Unitary Fermi gas
Phase diagram and Raman spectroscopy

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Outline

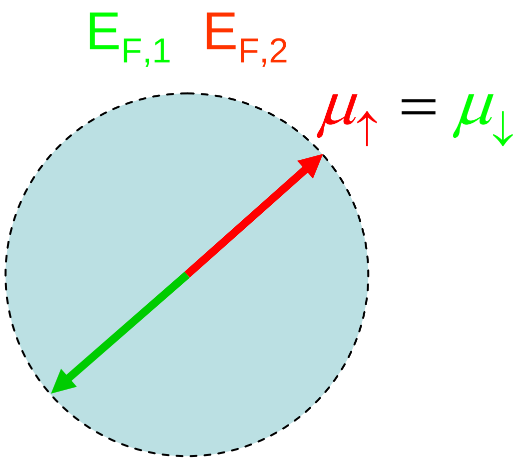
Phase diagram of strongly interacting fermions at unitarity
With unbalanced spin populations

F. Chevy, Phys. Rev. A 74, 063628 (2006),
and cond-mat 0701350

Measurement of single particle excitations in strongly interacting Fermi gas by stimulated Raman spectroscopy

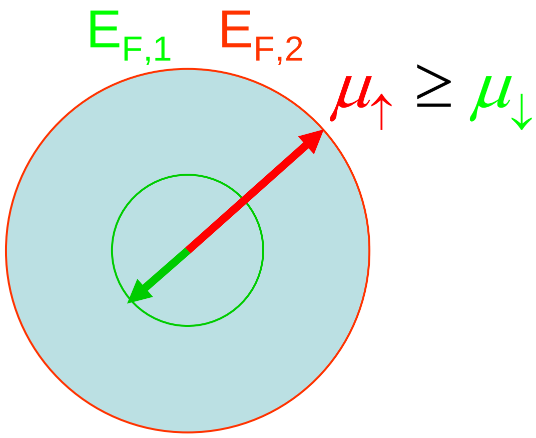
T. L. Dao, A. Georges, J. Dalibard, C.S., I. Carusotto, cond-mat 0611206

Motivation



Attractive Fermi gas with equal spin population
$\Rightarrow$ BCS theory, pairing at edge of Fermi surface

What is the nature and existence of superfluidity when spin population is imbalanced ?
Mismatched density and/or pairing with different masses



Ex:
Superconductors in magnetic field or quark matter
Cold gases: Mit and Rice expt

$$E_{F,i} = \frac{\hbar^2 k_{F,i}^2}{2m_i} = \frac{\hbar^2}{2m_i}\left(6\pi^2 n_i\right)^{2/3}$$

Overview of Theoretical scenarios

Chandrasekhar and Clogston: stability of the paired state : $\mu_\uparrow > \mu_\downarrow$

Conversion of a particle: $\downarrow \rightarrow \uparrow$

Decrease the grand potential $H - \mu_\uparrow N_\uparrow - \mu_\downarrow N_\downarrow$: $\mu_\uparrow - \mu_\downarrow$

Cost of pair breaking: Δ

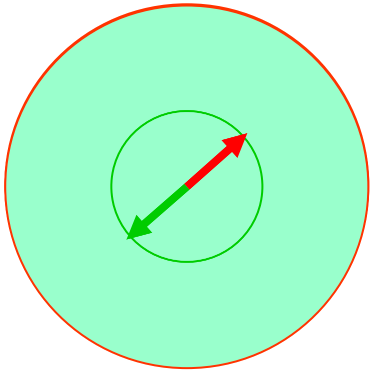
$\Rightarrow$ Paired state stable for $\mu_\uparrow - \mu_\downarrow < \Delta$

And beyond?

Polarized phase : One spin species (Carlson, PRL **95**, 060401 (2005))

FFLO Phase (Fulde Ferrell Larkin Ovchinikov) : pairing in $\mathbf{k}_\uparrow - \mathbf{k}_\downarrow \neq 0$
(C. Mora et R. Combescot, PRB **71**, 214504 (2005))

Sarma phase (internal gap) : pairing in $\mathbf{k}_\uparrow - \mathbf{k}_\downarrow = 0$
⬇opening of a gap in the Fermi sea of majority species. (Liu, PRL **90**, 047002 (2003))



Avalanche of recent publications !

P. Pieri and G.C. Strinati cond-mat/0512354 : diagrammatic method
Extrapolation from BEC regime

W. Yi and L.-M. Duan, cond-mat/0601006 : BCS at finite temperature

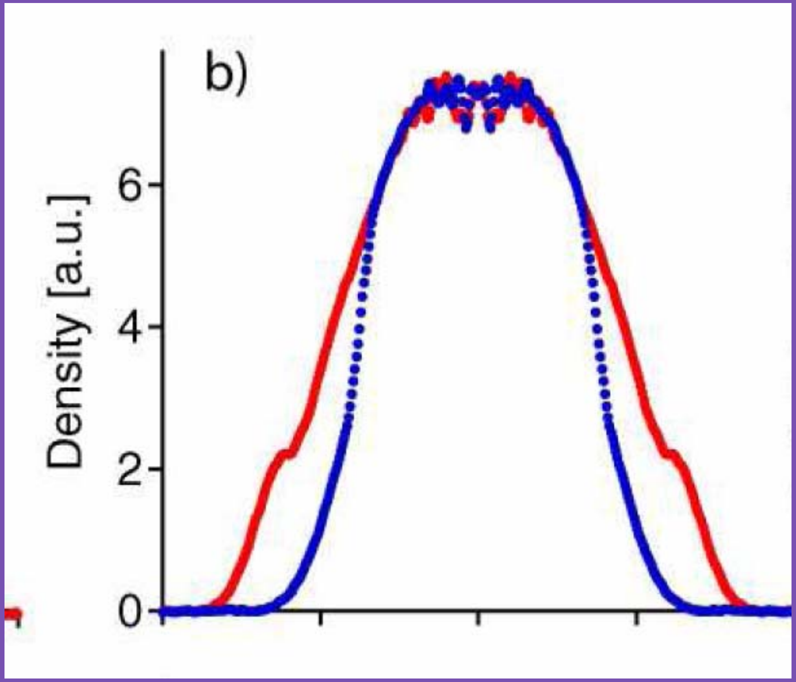
M. Haque and H.T.C. Stoof, cond-mat/0601321 : BCS at T=0

T.N. de Silva and E.J. Mueller, cond-mat/0601314 : BCS at T=0

D. Sheehy, L. Radzihovsky, PRL 06
A. Bulgac, M. McNeil Forbes '06
K. Levin et al., 06
M. Parish, Nature Physics 3 '07
………

Assumptions:
1) Unitarity: universal parameter $\mu = (1+\beta)\ E_F = \xi E_F$ known
2) Grand canonical description, Local density approx,
3) T=0 approach

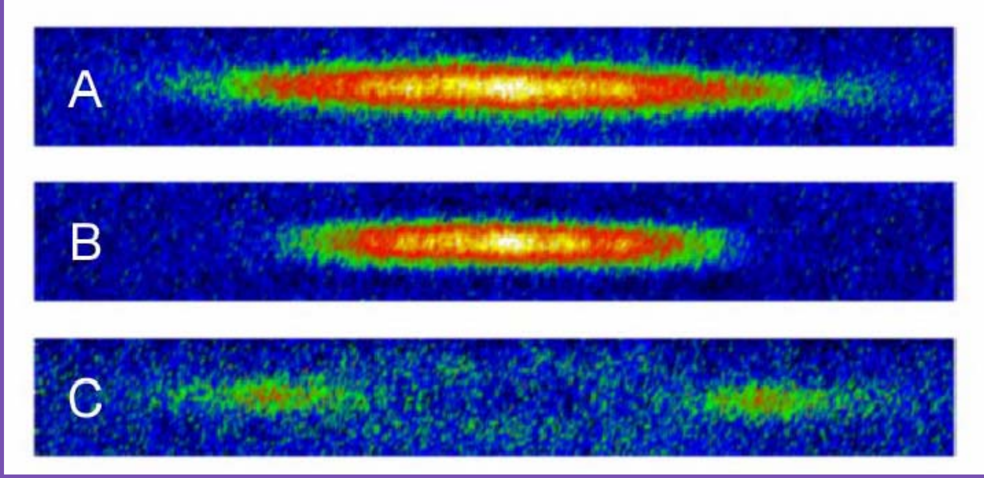
Experimental results



MIT: 3 phases
- Fully paired superfluid core
-Intermediate mixture
-Fully polarized rim

M.W. Zwierlein, *et al.*, Science, **311** (2006) 492.

