Bose gas in Flatland

Berezinskii-Kosterlitz-Thouless Physics in an Atomic Gas

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CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE

1530
Collège de France

Outline of the talk

Bose gases in 2D

Berezinskii-Kosterlitz-Thouless transition

Homogeneous vs. trapped & ideal vs. interacting gas

Critical point of an interacting 2D gas

BEC vs. BKT

Vortices and quasi-long-range coherence

Long-range order in reduced dimensionality

more vulnerable to fluctuations, disorder…

c.f. classical transport:

1D - impossible

2D - marginal

3D - easy

BEC, coherence, and superfluidity in 2D

Homogeneous 2D Bose fluid in the thermodynamic limit

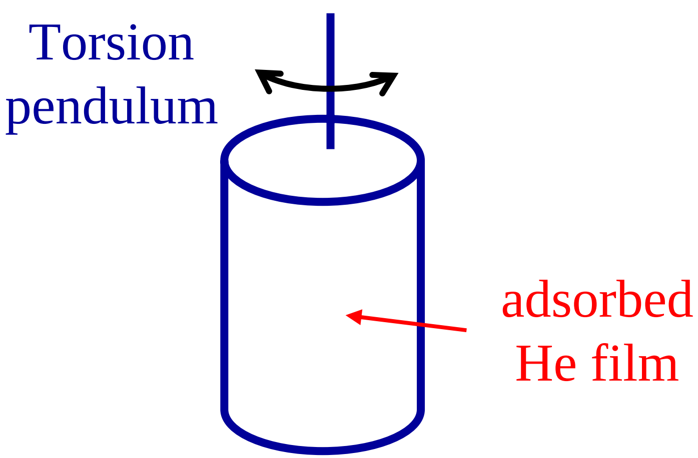
No BEC in an ideal gas

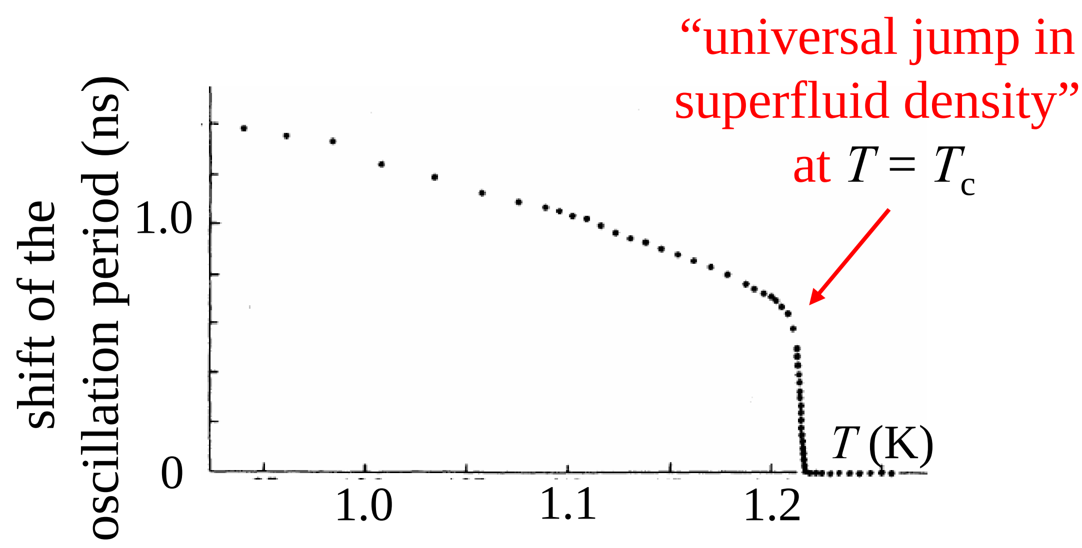
No true long-range order in an interacting gas at finite T

(Mermin-Wagner-Hohenberg theorem)

But still a superfluid transition at finite T

Bishop and Reppy (1978), superfluidity in liquid He films :





Berezinskii & Kosterlitz – Thouless (1971-73)

