Tomonaga-Luttinger Liquids in Cold Atomic Gases.

A Fresh Look at Some Old and "New" Problems: Hubbard Models and Non-equilibrium

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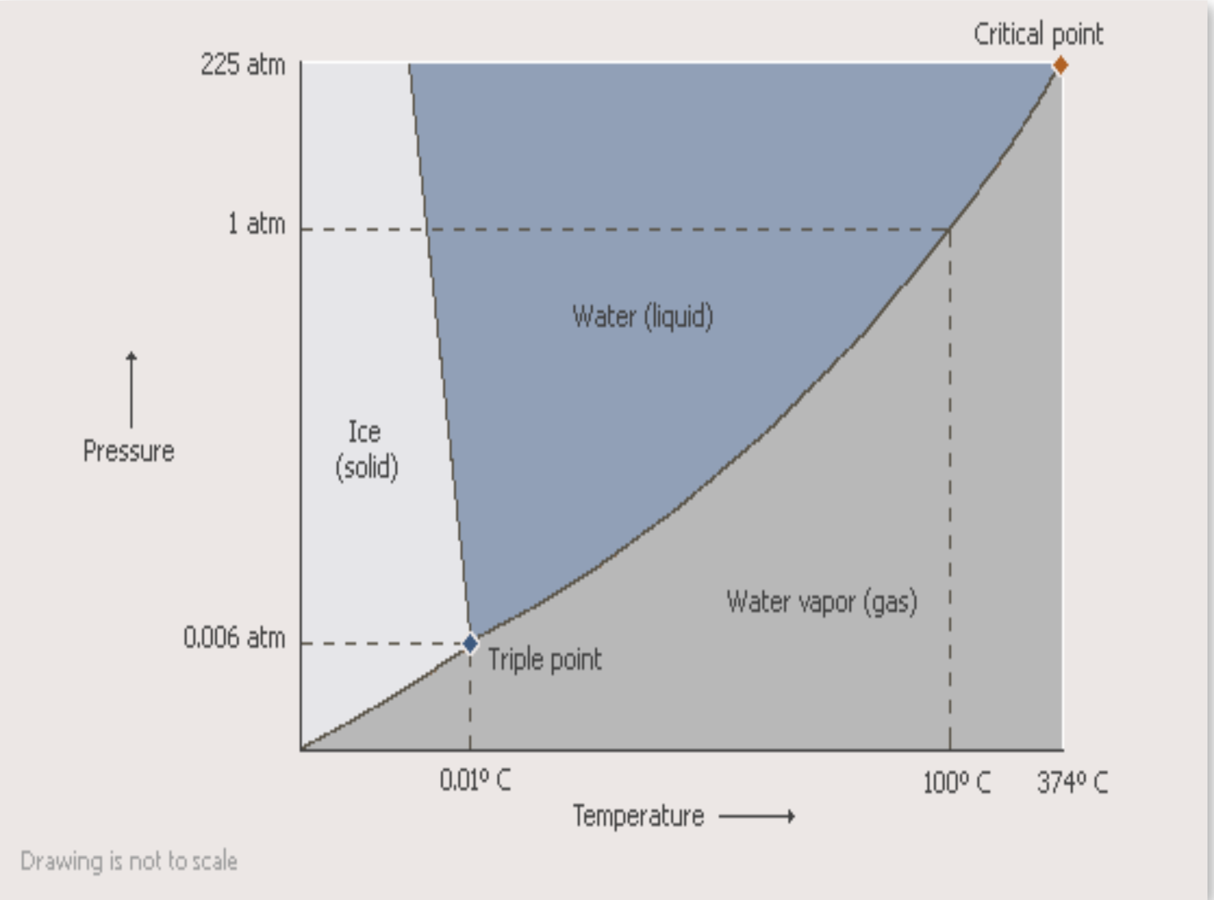
What is “new”?

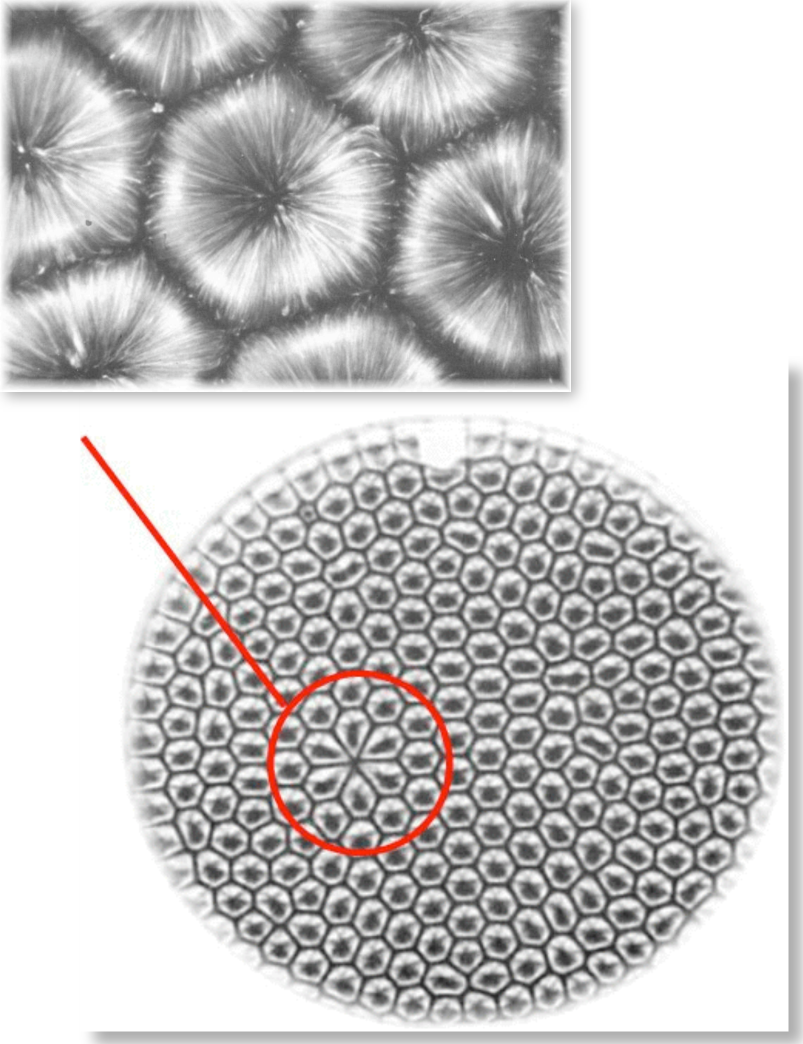
Non-equilibrium steady states

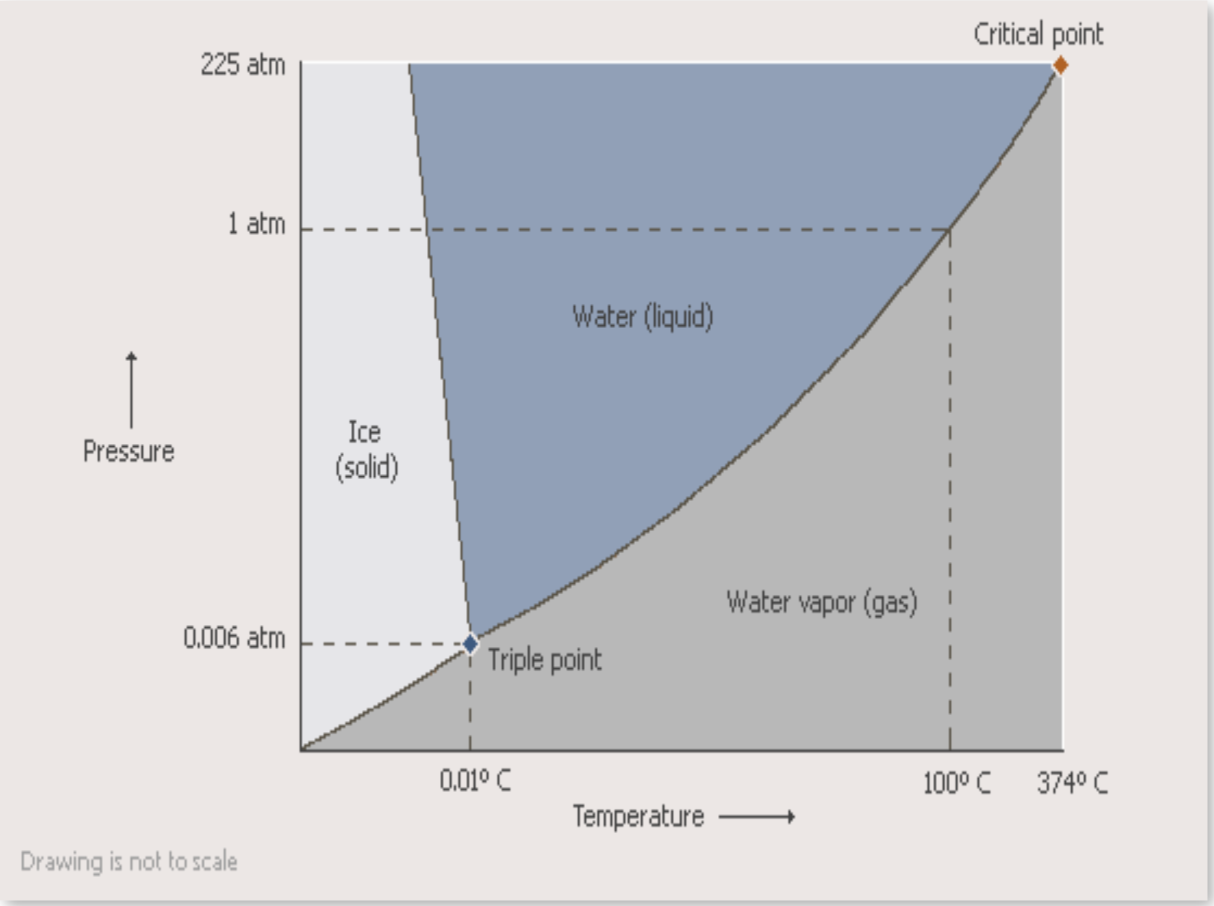


Equilibrium phase diagram of H_2O

Non-equilibrium steady states

Rayleigh-Bénard convection cells





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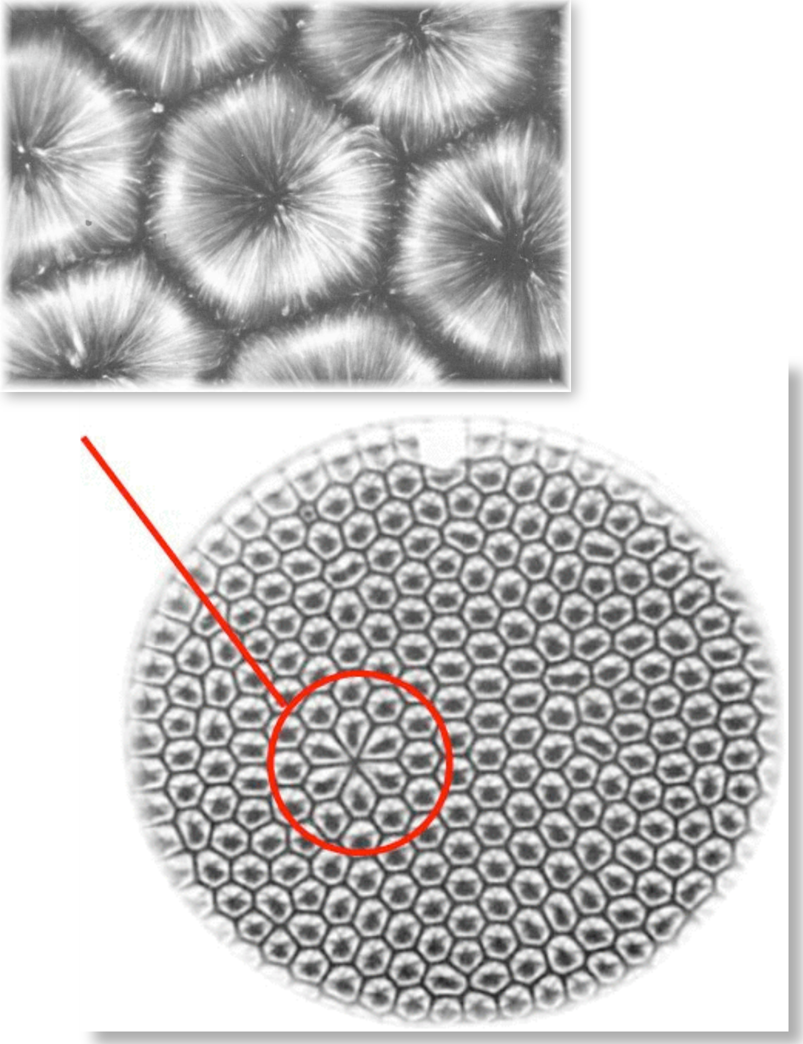
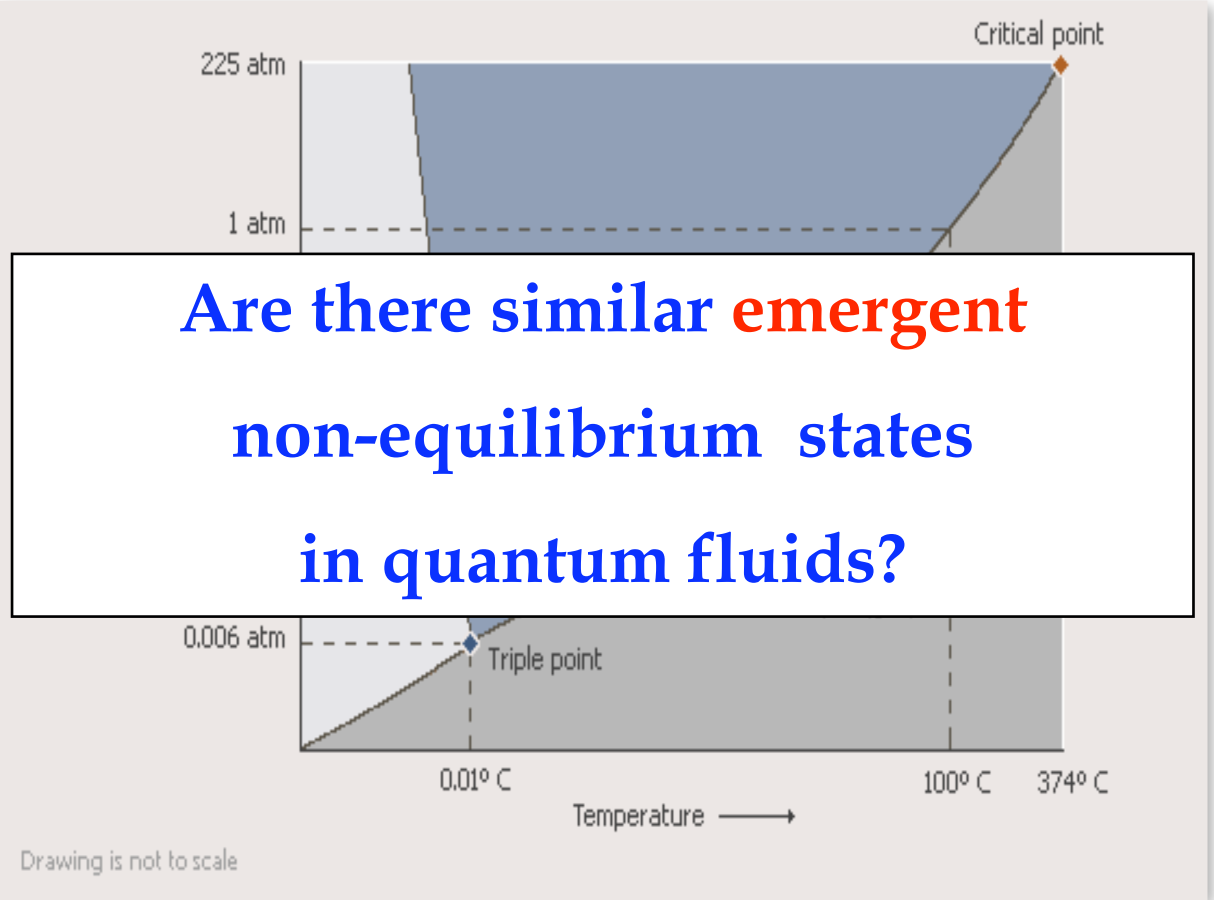


Are there similar emergent

non-equilibrium states

in quantum fluids?



