Signatures of Non-Equilibrium Cooper Pairing in Ultra-Cold Fermi Gases

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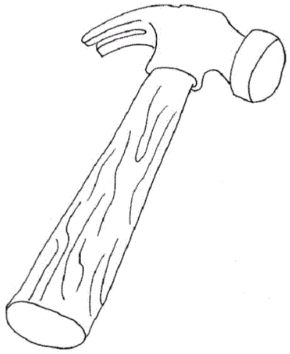
Nonadiabatic dynamics of fermionic condensates?



Fermionic condensate Δ_{eq}

?

$$\Delta(t) = ?$$

perturbation time $\leq \Delta_{eq}^{-1}$ « energy relaxation time τ_ε

(nonadiabatic regime)

~~Conventional nonequilibrium Superconductivity~~

~~Time-dependent Ginzburg-Landau equation~~

~~Boltzmann eqn + selfconsistency eqn for the order parameter~~

} **Adiabatic**

How to describe nonadiabatic dynamics at times of the order of $1/\Delta$?

perturbation time $\leq \Delta_{eq}^{-1}$ « energy relaxation time τ_ε

Difficulty – far from equilibrium.

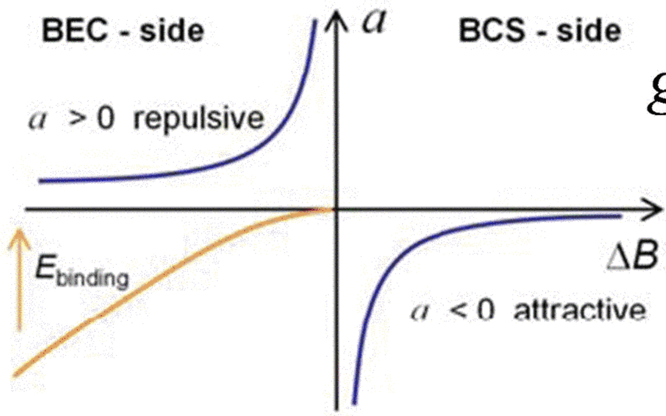
How to describe nonadiabatic dynamics at times of the order of $1/\Delta$?

P.W. Anderson, 1958
V.P. Galaiko, 1972
A. F. Volkov & Sh.M. Kogan, 1974
Yu.M. Galperin, V.I. Kozub & B.Z. Spivak, 1981
V.S. Shumeiko, Doctoral Thesis, 1990

Conventional superconductors are in adiabatic regime

$$\Delta_{eq}^{-1} \approx 100 \text{ ps (Al)}$$

Ultra-cold fermions are in nonadiabatic regime



$$g = k_F \, |a| / \pi v_F$$

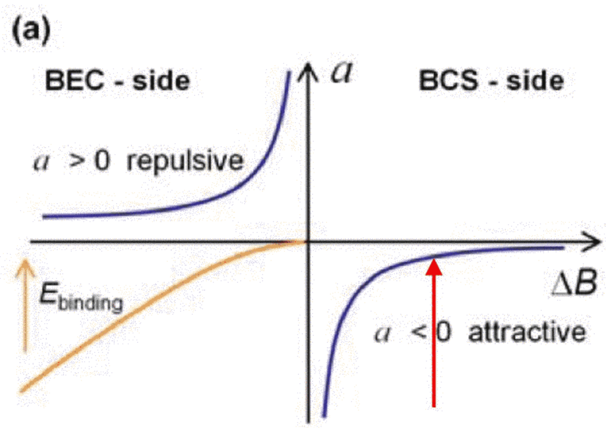
Greiner, Regal &Jin (JILA, ^{40}K)

$g \propto$ scatt. length, $g_i \to g_f$

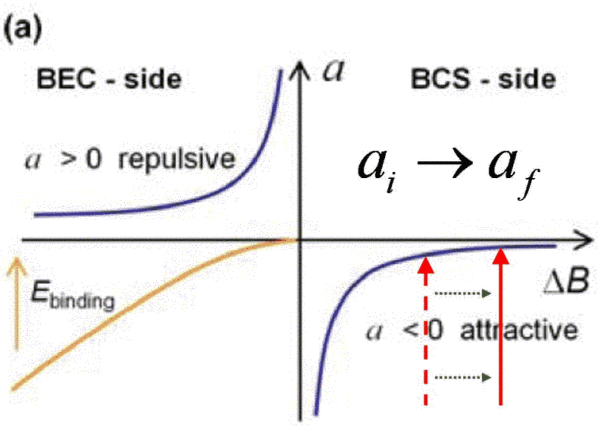
Abrupt change of the BCS coupling constant

^6Li: pert. time « 10 μs, Δ_{eq}^{-1} « 50 μs, τ_ε « 500 μs

Problem

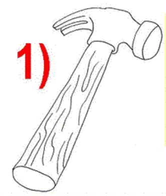


Problem



$$g = k_F \, |a| / \pi \nu_F$$

$$a_i \to a_f$$

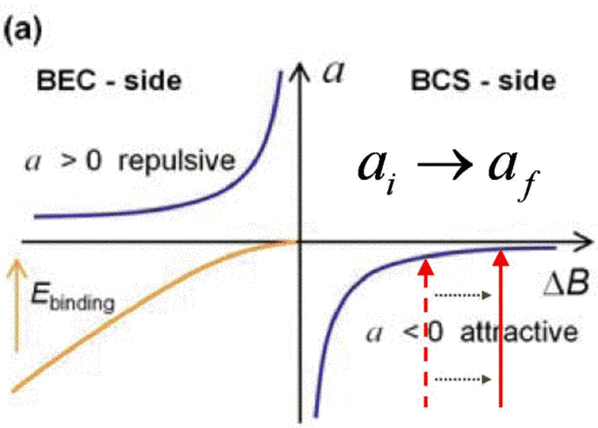


1) $g_i \to g_f$

2) $\Psi_{cond}(t=0) = |\text{Ground state with wrong coupling}\rangle$

3) $\Psi_{cond}(t>0) = ?$ $t \sim \Delta^{-1}$, weak coupling

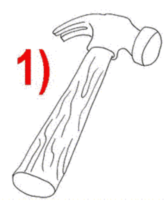
Problem



$$g = 2k_F \, |a| / \pi \nu_F$$

$$a_i \to a_f$$

$$\Delta_{eq} = 2E_F e^{-1/g\nu_F}$$



1) $\Delta_i \to \Delta_f$

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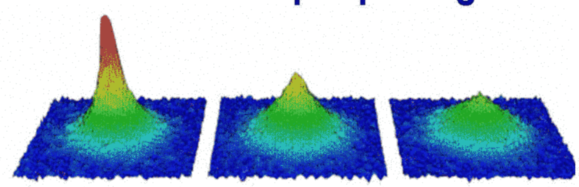
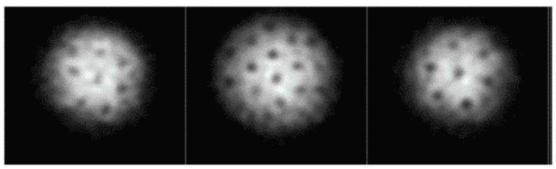
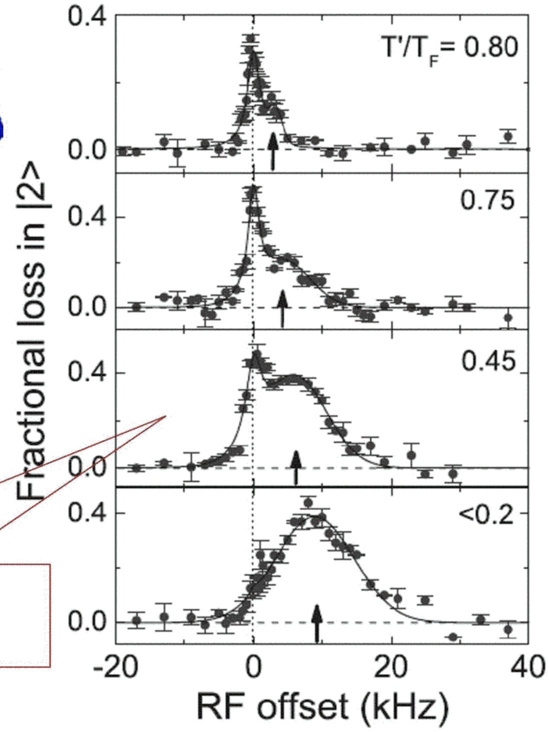
Cooper pairing in cold atomic fermions



Optical images of condensate. Regal, Greiner & Jin '04.



