Crystallization and trimer formation in Fermi mixtures

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Outline

- Introduction.
- Molecules in Fermi gases. Collisional relaxation
- Molecules in Ferm mixtures. Trimer states
- Crystalline phase and quantum transitions
- Stability of the crystalline phase
- Conclusions

Collaborations: D.S. Petrov, C. Salomon (ENS), G.Astrakharchik (Barcelona)

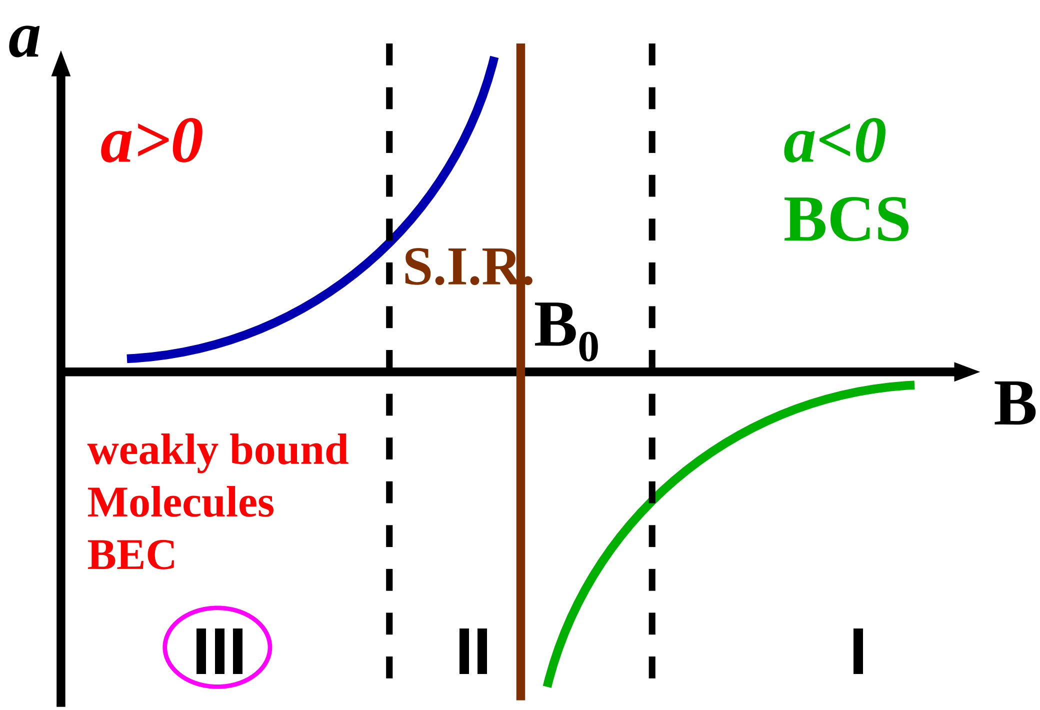
Two-component Fermi gases. Experiments ^{40}K ^{6}Li

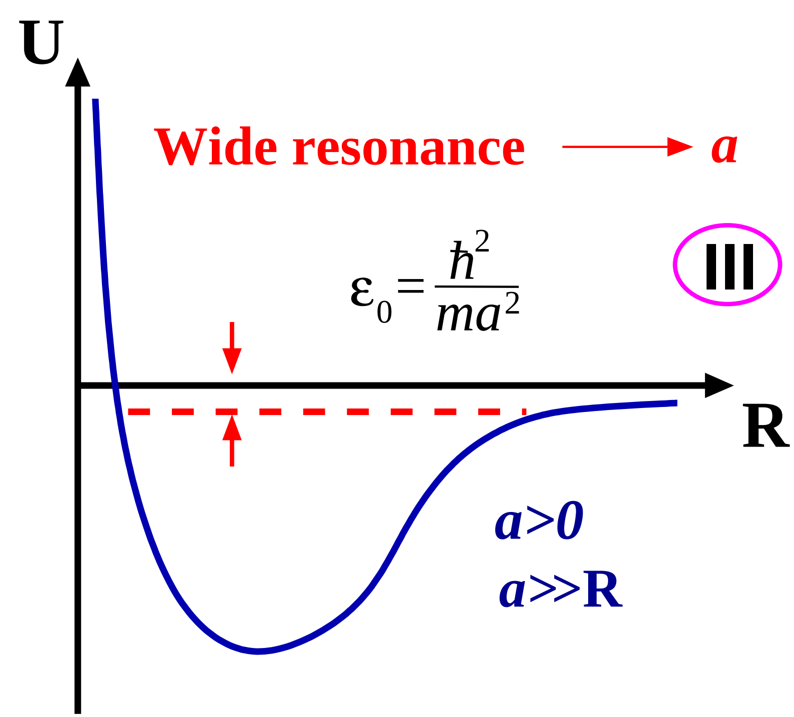
Dilute limit $nR_e^3 \ll 1$ Ultracold limit $\Lambda_T \gg R_e$