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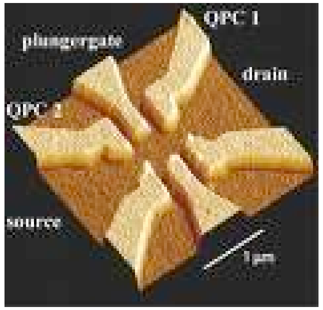
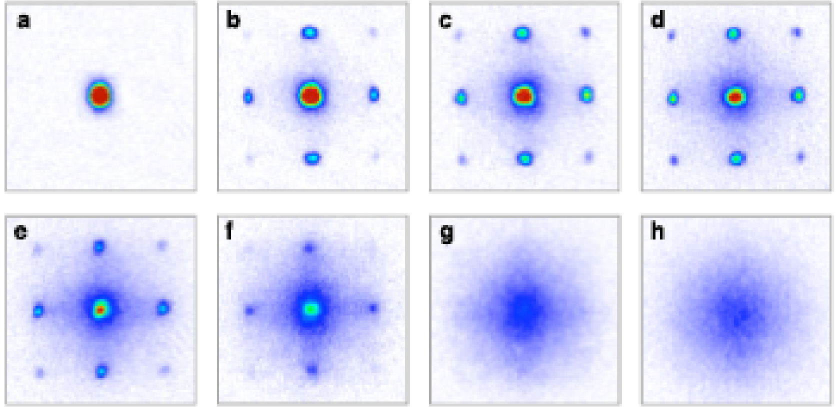
Quantum Fluids out of equilibrium



Problems with solid state/liquid He quantum fluids:

- Not easily tunable
- Quantum decoherence is a killer



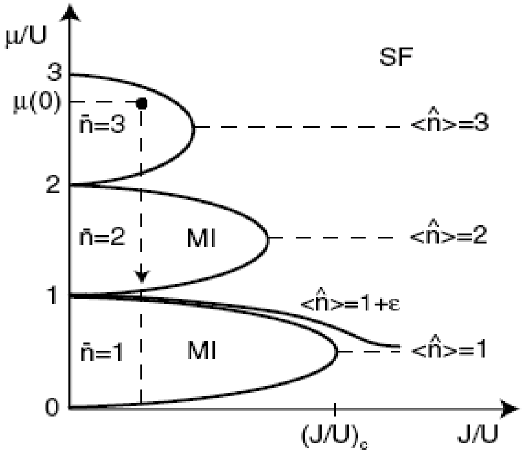
Cold atoms in an optical lattice

[D. Jaksch *et al.* PRL 81 (1998)]
[M Greiner *et al.* Nature, 415 (2002)]

Interacting bosons on a lattice

[MPA Fisher *et al.* PRB 40 (1989)]

$$H_{\mathrm{BH}} = -J \sum_{\langle \mathbf{R},\mathbf{R}'\rangle} b^{\dagger}_{\mathbf{R}} b_{\mathbf{R}'} + U \sum_{\mathbf{R}} n_{\mathbf{R}}(n_{\mathbf{R}} - 1)$$



Absence of thermalization in 1DBG

nature

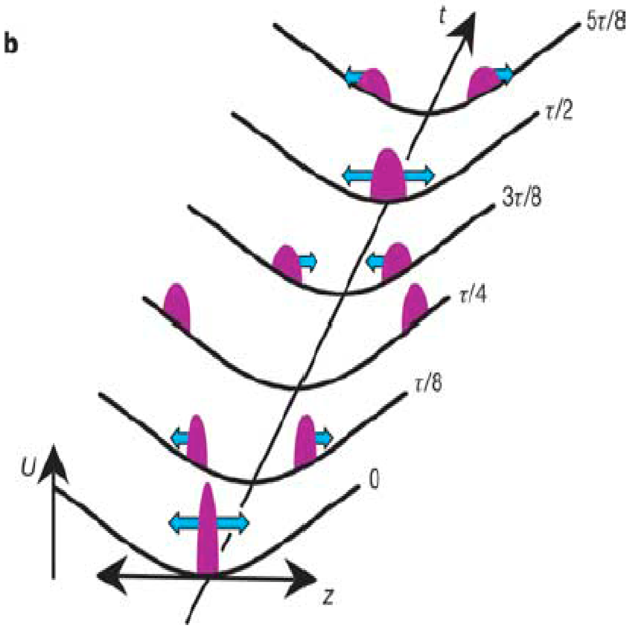
LETTERS

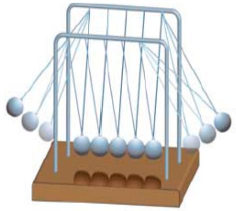
[T Kinoshita, T Wenger & D Weiss, Nature (2006)]

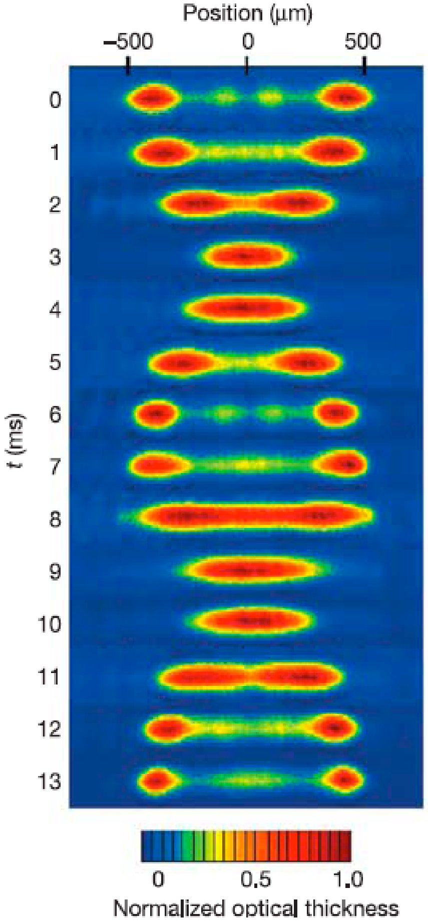
A quantum Newton's cradle

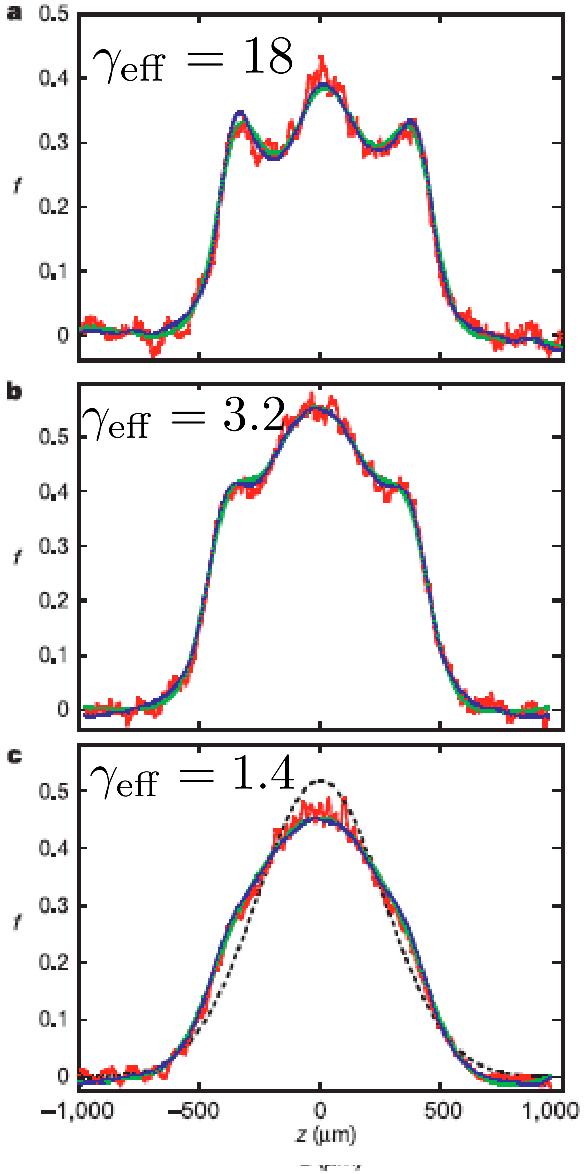
Toshiya Kinoshita[1], Trevor Wenger[1] & David S. Weiss[1]

$$\begin{aligned} p_1 + p_2 &= p'_1 + p'_2, \\ \frac{p_1^2}{2M} + \frac{p_2^2}{2M} &= \frac{p'^2_1}{2M} + \frac{p'^2_2}{2M}, \\ p_1 = p'_1,\ p_2 = p'_2, \quad & \quad p_1 = p'_2,\ p_2 = p'_1 \end{aligned}$$









A Statistical description of non-equilibrium states?

Quench at t = 0 (sudden approximation):

$$|\Phi(t>0)\rangle = e^{-iH_f\ t/\hbar}|\Phi(0)\rangle = e^{-iH_f\ t/\hbar}|\underset{\uparrow}{\Phi_0}\rangle$$

Not an eigenstate of *Hf* !!

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Operators after the quench:

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Does the system reach a (quasi-) stationary state? If so,

$$\bar{O} = \lim_{T\to+\infty}\lim_{t_0\to+\infty}\frac{1}{T}\int_{t_0}^{T+t_0} dt\, O(t)$$

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Is $\hat{\rho}_{\mathrm{quench}} = \rho_{\mathrm{eq}} = e^{-(H_f-\mu N)/T_{\mathrm{eff}}}$ **? (ergodic hypotesis)**

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Won’t be looking at the creation of defects (Stamper-Kurn, Damski,...)

Free expansion of hard-core bosons

[M Rigol, B Dunjko, V Yurovsky, and M Olshanii PRL 98 (2007)]

Integrable XY model

$$H = -J \sum_{\langle n,m \rangle} \sigma_m^+ \sigma_n^- = -\sum_p (2J \cos p) \; f^\dagger(p) f(p)$$

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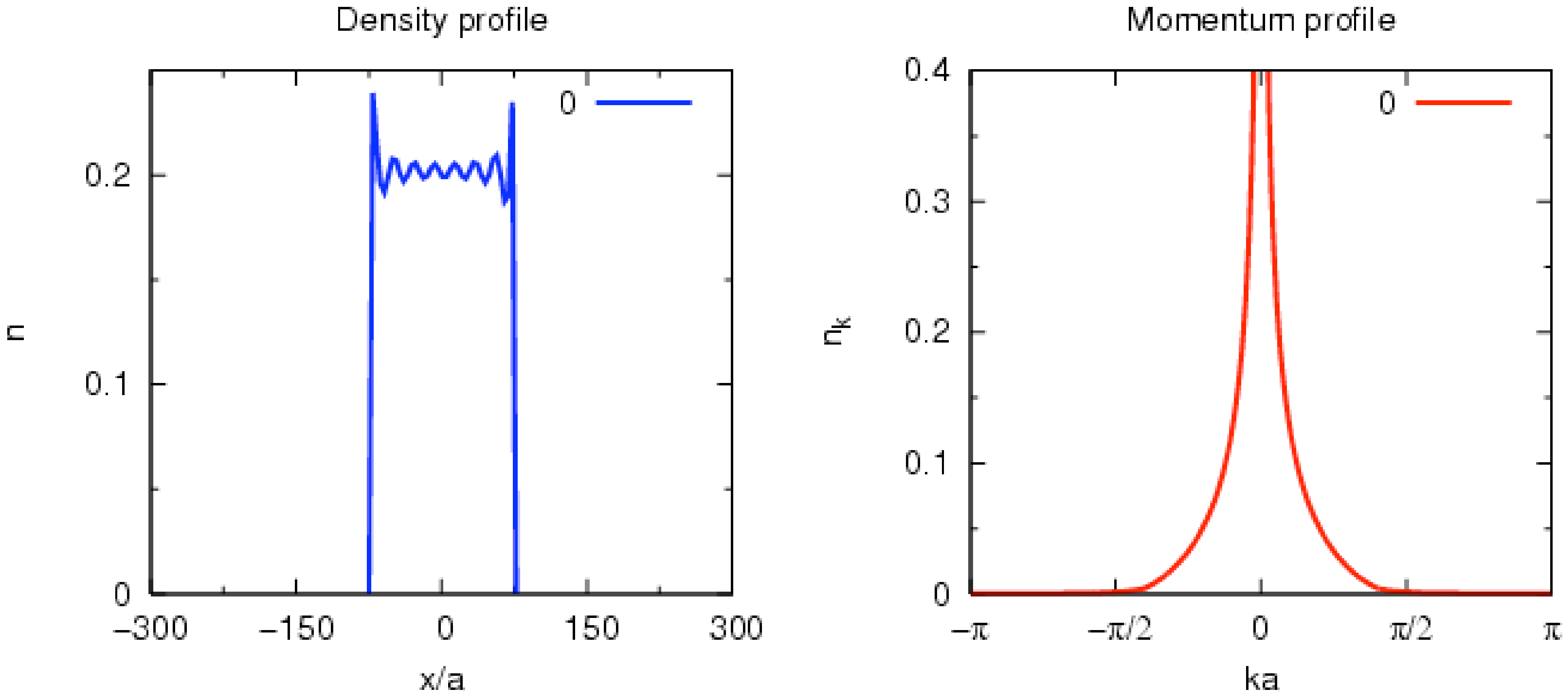

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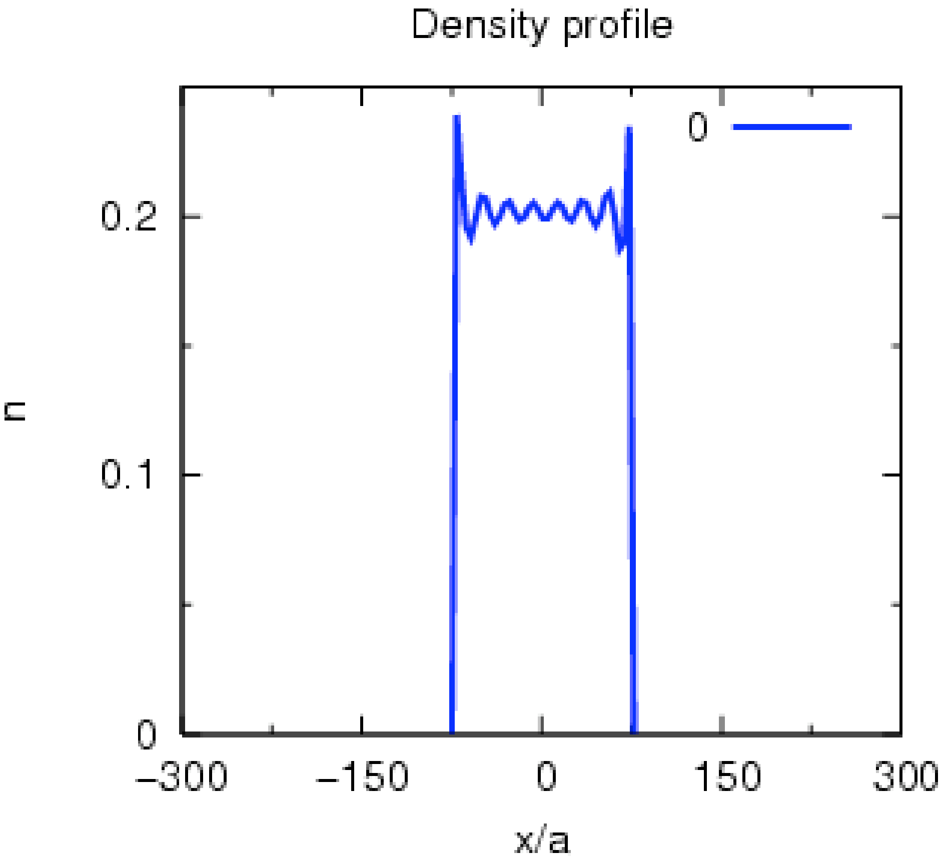

