Fluctuating quantum spin nematics

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Quantum magnetism of Mott insulators

Electronic Mott insulators - charges localize below some energy scale "U"

Active low energy degree of freedom - electron spin

Fate of local moments at low temperature??

Effective Hamiltonian $H_{eff} \approx J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + \underbrace{\cdots\cdots}_{\text{longer range interaction, ring exchange, etc.}}$

(Typically $J \sim \frac{t^2}{U} > 0$;

$t \sim$ electron hopping)

Mott insulators of S = 1 bosons

Yip '02

Imambekov et. al. '03

Focus on odd # of bosons per site

Low energy physics : $S=1$ quantum magnet

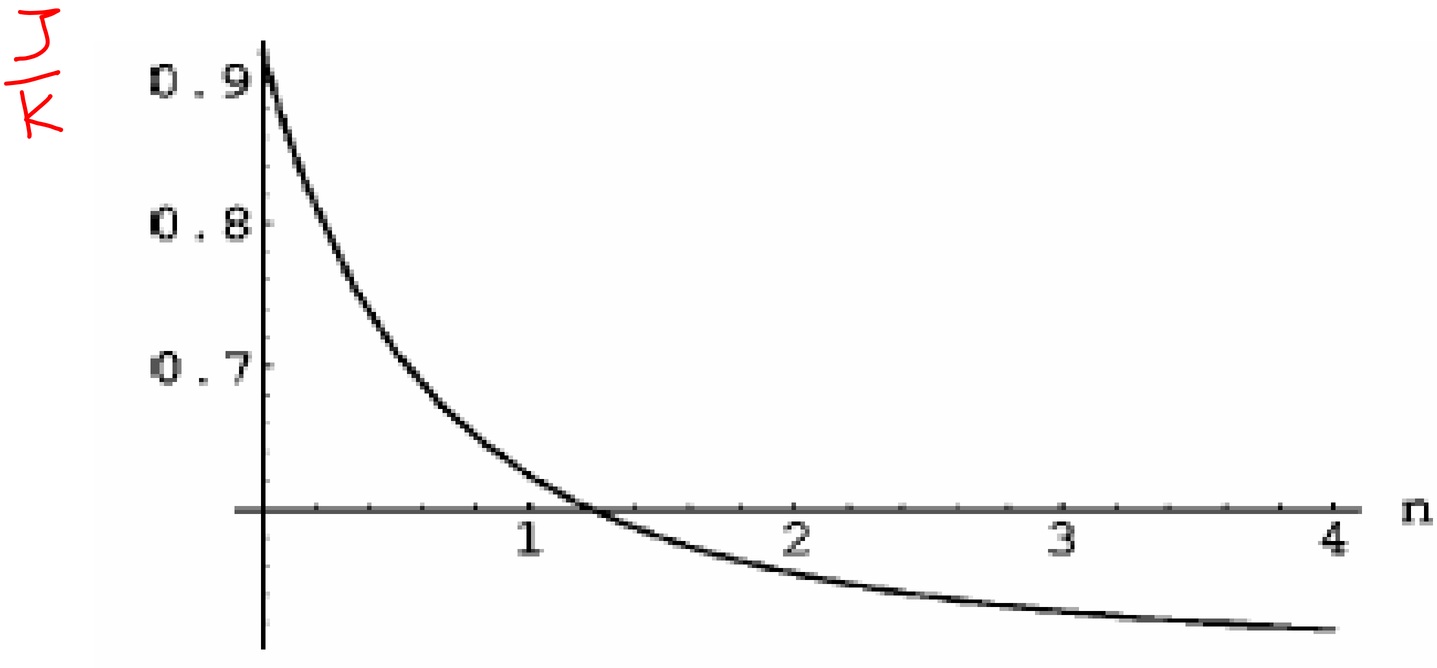
To leading order in t/U ,

$$H_{eff} \approx -J \sum_{\langle rr' \rangle} \vec{S}_r \cdot \vec{S}_{r'} - K \sum_{\langle rr' \rangle} \left(\vec{S}_r \cdot \vec{S}_{r'} \right)^2$$

Biquadratic term natural for bosons :

Tuning J/K in optical lattices of Na^{23}

Imambekov, Demler, Lukin '03



# of particles per site $N = 2n+1$

Why are bosons interesting for quantum magnetism?

