Noise and counting statistics in ultracold gases

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outline

(1) introduction

(2) full counting statistics of a superconducting beam splitter

(3) density correlations in ultracold Fermi gases

noise = information

Rolf Landauer: “The noise is the signal”

physical quantities like current fluctuate $\Rightarrow$ measurement contains information beyond the average value

Schottky formula

example: current in a vacuum tube (Schottky 1918)

discreteness of electron charge leads to shot noise
definition: noise power

$$S(\omega) = \int_{-\infty}^{\infty} \mathrm{d}t\, e^{i\omega t} \langle \Delta I(t) \Delta I(0) + \Delta I(0) \Delta I(t) \rangle$$

where $\Delta I = I - \bar{I}$

Schottky showed that $S(\omega = 0) = 2e\bar{I}$

random and independent emission of electrons from cathode
$\Rightarrow$ Poisson process $\Rightarrow \langle (N - \bar{N})^2 \rangle = \bar{N}$

Schottky formula 2

hence, by measuring both

the average current $\bar{I}$

and the noise power $S = 2e\bar{I}$

we get additional information, viz., the electron charge!

full counting statistics

generalization of noise / cross-correlations to higher cumulants leads to the idea of full counting statistics

Levitov and Lesovik ‘93; Levitov, Lee, and Lesovik ‘96

Nazarov, Belzig, Kindermann, Bagrets, Samuelsson, Büttiker, Cuevas, Fazio, ... ‘99 – ‘07

idea: calculate probability $P(N)$ that N electrons have passed a certain cross section of the lead during time t_0

$\bar{I} = \frac{e\bar{N}}{t_0}$ where $\bar{N} = \sum_N P(N) N$

$$S(\omega = 0) = \frac{2e^2}{t_0} \langle (N - \bar{N})^2 \rangle$$

all higher moments/cumulants also determined by $P(N)$

experiment

experimental situation: noise power measurements ✓

higher moments: very difficult!

3^{rd} cumulant of the current through a tunnel junction

Reulet et al., PRL 2003, Reznikov et al., PRL 2005

FCS by counting single electron tunneling events:

Gustavsson et al., PRL 2006 (4^{th} cumulant!)

Fujisawa et al., Science 2006

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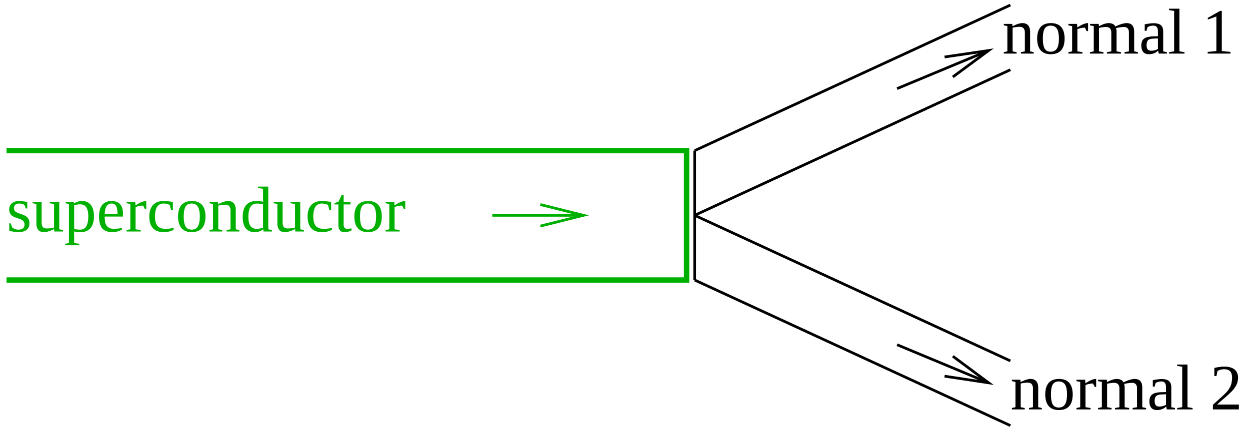
hybrid beamsplitter

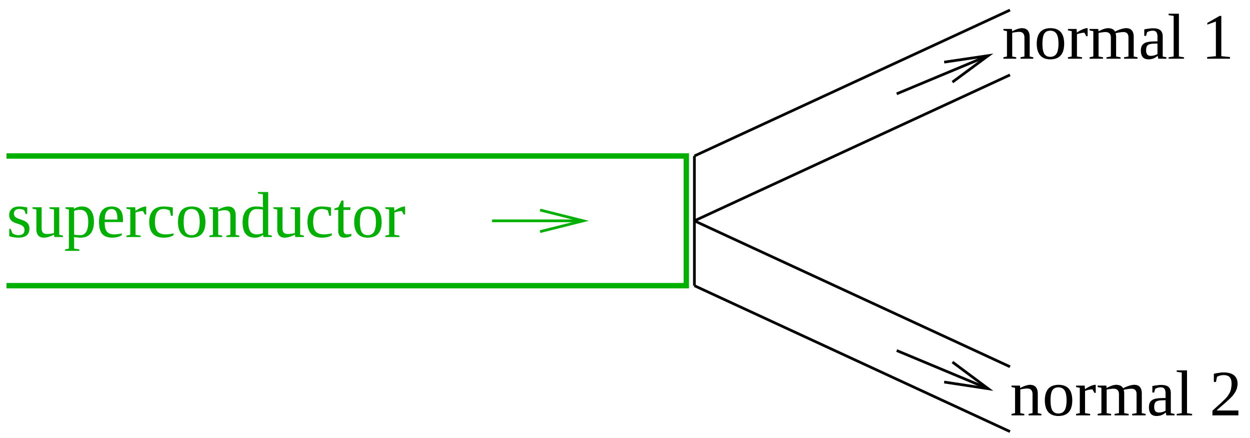
cross-correlations of currents in two normal-metal leads created by splitting a supercurrent



$\langle \Delta I_1 \Delta I_2 \rangle = 2 \int \mathrm{d}t \langle \Delta I_1(t) \Delta I_2(0) \rangle$ positive or negative ?

hybrid beamsplitter

cross-correlations of currents in two normal-metal leads created by splitting a supercurrent

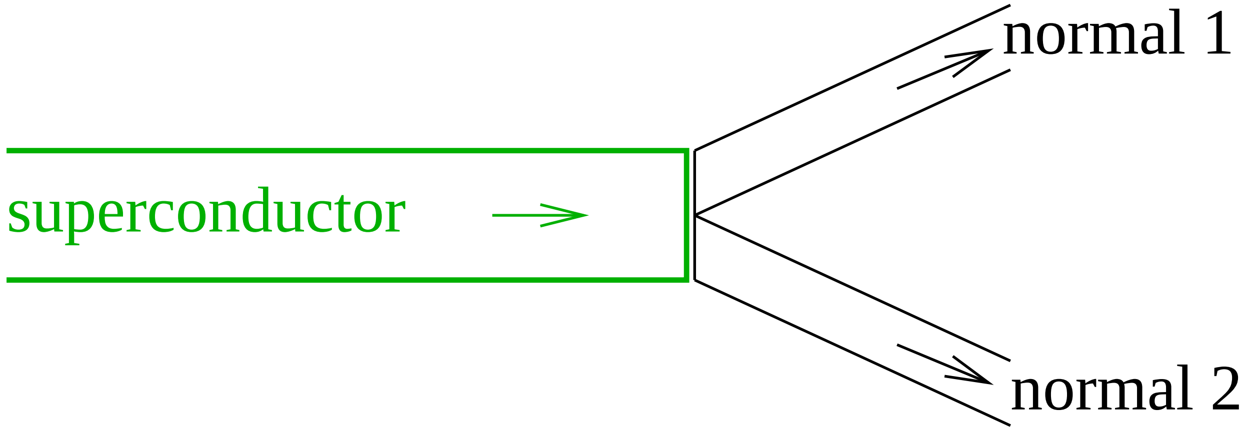


$\langle \Delta I_1 \Delta I_2 \rangle = 2 \int dt \langle \Delta I_1(t) \Delta I_2(0) \rangle$ positive or negative ?

thermal bosons $\Rightarrow$ bunching $\Rightarrow$ positive cross-correlations

hybrid beamsplitter

cross-correlations of currents in two normal-metal leads created by splitting a supercurrent



$\langle \Delta I_1 \Delta I_2 \rangle = 2 \int \mathrm{d}t \langle \Delta I_1(t) \Delta I_2(0) \rangle$ positive or negative ?

thermal bosons $\Rightarrow$ bunching $\Rightarrow$ positive cross-correlations

thermal fermions $\Rightarrow$ antibunching $\Rightarrow$ negative cross-corr.