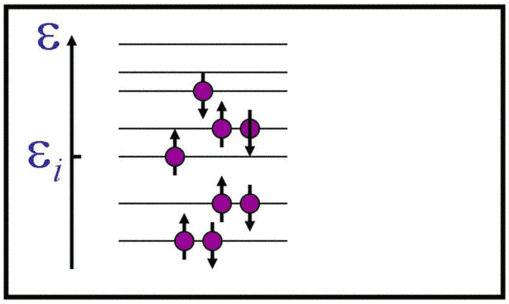
Vortex lattice. Zwierlein et. al. '05



Direct measurement of Δ on BCS side. Chin et. al. '04.

Paired fermions

BCS Hamiltonian. Finite systems



$$\hat{H}_0 = \sum_j \varepsilon_j n_j \qquad \hat{H} = \hat{H}_0 + \hat{H}_{\text{int}}$$

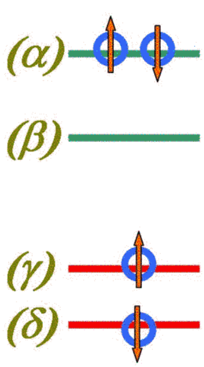
Pair up time-reversed states P. W. Anderson (1959) $|j\uparrow\rangle \quad |j\downarrow\rangle$

$$\hat{H}_{\text{int}} = -g\sum_{i,j} c^+_{i\uparrow}c^+_{i\downarrow}c_{j\uparrow}c_{j\downarrow}$$

At times $t \ll \Delta^{-1} \ll \tau_\varepsilon \approx \varepsilon_F / \Delta^2$ and weak coupling can neglect other interactions

---

$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}} \qquad \hat{H}_0 = \sum_j \varepsilon_j n_j \qquad \hat{H}_{\text{int}} = -g\sum_{i,j} c^+_{i\uparrow}c^+_{i\downarrow}c_{j\uparrow}c_{j\downarrow}$$



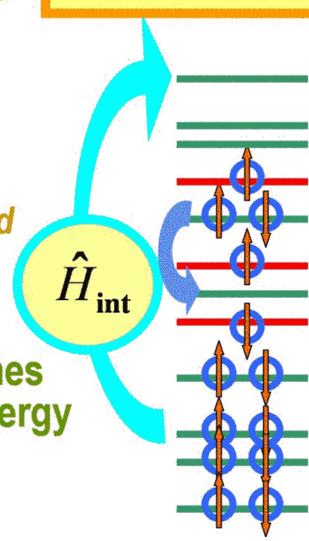
(α) double occupied

(β) empty

(γ) (δ) single occupied

$c^+_{i\uparrow}c^+_{i\downarrow}c_{j\uparrow}c_{j\downarrow}$ mixes (α) and (β)

at the same time $c^+_{j\uparrow}c^+_{j\downarrow}c_{q\uparrow}c_{q\downarrow}$ $(\gamma,\delta) = 0$



This single-occupied states are not effected by the interaction.

They are blocked

The Hilbert space splits into two independent Hilbert subspaces

Blocking effect.

(V.G.Soloviev, 1961)

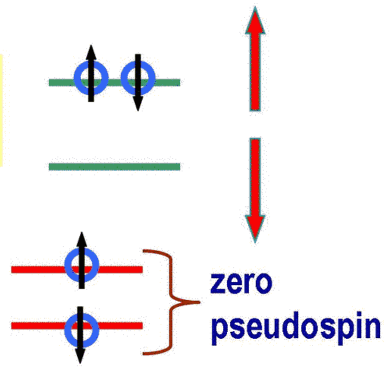
Correct only for times shorter than the energy relaxation time τ_ε

Anderson pseudospins

$$H_{BCS} = \sum_j \varepsilon_j n_j - g \sum_{i,j} c^+_{j\uparrow} c^+_{j\downarrow} c_{i\downarrow} c_{i\uparrow}$$

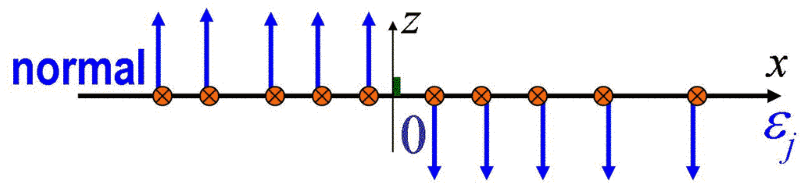
P. W. Anderson (1958).

$$K^z_j = \frac{n_j - 1}{2}; \quad K^+_j = c^+_{j\uparrow} c^+_{j\downarrow}; \quad K^-_j = c_{j\downarrow} c_{j\uparrow}$$



$$H_{BCS} = \sum_j 2\varepsilon_j K^z_j - g \sum_{i,j} K^+_i K^-_j$$

ground states:



Anderson pseudospins

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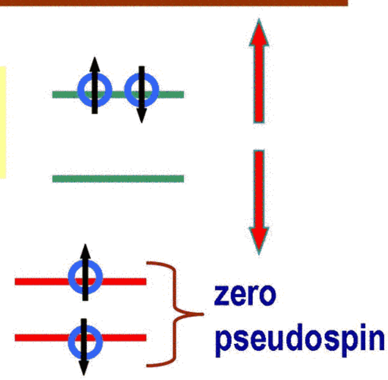

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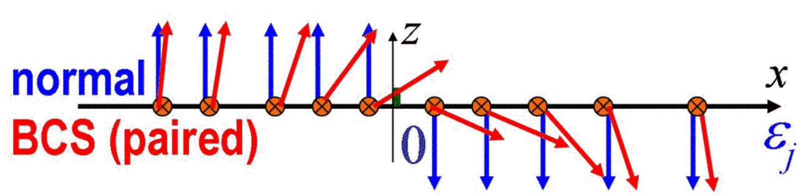


Excitations

$$H_{BCS} = \sum_j 2\varepsilon_j K_j^z - g\sum_{i,j} K_i^+ K_j^-$$

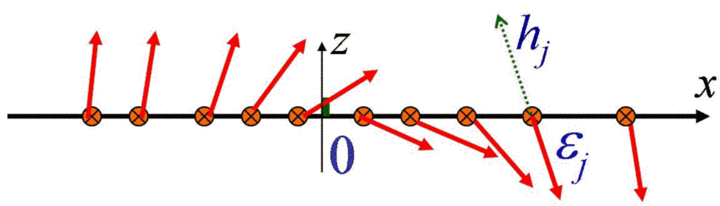
mean-field $H_{BCS} = \sum_j \vec{h}_j \vec{K}_j \qquad \vec{h}_j = (-2\Delta,\ 0,\ 2\varepsilon_j)$

$$\Delta = g\sum_j \langle K_j^x \rangle$$

$$|\vec{h}_j| = 2\sqrt{\varepsilon_j^2 + \Delta^2}$$

Paired ground state

BCS (1958)



$$E_0 = \sum_j \left(-|\vec{h}_j|/2\right) = \sum_j \left(-\sqrt{\varepsilon_j^2 + \Delta^2}\right)$$

Excitations

$$H_{BCS} = \sum_j 2\varepsilon_j K_j^z - g\sum_{i,j} K_i^+ K_j^-$$

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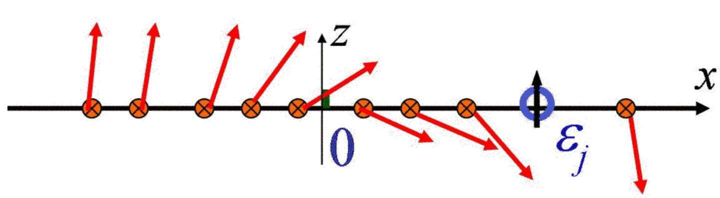
1. single-particle excitations

BCS (1958)



$$E_{\mathrm{ex}} = \sqrt{\varepsilon_j^2 + \Delta^2} \qquad S = 1/2 \qquad Q = e$$

Excitations

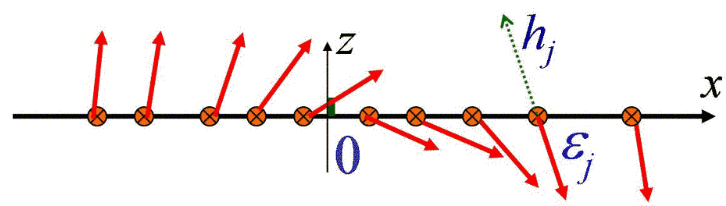
$$H_{BCS} = \sum_j 2\varepsilon_j K_j^z - g \sum_{i,j} K_i^+ K_j^-$$

mean-field $H_{BCS} = \sum_j \vec{h}_j \vec{K}_j \qquad \vec{h}_j = (-2\Delta,\ 0,\ 2\varepsilon_j)$

$$\Delta = g \sum_j \langle K_j^x \rangle \qquad |\vec{h}_j| = 2\sqrt{\varepsilon_j^2 + \Delta^2}$$

2. Excited pairs (pseudospin flips)

BCS (1958)



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2. Excited pairs (pseudospin flips)