Quantum degeneracy $\rightarrow$ JILA 1998 ^{40}K

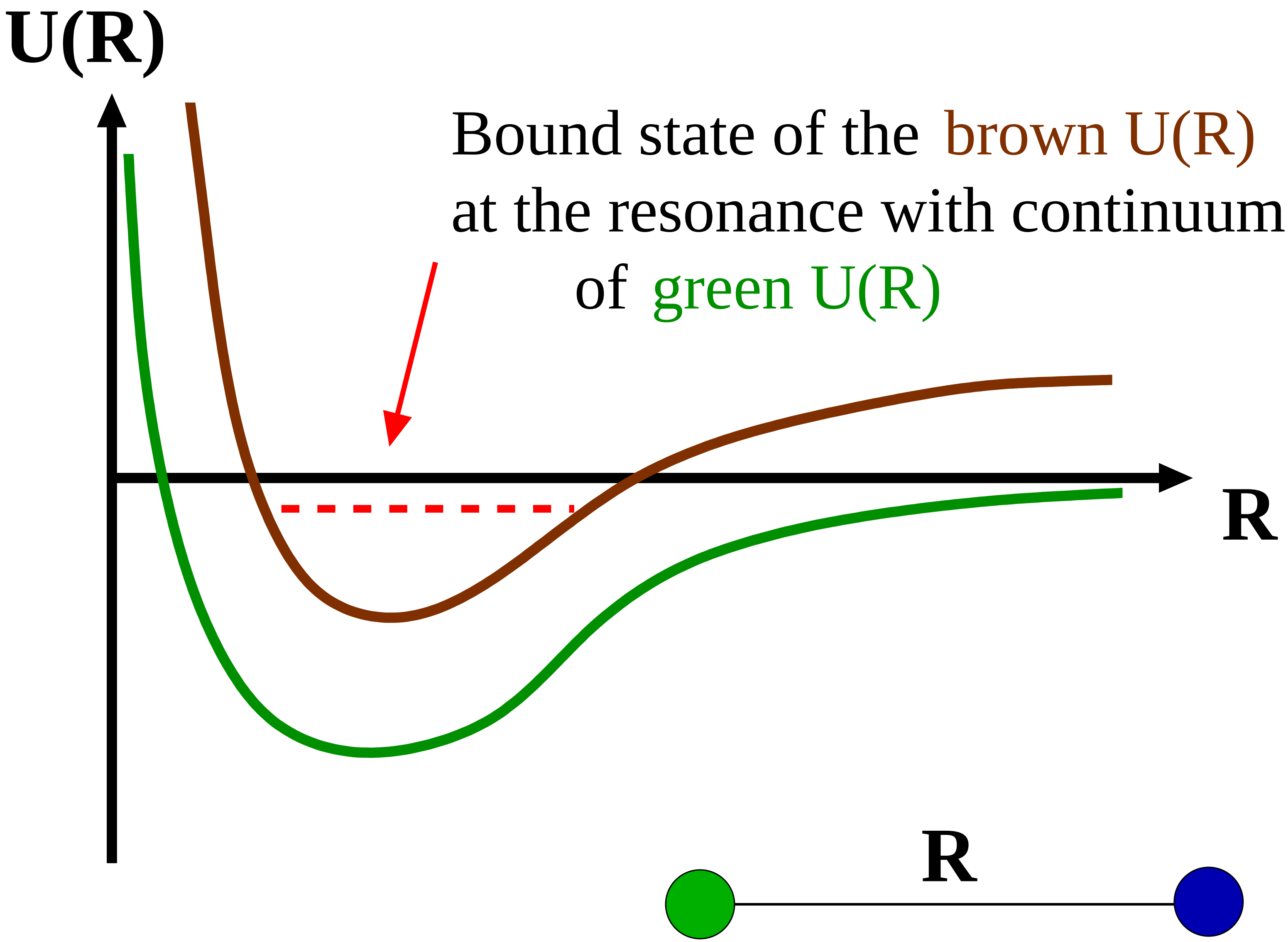
At present $n \sim 10^{13} - 10^{14}\mathrm{cm}^{-3}$; $T \sim 1\mu\mathrm{K}$

JILA, LENS   Innsbruck,MIT,ENS,Rice,Duke, ETH, Hamburg, Tuebingen, Toronto





Feshbach resonance

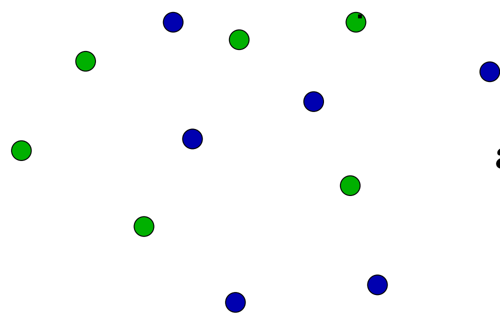


Strongly interacting regime

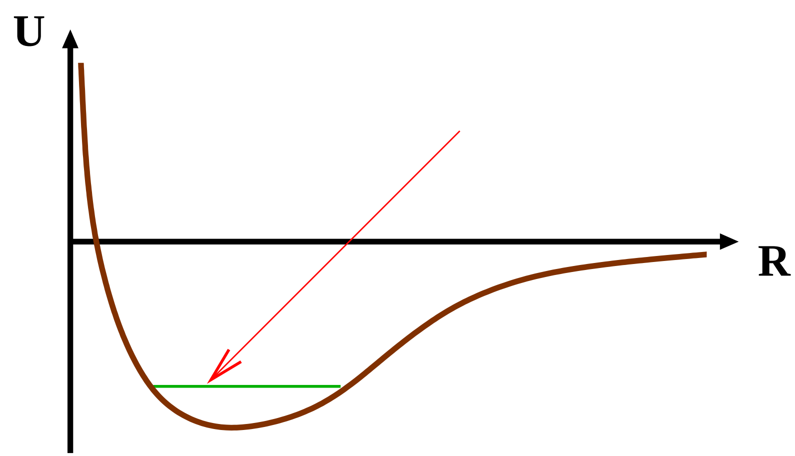
$T = 0 \quad k_F|a| \gg 1 \quad \rightarrow$ Only one distance scale $n^{-1/3}$

