Unitary Fermi gas
Phase diagram and Raman spectroscopy

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Phase diagram of strongly interacting fermions at unitarity
With unbalanced spin populations
F. Chevy, Phys. Rev. A 74, 063628 (2006),
and cond-mat 0701350

Measurement of single particle excitations in strongly
interacting Fermi gas by stimulated Raman spectroscopy
T. L. Dao, A. Georges, J. Dalibard, C.S., I. Carusotto, cond-mat 0611206
Motivation

Attractive Fermi gas with equal spin population
$\Rightarrow$ BCS theory, pairing at edge of Fermi surface

What is the nature and existence of superfluidity when spin population is imbalanced?
Mismatched density and/or pairing with different masses

Ex:
Superconductors in magnetic field or quark matter
Cold gases: Mit and Rice expt

$$E_{F,i} = \frac{\hbar^2 k_{F,i}^2}{2m_i} = \frac{\hbar^2}{2m_i} \left(6\pi^2 n_i\right)^{2/3}$$
Overview of Theoretical scenarios

Chandrasekhar and Clogston: stability of the paired state: $\mu_\uparrow > \mu_\downarrow$

Conversion of a particle: $\downarrow \rightarrow \uparrow$
Decrease the grand potential $H - \mu_\uparrow N_\uparrow - \mu_\downarrow N_\downarrow : \mu_\uparrow - \mu_\downarrow$
Cost of pair breaking: $\Delta$
$\Rightarrow$ Paired state stable for $\mu_\uparrow - \mu_\downarrow < \Delta$

And beyond?

Polarized phase: One spin species (Carlson, PRL 95, 060401 (2005))

FFLO Phase (Fulde Ferrell Larkin Ovchiniakov): pairing in $k_\uparrow - k_\downarrow \neq 0$
(C. Mora et R. Combescot, PRB 71, 214504 (2005))

Sarma phase (internal gap): pairing in $k_\uparrow - k_\downarrow = 0$
Opening of a gap in the Fermi sea of majority species. (Liu, PRL 90, 047002 (2003))
Avalanche of recent publications!

P. Pieri and G.C. Strinati cond-mat/0512354 : diagrammatic method
Extrapolation from BEC regime
W. Yi and L.-M. Duan, cond-mat/0601006 : BCS at finite temperature
M. Haque and H.T.C. Stoof, cond-mat/0601321 : BCS at T=0
T.N. de Silva and E.J. Mueller, cond-mat/0601314 : BCS at T=0
D. Sheehy, L. Radzihovsky, PRL 06
A. Bulgac, M. McNeil Forbes ’06
K. Levin et al., 06
M. Parish, Nature Physics 3 ’07

Assumptions:
1) Unitarity: universal parameter $\mu = (1 + \beta) \ EF = \xi EF$ known
2) Grand canonical description, Local density approx,
3) T=0 approach
Experimental results

MIT: 3 phases
- Fully paired superfluid core
- Intermediate mixture
- Fully polarized rim


Rice: 2 phases
- Fully paired superfluid core
- Fully polarized rim


G. Partridge et al., Cond-mat 0608455
MIT experiment
(Science Express, December 22, 2005)

Superfluidity observed in Time of flight
Loss of superfluidity for large
Spin population imbalance
Why Unitarity?

**Dimensional analysis:** for any *intensive* physical quantity $Q$ (density, pressure...)

$$Q[V, \mu_\uparrow, \mu_\downarrow, m, \hbar, a] = Q_0[\mu_\uparrow, m, \hbar] f (\mu_\downarrow / \mu_\uparrow, 1/k_{F\uparrow} a)$$

$q_0$: value for the ideal Fermi gas;

$$\mu_\uparrow = \hbar^2 k_{F\uparrow}^2 / 2m$$

At Feshbach resonance, $a = \infty \Rightarrow Q/Q_0 = g(\mu_\downarrow / \mu_\uparrow)$ only!
Application: universal equation of state of the balanced Fermi gas

