BEC-BCS crossover, phase transitions and phase separation in polarized resonantly-paired superfluids

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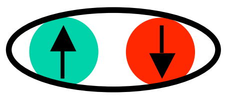
Overview

- Bose-Einstein condensation (BEC) of dilute vapors of alkali atoms
  - All bosons in same quantum state
  - Superfluidity
  - Condensed matter physics in an atomic physics setting

Anderson et al Science 95
Davis et al PRL 95

- Recent interest: condensation of two types of atomic fermion



[e.g., Regal et al PRL 2004; Zwierlein et al PRL 2004,…]

  - Fermionic superfluid

- Relies on strong attraction between fermions: Feshbach resonance
  - Novel experimental knob: Tune interaction strength
  - Crossover from BEC to BCS superfluidity

Bardeen, Cooper, Schrieffer 1957

- Recent Work: Apply spin polarization to fermion superfluid
  - Usual case: Equal numbers of ↑ and ↓
  - Polarization: More ↑ than ↓
  - Exotic phases, phase separation

Next: Fermionic superfluids…

Fermionic pairing of cold atoms

Regal et al PRL 04; Zwierlein et al ibid; Kinast et al ibid, Bartenstein et al ibid, Bourdel et al ibid; Partridge et al ibid 05…

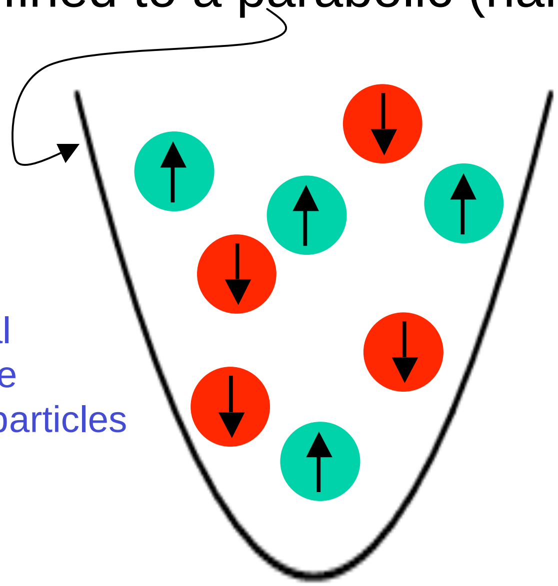
- Fermionic superfluidity: atomic fermions ^{40}K, ^{6}Li

  Ultracold: ~10-100 nK

  Dilute: ~ 10^{10}-10^{13} cm^{-3}

- Confined to a parabolic (harmonic) trap -- typically optical

Atoms attracted to laser beam



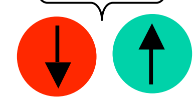
n.b.: actual exp'ts have ~10^5-10^6 particles

– Pairing of two distinguishable fermion species

– "Spin" -- different atomic hyperfine states

(e.g. ^{40}K: f=9/2, m_f = 7/2,9/2)

total atom spin



- Novel feature: Interactions experimentally tunable

  – Feshbach resonance: interactions enhanced by applied magnetic field B

s-wave scattering length

$$a_s \propto -\frac{1}{B - B_0}$$

B_0 "resonance position"

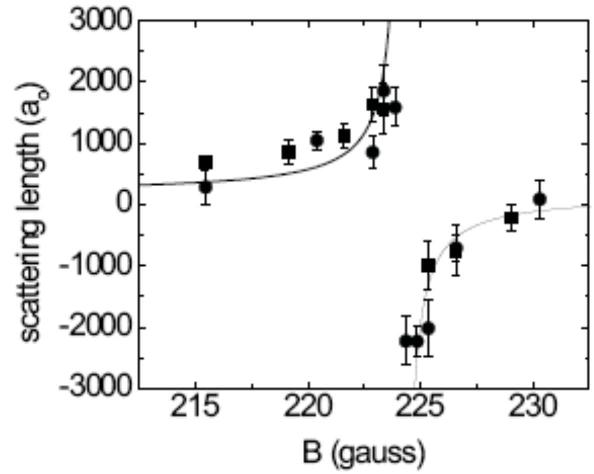
Next: Feshbach resonance

One channel model of Feshbach Resonance

- Near resonance:

fermion scattering length $a_s \propto -\frac{1}{B - B_0}$

e.g.,Regal & Jin PRL 90, 230404 (2003)



- One-channel model:

$$H = \sum_{p,\sigma} (\varepsilon_p - \mu) c^\dagger_{p\sigma} c_{p\sigma} + g \sum_{p,q,k} c^\dagger_{k\uparrow} c^\dagger_{p\downarrow} c_{k+q\downarrow} c_{p-q\uparrow}$$

two species of fermion $(\sigma = \uparrow, \downarrow)$

Strong attractive interactions $g < 0$

- Reproduces correct two-body physics
  - Vacuum scattering length: $a_s \propto \frac{1}{|g| - 2\pi^2 / m\Lambda}$ (Λ UV cutoff)

Holland et al PRL 01
Ohashi & Griffin PRL 02
Andreev et al PRL 04

- Gas at density n: Mean-field theory based on BCS wavefunction
  - Quantitatively valid for weak coupling (small g)
  - Qualitatively valid for any coupling

Next: BEC-BCS crossover

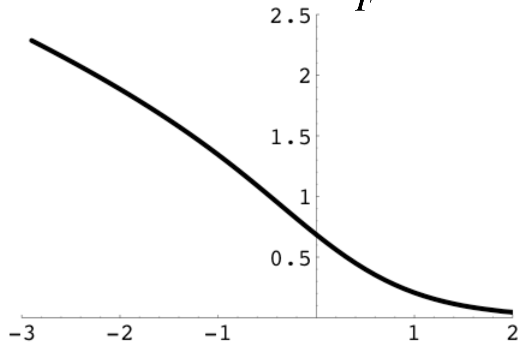
BEC-BCS crossover: Mean-field theory

Leggett 1980
Nozieres & Schmitt Rink 1985
Sa De Melo et al 1993

- Theory: smooth crossover between BEC and BCS limits
  - Assume: Variational BCS ground state characterized by the pairing $\Delta = \langle c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} \rangle$
  - Minimize variational ground-state energy

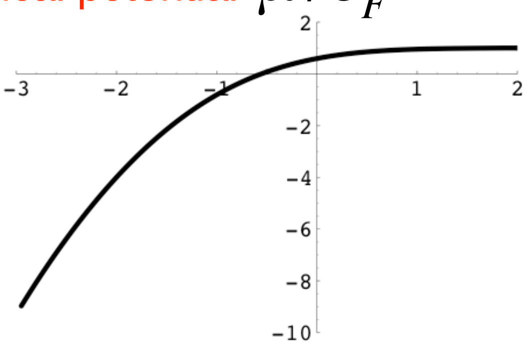
← BEC   BCS →

Pairing Δ / ε_F

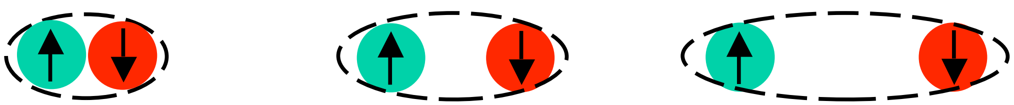


$-\frac{1}{k_F a_s} \propto B - B_0$

Chemical potential μ / ε_F



$-\frac{1}{k_F a_s}$



Positive detuning: BCS regime $(a_s < 0)$

– Weakly attractive interactions

$$\left.\begin{matrix} \mu > 0 \\ \Delta << \varepsilon_F \end{matrix}\right\} \Rightarrow \text{Neutral BCS superconductor!!}$$