Only one energy scale $E_F \sim \hbar^2 n^{2/3}/m$

Universal thermodynamis (J. Ho)



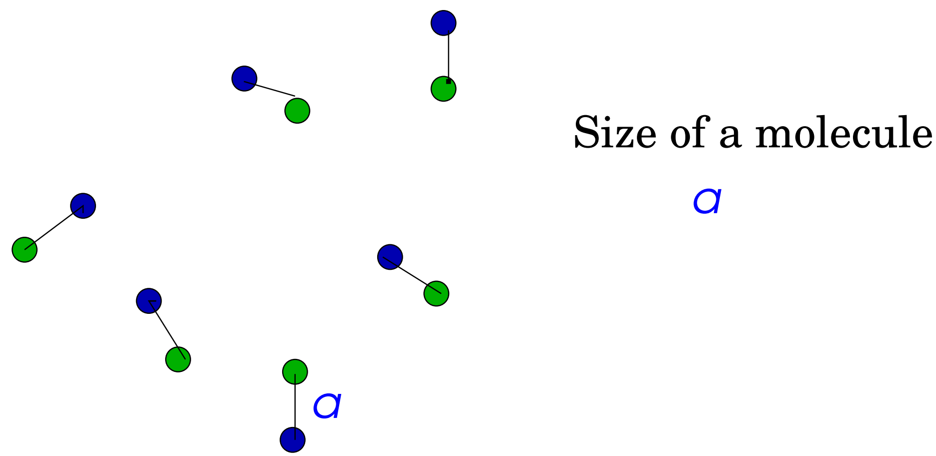
Mean separation between atoms ~ $n^{1/3}$



Interatomic potential $U \Rightarrow$ 3-body recombination into deep bound states

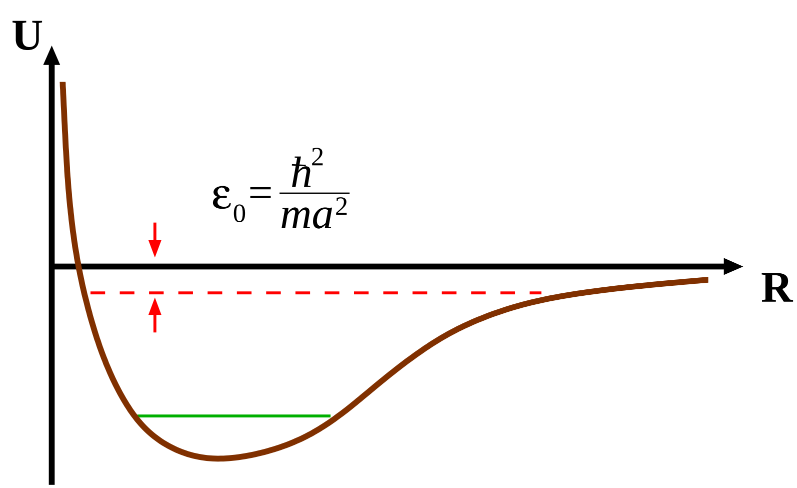
Gas of bosonic molecules (dimers)

Region III $(a > 0) \Rightarrow$ gas of weakly bound bosonic molecules



$na^3 \ll 1 \quad \Rightarrow$ weakly interacting Bose gas

Weakly bound dimers $\rightarrow$ The highest rovibrational state $\Rightarrow$ Collisional relaxation



($\tau \sim$ 1ms for Rb_2 at $n \sim 10^{13}$cm^{-3})

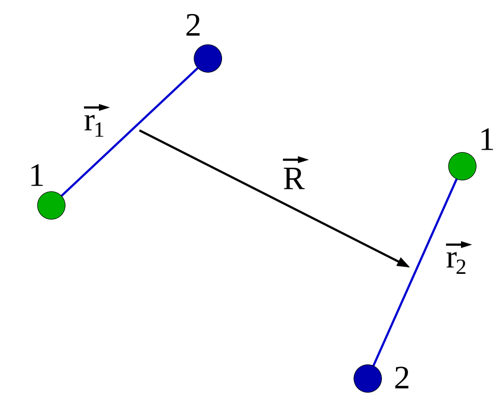
Weakly interacting gas of bosonic dimers

Elastic interaction   BEC stability   "Old answer" $\rightarrow 2a$

4-body problem   Exact solution for $a \gg R_e$ (Petrov et al 2003)

$\Psi \quad \rightarrow \quad$ 9 variables

Zero-range approximation



$$\Psi_{r_1 \rightarrow 0} \;\rightarrow f(\vec{r}_2, \vec{R})(1/4\pi r_1 - 1/4\pi a)$$

Integral equation for f $k \rightarrow 0$ s-wave scattering;   3 variables

$$R \rightarrow \infty \quad \Psi = \phi_0(r_1)\phi_0(r_2)(1 - a_{dd}/R); \;\; \phi_0(r) = \frac{1}{\sqrt{2\pi a}} \exp(-r/a)$$

$$a_{dd} = 0.6a$$

Monte Carlo (Giorgini/Astracharchik, 2004)