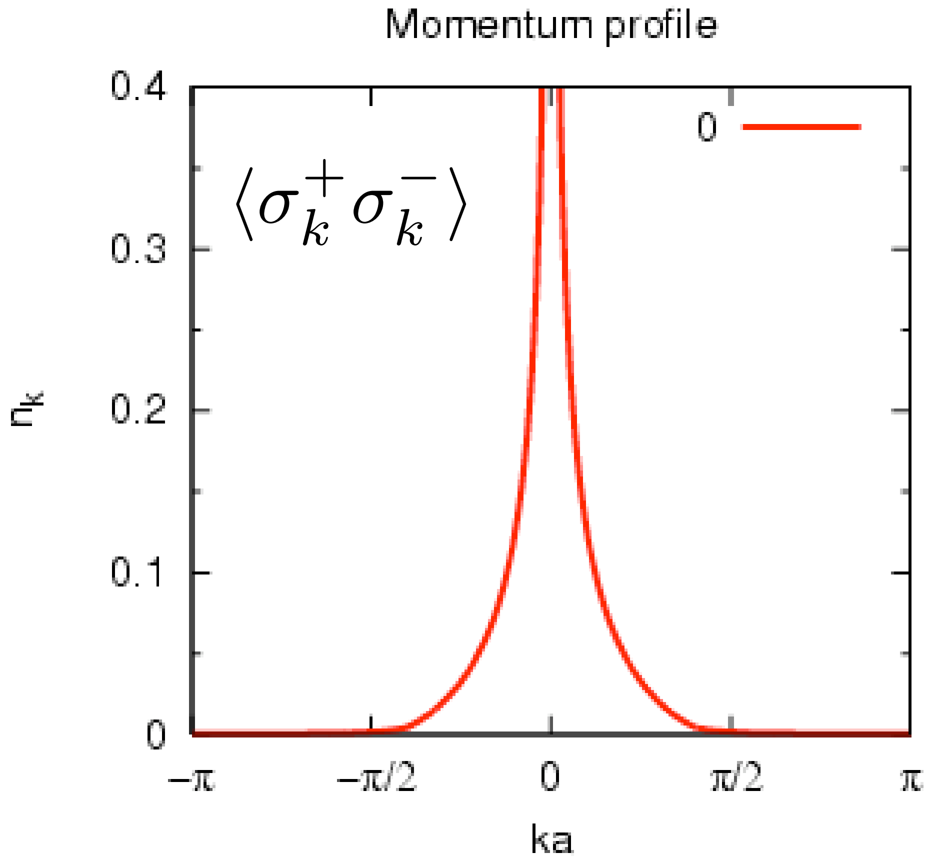

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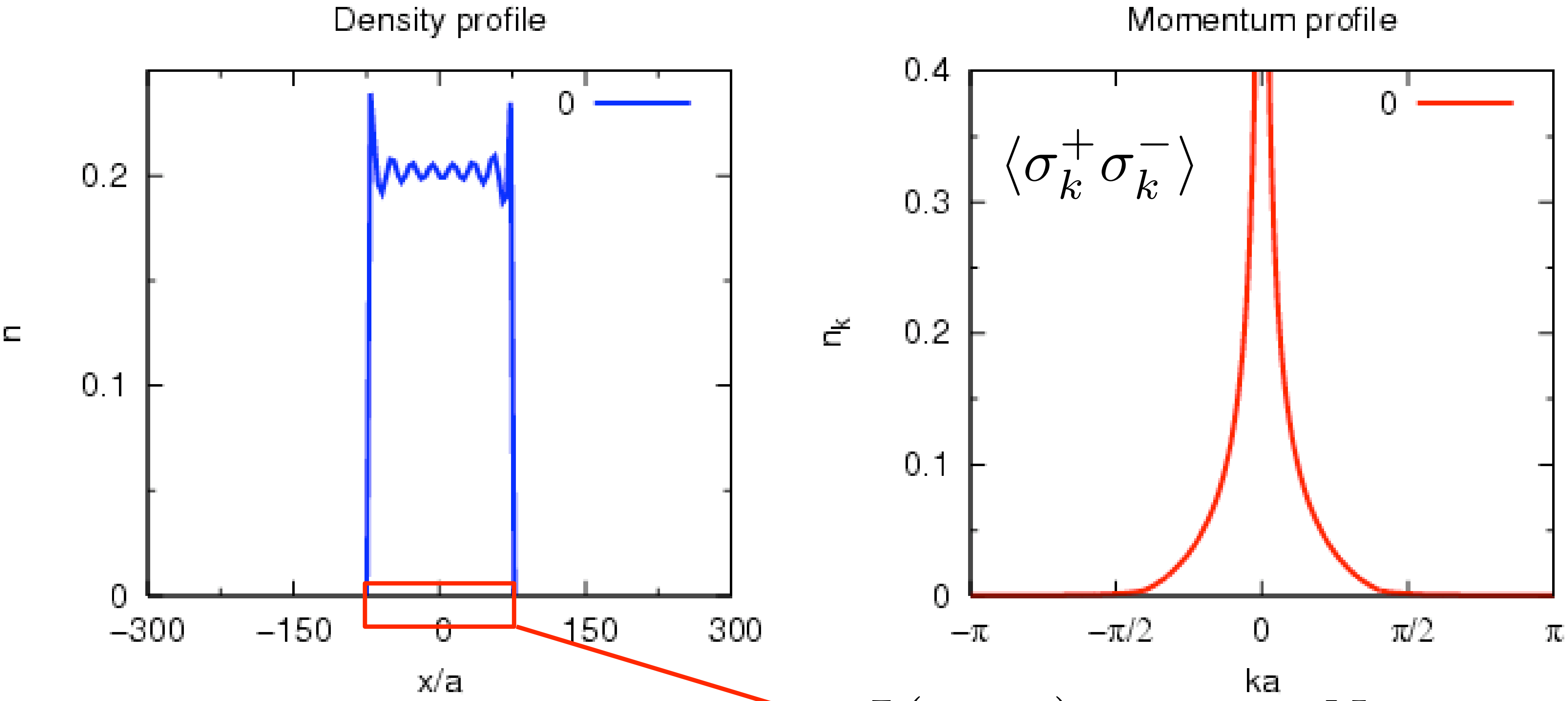

$$L(t=0) = 150\,a, N = 30$$

$$|\Phi(t=0)\rangle$$

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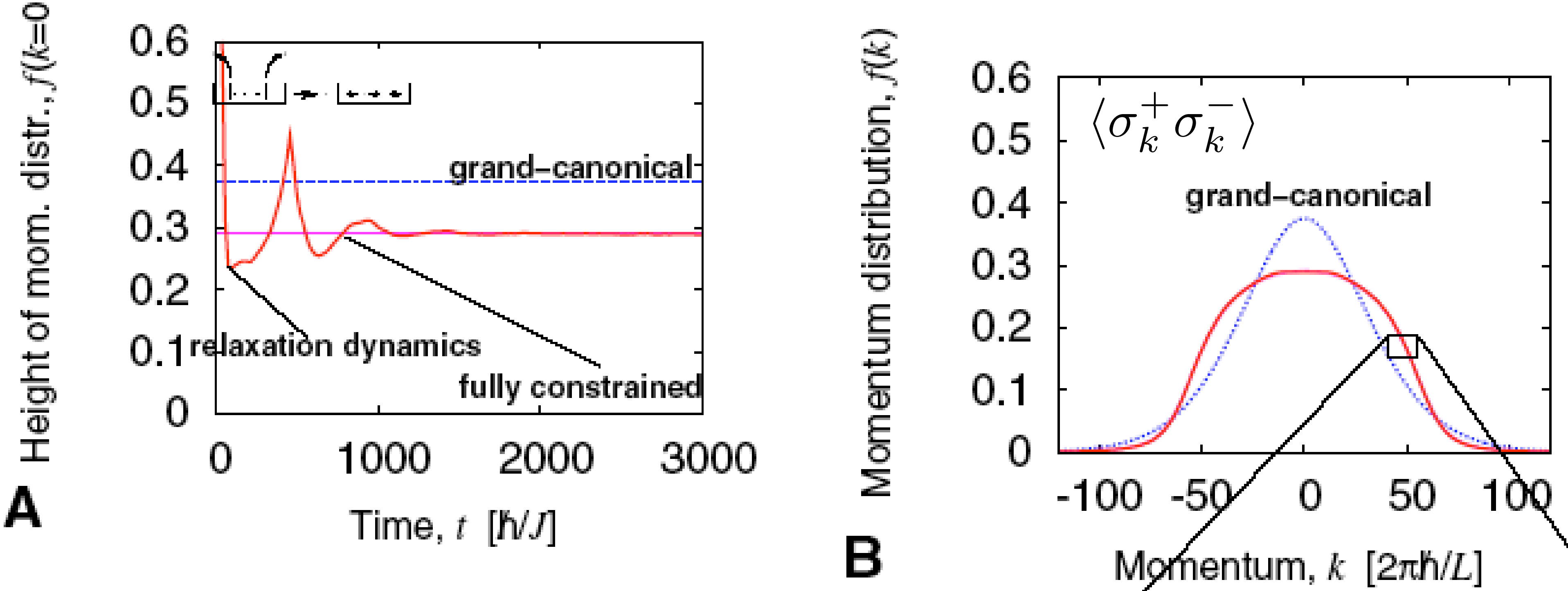
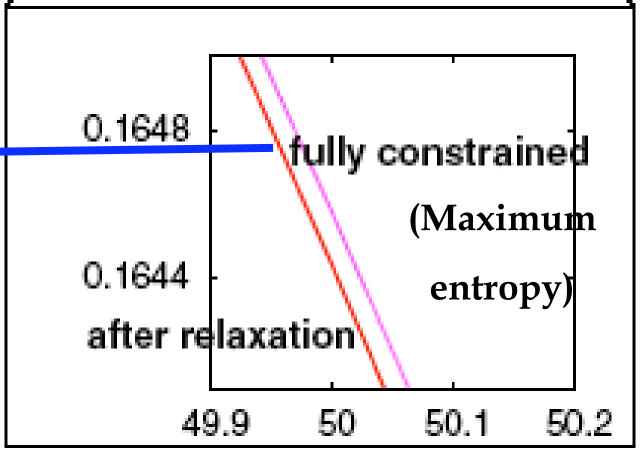


Maximum entropy (generalized Gibbs)

$$\rho_{\mathrm{gG}}(\{\lambda_p\}) = \frac{e^{\sum_p \lambda_p I_p}}{Z_{\mathrm{gG}}}, \quad I_p = f^\dagger(p) f(p)$$

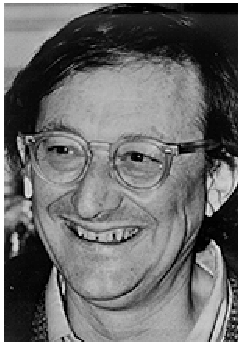
$$\langle I_p\rangle_{\mathrm{gG}} = \mathrm{Tr}\, \rho_{\mathrm{gG}} I_p = \langle \Phi(t=0)| f^\dagger(p) f(p) |\Phi(t=0)\rangle$$



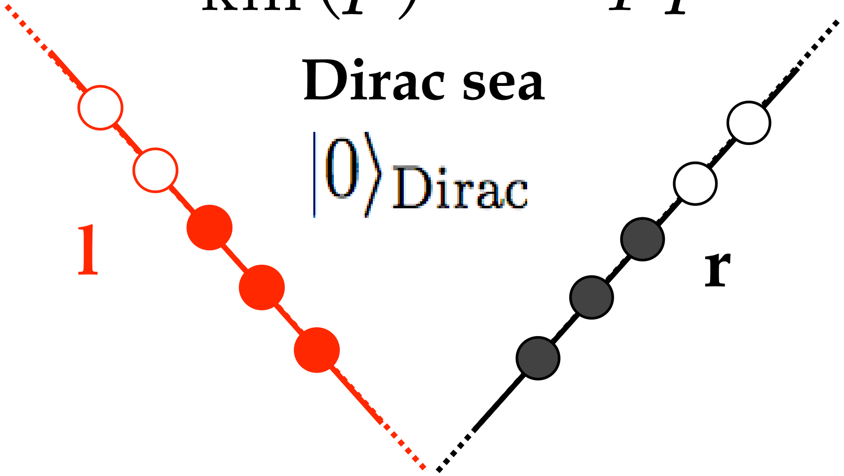
[E. T. Jaynes, PR 106 / 108 (1957)]

The Luttinger model (LM)

Joaquin M Luttinger



$$\epsilon_{\text{kin}}(p) = v_F p$$



Dirac sea

$|0\rangle_{\text{Dirac}}$

“Infinite storey hotel”

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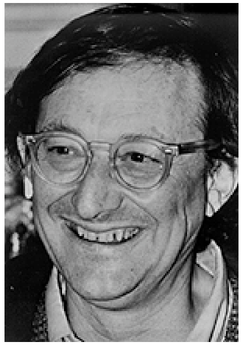
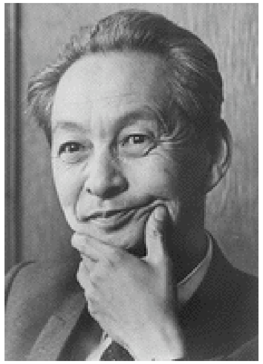


朝永振一郎



Daniel C. Mattis & Elliot H. Lieb

[J. Math. Phys. (N.Y.) 6 (1965)]

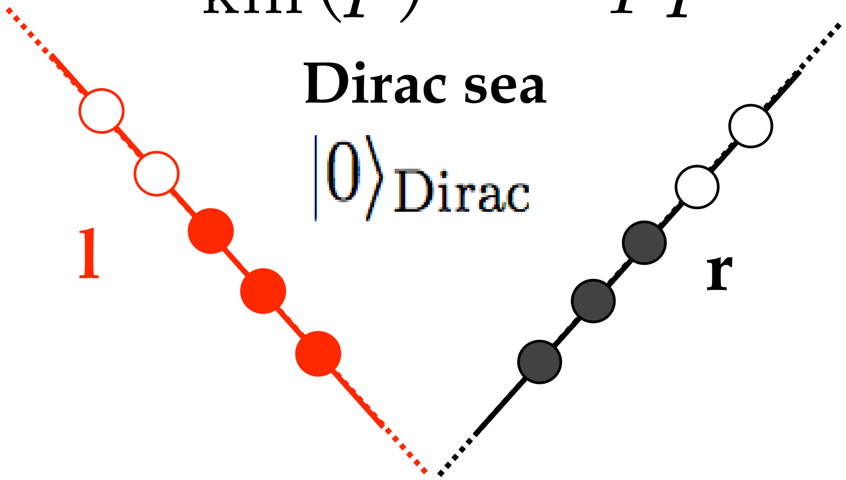
$$\boxed{[\rho_\alpha(q), \rho_\beta(-q')] = \frac{qL}{2\pi}\delta_{q,q'}\delta_{\alpha,\beta} \quad (\alpha, \beta = r, l)}$$

‘Anomalous’ commutation relations

$$\epsilon_{\rm kin}(p) = v_F p$$

Dirac sea

$$|0\rangle_{\rm Dirac}$$



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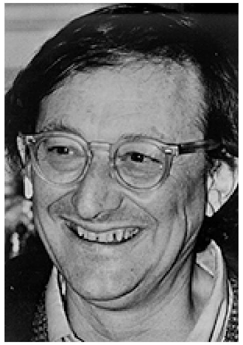