1. Large biquadratic term

$\Rightarrow$ easy access to phenomena not so commonly seen in electronic quantum magnets

2. Possible route to exotic phenomena (quantum spin liquids, Landau-forbidden quantum phase transitions, ...) in natural models

The problem

$$H = -\sum_{\langle rr'\rangle} J_{rr'}\, \vec{S}_r \cdot \vec{S}_{r'} \; - \sum_{\langle rr'\rangle} K_{rr'} \left(\vec{S}_r \cdot \vec{S}_{r'}\right)^2$$

?? Properties in various dimensions ??

$J \gtrsim K$: Ferromagnetic order

More interesting physics when $\left| J/K \right| \lesssim o(1)$

natural for bosonic Mott insulators.

Spin nematic ordering in d = 2,3

Mean field theory : Biquadratic term

(Chen, Levy '73)

favors spin nematic order

$$Q_{\alpha\beta} = \left\langle \frac{S_\alpha S_\beta + S_\beta S_\alpha}{2} - \frac{2\delta_{\alpha\beta}}{3} \right\rangle \neq 0$$

though $\langle \vec{S} \rangle = 0$.

Confirmed through Monte Carlo calculations.

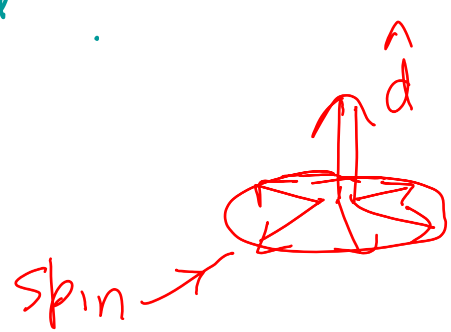
(Harada, Kawashima '02, '03)

Physics of the spin nematic

Spontaneous single-ion anisotropy without magnetic ordering

Breaks spin rotation but not time reversal

Spins fluctuate in a single plane perpendicular to a spontaneous hard axis $\hat{d}$.



??``Disordered spin nematic" in one dimension??

In $d=1$ spin nematic order is unstable to quantum fluctuations.

Old proposal: Featureless gapped paramagnetic ground state ("disordered spin nematic")

(Chubukov '91)

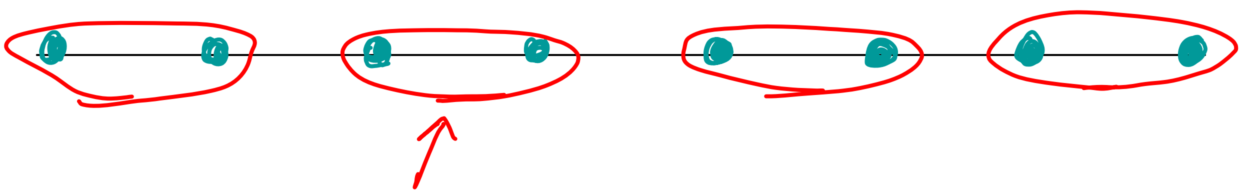
- analagous to Haldane phase of antiferro $S=1$ chains.

(Lauchli et. al. '05)

However no evidence for such a state in numerics!

Dimer order in d= 1

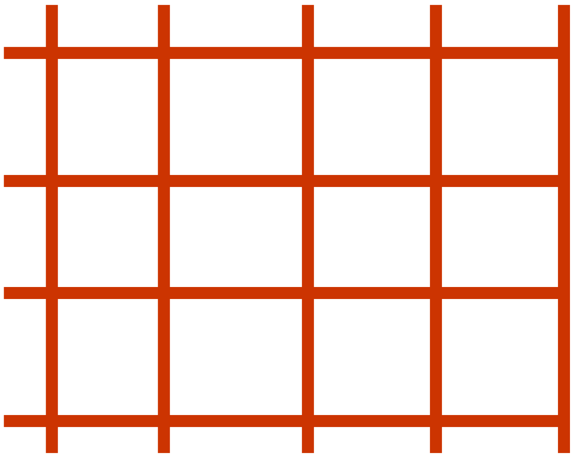
- True ground state: a dimerized valence bond solid paramagnet