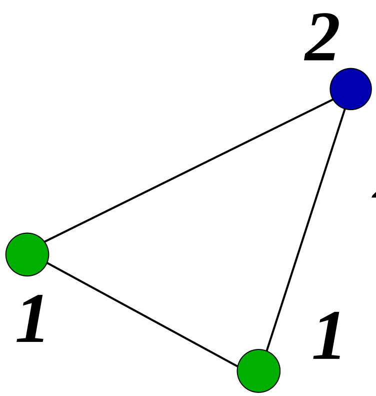
Diagrammatic approach (M.Kagan et al,2005; Gurarie et al,2006)

Atom-dimer collisions

Size $\rightarrow$

Weakly bound dimer $\sim a$

Deep bound state $\sim R_e$ (50 Å) $\ll a$

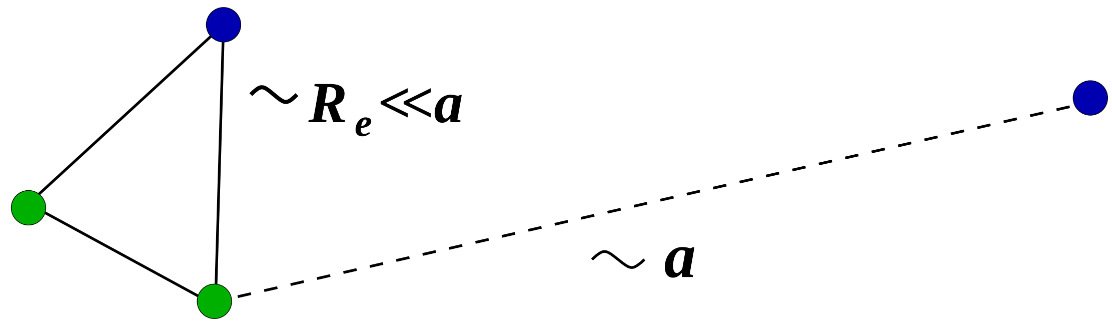


$\sim R_e$ 2 particles are identical fermions

Pauli principle

$$\alpha_{rel} \sim (k_{eff} R_e)^{2?} \sim (R_e/a)^{2?}$$

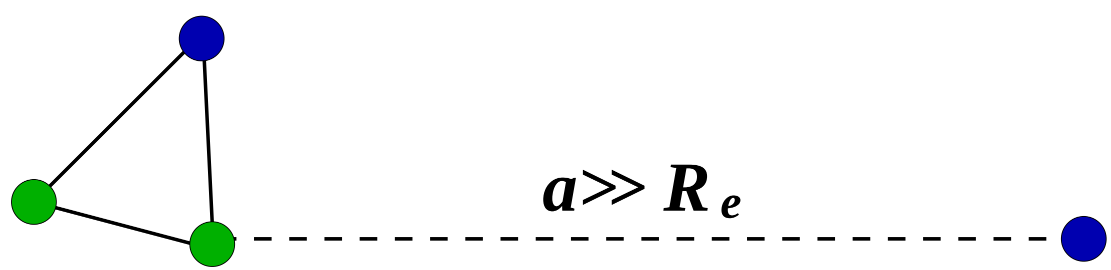
Molecule-molecule relaxation collisions



$$\alpha_{rel} = C \frac{\hbar R_e}{m} \left(\frac{R_e}{a} \right)^s ; \quad s = 2.55$$

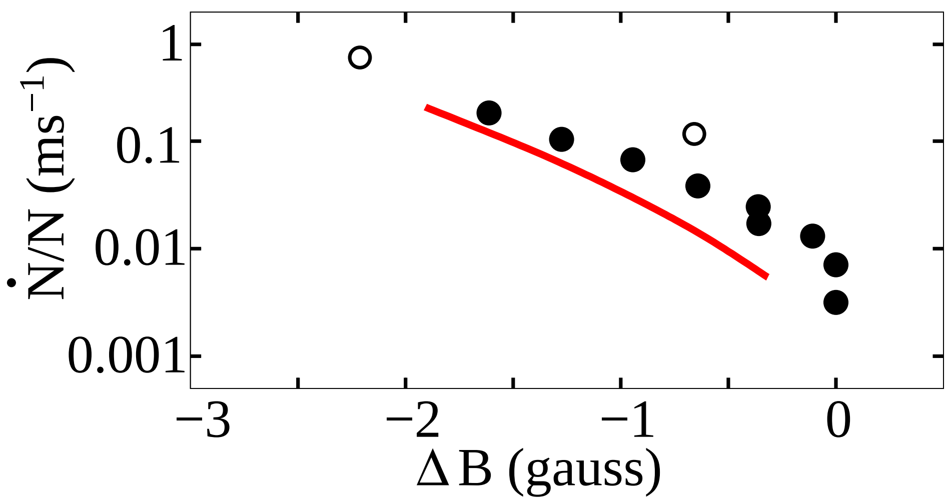
$\tau \sim (\alpha_{rel} n)^{-1} \sim$ seconds (Petrov et al 2003)

