Fermi surface change across quantum phase transitions

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Talk online at http://sachdev.physics.harvard.edu
Consider a system of bosons and fermions at non-zero density, and $N$ particle-number (U(1)) conservation laws.

Then, for each conservation law there is a “Luttinger” theorem constraining the momentum space volume enclosed by the locus of gapless single particle excitations, \textit{unless}:

- there is a broken translational symmetry, and there are an integer number of particles per unit cell for every conservation law;
- there is a broken U(1) symmetry due to a boson condensate – then the associated conservation law is excluded;
- the ground state has “topological order” and fractionalized excitations.
Outline

A. Bose-Fermi mixtures
   *Depleting the Bose-Einstein condensate in trapped ultracold atoms*

B. Fermi-Fermi mixtures
   *Normal states with no superconductivity*

C. The Kondo Lattice
   *The heavy Fermi liquid (FL) and the fractionalized Fermi liquid (FL*)*

D. Deconfined criticality
   *Changes in Fermi surface topology*
A. Bose-Fermi mixtures
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Mixture of bosons $b$ and fermions $f$

(e.g. $^7$Li+$^6$Li, $^{23}$Na+$^6$Li, $^{87}$Rb+$^{40}$K)

Tune to the vicinity of a Feshbach resonance associated with a molecular state $\psi$

Conservation laws:

\[
b^\dagger b + \psi^\dagger \psi = N_b
\]

\[
f^\dagger f + \psi^\dagger \psi = N_f
\]
Phases

\[ \frac{k^2}{2} \]

\[ f \]

\[ b \]

\[ \tilde{\Lambda} \]

2 FS, no BEC  |  2 FS + BEC  |  1 FS + BEC

Detuning °
Phase diagram

\[ \langle b \rangle = 0 \]

- 2 FS, no BEC
- 2 FS + BEC
- \( \langle b \rangle \neq 0 \)
- 1 FS + BEC
2 FS, no BEC phase

“molecular” Fermi surface

\[
\text{Volume} = N_b
\]

\[
\langle b \rangle = 0
\]

“atomic” Fermi surface

\[
\text{Volume} = N_f - N_b
\]

2 Luttinger theorems; volume within both Fermi surfaces is conserved
Phase diagram

- \langle b \rangle = 0
- \langle b \rangle \neq 0

- 2 FS, no BEC
- 2 FS + BEC
- 1 FS + BEC
2 FS + BEC phase

“molecular” Fermi surface

“atomic” Fermi surface

\[ \langle b \rangle \neq 0 \]

Total volume = \( N_f \)

1 Luttinger theorem; only total volume within Fermi surfaces is conserved
1 FS + BEC phase

“atomic” Fermi surface

\[ \langle b \rangle \neq 0 \]

Total volume = \( N_f \)

1 Luttinger theorem; only total volume within Fermi surfaces is conserved
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Mixture of fermions $f_{\downarrow}$ and $f_{\uparrow}$

Tune to the vicinity of a Feshbach resonance associated with a Cooper pair $\Delta$

Conservation laws:

\[
\begin{align*}
    f_{\downarrow}^\dagger f_{\downarrow} + \Delta^\dagger \Delta &= N_{\downarrow} \\
    f_{\uparrow}^\dagger f_{\uparrow} + \Delta^\dagger \Delta &= N_{\uparrow}
\end{align*}
\]
\( \mu \) chemical potential; \( h \) "magnetic" field; \( \nu \) detuning.
μ chemical potential; \( h \) "magnetic" field; \( ν \) detuning
$\mu$ chemical potential; $h$ "magnetic" field; $\nu$ detuning
2 FS, normal state

Volume $= N_{\downarrow}$

$\langle \Delta \rangle = 0$

Volume $= N_{\uparrow}$

2 Luttinger theorems; volume within both Fermi surfaces is conserved
1 FS, normal state

 minority Fermi surface

 $\langle \Delta \rangle = 0$

 $N_{\downarrow} = 0$

 majority Fermi surface

 Volume $= N_{\uparrow}$

 2 Luttinger theorems; volume within both Fermi surfaces is conserved
Superfluid

minority Fermi surface

majority Fermi surface

\[ \langle \Delta \rangle \neq 0 \]

\[ \text{Volume}_\uparrow - \text{Volume}_\downarrow = N_\uparrow - N_\downarrow \]

