



Strongly interacting cold atoms

No $\backslash \text{cl} \text{N} [] \text{b}^{\wedge} _ \text{p}$

Talks online at <http://sachdev.physics.harvard.edu>



Outline

Strongly interacting cold atoms

1. Quantum liquids near unitarity:
from few-body to many-body physics
 - (a) Tonks gas in one dimension
 - (b) Paired fermions across a Feshbach resonance

2. Optical lattices
 - (a) Superfluid-insulator transition
 - (b) Quantum-critical hydrodynamics via mapping to quantum theory of black holes.
 - (c) Entanglement of valence bonds

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Strongly interacting cold atoms

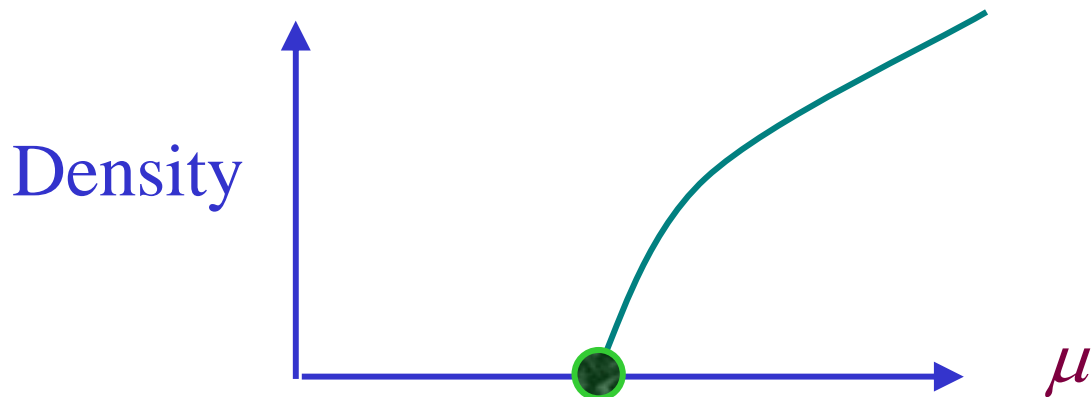
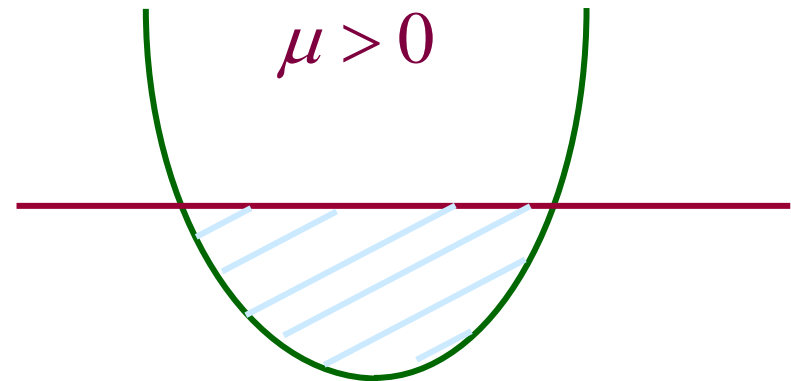
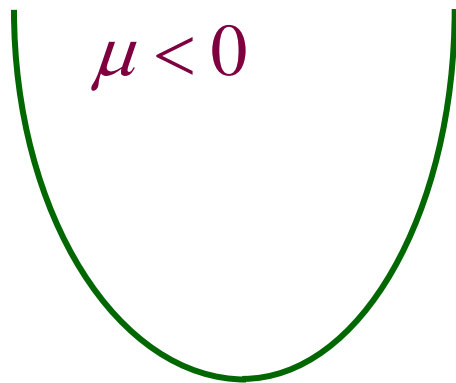
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Fermions with repulsive interactions

$$H = \sum_k (\varepsilon_k - \mu) c_{k\sigma}^\dagger c_{k\sigma}$$

+ short-range repulsive interactions of strength u



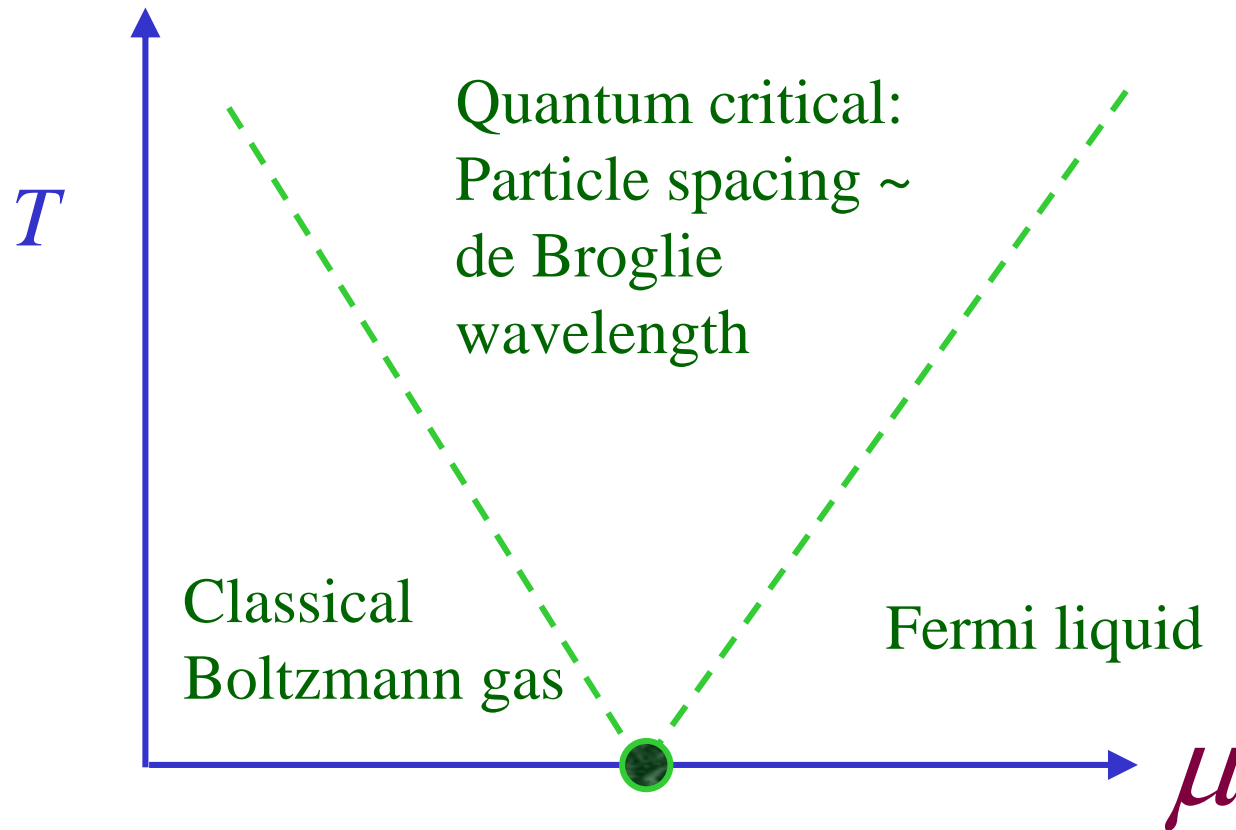
Fermions with repulsive interactions

Characteristics of this ‘trivial’ quantum critical point:

- Zero density critical point allows an elegant connection between few body and many body physics.
- No “order parameter”. “Topological” characterization in the existence of the Fermi surface in one state.
- No transition at $T > 0$.
- Characteristic crossovers at $T > 0$, between quantum criticality, and low T regimes.

Fermions with repulsive interactions

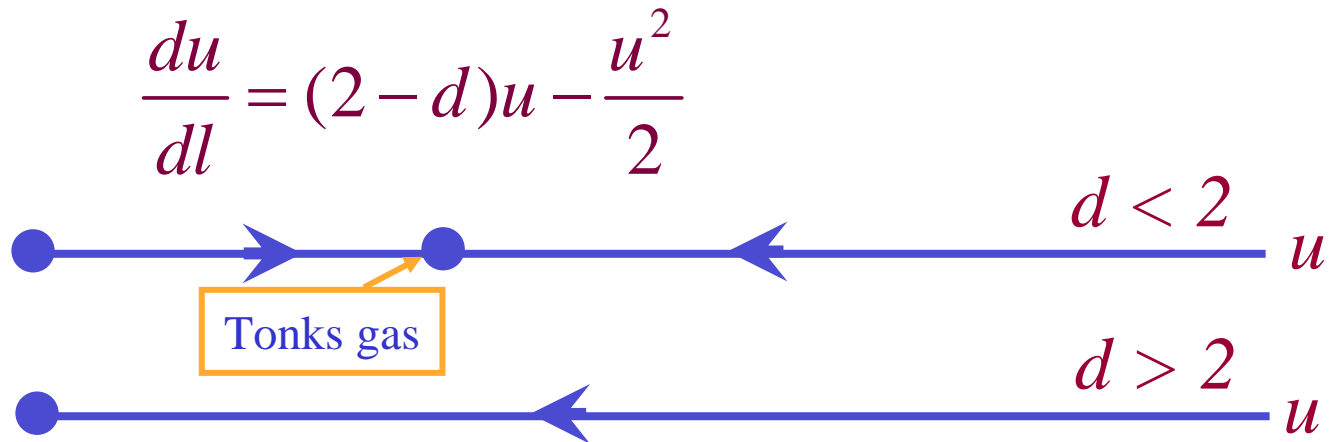
Characteristics of this 'trivial' quantum critical point:



Fermions with repulsive interactions

Characteristics of this ‘trivial’ quantum critical point:

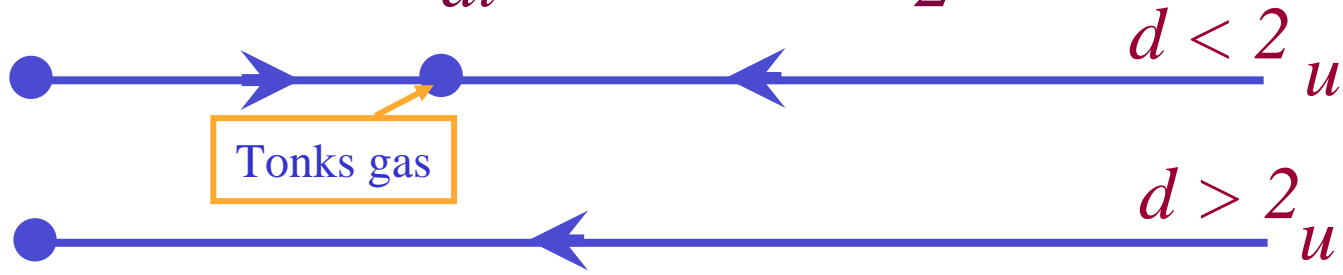
RG flow characterizing quantum critical point:



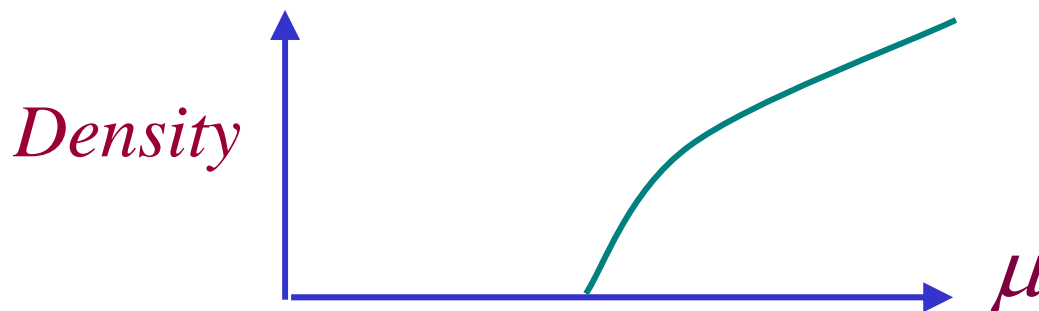
- $d > 2$ – interactions are irrelevant. Critical theory is the *spinful* free Fermi gas.
- $d < 2$ – universal fixed point interactions. In $d=1$ critical theory is the *spinless* free Fermi gas (Tonks gas).

Bosons with repulsive interactions

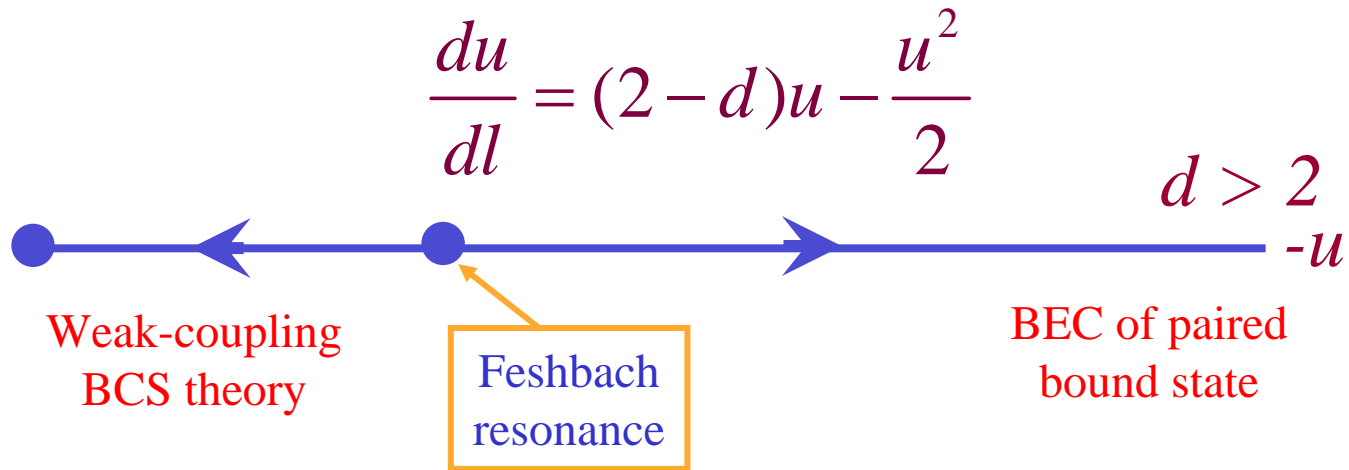
$$\frac{du}{dl} = (2-d)u - \frac{u^2}{2}$$



- Critical theory in $d = 1$ is also the *spinless free Fermi gas* (Tonks gas).
- The dilute Bose gas in $d > 2$ is controlled by the zero-coupling fixed point. Interactions are “dangerously irrelevant” and the density above onset depends upon bare interaction strength (Yang-Lee theory).

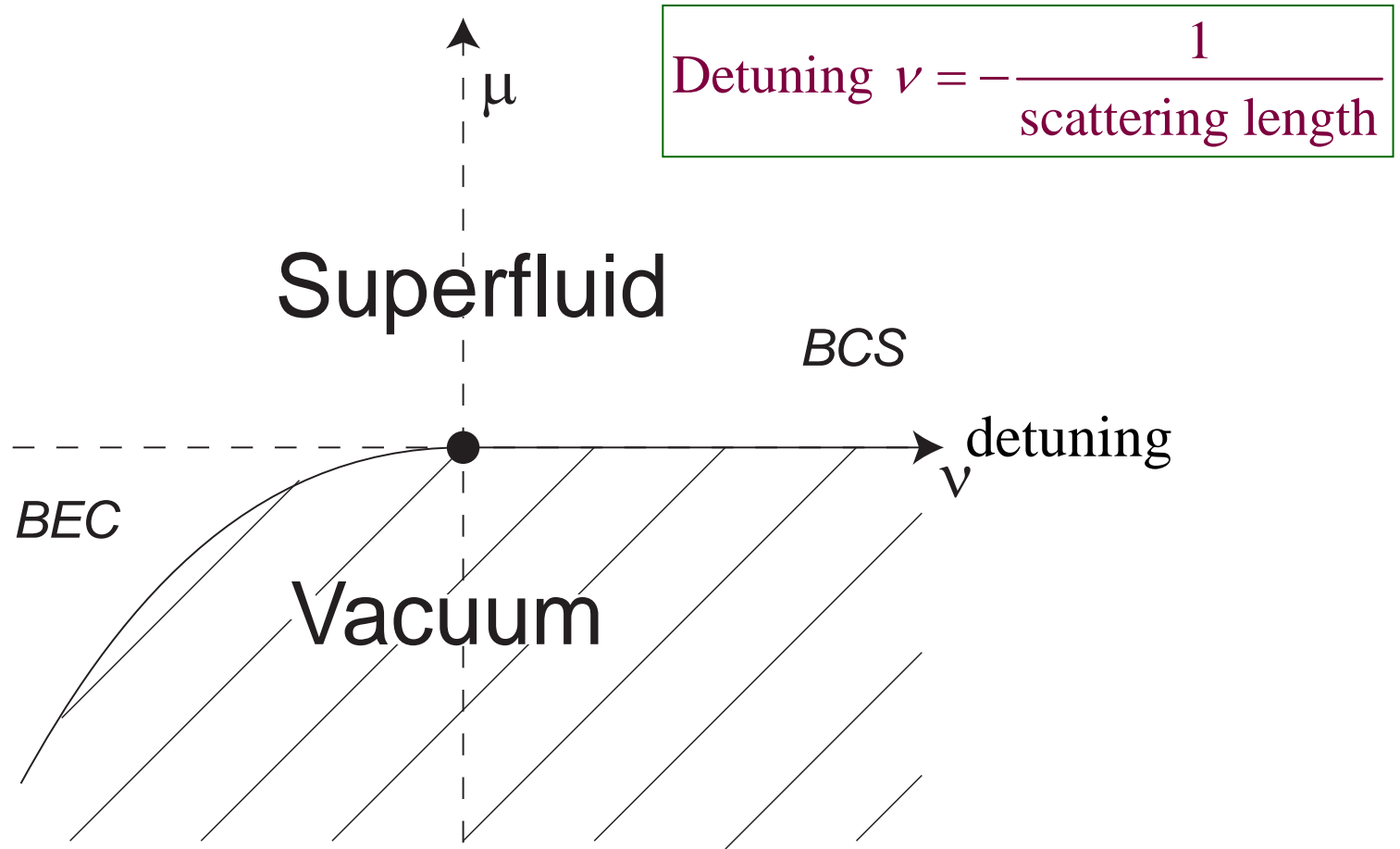


Fermions with attractive interactions

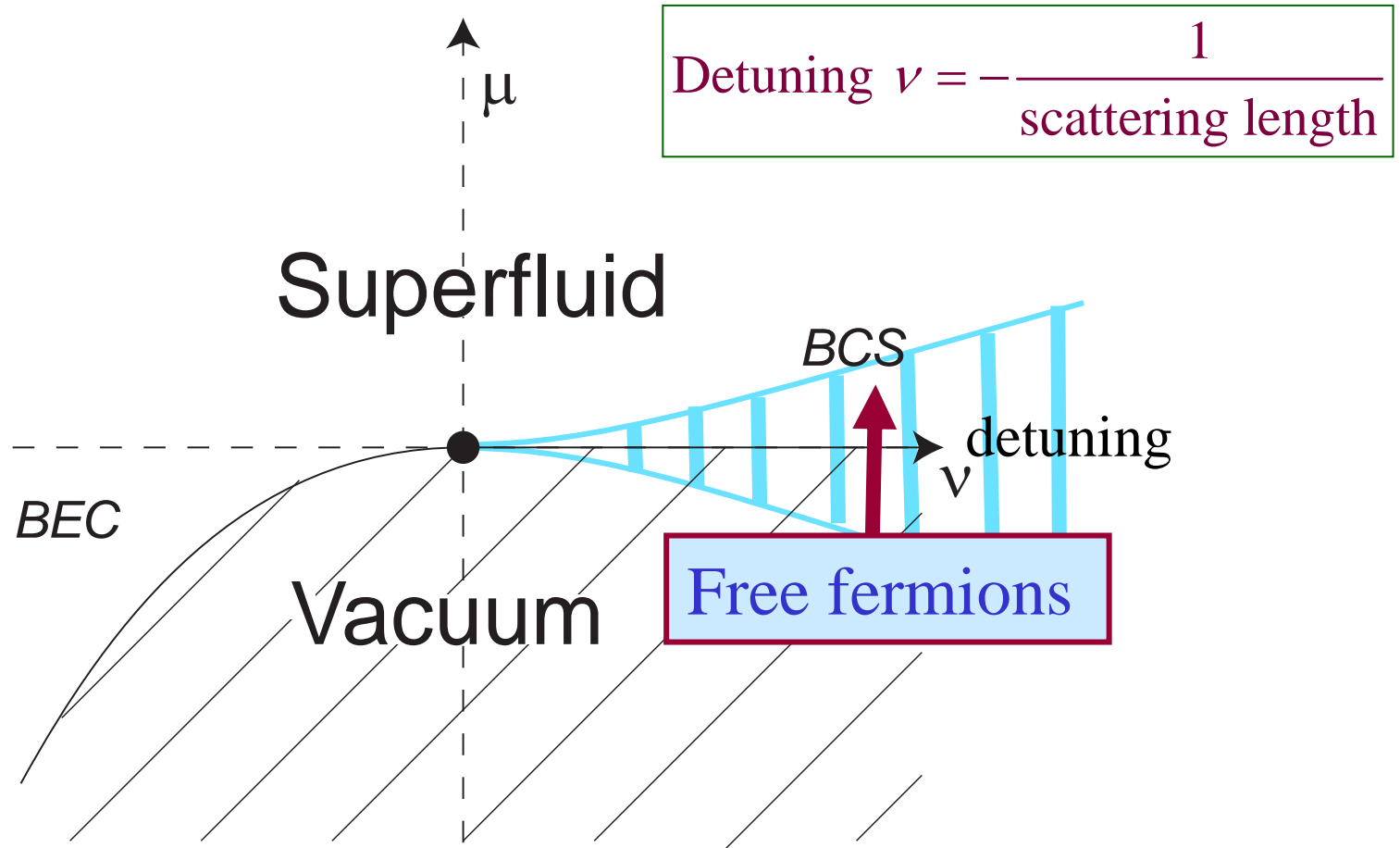


- Universal fixed-point is accessed by *fine-tuning* to a Feshbach resonance.
- Density onset transition is described by free fermions for weak-coupling, and by (nearly) free bosons for strong coupling. The quantum-critical point between these behaviors is the Feshbach resonance.