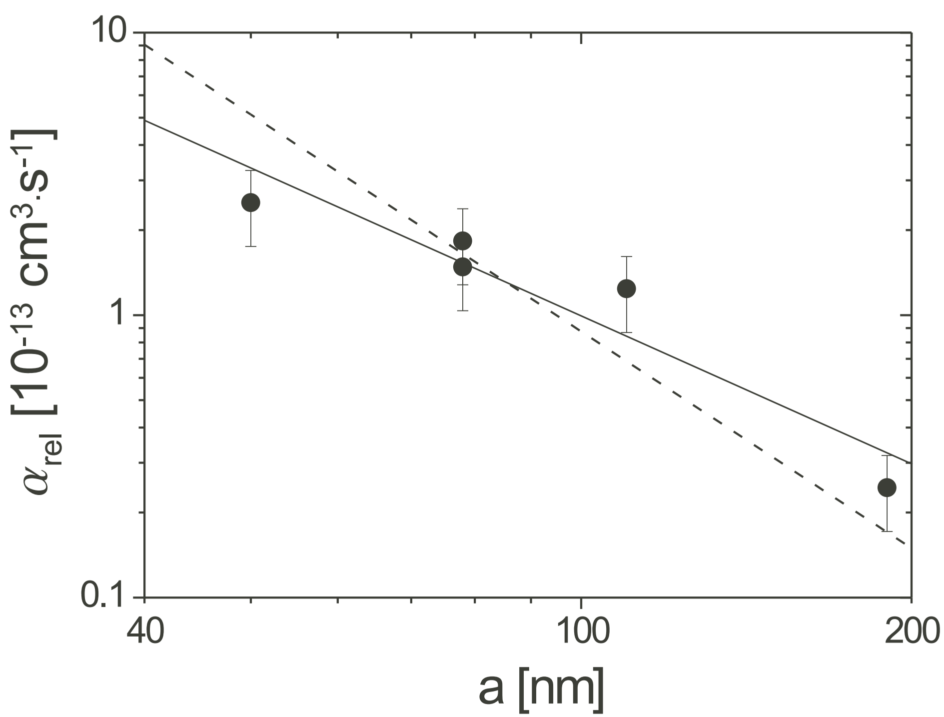
Molecules of bosonic atoms



Resonant enhancement $\alpha_{rel} \sim \hbar a/m$ $\tau < 1$ms

Suppressed collisional relaxation





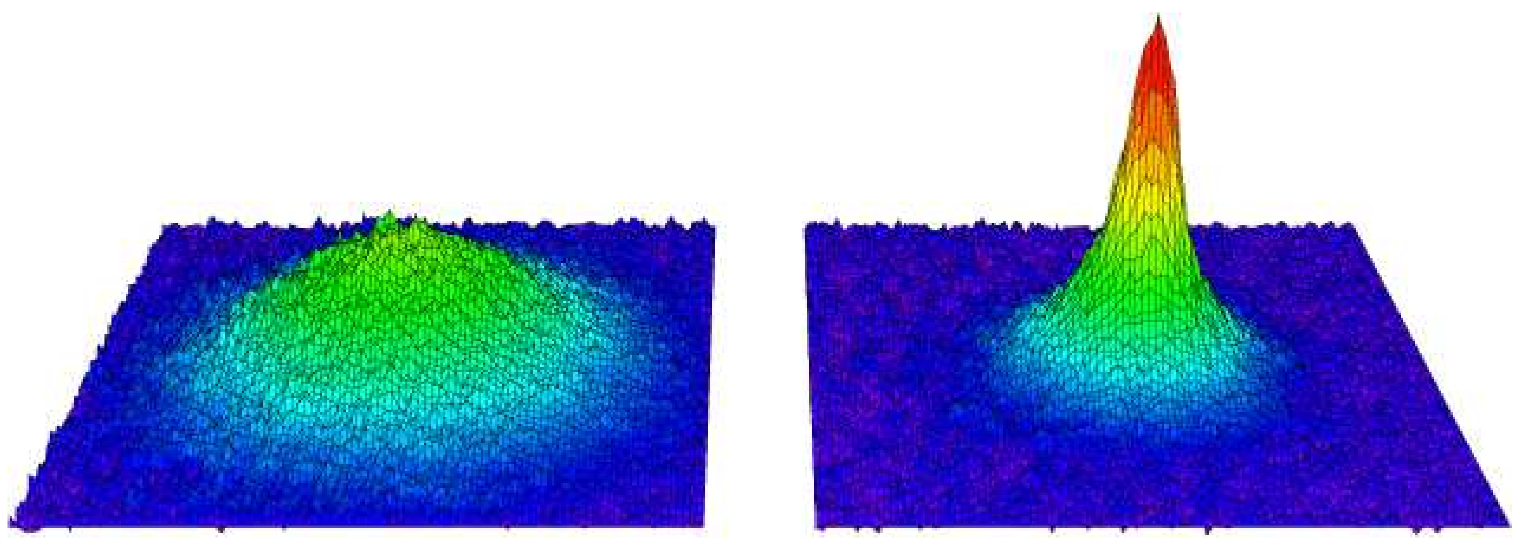
Bose-Einstein condensates of molecules

Suppressed relaxation **Fast elastic collisions** $a_{dd} = 0.6a$

$$^{6}\mathrm{Li}_2 \rightarrow \frac{\alpha_{rel}}{\alpha_{el}} \leq 10^{-4}$$

Efficient evaporative cooling $\rightarrow$ BEC

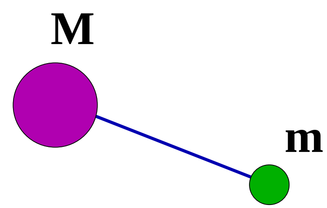
JILA, Innsbruck, MIT, ENS, Rice



Molecules in Fermi mixtures

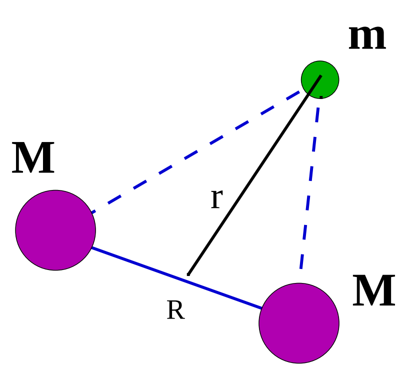
Heavy and light fermions $^{6}\mathrm{Li}^{40}\mathrm{K}$ $^{6}\mathrm{Li}^{87}\mathrm{Sr}$