singlet formed from the 2 spins

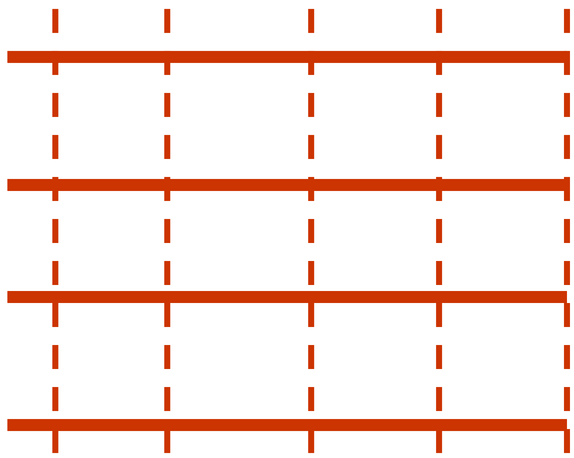
Dimer order spontaneously breaks lattice translation symmetry

Towards one dimension: killing the spin nematic



Towards one dimension: killing the spin nematic

(Harada, Kawashima, Troyer '07)



Weaken vertical bonds by a factor λ relative to horizontal ones

$\lambda \searrow \Rightarrow$ Increase quantum fluctuations

Eventually lose nematic order beyond some critical λ.

Phase diagram

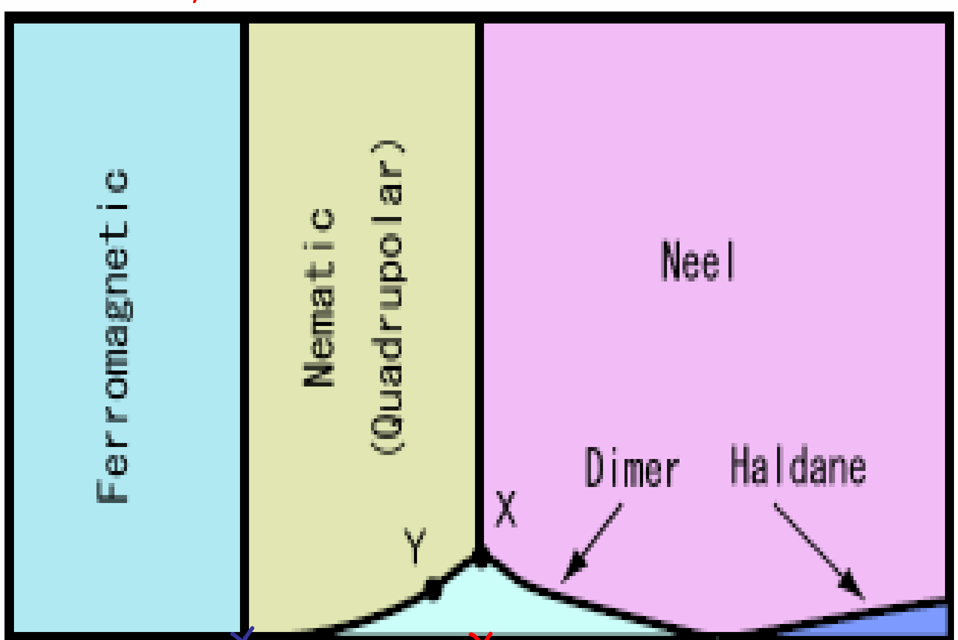
Harada, Kawashima, Troyer '07

$K_{vert.} = \lambda K_{hor.}$

$J_{vert.} = \lambda J_{hor.}$

Natural regime for Mott insulator in ^{23}Na

$\lambda = 1$



$\lambda = 0$

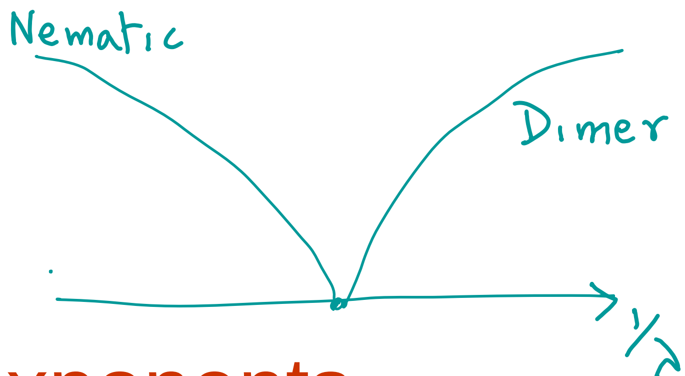
$J/K = 1$ $\quad$ $J/K = 0$ $\quad$ J/K

Quantum phase transition out of the spin nematic

Harada et. al. '07

- Nematic-dimer transition seems to be second order despite their distinct broken symmetries

(Landau-forbidden)



Estimates for some critical exponents available.