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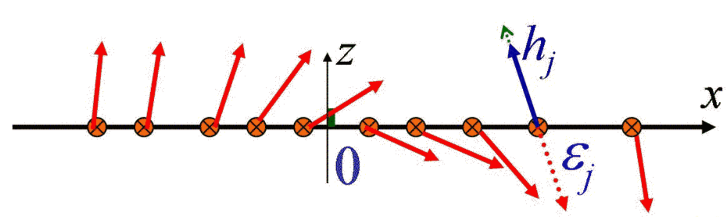


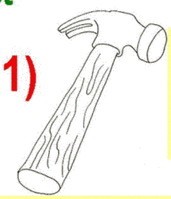
How to detect!?

$$E_{\mathbf{ex}} = 2\sqrt{\varepsilon_j^2 + \Delta^2} \qquad S = 0 \qquad Q = 0$$

Time evolution in non-adiabatic regime

$$H_{BCS} = \sum_j 2\varepsilon_j K_j^z - g \sum_{i,j} K_i^+ K_j^-$$

single-particle levels — coupling const

1)  $g_i \to g_f$

2) $\Psi_{cond}(t=0) = |\text{Ground state with wrong coupling } g_i\rangle$

3) $\Psi_{cond}(t>0) = ?$ i.e. solve time-dependent Shrodinger equation for *H*

Time evolution in non-adiabatic regime

$$H_{BCS} = \sum_j 2\varepsilon_j K_j^z - g \sum_{i,j} K_i^+ K_j^- \qquad \Delta = g\sum_j \langle K_j^x \rangle$$

$$\frac{d\mathbf{K}_j}{dt} = i\left[H_{BCS}, \mathbf{K}_j\right] = \left(-2g\sum_j K_j^x,\ -2g\sum_j K_j^y,\ 2\varepsilon_j\right) \times \mathbf{K}_j$$

Nonlinear, many-body, far from equilibrium – normally would be intractable analytically

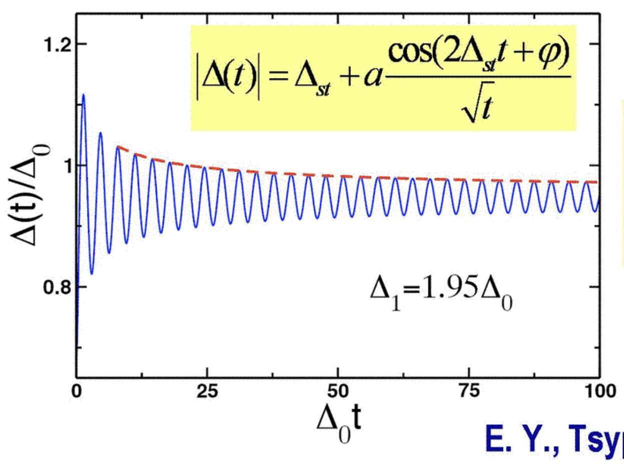
H_{BCS} is integrable (Richardson &Sherman (1964), Gaudin (1972)) – infinitely many integrals of motion.

Exact solution for dynamics in $n \to \infty$ limit (E.Y., Altshuler, Kuznetsov , Enolskii *PRB* (2004))

I. "Perturbative" regime $e^{-\pi/2} < \frac{\Delta_f}{\Delta_i} < e^{\pi/2}$

$g_i = g_f$ → $|\Delta(t)| = \Delta_i = \Delta_f$

This type of behavior persists for $e^{-\pi/2} < \frac{\Delta_f}{\Delta_i} < e^{\pi/2}$



$|\Delta(t)| \to \Delta_{st}$

$$\Delta_{st} = \Delta_i \cos\eta < \Delta_f$$

$$\exp[-\eta\tan(\eta/2)] = \frac{\Delta_f}{\Delta_i}$$

$|\eta| \le \pi/2$

E. Y., Tsyplyatyev, Altshuler *PRL* (2006)

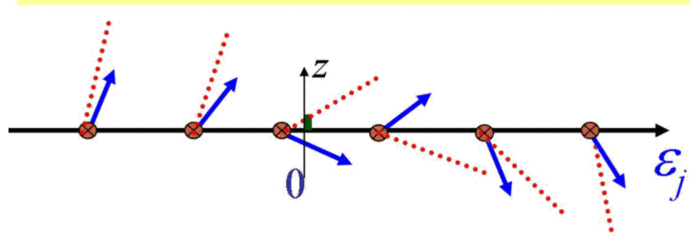
I. "Perturbative" regime $e^{-\pi/2} < \frac{\Delta_f}{\Delta_i} < e^{\pi/2}$

$|\Delta(t)| \to \Delta_{st}$ **What is the nature of the steady state?**

Each spin rotates in a constant magnetic field

$$\frac{d\mathbf{K}_j}{dt} = (-2\Delta_{st}, 0, 2\varepsilon_j) \times \mathbf{K}_j \qquad \omega(\varepsilon) = 2\sqrt{\varepsilon^2 + \Delta_{st}^2}$$

$$\Delta(t) = g\sum_j \langle K_j^x \rangle = \text{const} + \int_{-\infty}^{\infty} a(\omega)\cos\omega t\, d\varepsilon \to \text{const}$$



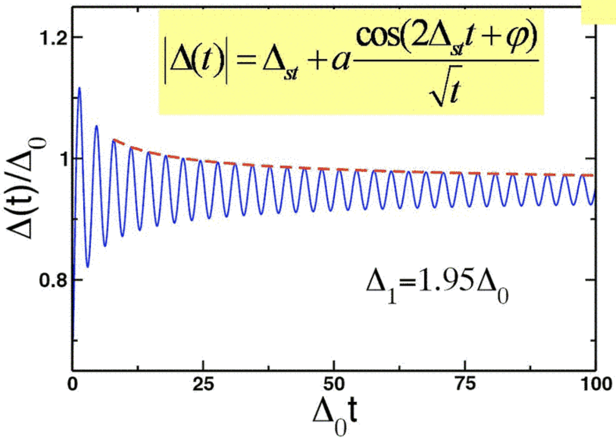
Dephasing similar to inhomogeneous line broadening in NMR

I. "Perturbative" regime $e^{-\pi/2} < \frac{\Delta_f}{\Delta_i} < e^{\pi/2}$

$$|\Delta(t)| \to \Delta_{st}$$

$$\Delta_{st} = \Delta_i \cos\eta \quad < \Delta_f$$

$$\exp\left[-\eta \tan(\eta/2)\right] = \frac{\Delta_f}{\Delta_i}$$



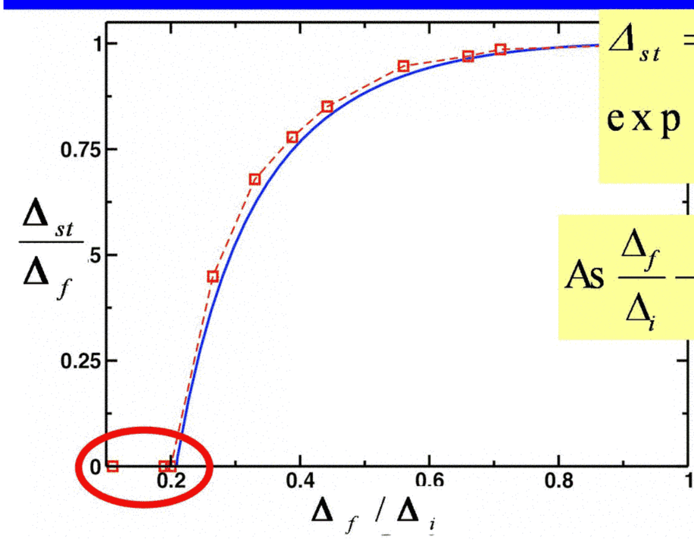
$$\omega(\varepsilon) = 2\sqrt{\varepsilon^2 + \Delta_{st}^2}$$

$$d\varepsilon = \frac{\omega\, d\omega}{2\sqrt{\omega^2 - (2\Delta_{st})^2}}$$

Decay law set by sqrt singularity

Non-stationary analog of square root singularity

Dynamical "phase transition" at $\left(\frac{\Delta_f}{\Delta_i}\right)_c = e^{-\pi/2} \approx 0.21$



$$\Delta_{st} = \Delta_i \cos\eta \quad < \Delta_f$$

$$\exp\left[-\eta \tan(\eta/2)\right] = \frac{\Delta_f}{\Delta_i}$$

As $\frac{\Delta_f}{\Delta_i} \to e^{-\pi/2}$ $\quad \eta \to \frac{\pi}{2}$ and $\Delta_{st} \to 0$!!

$E_n - E_{sf} \propto \Delta_f^2$ decreases

$E_i - E_{sf}$ increases

For $\frac{\Delta_f}{\Delta_i} \le e^{-\pi/2} \approx 1/5, \quad \Delta_{st} = 0$!!