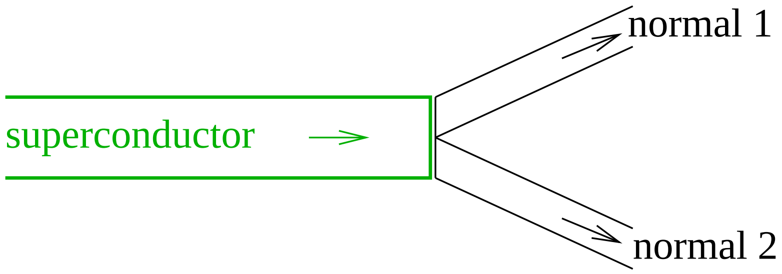
Büttiker PRB 1992:

zero-frequency cross-correlations in

- non-interacting
- normal-metal multi-terminal structures

are always negative!

...doesn't cover our hybrid beam splitter...



...so let's calculate its statistics

cross-correlations

calculate the probability $P(N_1, N_2; t_0)$ to count

$$\left.\begin{array}{lll} N_1 & \text{electrons passing lead} & 1 \\ N_2 & \text{electrons passing lead} & 2 \end{array}\right\} \quad \text{during time } t_0$$

$P(N_1, N_2; t_0)$ determines ALL moments of I_1 and I_2, in particular the cross-correlations $\langle I_1 I_2 \rangle$.

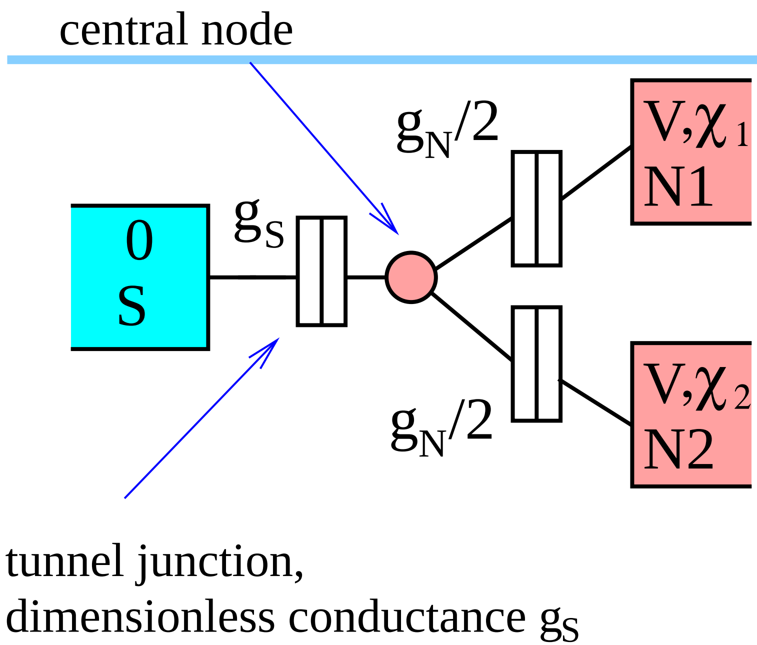
cumulant generating function

$$\exp(-S(\chi_1,\chi_2)) := \sum_{N_1,N_2} P(N_1,N_2)\exp(i\chi_1 N_1 + i\chi_2 N_2)$$

χ_i: “counting fields”

example: noise or cross-correlations of two currents:

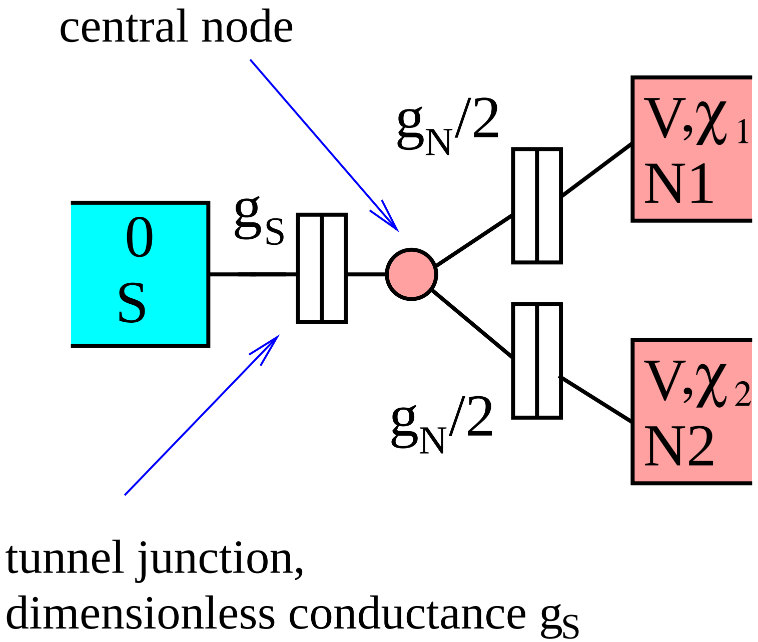
$$\langle \Delta I_1 \Delta I_2 \rangle = \frac{2e^2}{t_0}\,\frac{\partial^2 S(\chi_1,\chi_2)}{\partial\chi_1\partial\chi_2}\bigg|_{\chi_1=\chi_2=0}$$



Nazarov 1998

1. draw system as a ‘discretized’ electric circuit
2. temperature, potential, counting field χ_i determine 4×4 matrix Keldysh Green’s function in each contact
3. Kirchhoff’s law for matrix currents determines Green’s function on central node
4. matrix currents $\Rightarrow$ physical currents $\Rightarrow$ cumulant-generating function $S(\chi_1, \chi_2)$

cumulant-generating function



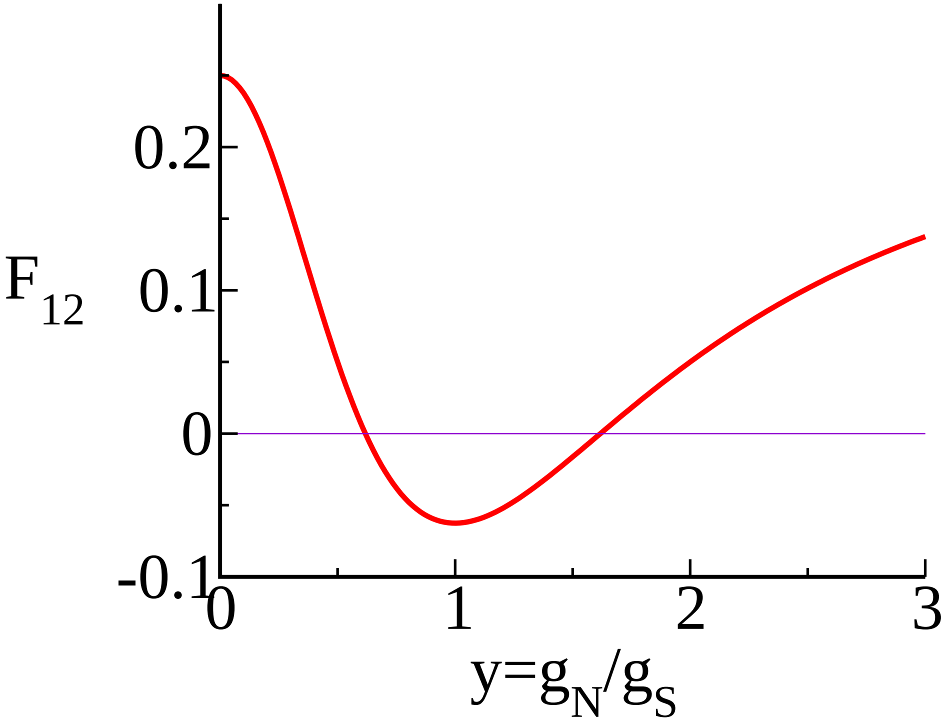
$$p := \frac{g_S g_N}{g_S^2 + g_N^2}$$

$$S(\chi_1, \chi_2) = -\frac{V t_0 \sqrt{g_S^2 + g_N^2}}{\sqrt{2} e} \sqrt{1 + \sqrt{1 + p^2 (e^{i\chi_1} + e^{i\chi_2})^2 - 4p^2}}$$