Questions

1. Why no featureless ``disordered spin nematic" in the 1d chain?

2. Mechanism for appearance of dimer order when nematic ordering is destroyed?

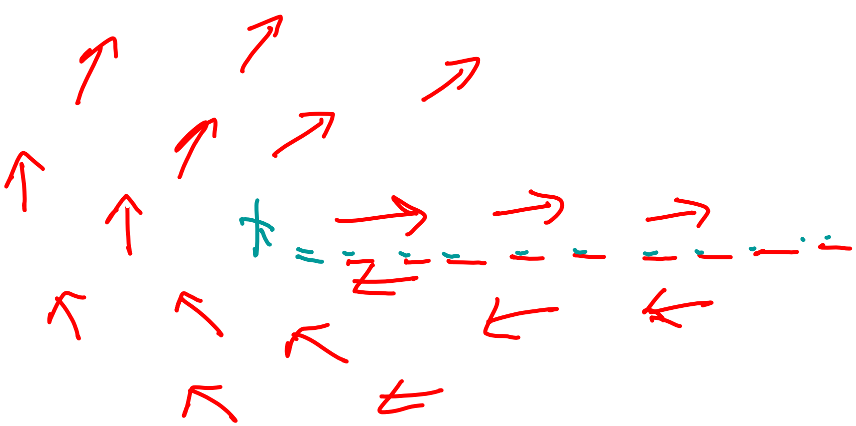
3. Theory of possible Landau-forbidden spin nematic- dimer transition?

Topological defects of the spin nematic

Spontaneous hard axis $\hat{d} \in S^2/Z_2$

(as $\hat{d}$ & $-\hat{d}$ are the same state)

$\Rightarrow$ Point Z_2 vortices ("disclinations") in $d = 2$.



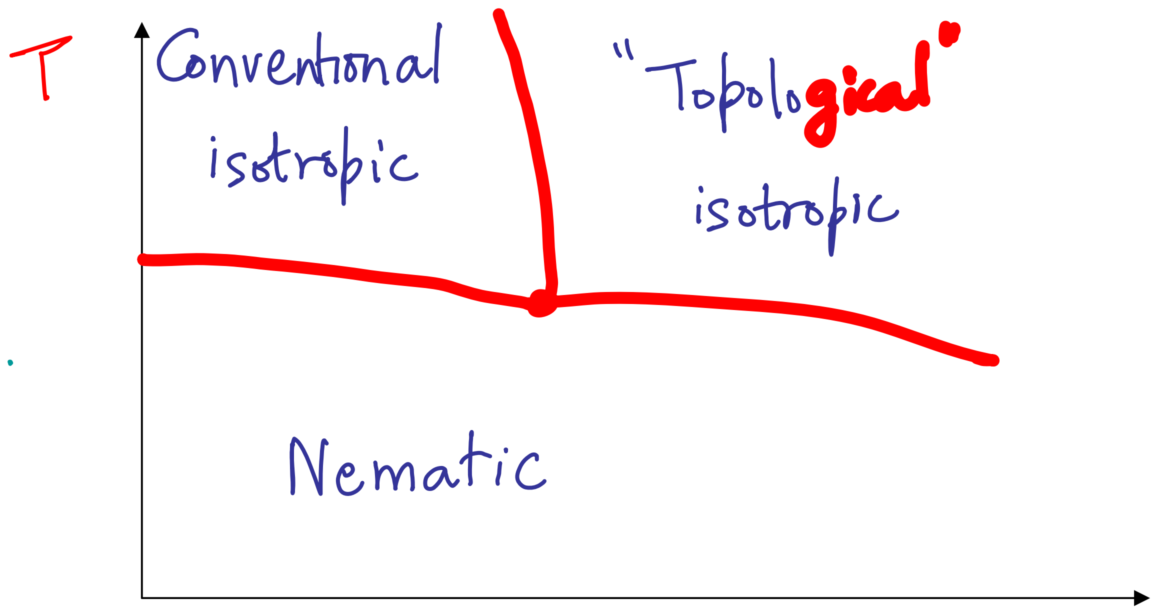
$\hat{d}$ winds to $-\hat{d}$

around such a vortex.