E. Y. & Dzero: *PRL* (2006)

Barankov & Levitov *PRL* (2006)

$$|\Delta(t)| \to 0$$

Each spin rotates in a constant magnetic field along z axis

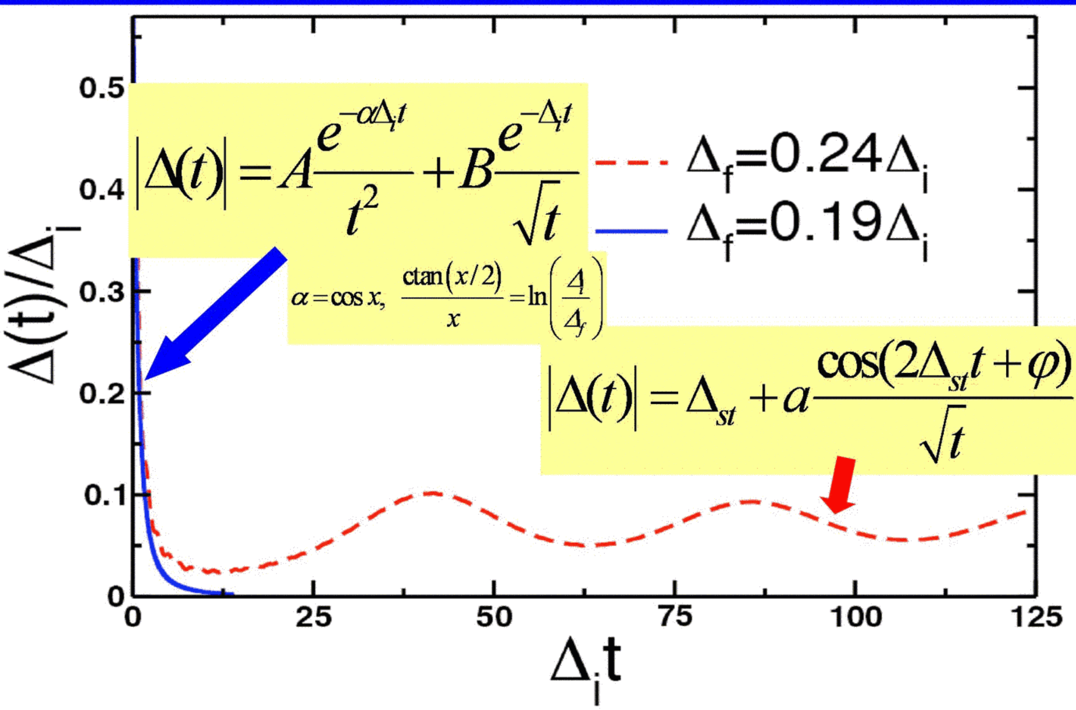
$$\frac{d\mathbf{K}_j}{dt} = (0, 0, 2\varepsilon_j) \times \mathbf{K}_j \qquad \omega(\varepsilon) = 2\varepsilon$$

$$\Delta(t) = g\sum_j \left\langle K_j^x \right\rangle = \int_{-\infty}^{\infty} a(\omega)\cos\omega t \, d\varepsilon \propto e^{-\alpha\Delta_i t}$$

Linear dispersion, no square root singularity

Exponential decay

Dynamical "phase transition" at $\left(\frac{\Delta_f}{\Delta_i}\right)_c = e^{-\pi/2} \approx 0.21$

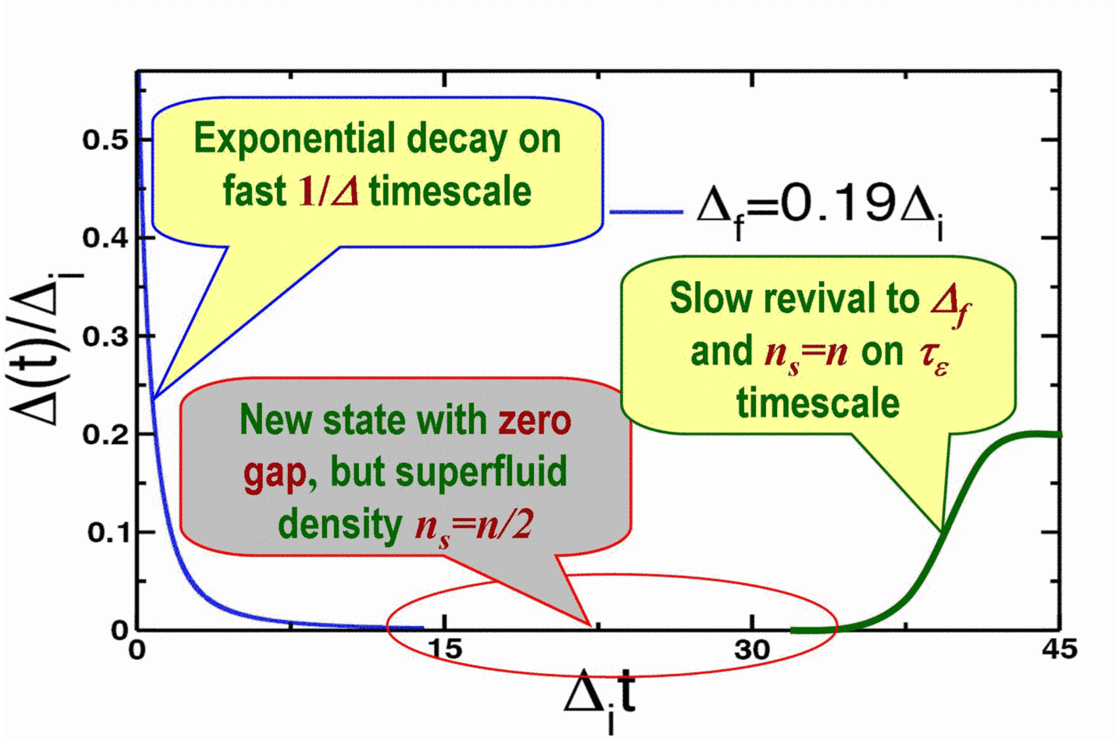


$$|\Delta(t)| = A\frac{e^{-\alpha\Delta_i t}}{t^2} + B\frac{e^{-\Delta_i t}}{\sqrt{t}}$$

$$\alpha = \cos x, \quad \frac{\mathrm{ctan}(x/2)}{x} = \ln\left(\frac{\Delta_i}{\Delta_f}\right)$$

$$|\Delta(t)| = \Delta_{st} + a\frac{\cos(2\Delta_{st} t + \varphi)}{\sqrt{t}}$$

II. Gapless regime $\Delta_f / \Delta_i < e^{-\pi/2} \approx 1/5$



What happens when Δ_f / Δ_i is increased?

$\Delta_f / \Delta_i = \infty$, i.e $\Delta_i = 0$

$$\frac{d\mathbf{K}_j}{dt} = \left(-2g\sum_j K_j^x,\ -2g\sum_j K_j^y,\ 2\varepsilon_j \right) \times \mathbf{K}_j$$

Linear analysis around the normal state

normal modes $\omega_j = 2\varepsilon_j$ unstable mode $\omega_0 = i\Delta_f$

$$|\Delta(t)| = \int_{-E_F}^{E_F} A(\varepsilon)\cos(2\varepsilon t + \varphi)d\varepsilon + A_0 e^{\Delta_f t} \approx A_0 e^{\Delta_f t}$$

What happens when Δ_f / Δ_i is increased?

$$\Delta_f / \Delta_i > e^{\pi/2}$$

$$\frac{d\mathbf{K}_j}{dt} = \left(-2g\sum_j K_j^x,\ -2g\sum_j K_j^y,\ 2\varepsilon_j \right) \times \mathbf{K}_j$$