exact expression for the cumulant-generating function

cross-correlations

$$F_{12} = \frac{\langle \Delta I_1 \Delta I_2 \rangle}{2e\langle I \rangle} = \frac{e}{t_0 \langle I \rangle} \left. \frac{\partial^2 S(\chi_1, \chi_2)}{\partial \chi_1 \partial \chi_2} \right|_{\chi_1 = \chi_2 = 0}$$



J. Börlin, W. Belzig, and CB, PRL 2002

P. Samuelsson and M. Büttiker, PRL 2002

T. Martin, Phys. Lett. '96

J. Torres and T. Martin, EPJB '99

- positive cross-correlations for $y = g_N/g_S \ll 1$ or $\gg 1$
- negative cross-correlations around $y = 1$

interpretation

For $y = g_N/g_S \ll 1$ or $\gg 1$, expand $S(\chi_1, \chi_2)$:

$$S(\chi_1, \chi_2) \sim e^{2i\chi_1} + e^{2i\chi_2} + 2e^{i(\chi_1+\chi_2)}$$

Poisson statistics

$\Rightarrow$ uncorrelated pair tunneling events

$\Rightarrow$ first two terms do NOT contribute to cross-correlations

positive contribution of third term

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idea

use noise correlations / statistics of density fluctuations in absorption images to get information on the many-body nature of an ultracold atom system

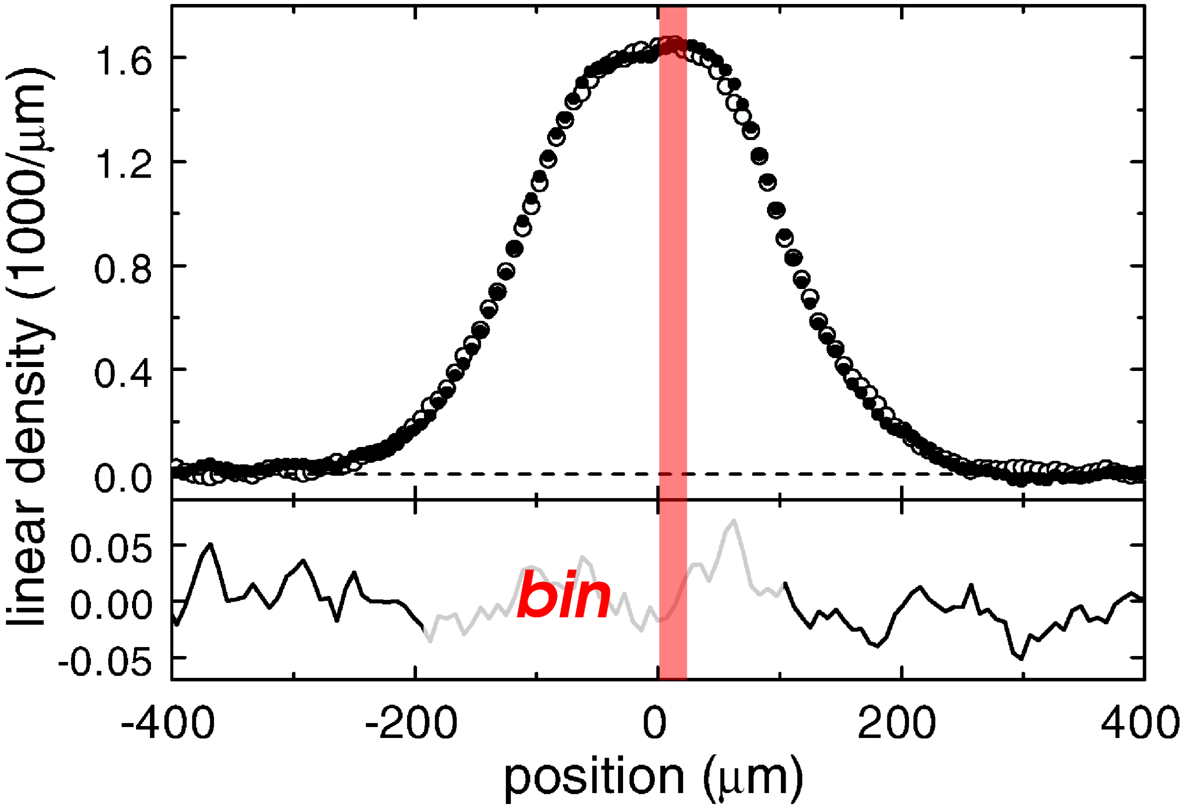
E. Altman, E. Demler, and M. Lukin, PRA 2004:

“Probing many-body states of ultracold atoms via noise correlations”

see also A. Lamacraft, PRA 2006

density fluctuations

M. Bartenstein et al., PRL 92, 120401 (2004)



- axial profile, averaged over 50 experiments
- bin size $\approx 10\mu$m (imaging resolution)

correlation/noise experiments

S. Fölling et al., Nature 2005

“Spatial quantum noise interferometry in expanding ultracold atom clouds”

M. Greiner, C.A. Regal, J.T. Stewart, and D.S. Jin, PRL 2005 (experiment)

“Probing pair-correlated fermionic atoms through correlations in atom shot noise”

A. Öttl , S. Ritter, M. Köhl, and T. Esslinger, PRL 2005

“Correlations and counting statistics of an atom laser”

I.B. Spielman et al., PRL 2007

“Mott-Insulator Transition in a Two-Dimensional Atomic Bose Gas”

experiment by Greiner et al.