Killing nematic order – role of defects

Insights from theory of classical nematics.

(Lammert et al, '93)



"Disclination core energy"

Killing nematic order – role of defects

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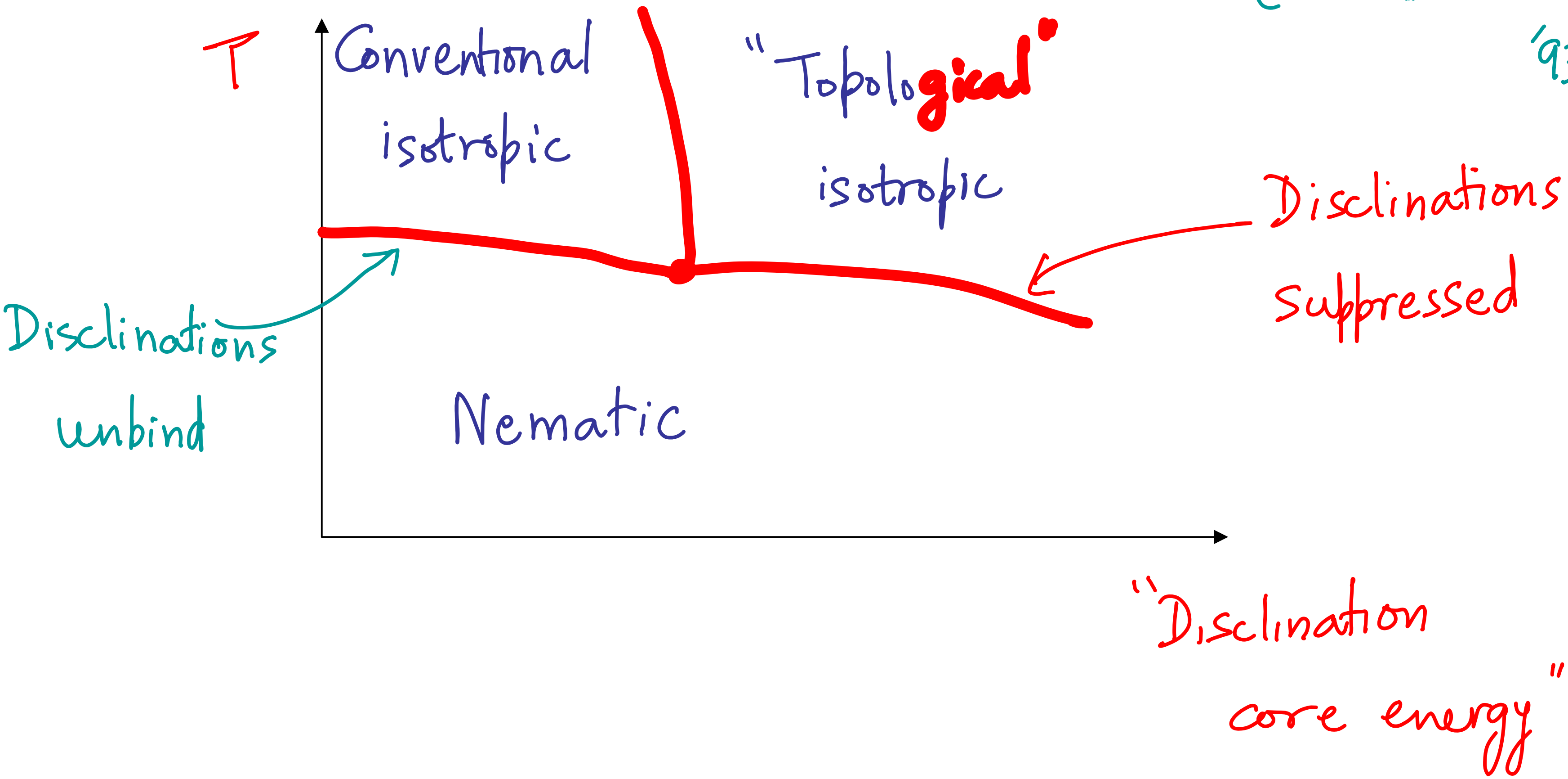
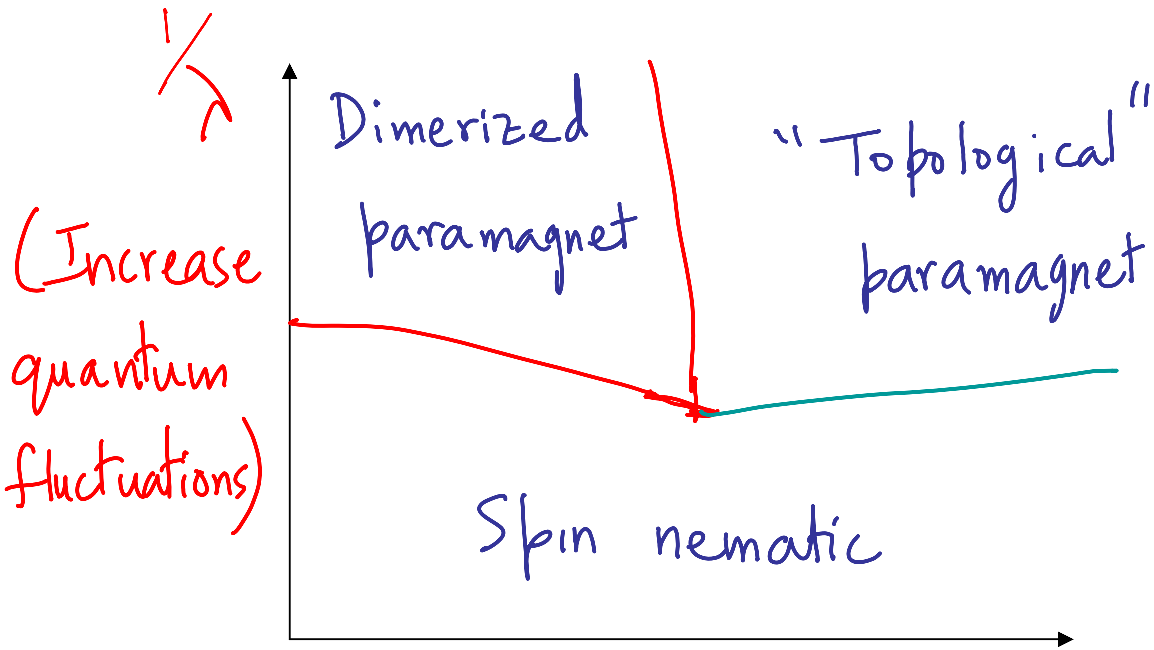