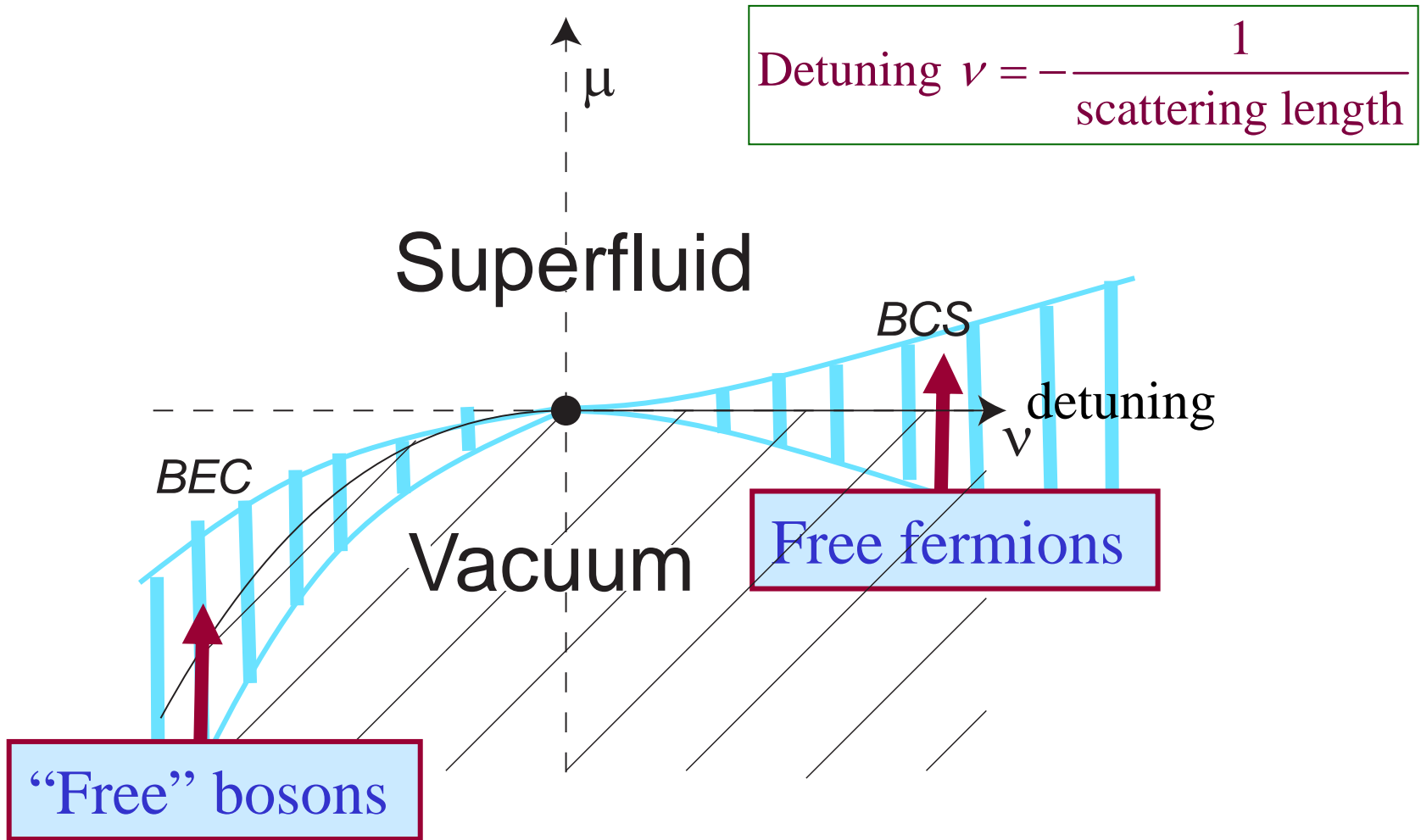
Fermions with attractive interactions



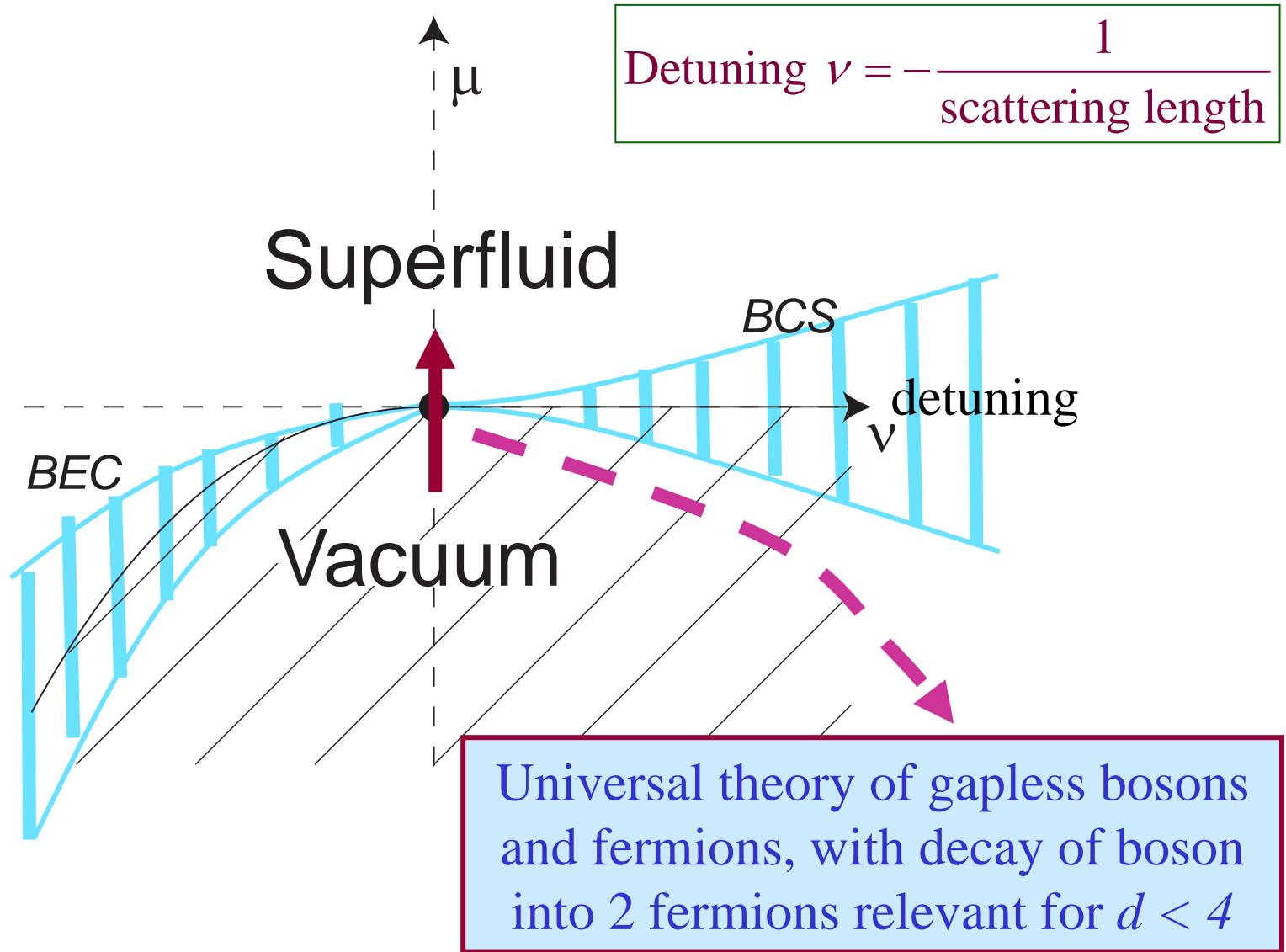
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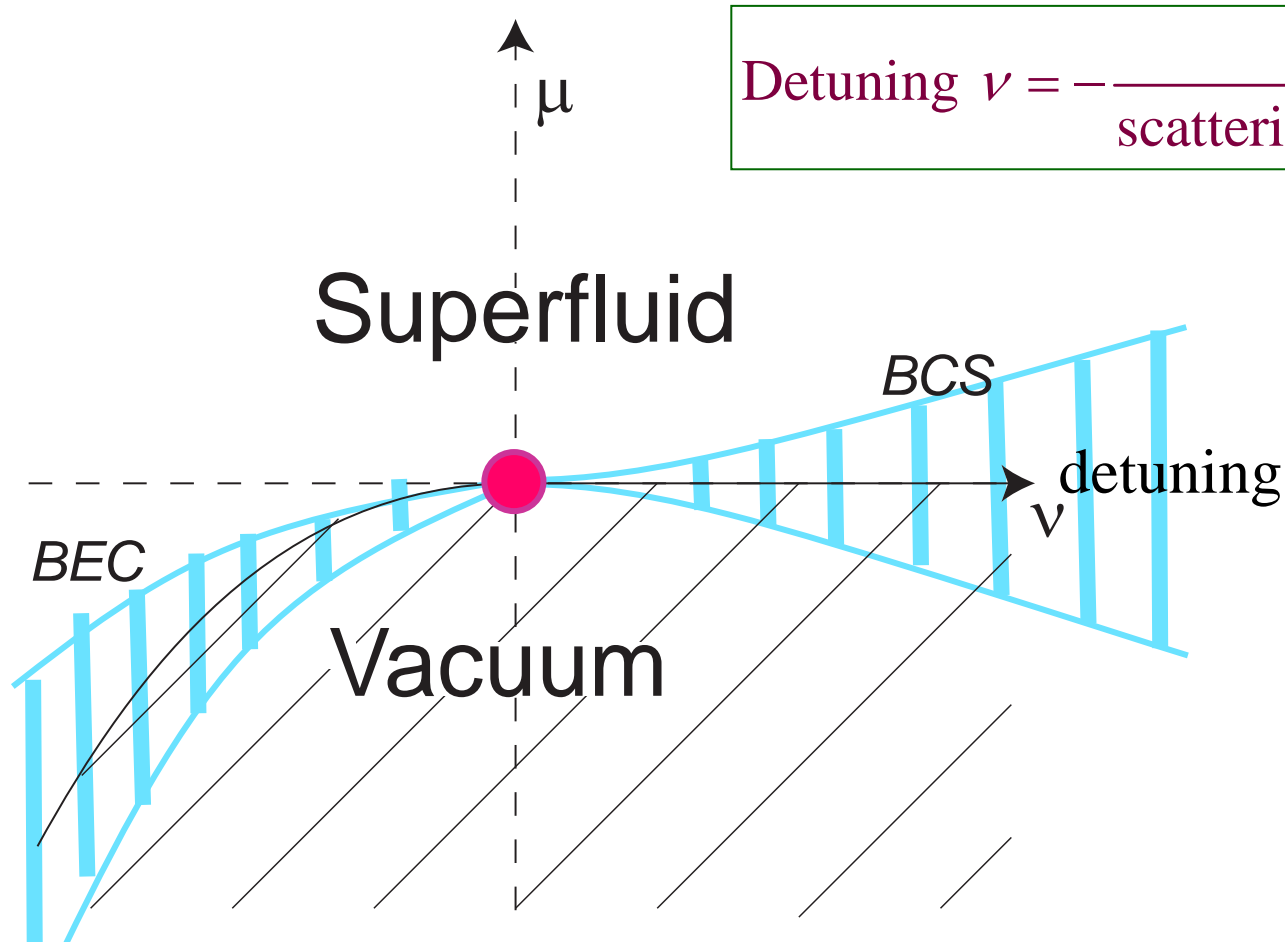
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Fermions with attractive interactions



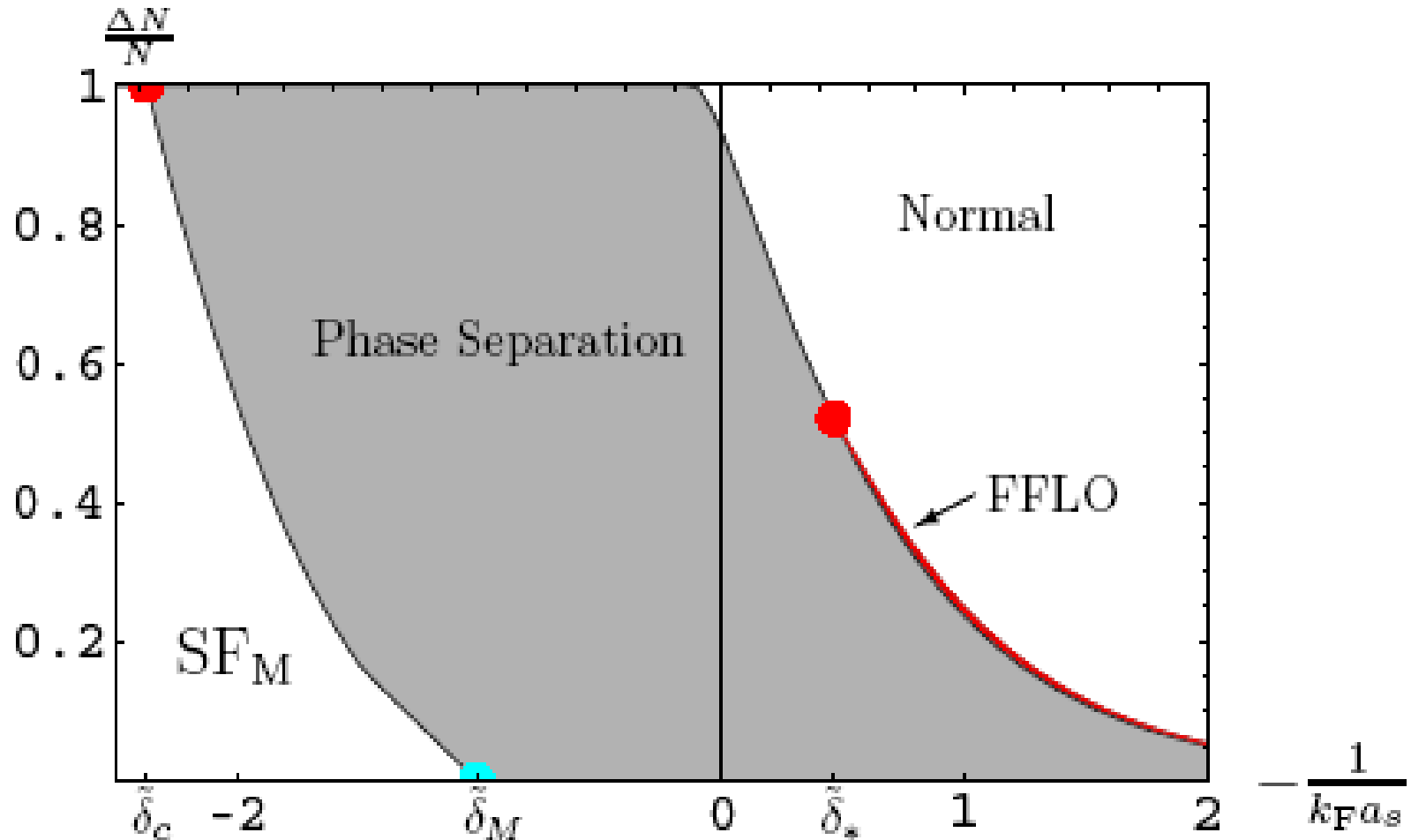
Fermions with attractive interactions



Quantum critical point at $\mu=0, \nu=0$, forms the basis of the theory of the BEC-BCS crossover, including the transitions to FFLO and normal states with unbalanced densities

Fermions with attractive interactions

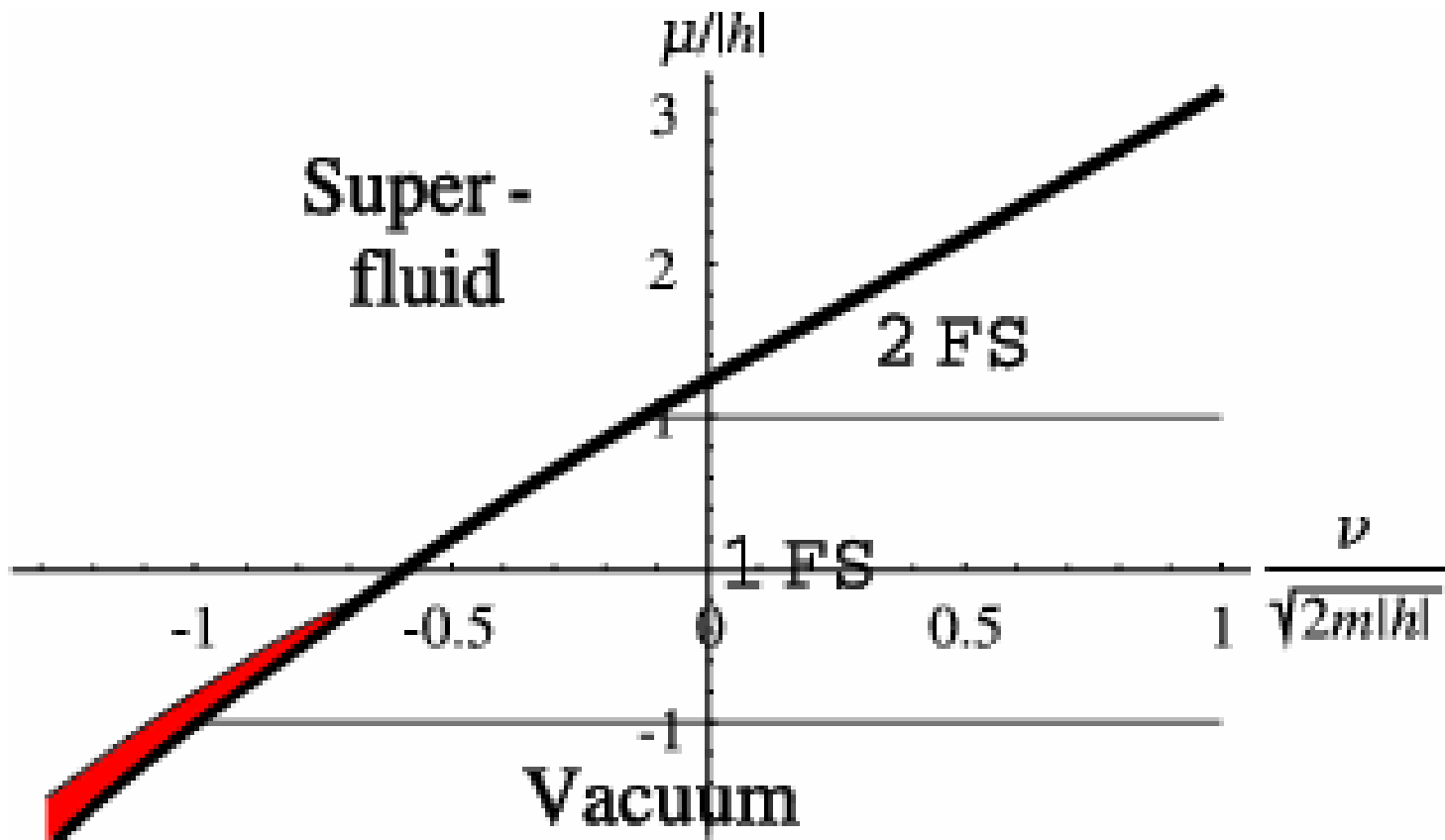
Universal phase diagram



D. E. Sheehy and L. Radzihovsky, *Phys. Rev. Lett.* **95**, 130401 (2005)

Fermions with attractive interactions

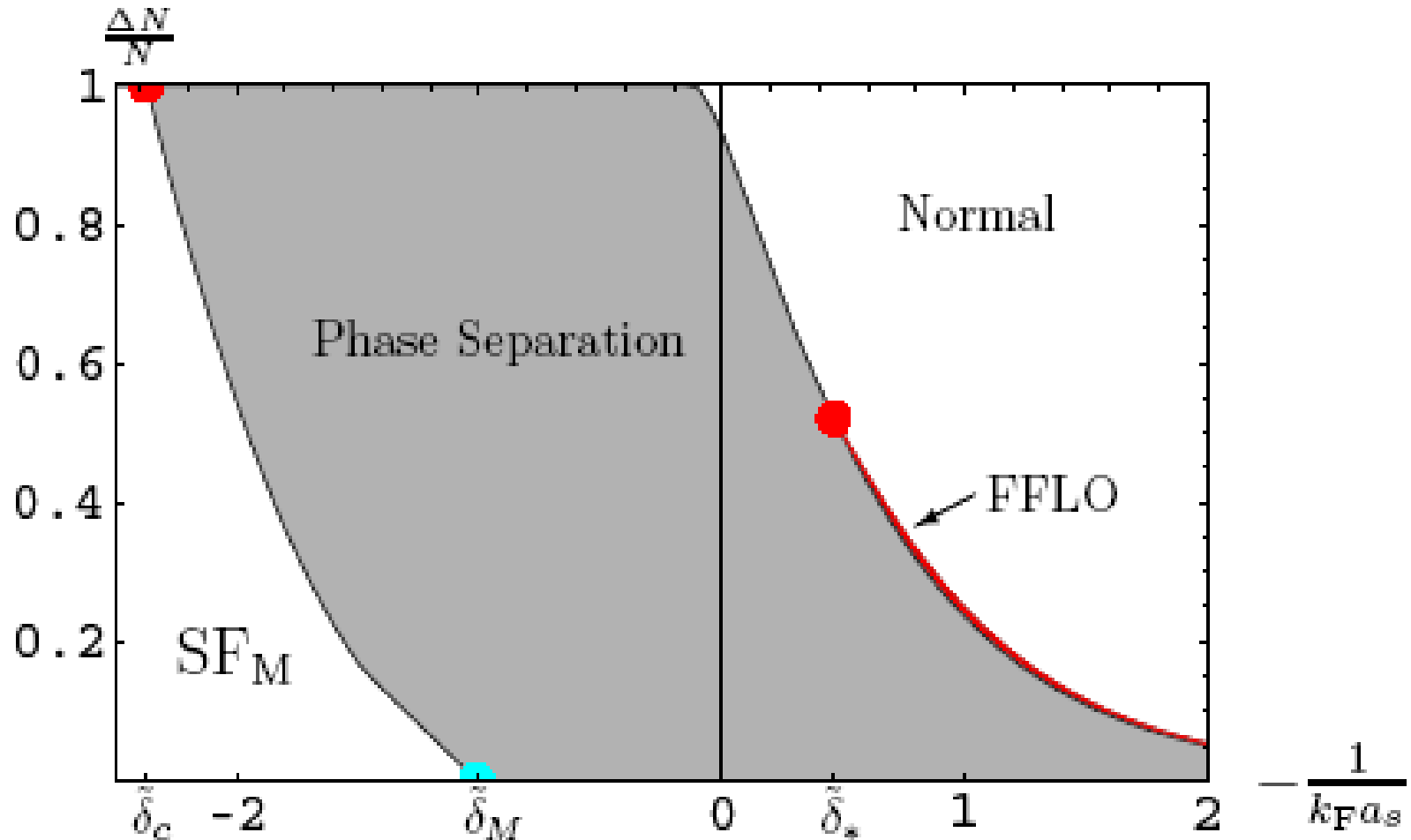
Universal phase diagram



h – Zeeman field

Fermions with attractive interactions

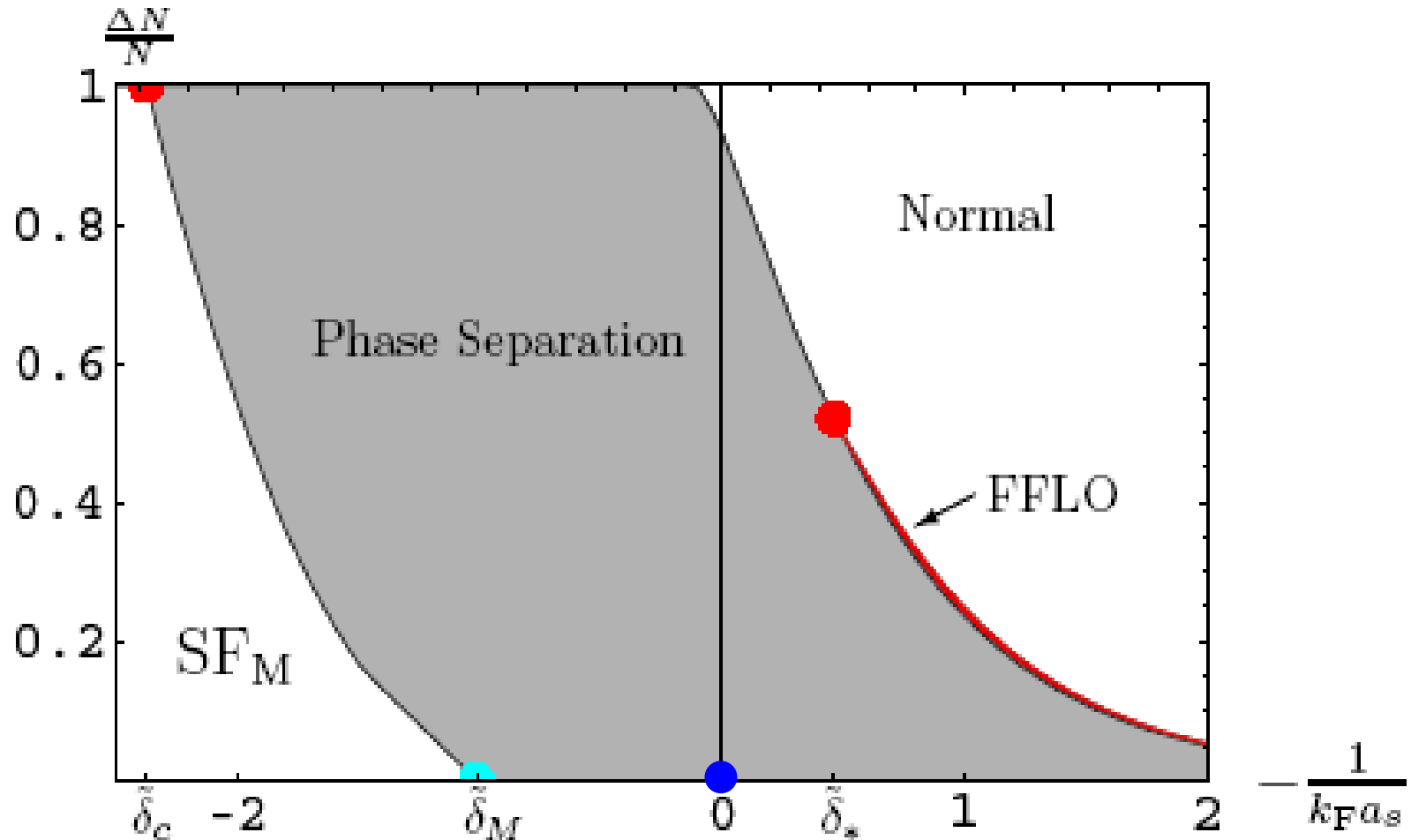
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Fermions with attractive interactions

Universal phase diagram



D. E. Sheehy and L. Radzihovsky, *Phys. Rev. Lett.* **95**, 130401 (2005)

Fermions with attractive interactions

Ground state properties at unitarity and balanced density

$$\frac{\Delta}{\varepsilon_F} = \sqrt{\varepsilon} + \frac{\varepsilon^{3/2}}{8} \ln(\varepsilon) - 0.3947 \varepsilon^{3/2}$$

Expansion in $\varepsilon=4$ -d

Y. Nishida and D.T. Son,

Phys. Rev. Lett. **97**, 050403 (2006)

$$\frac{\mu}{\varepsilon_F} = \frac{\varepsilon^{3/2}}{2} + \frac{\varepsilon^{5/2}}{16} \ln(\varepsilon) - 0.0246 \varepsilon^{5/2}$$

$$\frac{\Delta}{\varepsilon_F} = 0.6864 - \frac{0.163}{N}$$

Expansion in $1/N$ with $\text{Sp}(2N)$ symmetry

M. Y. Veillette, D. E. Sheehy, and L. Radzihovsky

Phys. Rev. A **75**, 043614 (2007)

$$\frac{\mu}{\varepsilon_F} = 0.5906 - \frac{0.312}{N}$$

$$\frac{\Delta}{\varepsilon_F} = 0.54$$

Quantum Monte Carlo

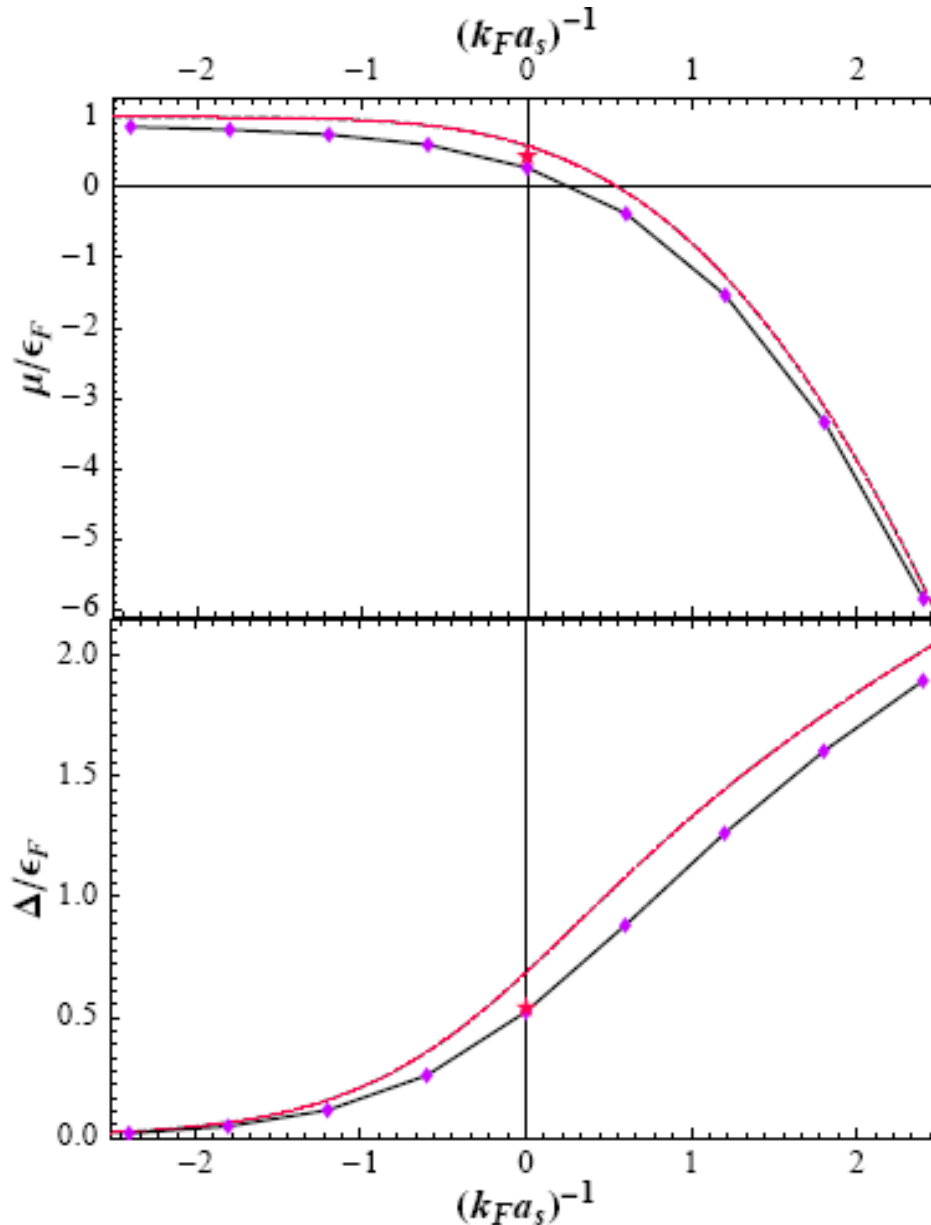
J. Carlon, S.-Y. Chang, V.R. Pandharipande, and

K.E. Schmidt, *Phys. Rev. Lett.* **91**, 050401 (2003).

$$\frac{\mu}{\varepsilon_F} = 0.44$$

Fermions with attractive interactions

Ground state properties near unitarity and balanced density



Expansion in $1/N$
with $Sp(2N)$
symmetry
M. Y. Veillette,
D. E. Sheehy, and
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Phys. Rev. A **75**, 043614
(2007)

Quantum Monte
Carlo
J. Carlon, S.-Y. Chang,
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K.E. Schmidt, *Phys. Rev.
Lett.* **91**, 050401 (2003).

Fermions with attractive interactions

Finite temperature properties at unitarity and balanced density

$$\frac{\varepsilon_F}{T_c} = 2.104 + \frac{5.317}{N}$$

$$\frac{\mu}{T_c} = 1.504 + \frac{2.785}{N}$$

$$\frac{P/N}{(2m)^{3/2} T_c^{5/2}} = 0.132 + \frac{0.405}{N}$$

Expansion in $1/N$ with $\text{Sp}(2N)$ symmetry

M. Y. Veillette, D. E. Sheehy, and L. Radzihovsky

Phys. Rev. A **75**, 043614 (2007)

P. Nikolic and S. Sachdev,

Phys. Rev. A **75**, 033608 (2007).