M. Greiner et al., PRL 2005: noise in absorption images of ^{40}K

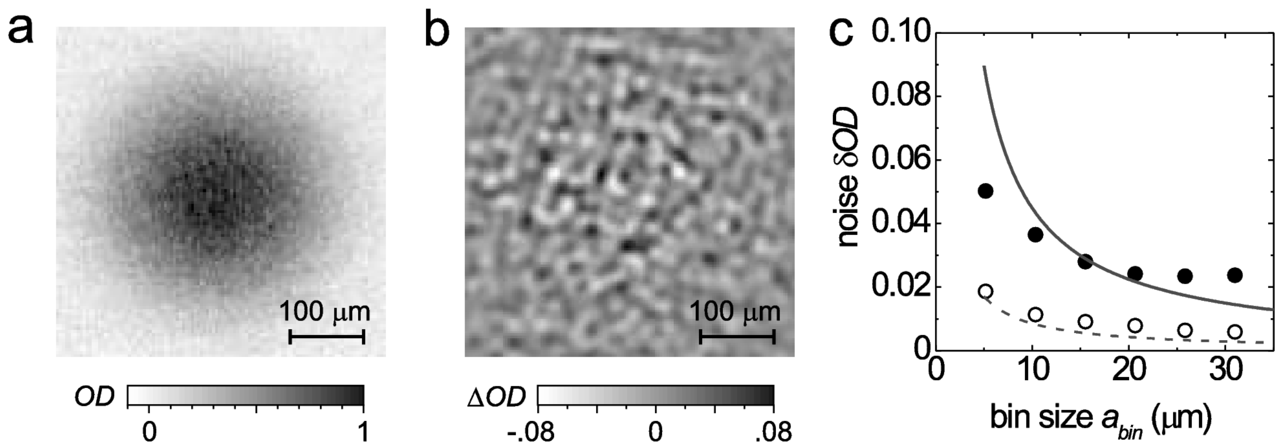
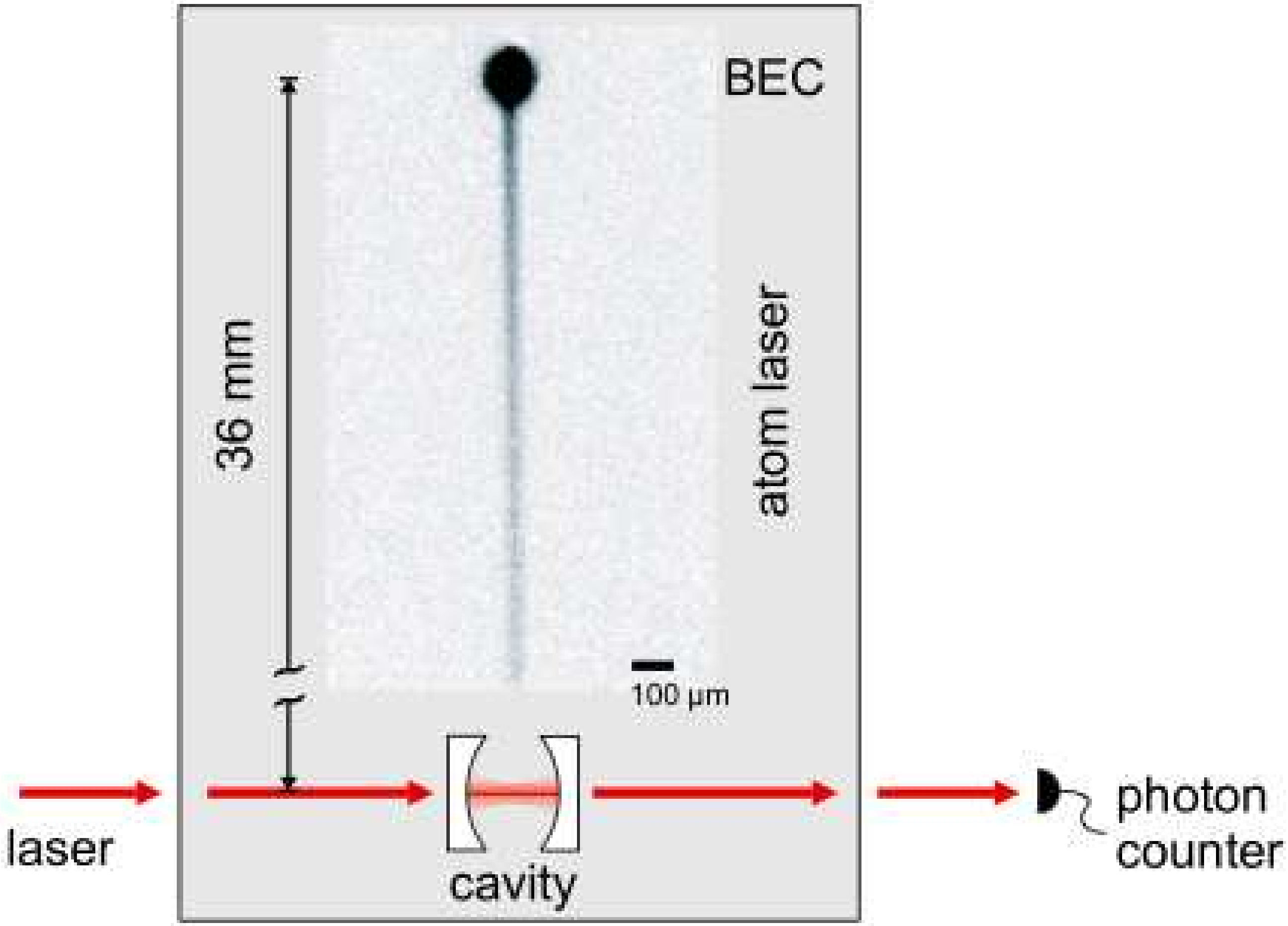


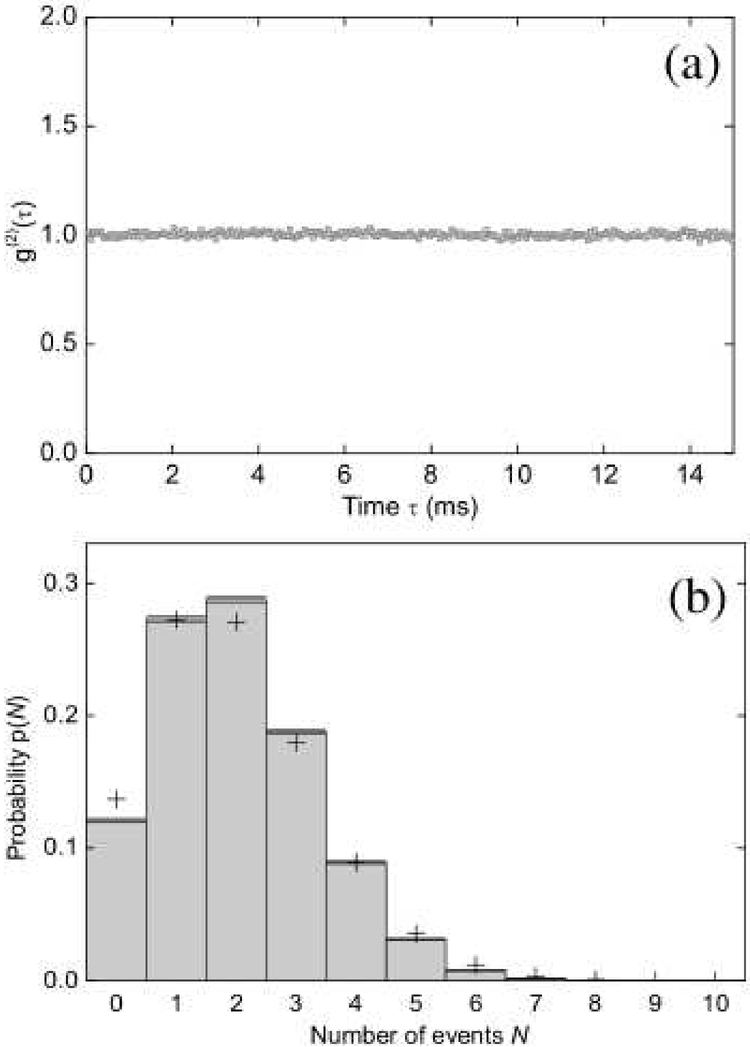
FIG. 1: Atom shot noise in a time-of-flight (TOF) absorption image. (a) One spin state of a weakly interacting, two-component, degenerate Fermi gas with 2.3×10^5 atoms per spin state is imaged after 19.2 ms of expansion. (b) The noise on the absorption image was extracted using a filter with an effective bin size of 15.5 microns. (c) The noise at the cloud center (•) is dominated by atom shot noise, while the noise at the edge of the image (◦) shows the photon shot noise. The noise in OD decreases when averaged over a larger bin size.