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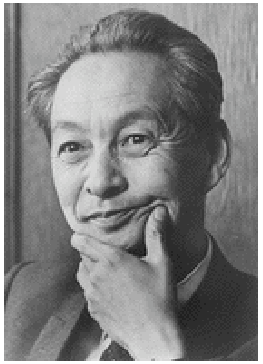

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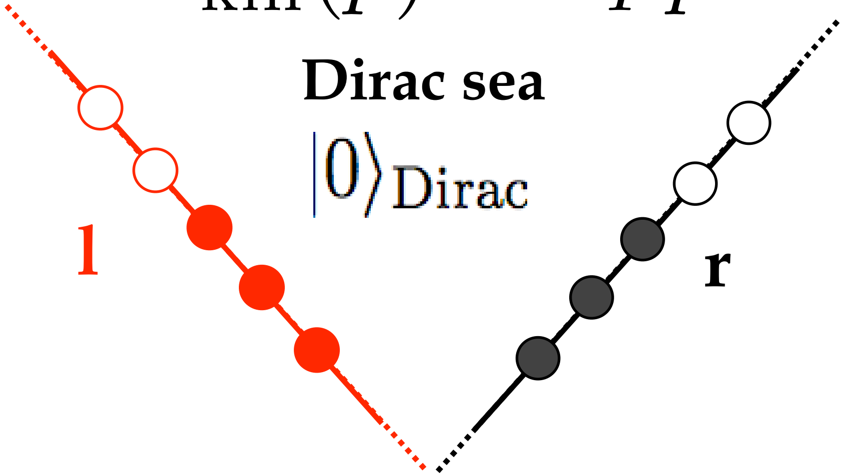
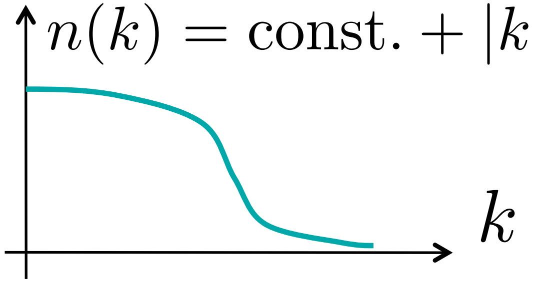


"Infinite storey hotel"

朝永-Luttinger Liquids (TLL)

$$\langle O(x)O(0)\rangle \sim x^{-\alpha}$$

$$n(k) = \text{const.} + |k - k_F|^\alpha \text{sgn}(k - k_F)$$



[F. D. M. Haldane, J. Phys. C 14 (1981)]

Quantum quench in the LM

$$H_{\text{kin}} = \sum_{q \neq 0} \hbar v_F |q| \, a^\dagger(q) a(q) \qquad H_{\text{LM}} = \sum_{q \neq 0} \hbar v |q| \, b^\dagger(q) b(q)$$

Non-interacting fermions (t ≤ 0)

Interacting fermions (t > 0)

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Equilibrium solution $b(q) = \cosh\varphi(q)\, a(q) + \sinh\varphi(q)\, a^\dagger(-q)$

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$$\begin{aligned} a(q,t) &= e^{iH_{LM}t/\hbar} a(q) e^{-iH_{LM}t/\hbar} = f(q,t)a(q) + g^*(q,t)a^\dagger(-q), \\ f(q,t) &= \cos v|q|t - i\sin v|q|t\ \cosh 2\varphi(q), \\ g(q,t) &= i\sin v|q|t \sinh 2\varphi(q) \end{aligned}$$

[MAC, PRL 97 (2006)]

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[MAC, PRL 97 (2006)]

One-particle density matrix

$$C_{\psi_r}(x, t > 0) = \langle 0 | e^{iH_{LM}t/\hbar} \psi^\dagger_r(x) \psi_r(0) e^{-iH_{LM}t/\hbar} | 0 \rangle_{\rm Dirac}$$

Quench in the LM

~ **Interaction range**

[MAC, PRL 97 (2006)]

Thermodynamic limit :

$$C_{\psi_r}(x, t > 0) = C^{\text{free}}_{\psi_r}(x) \left| \frac{R}{x} \right|^{\gamma^2} \left| \frac{x^2 - (2vt)^2}{(2vt)^2} \right|^{\gamma^2/2}$$

Quench in the LM

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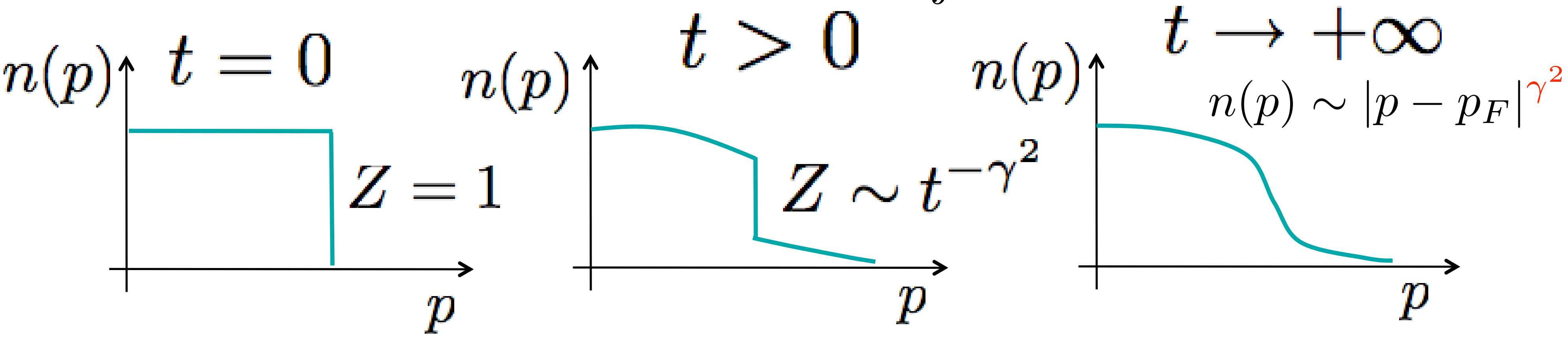
Momentum distribution at time *t* : $n(p,t) = \int dx\, e^{-ipx} C_{\psi_r}(x,t>0)$



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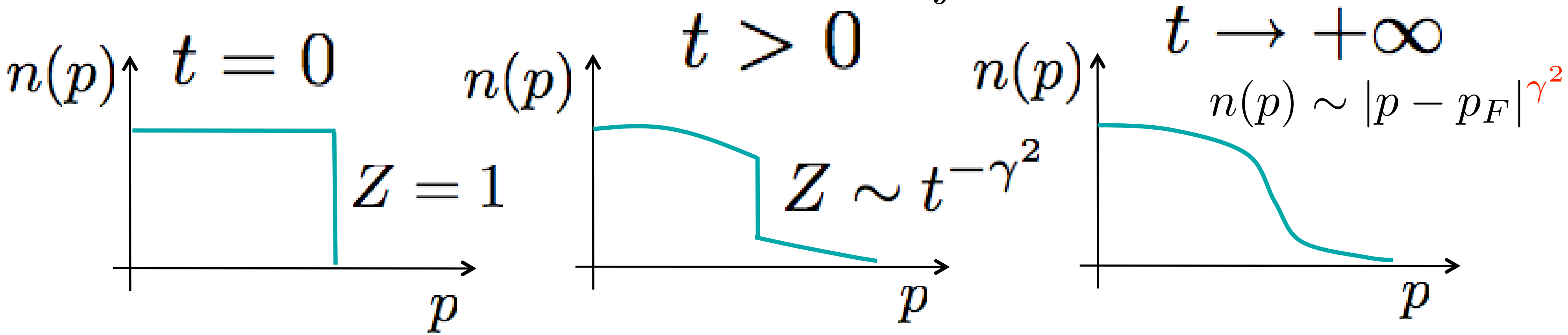

Non-equilibrium exponent : $\gamma^2 = \sinh^2 2\varphi > \gamma^2_{\text{eq}} = 2\sinh^2\varphi$

Quench in the LM

[MAC, PRL 97 (2006)]

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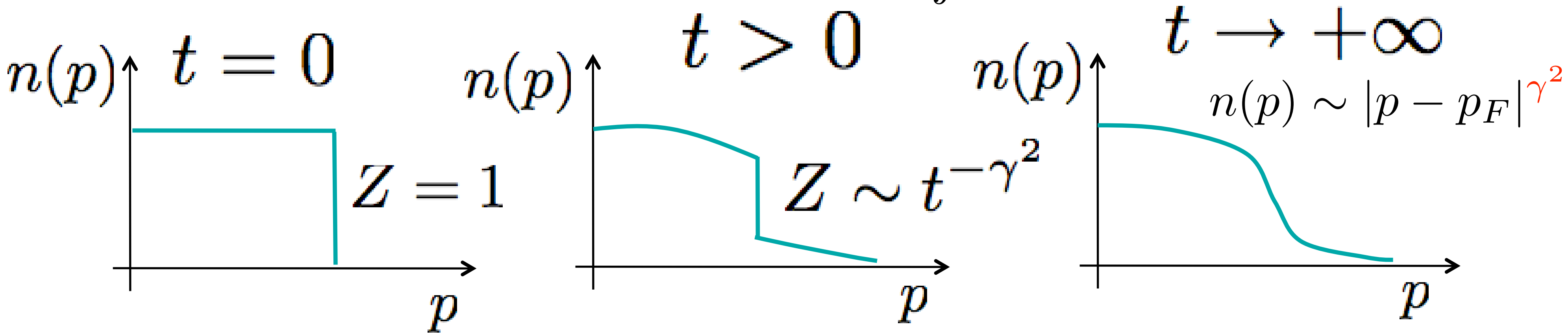

Non-equilibrium exponent : $\gamma^2 = \frac{1}{4}\left(K-K^{-1}\right)^2 > \gamma_{eq} = \frac{1}{2}\left(K-K^{-1}-2\right)$

Quench in the LM

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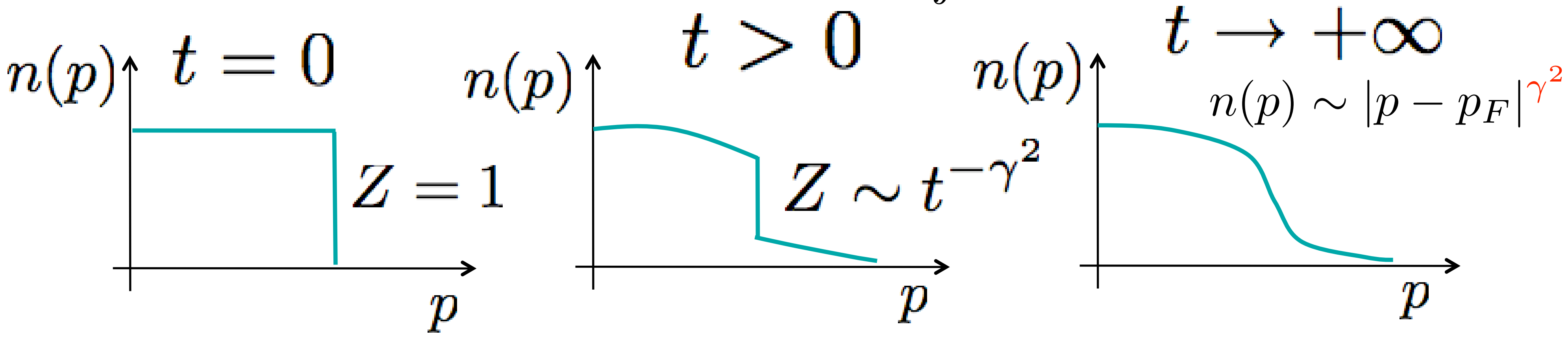

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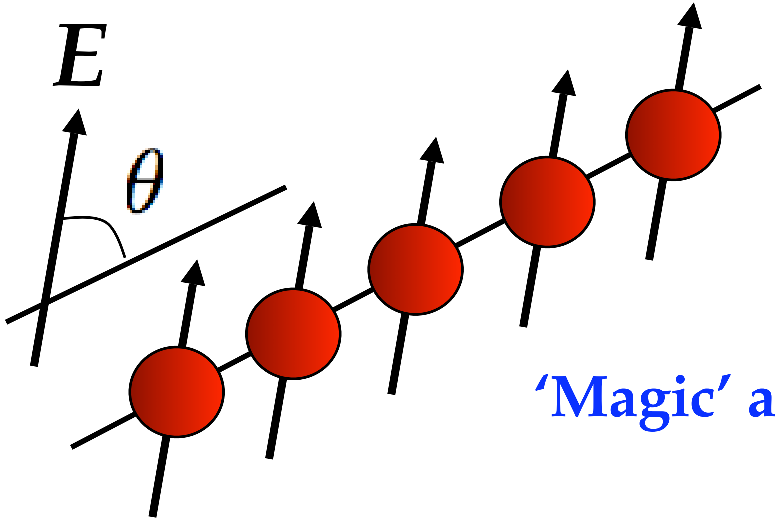
Maximum entropy : $\rho_{\text{gG}} = \frac{e^{\sum_q \lambda_q I_q}}{Z_{\text{gG}}}, \quad I_q = b^\dagger(q)b(q)$

$$\lim_{t\to+\infty} C_{\psi_r}(x,t) = C^{\text{gG}}_{\psi_r}(x) = \text{Tr}\,\rho_{\text{gG}}\,\psi^\dagger_r(x)\psi(0)$$

Relevant (to) experiments?

1D dipolar gas of (spin polarized) fermionic atoms/molecules

(not the LM but a Tomonaga-Luttinger liquid)



$$V_{\text{dip}}(x) \simeq \frac{1}{4\pi\epsilon_0}\frac{D^2(1-3\cos\theta)}{[x^2+R^2]^{3/2}}$$

'Magic' angle : $\theta_m = \cos^{-1}\left(\frac{1}{3}\right) \Rightarrow V_{\text{dip}} = 0$

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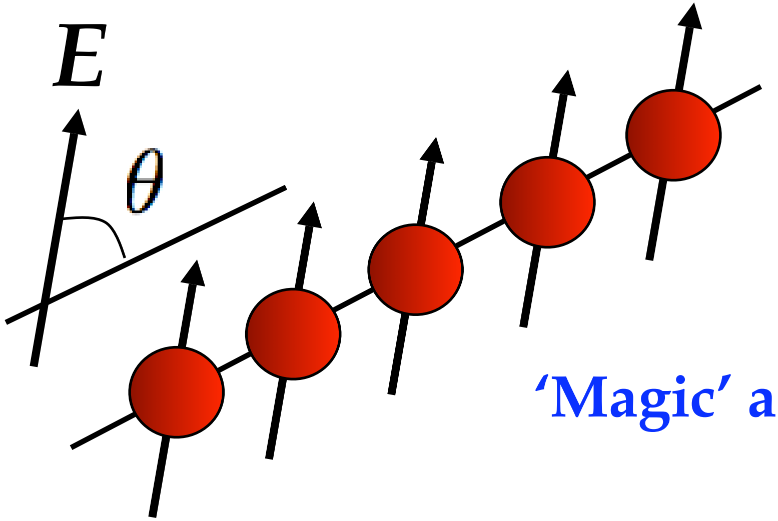
(not the LM but a Tomonaga-Luttinger liquid)



$$V_{\rm dip}(x) \simeq \frac{1}{4\pi\epsilon_0}\frac{D^2(1-3\cos\theta)}{[x^2+R^2]^{3/2}}$$

'Magic' angle : $\theta_m = \cos^{-1}\left(\frac{1}{3}\right) \Rightarrow V_{\rm dip} = 0$

Quench (very rapid change in the direction of E away from magic angle)

Relevant (to) experiments?