For instance: $Q = \text{density, balanced Fermi gas } (\mu_\uparrow = \mu_\downarrow )$

\[
n = \frac{1}{6\pi^2} \left( \frac{2m\mu_\uparrow}{\hbar^2} \right)^{3/2} \times \text{numerical factor}
\]

\[
\mu_\uparrow = \xi \frac{\hbar^2}{2m} \left( 6\pi^2 n \right)^{2/3} = \xi E_F
\]

### Determination of $\xi$

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Theory</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ENS ($^6\text{Li}$)</td>
<td>BCS</td>
<td>0.59</td>
</tr>
<tr>
<td>Rice ($^6\text{Li}$)</td>
<td>Astrakharchik</td>
<td>0.42(1)</td>
</tr>
<tr>
<td>JILA($^{40}\text{K}$)</td>
<td>Perali</td>
<td>0.455</td>
</tr>
<tr>
<td>Innsbruck ($^6\text{Li}$)</td>
<td>Carlson</td>
<td>0.42(1)</td>
</tr>
<tr>
<td>Duke ($^6\text{Li}$)</td>
<td>Haussmann</td>
<td>0.36</td>
</tr>
</tbody>
</table>
Universal phase diagram of the homogeneous unitary system


« Exact » eigenstates of the grand potential

- Single component ideal gas \( \Omega = \Omega_0 \)
- Fully paired superfluid

\(|\text{SF}\rangle_{\mu} \) eigenstate of the \textit{balanced} grand-potential

\[
\hat{\Omega}' = \hat{H} - \mu(\hat{N}_\uparrow + \hat{N}_\downarrow)
\]

\[
\hat{\Omega}'|\text{SF}\rangle_{\mu} = \Omega'|\text{SF}\rangle_{\mu} \quad \hat{\mathcal{N}}_\uparrow|\text{SF}\rangle_{\mu} = \hat{\mathcal{N}}_\downarrow|\text{SF}\rangle_{\mu} = N|\text{SF}\rangle_{\mu}
\]

\[
\hat{\Omega} = \hat{H} - \mu_\uparrow \hat{\mathcal{N}}_\uparrow - \mu_\downarrow \hat{\mathcal{N}}_\downarrow \quad \Rightarrow \hat{\Omega}|\text{SF}\rangle_{(\mu_\uparrow + \mu_\downarrow)/2} = \Omega'|\text{SF}\rangle_{(\mu_\uparrow + \mu_\downarrow)/2}
\]

\[
\hat{\Omega} = \hat{H} - \frac{\mu_\uparrow + \mu_\downarrow}{2}(\hat{\mathcal{N}}_\uparrow + \hat{\mathcal{N}}_\downarrow) - \frac{\mu_\uparrow - \mu_\downarrow}{2}(\hat{\mathcal{N}}_\uparrow - \hat{\mathcal{N}}_\downarrow)
\]
Universal phase diagram of the homogeneous unitary system (2)

\[
\begin{align*}
\Omega &= -PV \\
\text{d}P &= \sum_{\sigma=\uparrow,\downarrow} n_\sigma \text{d}\mu_\sigma
\end{align*}
\]

\[\Rightarrow \quad \text{Just need to know} \quad n(\mu)\]

\[
P = P_0 (1 + \eta)^{5/2} / (2\xi)^{3/2}
\]

\[
\begin{array}{c}
\eta = \mu_\downarrow / \mu_\uparrow \\
\eta_\alpha > \eta_c > \eta_\beta
\end{array}
\]

\[
\eta_c = (2\xi)^{3/5} - 1 \\
\approx -0.099
\]
Theoretical evidence for an intermediate phase

General properties of a mixed branch?