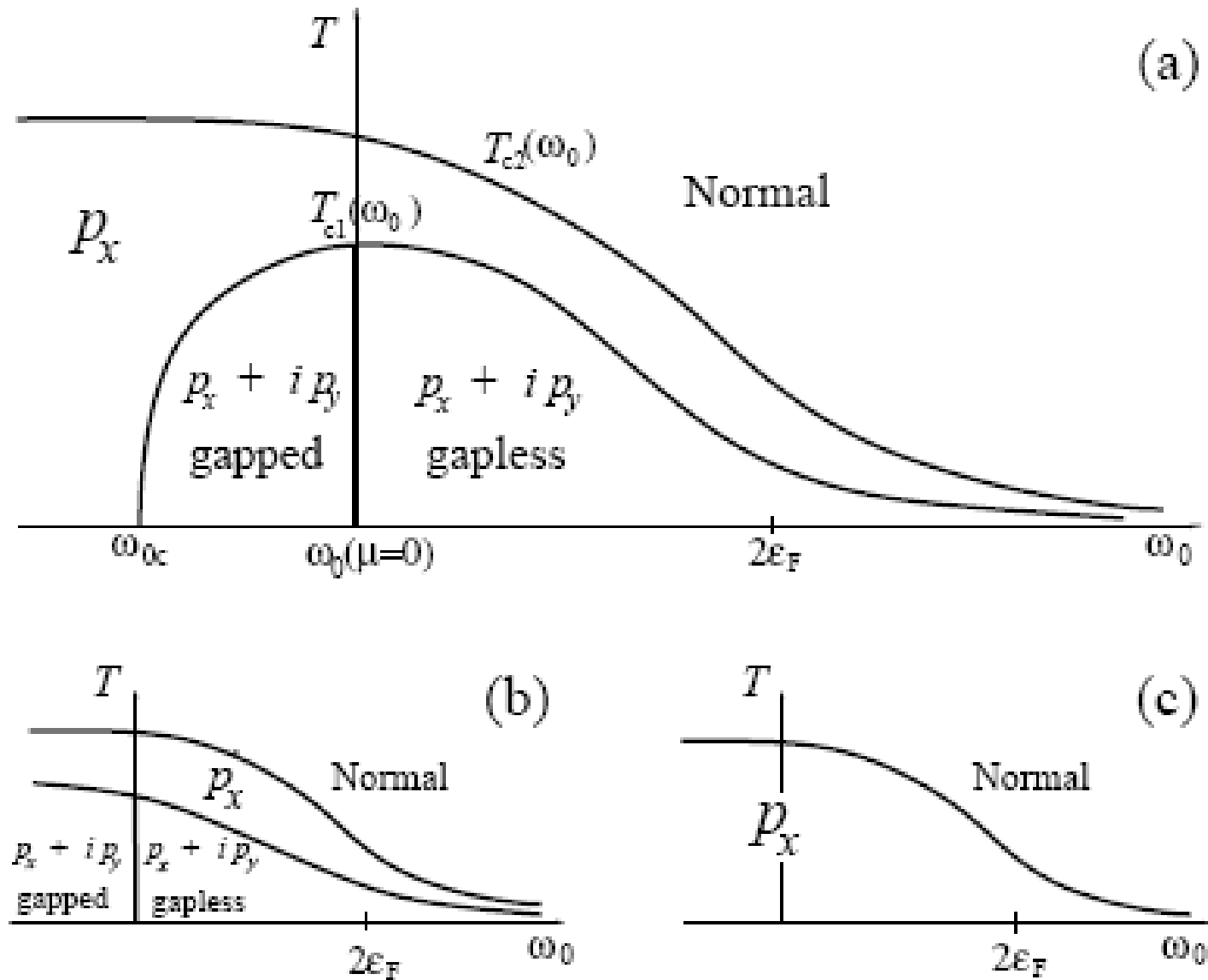
$$\frac{\varepsilon_F}{T_c} = 6.579$$

$$\frac{\mu}{T_c} = 3.247$$

$$\frac{P/N}{(2m)^{3/2} T_c^{5/2}} = 0.776$$

E. Burovski, N. Prokof'ev, B. Svistunov, and
M. Troyer, *New J. Phys.* **8**, 153 (2006)

Fermions with attractive interactions in p-wave channel



V. Gurarie, L. Radzihovsky, and A.V. Andreev, *Phys. Rev. Lett.* **94**, 230403 (2005)

C.-H. Cheng and S.-K. Yip, *Phys. Rev. Lett.* **95**, 070404 (2005)

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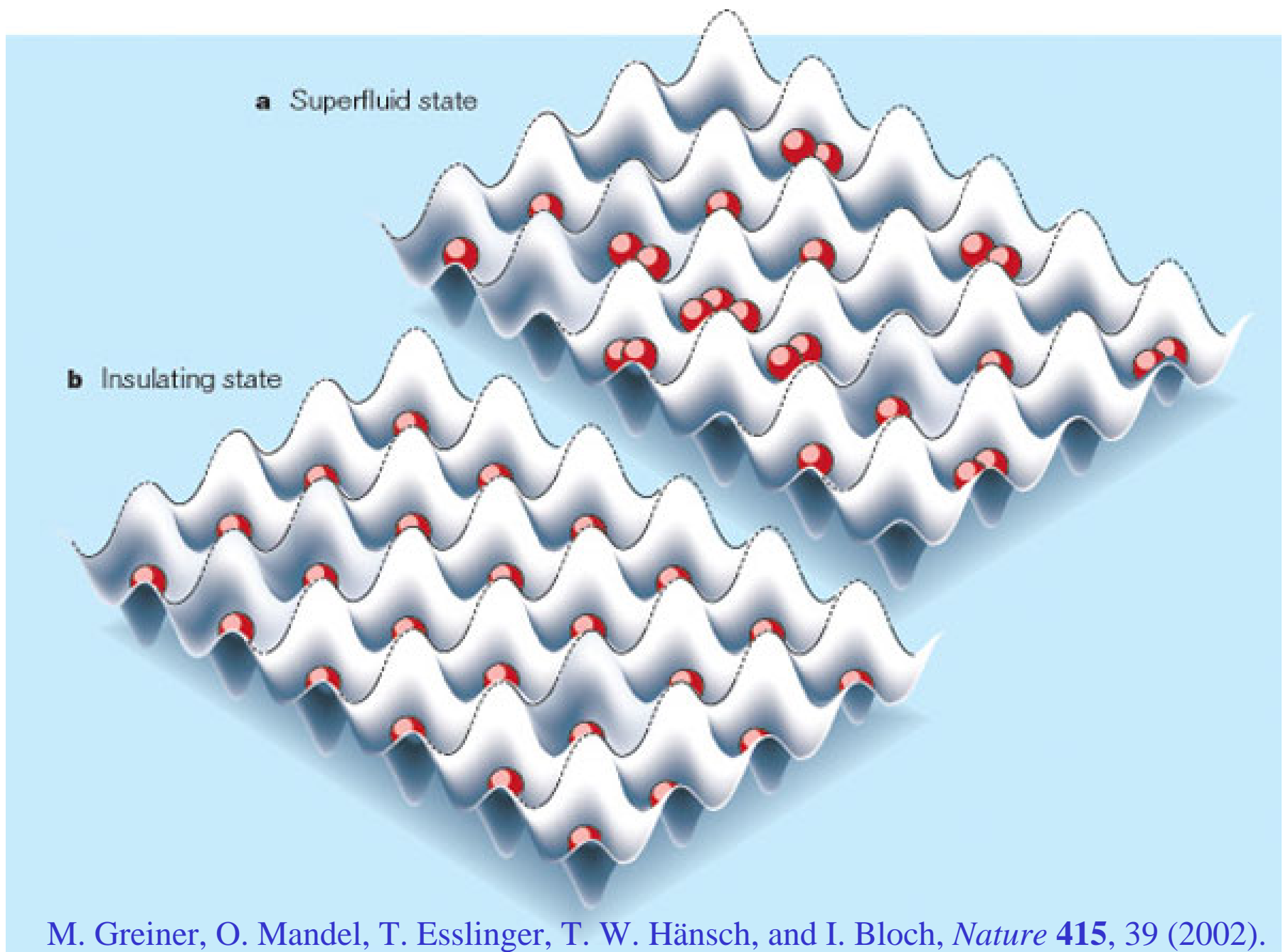
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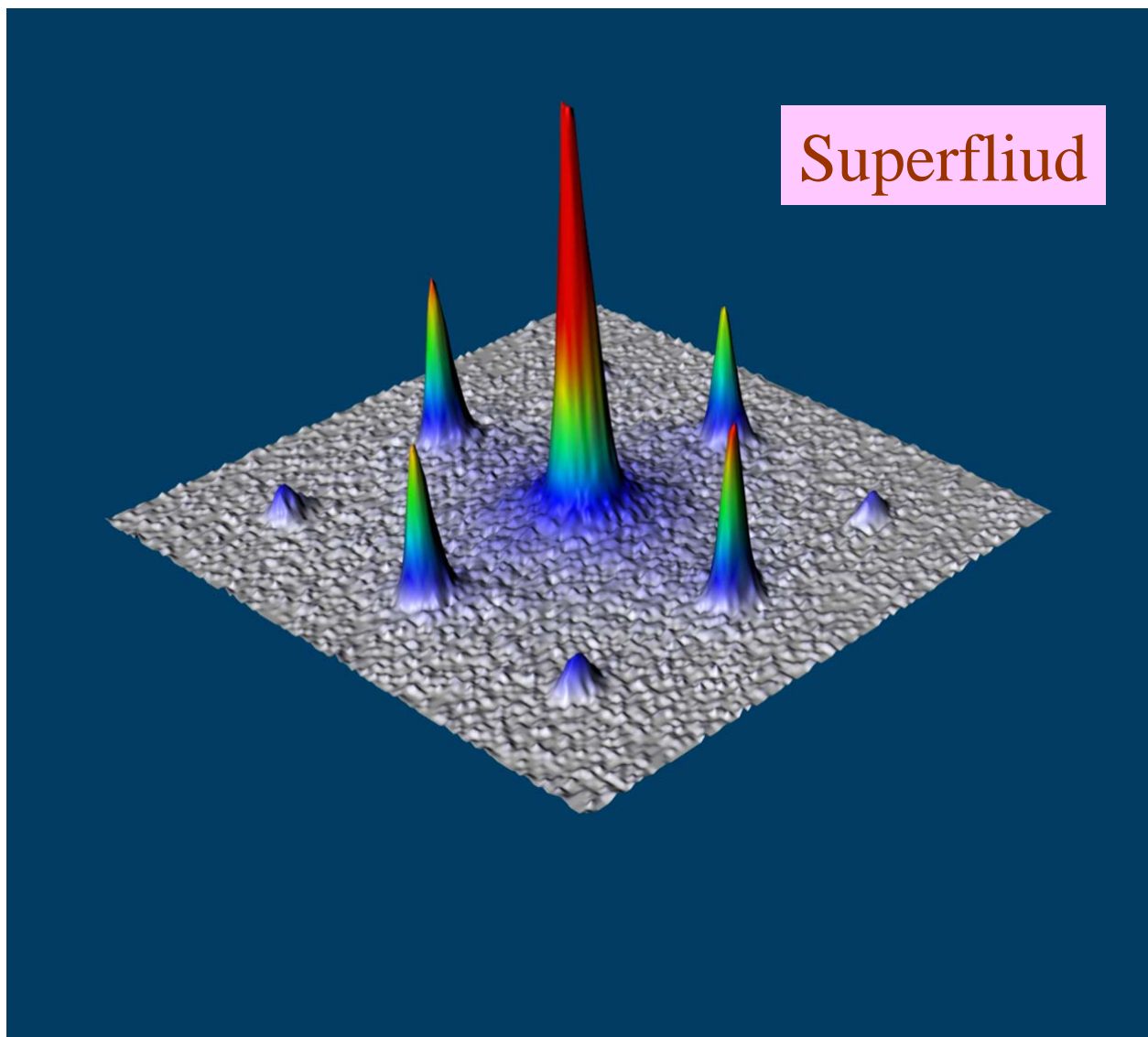
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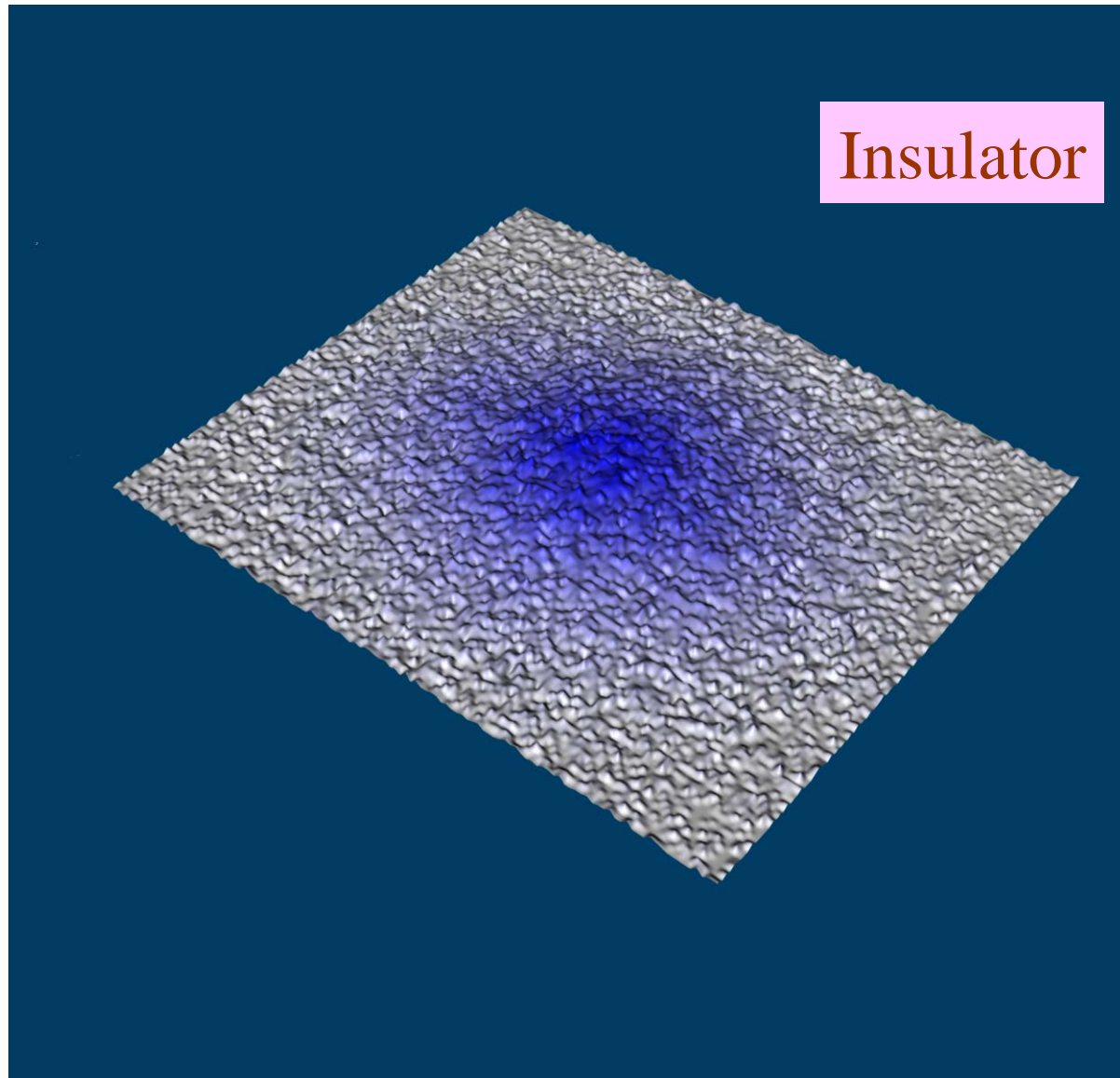
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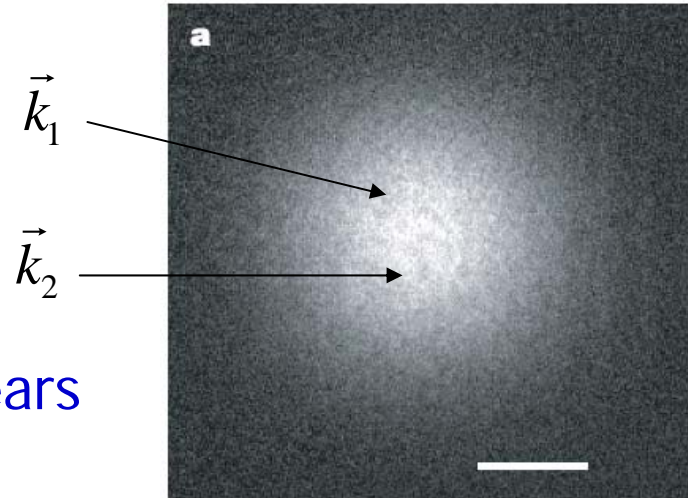
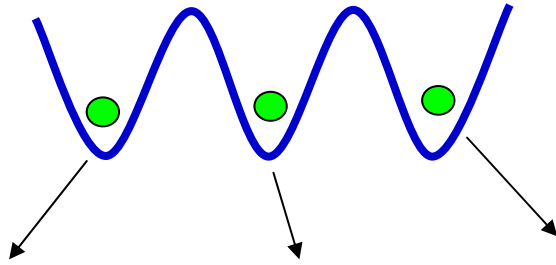
Velocity distribution of ^{87}Rb atoms



Velocity distribution of ^{87}Rb atoms



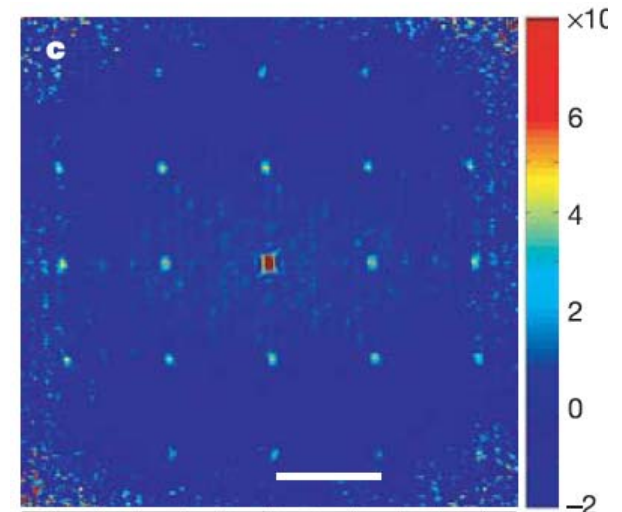
Noise correlation (time of flight) in Mott-insulators



- 1st order coherence $\langle n(\vec{k}) \rangle$ disappears in the Mott-insulating state.

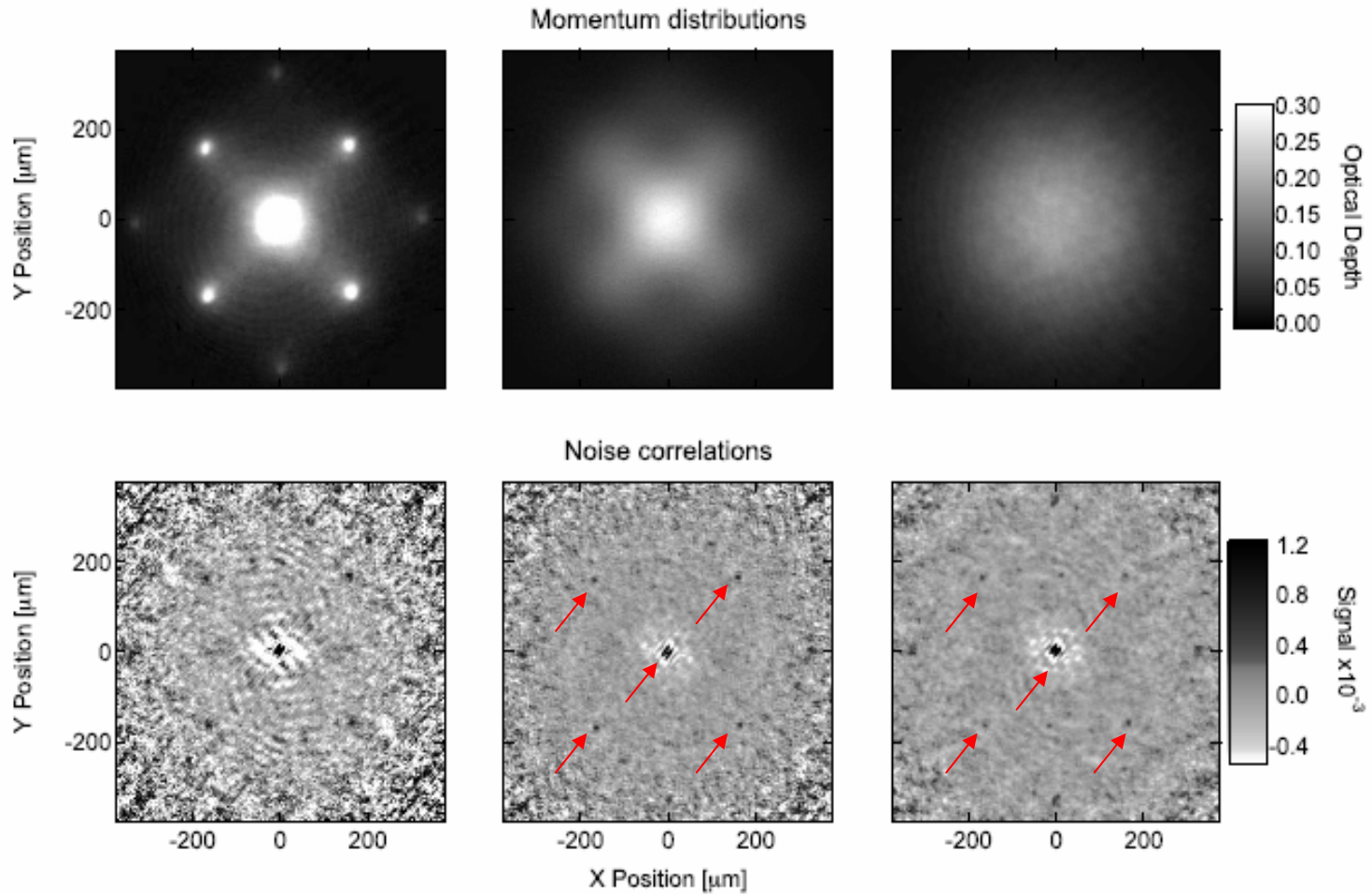
- Noise correlation function oscillates at reciprocal lattice vectors; bunching effect of bosons.

$$\langle n(\vec{k}_1)n(\vec{k}_2) \rangle - \langle n(\vec{k}_1) \rangle \langle n(\vec{k}_2) \rangle \propto \sum_{\vec{G}} \delta(\vec{k}_1 - \vec{k}_2 - \vec{G})$$



Folling et al., Nature 434, 481 (2005); Altman et al., PRA 70, 13603 (2004).

Two dimensional superfluid-Mott insulator transition



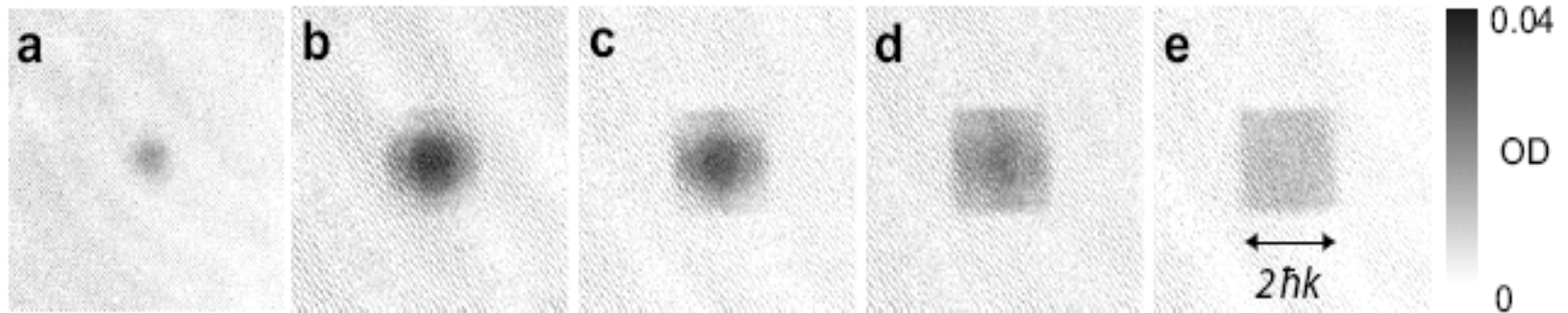
$$V / E_R = 12$$

$$V / E_R = 20$$

$$V / E_R = 21$$

Fermionic atoms in optical lattices

- Observation of Fermi surface. $^{40}\text{K} : |Fm\rangle = \left| \frac{99}{22} \right\rangle, \left| \frac{97}{22} \right\rangle$



Low density: metal

high density: band insulator

Esslinger et al.,
PRL 94:80403
(2005)

Fermions with near-unitary interactions in the
presence of a periodic potential

Fermions with near-unitary interactions in the presence of a periodic potential

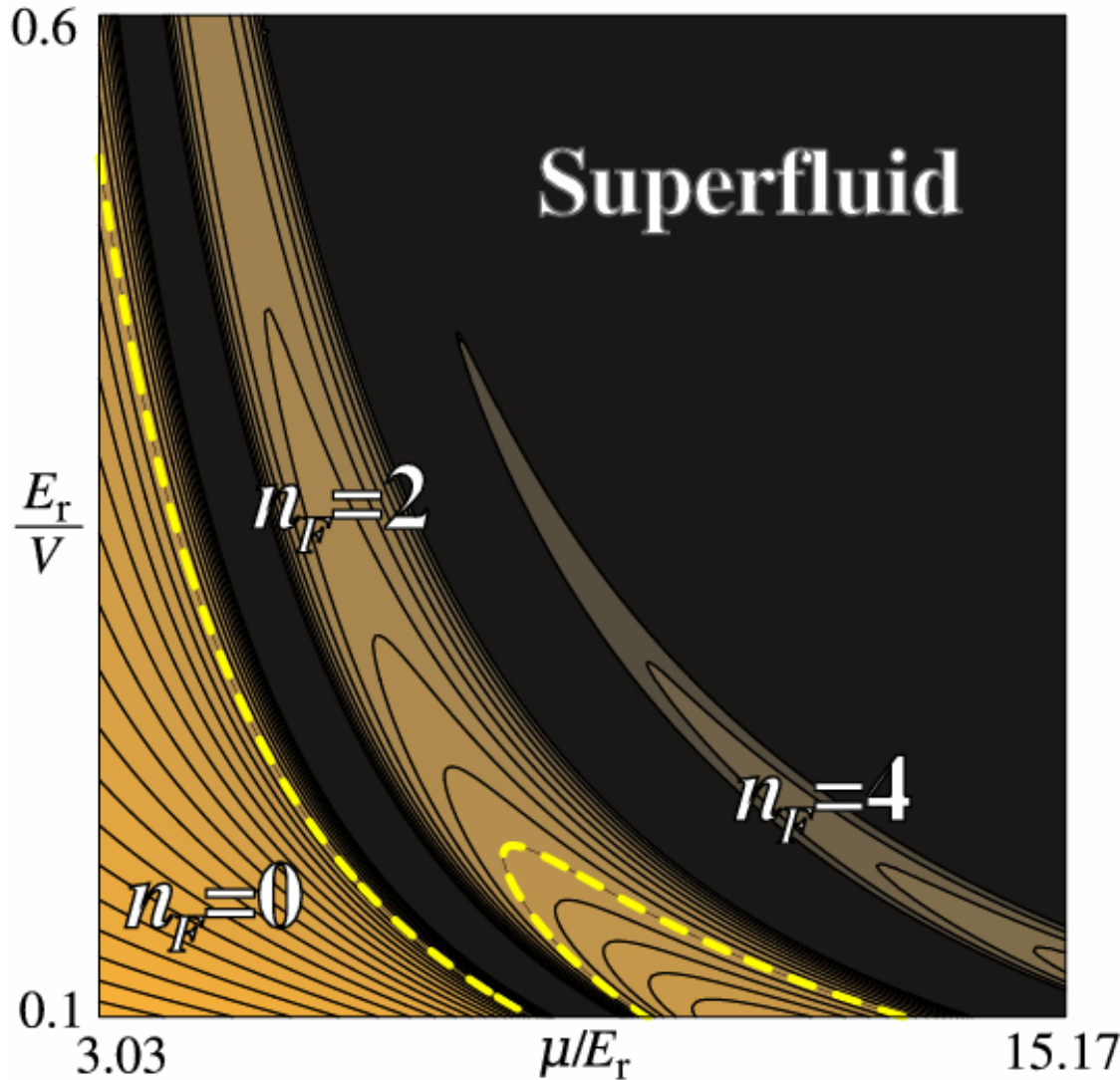
In the presence of a potential

$$V(\mathbf{r}) = V \left[\cos\left(\frac{2\pi x}{a_L}\right) + \cos\left(\frac{2\pi y}{a_L}\right) + \cos\left(\frac{2\pi z}{a_L}\right) \right]$$

there is a universal phase diagram determined by the ratio of 3 energy scales: V , the chemical potential μ , and the

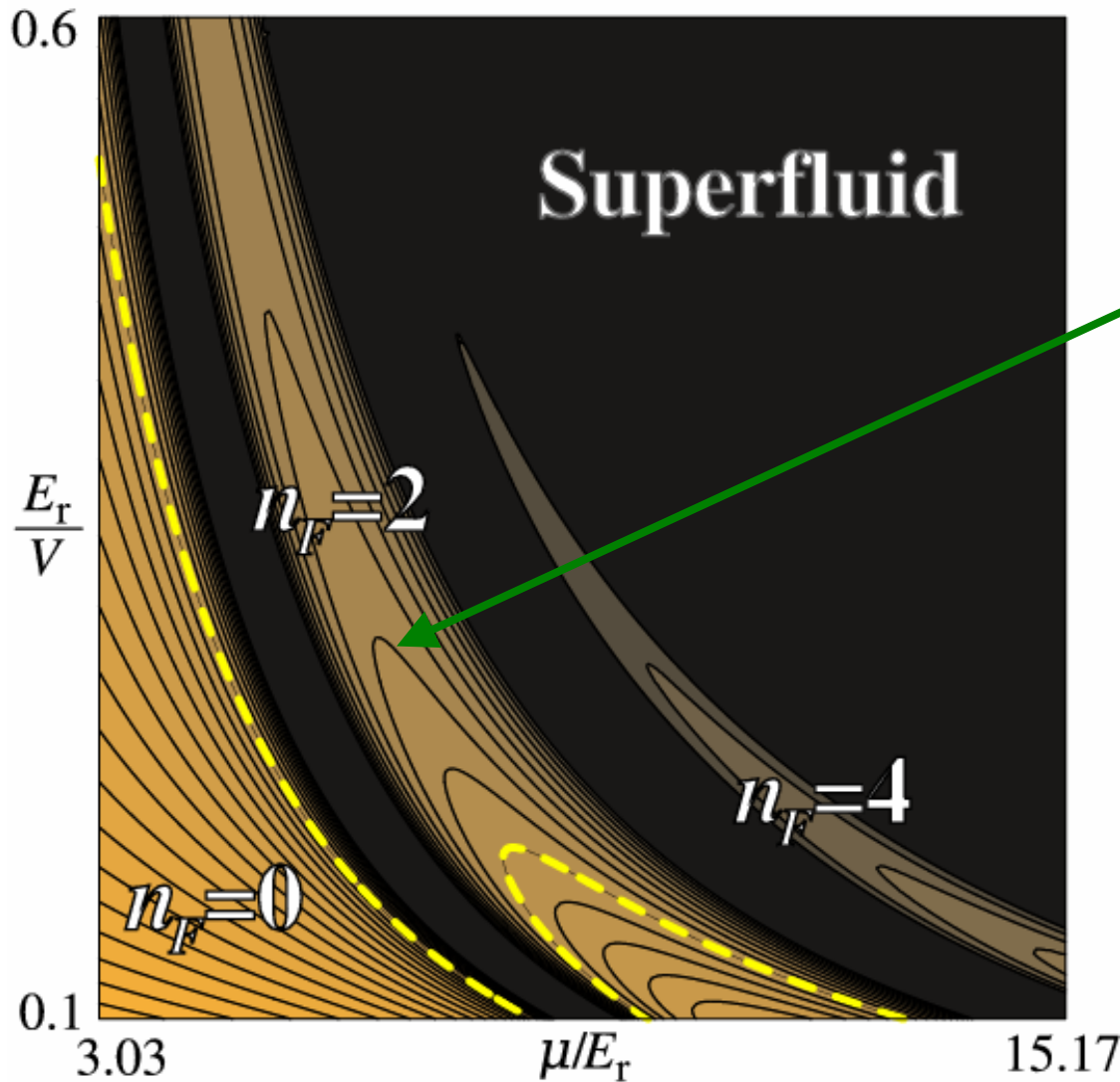
recoil energy $E_r = \frac{\hbar^2}{4ma_L^2}$