Cooper Pair size >> Interparticle spacing

Negative detuning: BEC regime $(a_s > 0)$

– Strong attractive interactions

$$\left.\begin{matrix} \mu < 0 \\ n_m \propto |\Delta|^2 \approx n/2 \end{matrix}\right\} \Rightarrow \text{Tightly bound Molecular BEC}$$

Molecule size << Interparticle spacing

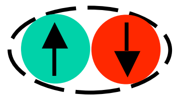
Next: Validity

Validity of BEC-BCS mean-field theory

- Quantitatively valid at large positive detuning (BCS) regime $a_s < 0,\ |a_s| << k_F^{-1}$
  - Fluct's around mean field theory grow with reduced detuning

$k_F \propto n^{1/3}$

- Asymptotic negative detuning (BEC) regime $a_s > 0,\ |a_s| << k_F^{-1}$
  - Gas of repulsive bosons



BCS wavef'n: $a_m = 2a_s$

Born approx. result

Exact analysis: $a_m = 0.6a_s$

Petrov et al PRL 04; Levinsen & Gurarie PRA 06

  - Confirms qualitative validity (and quantitative invalidity) of BCS w.f.

- Unitary regime: $|a_s| >> k_F^{-1}$ Universal: Only energy scale is $\varepsilon_F = \dfrac{k_F^2}{2m}$
  - No small parameter
  - Monte Carlo
  - Introduce artificial small parameter

Narrow resonance - Andreev et al PRL 2004

Epsilon expansion (spatial dimension) - Nishida & Son PRL 2006

N species of fermions: Large N limit - M.Y. Veillette, D.S., L. Radzihovsky 2007

also Nikolic & Sachdev 2007

Next: Exp'ts

Experiments: Smooth crossover & superfluidity

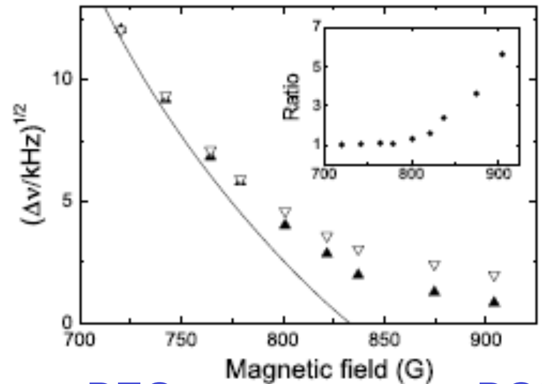
- Measure condensation directly: Occupation of lowest state

Regal et al PRL 2004
Zwierlein et al PRL 2004

- Measure binding energy of pairs/molecules

Increases with reduced detuning

Chin et al Science 2004
Partridge et al PRL 2005



BEC BCS

- Collective oscillations: consistent with superfluidity

Indirect measure of superfluidity

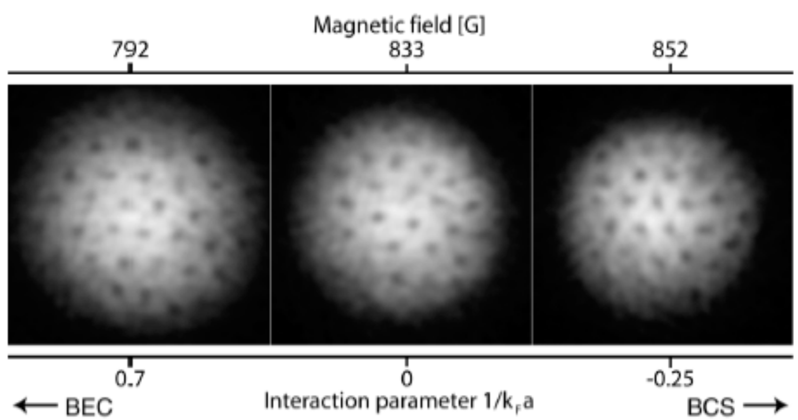
Kinast et al PRL 2004

- Rotation of cloud: Vortices across resonance

Zwierlein et al Nature 2005

Direct measure of superfluidity

Vortices in a Bose-Einstein condensate



Vortices in a neutral BCS superconductor

Next: Spin Polarization

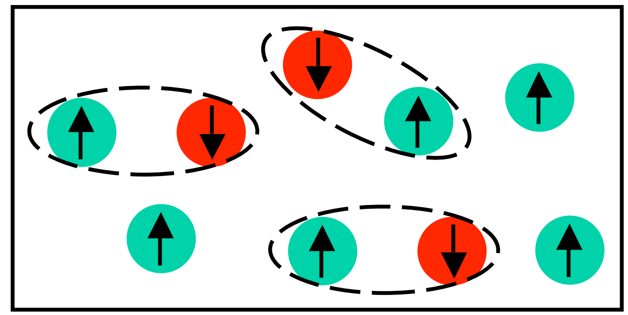
Applied spin polarization

- Recent work*: Explore changing relative number of $\downarrow$, $\uparrow$

Hard to do in condensed-matter settings!!

  – Additional experimental “knob” for cold-atom experiments

Polarization: $$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$



Aim: Extend phase diagram to polarized case $P \neq 0$

  – Analogous to applying a Zeeman magnetic field that favors more $\uparrow$ than $\downarrow$
    - “Pauli limiting” (Clogston limit) magnetic field in superconductors
    - FFLO state: Exotic inhomogeneous superconductor $\Delta(\mathbf{r}) \propto \cos[\mathbf{Q}\cdot\mathbf{r}]$
  – Strongly-interacting fermions with density imbalance
    - crystalline color superconductivity Alford et al PRD 01, Liu & Wilczek PRL 03
  – Smooth crossover fractured into rich phase diagram

Phase transitions, polarized superfluidity, polarized Fermi liquid, phase separation…

*Theory: DS & Leo Radzihovsky, PRL 06, Ann. Phys. 07, PRB 07,
Bedaque et al PRL 03, Carlson & Reddy PRL 05, Cohen PRL 05
Pao et al PRB 06, Son & Stephanov PRA 06, Chien et al PRL 06,
Parish et al Nat. Phys. 07,….

Exp’t: Zwierlein et al Science 06,
Partridge et al Science 06,…..