Quantum is different



"Disclination core energy"

Killing the nematic by condensing disclinations leads to dimer order due to quantum Berry phase effects

Berry phases of the defects

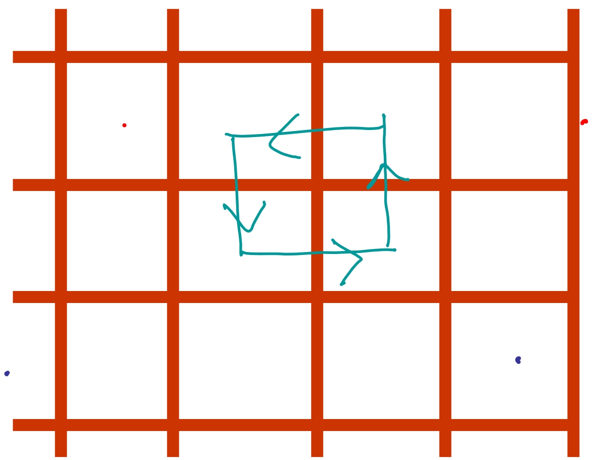
Consider single $S=1$ moment in time

varying hard axis $\mathcal{H} = (\hat{d}(t)\cdot\vec{S})^2$

Adiabatic evolution of $\hat{d}$ to $-\hat{d}$

$$\Rightarrow \quad |\psi_{gd}\rangle \longrightarrow -|\psi_{gd}\rangle$$

Berry phases of the defects



Drag disclination in loop enclosing a site

$\Rightarrow$ at that site $\hat{d} \to -\hat{d}$

$\Rightarrow$ acquire phase of π.

$\Rightarrow$ Defects move on sites of dual lattice with π-flux thru' each plaquette.