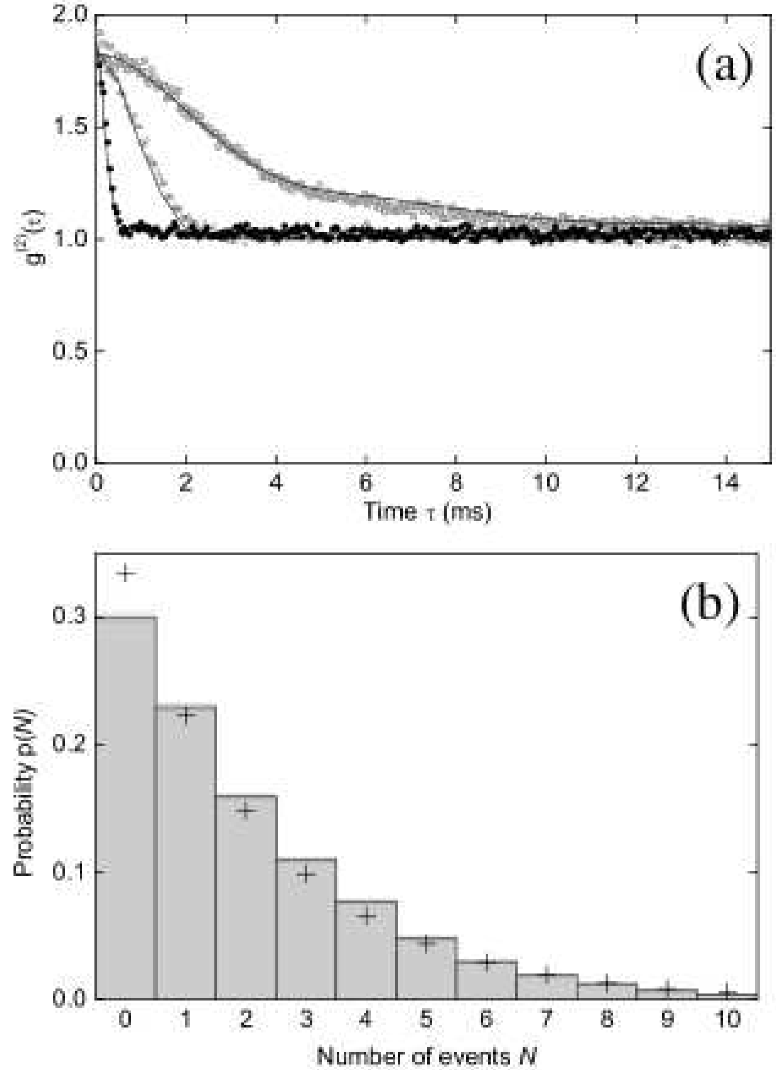
Esslinger counting exp.



Öttl et al., PRL 2005

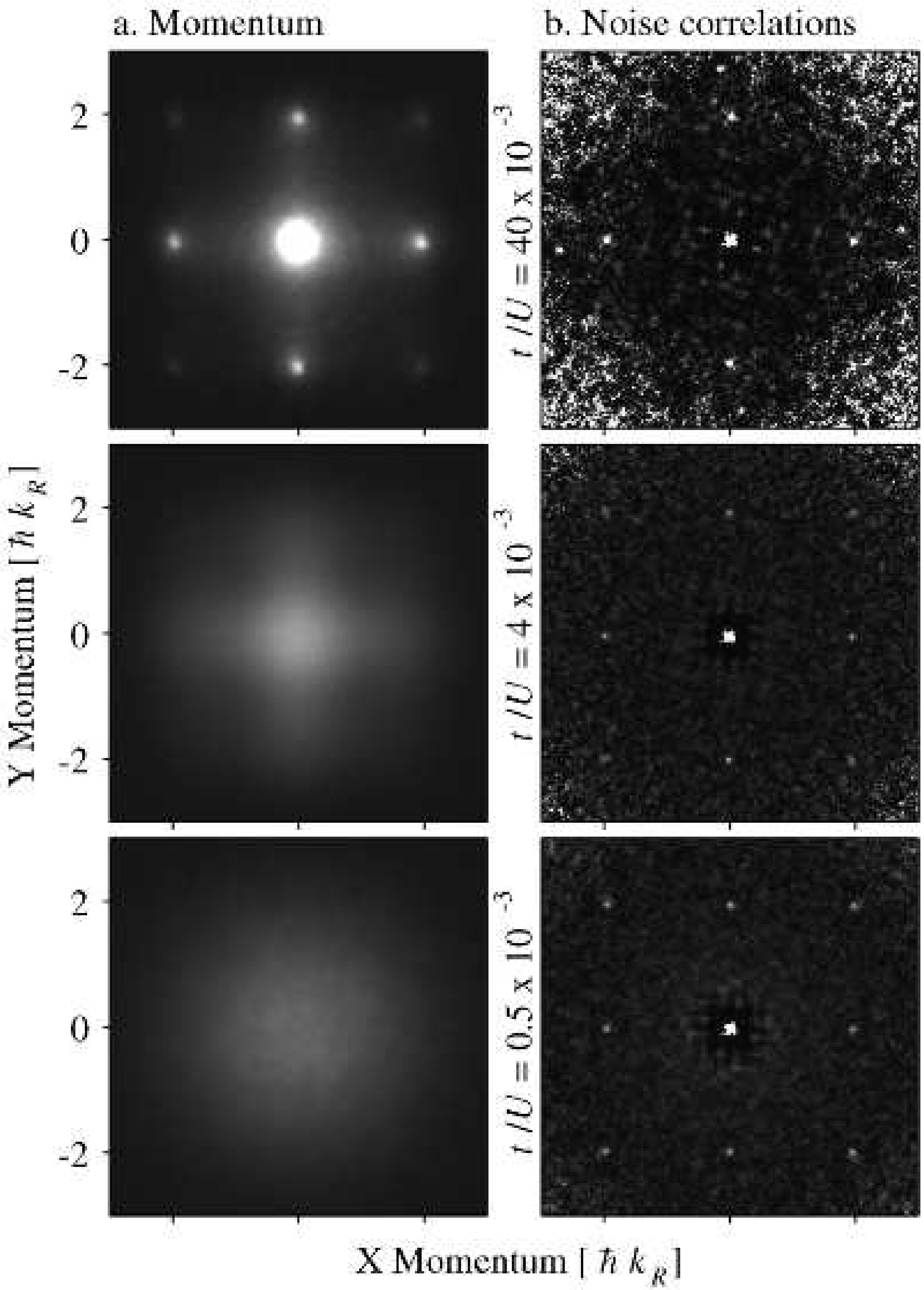
Esslinger counting exp.





atom laser (Poissonian) vs. (pseudo-)thermal beam (Bose)

Porto experiment

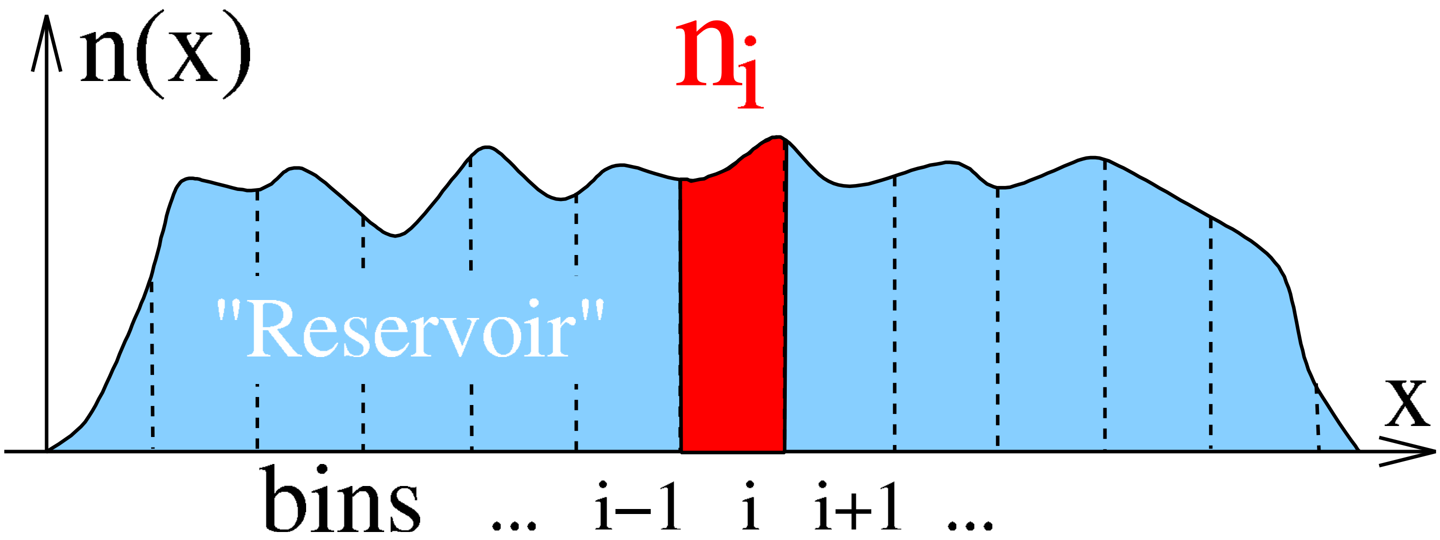


Spielman et al., PRL 2007

application to BEC-BCS transition

goal:
calculate counting statistics of atom number in a bin of a fermionic cloud of cold atoms with attractive interactions