Next: Model

Model of a polarized superfluid

Model: $H = \sum_{p,\sigma} (\varepsilon_p - \mu_\sigma) c^\dagger_{p\sigma} c_{p\sigma} + g \sum_{p,q,k} c^\dagger_{k\uparrow} c^\dagger_{p\downarrow} c_{k+q\downarrow} c_{p-q\uparrow}$

– Different chemical potentials ➡ $h = \mu_\uparrow - \mu_\downarrow$ induce (polarization) P

- BCS superconductor under applied Zeeman field h: First-order transition

Clogston PRL 1962; Sarma J. Phys. Chem. Sol. 1963; Bedaque et al PRL 03

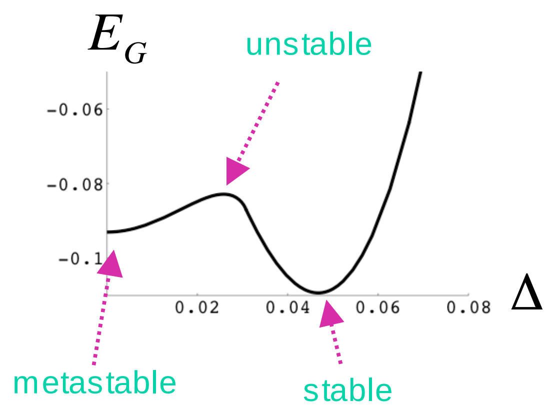
- Phenomenology of first-order transitions

Variational ground-state energy $E_G(\Delta) = \langle \Psi | H | \Psi \rangle$

– Phase separation, metastable, unstable solutions

– Cannot detect transitions locally

local criteria determines spinodals



- Must globally minimize ground-state energy (or free energy) to correctly obtain phase diagram

Sarma 1963: Solutions to gap equation that do not minimize E_G

$$0 = \frac{dE_G}{d\Delta}$$

Next: Global phase diagram

Global phase diagram

T=0, Uniform system

$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$

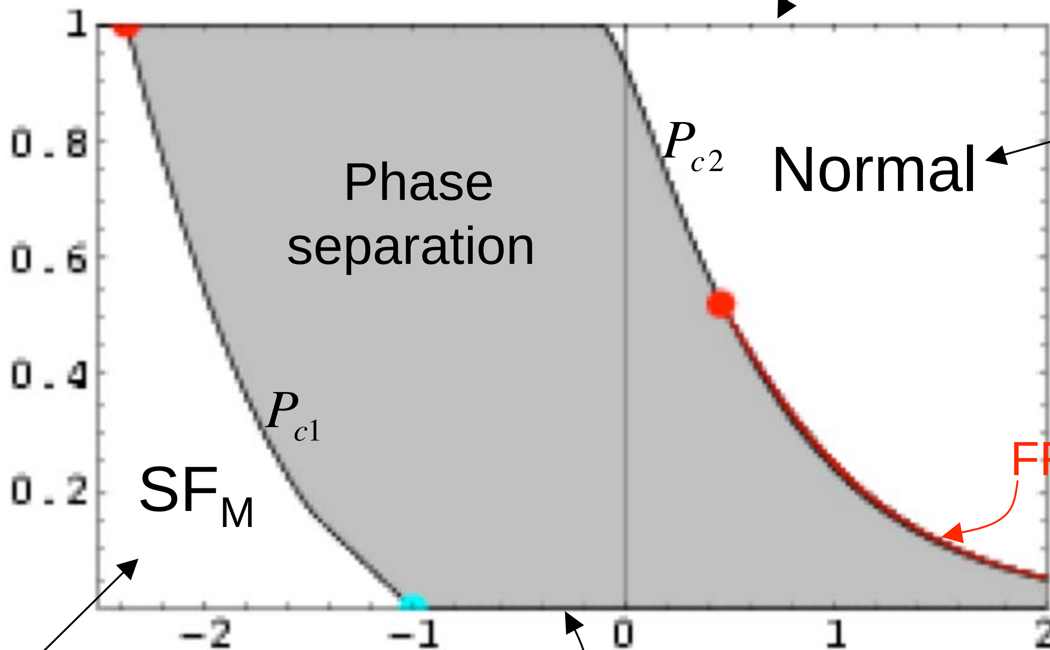
Top axis: $N_\downarrow = 0$

(Pure Normal)

P_{c1}, P_{c2} 1st order

$FFLO - N$ 2nd

Polarized Fermi gas



FFLO

$-\frac{1}{k_F a_s} \propto B - B_0$

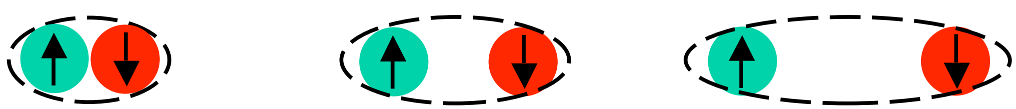
(detuning)

"Magnetic superfluid": Tightly-bound molecules and spin-polarized Fermi sea

BEC limit

BCS limit

Bottom axis: Smooth BEC-BCS crossover phase



See also: Gu et al cond-mat 06, Parish et al 07

Next: BCS regime

BCS regime zero-T phase diagram

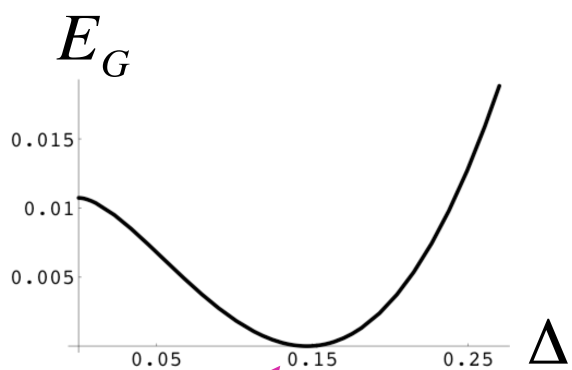
- Grand-canonical ensemble:
  - Impose $\mu_\uparrow, \mu_\downarrow$, minimize $E_G(\Delta) = \langle \Psi | H | \Psi \rangle$
  - Adjust $\mu_\uparrow, \mu_\downarrow$ to achieve correct $N_\uparrow, N_\downarrow$ where $N_\sigma = \langle \Psi | \hat{N}_\sigma | \Psi \rangle$
  - Certain regimes: cannot achive correct $N_\uparrow, N_\downarrow$

…Maxwell construction

True solution is phase separated in such regimes

- Positive-detuning BCS regime:

$h = 0:$

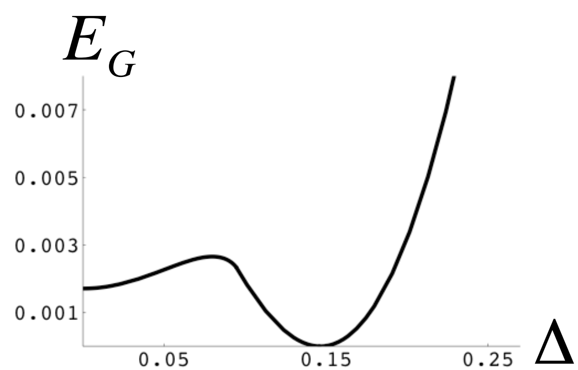


- Minimum at:

$$\Delta_0 \cong 8e^{-2}\mu \exp[\frac{\pi}{2k_F a_s}]$$

- Usual BCS pairing

$h \neq 0:$

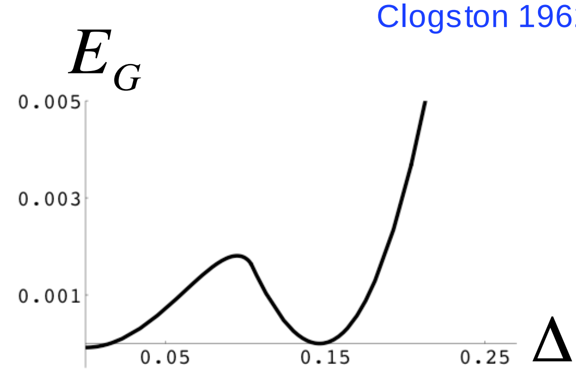


- Second local minimum @ $\Delta = 0$
- Global min. unaltered

BCS won't polarize! Sarma 1963

$h = h_c \cong \Delta_0 / \sqrt{2}:$

Clogston 1962

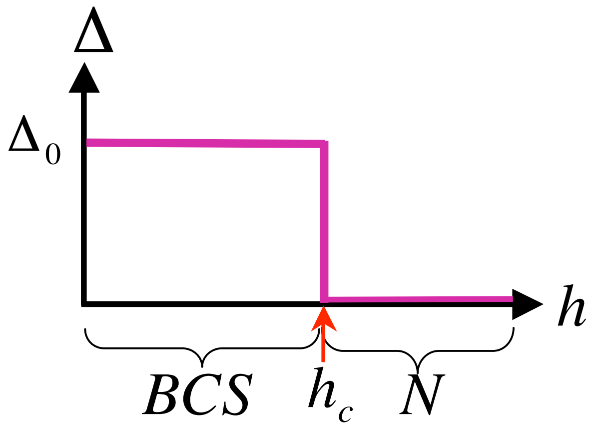


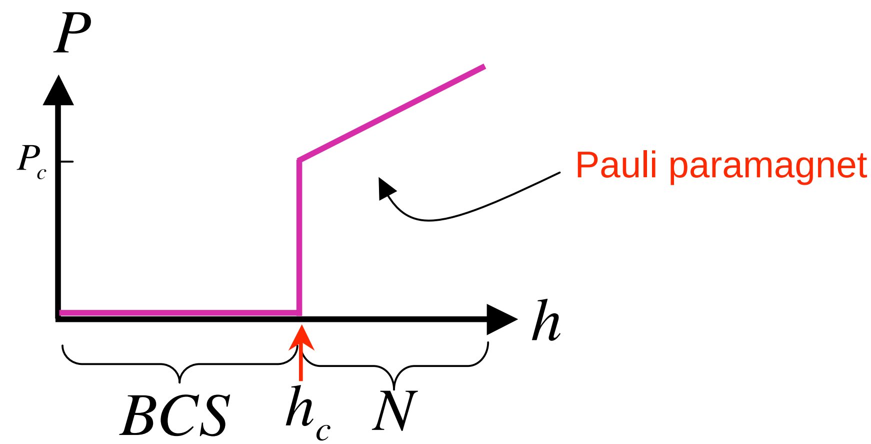
- Transition to state at $\Delta = 0$
- Unpaired N state stable!

Next: 1st-order transition

First-order BCS-to-N transition

- Polarization (P) and pairing (Δ) jump at first-order BCS-N transition





- Polarization $P<P_c$: cannot be attained for a homogeneous phase
- Phase separation to achieve imposed polarization

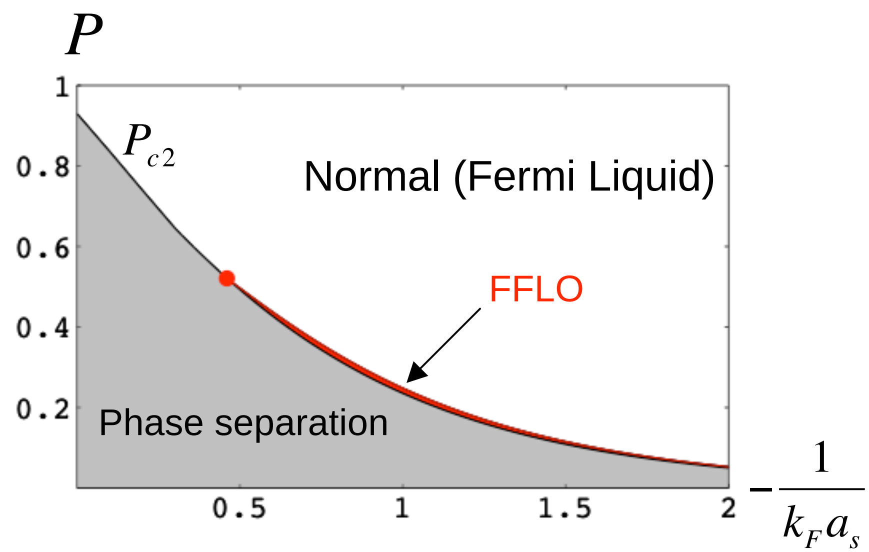
BCS	Normal
$n_\uparrow = n_\downarrow$	$n_\uparrow > n_\downarrow$

BCS regime phase diagram:

- Paired BCS phase: $P = 0$
- Phase sep. for arbitrarily small P
- Below solid line: N phase unstable via first-order transition

$$P_{c2} \propto \Delta_0$$

- Thin window of inhomogeneous FFLO state (FFLO-N continous trans.)

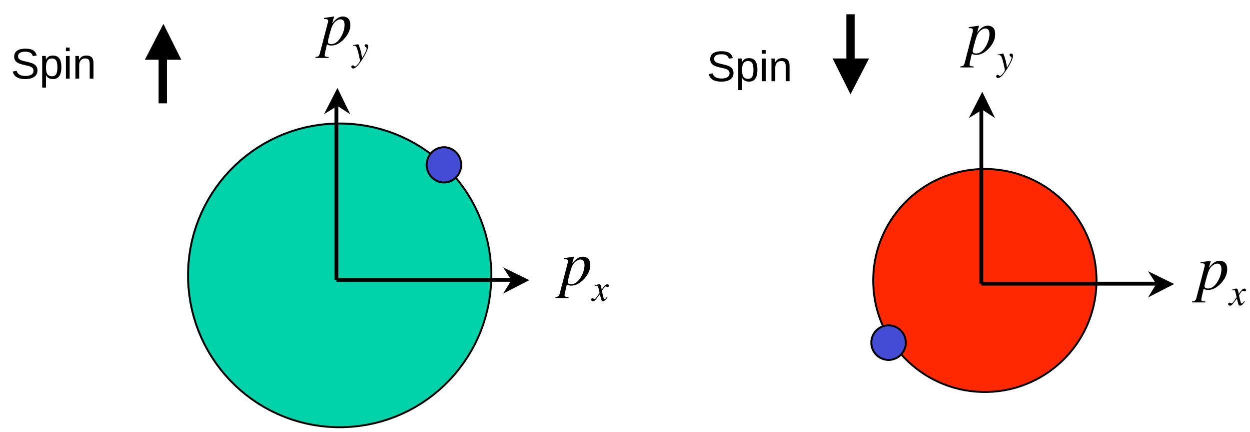


Next: FFLO

FFLO state

Fulde & Ferrell PR 1964;
Larkin & Ovchinnikov JETP 1965

- Excess spin ↑: Larger Fermi surface $p_{F\uparrow} \propto n_\uparrow^{1/3},\ p_{F\downarrow} \propto n_\downarrow^{1/3}$



- Pairing of low-energy states near Fermi surface: $Q \equiv p_{F\uparrow} - p_{F\downarrow}$

Cooper pairs
have finite momentum!