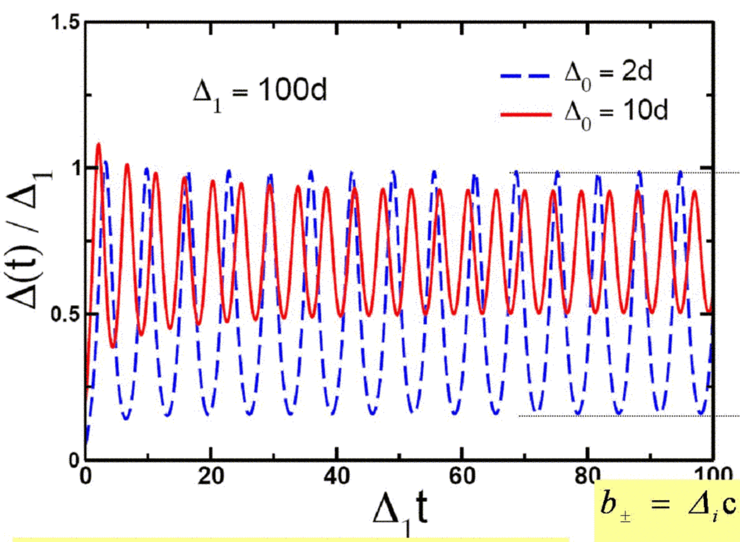
Nonlinear analysis

$$|\Delta(t)| = \int_{-E_F}^{E_F} A(\omega)\cos(\omega t + \varphi)d\omega + \sum_{n=1}^{\infty} B_n \cos(n\omega_0 t + \varphi_n)$$

vanishes for $t \ll 1/\Delta_f$ **periodic!**

$|\Delta(t)| \to$ periodic oscillations for $\Delta_f / \Delta_i > e^{\pi/2}$

Dynamical transition to oscillating order parameter



$$\Delta_+ = \frac{b_+ + b_-}{2}$$

$$\Delta_- = \frac{b_+ - b_-}{2}$$

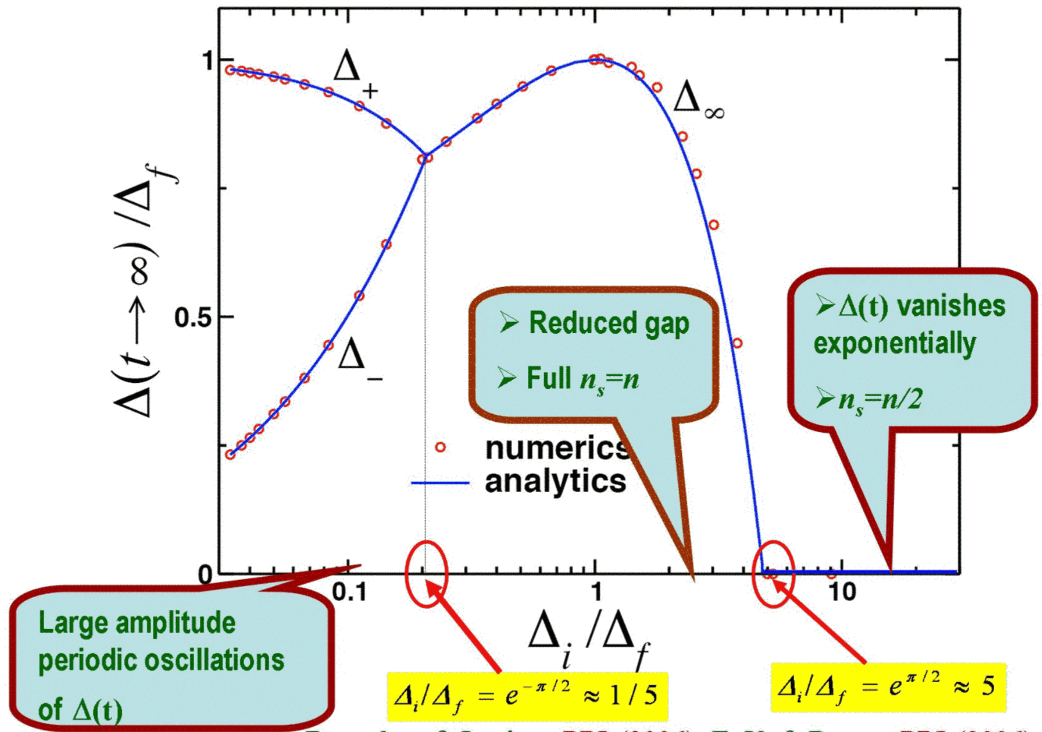
$$b_\pm = \Delta_i \cosh\eta_\pm < \Delta_f$$

$$\exp[\eta_+ \tanh(\eta_+/2)] = \frac{\Delta_f}{\Delta_i}$$

$$\exp[\eta_- \coth(\eta_-/2)] = \frac{\Delta_f}{\Delta_i}$$

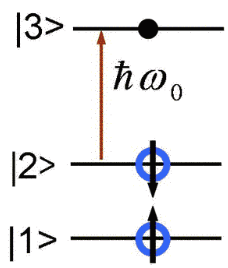
$$|\Delta(t)| \to \Delta_+ \mathrm{dn}[\Delta_+(t-t_0), k] \quad k = \Delta_+^2 / \Delta_-^2$$

Time evolution in non-adiabatic regime



Barankov & Levitov *PRL* (2006); E. Y. & Dzero: *PRL* (2006)

RF Spectroscopy

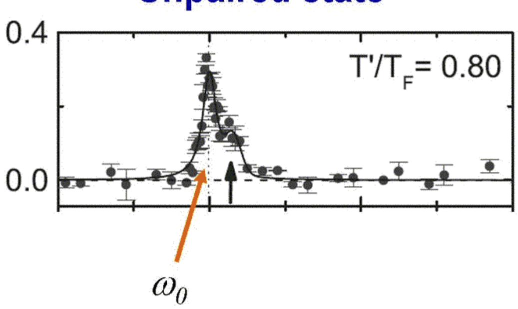


^6Li $|1\rangle = |F=1/2, m_F=1/2\rangle$ $|2\rangle = |F=1/2, m_F=-1/2\rangle$
$|3\rangle = |F=3/2, m_F=-3/2\rangle$

Measure loss of atoms in state |2>

$$I = -\frac{dN_2}{dt}$$

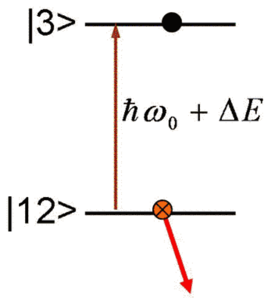
Unpaired state



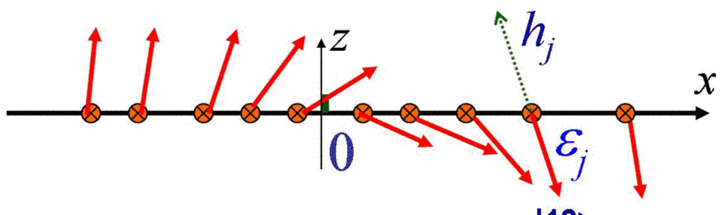
sharp peak at $\omega_0 = \dfrac{E_3 - E_2}{\hbar}$

RF Spectroscopy

Paired ground state

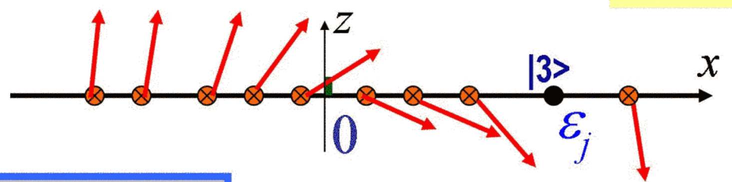


$E_{in} = -h_j/2 = -\sqrt{\varepsilon_j^2 + \Delta^2}$



$\Delta E = E_f - E_{in} = \varepsilon_j + \sqrt{\varepsilon_j^2 + \Delta^2}$

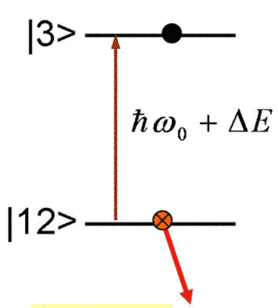
$E_f = \varepsilon_j$



$(\Delta E)_{min} = -\varepsilon_F + \sqrt{\varepsilon_F^2 + \Delta^2} \approx \Delta^2 / 2\varepsilon_F$

RF Spectroscopy

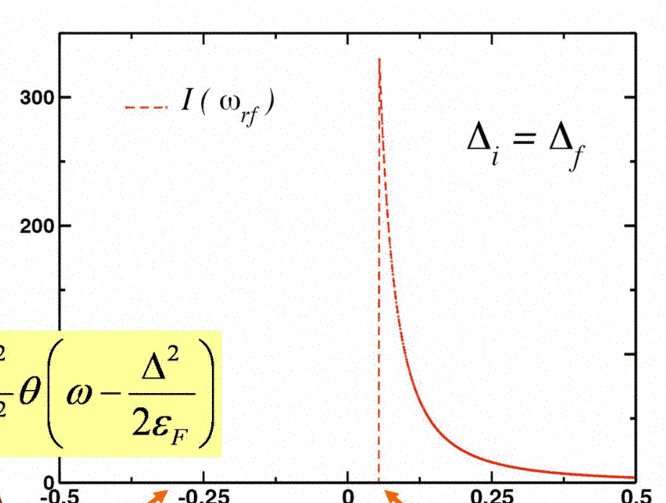
Paired ground state



$I = \frac{dN_2}{dt}$

$I_{eq}(\omega) = 2\pi |T|^2 \frac{\Delta^2}{\omega^2} \theta\left(\omega - \frac{\Delta^2}{2\varepsilon_F}\right)$