Universal phase diagram of fermions with near-unitary interactions in the presence of a periodic potential



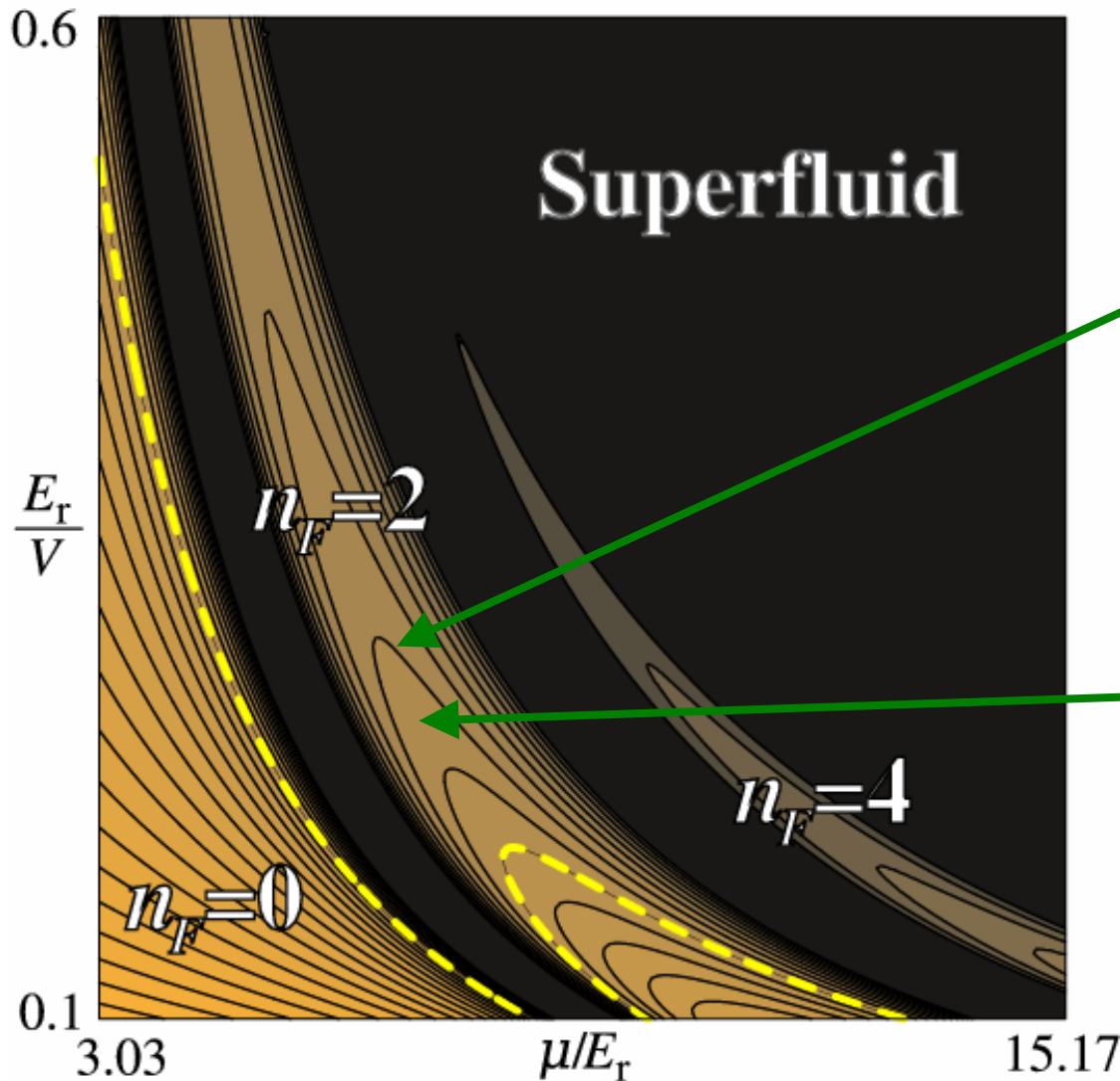
Expansion in $1/N$
with $Sp(2N)$
symmetry

Universal phase diagram of fermions with near-unitary interactions in the presence of a periodic potential



Boundaries to insulating phases for different values of va_L where v is the detuning from the resonance

Universal phase diagram of fermions with near-unitary interactions in the presence of a periodic potential



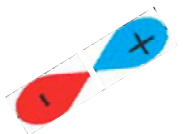
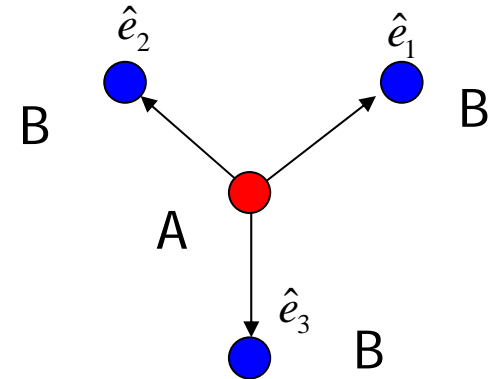
Boundaries to insulating phases for different values of νa_L where ν is the detuning from the resonance

Insulators have multiple band-occupancy, and are intermediate between band insulators of fermions and Mott insulators of bosonic fermion pairs

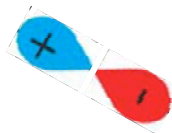
Artificial graphene in optical lattices

- Band Hamiltonian (σ -bonding) for spin-polarized fermions.

$$H_t = t_{//} \left\{ \sum_{\vec{r} \in A} [p_1^+(\vec{r}) p_1(\vec{r} + \hat{e}_1) + h.c.] \right. \\ \left. + [p_2^+(\vec{r}) p_1(\vec{r} + \hat{e}_2) + h.c.] \right. \\ \left. + [p_3^+(\vec{r}) p_3(\vec{r} + \hat{e}_3) + h.c.] \right\}$$



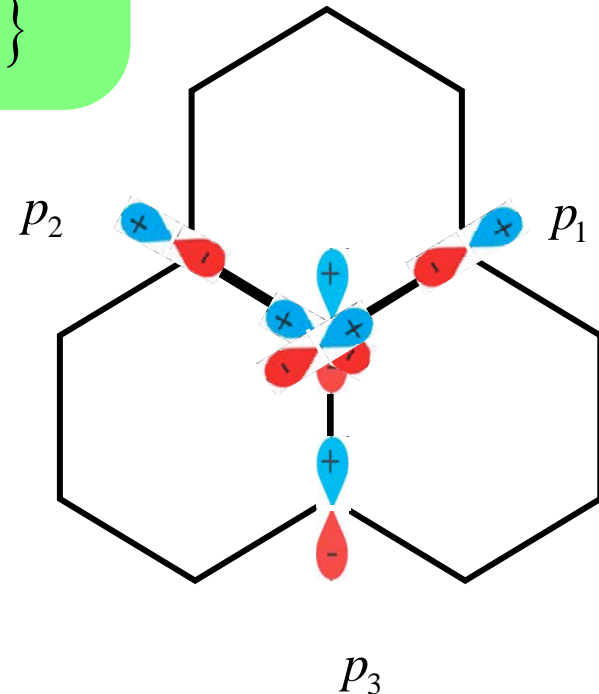
$$p_1 = \frac{\sqrt{3}}{2} p_x + \frac{1}{2} p_y$$



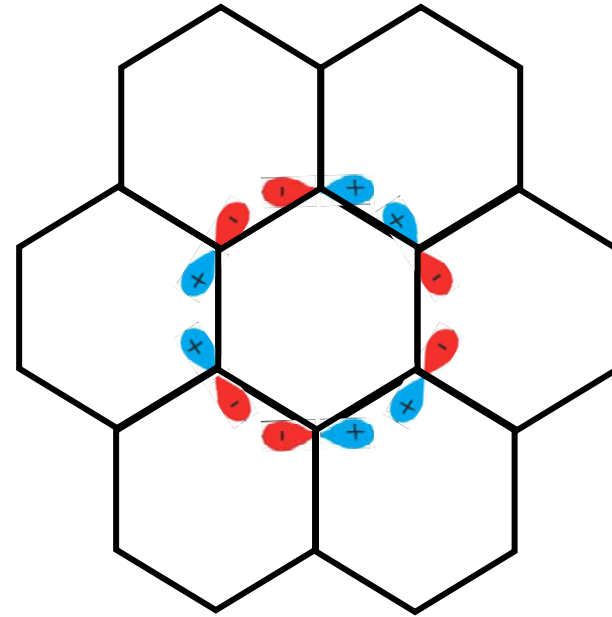
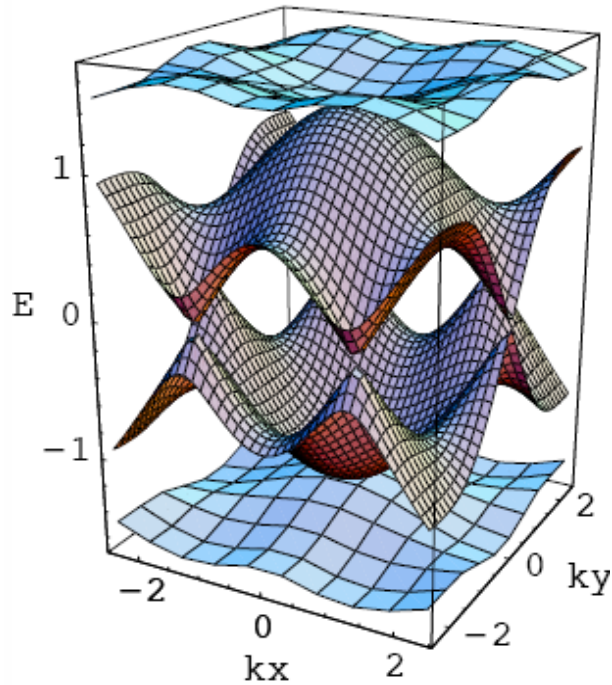
$$p_2 = \frac{\sqrt{3}}{2} p_x - \frac{1}{2} p_y$$



$$p_3 = -p_y$$



Flat bands in the entire Brillouin zone



- Flat band + Dirac cone.

- localized eigenstates.

Many correlated phases possible

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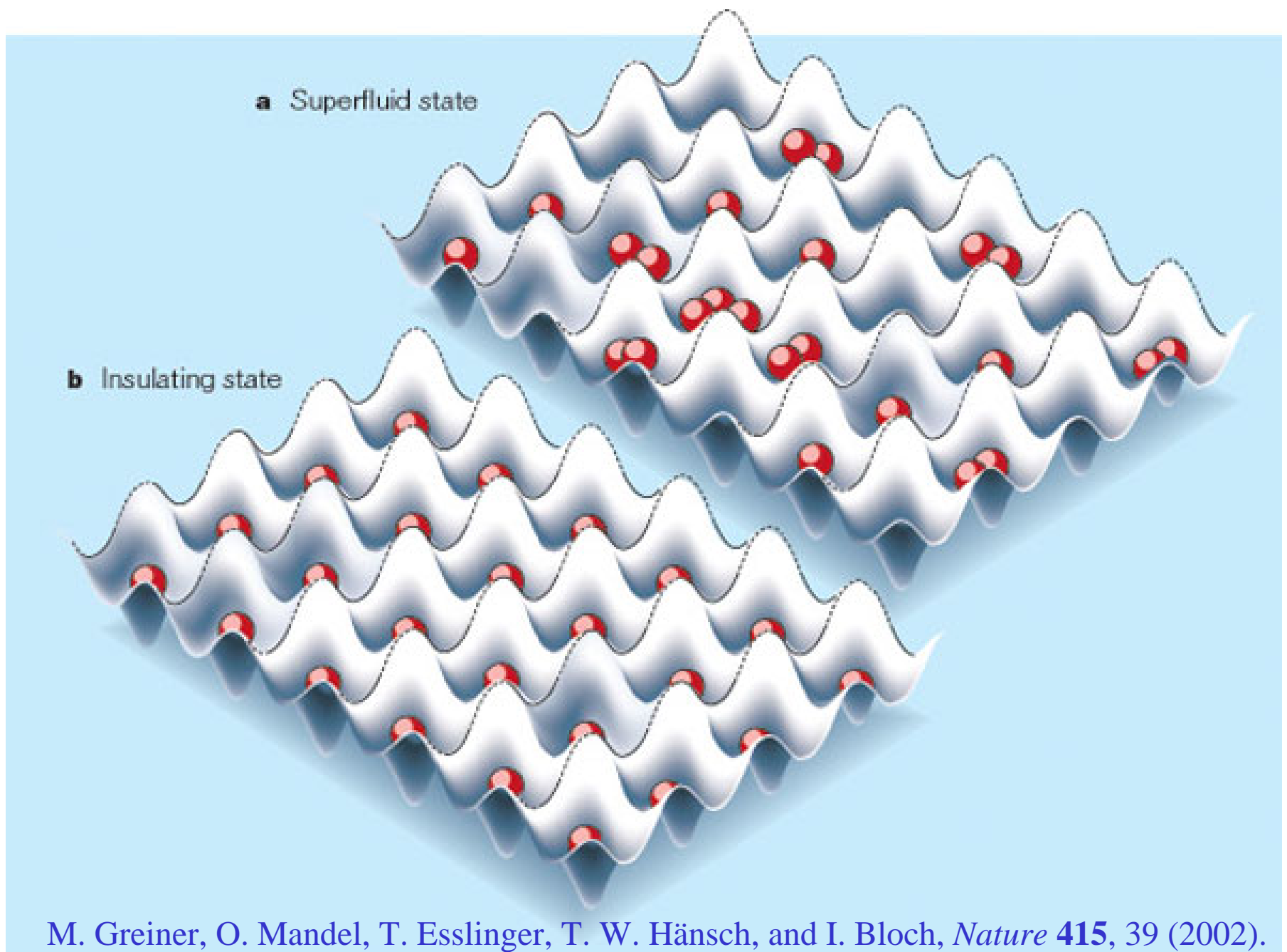
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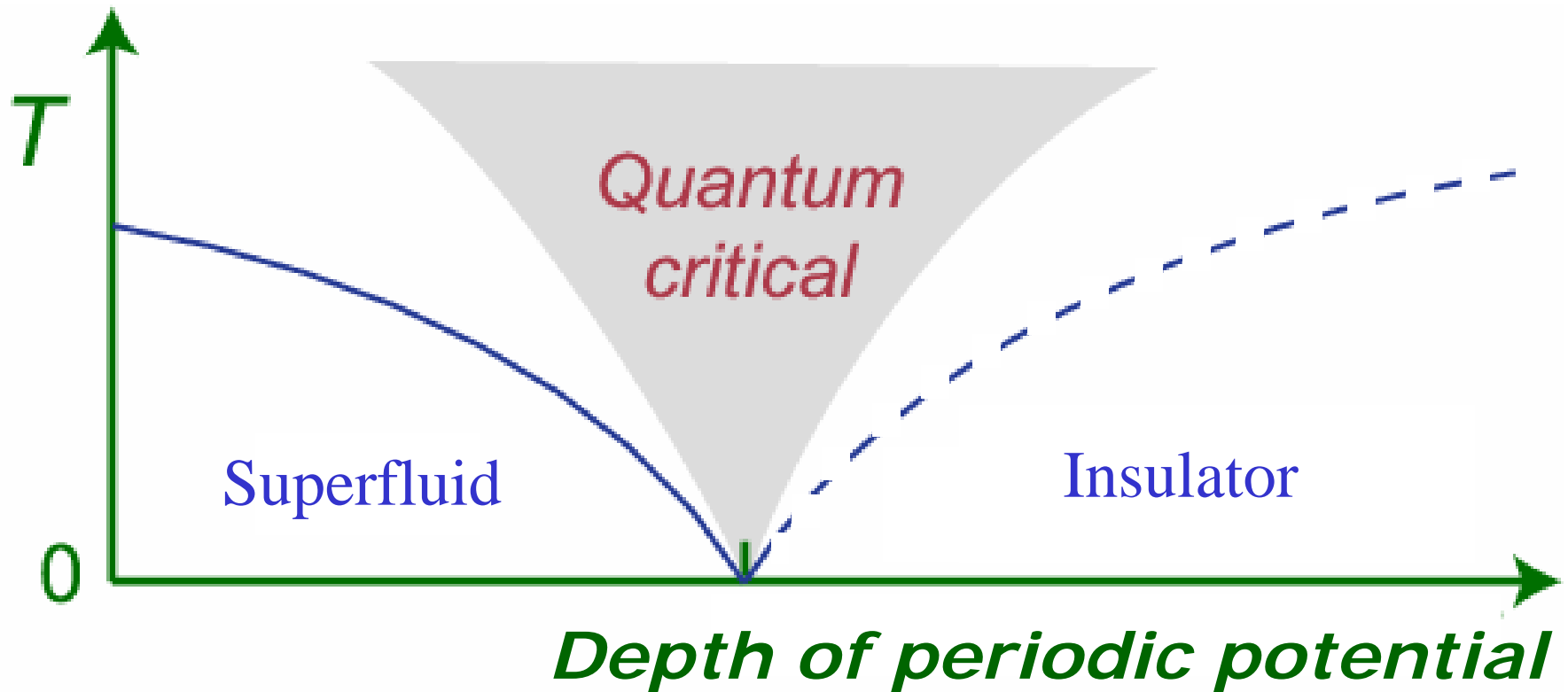
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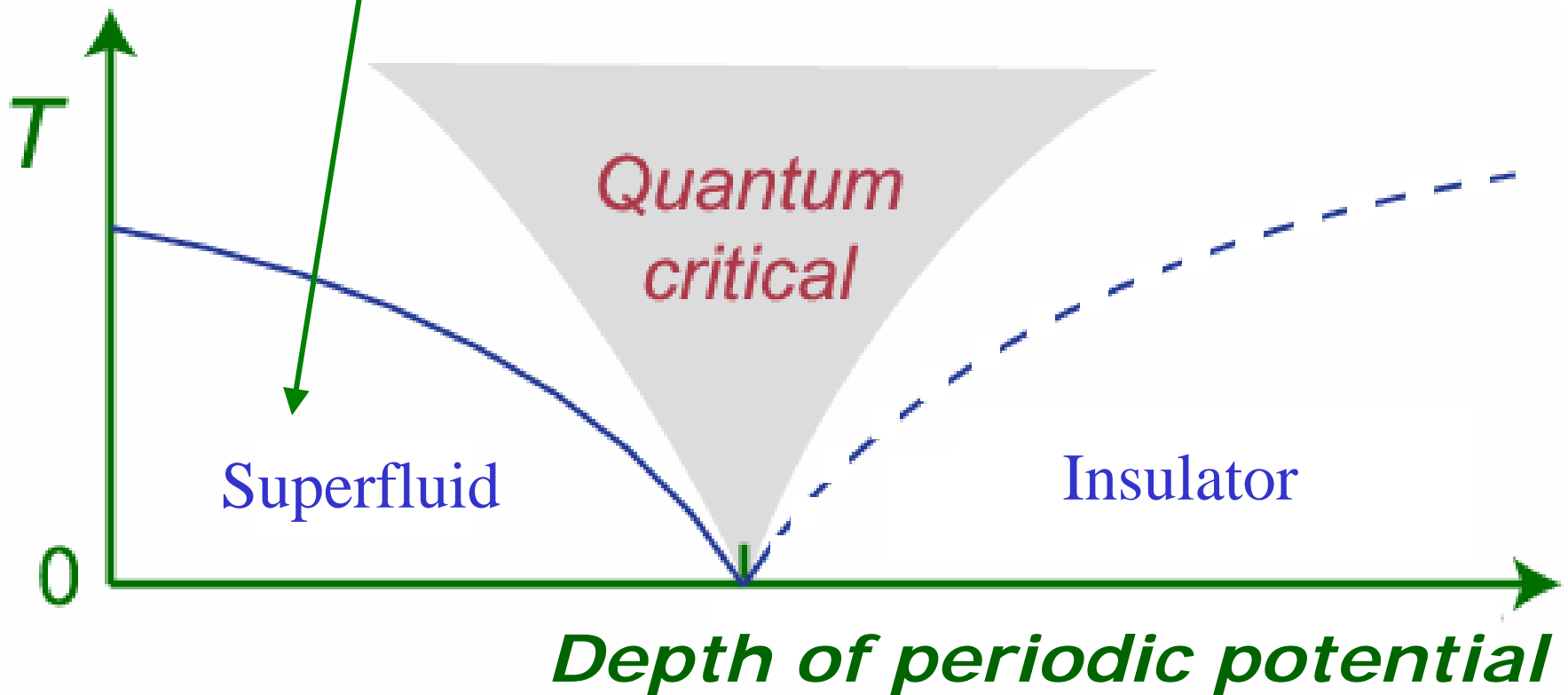


Non-zero temperature phase diagram



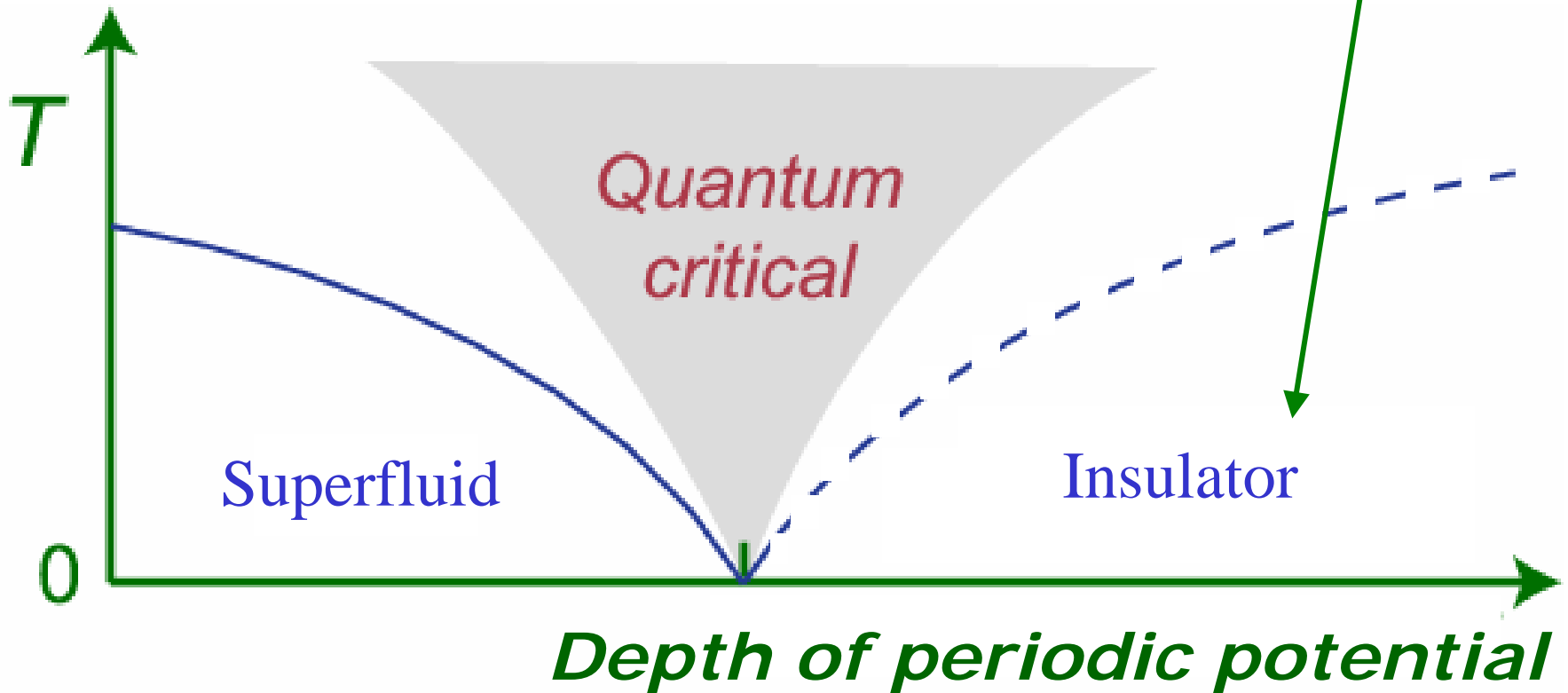
Non-zero temperature phase diagram

Dynamics of the classical
Gross-Pitaevski equation

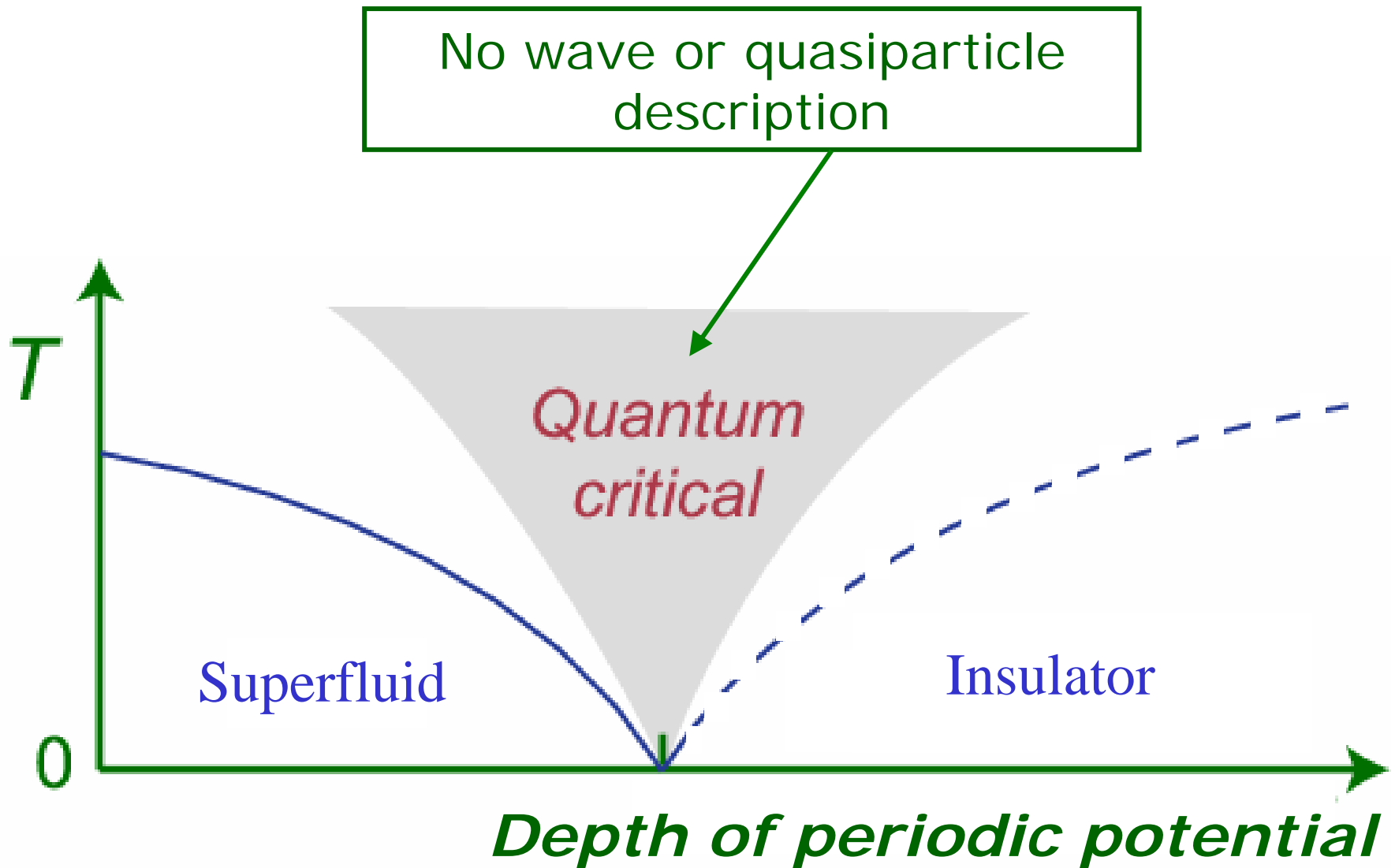


Non-zero temperature phase diagram

Dilute Boltzmann gas of
particle and holes



Non-zero temperature phase diagram



Resistivity of Bi films

Conductivity σ

$$\sigma_{\text{Superconductor}} (T \rightarrow 0) = \infty$$

$$\sigma_{\text{Insulator}} (T \rightarrow 0) = 0$$

$$\sigma_{\text{Quantum critical point}} (T \rightarrow 0) \approx \frac{4e^2}{h}$$