$$\Delta(\mathbf{r}) \propto \cos[\mathbf{Q} \cdot \mathbf{r}]$$

  – Breaks rotational and translational symmetry

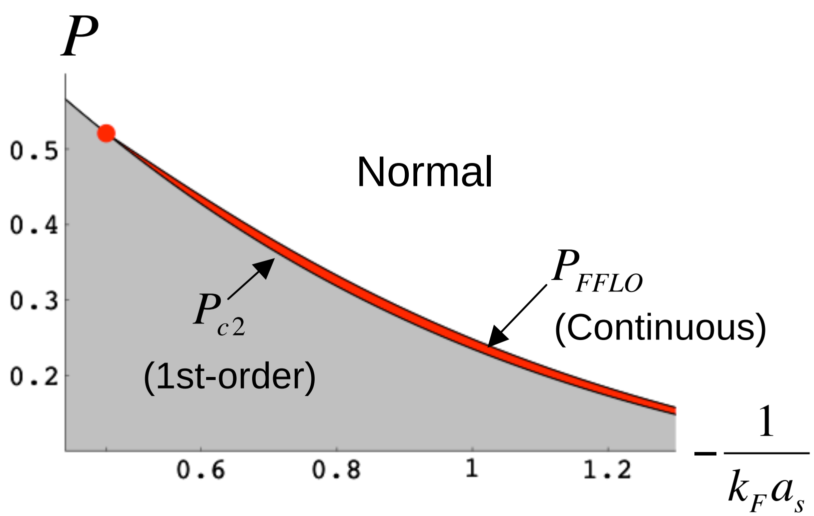
- Evaded observation in condensed-matter systems
  – Disorder
  – Coupling of physical magnetic field to orbital electron motion
  – Possibly observed in $CeCoIn_5$

Radovan et al Nature 2003
Bianchi et al PRL 2003

- Motivation: Observe FFLO in cold-atom experiment?
  – Perfectly clean; Purely Zeeman coupling
  – Spontaneous crystalline order observable in time-of-flight exp'ts

Predictions for FFLO regime

- Simplest FFLO-type state: $\Delta(\mathbf{r}) = \Delta_{\mathbf{Q}} \exp[i\mathbf{Q}\cdot\mathbf{r}]$
  - More generally: $\Delta(\mathbf{r}) = \sum_{\mathbf{Q}} \Delta_{\mathbf{Q}} \exp[i\mathbf{Q}\cdot\mathbf{r}]$ Bowers & Rajagopal PRD 02



- $P_{c2} < P_{FFLO}$ are $\sim \exp\left[-\frac{c}{k_F|a_s|}\right]$
- Large detuning: $\frac{P_{FFLO}}{P_{c2}} \cong 1.07$
- P_{FFLO} crosses P_{c2} at $-\frac{1}{k_F a_s} \approx 0.5$

  FFLO no longer globally stable

- Critical polarization $P_{FFLO} \approx \frac{3}{2}\eta\frac{\Delta}{\varepsilon_F}$

  weakly detuning-dependent

- FFLO wavevector: $Q \cong 2\eta\lambda\frac{\Delta}{\hbar v_F}$

Estimate using exp't parameters

Chin et al Science 2004

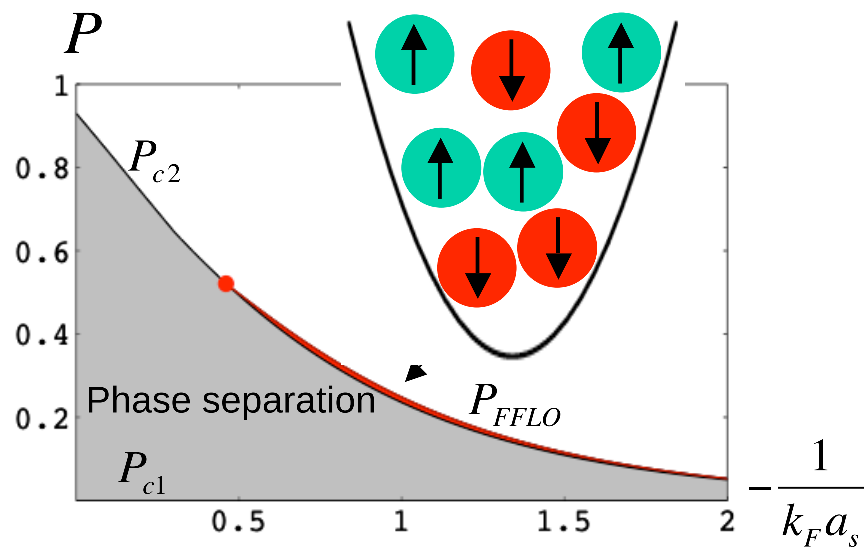
$P_{FFLO} \cong .05$

$Q^{-1} \cong 5\mu m$

- Phase separation: SF-FFLO coexistence underneath P_{FFLO}

  still observable in time of flight

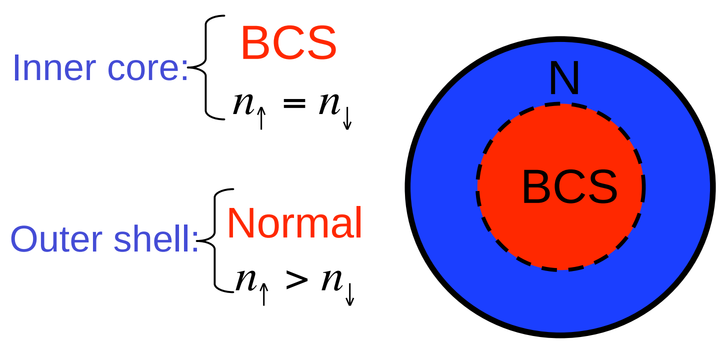
Mean-field theory predictions: Positive detuning



- $P_{c1}=0$: Phase Separation for any small P
  - Polarization above which system phase sep.
- $P_{c2}=0.93$ at unitarity
  - Polarization below which system phase sep.

$\mu_N = \mu_{BCS}$

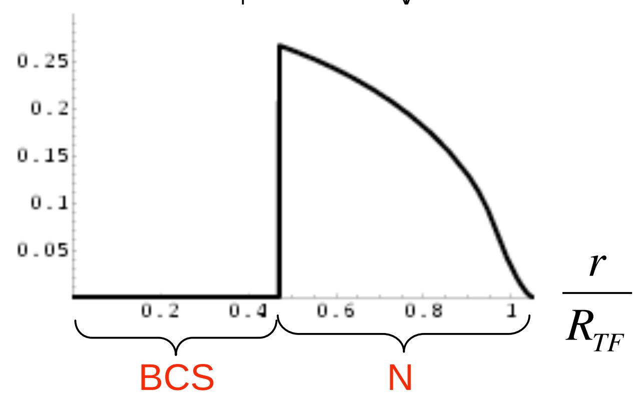
- Phase separated regime: Two phases in chemical equilibrium with different densities
  - Harmonic trap: higher density phase (superfluid) falls to the center



Calculation: Local density approx.

"Magnetization"

$$M(r) = n_\uparrow(r) - n_\downarrow(r)$$

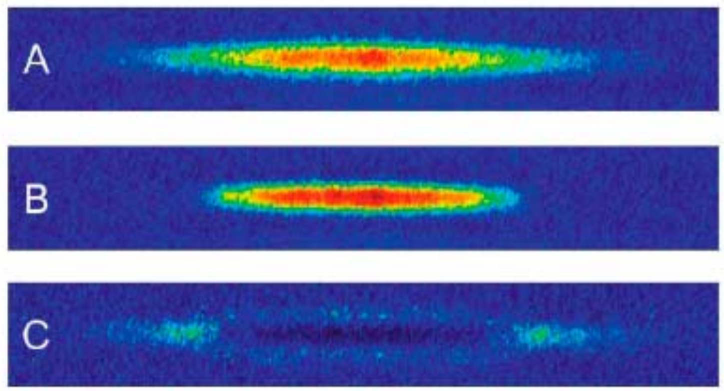


Shell structure: Imposed polarization goes to the edge, center paired!