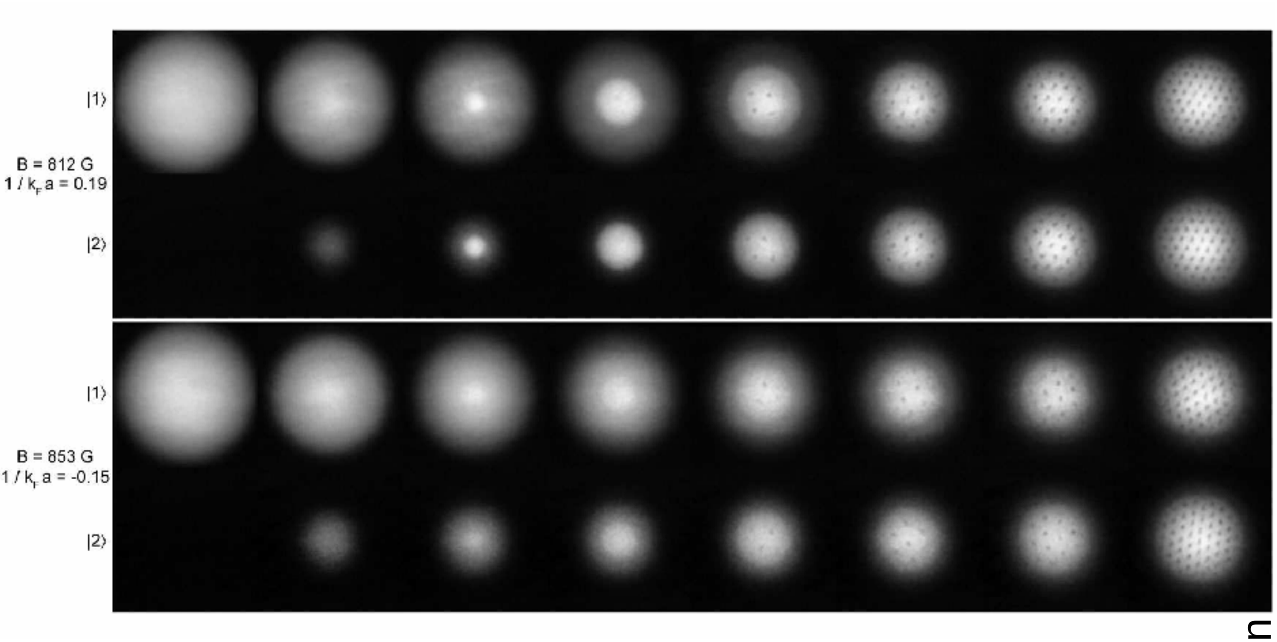
Rice: 2 phases
Fully paired superfluid core
Fully polarized rim



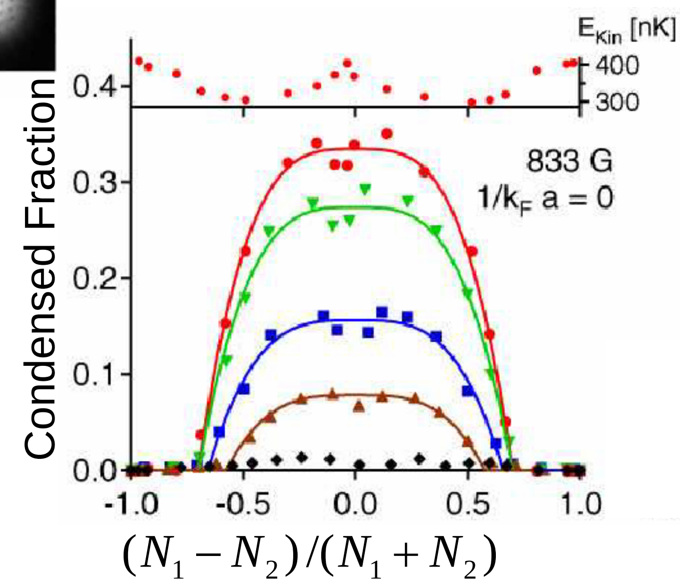
G. Partridge, W. Li , R.I. Kamar, Y.-A. Liao, R.G. Hulet, Science, **311** (2006) 503.

G. Partridge et al., Cond-mat 0608455

MIT experiment
(Science Express, December 22, 2005)



Superfluidity observed in Time of flight
Loss of superfluidity for large
Spin population imbalance



Why Unitarity?

Dimensional analysis: for any *intensive* physical quantity Q (density, pressure…)

$$Q[V,\mu_\uparrow,\mu_\downarrow,m,\hbar,a]=Q_0[\mu_\uparrow,m,\hbar]\,f(\mu_\downarrow/\mu_\uparrow,1/k_{F\uparrow}a)$$

Q_0: value for the ideal Fermi gas;

$$\mu_\uparrow=\hbar^2k_{F\uparrow}^2/2m$$

At Feshbach resonance, $a=\infty \Rightarrow Q/Q_0=g(\mu_\downarrow/\mu_\uparrow)$ only!

Application: universal equation of state of the balanced Fermi gas

For instance: *Q*=density, balanced Fermi gas ($\mu_\uparrow = \mu_\downarrow$)

$$n = \frac{1}{6\pi^2}\left(\frac{2m\mu_\uparrow}{\hbar^2}\right)^{3/2}$$ x numerical factor

$$\mu_\uparrow = \xi \frac{\hbar^2}{2m}\left(6\pi^2 n\right)^{2/3} = \xi E_F$$

Determination of ξ

Experiment	*ENS (^{6}Li)*	0.41(15)
	Rice (^{6}Li)	0.46(5)
	JILA(^{40}K)	0.46(10)
	Innsbruck (^{6}Li)	0.27(10)
	Duke (^{6}Li)	0.51(4)

Theory	*BCS*	0.59
	Astrakharchik	0.42(1)
	Perali	0.455
	Carlson	0.42(1)
	Haussmann	0.36