Step 1: calculate the energy $E$ of a single impurity atom immersed in a Fermi sea ($E = \mu_\downarrow$, with $n_\downarrow = 0^+$)

Step 2: $dP/d\mu_\sigma = n > 0$

$\eta_\beta < \eta_c$: the new branch is stable

$\eta_\beta > \eta_c$: the new branch is unstable
Variational upper bound for the N+1 body problem

\[ \hat{H} = \sum_{k\sigma} \varepsilon_k \hat{a}_{k\sigma}^{\dagger} \hat{a}_{k\sigma} + \frac{g_b}{\Omega} \sum_{k,k',q} \hat{a}_{k' - q, \uparrow}^{\dagger} \hat{a}_{k+q, \downarrow}^{\dagger} \hat{a}_{k, \downarrow} \hat{a}_{k, \uparrow} \]

\[ \varepsilon_k = \frac{\hbar^2 k^2}{2m} \quad \frac{1}{g_b} = \frac{m}{4\pi \hbar^2 a} - \frac{1}{\Omega} \sum_k \frac{1}{2\varepsilon_k} \]

One impurity: restrict the effect of interactions to the formation of a single particle-hole pair.

\[ |\Psi\rangle = \varphi_0 |0\rangle + \sum_{k,q} \varphi_{k,q} |k, q\rangle \]

\[ |0\rangle = \]

\[ |kq\rangle = \]

\[ \text{q-k} \]
Comparison with Monte-Carlo simulations (C. Lobo et al. PRL. 97, 200403 (2006))

Minimization of H with respect to $\varphi_0$ and $\varphi_{kq}$

$$E = \frac{1}{V} \sum_{q<k_F} \frac{m}{4\pi\hbar^2a} - \frac{1}{V} \sum_{k<k_F} \frac{1}{2\varepsilon_k} + \sum_{k>k_F} \left( \frac{1}{E-(\varepsilon_k+\varepsilon_{q-k}-\varepsilon_q)} - \frac{1}{2\varepsilon_k} \right) \frac{1}{2\varepsilon_k}$$

$$\varepsilon_k = \frac{\hbar^2 k^2}{2m}$$

For $a=\infty$, $E=-0.606\ E_F$, $\eta_\beta < -0.606 < \eta_c \sim -0.1$

Monte Carlo simulations: $\eta_\beta = -0.58(1)$
BCS theory: $\eta_\beta = 0$

Why such a good agreement?

Weak excitation of the pairs, even at unitarity

$$\sum_{k,q} |\varphi_{k,q}|^2 = 0.2$$
Three phase mixture in a trap: MIT
Interpretation of MIT’s experiment

Local density approximation in harmonic trap $V(r) \sim r^2$

\[ \mu_\uparrow(r) = \mu_\uparrow^0 - V(r) \]
\[ \mu_\downarrow(r) = \mu_\downarrow^0 - V(r) \]

$R_\uparrow$ : outer radius of the majority component \hspace{1cm} $\mu_\uparrow(R_\uparrow) = 0$
$R_\beta$ : outer radius of the minority component \hspace{1cm} $\mu_\downarrow(R_\beta) / \mu_\uparrow(R_\beta) = \eta_\beta$
$R_\alpha$ : radius of the superfluid core \hspace{1cm} $\mu_\downarrow(R_\alpha) / \mu_\uparrow(R_\alpha) = \eta_\alpha$

\[ \frac{R_\alpha}{R_\uparrow} = \sqrt{\frac{(R_\beta / R_\uparrow)^2 - q}{1 - q}} \hspace{1cm} q = \frac{\eta_\alpha - \eta_\beta}{1 - \eta_\beta} \]
Comparison with experimental data

Superfluid radius

$q=0.32$
Improved bounds for $\eta_\alpha$ and $\eta_\beta$

$q = 0.32$
$\eta_\alpha > -0.10$
$\eta_\beta < -0.60$

$-0.62 < \eta_\beta < -0.60$
$-0.10 < \eta_\alpha < -0.088$

$\eta = \frac{\mu_\downarrow}{\mu_\uparrow}$

BCS:
$\eta_\alpha = 0.1$
$\eta_\beta = 0$

Convexity of $P$:
real equation of state below
Experiment by Rice’s group: fully compatible with 2 phase scenario (no intermediate phase+LDA). No adjustable parameters.

For $P=0.7$, $q \approx 0.16$ differs from MIT and contradicts theoretical bounds: $q > 0.31$ set by $\eta_\alpha > -0.1$ and $\eta_\beta < 0.60$
Summary: part 1

What have we demonstrated?

3 homogeneous phases in the phase diagram of imbalanced unitary Fermi gases
LDA valid agreement with MIT

But a host of unanswered questions!

