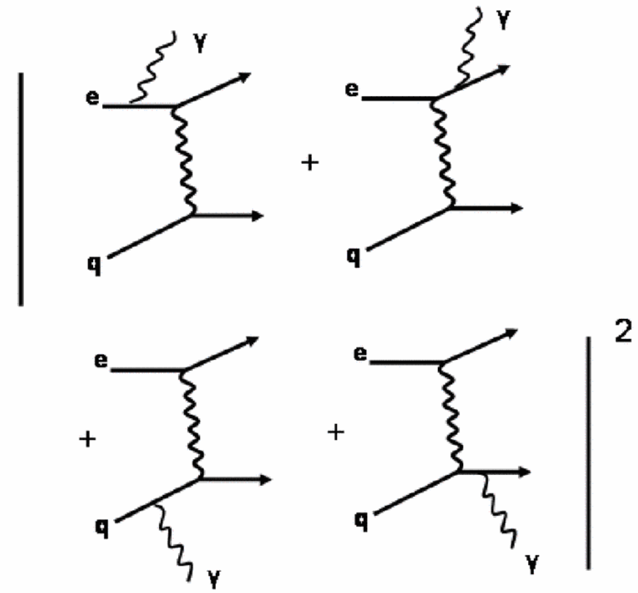
$\Rightarrow$ mass singularity when $\gamma \parallel q$

$$\frac{\alpha}{2\pi} \langle e_q^2 \rangle \ln\left(\frac{Q^2}{m_q^2}\right) \simeq 0.01$$

for $Q = 100$ GeV, $m_q = 10$ MeV, $\langle e_q^2 \rangle = 5/18$.

- such corrections included in standard QED radiative correction packages:

  HERACLES: Spiesberger et al., Comp. Phys. Comm. 69, 155 (1992)
  HECTOR: Arbuzov et al., hep-ph/9511434

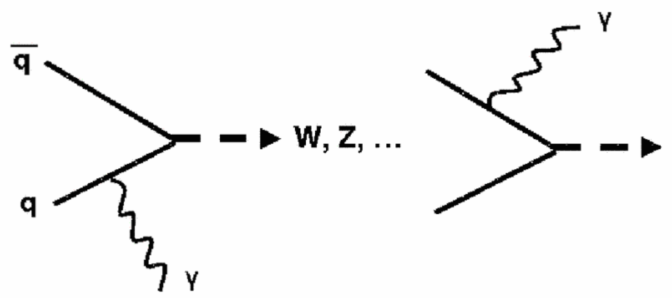


+ loop corrections

$$\Rightarrow \quad \frac{\alpha}{\pi}\left[\, \mathcal{C}_{\rm lept} \; + \; e_q^2\, \mathcal{C}_{\rm quark} \; + \; e_q\, \mathcal{C}_{\rm int} \,\right]$$

- Note: interference of leptonic and partonic radiative corrections finite as $m_q \to 0$
- Issue: exactly what EW corrections have been applied in extraction of structure function measurements in DIS experiments?

- above QED collinear singularities are *universal* and can be absorbed into pdfs, exactly as for QCD collinear singularities, leaving finite (as $m_q \to 0$) $\mathcal{O}(\alpha)$ QED corrections in coefficient functions



- relevant for existing electroweak correction calculations for processes at Tevatron, LHC, e.g. W, Z, WH, ...

— see for example U. Baur, S. Keller and D. Wackeroth, Phys. Rev. D59, 013002 (1999) for a full discussion of the formalism

QED–improved DGLAP equations

$$\frac{\partial q_i(x,\mu^2)}{\partial \log\mu^2} = \frac{\alpha_S}{2\pi}\int_x^1 \frac{dy}{y}\left\{P_{qq}(y)\; q_i(\frac{x}{y},\mu^2) + P_{qg}(y,\alpha_S)\; g(\frac{x}{y},\mu^2)\right\}$$
$$+ \frac{\alpha}{2\pi}\int_x^1 \frac{dy}{y}\left\{\tilde{P}_{qq}(y)\; e_i^2 q_i(\frac{x}{y},\mu^2) + P_{q\gamma}(y)\; e_i^2\gamma(\frac{x}{y},\mu^2)\right\}$$

$$\frac{\partial g(x,\mu^2)}{\partial \log\mu^2} = \frac{\alpha_S}{2\pi}\int_x^1 \frac{dy}{y}\left\{P_{gq}(y)\;\sum_j q_j(\frac{x}{y},\mu^2) + P_{gg}(y)\; g(\frac{x}{y},\mu^2)\right\}$$

$$\frac{\partial \gamma(x,\mu^2)}{\partial \log\mu^2} = \frac{\alpha}{2\pi}\int_x^1 \frac{dy}{y}\left\{P_{\gamma q}(y)\;\sum_j e_j^2\; q_j(\frac{x}{y},\mu^2) + P_{\gamma\gamma}(y)\;\gamma(\frac{x}{y},\mu^2)\right\}$$