$a > 0 \Rightarrow$ weakly bound molecules



Relaxation into deep bound states. What else ? $\longrightarrow$ Trimer states ?

$M \gg m \quad \longrightarrow$ Born-Oppenheimer picture



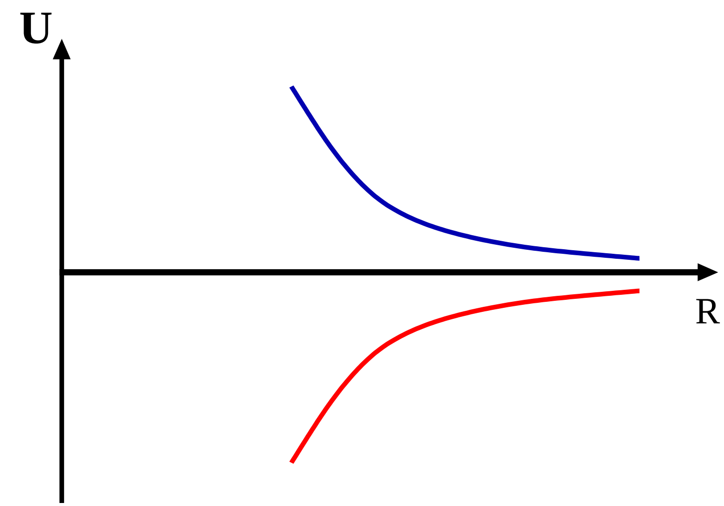
$r \ll a \rightarrow$ One bound state of a light atom with two fixed heavy ones

Mediated attractive potential $U(R) \approx -0.16\hbar^2/mR^2$

Trimer states

Pauli principle $\Rightarrow$ Centrifugal potential $U_c = 2\hbar^2/MR^2$

Mediated attraction competes with Pauli principle



$$U_{eff}(R) = U(R) + U_c(R)$$
$$= -0.16\hbar^2/mR^2 + 2\hbar^2/MR^2$$

$M/m > 13.6 \quad \rightarrow$ fall into center short-range physics

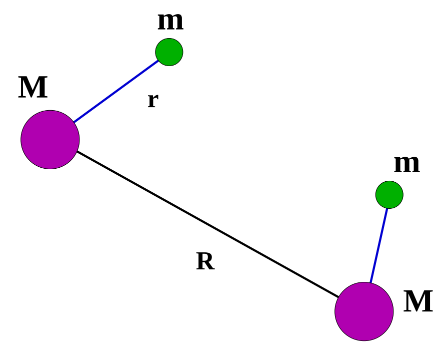
Many nodes of the wavefunction

Many (trimer) bound states

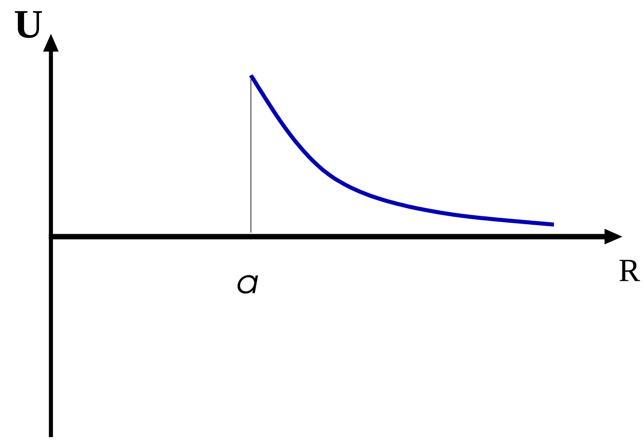
Long-range intermolecular repulsion

Molecules of heavy and light fermions Born-Oppenheimer picture

R $\gg$ a



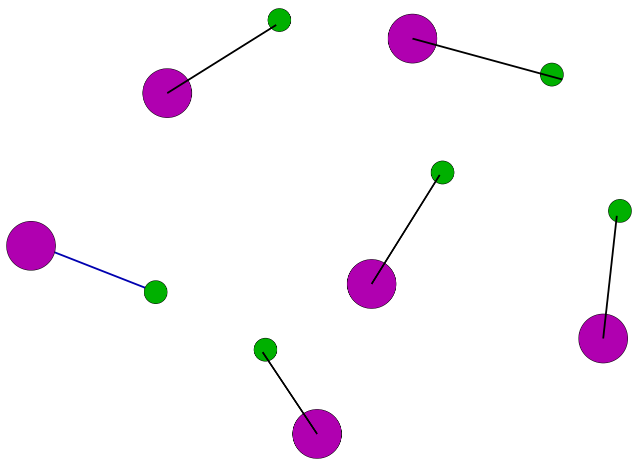
$$U(R) = 2\left(\frac{\hbar^2}{maR}\right)\exp(-2R/a)$$



$$P \sim \exp\left(-0.9\sqrt{\frac{M}{m}}\right)$$

$M >>> m \rightarrow$ Collisional stability independent of a

Many-body system of molecules



No interaction between light fermions

Born-Oppenheimer approach N lowest single-particle states for a light atom

Zero-range approximation for light-heavy interaction. Large inter-heavy distances $\Rightarrow$