D. B. Haviland, Y. Liu, and A. M. Goldman,
Phys. Rev. Lett. **62**, 2180 (1989)

M. P. A. Fisher, *Phys. Rev. Lett.* **65**, 923 (1990)

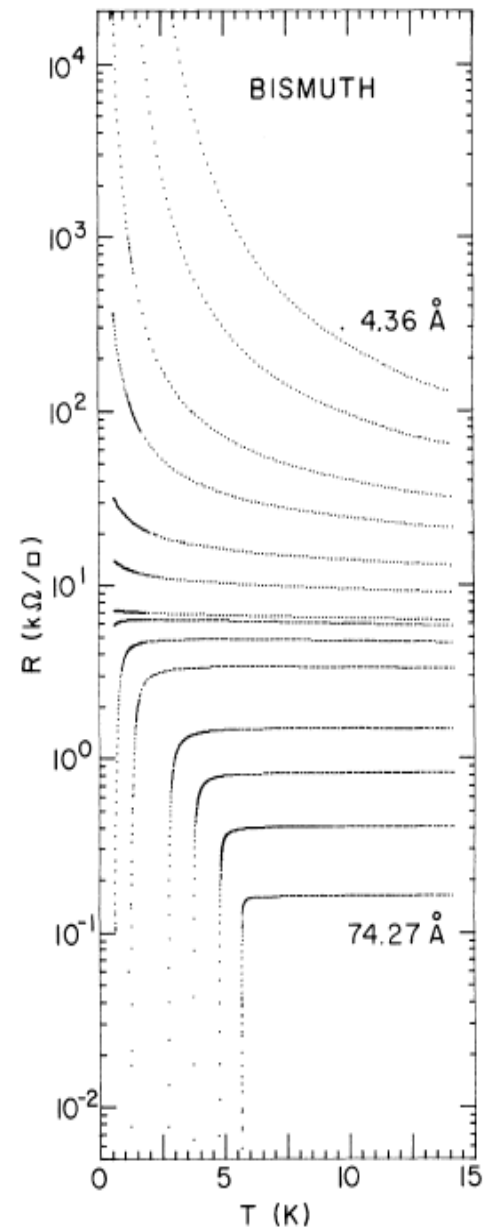
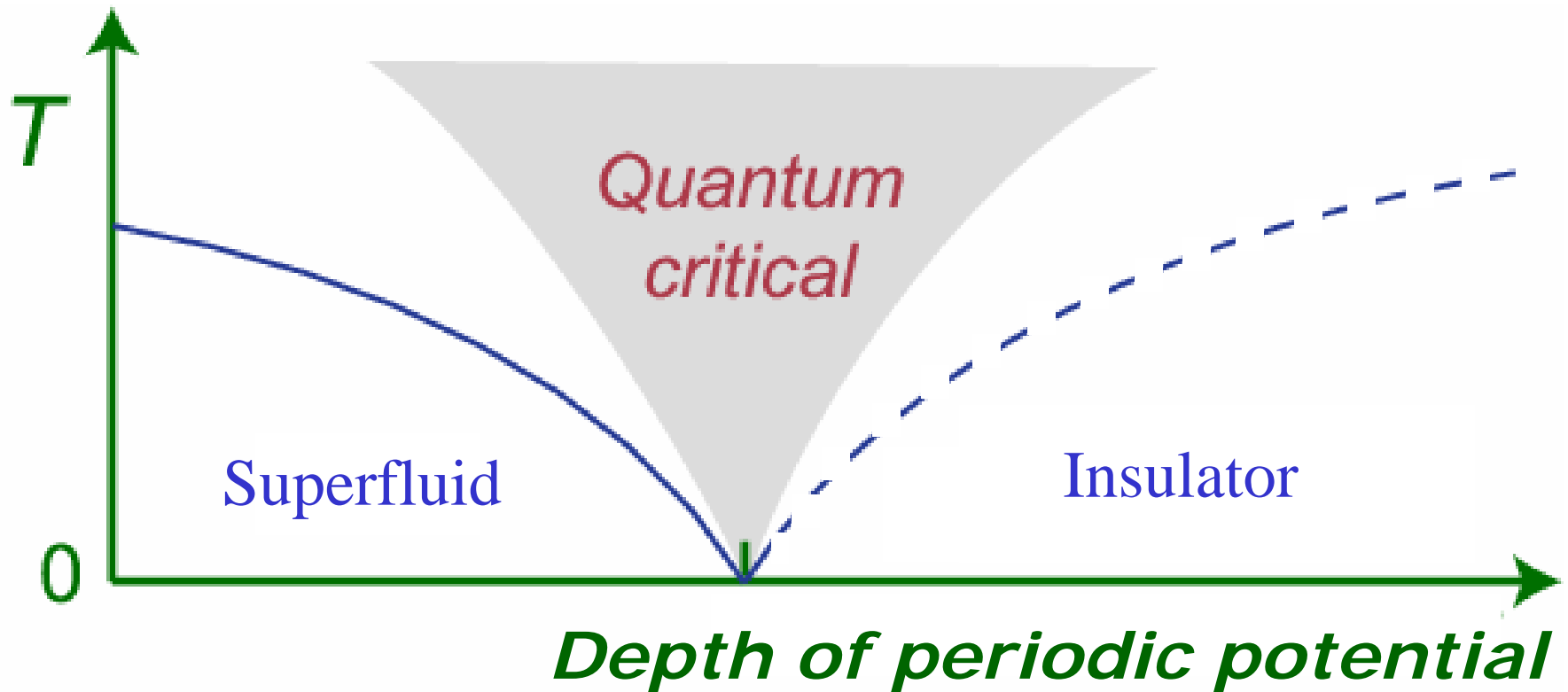
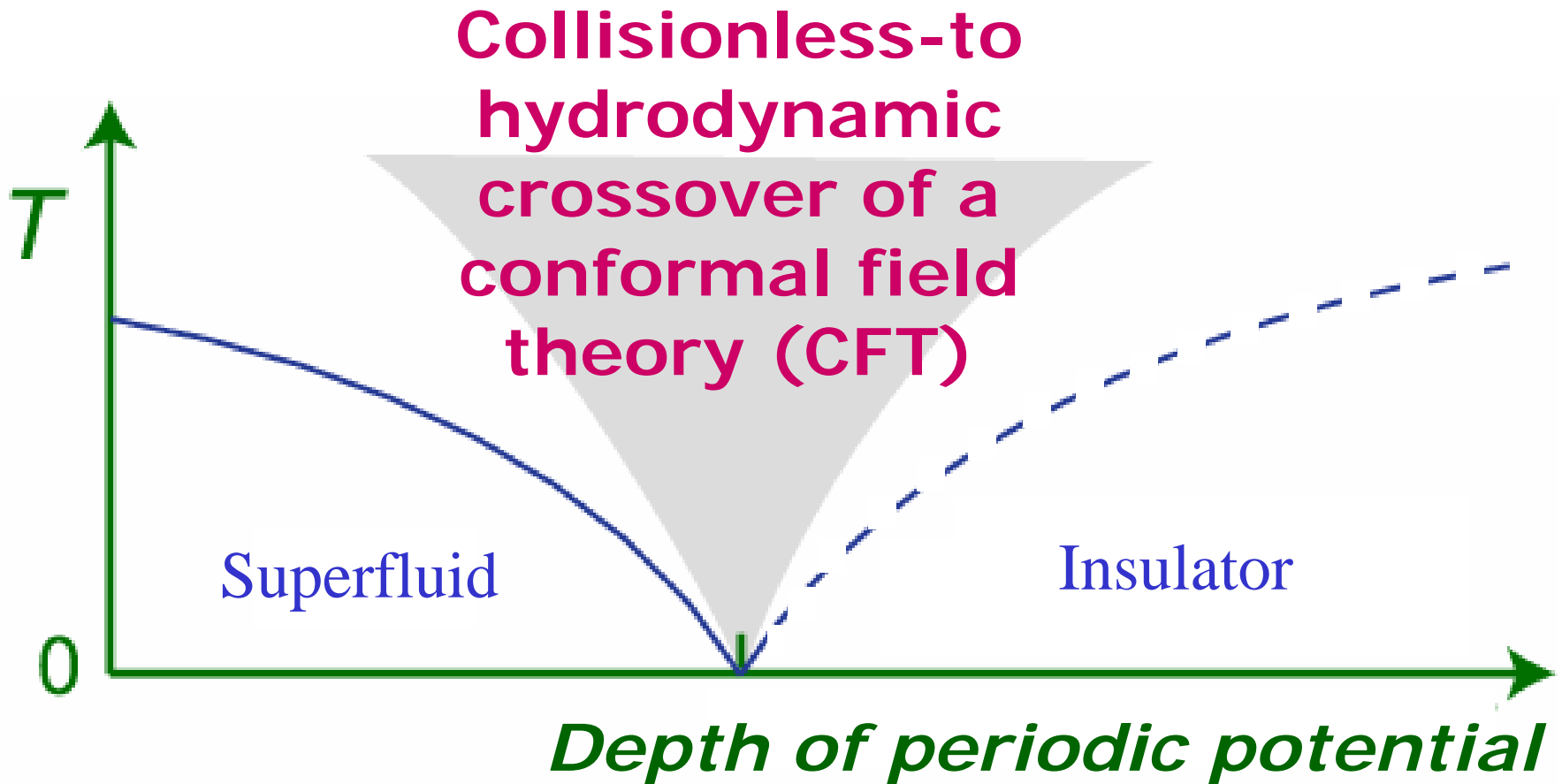


FIG. 1. Evolution of the temperature dependence of the sheet resistance $R(T)$ with thickness for a Bi film deposited onto Ge. Fewer than half of the traces actually acquired are shown. Film thicknesses shown range from 4.36 to 74.27 Å.

Non-zero temperature phase diagram

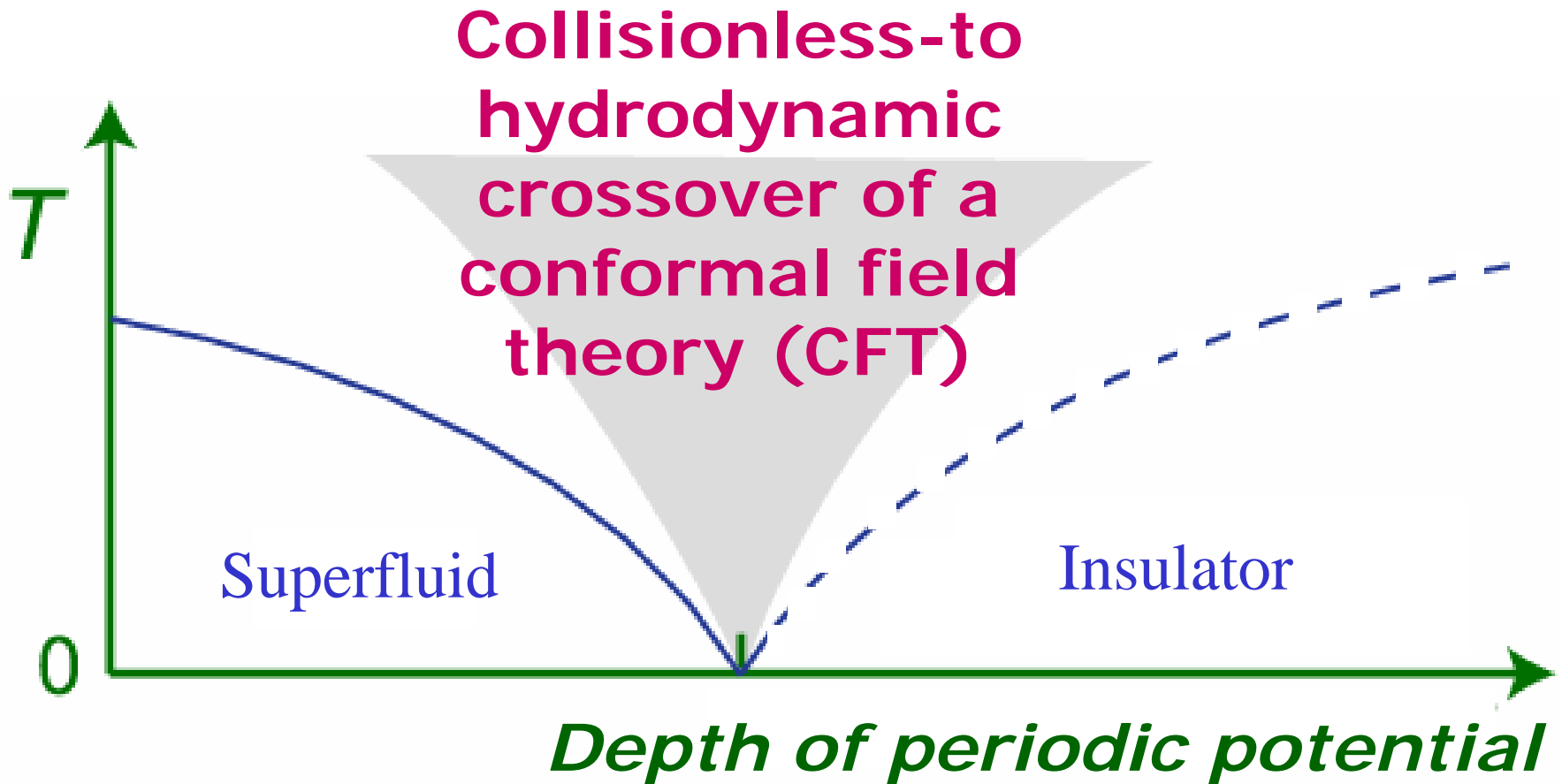


Non-zero temperature phase diagram



Non-zero temperature phase diagram

Needed: Cold atom experiments in this regime

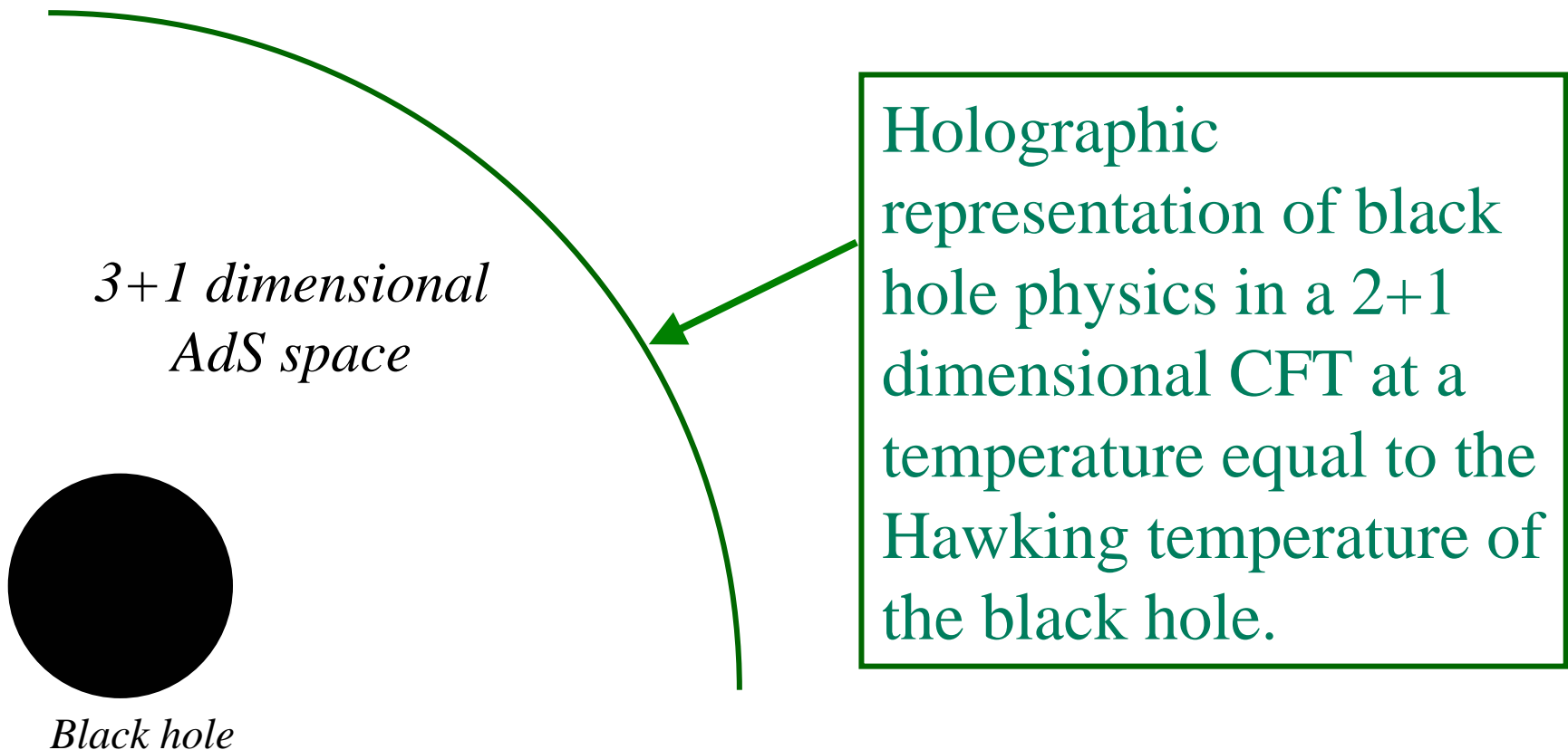


Hydrodynamics of a conformal field theory (CFT)

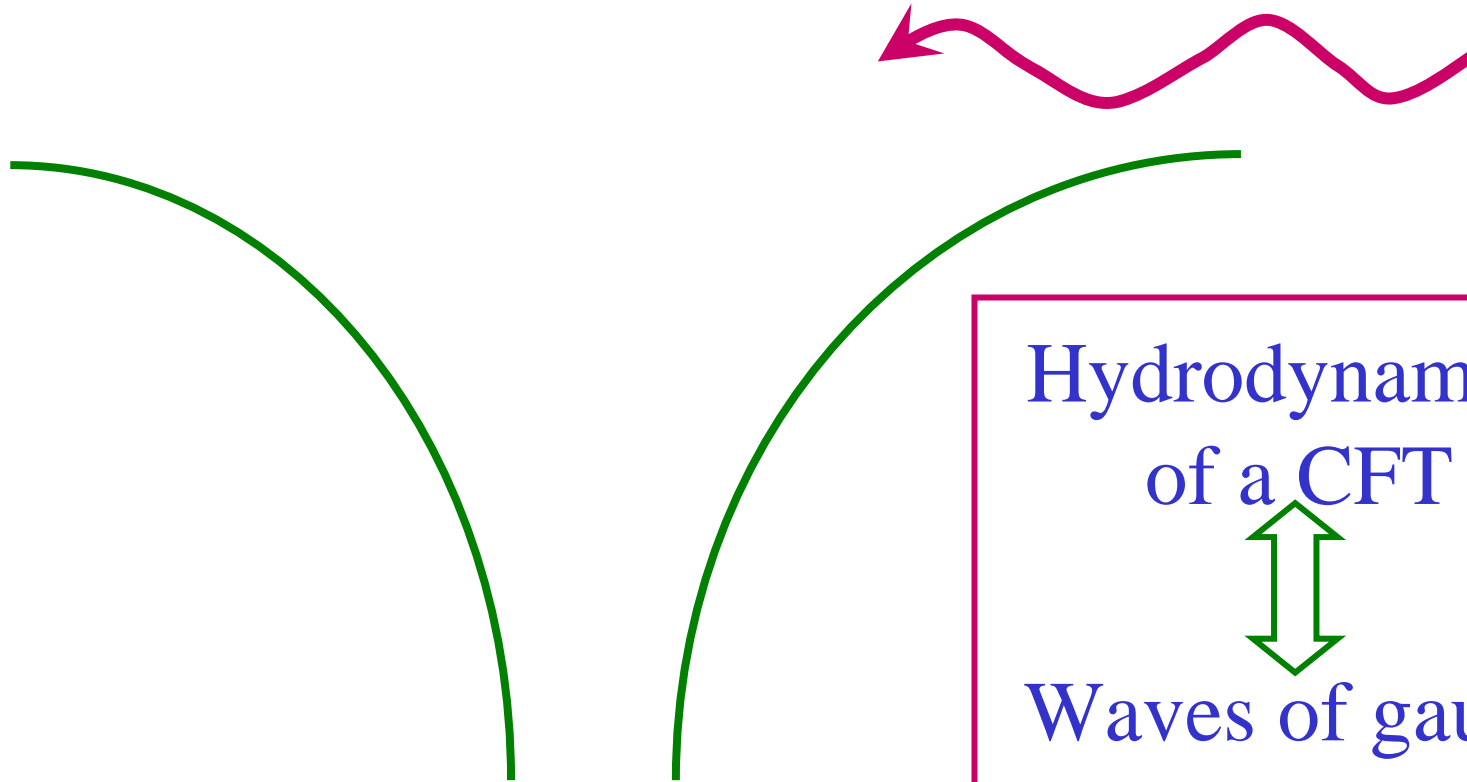
Maldacena's AdS/CFT correspondence relates the hydrodynamics of CFTs to the quantum gravity theory of the horizon of a black hole in Anti-de Sitter space.

Hydrodynamics of a conformal field theory (CFT)

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Hydrodynamics of a conformal field theory (CFT)



Hydrodynamics
of a CFT



Waves of gauge
fields in a curved
background

Hydrodynamics of a conformal field theory (CFT)

The scattering cross-section of the thermal excitations is universal and so transport coefficients are universally determined by $k_B T$

Charge diffusion constant

$$D_c = \Theta \frac{c^2}{k_B T}$$

Conductivity

$$\sigma = \Theta \frac{4e^2}{h}$$

Hydrodynamics of a conformal field theory (CFT)

For the (unique) CFT with a $SU(N)$ gauge field and 16 supercharges, we know the exact diffusion constant associated with a global $SO(8)$ symmetry:

Spin diffusion constant

$$D_s = \frac{3}{4\pi} \frac{c^2}{k_B T}$$

Spin conductivity

$$\sigma = \frac{N^{3/2}}{3\sqrt{2}\pi}$$

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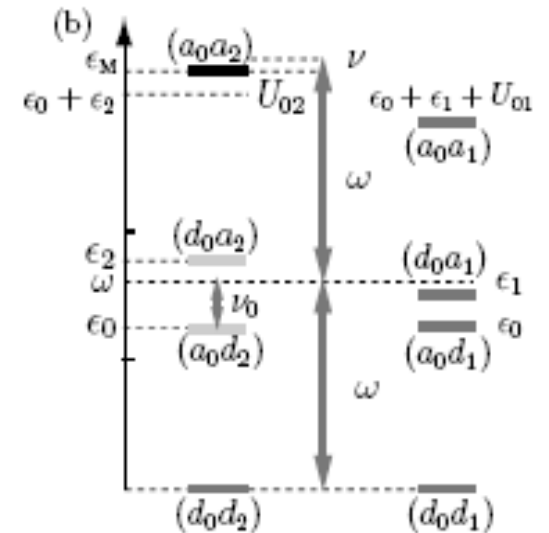
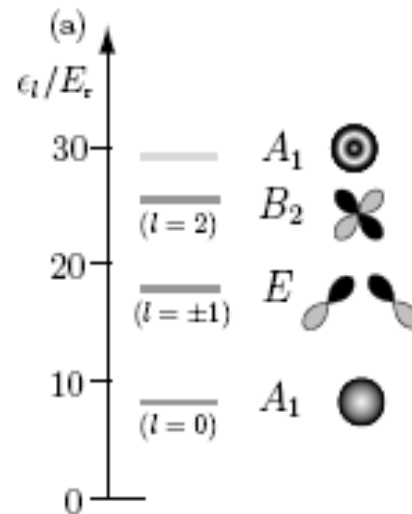
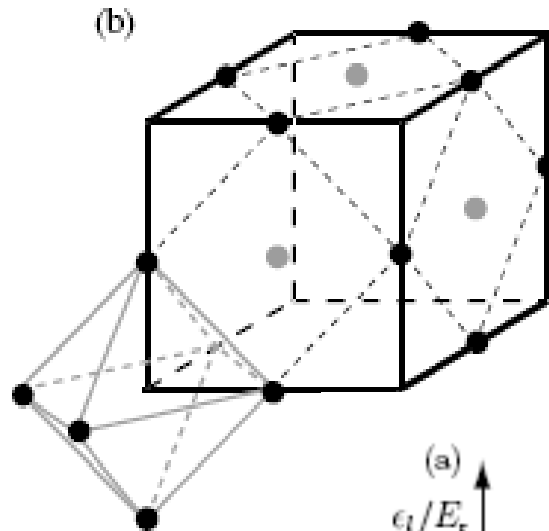
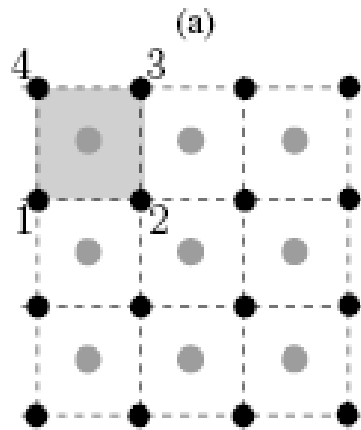
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 - (a) Tonks gas in one dimension
 - (b) Paired fermions across a Feshbach resonance

2. Optical lattices
 - (a) Superfluid-insulator transition
 - (b) Quantum-critical hydrodynamics via mapping to quantum theory of black holes.
 - (c) Entanglement of valence bonds

Ring-exchange interactions in an optical lattice using a Raman transition



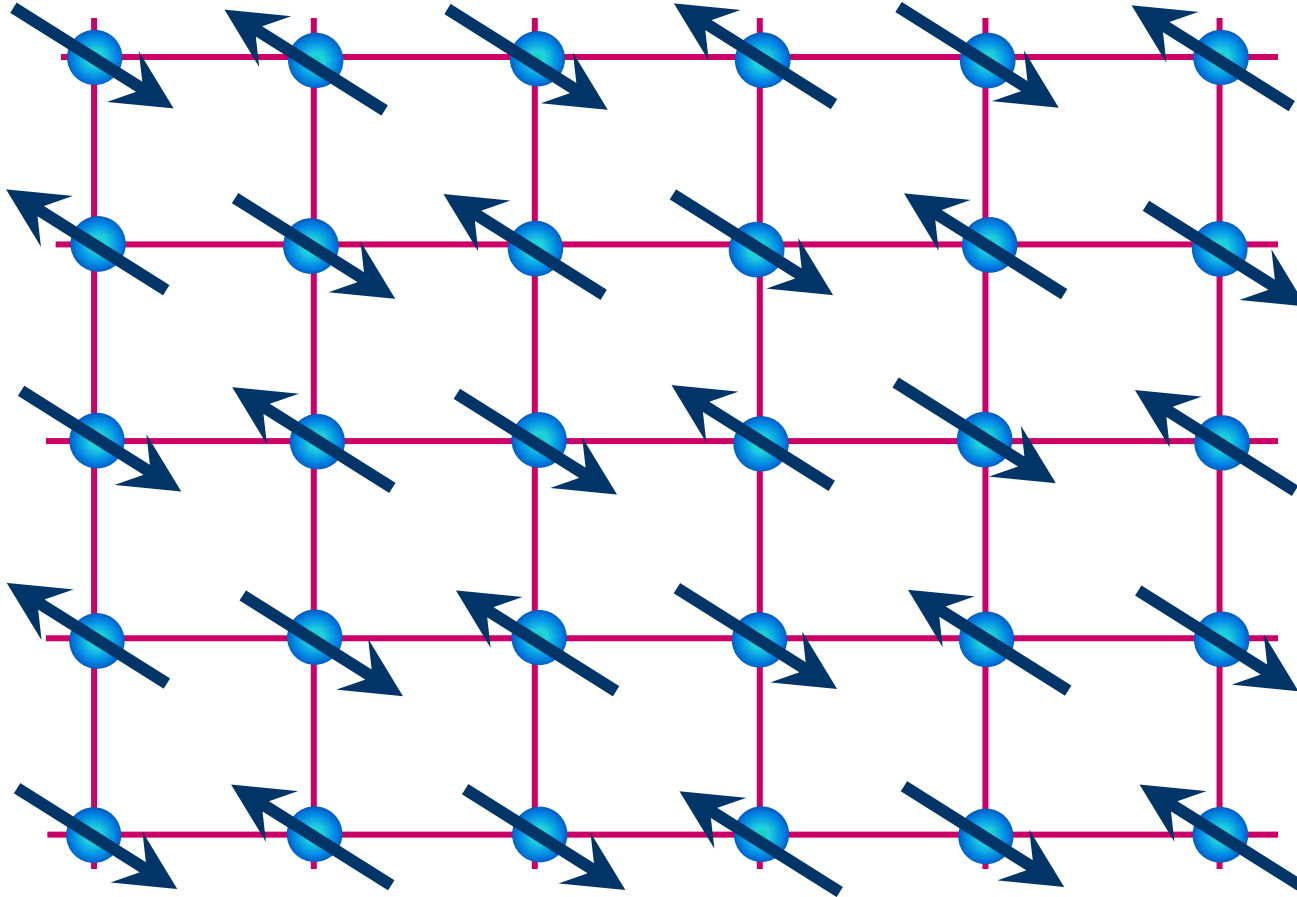
At each site, identify the states $|\uparrow\rangle$, $|\downarrow\rangle$, with the occupation number of a hard-core boson:

$$\begin{aligned}|\downarrow\rangle &= |0\rangle \\ |\uparrow\rangle &= b^\dagger|0\rangle\end{aligned}$$