Torma & Zoller *PRL* (2000)



no loss at negative ω

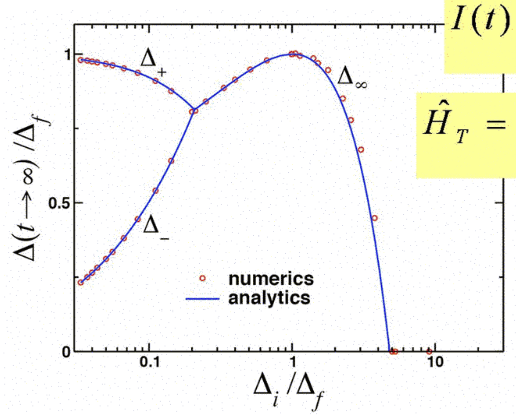
$(\Delta E)_{min} \approx \Delta^2 / 2\varepsilon_F$

RF Spectroscopy out of equilibrium

$$I(t) = -\frac{d\langle \hat{N}_2 \rangle}{dt} = i\int_{-\infty}^{t} \langle [\hat{H}_T(t'), \hat{I}(t)] \rangle dt'$$

$$\hat{H}_T = \sum_{ij} T_{ij} c^+_{i3} c_{i2} + h.c.$$

average w.r.t. $\Psi_{cond}(t)$



Exact solution for BCS dynamics

Can evaluate $I(t)$ exactly

I. "Perturbative" regime $e^{-\pi/2} < \frac{\Delta_f}{\Delta_i} < e^{\pi/2}$

$|\Delta(t)| \to \Delta_{st}$

$$I(\omega) = 2\pi |T|^2 \frac{\Delta^2}{\omega^2}\left[\sin^2\chi(\omega)\, \theta\left(\omega - \Delta^2/2\varepsilon_F\right) + \cos^2\chi(\omega)\theta\left(-\omega-\Delta^2/2\varepsilon_F\right)\right]$$

$$I_{eq}(\omega) = 2\pi |T|^2 \frac{\Delta^2}{\omega^2}\theta\left(\omega - \frac{\Delta^2}{2\varepsilon_F}\right)$$

$$\sin^2 2\chi(\epsilon) = \frac{G(\epsilon)}{2\pi^2} - \sqrt{\frac{G^2(\epsilon)}{4\pi^4} - \frac{4\beta^2\Delta_i^2}{\pi^2(\epsilon^2+\Delta_i^2)}},$$

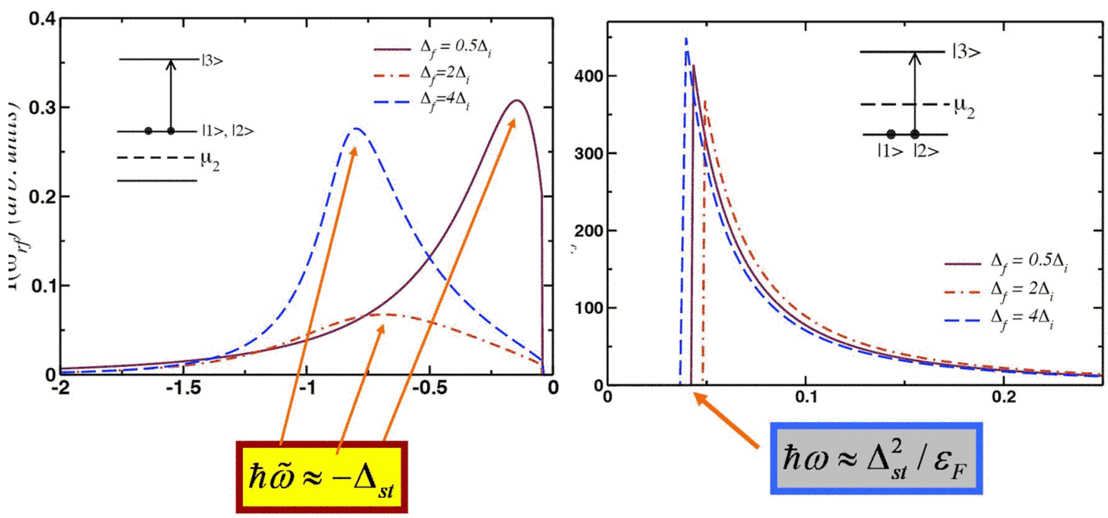
where $\beta = \ln(\Delta_i/\Delta_f)$ and

$$G(\epsilon) = \pi^2 + 4\left[\sinh^{-1}(\epsilon/\Delta_i)\right]^2 + \frac{8\beta\epsilon}{\sqrt{\epsilon^2+\Delta_i^2}}\sinh^{-1}(\epsilon/\Delta_i) + 4\beta^2$$

I. "Perturbative" regime $e^{-\pi/2} < \frac{\Delta_f}{\Delta_i} < e^{\pi/2}$

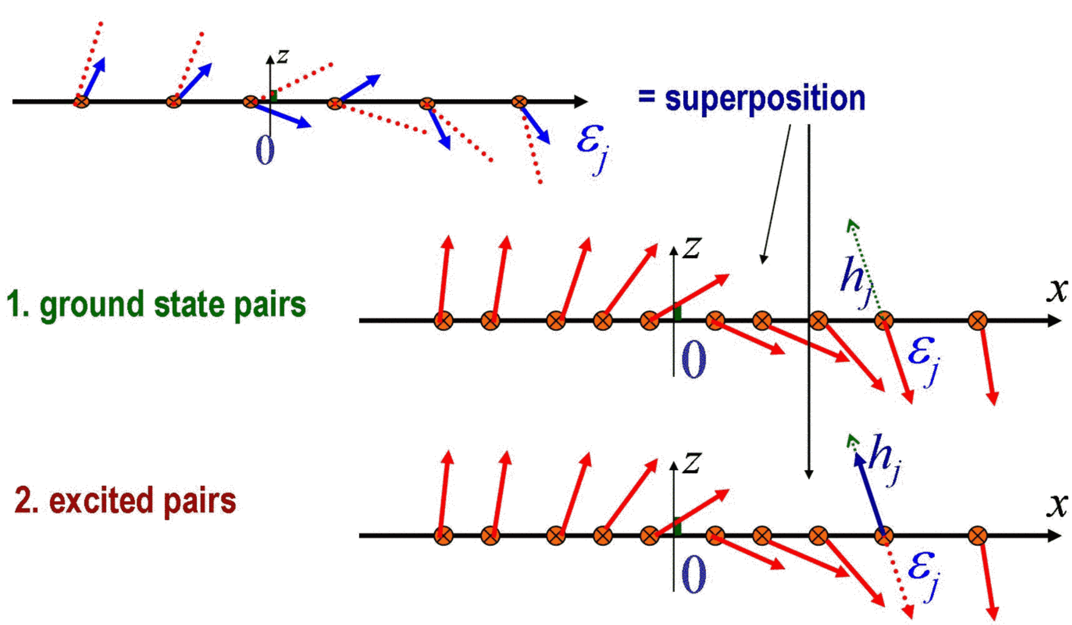
$|\Delta(t)| \to \Delta_{st}$

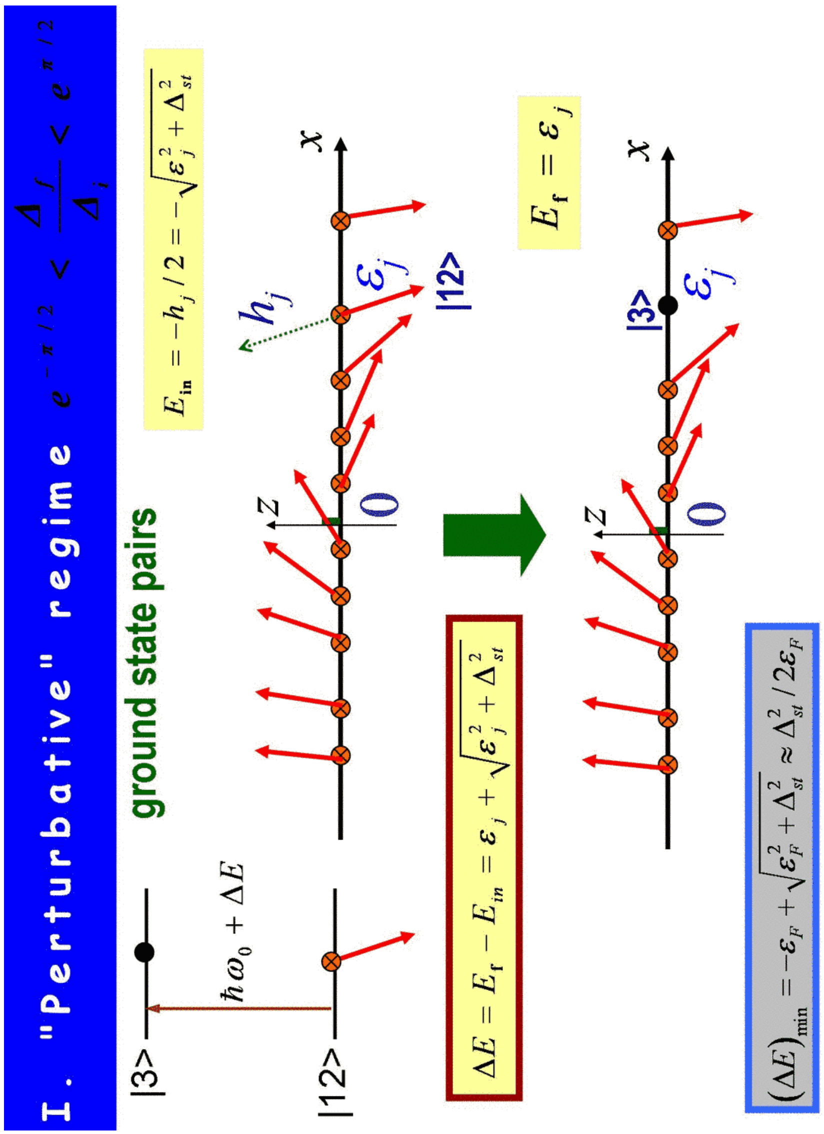
$$I(\omega) = 2\pi |T|^2 \frac{\Delta^2}{\omega^2}\left[\sin^2\chi(\omega)\,\theta\left(\omega - \Delta^2/2\varepsilon_F\right) + \cos^2\chi(\omega)\theta\left(-\omega - \Delta^2/2\varepsilon_F\right)\right]$$