Universal phase diagram of the homogeneous unitary system

(F. **Chevy, PRA Phys. Rev. A 74, 063628 (2006),**

A. Bulgac, M. McNeil Forbes, cond-mat/0606043)

« Exact » eigenstates of the grand potential

- Single component ideal gas $\Omega = \Omega_0$
- Fully paired superfluid

$\left|\mathrm{SF}\right\rangle_\mu$ eigenstate of the *balanced* grand-potential

$$\hat{\Omega}' = \hat{H} - \mu(\hat{N}_\uparrow + \hat{N}_\downarrow)$$

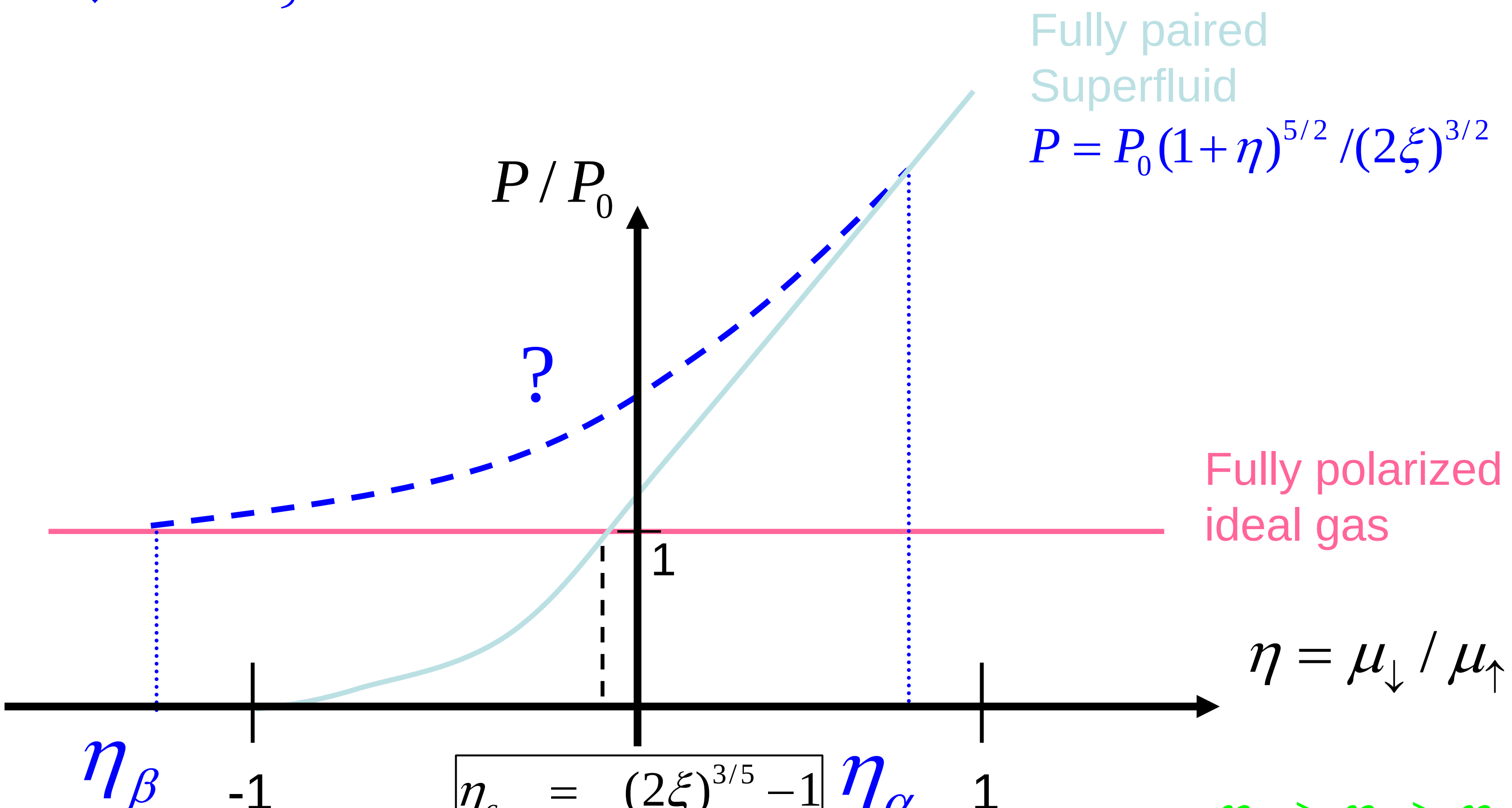
$$\hat{\Omega}'\left|\mathrm{SF}\right\rangle_\mu = \Omega'\left|\mathrm{SF}\right\rangle_\mu \qquad \hat{N}_\uparrow\left|\mathrm{SF}\right\rangle_\mu = \hat{N}_\downarrow\left|\mathrm{SF}\right\rangle_\mu = N\left|\mathrm{SF}\right\rangle_\mu$$

$$\hat{\Omega} = \hat{H} - \mu_\uparrow\hat{N}_\uparrow - \mu_\downarrow\hat{N}_\downarrow \quad \Rightarrow \hat{\Omega}\left|\mathrm{SF}\right\rangle_{(\mu_\uparrow+\mu_\downarrow)/2} = \Omega'\left|\mathrm{SF}\right\rangle_{(\mu_\uparrow+\mu_\downarrow)/2}$$

$$\hat{\Omega} = \hat{H} - \frac{\mu_\uparrow + \mu_\downarrow}{2}(\hat{N}_\uparrow + \hat{N}_\downarrow) - \frac{\mu_\uparrow - \mu_\downarrow}{2}(\hat{N}_\uparrow - \hat{N}_\downarrow)$$

Universal phase diagram of the homogeneous unitary system (2)

$$\left.\begin{aligned} \Omega &= -PV \\ dP &= \sum_{\sigma=\updownarrow} n_\sigma d\mu_\sigma \end{aligned}\right\} \Rightarrow$$ Just need to know $n(\mu)$



$$\begin{aligned} \eta_c &= (2\xi)^{3/5} - 1 \\ &\approx -0.099 \end{aligned}$$

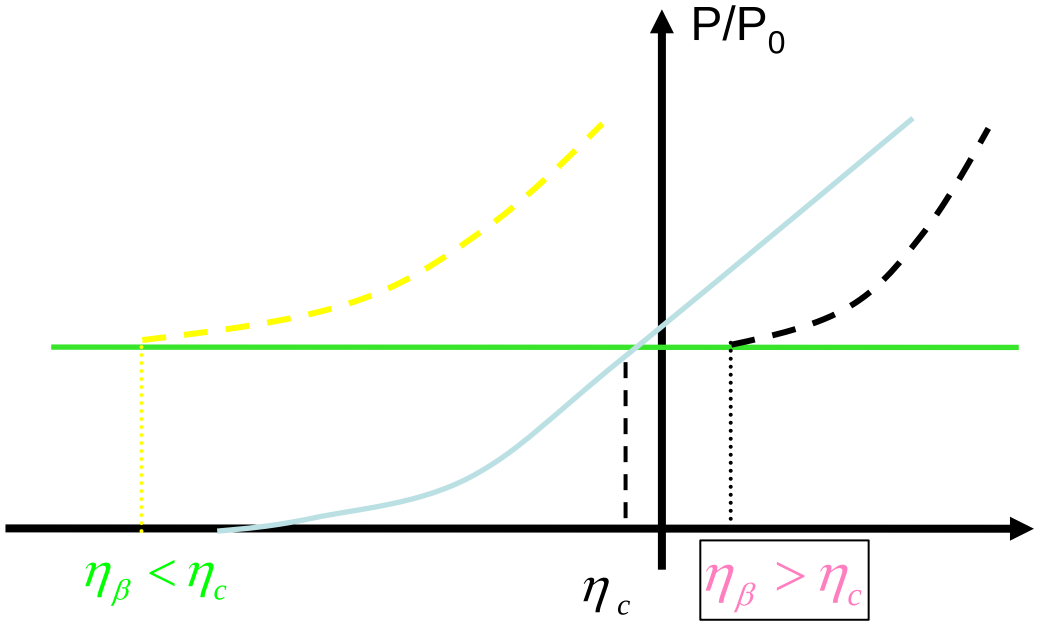
$$\eta_\alpha > \eta_c > \eta_\beta$$

Theoretical evidence for an intermediate phase

General properties of a mixed branch?