Then the spin operators map as follows

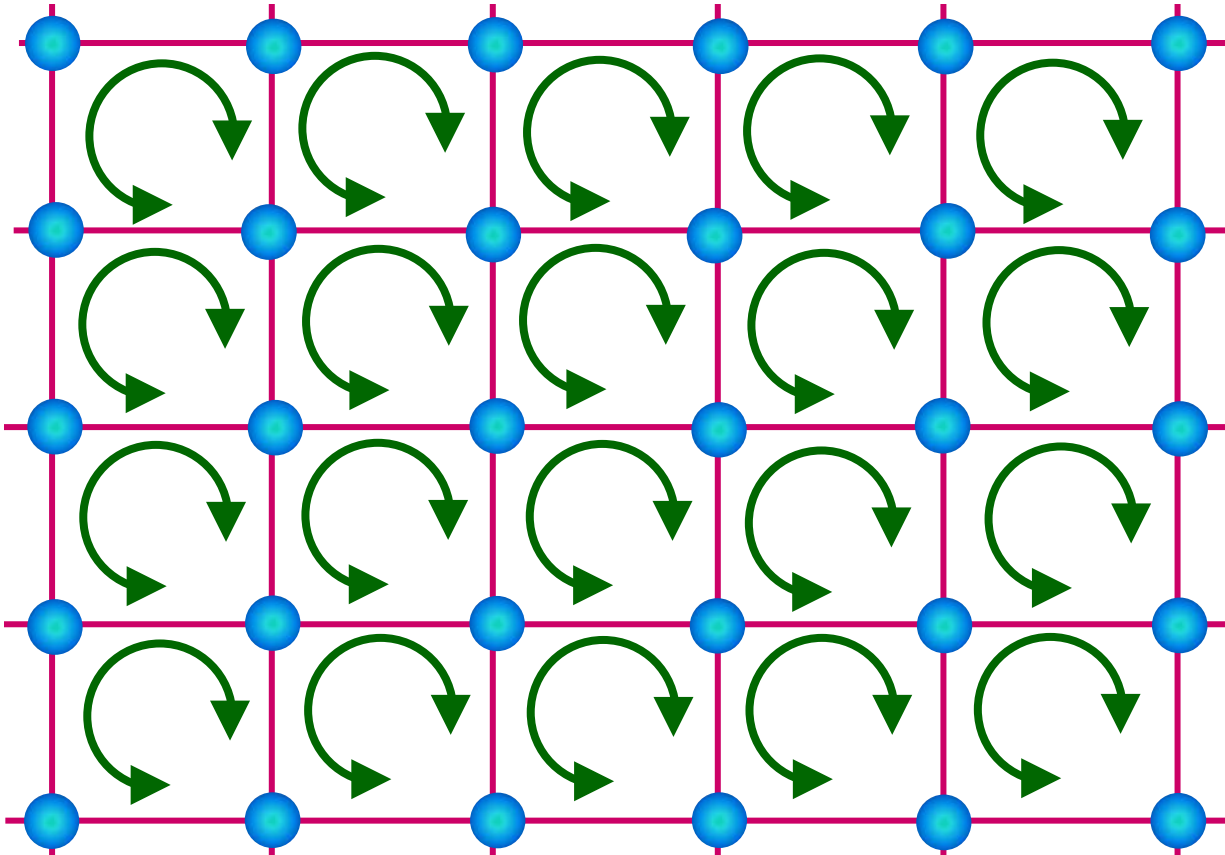
$$\begin{aligned}S_z &= b^\dagger b - 1/2 \\ S_+ &= b^\dagger \\ S_- &= b\end{aligned}$$

Antiferromagnetic (Neel) order in the insulator

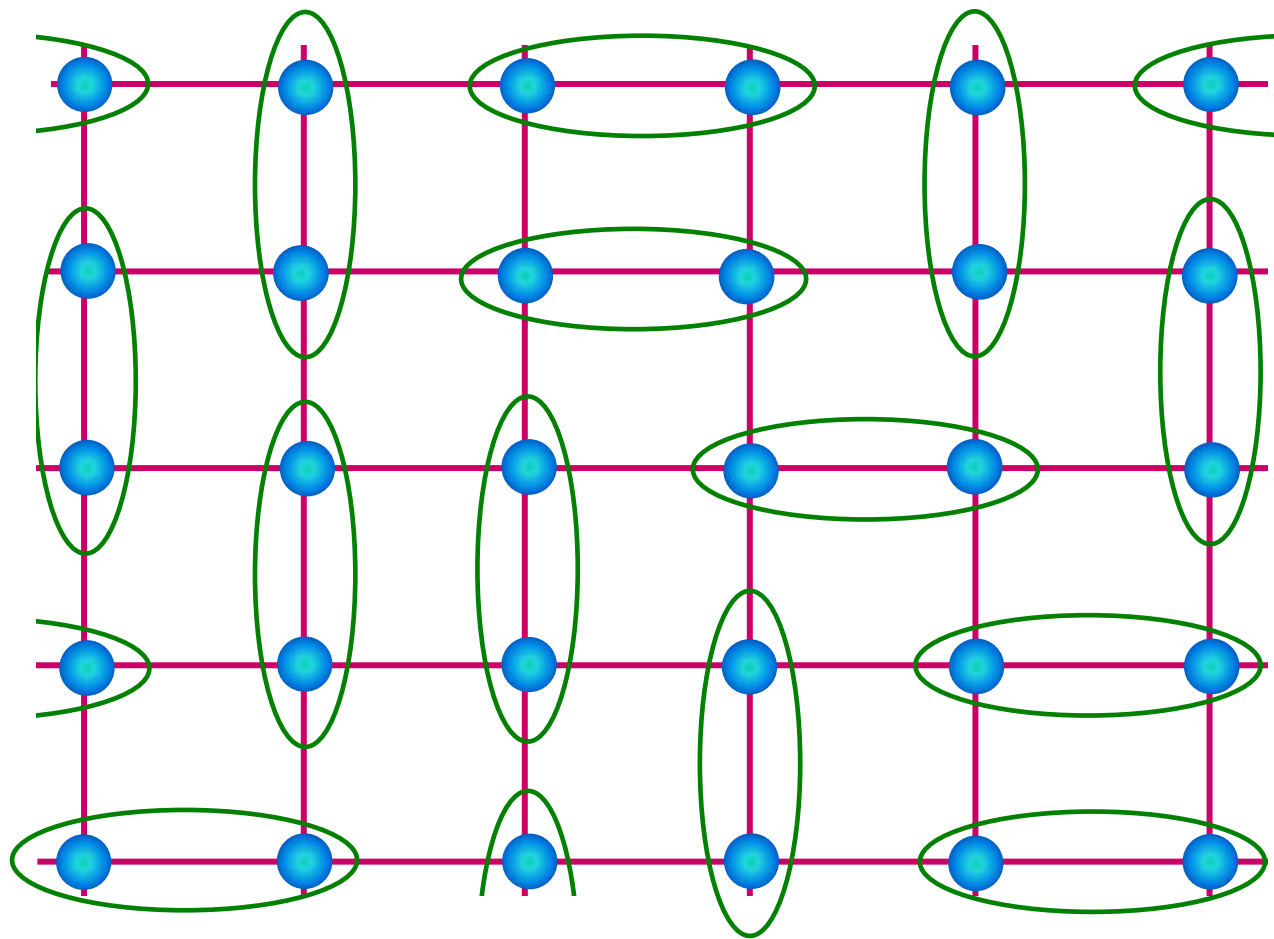


$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \quad ; \quad \vec{S}_i \Rightarrow \text{spin operator with } S=1/2$$

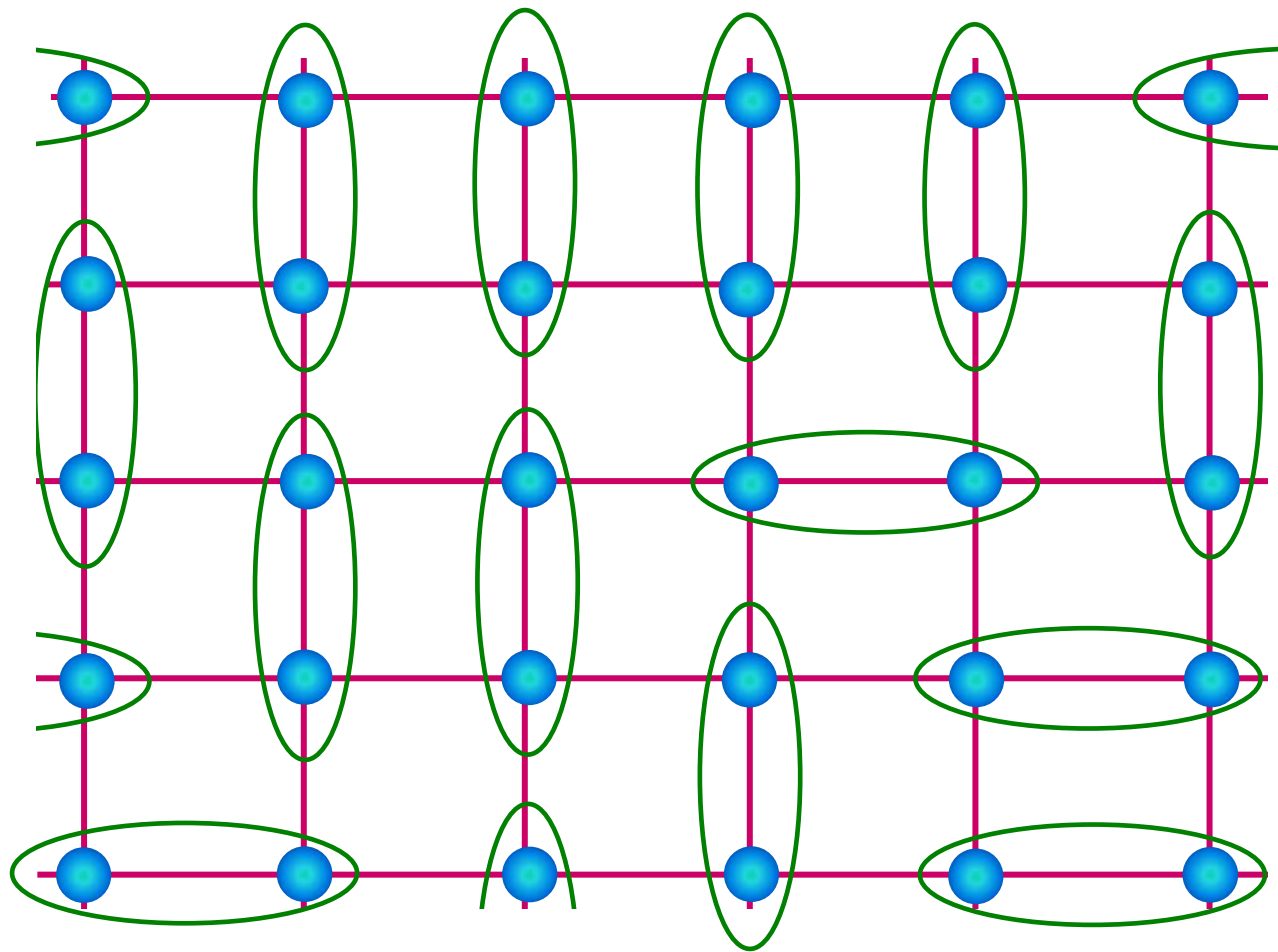
Induce formation of valence bonds by
e.g. ring-exchange interactions



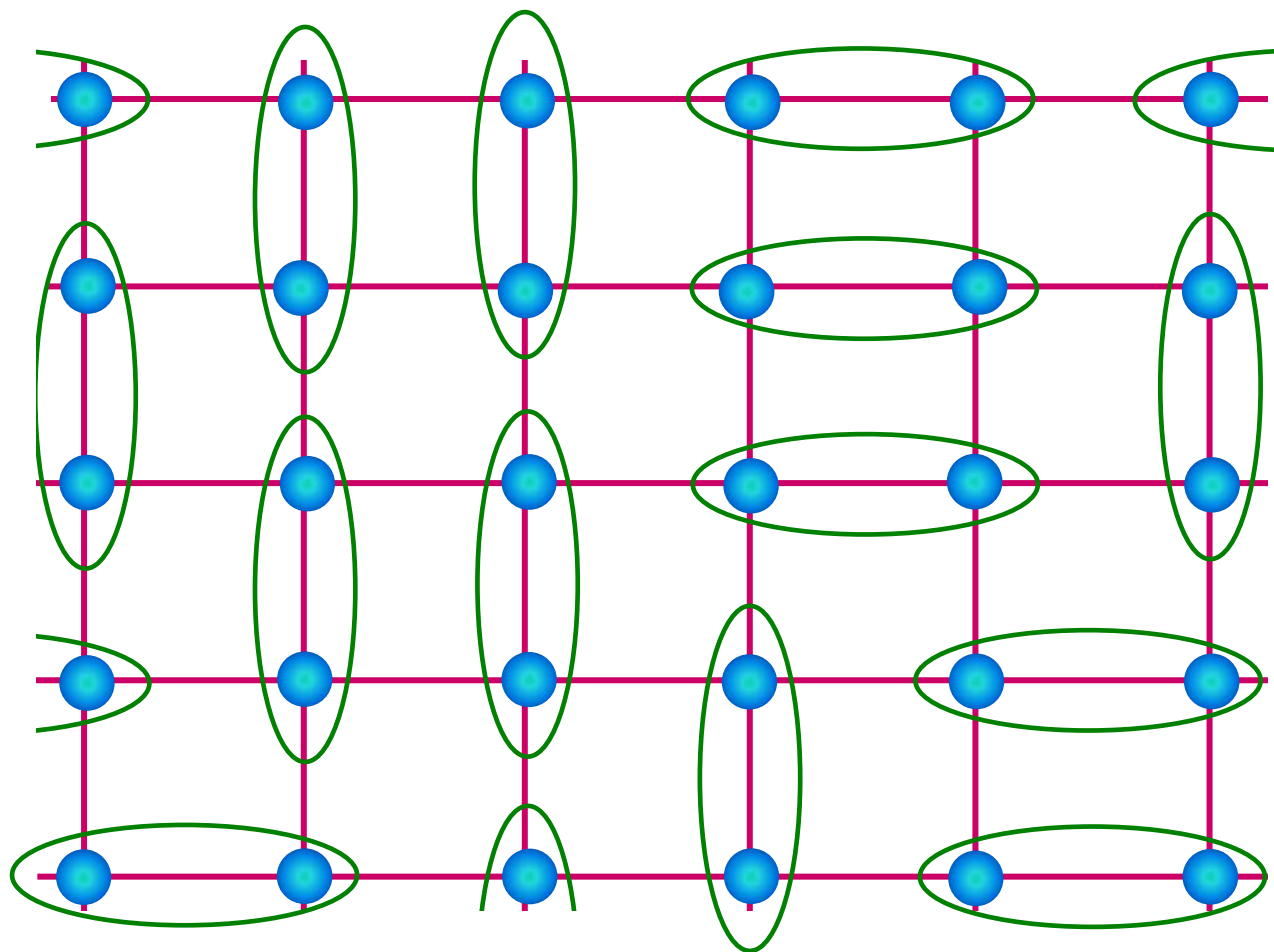
$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + K \sum_{\square} 4\text{-spin exchange}$$



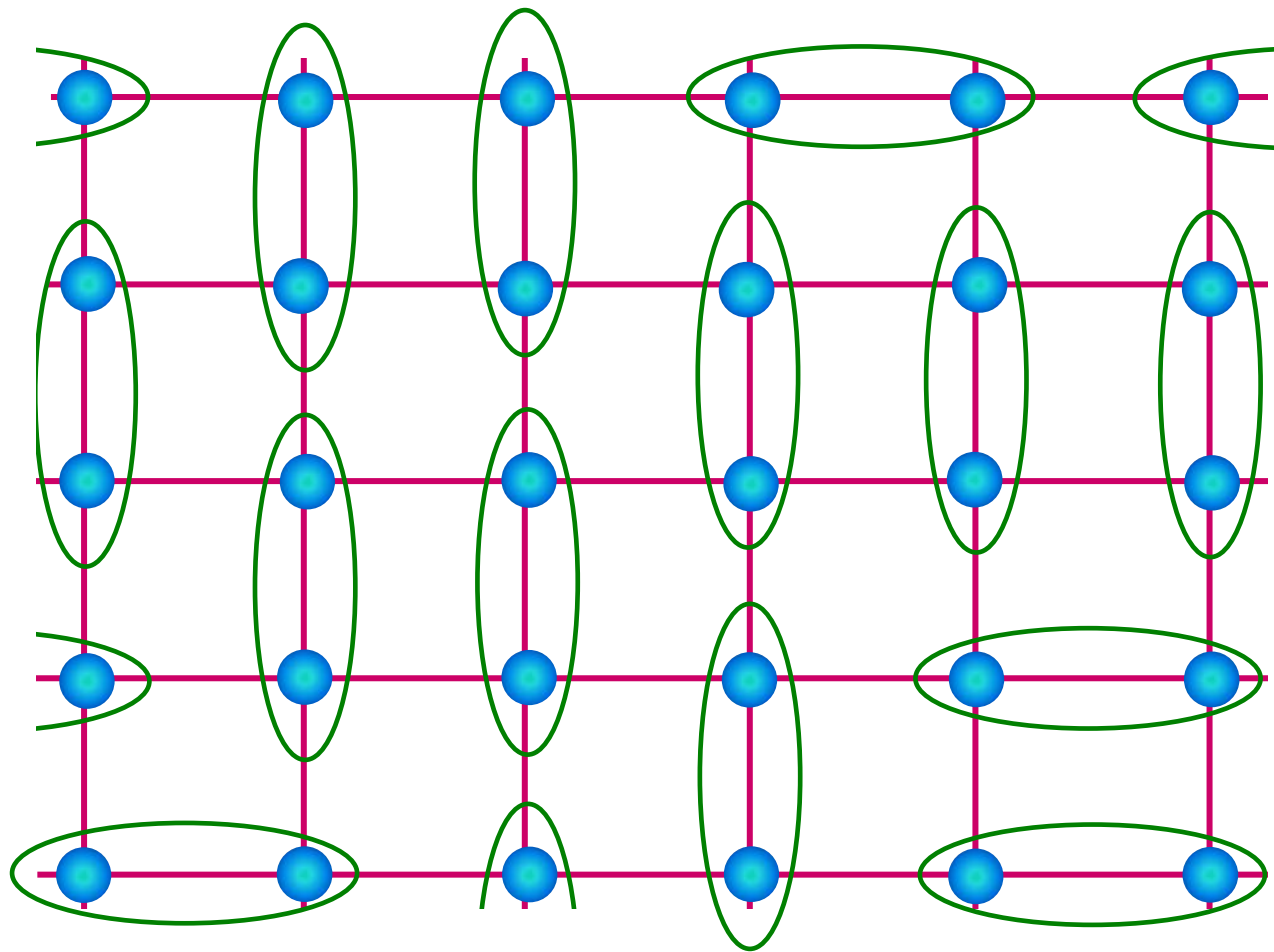
$$\text{Oval} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$



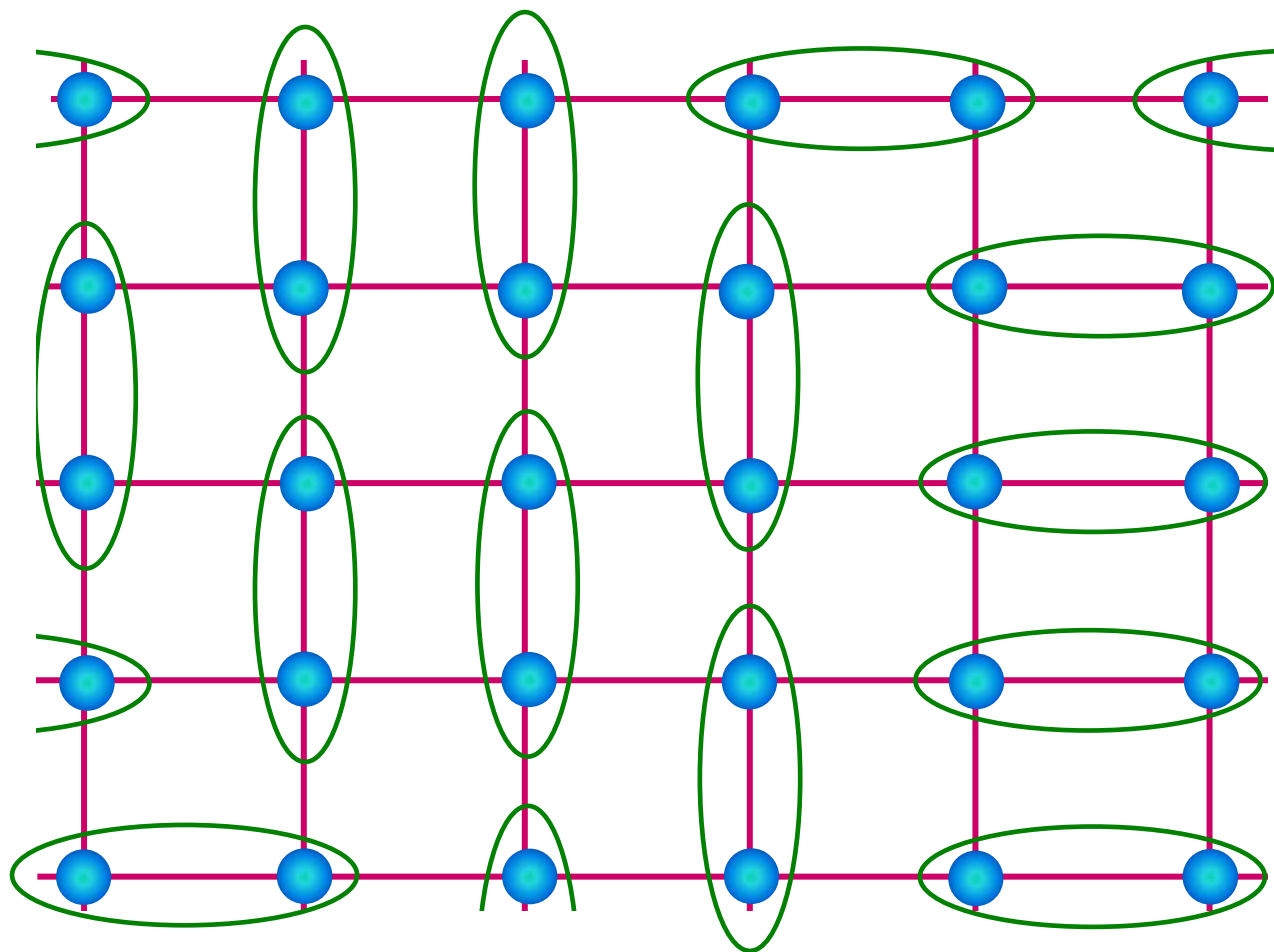
$$\text{Oval} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$



$$\begin{array}{c}
 \text{Oval with two blue spheres} \\
 = \\
 \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)
 \end{array}$$

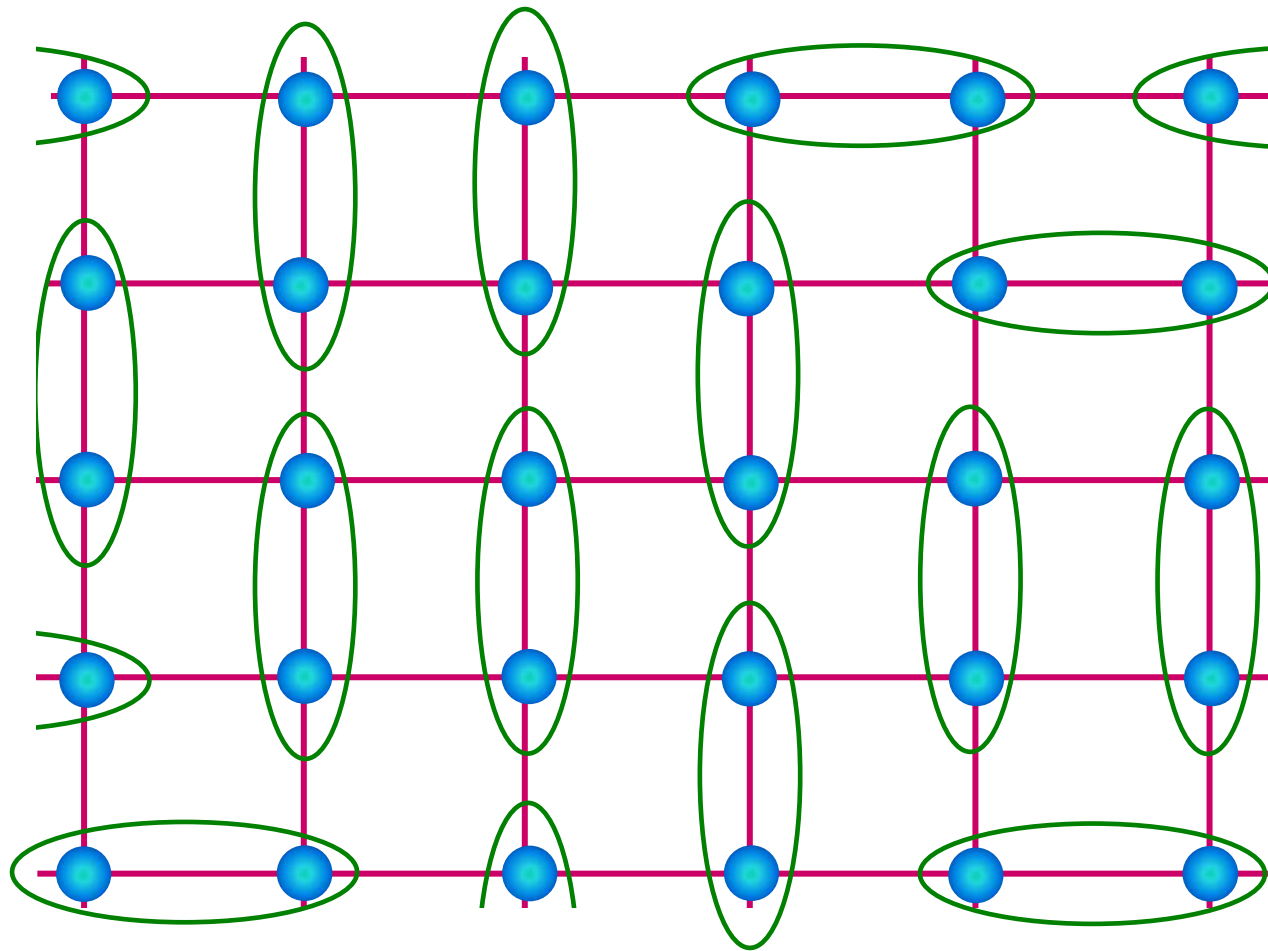


$$\text{Oval} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$



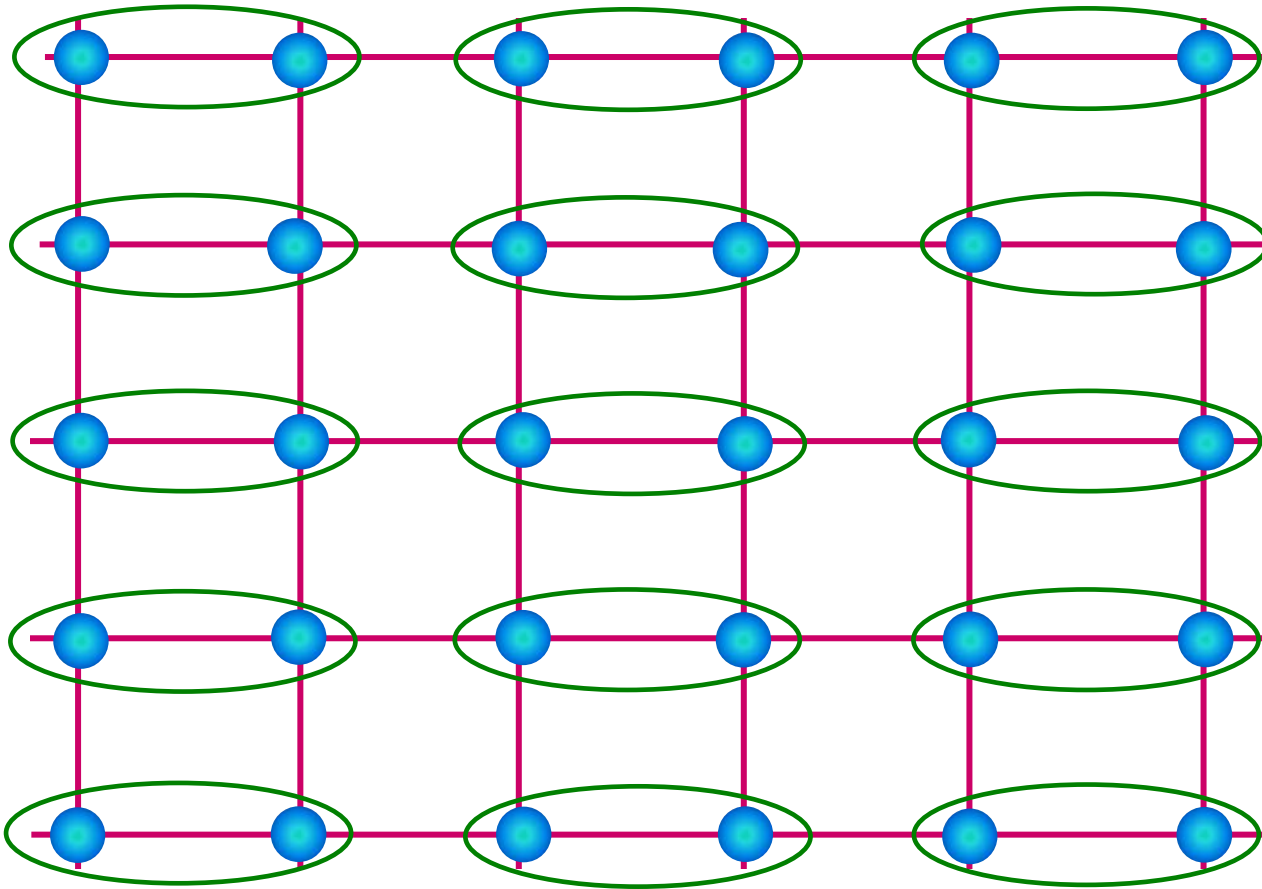
$$\begin{array}{c}
 \text{[Green Oval with 2 Blue Spheres]} = \\
 \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)
 \end{array}$$