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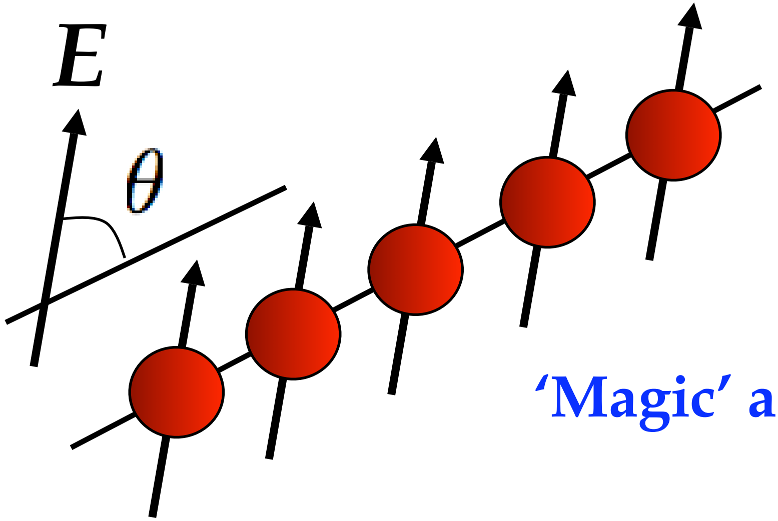

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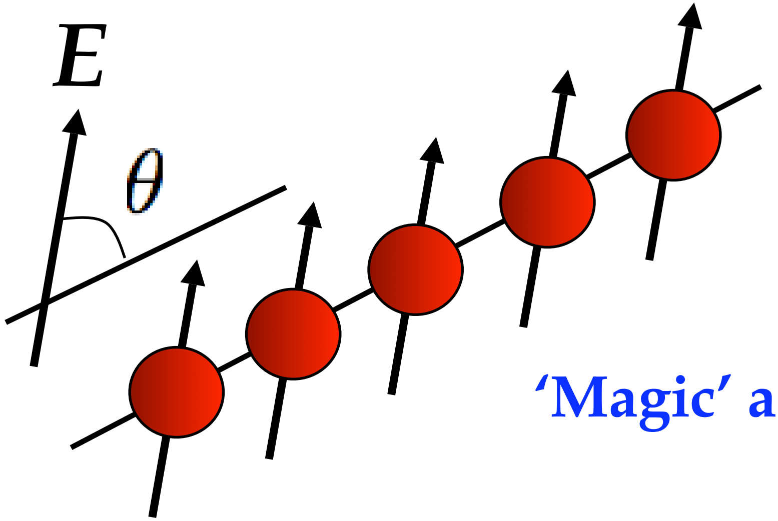

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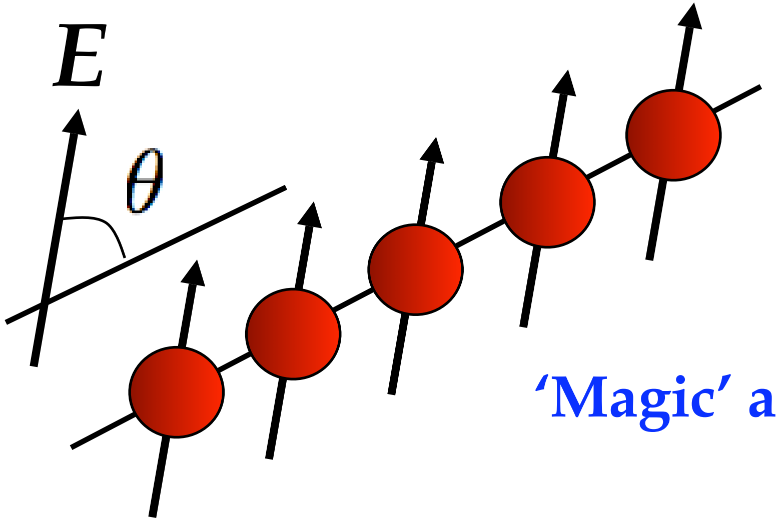

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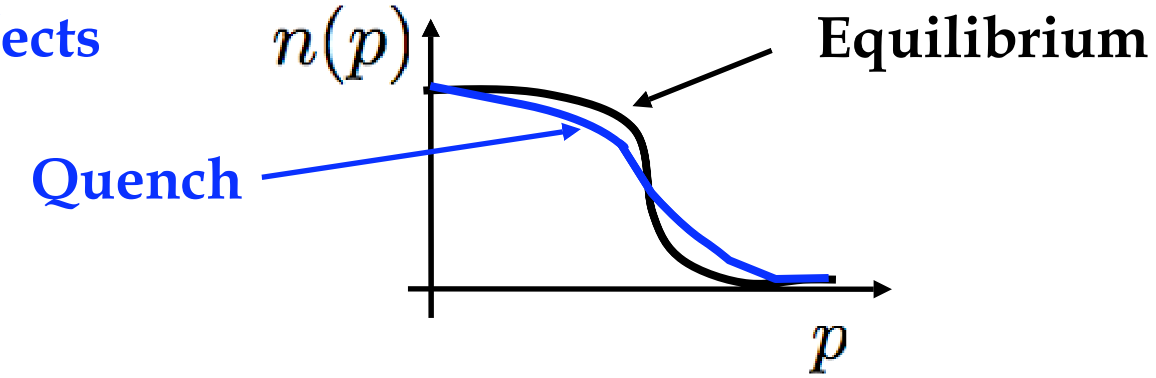
$$t_{\rm Relax} \simeq \frac{\hbar}{T}$$



[MAC, PRL 97 (2006)]

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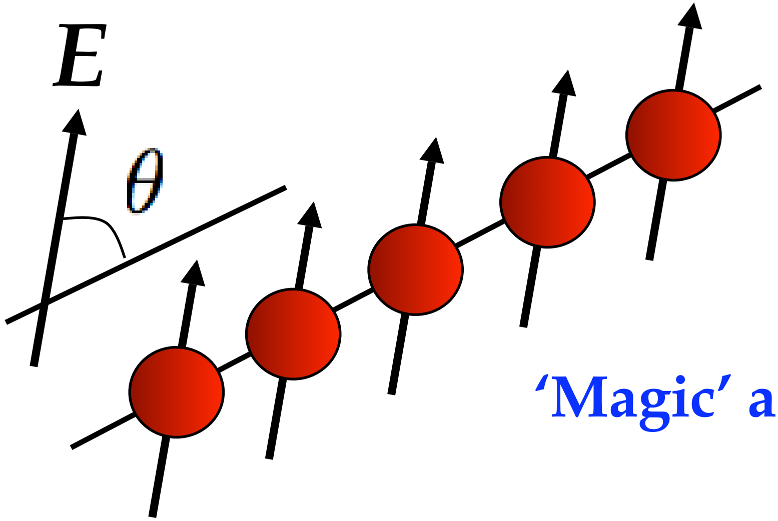

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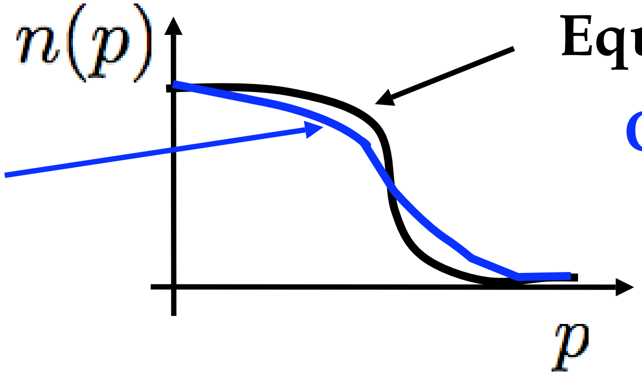


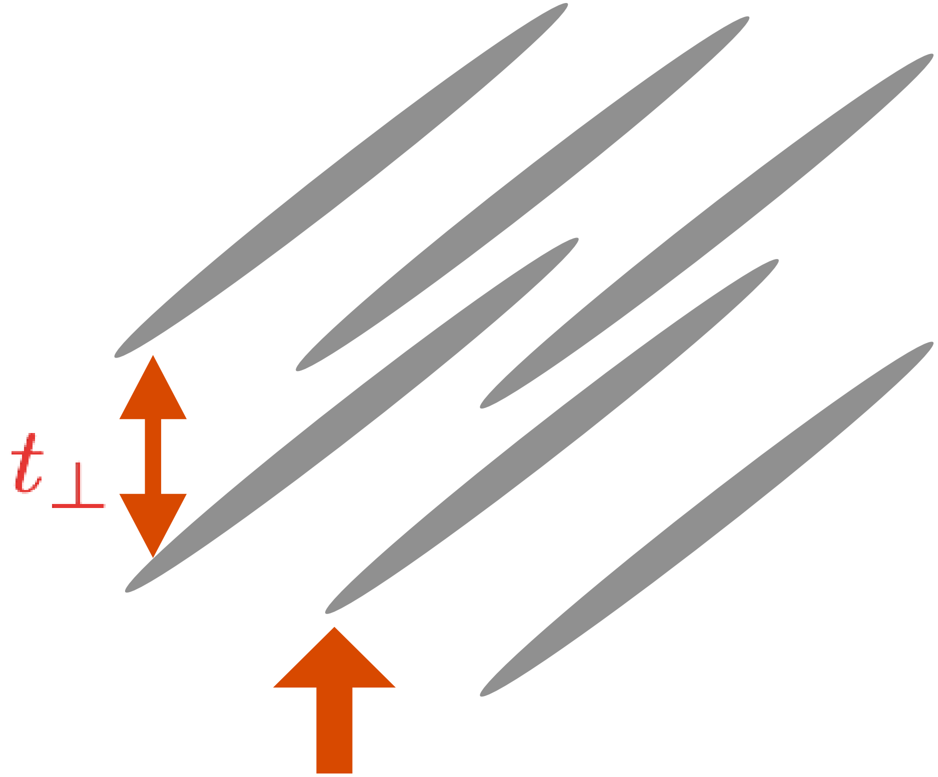
Other probes: Noise

[A Polkovnikov *et al* PNAS 103 (2006)]

[MAC, PRL 97 (2006)]

What is old (but different)?

Asymetric 1D Hubbard model



Deep 2D optical lattice $\min\{t_\uparrow, t_\downarrow\} \gg t_\perp$

[T Stöferle, H Moritz, C Schori M Köhl, and T Esslinger, PRL 92 (2004)]

Asymetric 1D Hubbard model

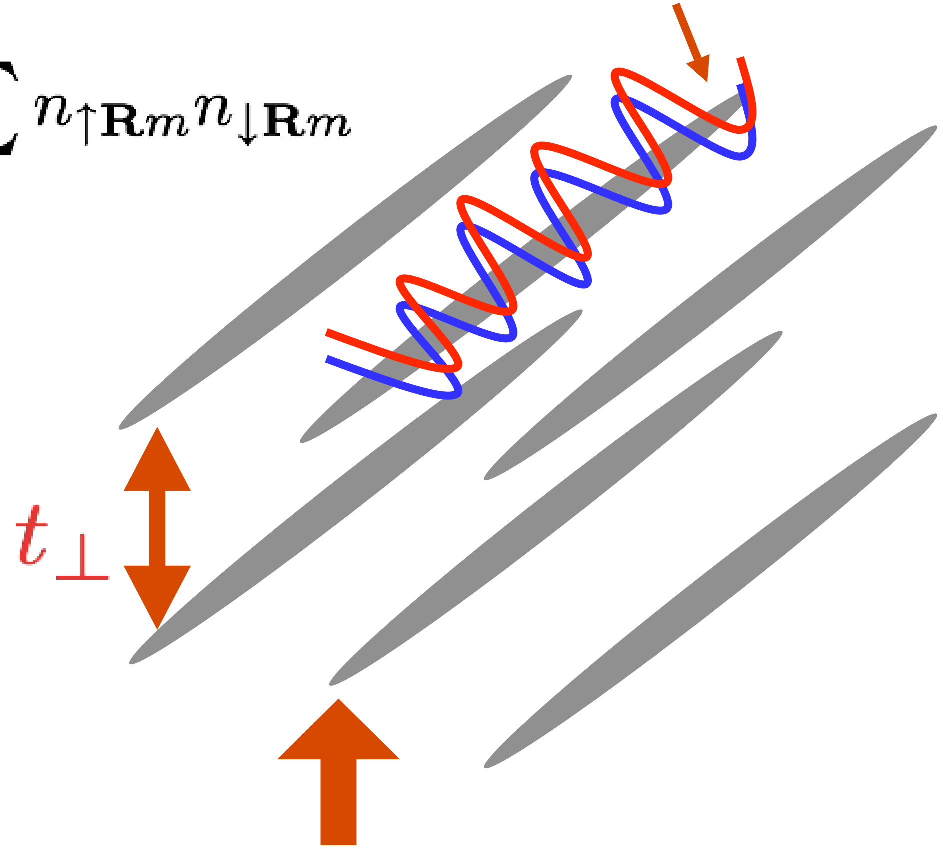
[V Liu, F Wilczek, and P Zoller PRA 70 (2004)]

Internal-state dependent optical lattice

[O Mandel et al. PRL 91 (2003)]

or two different atom species (^{6}Li + ^{40}K)

$$H_{\mathbf{R}} = -\sum_{\langle m,n\rangle\sigma} t_\sigma\, c^\dagger_{\sigma\mathbf{R}m} c_{\sigma\mathbf{R}n} + U \sum_m n_{\uparrow\mathbf{R}m} n_{\downarrow\mathbf{R}m}$$



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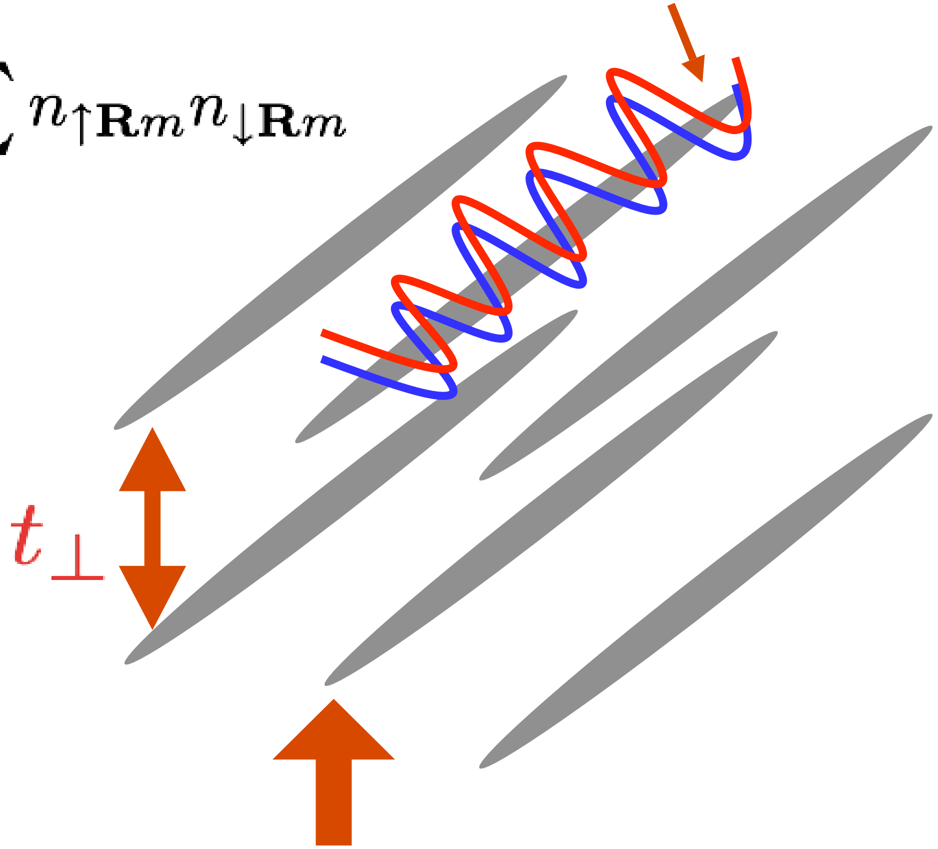
Constant N↑, N↓
(Canonical Ensemble)