Nature of the MIT/Rice's discrepancy (surface tension ?)
Microscopic nature of the intermediate phase?
Superfluid nature of the intermediate phase?

Dynamical properties, collective modes?
Extension to the BEC-BCS crossover (in progress)
Response to RF excitation (MIT experiment, Schunck et al. cond-mat/0702066)
Part 2. Measuring one-particle excitations
In Fermi gases using Raman spectroscopy

T-L Dao, A. Georges, J. Dalibard, C. Salomon, I. Carusotto, Cond-mat/0611206

In Fermi liquid theory: low energy excitations are build out of quasiparticles
Dispersion relation on a given point of the Fermi surface:

\[ \xi_k \sim v_F(k_F) + (k - k_F) + \ldots \]

Lifetime: \( \Gamma_k^{-1} \)

Fermi surface: excitation energy vanishes: \( \xi_{k_F} = 0 \)

Normal phase in Cuprate SC show strong deviations, anisotropic behavior.

Probe directly one particle correlator:

\[ \langle \psi^\dagger(r, t) \psi(r', t') \rangle \]

Bragg spectroscopy or noise corr.:

\[ \langle \psi^\dagger(r, t) \psi(r, t) \psi^\dagger(r', t') \psi(r', t') \rangle \]

Stimulated Raman spectroscopy: widely used for Bose systems

Very interesting for fermions:
Probing Fermi surface of strongly interacting fermions (Time of Flight not adequate)
Momentum-resolved quasiparticles excitations
Two-photon Raman excitation

Interacting fermions: \( |\alpha >, |\alpha' >\)
For instance \( ^6\text{Li} \) near F. Resonance

Third state empty: \( |\beta >\)
No interaction with \( |\alpha >, |\alpha' >\)

Similar to ARPES in cond. Matter.

\[
R(q, \Omega) \sim \int_{-\infty}^{+\infty} dt \int dr dr' e^{i[\Omega t - q \cdot (r - r')] \times g_{\beta}(r, r'; t) \left\langle \psi_{\alpha}^\dagger (r, t) \psi_{\alpha} (r', 0) \right\rangle
\]

Selectivity in \( q = k_1 - k_2 \)
Selectivity in energy
Interacting Fermions in homogeneous 2D square Lattice

Raman resonance condition \( \varepsilon_{k+q,\beta} - \xi_k = \Omega \)

Threshold in \( \Omega: \quad (\omega_1 - \omega_2)_T = \varepsilon^0_\beta - \mu \sim \varepsilon^0_\beta - \varepsilon^0_\alpha \)

\( \Omega \) close to threshold for \( q = -k_F \)

Shell surrounding Fermi Surface

d of width \( \sqrt{2M\Delta\Omega} \sim \sqrt{2Mv_F(k_F)(q+k_F)} \)

Non interacting fermions

\[
\xi_k = -2t_\alpha (\cos k_x + \cos k_y) - \mu
\]

Model pseudo-gap with d-wave symmetry

\[
\Delta_k = \Delta_0 (\cos k_x - \cos k_y)
\]

\( \Delta_0 = 0.1t_\alpha \)
\( N_\alpha = 0.45 \)
\( \Gamma_0 = 0.05t_\alpha \)
\( \Gamma_1 = 0.4t_\alpha \)
Rate $R(q, \Omega)$ and spectral function $A(k, \Omega)$

For fixed value of $q$, chosen near Nodal line or Anti-nodal Scan Raman detuning
In Harmonic Trap

LDA: $\mu(R) = \mu_0 - M \omega_0^2 R^2 / 2$

Non-interacting fermions  Model d-wave pseudogap state
New experimental setup

Enlarged glass cell
New laser sources: 120 mW diodes operating at 75 degree C
New Zeeman slower
More stable Ioffe-Pritchard trap
120 Watt far detuned optical trap (Fiber laser)
Access for 3D optical lattice
3 $10^{10}$ $^7$Li atoms in MOT  
expected increase of x10 in $^6$Li number
Ongoing: Transfer into magnetic trap
Thank you for your attention!