atom number fluctuations



- assumption: $\boldsymbol{N_{\mathrm{total}} \gg N_{\mathrm{bin}} \gg 1}$
- atoms outside a bin serve as reservoir:
  $\Rightarrow$ grand-canonical treatment

full counting statistics

systematic treatment of fluctuations:

probability to find N particles in the system = bin:

$$P(N) = \langle \delta(\hat{N} - N) \rangle$$

evaluated in thermal ensemble or ground state

full counting statistics 2

characteristic function:

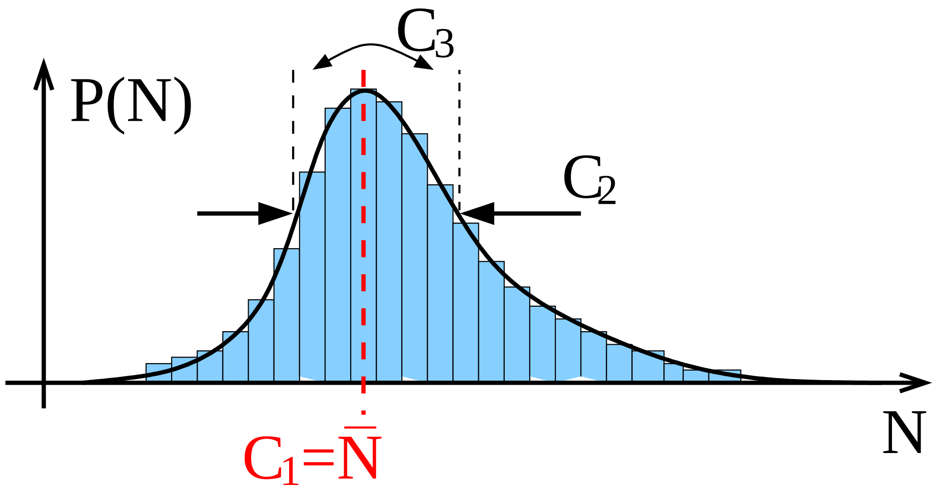
$$e^{-S(\chi)} = \sum_N e^{iN\chi} P(N) = \langle e^{i\hat{N}\chi} \rangle$$

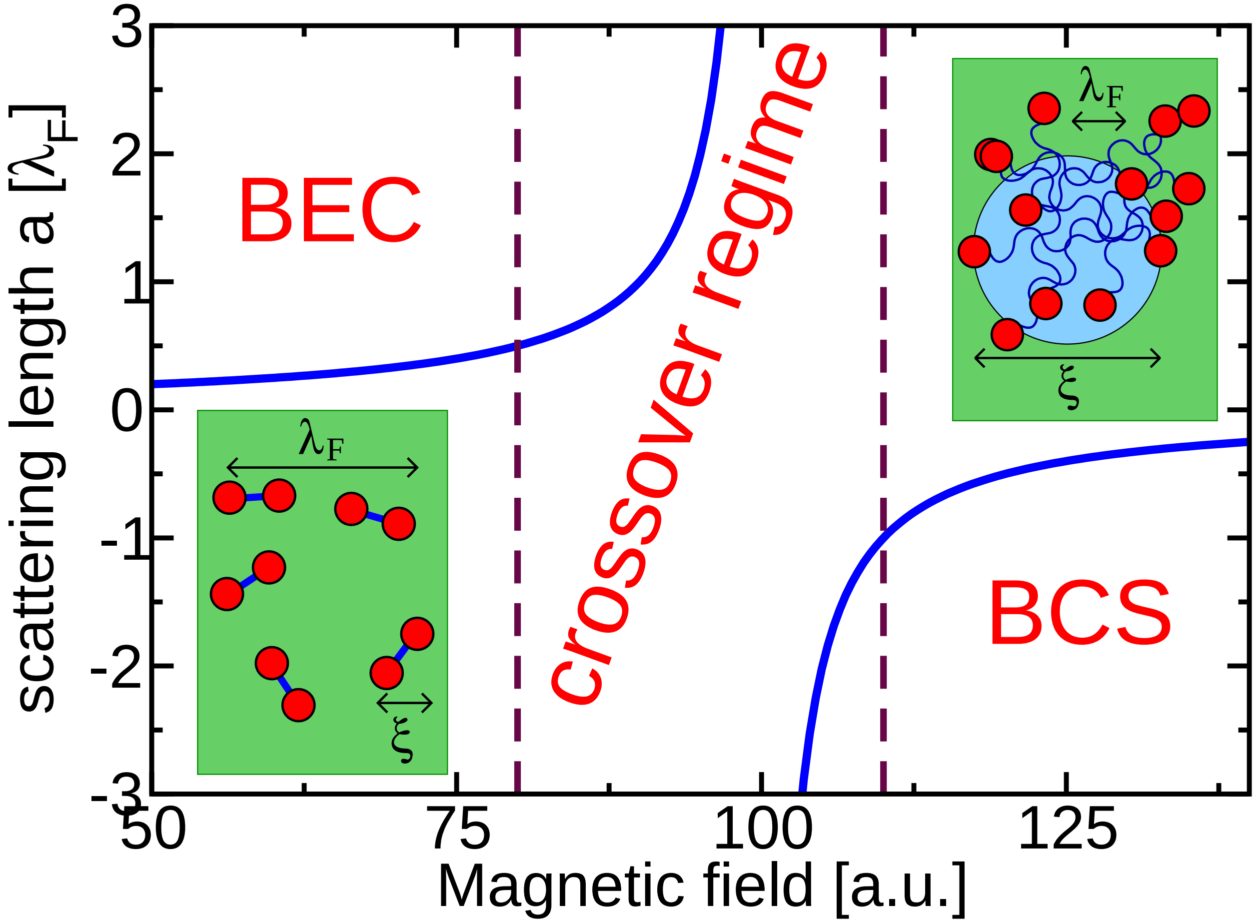
cumulant generating function:

$$S(\chi) = -\sum_{r=0}^{\infty} \frac{C_r}{r!} (i\chi)^r$$

$$C_n = -\left(-i\frac{\partial}{\partial\chi}\right)^n S(\chi)\Big|_{\chi=0}$$

$$C_1 = \langle \hat{N} \rangle$$
$$C_2 = \langle (\hat{N} - \langle \hat{N} \rangle)^2 \rangle$$
$$C_3 = \langle (\hat{N} - \langle \hat{N} \rangle)^3 \rangle$$





experimentally accessible by tuning magnetic field:
Feshbach resonance

mean-field description of crossover

BCS wavefunction

$$|\mathrm{BCS}\rangle = \prod_{\boldsymbol{k}} (u_{\boldsymbol{k}} + v_{\boldsymbol{k}} c^{\dagger}_{\boldsymbol{k}\uparrow} c^{\dagger}_{-\boldsymbol{k}\downarrow})|0\rangle$$