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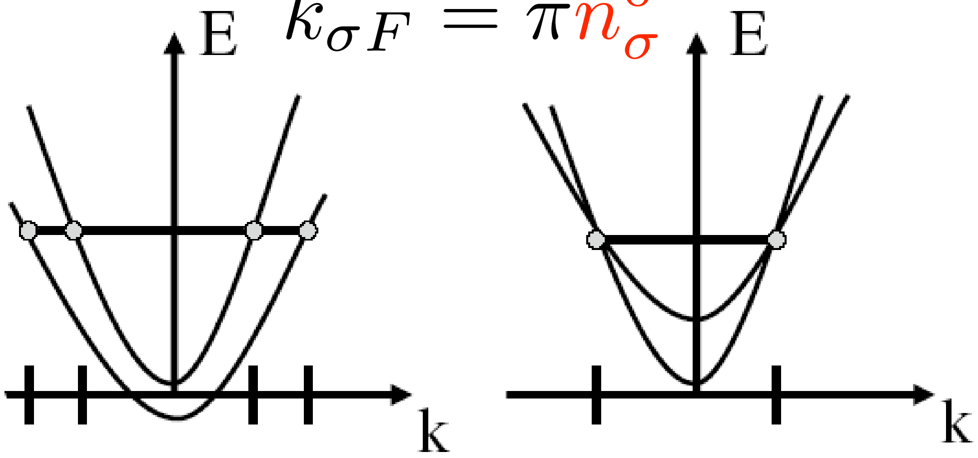
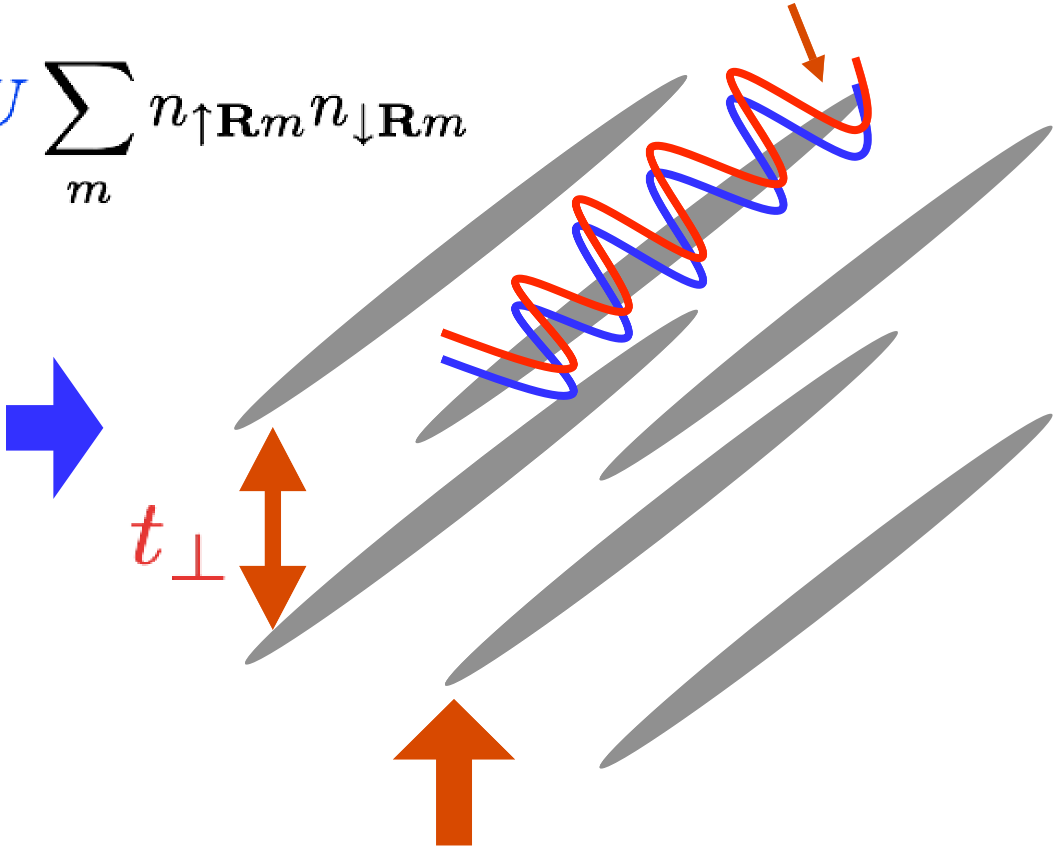
$$k_{\sigma F} = \pi n_\sigma^0$$





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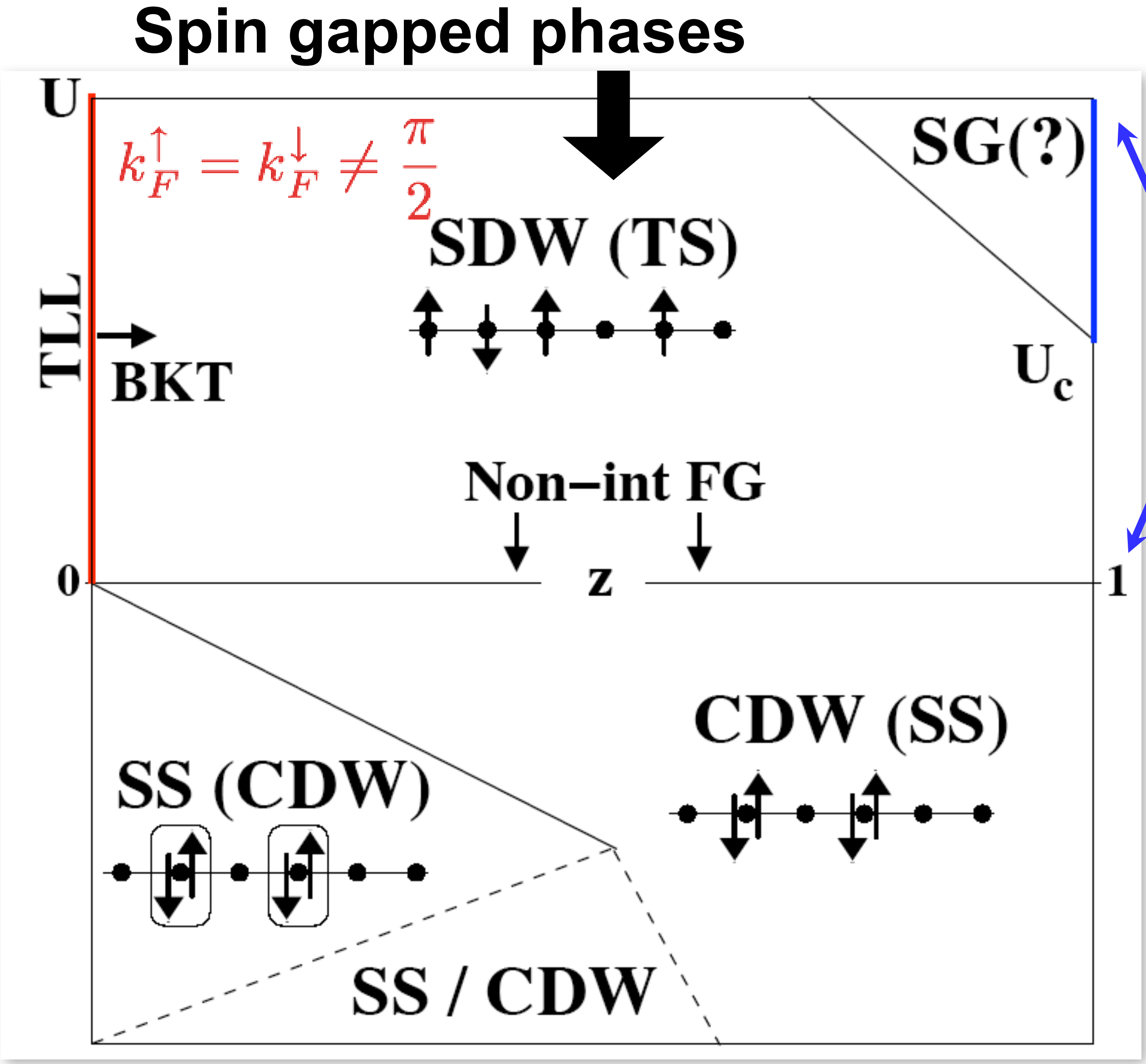
Schematic Phase Diagram

[MAC, AF Ho & T Giamarchi, PRL <u>95</u> (2005)]



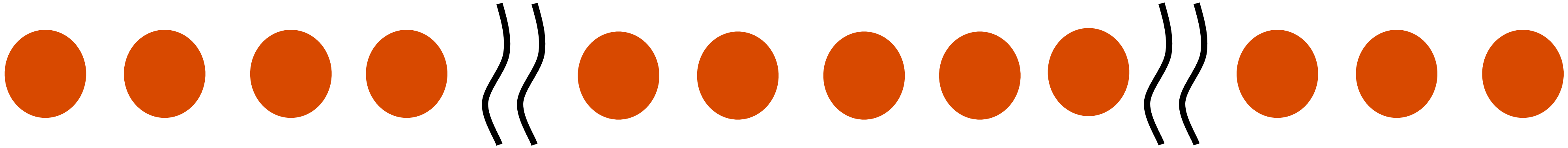
Falikov-Kimball Model

$$z = \frac{|t_{\uparrow} - t_{\downarrow}|}{t_{\uparrow} + t_{\downarrow}}$$

Explanation of Phase Diagram

“Almost crystaline” order in 1D

It looks like a crystal on a short distance scale, but it is disordered on a large distance scale. Periodic regions of all sizes.



Adiabatic approximation $(t_\uparrow \gg t_\downarrow)$

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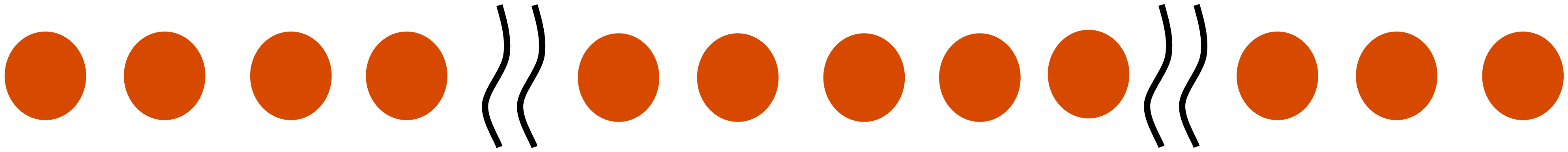

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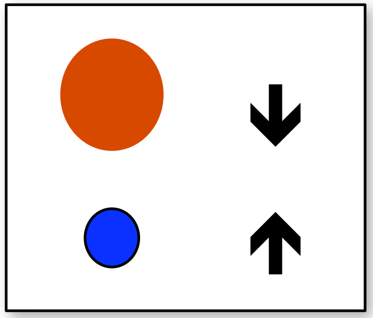

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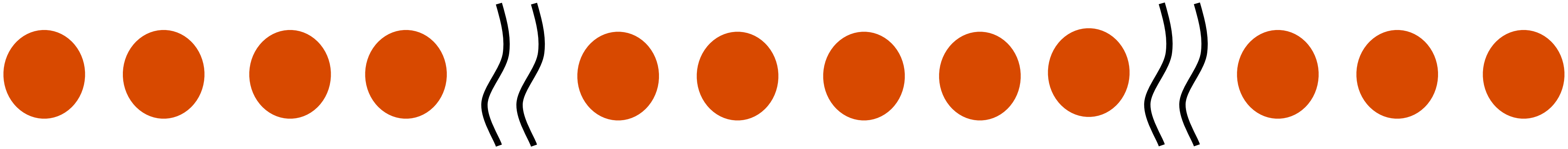

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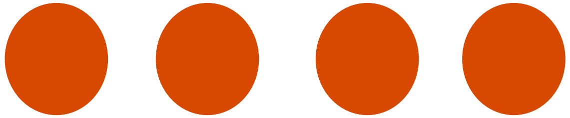

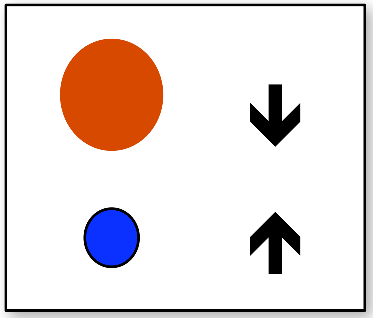

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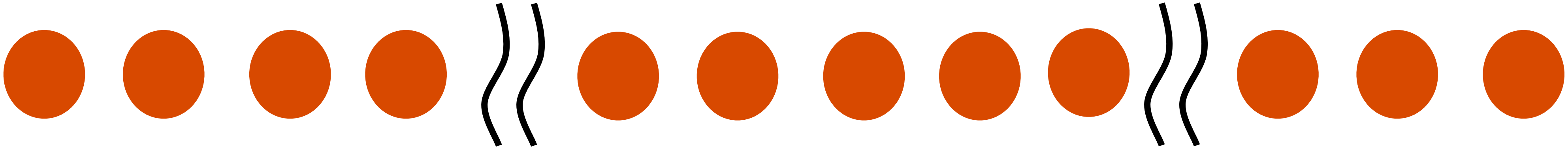

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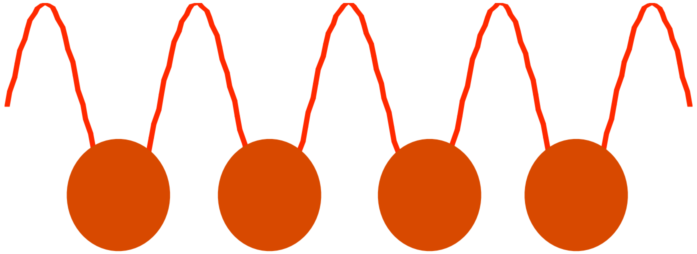

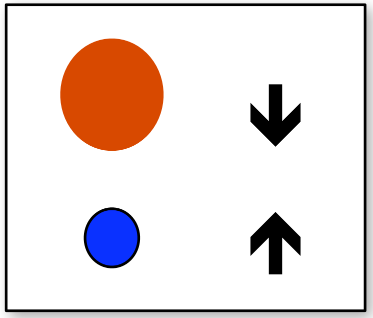

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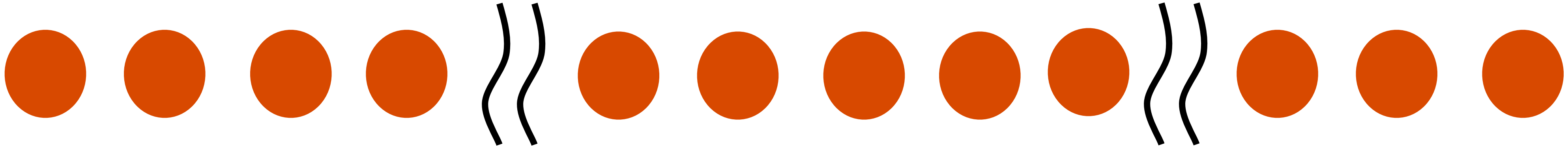

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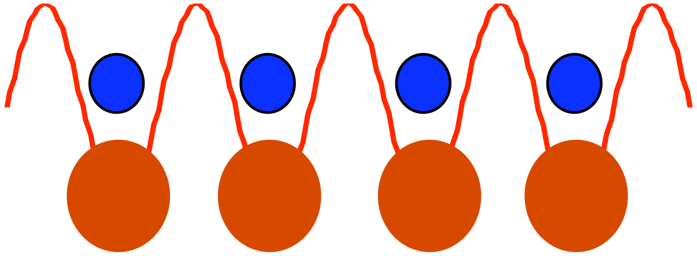