Berry phases and dimerization

Implication: Defects transform nontrivially under lattice translations

$\Rightarrow$ if defects condense

(a) spin nematic order is destroyed

(b) lattice translation symmetry is broken leading to dimer order. (Similar conclusion in $d=1$)

Theory for the nematic-dimer transition

$$H_{eff} = H[\hat{d}] + H_{defect} + H_{statistical}$$

$H[\hat{d}] \rightarrow$ dynamics of hard axis $\hat{d}$

$H_{defect} \rightarrow Z_2$ disclinations in background π-flux per plaquette

$H_{statistical}$: impose $\hat{d}$ acquires phase of π on going around a defect

Continuum field theory for nematic-dimer transition-I

H_{eff} not convenient due to statistical interaction

"Duality" techniques $\Rightarrow$ reformulate as

$$S = S_0[\vec{D}, a_\mu] + S_{monopole}$$ ("anisotropic $NCCP^2$ model")

$\vec{D}$ = 3-component complex unit vector

a_μ = U(1) gauge field coupled minimally to $\vec{D}$

$S_{monopole} \sim$ doubled space-time monopoles of U(1) gauge field

Continuum field theory for the nematic-dimer transition-II

Nematic order parameter $Q_{\alpha\beta} \sim \left\langle \frac{D^*_\alpha D_\beta + D^*_\beta D_\alpha}{2} - \frac{\delta_{\alpha\beta}}{3} \right\rangle$

Dimer order parameter $\sim \langle$ Single monopole operator $\rangle$

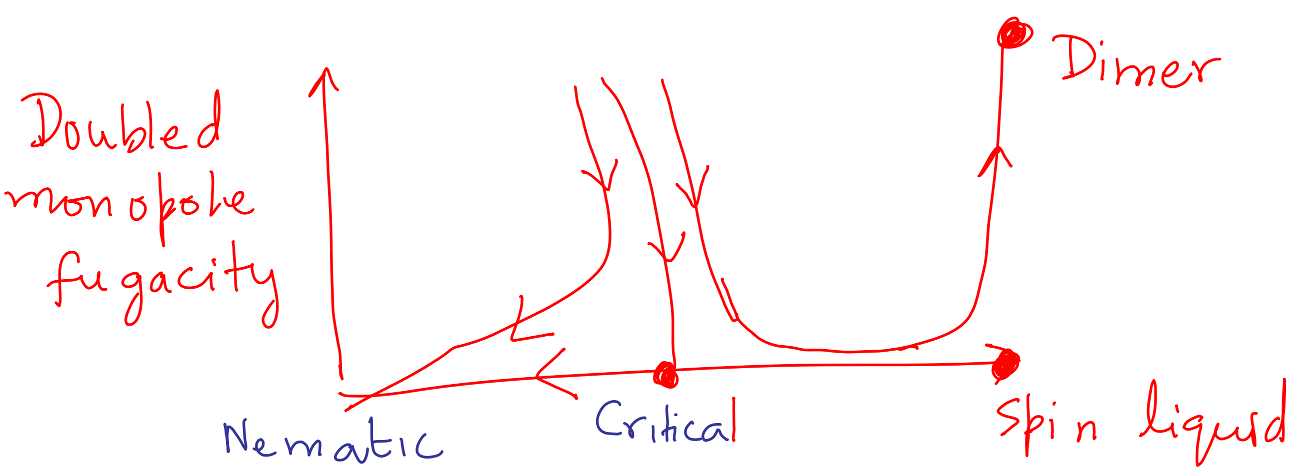
Transition described by condensation of $\vec{D}$:

Crude estimate of instanton scaling dimensions

$$S = S_0[\vec{D}, a_\mu] + S_{\text{monopole}}$$

Crude estimate suggests S_{monopole} may be irrelevant!

$\Rightarrow$ 2^{nd} order nematic–dimer transition with a "deconfined" quantum critical point.



Consequences for numerics

Most spectacular :- Enlarged symmetry at the critical point :

Dimer order parameter → XY-like near critical point

⇒ Power law for vertical dimer order with same exponent as horizontal dimer order!

Future issues

Most important – direct simulation of the action $S_0[\vec{D}, a_\mu]$ to

(a) test for irrelevance of doubled monopoles

(b) calculate critical exponents.

Summary

Bosonic Mott insulators with spin:

Natural model quantum magnets with potentially exotic quantum phase transitions (and quantum phases).