Step 1: calculate the energy E of a single impurity atom immersed in a Fermi sea (E= $\mu_\downarrow$, with $n_\downarrow = 0^+$)

Step 2: dP/dμ_σ= n > 0



$\eta_\beta < \eta_c$: the new branch is stable

$\eta_\beta > \eta_c$: the new branch is unstable

Variational upper bound for the N+1 body problem

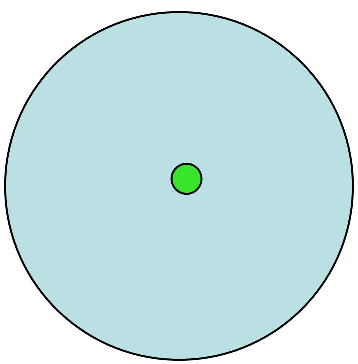
$$\hat{H} = \sum_{\mathbf{k}\sigma} \varepsilon_{\mathbf{k}} \hat{a}^{\dagger}_{\mathbf{k}\sigma} \hat{a}_{\mathbf{k}\sigma} + \frac{g_b}{\Omega} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} \hat{a}^{\dagger}_{\mathbf{k}'-\mathbf{q},\uparrow} \hat{a}^{\dagger}_{\mathbf{k}+\mathbf{q}\downarrow} \hat{a}_{\mathbf{k}\downarrow} \hat{a}_{\mathbf{k}'\uparrow}$$

$$\varepsilon_{\mathbf{k}} = \frac{\hbar^2 k^2}{2m} \qquad\qquad \frac{1}{g_b} = \frac{m}{4\pi\hbar^2 a} - \frac{1}{\Omega} \sum_{\mathbf{k}} \frac{1}{2\varepsilon_{\mathbf{k}}}$$

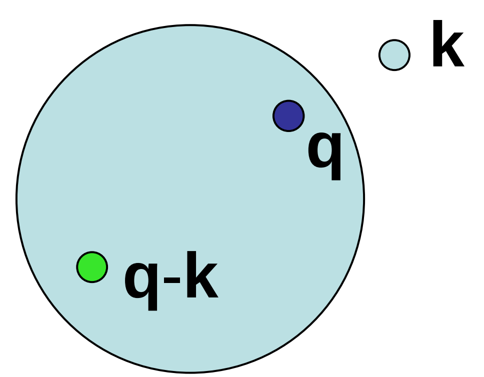
One impurity: restrict the effect of interactions to the formation of a *single particle-hole pair*.

$$|\Psi\rangle = \varphi_0 |0\rangle + \sum_{\mathbf{k},\mathbf{q}} \varphi_{\mathbf{k},\mathbf{q}} |\mathbf{k},\mathbf{q}\rangle$$

$|0\rangle =$



$|\mathbf{kq}\rangle =$



Comparison with Monte-Carlo simulations (C. Lobo et al. PRL. **97**, 200403 (2006))

Minimization of H with respect to φ_0 and φ_{kq}

$$E = \frac{1}{V}\sum_{q<k_F} \frac{1}{\frac{m}{4\pi\hbar^2 a} - \frac{1}{V}\sum_{k<k_F}\frac{1}{2\varepsilon_k} + \sum_{k>k_F}\left(\frac{1}{E-\left(\varepsilon_k+\varepsilon_{q-k}-\varepsilon_q\right)} - \right)\frac{1}{2\varepsilon_k}} \qquad \varepsilon_k = \frac{\hbar^2 k^2}{2m}$$

For a= ∞ , E=-0.606 E_{F} ⇒ η_β < - 0.606 < η_c ~ -0.1

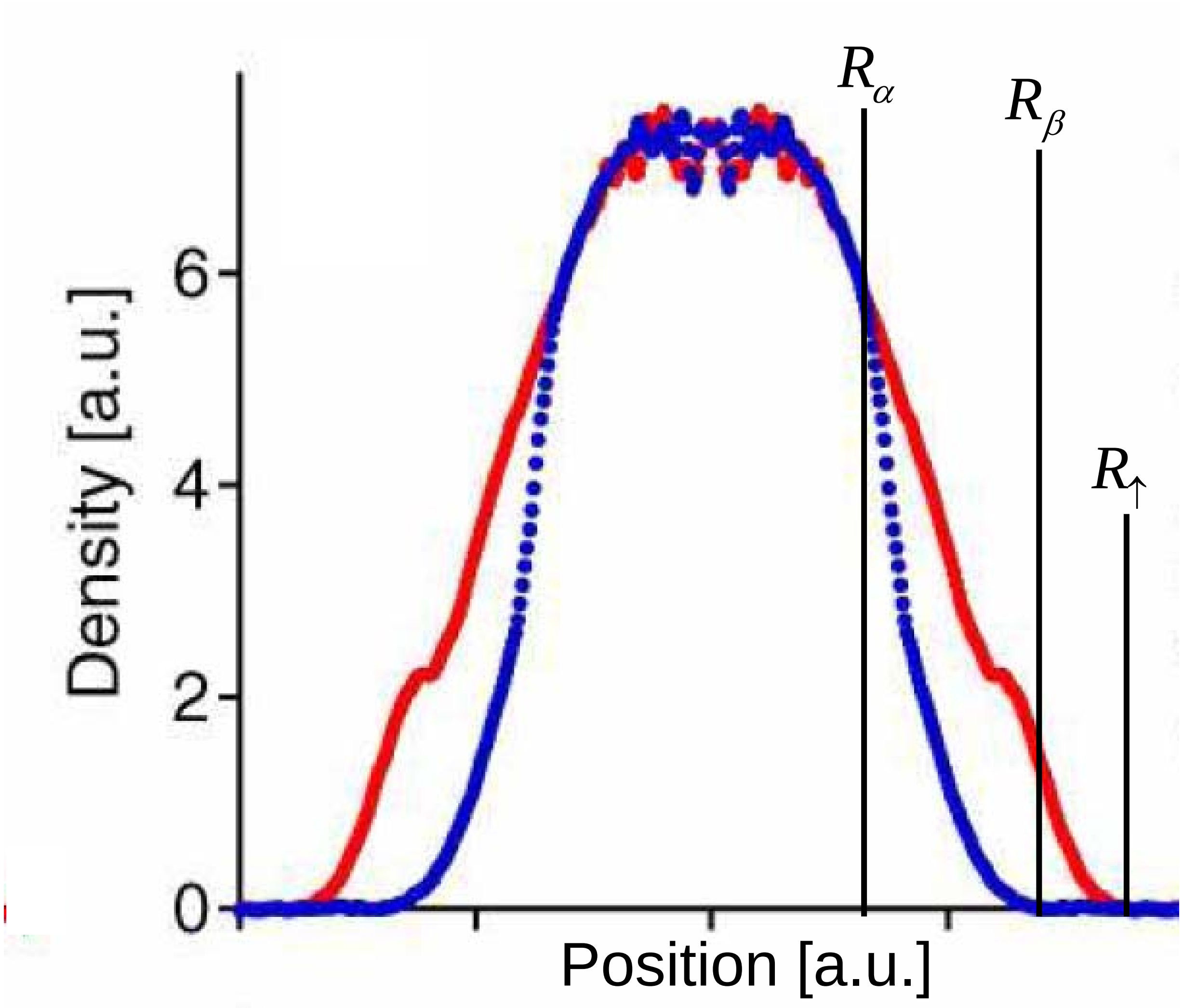
Monte Carlo simulations : η_β = - 0.58(1)
BCS theory: η_β = 0

Why such a good agreement?