Eagles 69, Leggett 80, Randeria et al. 90

variational approach yields

$$v_{\boldsymbol{k}}^2 = 1 - u_{\boldsymbol{k}}^2 = \frac{1}{2}\left(1 - \frac{\epsilon_{\boldsymbol{k}} - \mu}{\sqrt{(\epsilon_{\boldsymbol{k}} - \mu)^2 + \Delta^2}}\right)$$

self-consistency condition

Δ and μ determined by self-consistency equations:

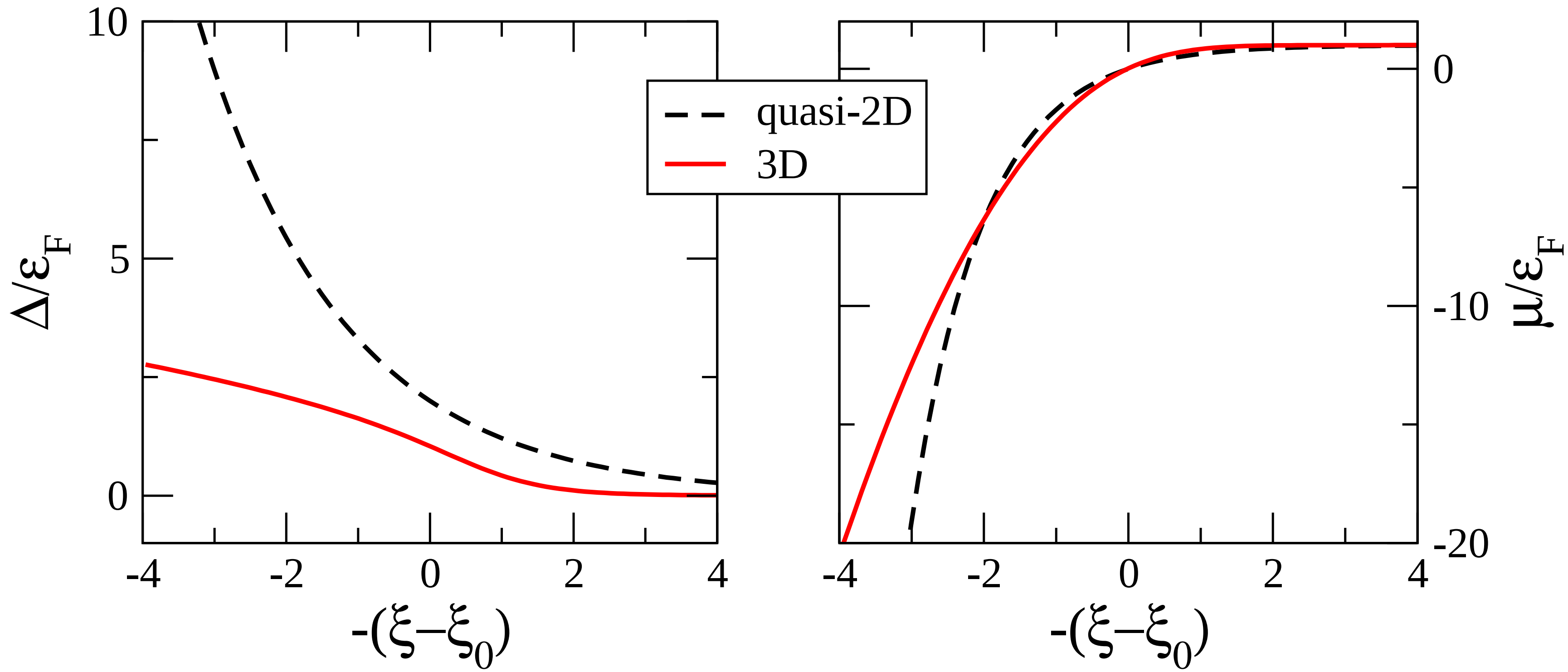
order parameter (interaction constant λ)

$$\Delta = -\lambda \sum_{k} u_k v_k$$

average particle number

$$\bar{N} = \langle \hat{N} \rangle = 2 \sum_{k} v_k^2$$

self-consistent solution



$$\xi = 1/k_F a$$

$-(\xi - \xi_{\mu=0}) \ll 0 \Rightarrow$ BEC-limit

$-(\xi - \xi_{\mu=0}) \gg 0 \Rightarrow$ BCS-limit

cumulant generating function

for a single $\boldsymbol{k}$-state (i.e. pair of states with $\boldsymbol{k}\uparrow, -\boldsymbol{k}\downarrow$):

$$\begin{aligned} e^{-S_{\boldsymbol{k}}(\chi)} &= \langle \mathrm{BCS}| e^{i\chi(\hat{n}_{\boldsymbol{k}\uparrow}+\hat{n}_{-\boldsymbol{k}\downarrow})} |\mathrm{BCS}\rangle \\ &= u_{\boldsymbol{k}}^2 + v_{\boldsymbol{k}}^2 e^{2i\chi} \end{aligned}$$

combining all states:

$$S(\chi) = \sum_{\boldsymbol{k}} S_{\boldsymbol{k}}(\chi) = -\sum_{\boldsymbol{k}} \ln[1 + v_{\boldsymbol{k}}^2(e^{2i\chi} - 1)]$$

general result in the BEC-BCS crossover regime

(for given Δ and μ)

counting statistics in 2D/3D

analytic expression in 2D:

$$S(\chi) = \bar{N}\frac{\Delta}{\epsilon_F}\cos(\chi)\mathrm{atan}(\frac{\epsilon_F}{\Delta}e^{i\chi}) + \bar{N}\frac{\mu}{\epsilon_F}\ln[1 + v_0^2(e^{2i\chi} - 1)]$$

numerical evaluation in 3D

$\bar{N}$ = average number of particles per bin,

$v_0^2 = \frac{1}{2}(1 + \frac{\mu}{\sqrt{\Delta^2 + \mu^2}})$

limiting cases: BEC-regime

BEC-limit: $\mu/\epsilon_F \ll -1$; $v_k^2 \ll 1$

$$S(\chi) \approx \sum_k v_k^2 (e^{2i\chi} - 1) = -\frac{\bar{N}}{2}(e^{2i\chi} - 1)$$

- Poissonian statistics of strongly bound pairs
- number statistics corresponds to Bose condensate of molecules

compare with free condensate

$|\psi\rangle = (a_0^\dagger)^{N_{\rm tot}}|\text{vac}\rangle$ where $a_k = b_k + c_k$,

$$b_k = \int_{V_{\rm bin}} d^3r\; e^{ikr}\Psi(r)\,, \quad c_k = \int_{V\setminus V_{\rm bin}} d^3r\; e^{ikr}\Psi(r)\,,$$

bin number operator $\hat{N} := \sum_k b_k^\dagger b_k$

$$S(\chi) = \langle e^{i\hat{N}\chi}\rangle = -N_{\rm tot}\ln\left[1 + \frac{V_{\rm bin}}{V}(e^{i\chi}-1)\right]$$

For $V_{\rm bin}/V \ll 1$

$S(\chi) \approx -\bar{N}(e^{i\chi}-1)$ ✓

$\bar{N} = N_{\rm tot}V_{\rm bin}/V$

limiting cases: BCS-regime

BCS-limit: $\mu/\epsilon_F \approx 1$; $\Delta \ll \epsilon_F$

$$S(\chi) = -i\bar{N}\chi - \pi\bar{N}D\frac{\Delta}{8\epsilon_F}(e^{i\chi} + e^{-i\chi} - 2)$$