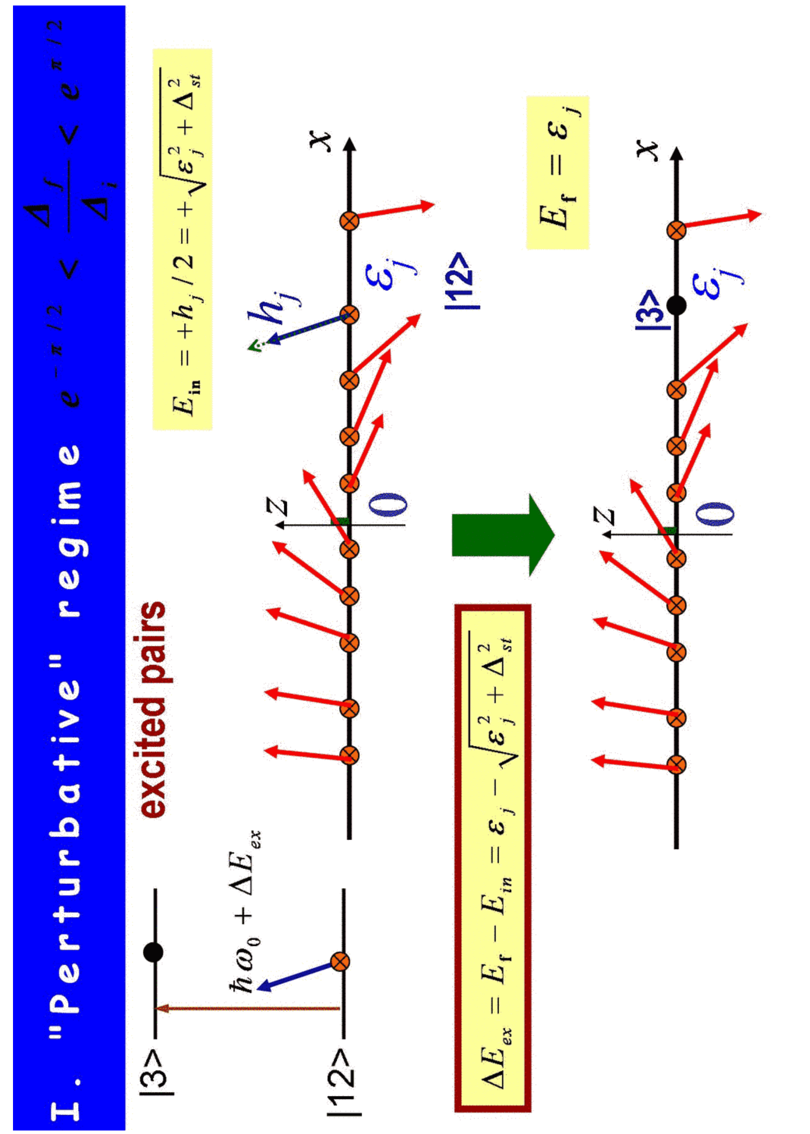


I. "Perturbative" regime $e^{-\pi/2} < \frac{\Delta_f}{\Delta_i} < e^{\pi/2}$

Each spin rotates in a constant magnetic field



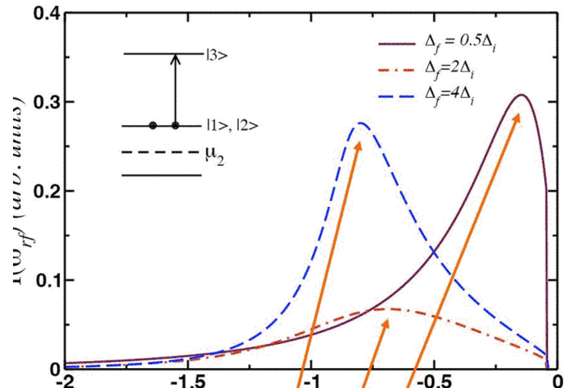

I. "Perturbative" regime $e^{-\pi/2} < \frac{\Delta_f}{\Delta_i} < e^{\pi/2}$
ground state pairs
|3>
|12>
$\hbar\omega_0 + \Delta E$
$E_{\rm in} = -h_j/2 = -\sqrt{\varepsilon_j^2 + \Delta_{st}^2}$
x
Z
0
h_j
ε_j
|12>
$E_{\rm f} = \varepsilon_j$
|3>
$\Delta E = E_{\rm f} - E_{in} = \varepsilon_j + \sqrt{\varepsilon_j^2 + \Delta_{st}^2}$
$(\Delta E)_{\min} = -\varepsilon_F + \sqrt{\varepsilon_F^2 + \Delta_{st}^2} \approx \Delta_{st}^2 / 2\varepsilon_F$

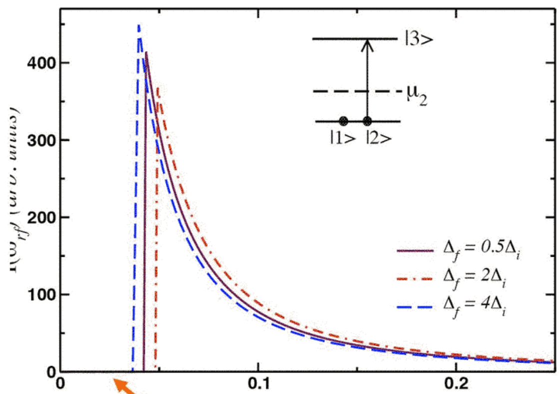

I. "Perturbative" regime $e^{-\pi/2} < \frac{\Delta_f}{\Delta_i} < e^{\pi/2}$
excited pairs
|3>
|12>
$\hbar\omega_0 + \Delta E_{ex}$
$E_{\rm in} = +h_j/2 = +\sqrt{\varepsilon_j^2 + \Delta_{st}^2}$
x
Z
0
h_j
ε_j
|12>
$E_{\rm f} = \varepsilon_j$
|3>
$\Delta E_{ex} = E_{\rm f} - E_{in} = \varepsilon_j - \sqrt{\varepsilon_j^2 + \Delta_{st}^2}$

I. "Perturbative" regime $e^{-\pi/2} < \frac{\Delta_f}{\Delta_i} < e^{\pi/2}$

$|\Delta(t)| \to \Delta_{st}$

for $\Delta_{st} = 0$ - one peak as in normal state

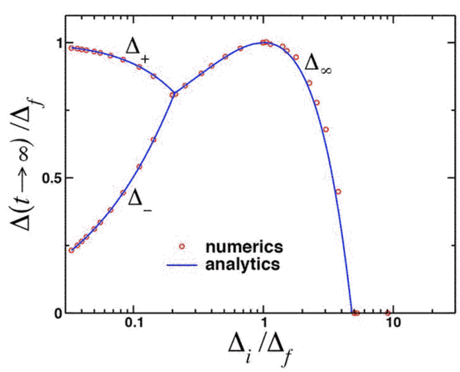




$\hbar\tilde{\omega} \approx -\Delta_{st}$

$\hbar\omega \approx \Delta_{st}^2 / 2\varepsilon_F$

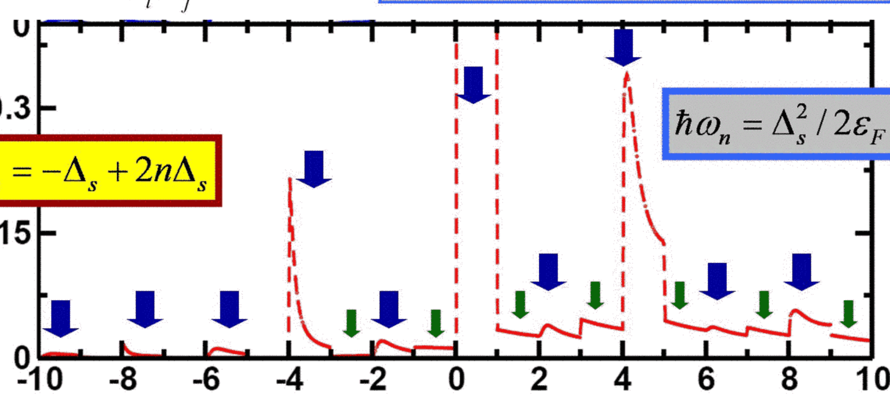
Regime of oscillating order parameter



$\Delta(t) \to \Delta_+ \mathrm{dn}[\Delta_+(t-t_0), k] \quad k = \Delta_+^2/\Delta_-^2$