Weak excitation of the pairs, even at unitarity

$$\sum_{k,q}\left|\varphi_{k,q}\right|^2 = 0.2$$

Three phase mixture in a trap: MIT



Interpretation of MIT's experiment

Local density approximation in harmonic trap V(r) ~ r^2

$$\mu_{\uparrow}(\mathbf{r}) = \mu_{\uparrow}^{0} - V(\mathbf{r})$$

$$\mu_{\downarrow}(\mathbf{r}) = \mu_{\downarrow}^{0} - V(\mathbf{r})$$

$R_{\uparrow}$: outer radius of the majority component $\mu_{\uparrow}(R_{\uparrow}) = 0$

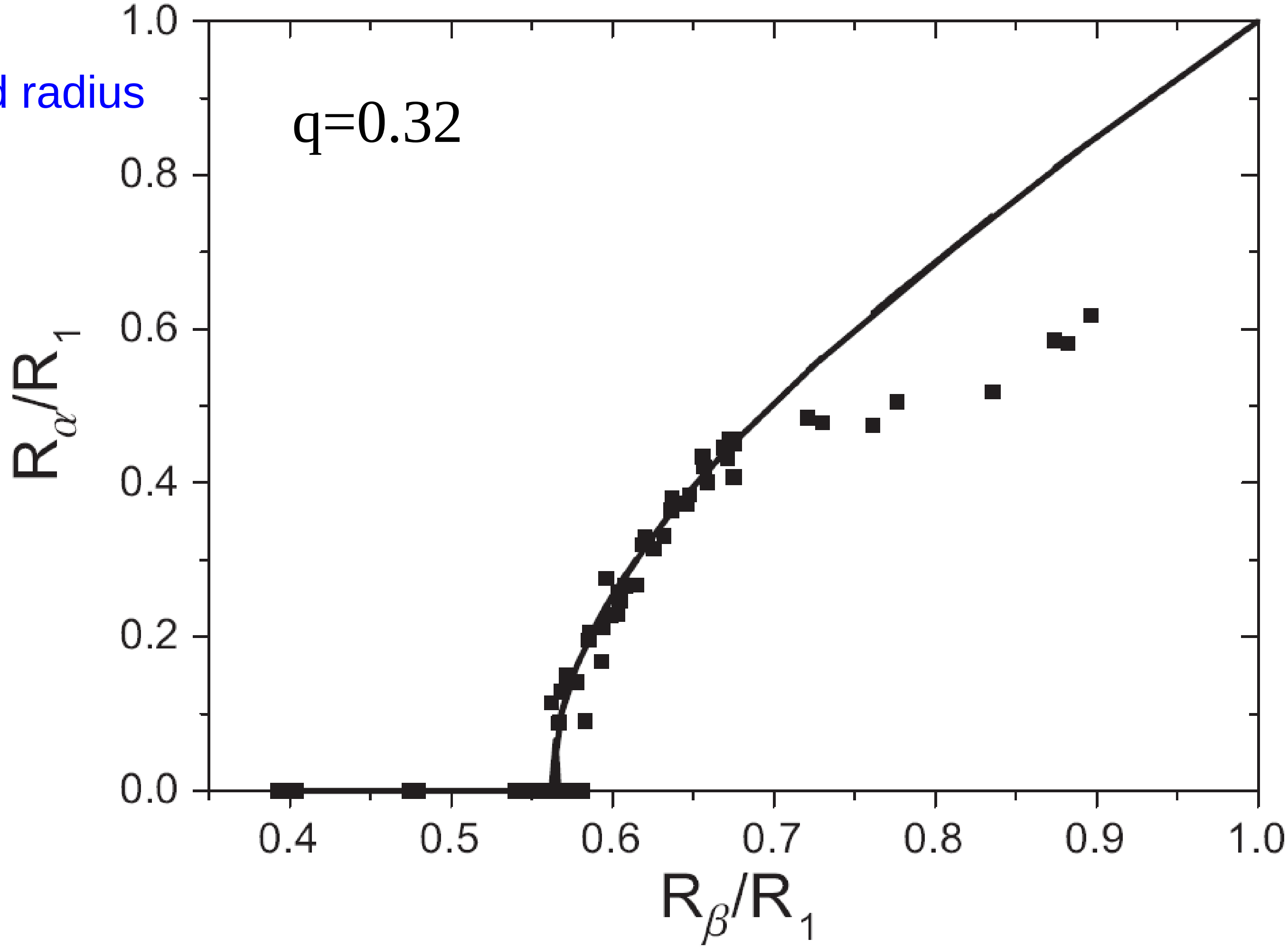
R_{β} : outer radius of the minority component $\mu_{\downarrow}(R_{\beta}) / \mu_{\uparrow}(R_{\beta}) = \eta_{\beta}$

R_{α} : radius of the superfluid core $\mu_{\downarrow}(R_{\alpha}) / \mu_{\uparrow}(R_{\alpha}) = \eta_{a}$

$$\frac{R_{\alpha}}{R_{\uparrow}} = \sqrt{\frac{(R_{\beta}/R_{\uparrow})^2 - q}{1-q}} \qquad q = \frac{\eta_{\alpha} - \eta_{\beta}}{1-\eta_{\beta}}$$

Comparison with experimental data
(M. Zwierlein et al., Nature, 442,54 (2006))

Superfluid radius



Minority atom radius

Improved bounds for η_α and η_β

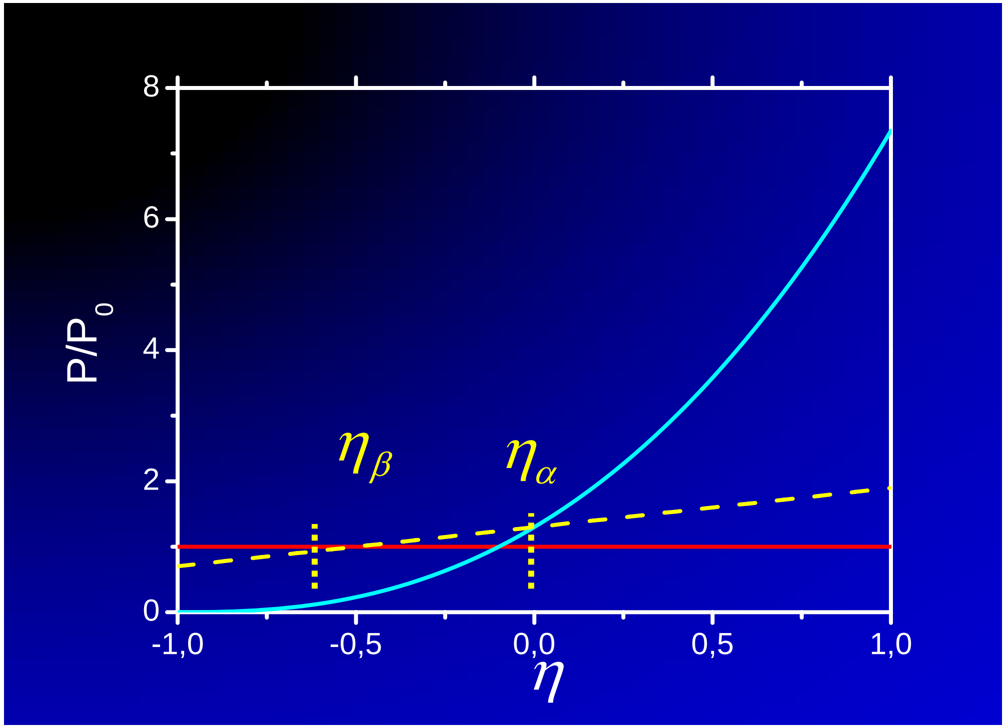
q=0.32
η_α>-0.10
η_β<-0.60

$$\boxed{\begin{array}{c} -0.62 < \eta_\beta < -0.60 \\ -0.10 < \eta_\alpha < -0.088 \end{array}}$$