at leading order in α_S and α, where

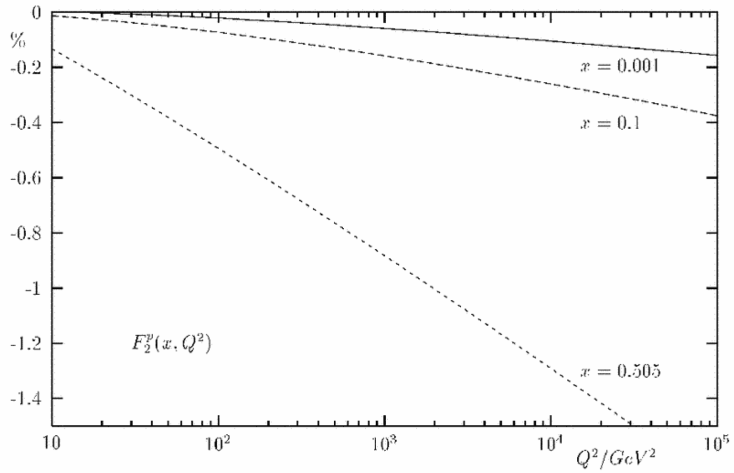
$$\tilde{P}_{qq} = C_F^{-1}P_{qq}, \qquad P_{\gamma q} = C_F^{-1}P_{gq},$$
$$P_{q\gamma} = T_R^{-1}P_{qg}, \qquad P_{\gamma\gamma} = -\frac{2}{3}\sum_i e_i^2\;\delta(1-x)$$

and momentum is conserved:

$$\int_0^1 dx\; x\left\{\sum_i q_i(x,\mu^2) + g(x,\mu^2) + \gamma(x,\mu^2)\right\} = 1$$

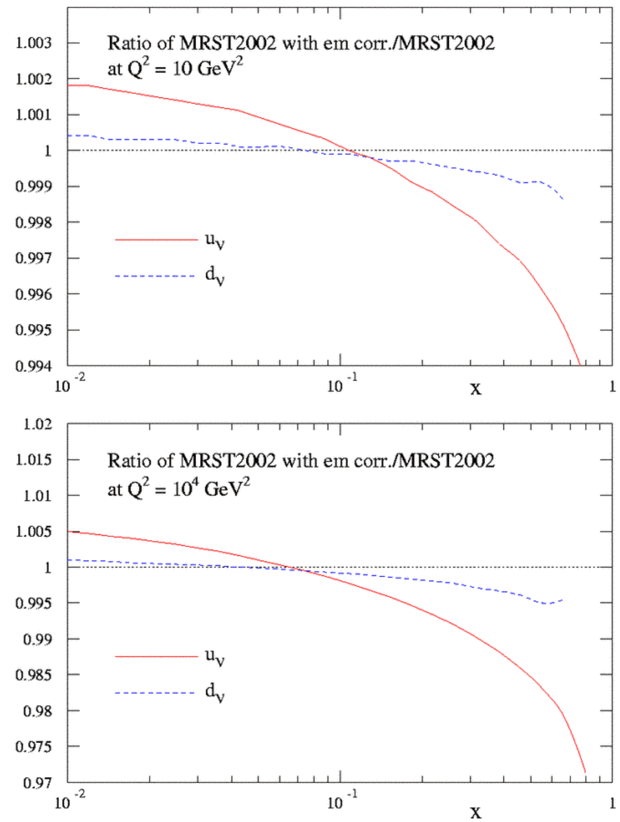
Note: In principle could introduce *different* factorisation scales for QCD, QED subtraction, thus $q(x,\mu^2_{F\mathrm{qcd}},\mu^2_{F\mathrm{qed}})$ etc with DGLAP equations for evolution with respect to each scale

- first quantitative estimate of effect on pdfs by Spiesberger, Phys. Rev. D52, 4936 (1995)



- new study by MRST in progress

Effect of including em corrections to valence quark evolution



comments

- effect on quark distributions is entirely negligible at small x where gluon contribution dominates DGLAP evolution
- at large x, effect only becomes noticeable (percent order) at very large Q^2, where it is equivalent to a slight shift in α_S:

$$\Delta\alpha_S(M_Z^2) \simeq +0.0003$$

cf. world average (global pdf fit) error of

$$\alpha_S^{\rm NLO}(M_Z^2) = 0.1165 \pm 0.002\ \text{(expt.)}\ \pm 0.003\ \text{(theory)}$$

(MRST, hep-ph/0308087)

- dynamic generation of photon parton distribution $\gamma(x, Q^2)$

concluding remarks

- pdf uncertainties: several groups now producing $f(x) \pm \delta f(x)$, but need to understand better the differences between various pdf sets — presumably due to different theoretical assumptions used in the fits (including choice of data fitted)
- focus on $\delta\sigma_{\rm pdf} = \delta\sigma_{\rm pdf,exp} \oplus \delta\sigma_{\rm pdf,th}$ at LHC, Tevatron
- does the DGLAP DIS fit at small x show evidence of higher-order contributions?
- $\sin^2\theta_W$ from νN scattering: subtle effects in parton structure could explain all or part of the apparent discrepancy $\Rightarrow$ more work needed
- QED effects in pdfs: needed for electroweak corrections to hadron collider cross sections — formalism exists for incorporation in existing global fits