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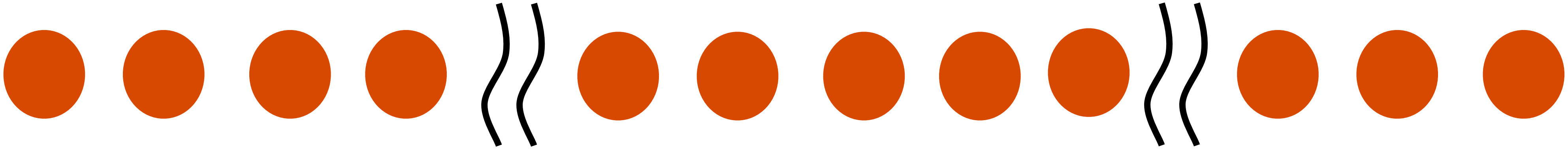

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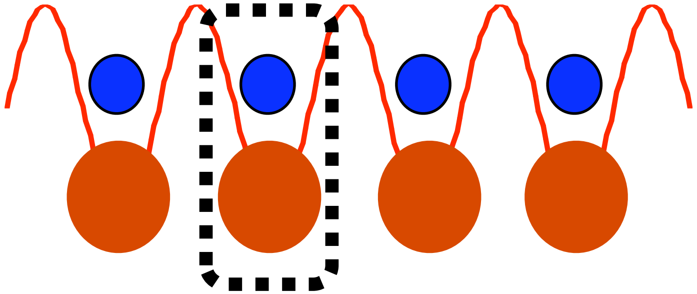

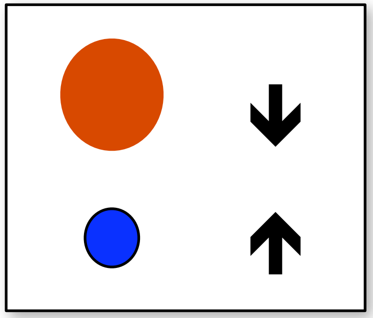

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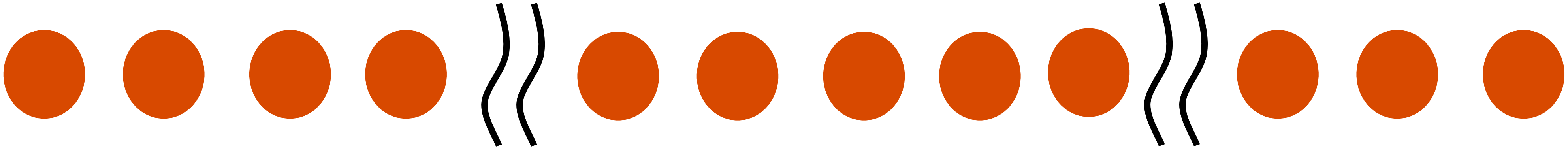

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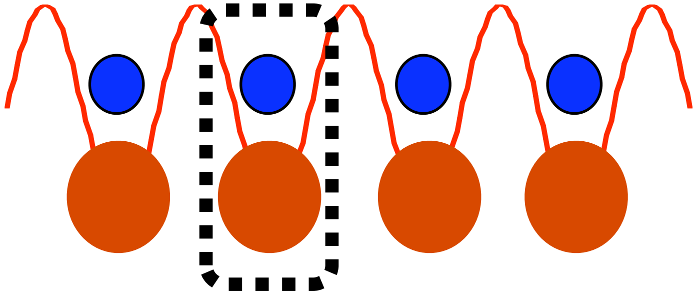
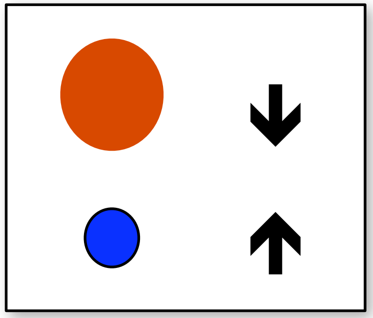


SS / CDW



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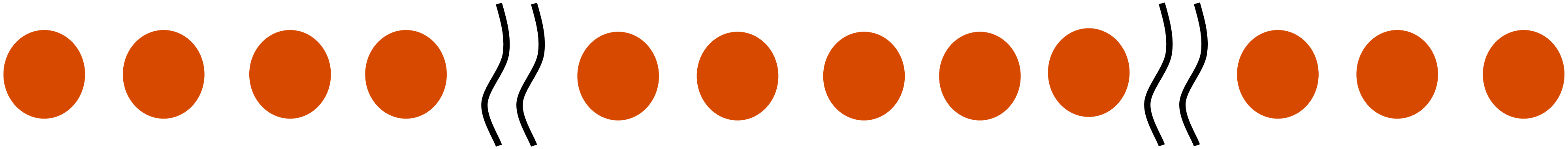

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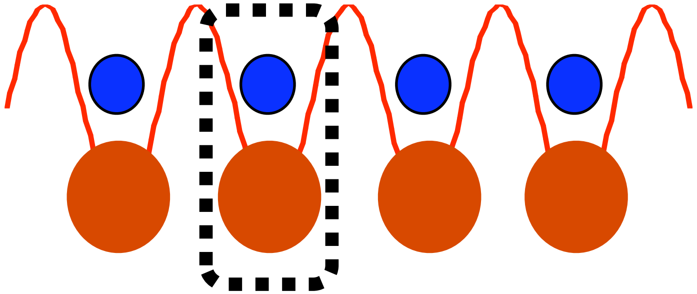

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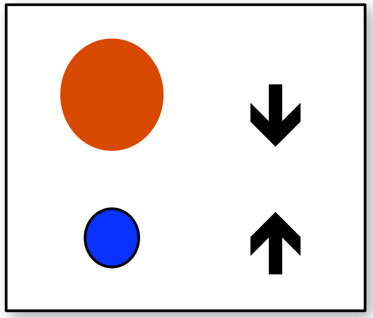

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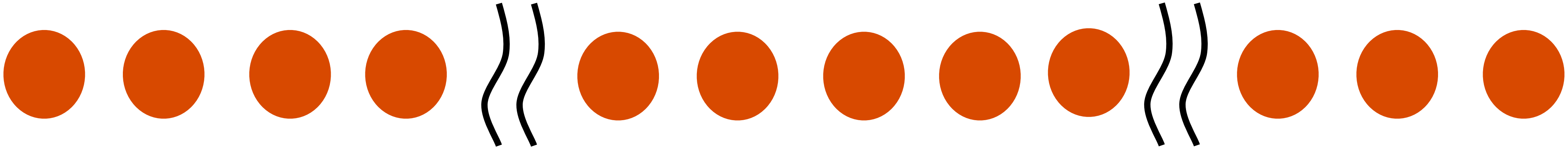

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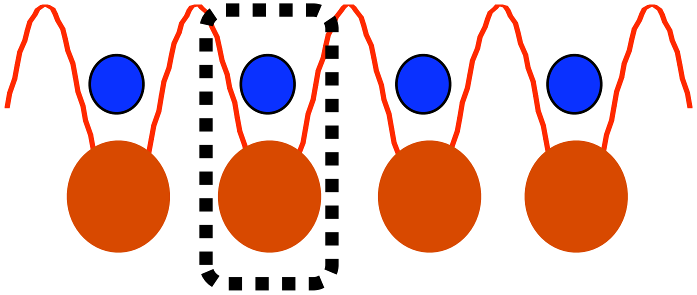

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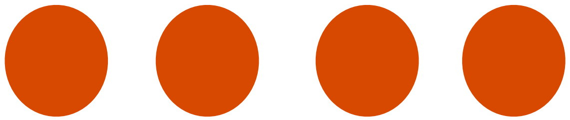

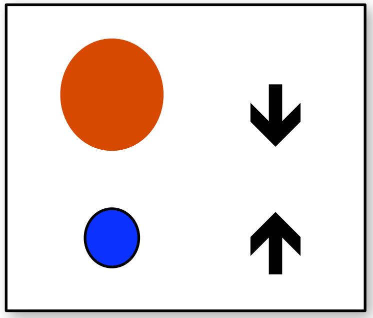

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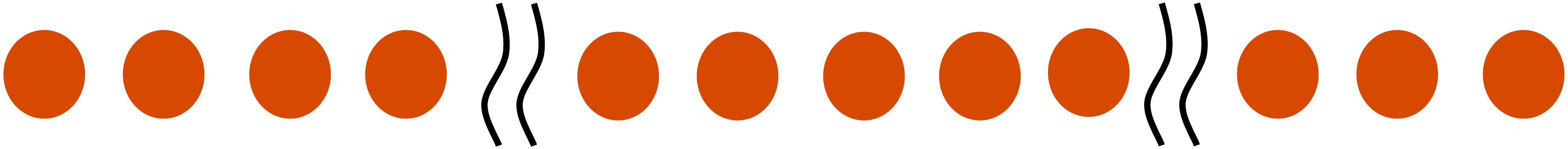

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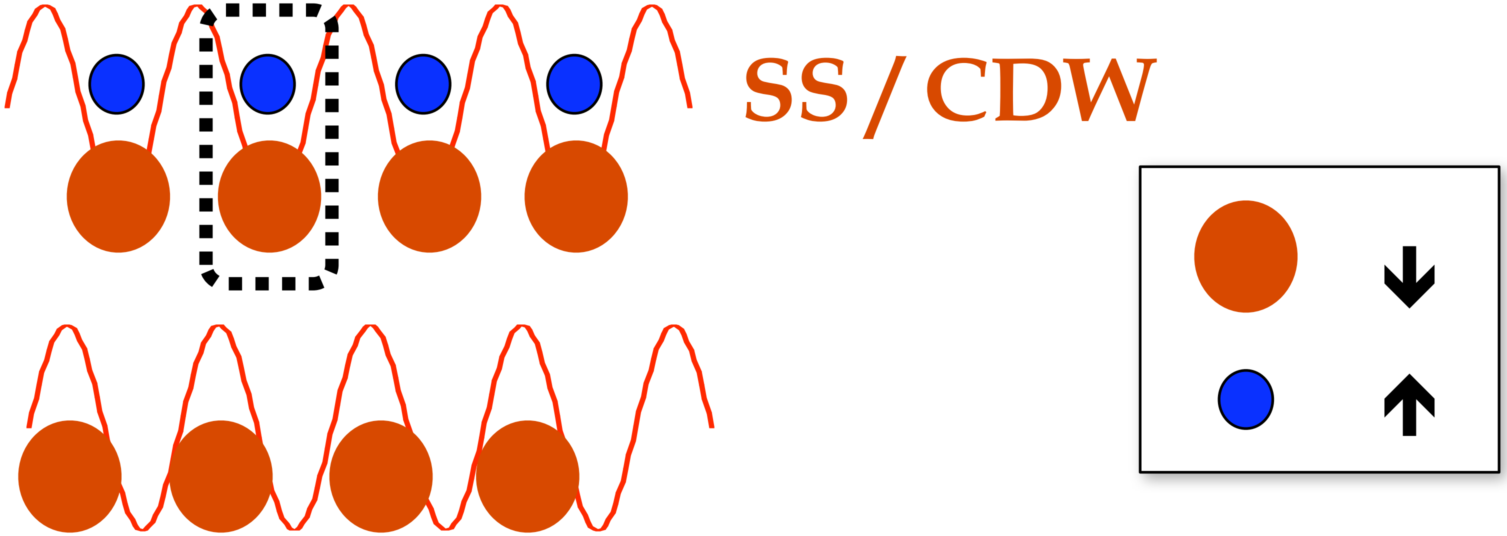

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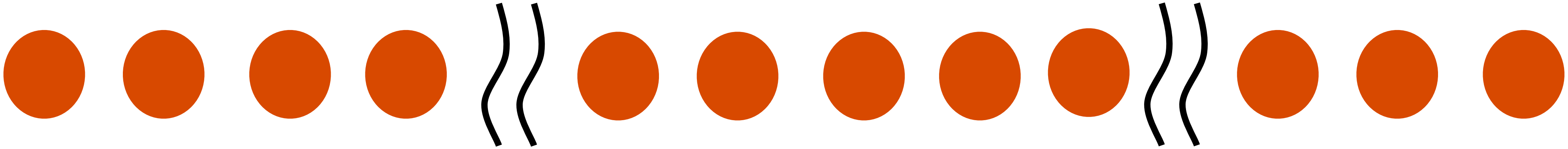

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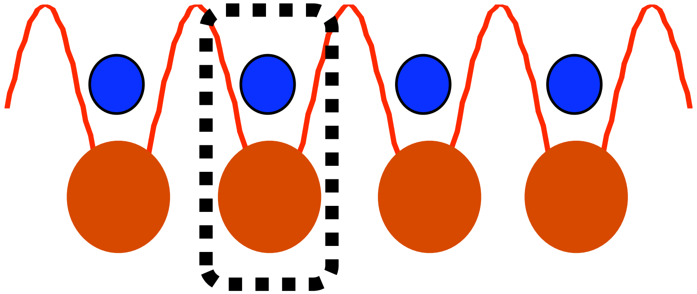

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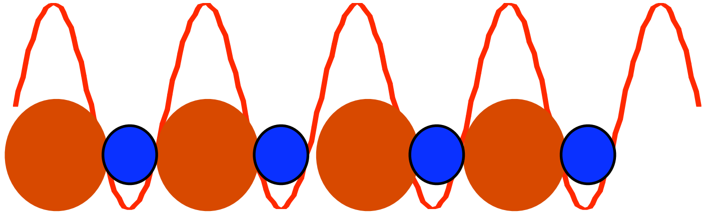

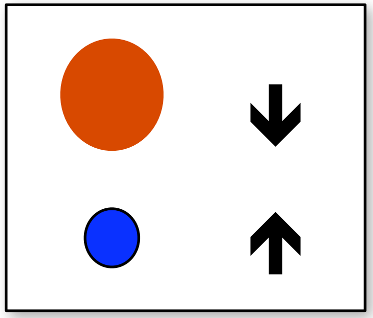

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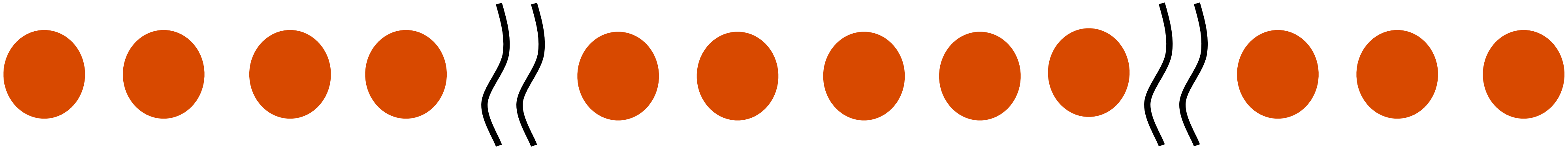

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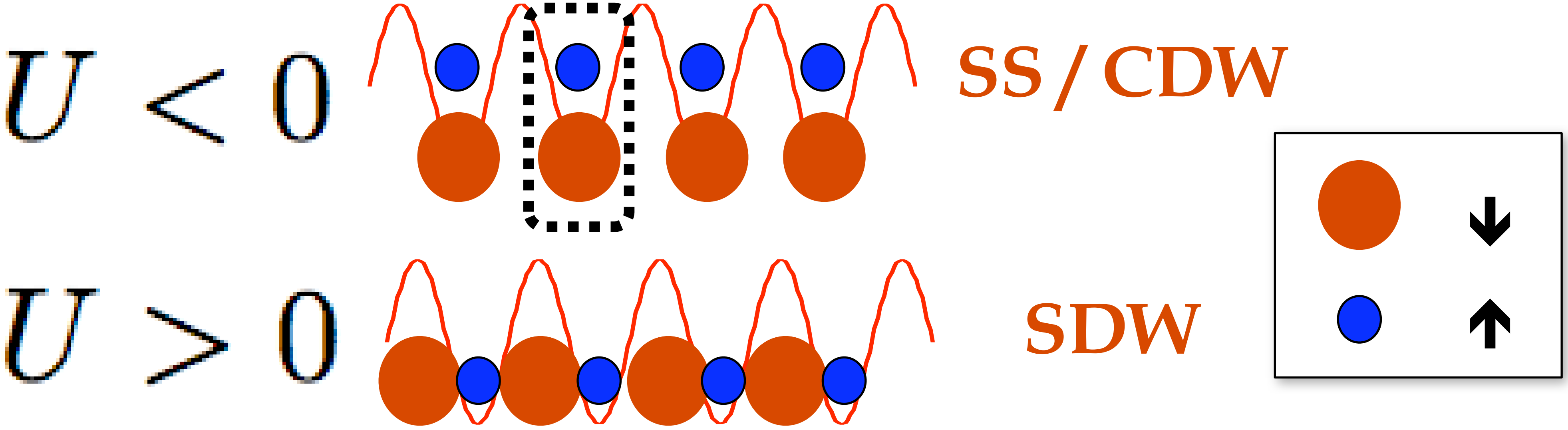