$$\eta = \frac{\mu_\downarrow}{\mu_\uparrow}$$

BCS: $\eta_\alpha = 0.1$
$\eta_\beta = 0$

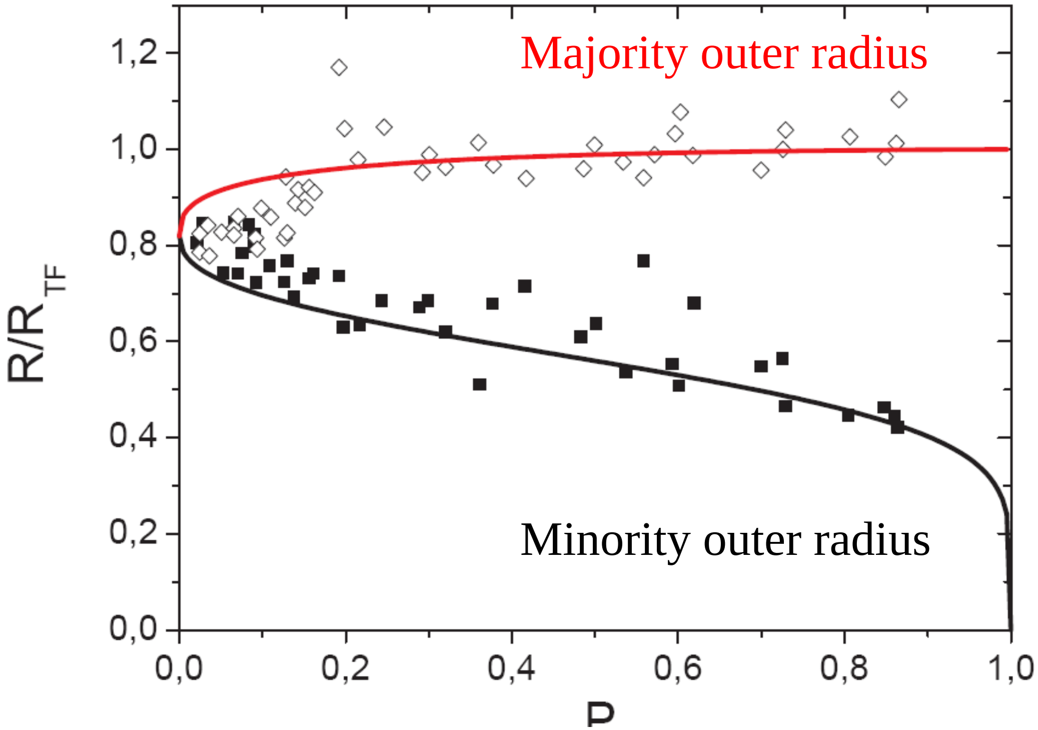
Convexity of P:
real equation of state below



Unresolved mystery: Rice expt

Partridge et al. Science **311**, 503 (2006)

Experiment by Rice's group: fully compatible with 2 phase scenario (no intermediate phase+LDA). No adjustable parameters.



$(N_1-N_2)/(N_1+N_2)$

For P=0.7, q~0.16 differs from MIT and contradicts theoretical bounds: q > 0.31 set by $\eta_\alpha > -0.1$ and $\eta_\beta < 0.60$

Summary: part 1

What have we demonstrated?

3 homogeneous phases in the phase diagram of imbalanced unitary Fermi gases
LDA valid ⬇ agreement with MIT

But a host of unanswered questions!

Nature of the MIT/Rice's discrepancy (surface tension ?)
Microscopic nature of the intermediate phase?
Superfluid nature of the intermediate phase?

Dynamical properties, collective modes?
Extension to the BEC-BCS crossover (*in progress*)
Response to RF excitation (MIT experiment, *Schunck et al.* ***cond-mat/0702066***)

Part 2. Measuring one-particle excitations In Fermi gases using Raman spectroscopy

T-L Dao, A. Georges, J. Dalibard, C. Salomon, I. Carusotto, Cond-mat/0611206

In Fermi liquid theory: low energy excitations are build out of quasiparticles
Dispersion relation on a given point of the Fermi surface:

$\xi_k \sim \mathbf{v_F}(k_F).(\mathbf{k}-\mathbf{k_F})+....$ Lifetime: Γ_k^{-1}

Fermi surface: excitation energy vanishes: $\xi_{k_F} = 0$

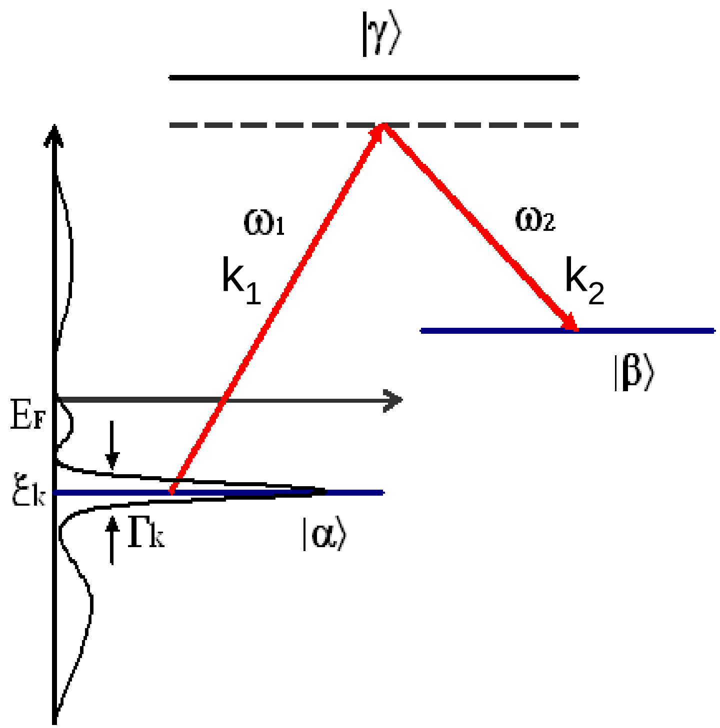
Normal phase in Cuprate SC show strong deviations, anisotropic behavior.

Probe directly one particle correlator: $\langle \psi^\dagger(r,t)\,\psi(r',t')\rangle$

Bragg spectroscopy or noise corr. : $\langle \psi^\dagger(r,t)\,\psi(r,t)\,\psi^\dagger(r',t')\,\psi(r',t')\rangle$

Stimulated Raman spectroscopy: widely used for Bose systems
Very interesting for fermions:
Probing Fermi surface of strongly interacting fermions (Time of Flight not adequate)
Momentum-resolved quasiparticles excitations

Two-photon Raman excitation



Interacting fermions: $|\alpha>, |\alpha'>$
For instance ^{6}Li near F. Resonance

Third state empty: $|\beta>$

No interaction with $|\alpha>, |\alpha'>$

Similar to ARPES in cond. Matter.

$$R(\mathbf{q},\Omega) \sim \int_{-\infty}^{+\infty} dt \int d\mathbf{r} d\mathbf{r}' e^{i[\Omega t - \mathbf{q}.(\mathbf{r}-\mathbf{r}')]} \times g_\beta(\mathbf{r},\mathbf{r}';t) \left\langle \psi_\alpha^\dagger(\mathbf{r},t)\,\psi_\alpha(\mathbf{r}',0) \right\rangle$$