Entangled liquid of valence bonds (Resonating valence bonds – RVB)



$$\text{oval} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

Valence bond solid (VBS)

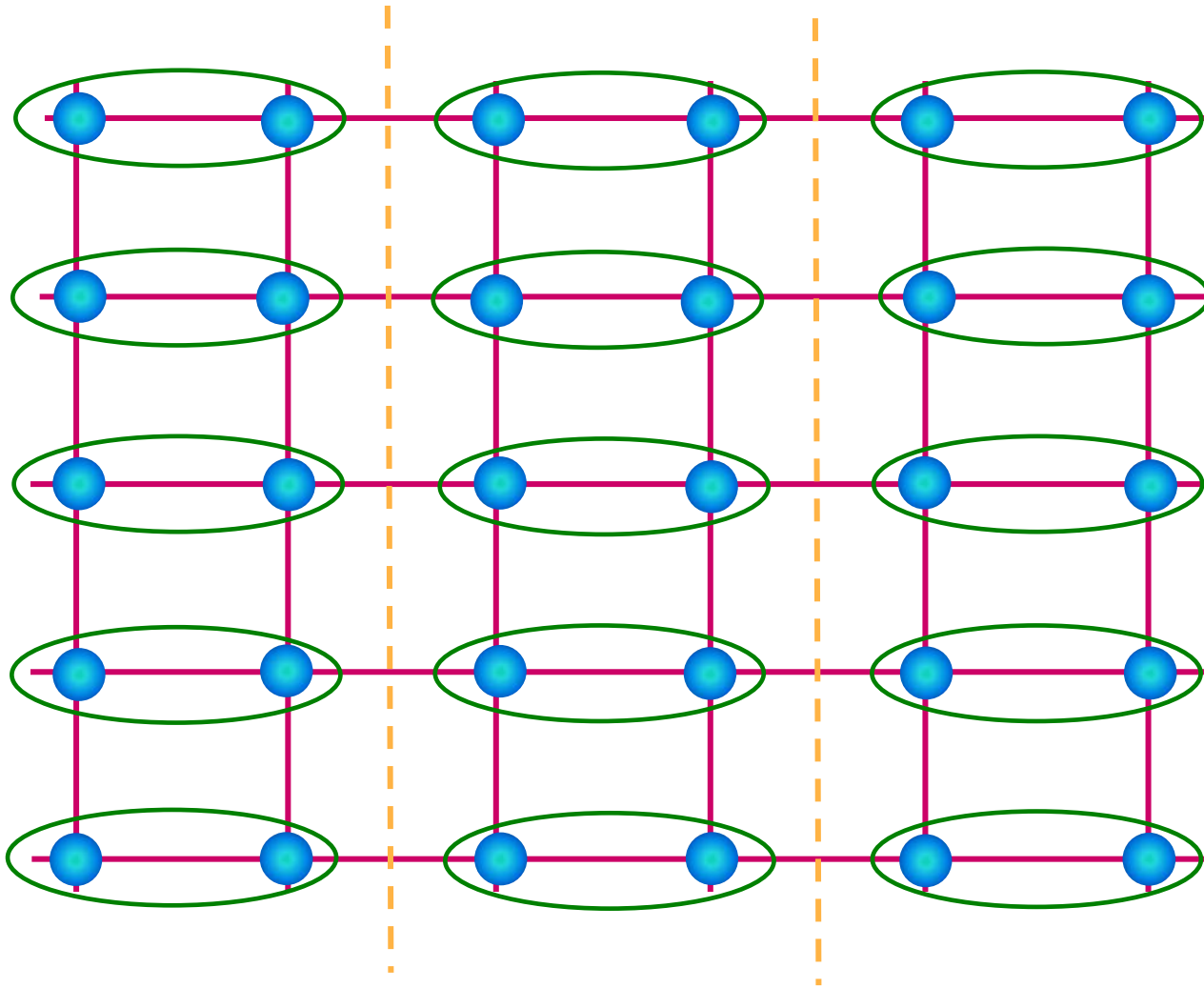


$$\text{green oval with two blue spheres} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

N. Read and S. Sachdev,
Phys. Rev. Lett. **62**, 1694
(1989).

R. Moessner and S. L.
Sondhi, *Phys. Rev. B* **63**,
224401 (2001).

Valence bond solid (VBS)

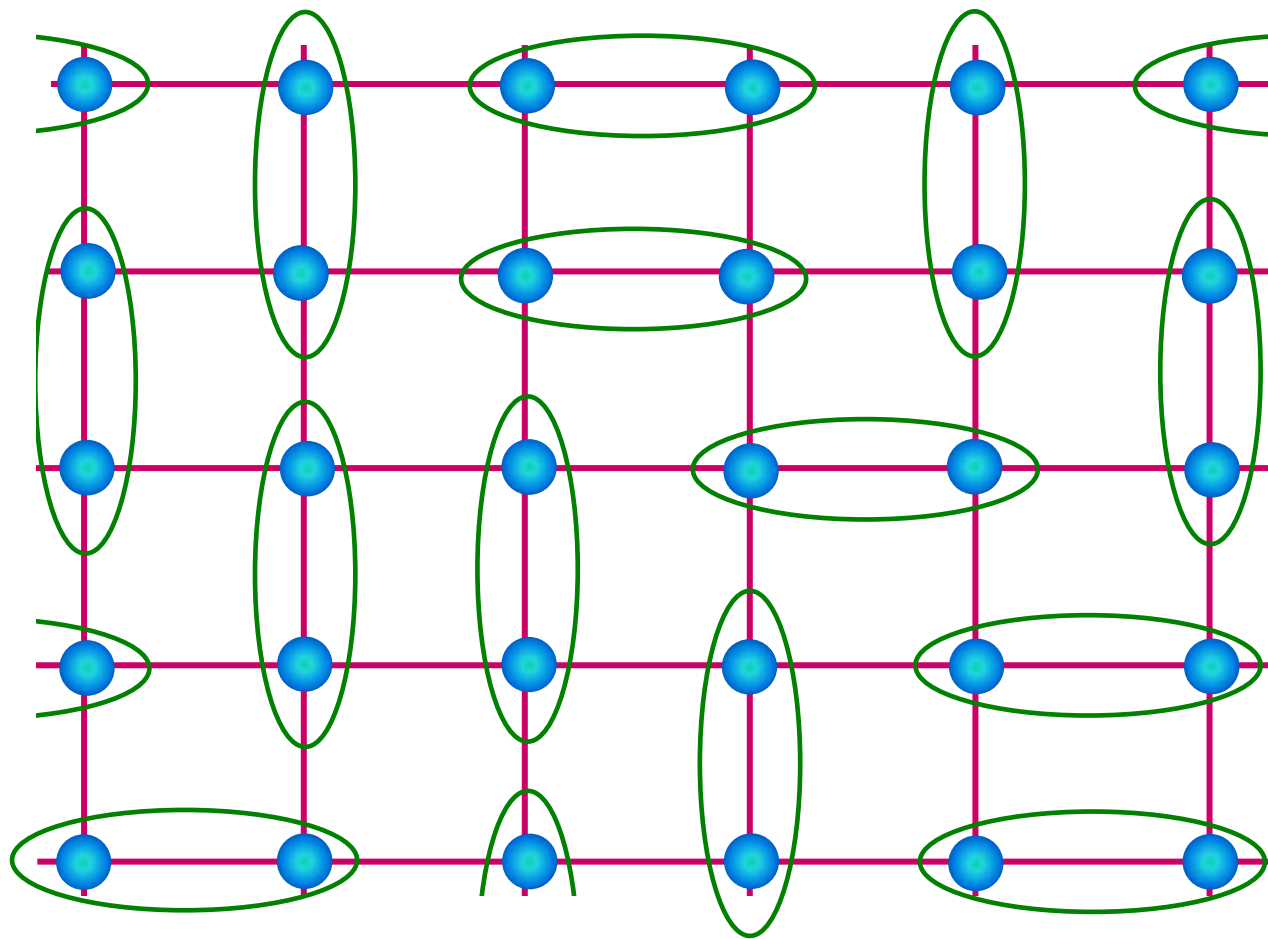


$$\text{[Green oval with two blue spheres]} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

N. Read and S. Sachdev,
Phys. Rev. Lett. **62**, 1694
(1989).

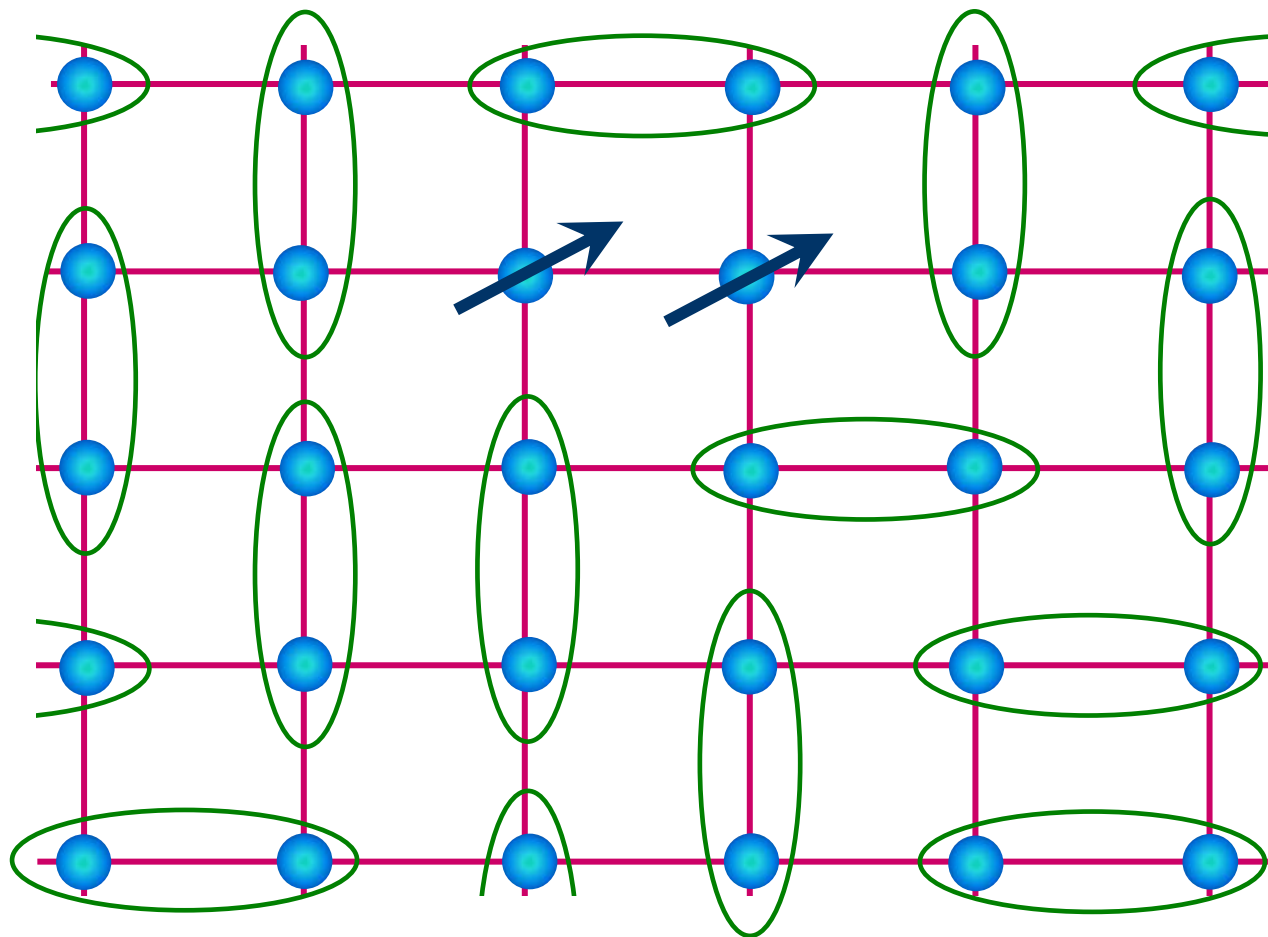
R. Moessner and S. L.
Sondhi, *Phys. Rev. B* **63**,
224401 (2001).

Excitations of the RVB liquid



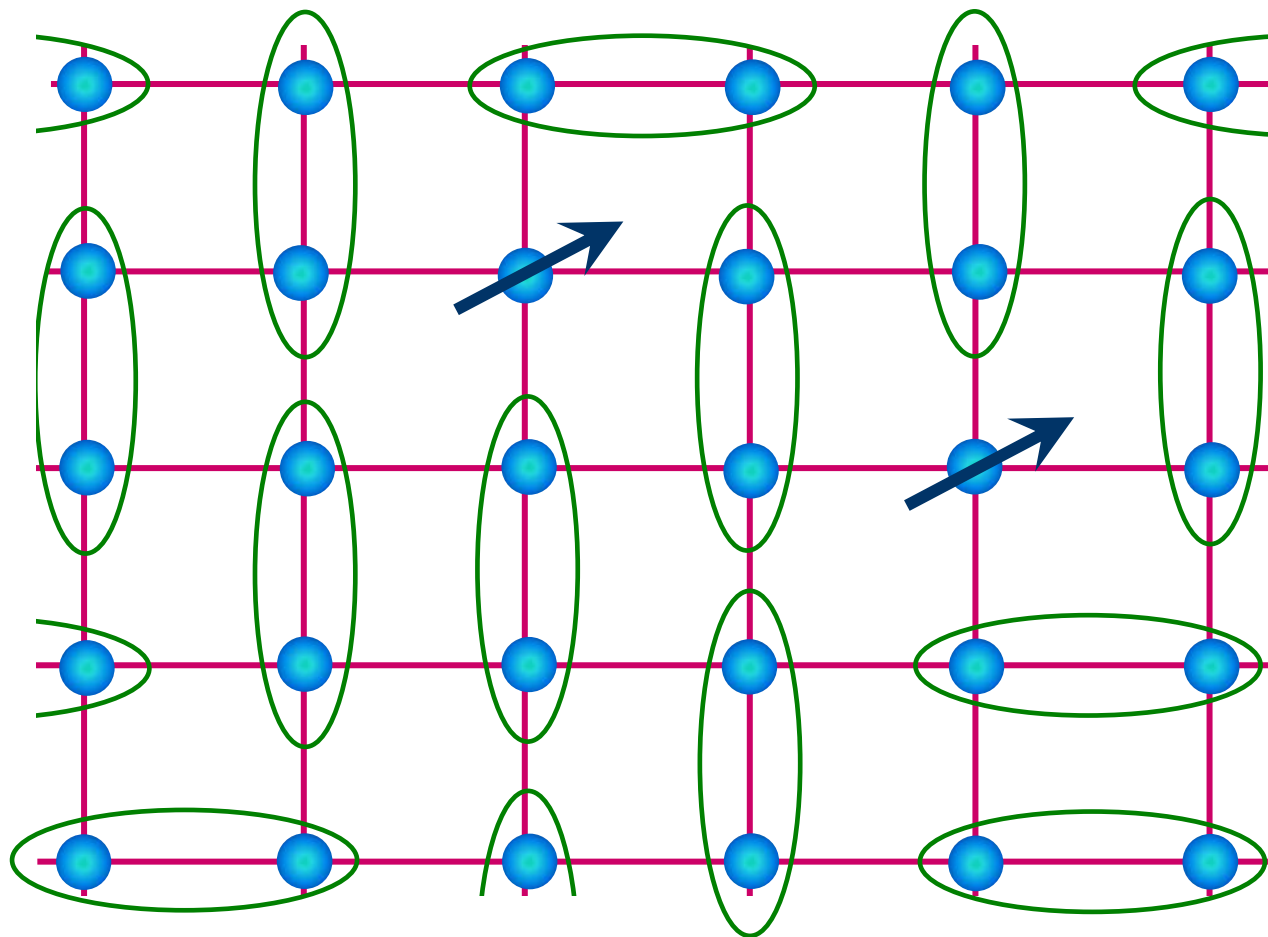
$$\text{Oval} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

Excitations of the RVB liquid



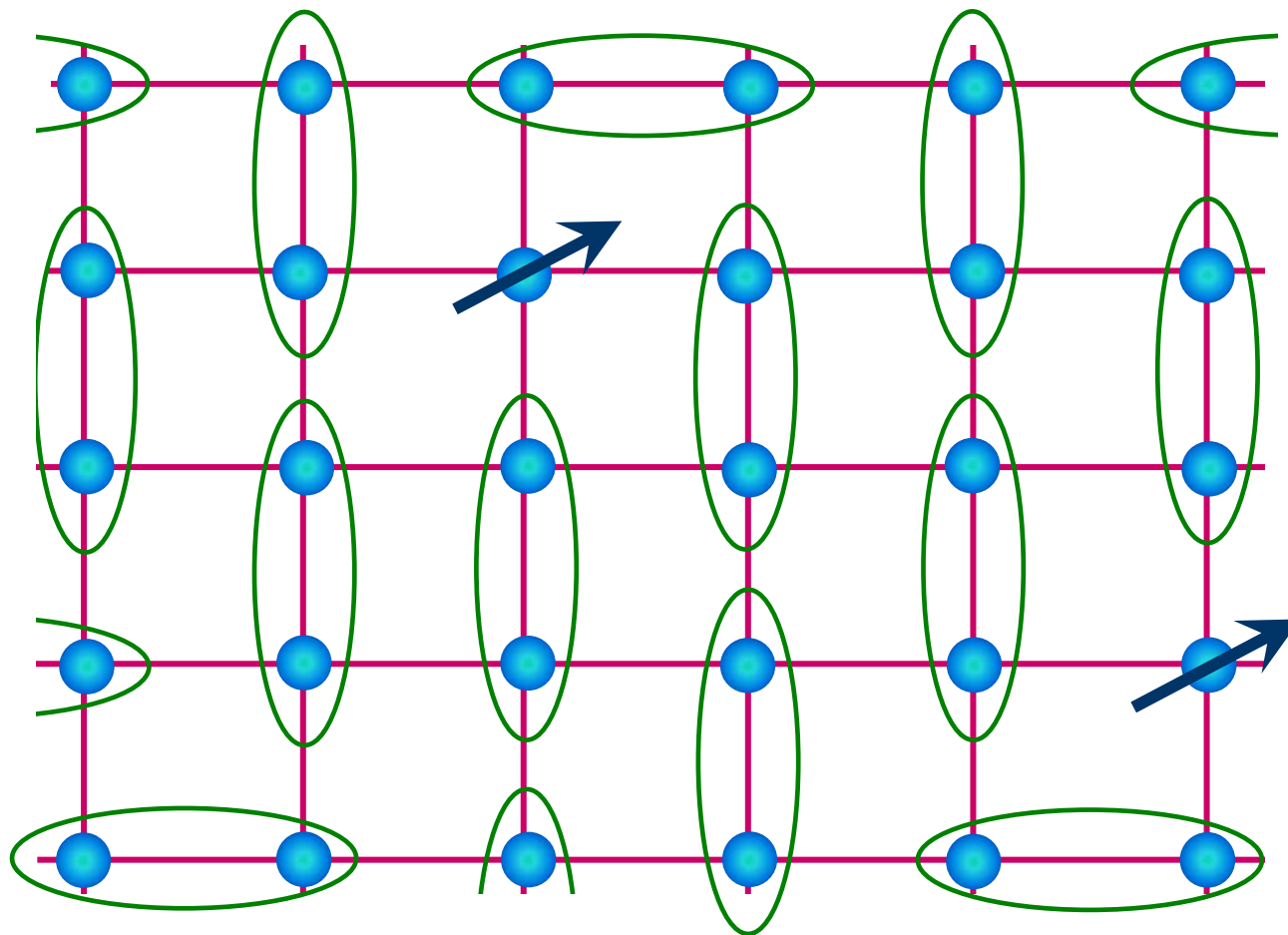
$$\text{Oval} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

Excitations of the RVB liquid



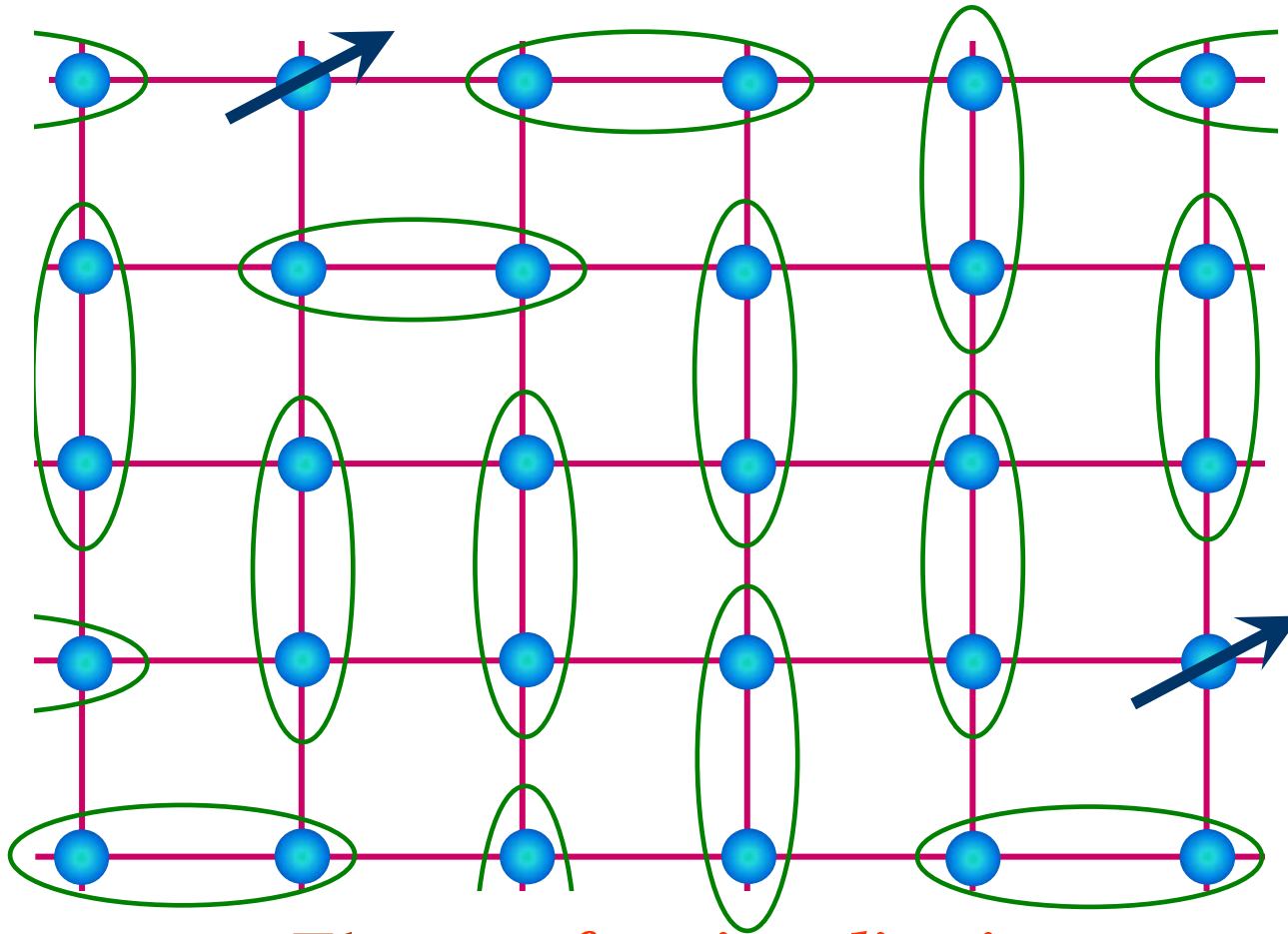
$$\text{Oval} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

Excitations of the RVB liquid



$$\text{Oval} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

Excitations of the RVB liquid

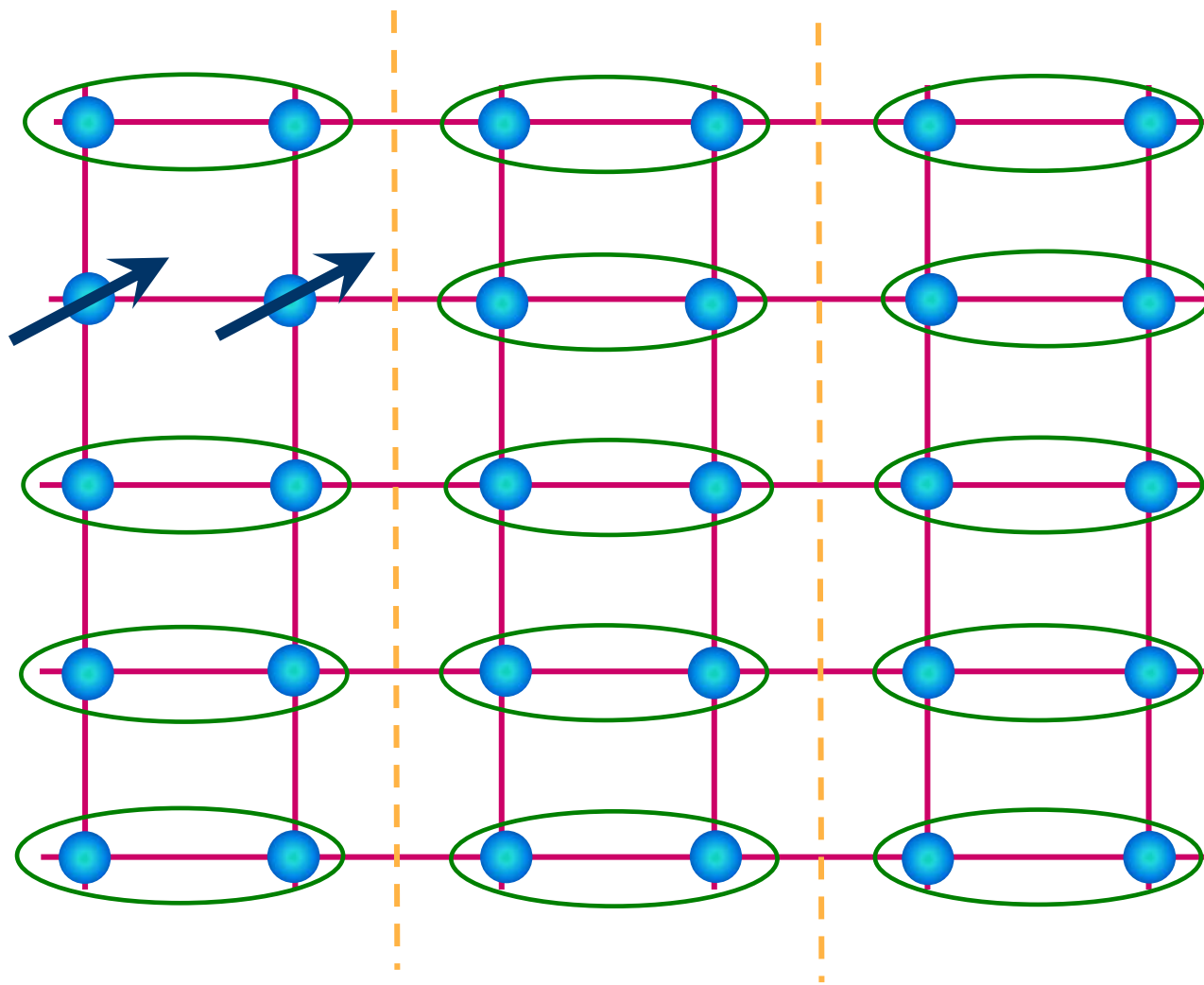


$$\text{Oval} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

Electron *fractionalization*:

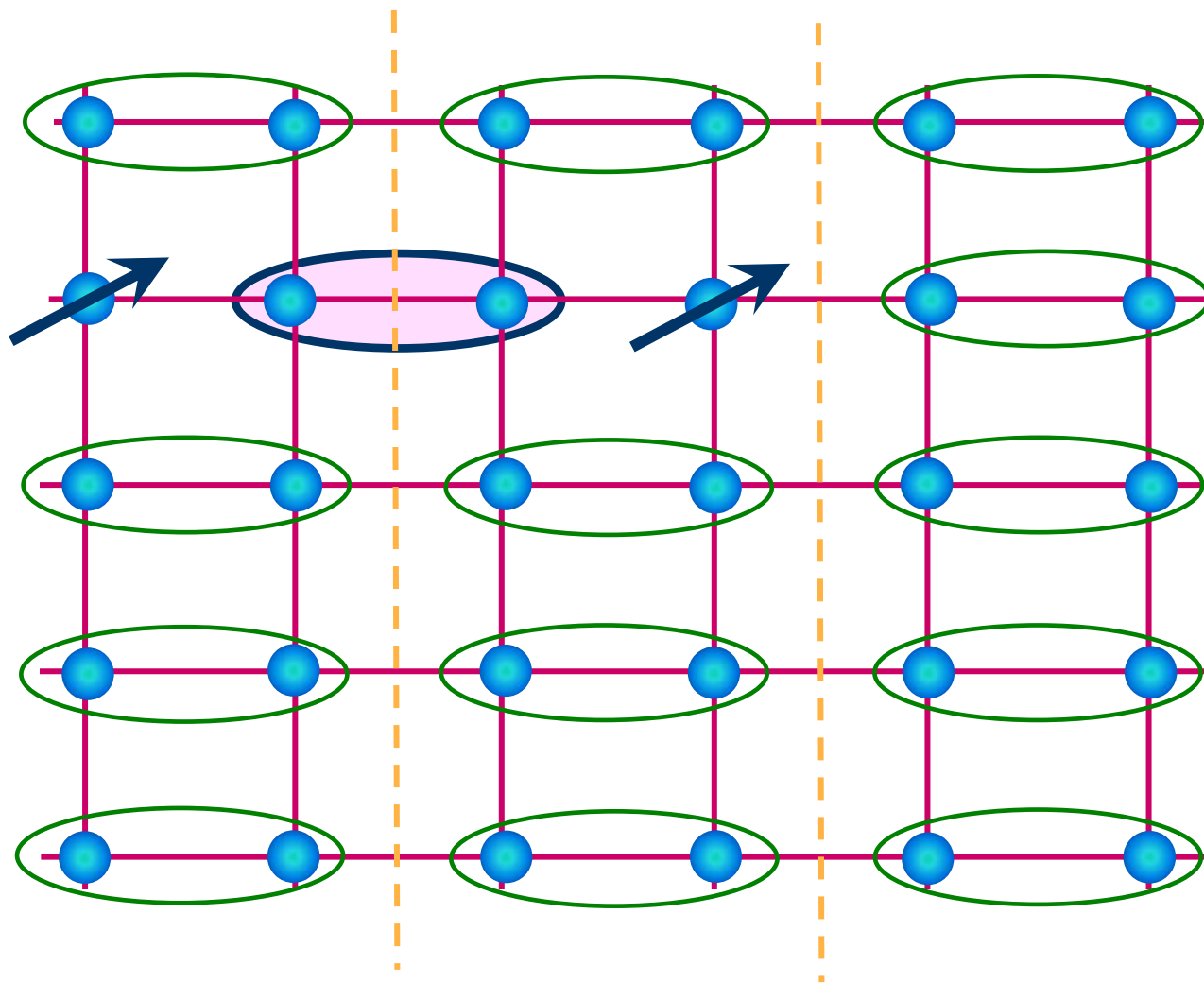
Excitations carry spin $S=1/2$ but no charge

Excitations of the VBS



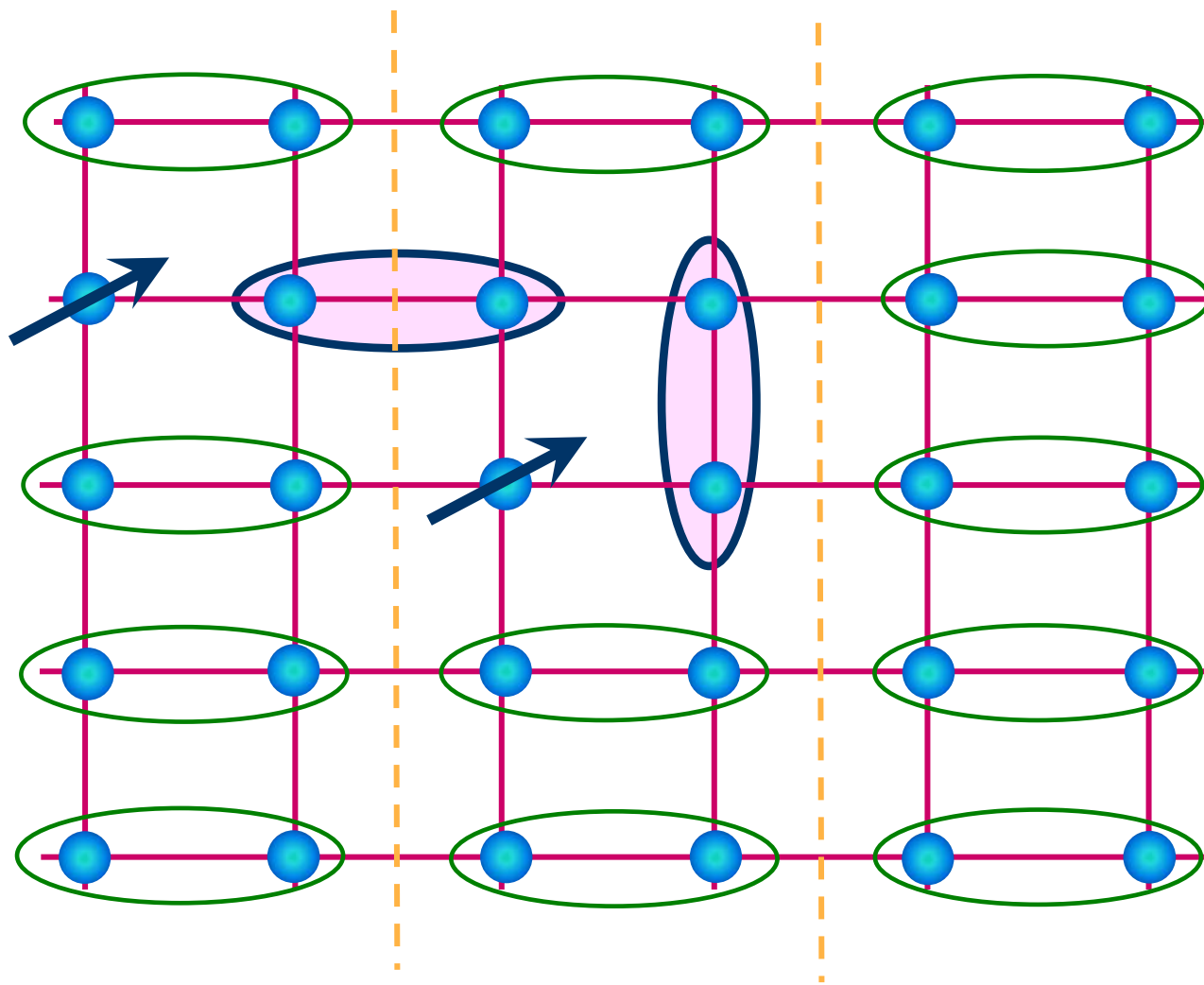
$$\text{Oval} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

Excitations of the VBS



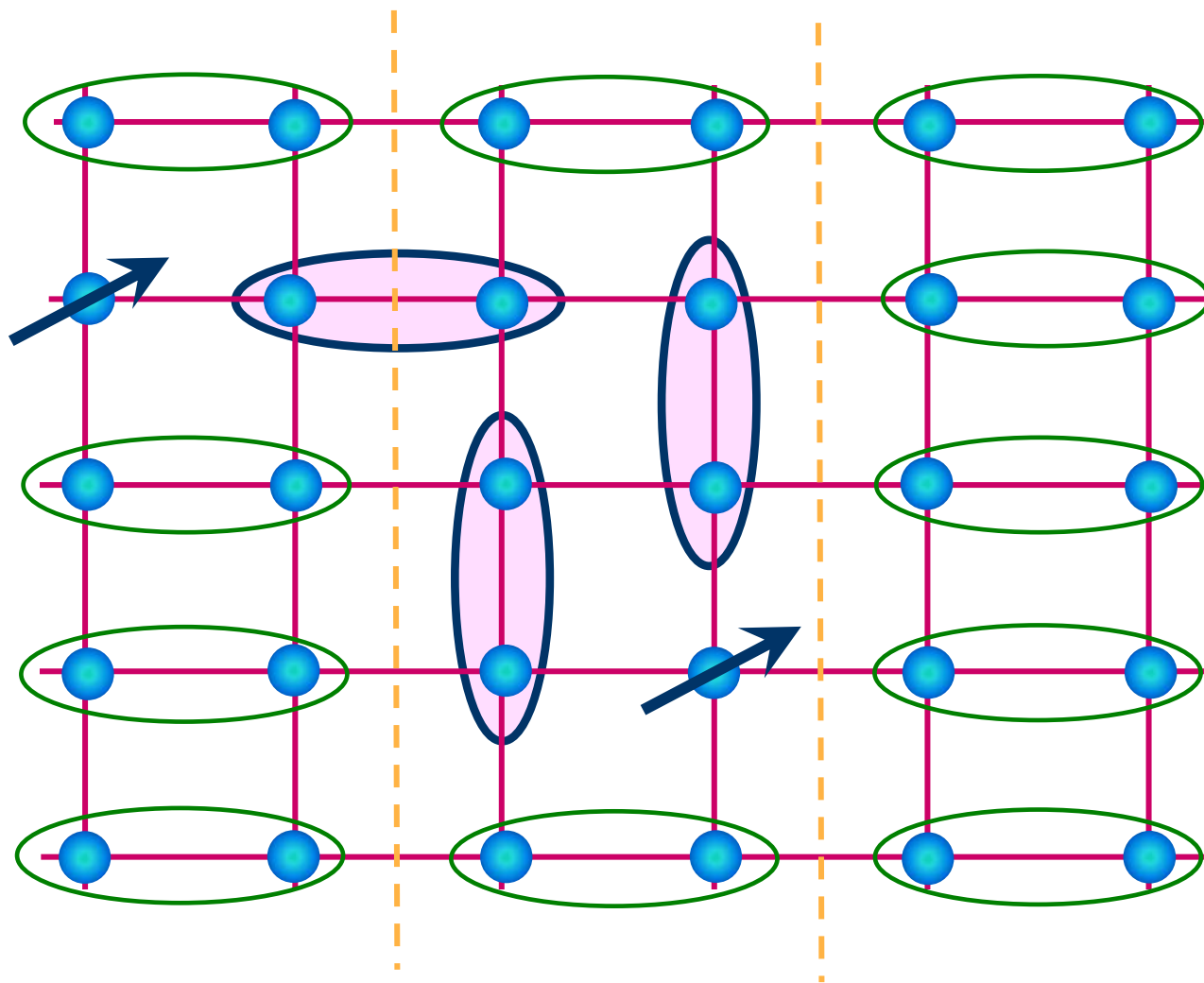
$$\text{[Diagram of two blue spheres in a green oval]} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

Excitations of the VBS



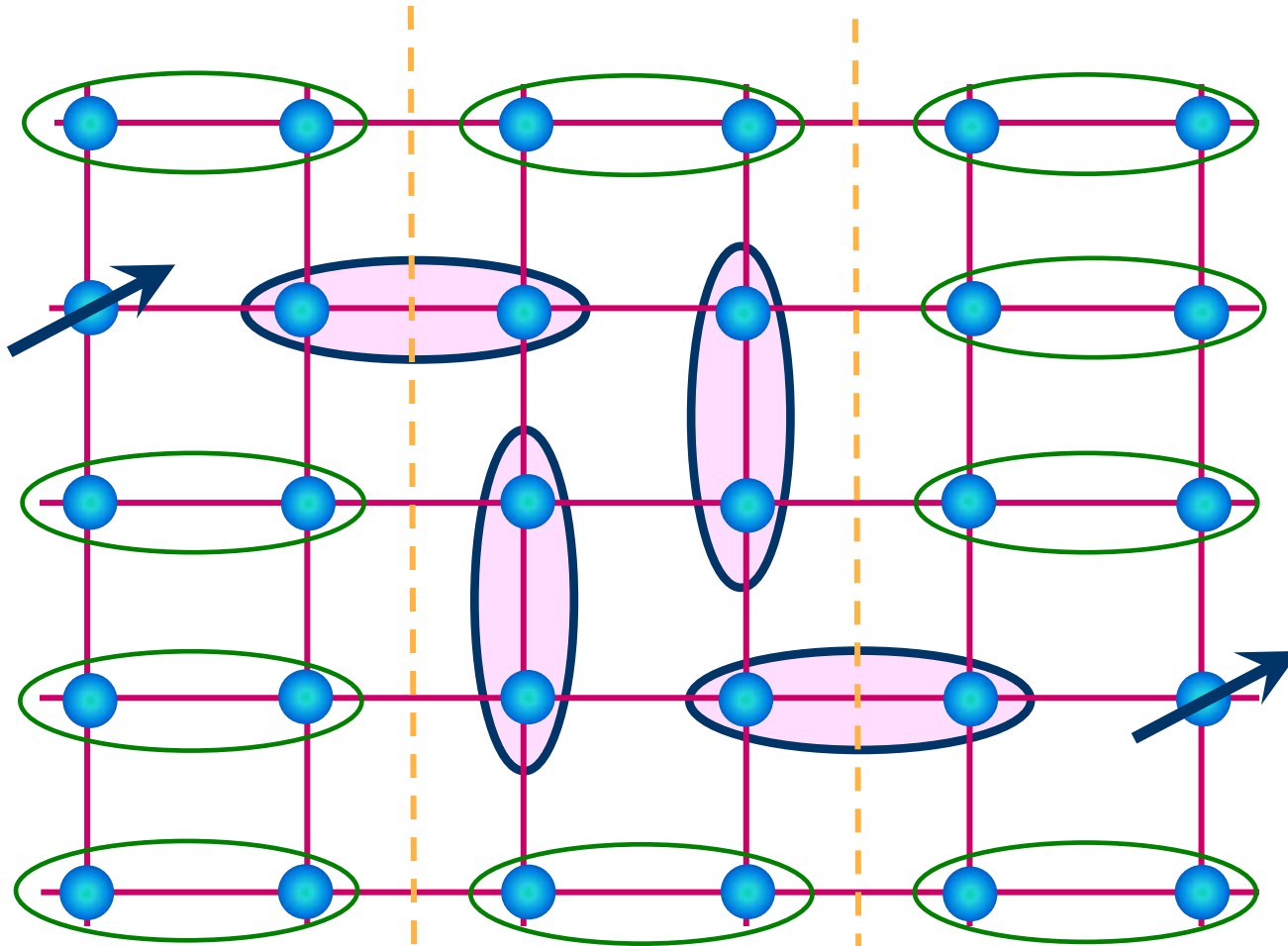
$$\text{green oval} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

Excitations of the VBS



$$\text{Oval} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

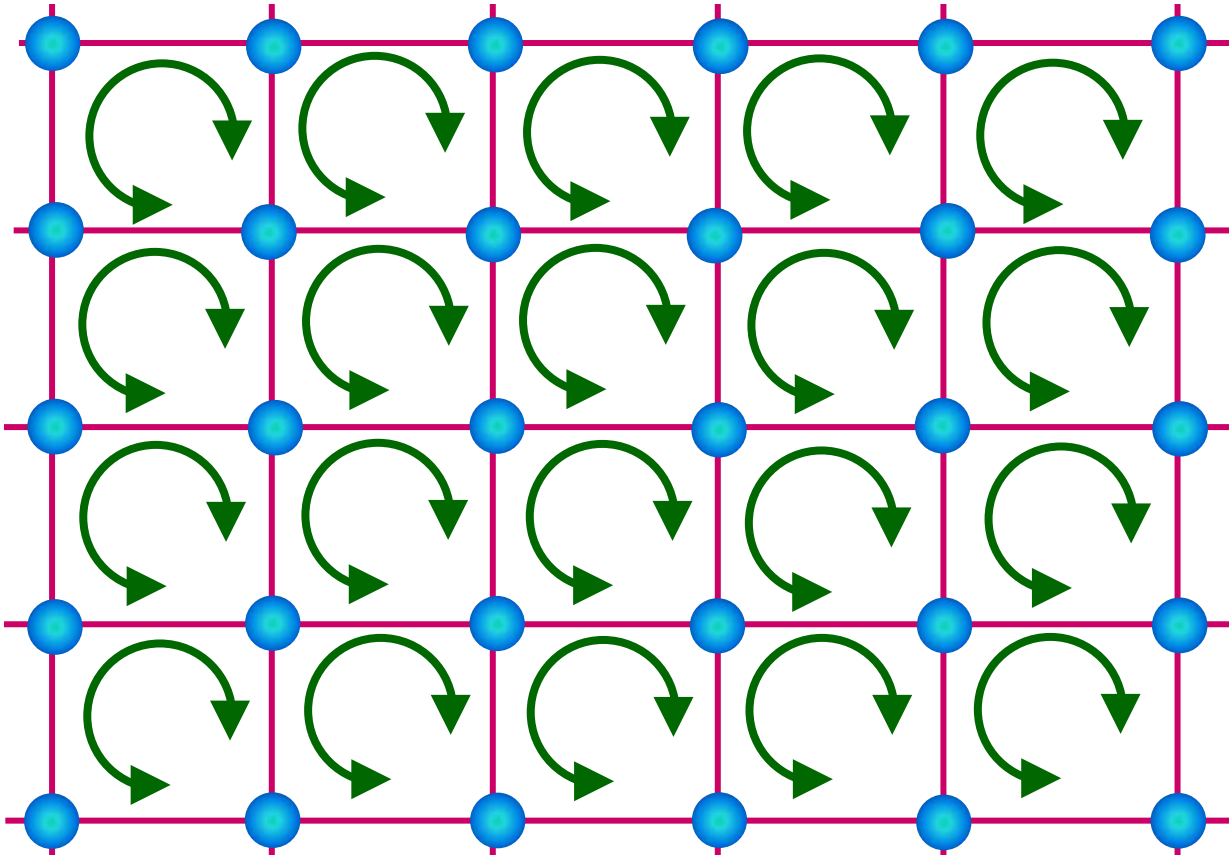
Excitations of the VBS



$$\text{Oval with two blue spheres} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

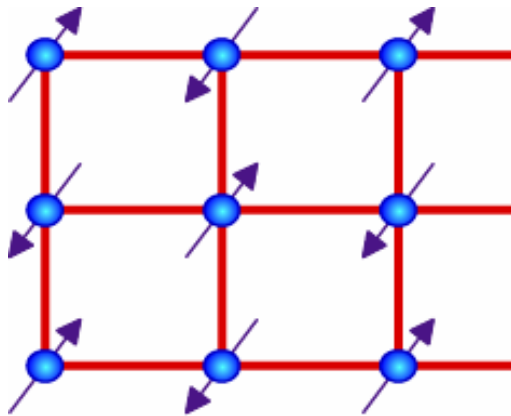
Free spins are unable to move apart:
no fractionalization, but *confinement*

Phase diagram of square lattice antiferromagnet

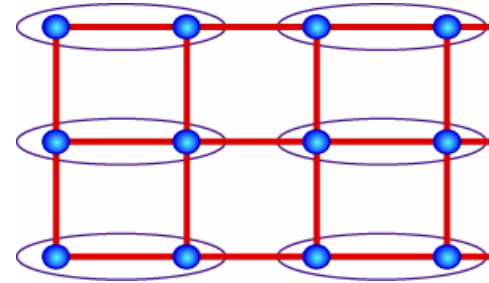


$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + K \sum_{\square} \text{4-spin exchange}$$

Phase diagram of square lattice antiferromagnet



Neel order

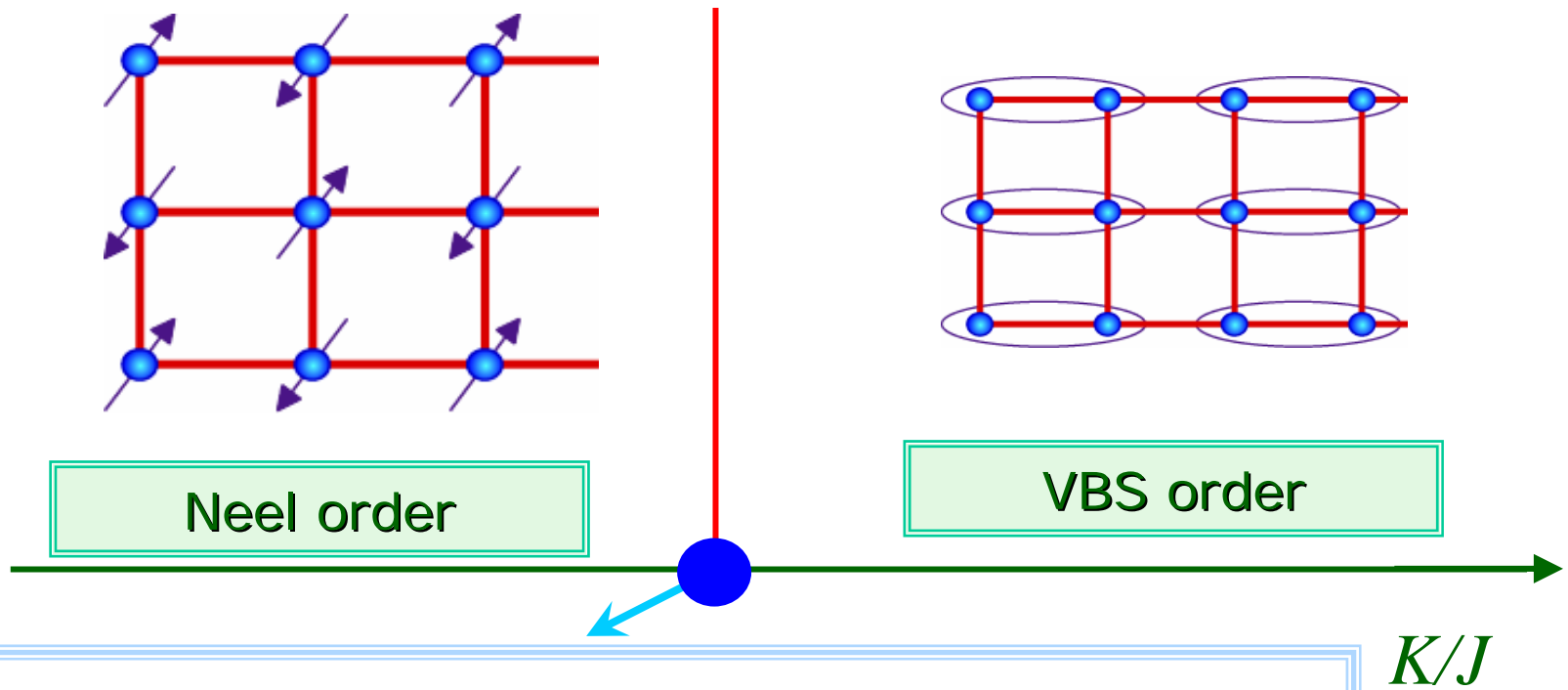


VBS order

K/J

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + K \sum_{\square} 4\text{-spin exchange}$$

Phase diagram of square lattice antiferromagnet



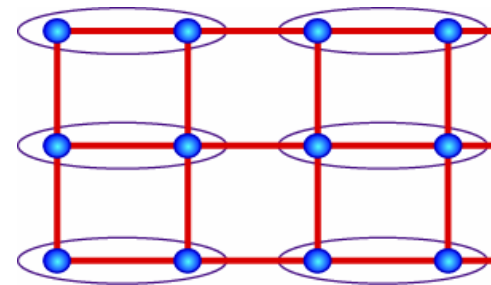
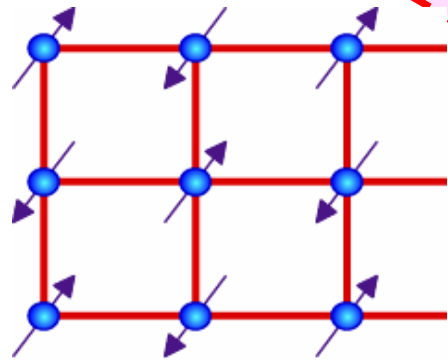
RVB physics appears at the quantum critical point which has fractionalized excitations: “deconfined criticality”

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + K \sum_{\square} 4\text{-spin exchange}$$

Temperature, T

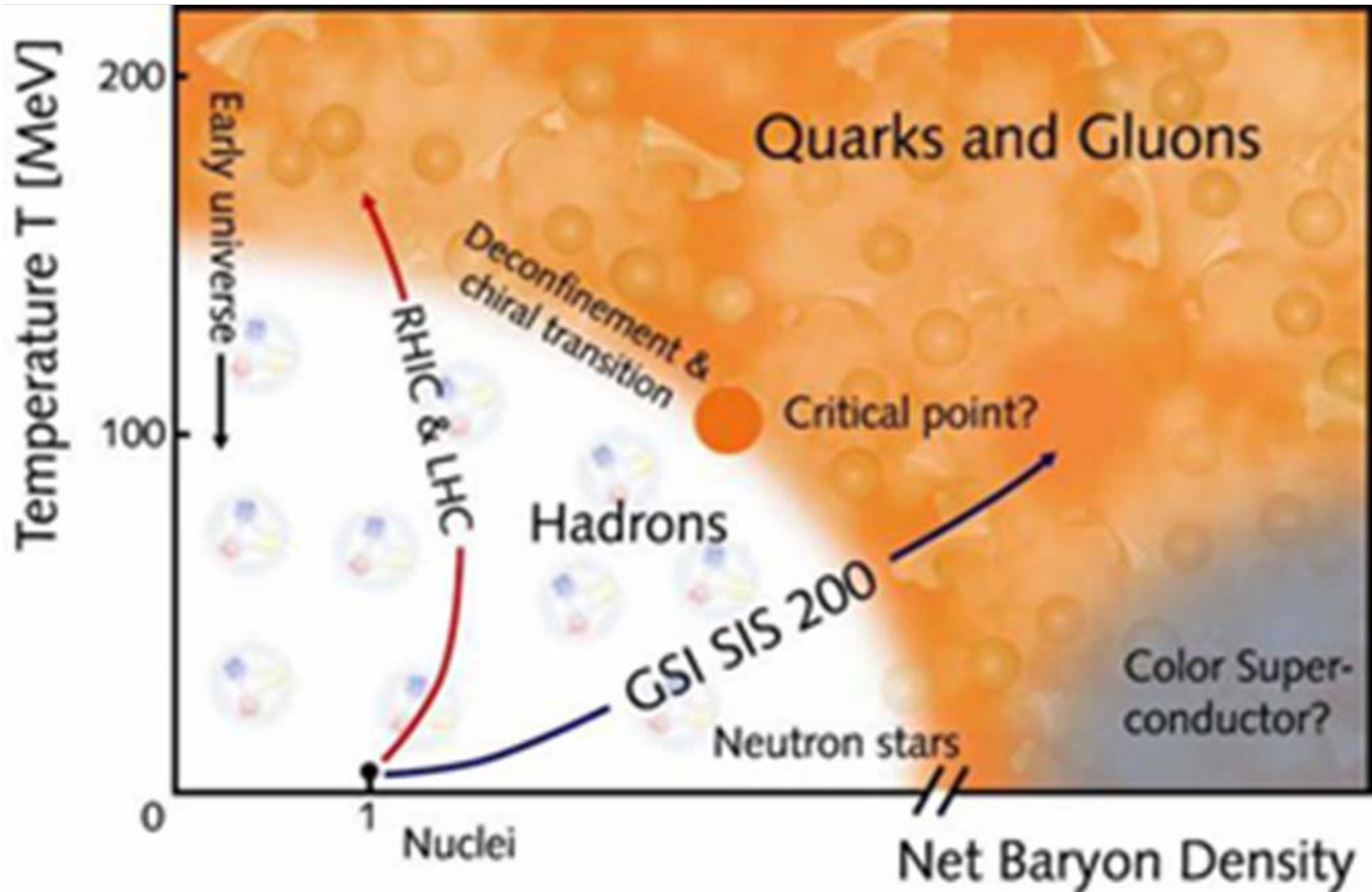
Quantum criticality of
fractionalized
excitations

0



K/J

Phases of nuclear matter



Conclusions

- Rapid progress in the understanding of quantum liquids near unitarity
- Rich possibilities of exotic quantum phases in optical lattices
- Cold atom studies of the entanglement of large numbers of qubits: insights may be important for quantum cryptography and quantum computing.
- Tabletop “laboratories for the entire universe”: quantum mechanics of black holes, quark-gluon plasma, neutrons stars, and big-bang physics.