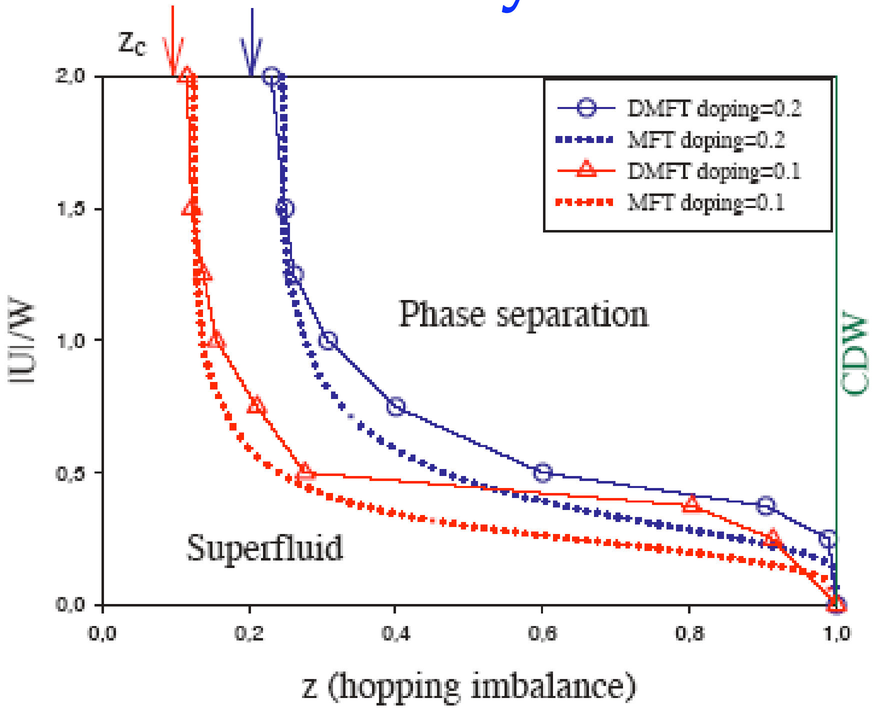
$U < 0$ SS / CDW

$U > 0$ SDW

What about d > 1?

[TL Dao, A Georges, and M Capone, arxiv/0407.2260]

DMFT U < 0 Asymmetric Hubbard



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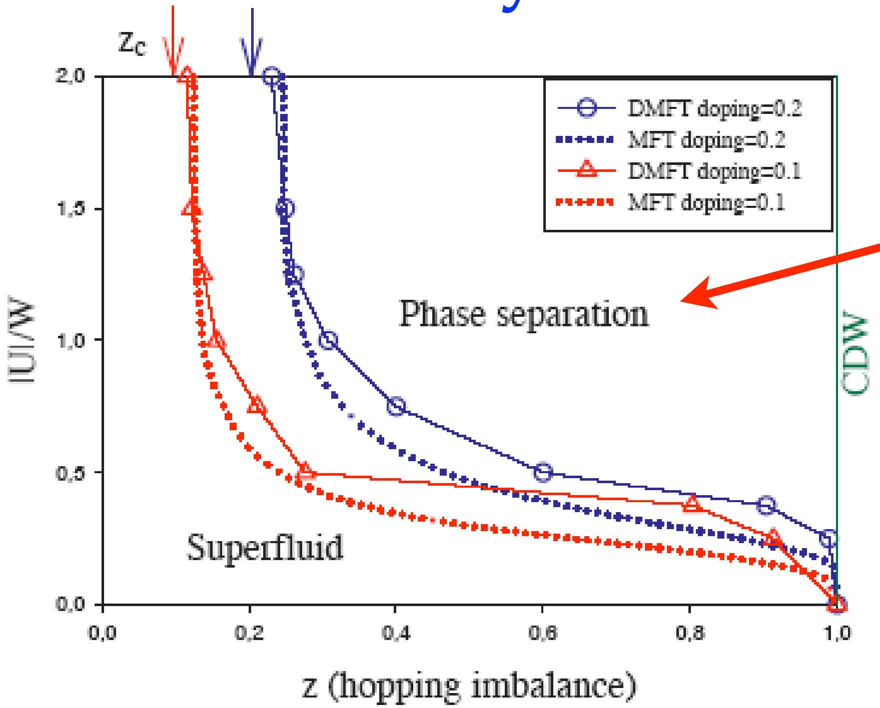

Coexistence of SF and CDW

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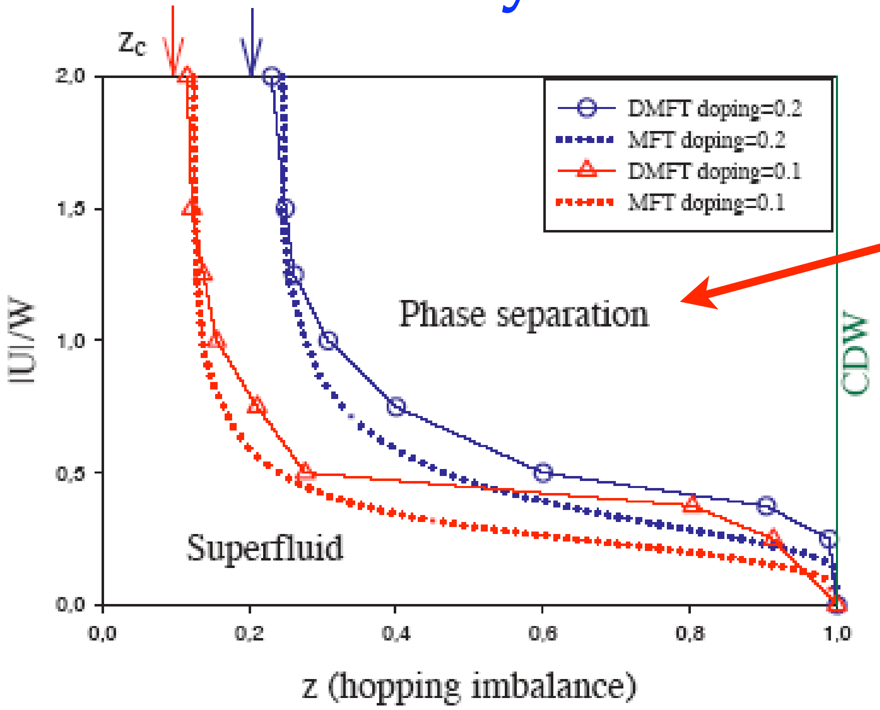
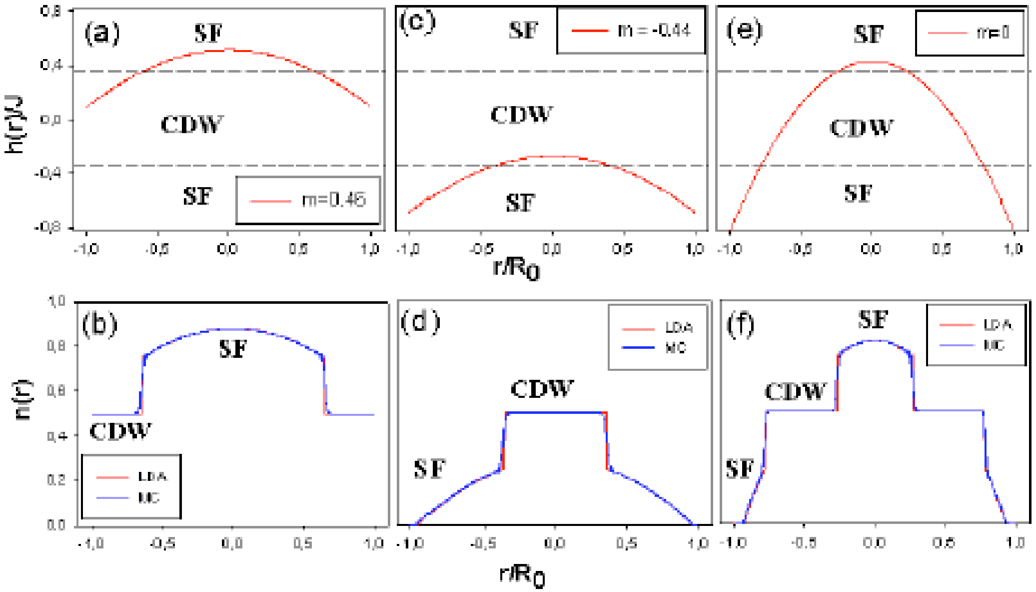


Coexistence of SF and CDW

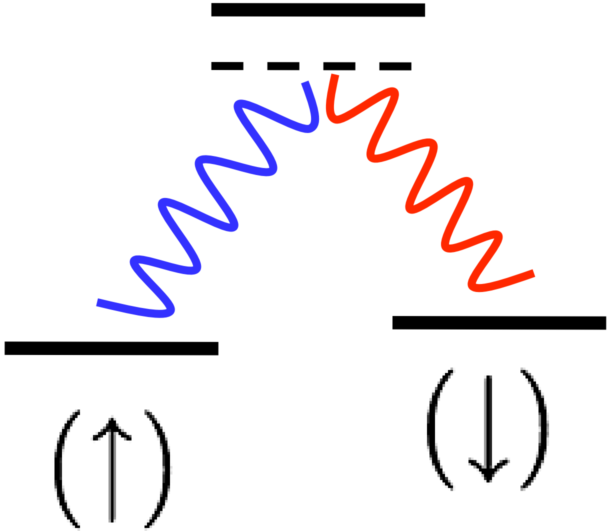
Harmonic trap (LDA)



Detecting the spin gap

A Raman laser induces transitions between hyperfine states

[HP Buchler *et al* PRL 93 (2004)]



$$S^{+-}(t) = \langle S_T^+(t) S_T^-(0) \rangle$$

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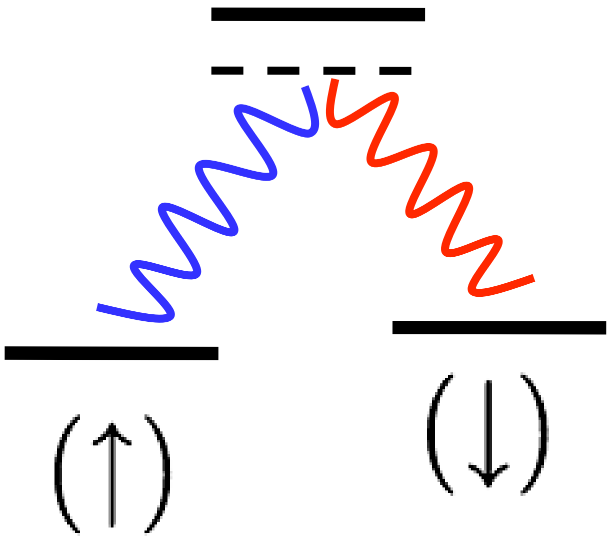
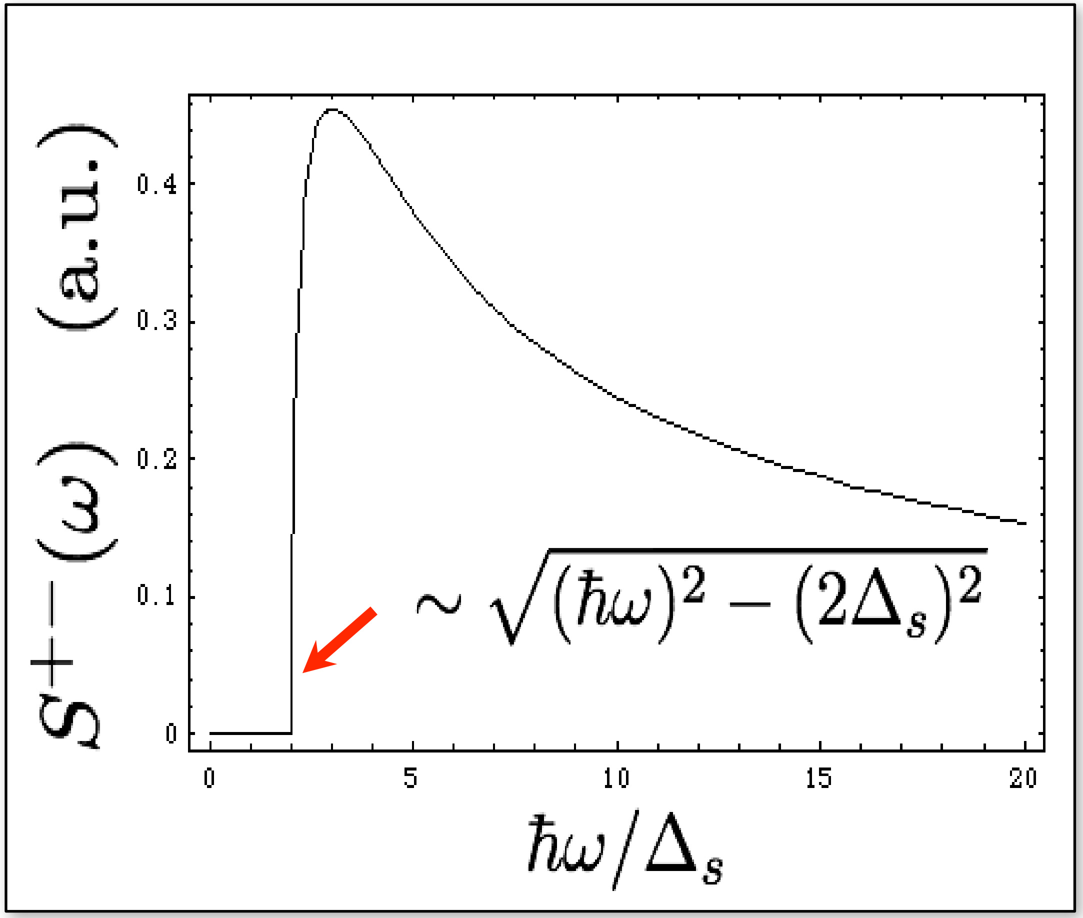
A Raman laser induces transitions between hyperfine states

[HP Buchler *et al* PRL 93 (2004)]



$$S^{+-}(t) = \langle S_T^{+}(t) S_T^{-}(0) \rangle$$

[MAC, AF Ho & T Giamarchi, PRL 95 (2005)]



Conclusions

- In cold atomic gases “conserved” quantities can yield physics different from cond-mat systems.
- Systems will not typically exhibit thermalization after a quench.
- Non-equilibrium stationary states can have properties that are different from their equilibrium properties. E.g. some observables of the LM have different critical indices.
- Absence of relaxation of the magnetization can yield different (spin gapped) zero-temperature phases in asymmetric 1D Hubbard models.

Thanks to...

Andrew F. Ho (see talk in this workshop)
A. Iucci
T. Giamarchi (see talk in this workshop)
for collaborations and discussions,

Antoine Georges
Victor Gurarie
Jason Ho
Brad Marston
Alejandro Muramatsu
Shura Nersesyan
Marcos Rigol
Gora Shlyapnikov
Masahito Ueda
for discussions,

and you, for your attention