1 Luttinger theorem; difference volume within both Fermi surfaces is conserved
Magnetized Superfluid

\[ \langle \Delta \rangle \neq 0 \]

Volume_{\uparrow} - Volume_{\downarrow} = N_{\uparrow} - N_{\downarrow}

1. Luttinger theorem; difference volume within both Fermi surfaces is conserved
Sarma (breached pair) Superfluid

minority Fermi surface

majority Fermi surface

\[ \langle \Delta \rangle \neq 0 \]

\[ \text{Volume}_{\uparrow} - \text{Volume}_{\downarrow} = N_{\uparrow} - N_{\downarrow} \]

1. Luttinger theorem; difference volume within both Fermi surfaces is conserved.
Any state with a density imbalance must have at least one Fermi surface
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The Kondo lattice

Local moments $f_\sigma$

\[ H_K = \sum_{i<j} t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + J_K \sum_i c_{i\sigma}^{\dagger} \bar{\tau}_{\sigma\sigma} c_{i\sigma} \cdot \vec{S}_{fi} + J \sum_{\langle ij \rangle} \vec{S}_{fi} \cdot \vec{S}_{fj} \]

Conduction electrons $c_\sigma$

Number of $f$ electrons per unit cell = $n_f = 1$

Number of $c$ electrons per unit cell = $n_c$
Define a bosonic field which measures the hybridization between the two bands:

\[ b_i \sim \sum_\sigma c_{i\sigma}^\dagger f_{i\sigma} \]

Analogous to Bose-Fermi mixture problem:
\[ c_{i\sigma} \] is the analog of the "molecule" \( \psi \)

Conservation laws:
\[
\begin{align*}
  &f_{\sigma}^\dagger f_{\sigma} + c_{\sigma}^\dagger c_{\sigma} = 1 + n_c \quad \text{(Global)} \\
  &f_{\sigma}^\dagger f_{\sigma} + b^\dagger b = 1 \quad \text{(Local)}
\end{align*}
\]

Main difference: second conservation law is \textit{local} so there is a U(1) gauge field.
If the $f$ band is dispersionless in the decoupled case, the ground state is always in the 1 FS FL phase.
A bare $f$ dispersion (from the RKKY couplings) allows a 2 FS FL phase.
The $f$ band “Fermi surface” realizes a spin liquid (because of the local constraint)
Another perspective on the FL* phase

Local moments \( f_\sigma \)

\[
H = \sum_{i<j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_i \left( J_K c_{i\sigma}^\dagger \vec{\tau}_{\sigma\sigma} c_{i\sigma} \cdot \vec{S}_{fi} \right) + \sum_{i<j} J_H (i, j) \vec{S}_{fi} \cdot \vec{S}_{fj}
\]

Determine the ground state of the quantum antiferromagnet defined by \( J_H \), and then couple to conduction electrons by \( J_K \)

Choose \( J_H \) so that ground state of antiferromagnet is a \( Z_2 \) or U(1) spin liquid
Influence of conduction electrons

Local moments $f_\sigma$

At $J_K = 0$ the conduction electrons form a Fermi surface on their own with volume determined by $n_c$.

Perturbation theory in $J_K$ is regular, and so this state will be stable for finite $J_K$.

So volume of Fermi surface is determined by $\binom{n_c+n_f-1}{n_c} = n_c \pmod{2}$, and does not equal the Luttinger value.

The (U(1) or $Z_2$) FL* state
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Phase diagram of S=1/2 square lattice antiferromagnet

Neel order

\( \tilde{\phi} \sim z^*_{\alpha} \tilde{\sigma}_{\alpha\beta} z_{\beta} \neq 0 \)

(Higgs)

VBS order \( \Psi_{VBS} \neq 0 \),

\( S = 1/2 \) spinons \( z_\alpha \) confined,

\( S = 1 \) triplon excitations

Deconfined critical point described by a theory of spinons

\[
S_{\text{critical}} = \int d^2x d\tau \left[ |(\partial_\mu - iA_\mu)z_\alpha|^2 + s |z_\alpha|^2 + \frac{u}{2} (|z_\alpha|^2)^2 + \frac{1}{4e^2} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \right]
\]

Landau-forbidden transition between phases which break “unrelated” symmetries
Holon metal
Area = $\frac{\delta}{4}$

Area = $\frac{\delta}{8}$