Narrow band of N light-atom states, by $\sim \epsilon_0$ below the continuum

Total energy $E = -N\epsilon_0 + (1/2)\sum_{i,j} U(R_{ij})$

$\epsilon_0 = \hbar^2\kappa_0^2/2m \Rightarrow$ molecular binding energy, $\kappa_0^{-1} \rightarrow$ molecular size

$$U_{3D}(R) = 4\epsilon_0[1 - 2(\kappa_0 R)^{-1})]\exp(-2\kappa_0 R); \quad (1/\kappa_0 R)\exp(-\kappa_0 R) \ll 1$$

$$U_{2D}(R) = 4\epsilon_0[\kappa_0 R K_0(\kappa_0 R) K_1(\kappa_0 R) - K_0^2(\kappa_0 R)]; \quad K_0(\kappa_0 R) \ll 1$$

$R \approx 2/\kappa_0$ or larger

Phase diagram

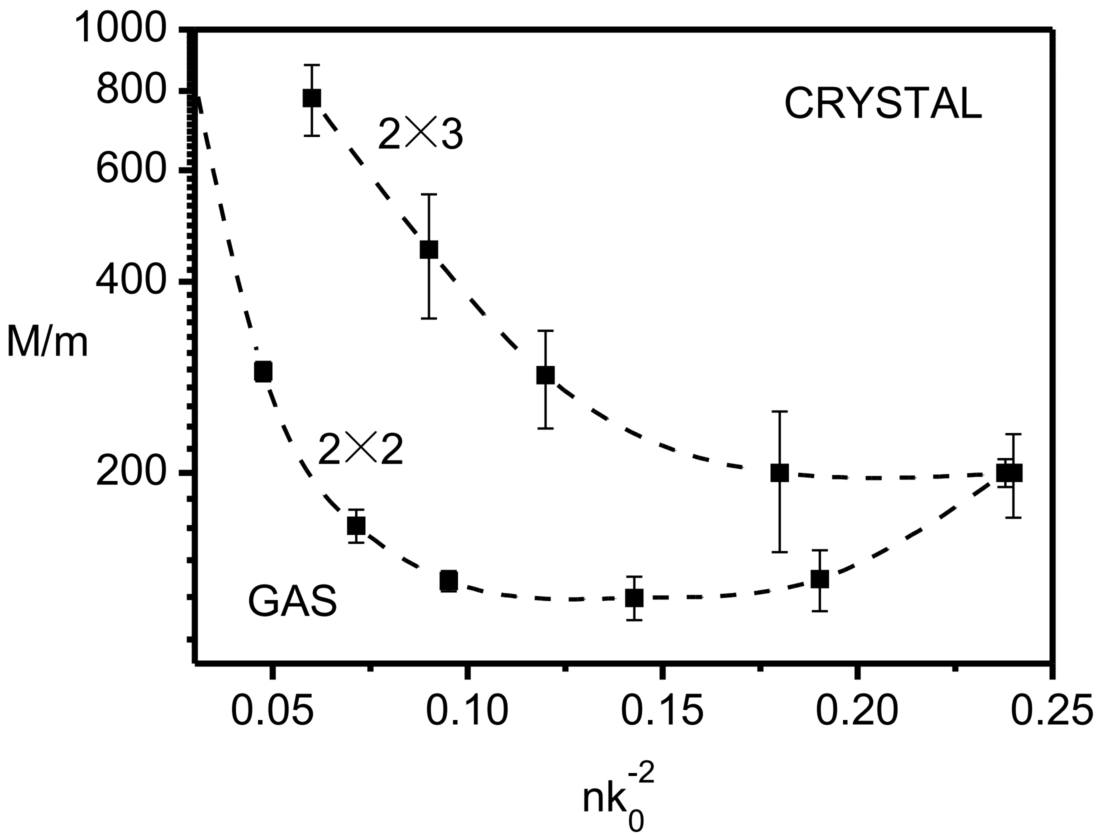
2D motion of heavy atoms

$$H = -(\hbar^2/2M)\sum_i \Delta_{R_i} + (1/2)\sum_{i,j} U(R_{i,j})$$

$(M/m) > (M/m)_c \rightarrow$ crystalline phase

2D motion of light atoms $\Rightarrow$ $(M/m)_c = 120$ triangular lattice

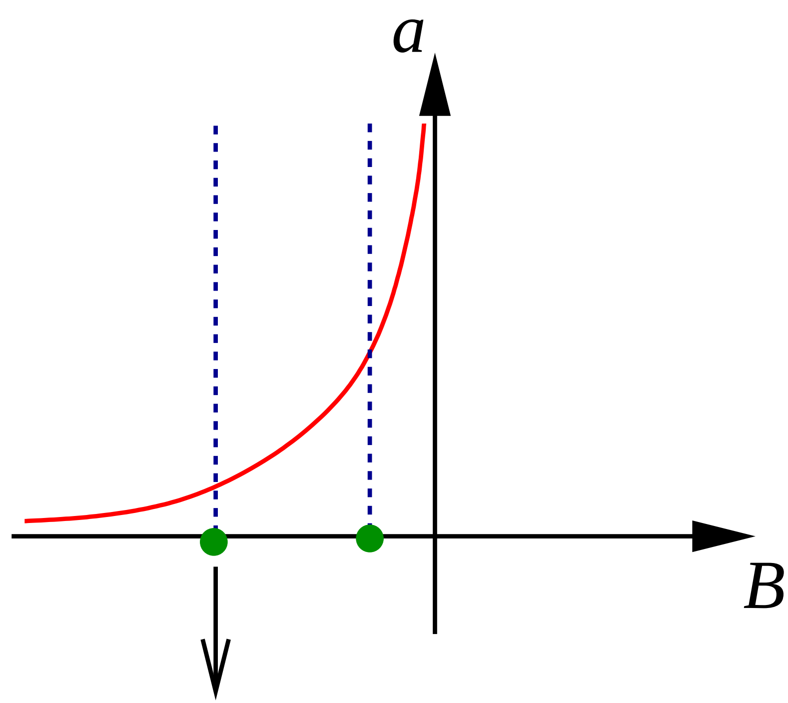
3D motion of light atoms $\Rightarrow$ $(M/m)_c = 200$ triangular lattice



Quantum transitions

$$\frac{M}{m} > \left(\frac{M}{m}\right)_c \quad \text{and } n \text{ fixed}$$

Increase a



depends on $\frac{M}{m}$ but always $na^3 \ll 1$

first-order transition

Realization of the crystalline phase

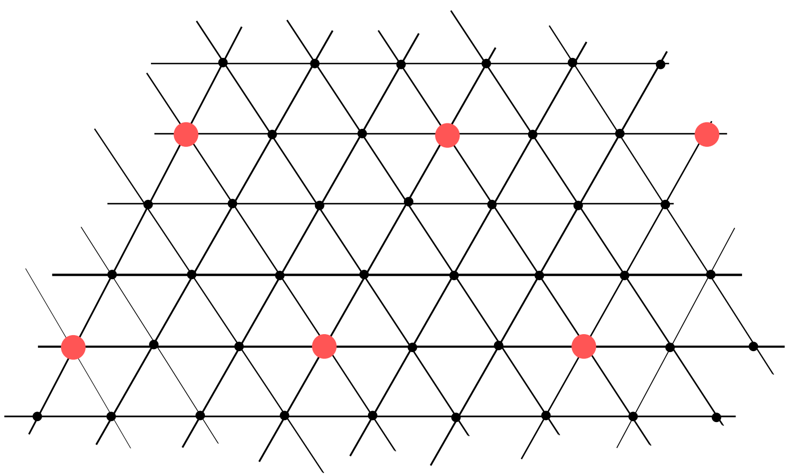