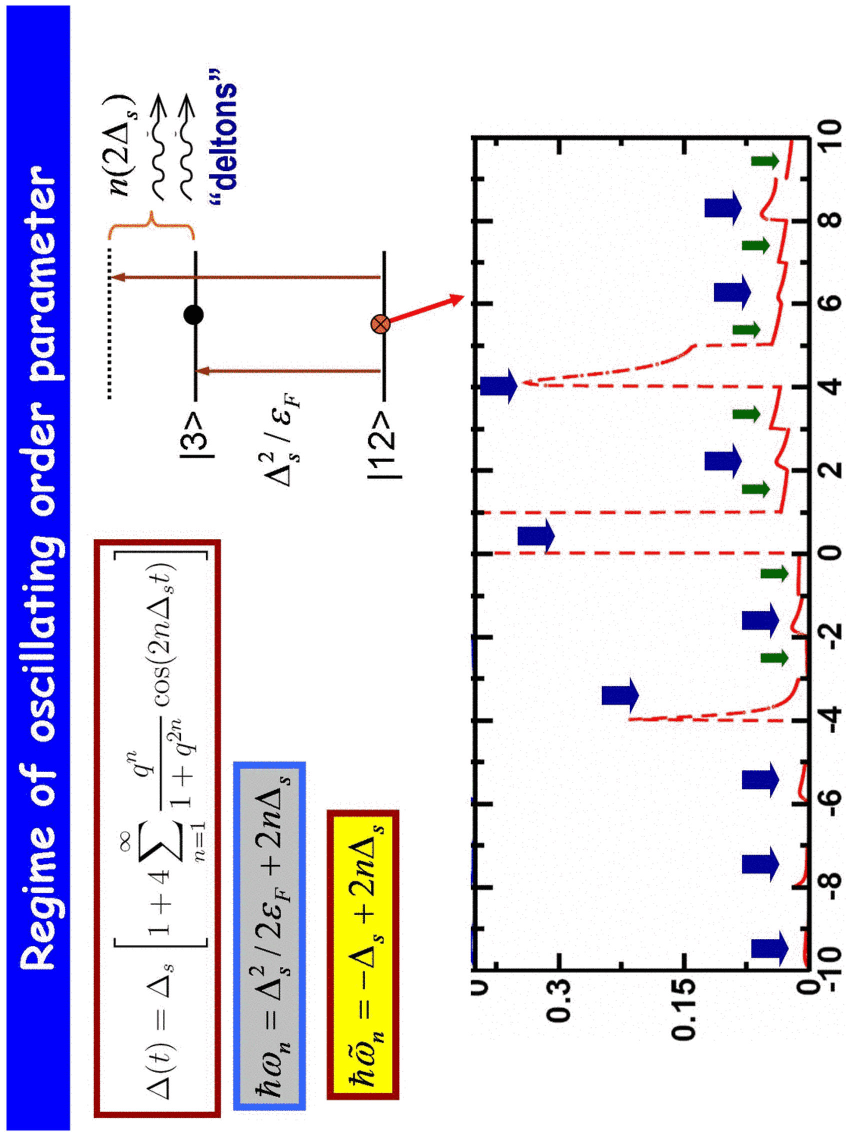
$$\Delta(t) = \Delta_s \left[1 + 4 \sum_{n=1}^{\infty} \frac{q^n}{1+q^{2n}} \cos(2n\Delta_s t) \right]$$

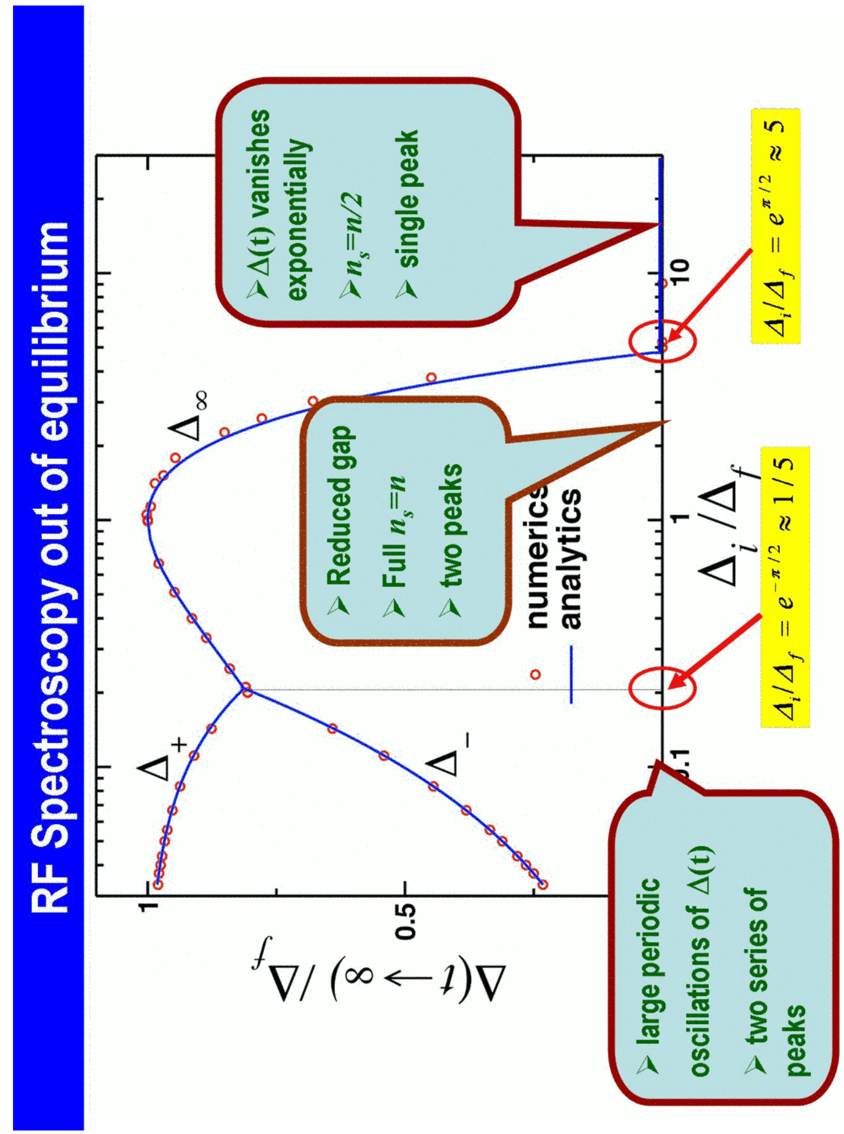
two series of equidistant peaks!



$\hbar\tilde{\omega}_n = -\Delta_s + 2n\Delta_s$

$\hbar\omega_n = \Delta_s^2 / 2\varepsilon_F + 2n\Delta_s$


Regime of oscillating order parameter
$\Delta(t) = \Delta_s \left[1 + 4\sum_{n=1}^{\infty} \frac{q^n}{1+q^{2n}} \cos(2n\Delta_s t) \right]$
$\hbar\omega_n = \Delta_s^2 / 2\varepsilon_F + 2n\Delta_s$
$\hbar\tilde{\omega}_n = -\Delta_s + 2n\Delta_s$
$n(2\Delta_s)$
"deltons"
|3>
Δ_s^2/ε_F
|12>


RF Spectroscopy out of equilibrium
Δ(t) vanishes exponentially
$n_s = n/2$
single peak
Reduced gap
Full $n_s = n$
two peaks
large periodic oscillations of Δ(t)
two series of peaks
numerics
analytics
Δ_∞
Δ_+
Δ_-
$\Delta(t\to\infty)/\Delta_f$
Δ_i/Δ_f
$\Delta_i/\Delta_f = e^{\pi/2} \approx 5$
$\Delta_i/\Delta_f = e^{-\pi/2} \approx 1/5$