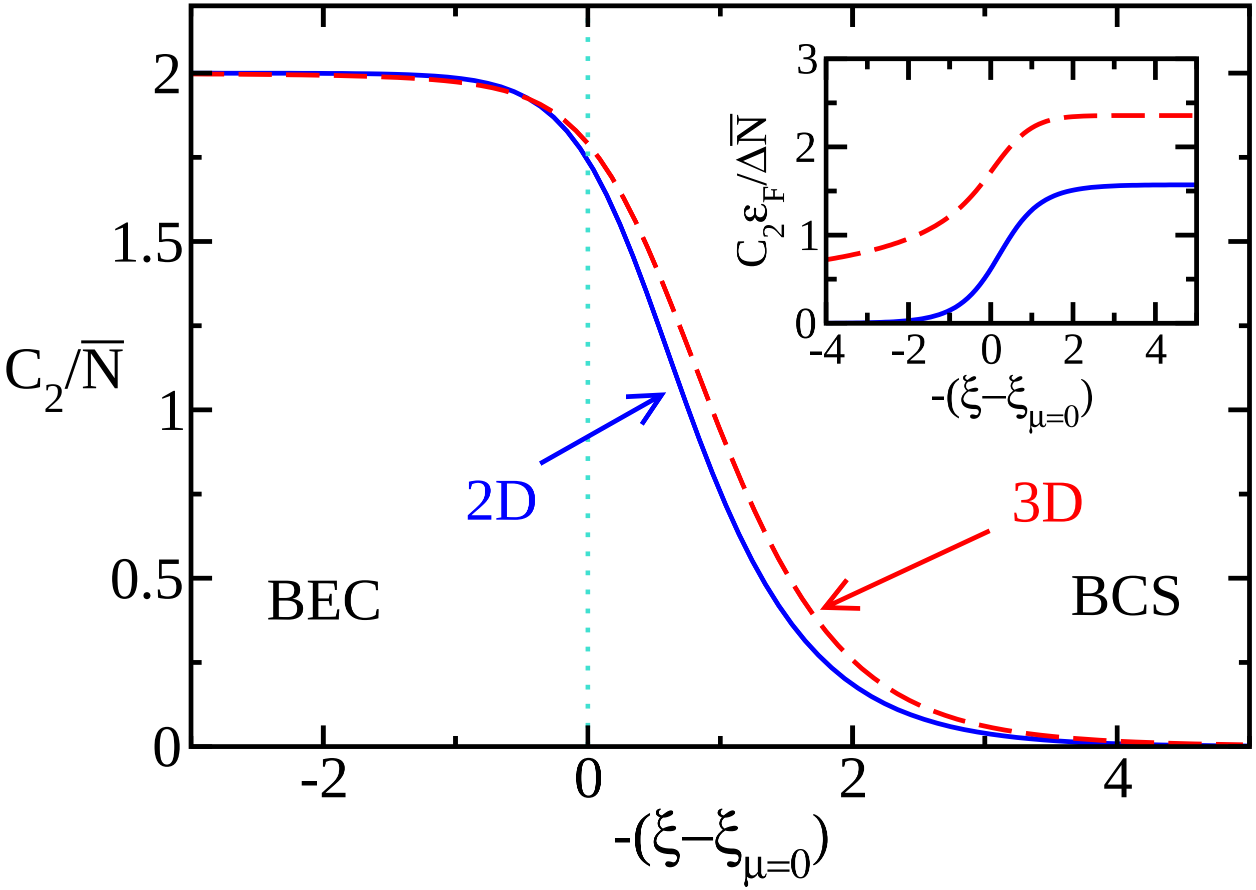
- mean value ($\sim i\chi$) dominates FCS
- small fluctuations: only $\sim \dfrac{\Delta}{\epsilon_F}\bar{N}$ particles fluctuate
- all odd cumulants vanish (except $C_1 = \bar{N}$)

compare with free Fermi gas for $k_B T \ll \epsilon_F$:

$$S(\chi) = -i\tilde{\chi}\bar{N} - (Dk_BT/4\epsilon_F)\bar{N}\tilde{\chi}^2$$

variance $\sim T/\epsilon_F$ (see also Castin, cond-mat/0612613)

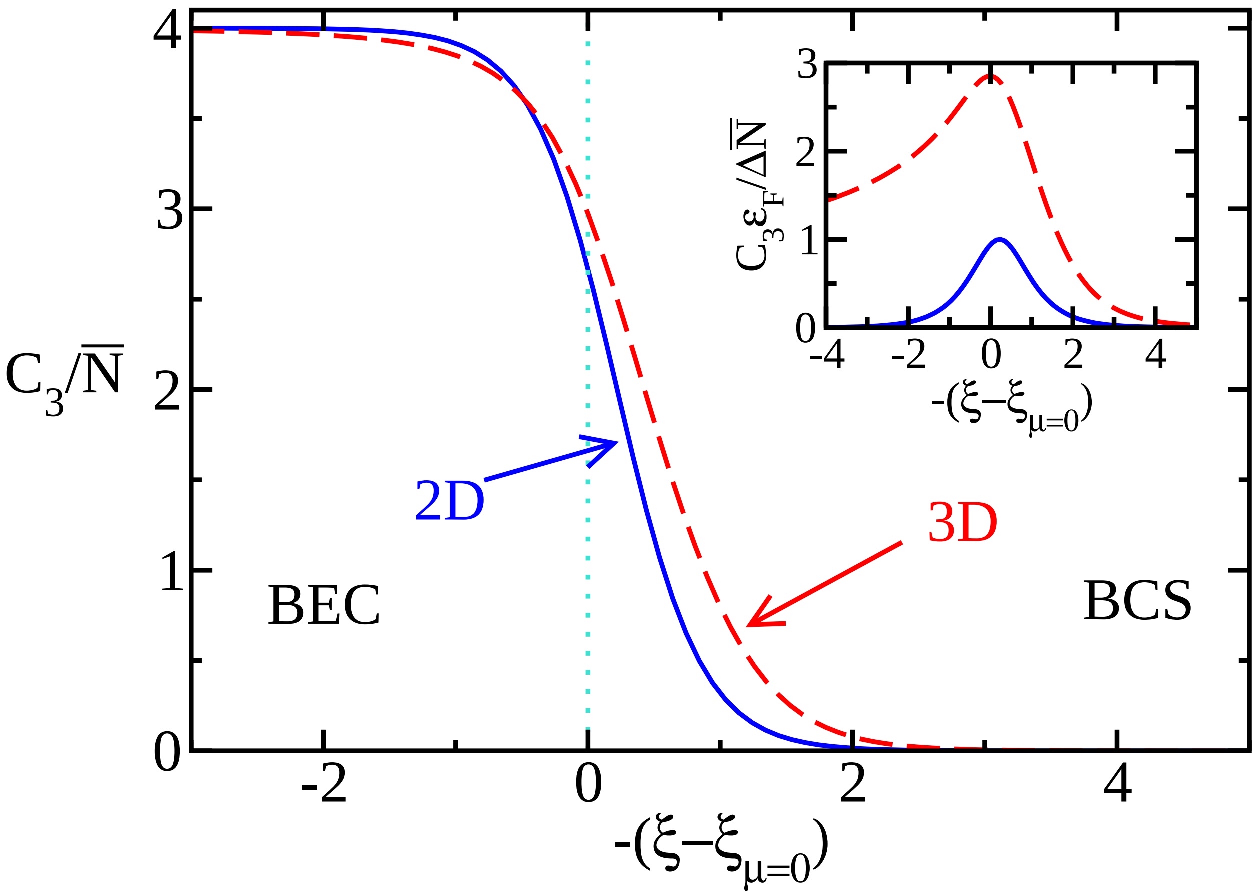
crossover regime: 2^{nd} cumulant



$\xi = 1/k_F a$

- noise strongly reduced in BCS-limit
- $\Delta/\epsilon_F = 4C_2/\pi\bar{N}D$ can be determined from C_2

crossover regime: 3^{rd} *cumulant*



$$\xi = 1/k_F a$$

- C_3 vanishes faster in 2D than in 3D (due to constant density of states in 2D $\Rightarrow$ particle-hole symmetry)

conclusion

- full counting statistics reveals additional information on many-body systems
- full counting statistics of fermionic atomic clouds at the BEC-BCS crossover

open questions:

- better models for crossover regime
- finite-temperature effects
- finite-trap effects