Evidence for shell structure in phase separation regime

- Partridge et al Science 2006: Density data
  - Integrated in one direction
  - Highly prolate trapping potential

$$N_\uparrow = 8.6 \times 10^4$$
$$N_\downarrow = 6.5 \times 10^4$$



$n_\uparrow(r)$

$n_\downarrow(r)$

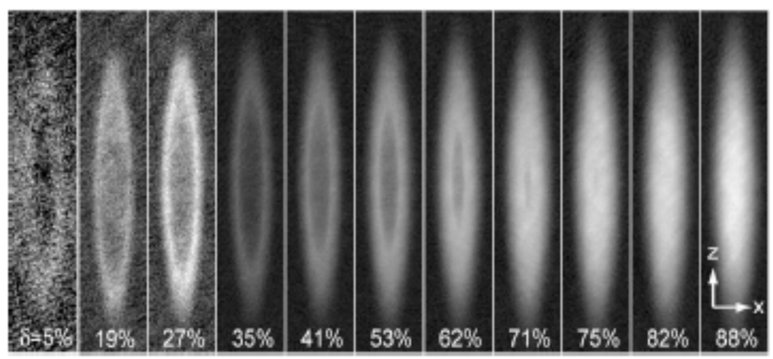
$M(r)$

Excess polarization pushed to boundary!

- Shin et al PRL 2006: Shrinking BCS core with increasing polarization

Plots: Integrated Magnetization

$$M(r) = n_\uparrow(r) - n_\downarrow(r)$$



$\approx P_{c2}$

- Quantitative understanding: Go beyond local density approximation to handle trap

Kinnunen et al, Yi & Duan, Chevy, De Silva & Mueller, Imambekov et al, …

Next: BEC regime

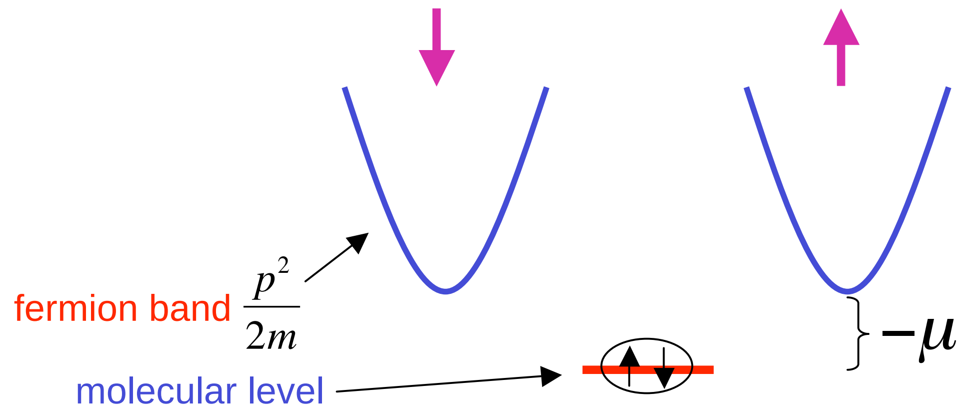
BEC superfluid under applied h

- $h = 0$: BEC superfluid
  - Paired molecular bosons
  - Fermions gapped: $\mu < 0$

$$\mu = -\frac{\hbar^2}{2ma_s^2}$$

Mol. binding energy

  - vacuum of fermions; BEC of pairs



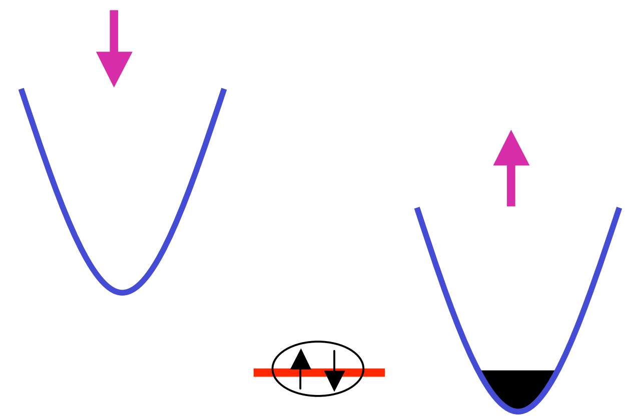
- Apply h to induce polarization P

$$\mu_\uparrow = \mu + h$$
$$\mu_\downarrow = \mu - h$$

Tilt Fermion bands!

  - Upper fermion band dips below mol. level
  - Pairs break up into spin-↑ fermions
  - Coherent mixture of BEC and spin-↑ Fermi gas

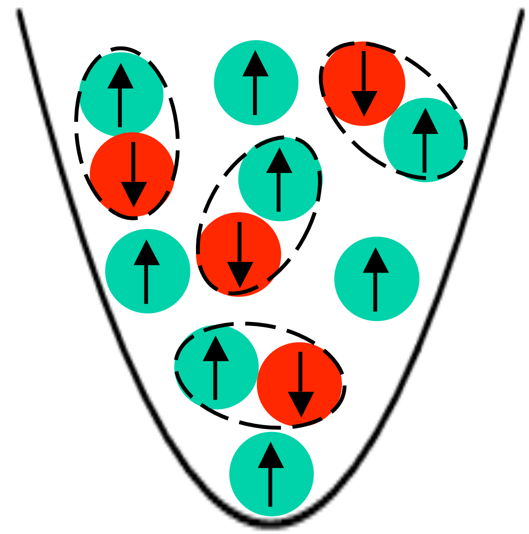
Magnetic superfluid (SF_M)



Next: SF_M

Magnetic superfluid (SF_M) phase

- Negative detuning: BEC tolerates small polarization!
  - Unlike BCS regime
  - Minority spins pair; excess majority form Fermi sea



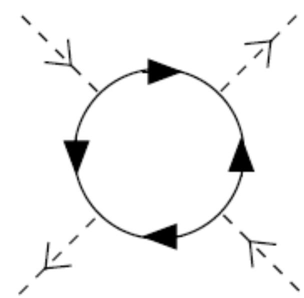
- Spin-up fermion & BEC are miscible fluids
  - Analogous to ^{3}He-^{4}He mixtures!

fermion boson

- First-order transition to phase sep. with increasing P
  - Compute molecular scattering length vs. h

$$a_m(h) = 2a_s F(h/|\mu|)$$

$$F(0) = 1$$



  - $F(x)$ vanishes at $x = 1.30$

- Stability requires $a_m > 0$
  - Instability at: $h_{c1} \approx 1.30 \frac{\hbar^2}{2ma_s^2}$

determined more accurately by minimizing E_G

BEC regime phase diagram

- SFM phase: First-order transition into regime of phase separation

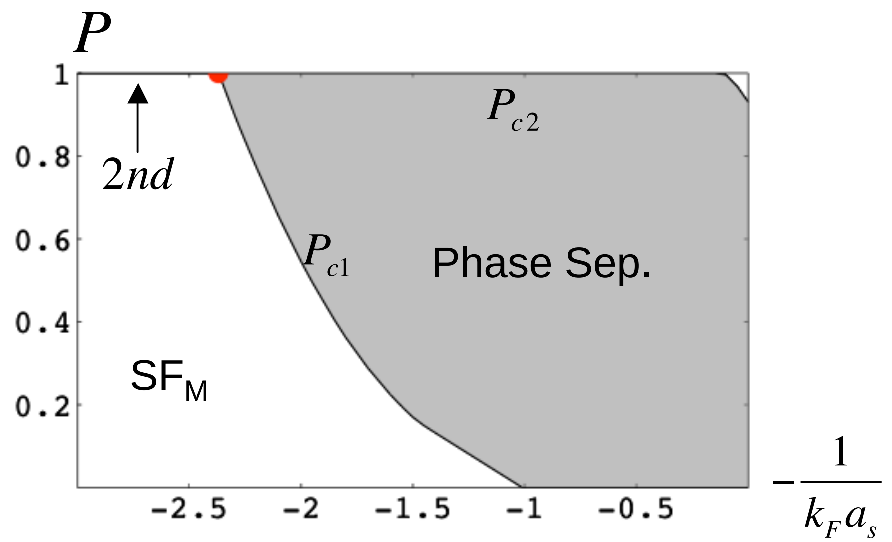
– P_{c1} : first-order trans. to phase sep.