Selectivity in **q= k$_1$-k$_2$**
Selectivity in energy

Interacting Fermions in homogeneous 2D square Lattice

Raman resonance condition $\varepsilon_{\mathbf{k+q},\beta} - \xi_{\mathbf{k}} = \Omega$

Threshold in Ω: $(\omega_1 - \omega_2)_T = \varepsilon_\beta^0 - \mu \sim \varepsilon_\beta^0 - \varepsilon_\alpha^0$

Ω close to threshold for **q** = - $\mathbf{k}_F$

Shell surrounding Fermi Surface

of width $\sqrt{2M\Delta\Omega} \sim \sqrt{2M\mathbf{v}_F(\mathbf{k}_F).(\mathbf{q}+\mathbf{k}_F)}$

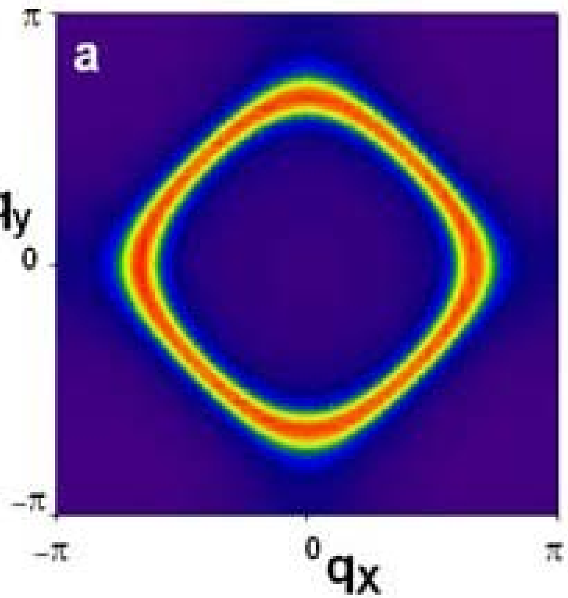
Non interacting fermions

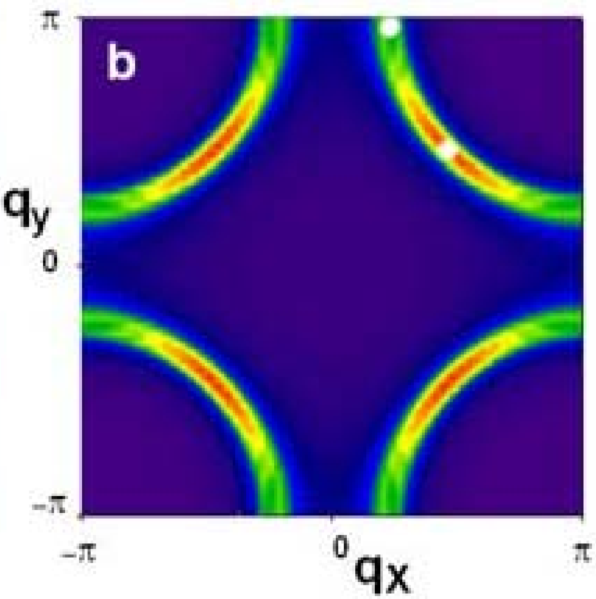
$\xi_{\mathbf{k}} = -2t_\alpha(\cos k_x + \cos k_y) - \mu$

Model pseudo-gap with d-wave symmetry

$\Delta_{\mathbf{k}} = \Delta_0(\cos k_x - \cos k_y)$

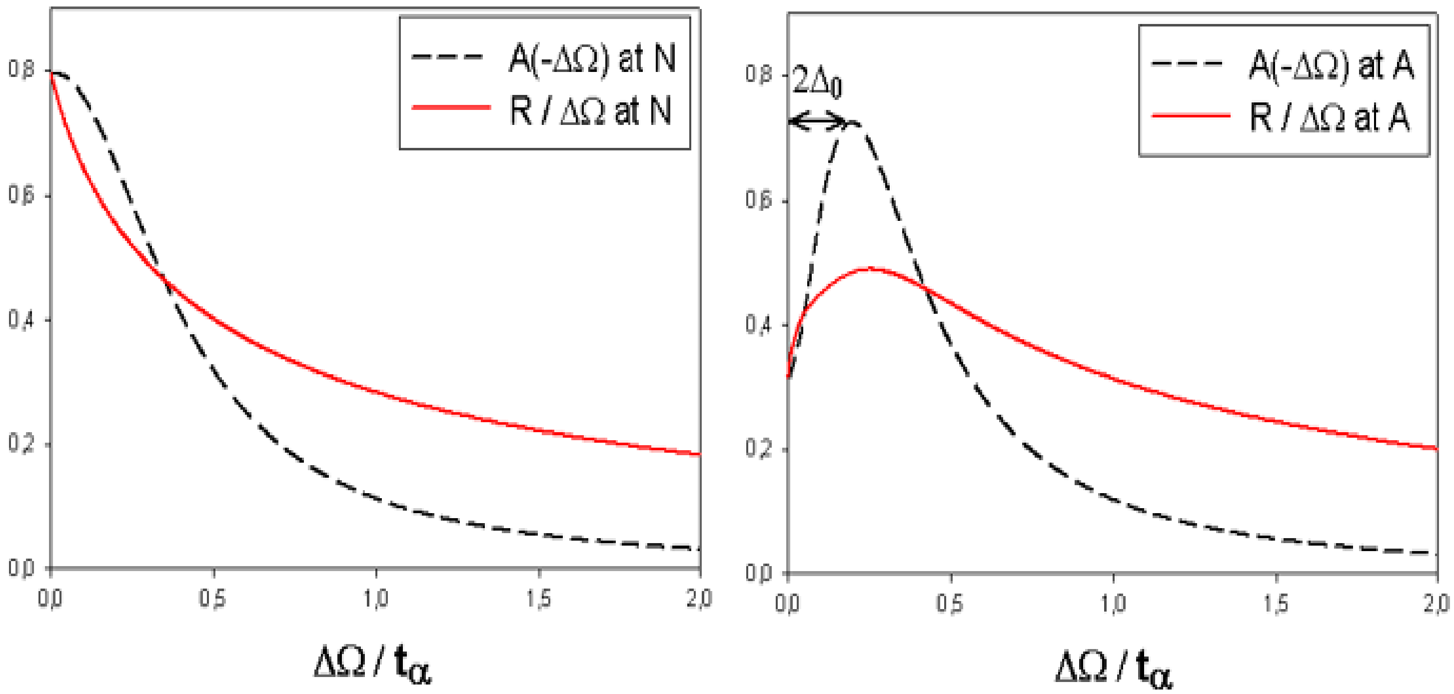
Lorentzian spectral Function A(k, Ω) uniform in k-space