$$\frac{M}{m} \approx 200 \quad \text{or} \quad \frac{M}{m} \approx 200 \quad \rightarrow \text{no gas phase possible}$$

How to obtain the crystalline phase?

Optical lattice for heavy fermions

Small filling factor $\Rightarrow$ Increase of M/m

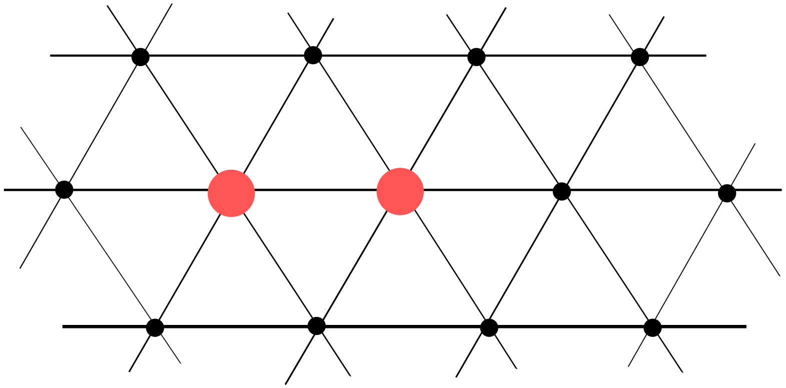
Increase of M by a factor of 20 or more is possible



Formation of a superlattice

Stability of the crystalline phase

Relaxation into deep bound states



heavy atoms in neighboring sites $\Rightarrow P_1 \sim nL^2 \exp(-\sqrt{M_*/m})$

jump to one and the same site $\Rightarrow P_2 \sim (t/U_0)^2; \;\; t = \hbar^2/M_* L^2, \;\; U_0 \sim \hbar\omega_l = \hbar^2/Ml^2$

undergo relaxation process $\Rightarrow \tau_0^{-1} \sim (\hbar/Ml)(1/l^3)$ at worst

Relaxation rate $\tau^{-1} \sim P_1 P_2 \tau_0^{-1} \sim nL^2 (M/M_*)^2 (l/L)^2 (\hbar/M) \exp(-\sqrt{M_*/m})$

τ exceeds 10s even for $n \sim 10^9$cm^{-2}

Formation of trimer states (2 heavy and 1 light atom)

4-body problem in a lattice $\Rightarrow$ τ can range from 0.1 to 100s for $n \sim 10^9$ cm^{-2}

Conclusions

- Remarkable physics of weakly bound molecules in cold Fermi gases
- Novel physics of molecular collisional stability in mixtures of Fermi gases
- Possibilities to create new macroscopic quantum systems