– Deep BEC: 2nd-order trans. to fully polarized

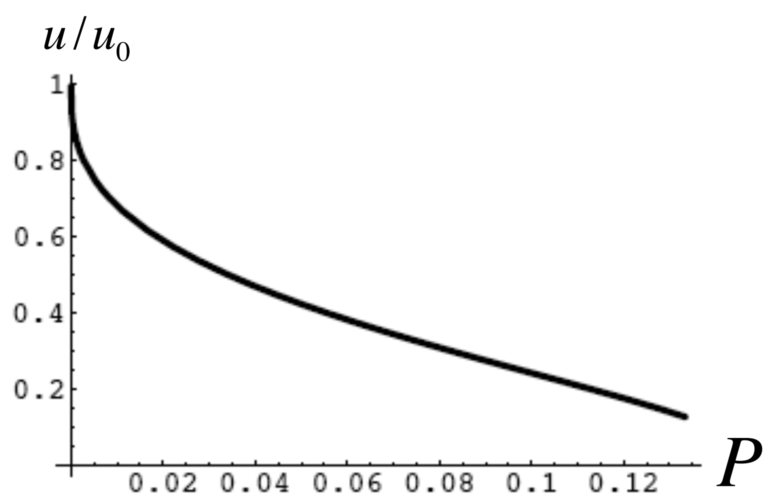
– Red dot: tricritical point

– $P_{c2} = 1$ except close to unitarity

Add one spin $\downarrow$ to Fermi sea of $\uparrow$: Forms Pair



- Bogoliubov sound velocity in SF_M: Driven to zero near transition at P_{c1}



– Relation between a_m and sound vel.

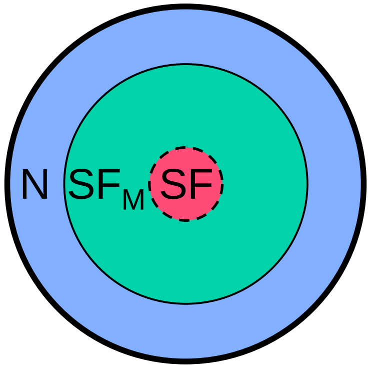
– Transition to phase sep. precedes vanishing

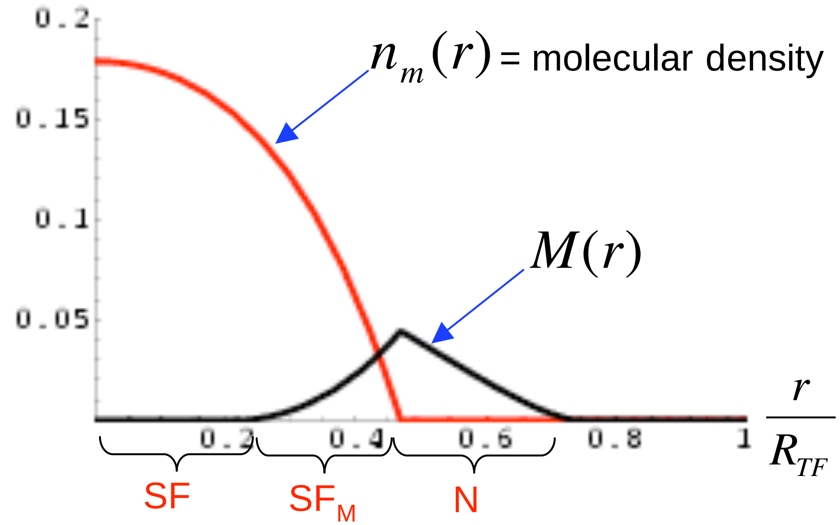
Next: LDA in BEC regime

Magnetic superfluid in a trap: LDA

Small P : Interior SF_M shell

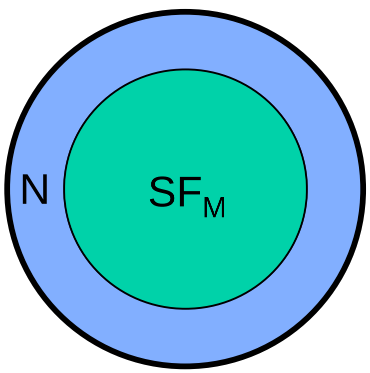
Three shells:

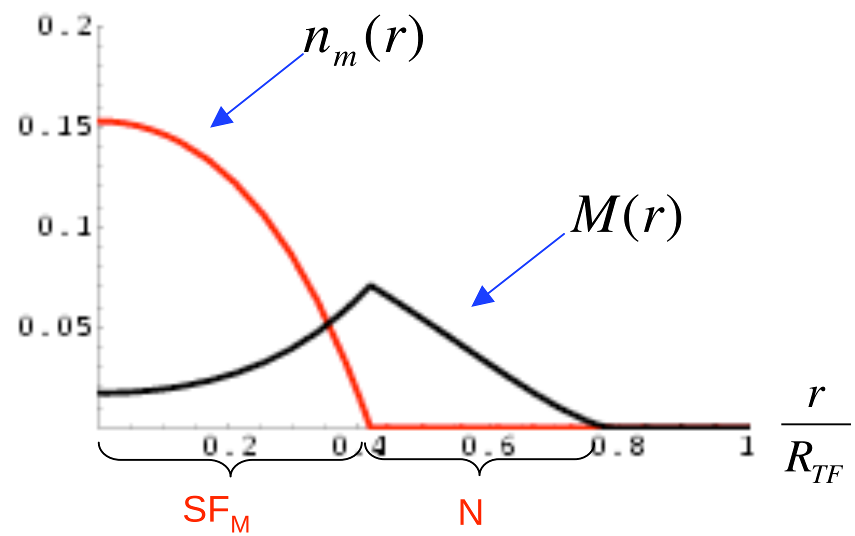




Large P : SF_M surrounded by Normal

Two shells:





Global phase diagram

D.S. & L. Radzihovsky, PRL 2006, Ann. of Phys. 2007, PRB 2007

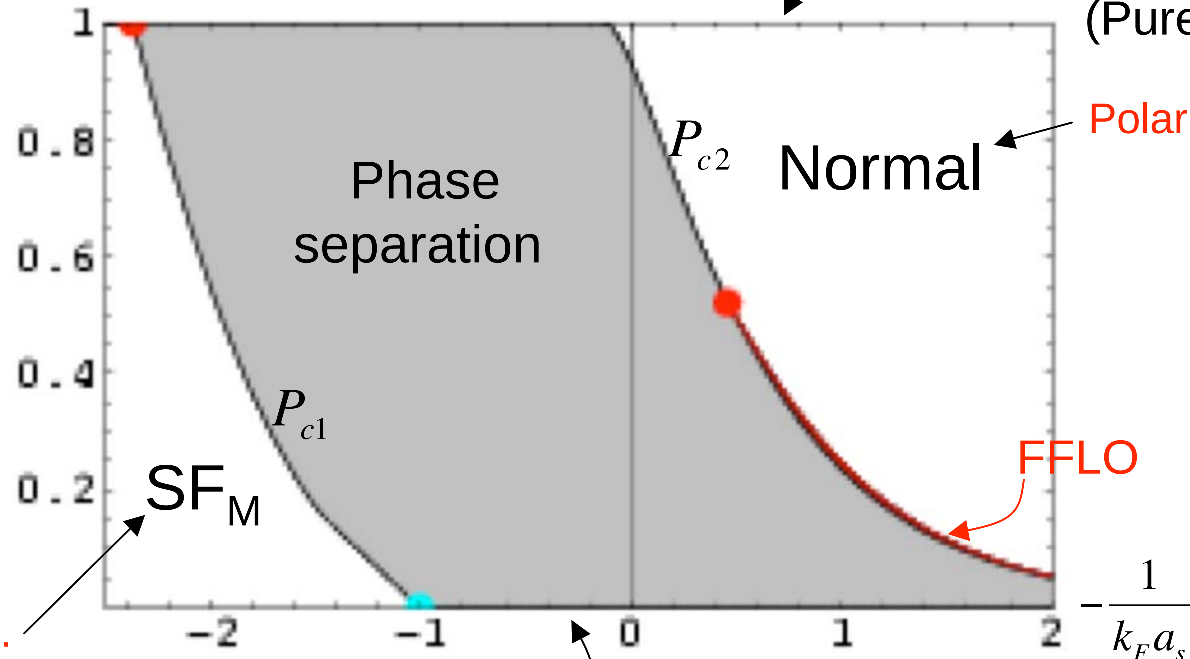
$$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$

P_{c1}, P_{c2} 1st order

$FFLO - N$ 2nd

Top axis: $N_\downarrow = 0$ (Pure Normal)

Polarized Fermi gas



(detuning)

"Magnetic superfluid": Tightly-bound molecules and spin-polarized Fermi sea

BEC limit

BCS limit

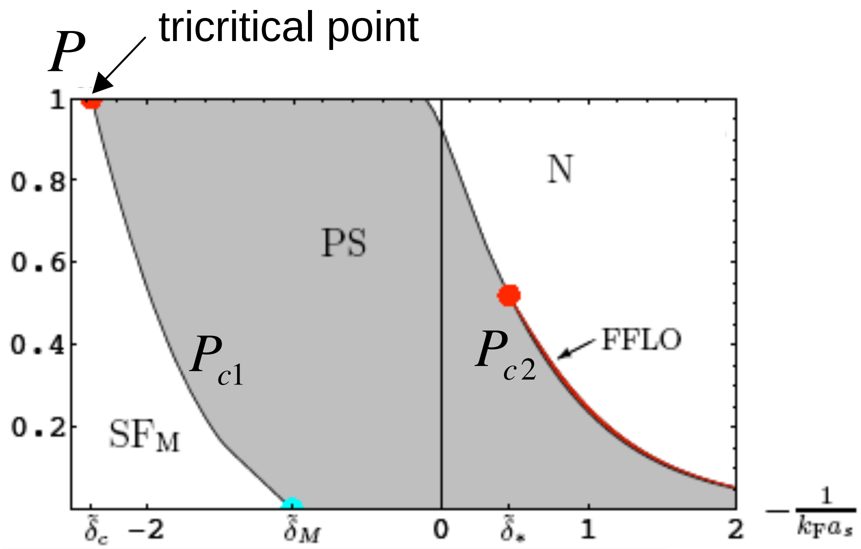
Bottom axis: Smooth BEC-BCS crossover phase

- Recent other work: Finite T: Parish et al, Chien et al…
  Including trap: Kinnunen et al, Duan et al, …
  Dynamics of phase separation: Lamacraft et al

Next: Tricritical

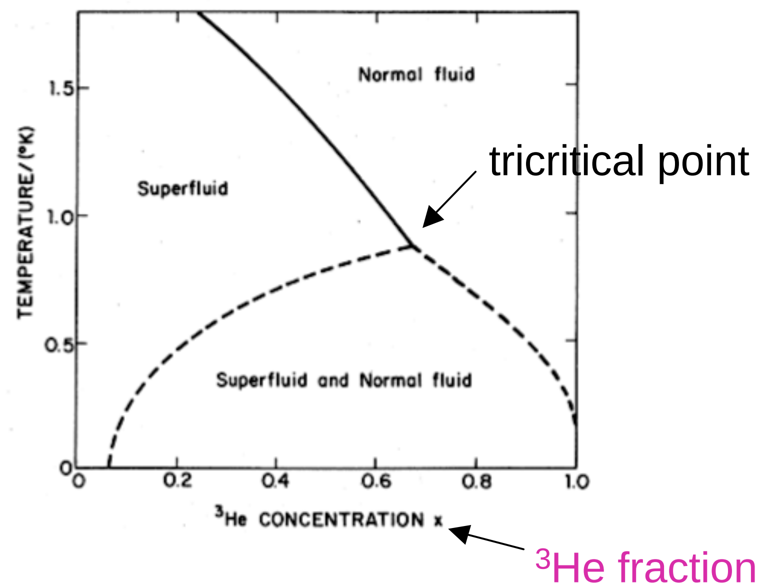
Tricritical point at nonzero T

Extension to finite T: (Parish et al, Chien et al)

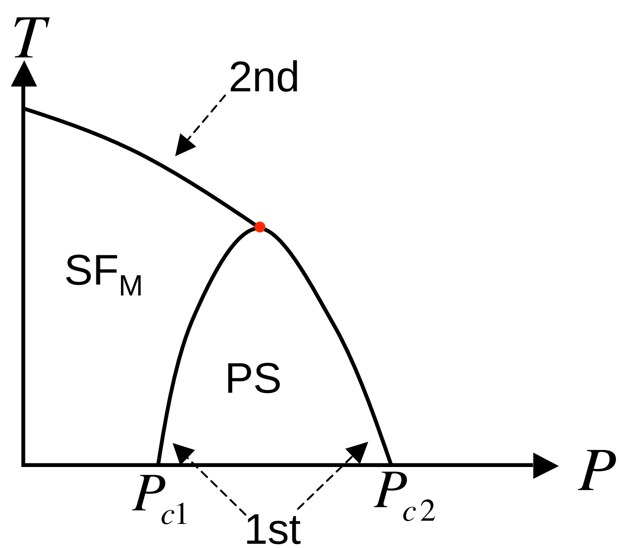


^{3}He-^{4}He mixtures

Graf, Lee, Reppy 67, et al.



By analogy:



Other possibilities:

Blume Emery Griffiths 70

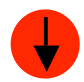
Concluding remarks

- Cold-atom experiments studying superconductivity of paired fermions

New context for strongly-correlated condensed-matter physics

- Other examples of interacting fermion systems: Superconductors, quark matter, nuclear matter, …

- Experiments already observed crossover between BEC and BCS states

- Different numbers of  ,  : Simple crossover “fractured”

  – Phase transitions
  – Phase separation
  – Magnetic superfluidity
  – Fulde-Ferrell-Larkin-Ovchinnikov states

- Future work:
  – Finite-T phase diagram near unitary point
  – Vicinity of the tricritical point
  – Strongly coupled normal state (recent MIT experiments)
  – Experimental signatures of FFLO and SF_M