Δ_0= 0.1t_α
N_α=0.45
Γ_0= 0.05 t_α
Γ_1= 0.4 t_α

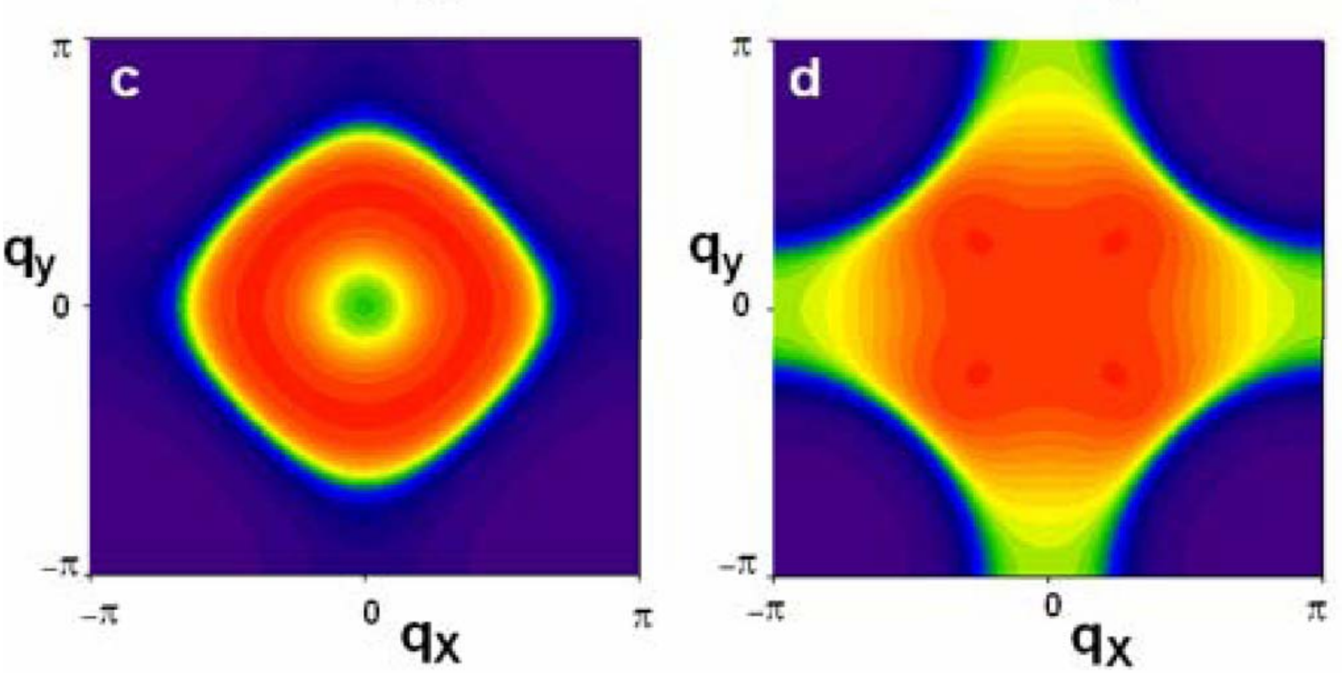
Rate R(q, Ω) and spectral function A (k, Ω)



For fixed value of q, chosen near Nodal line or Anti-nodal
Scan Raman detuning

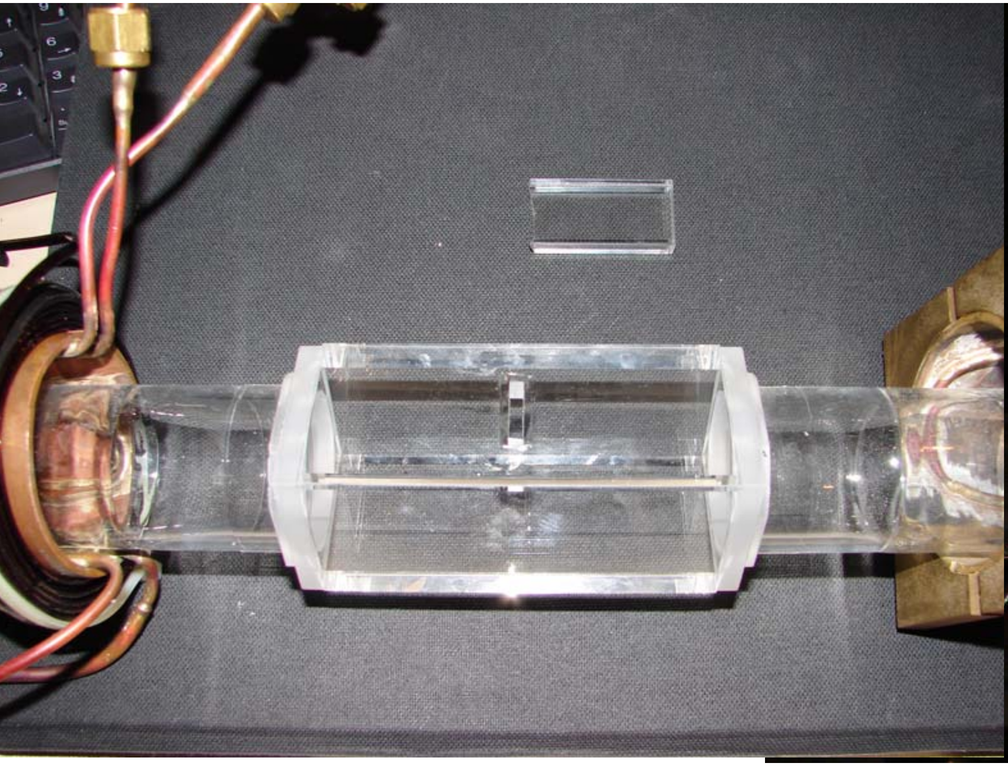
In Harmonic Trap

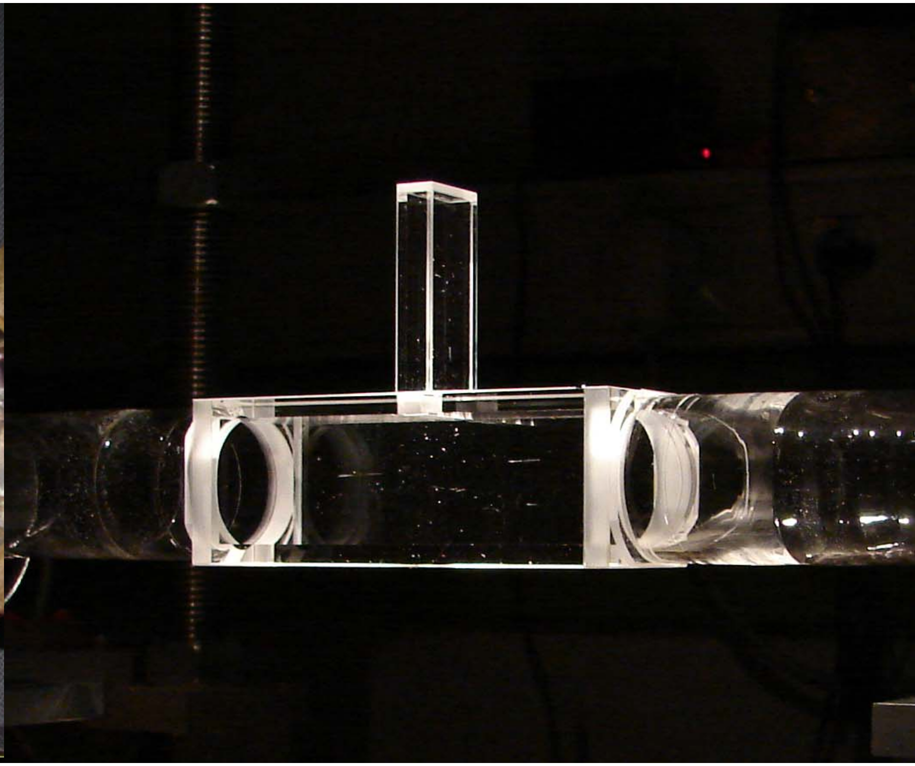
LDA: $$\mu(R) = \mu_0 - M\omega_0^2 R^2 / 2$$



Non-interacting fermions Model d-wave pseudogap state

New experimental setup





Enlarged glass cell
New laser sources: 120 mW diodes operating at 75 degree C
New Zeeman slower
More stable Ioffe-Pritchard trap
120 Watt far detuned optical trap (Fiber laser)
Access for 3D optical lattice
3 10^{10} ^{7}Li atoms in MOT → expected increase of x10 in ^{6}Li number
Ongoing: Transfer into magnetic trap

Thank you for your attention!