Phase transition without spontaneous symmetry breaking

0 — *superfluid* — T_c — *normal* — T

algebraic decay of g_1 | *exponential decay of g_1*

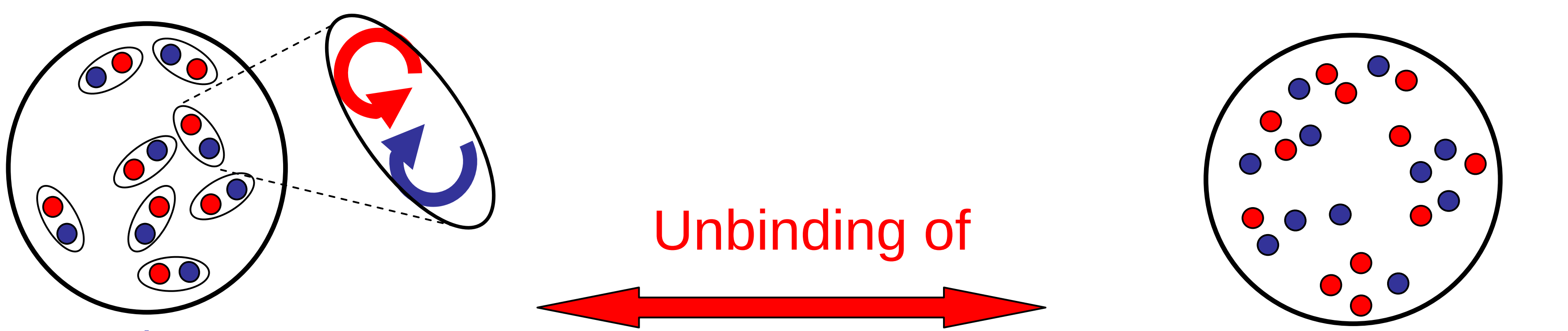
$$g_1(r) \propto \frac{1}{r^\alpha}$$

$$n_{\text{superfluid}}\lambda^2 = 4$$

(λ – thermal wavelength)

$$\alpha = \frac{1}{n_s \lambda^2}$$

$$g_1(|\vec{r} - \vec{r}\,'|) = \langle \psi^*(\vec{r})\psi(\vec{r}\,') \rangle$$



Bound vortex-antivortex pairs

Proliferation of free vortices

(Ideal gas) In a harmonic trap…

Homogeneous system:

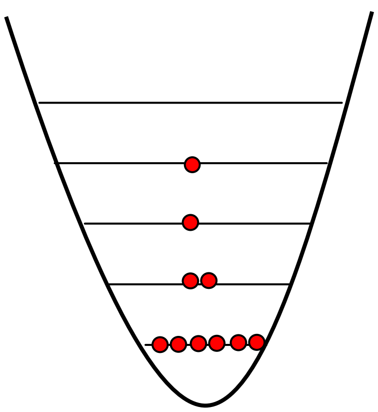
3D: BEC occurs when the phase space density reaches $n\lambda^3 = 2.6$

2D: no BEC for any phase space density $n\lambda^2$

In a harmonic trap:

3D: BEC occurs when $N = 1.2 \left(\frac{k_B T}{\hbar\omega}\right)^3$

2D: BEC occurs when $N = 1.6 \left(\frac{k_B T}{\hbar\omega}\right)^2$



Does harmonic trapping make 2D boring?

What about interactions?

The effect of (weak) interactions on BEC

3D harmonic trap:

Repulsive interactions slightly decrease the central density, for given N and T

For an ideal gas, the central density at condensation point is:

$$N = 1.2 \left(\frac{k_B T}{\hbar\omega}\right)^3 \quad \Longleftrightarrow \quad n(0)\lambda^3 = 2.6$$

(semi-classical)

Just put in a bit more atoms to obtain the needed $n(0)$

2D harmonic trap:

The same procedure completely fails:

$$N = 1.6 \left(\frac{k_B T}{\hbar\omega}\right)^2 \quad \Longleftrightarrow \quad n(0)\lambda^2 = \infty$$

where $n(r) = \int \rho(r,p) \frac{d^2p}{h^2}$ $\qquad \rho(r,p) = \left[e^{\beta\left(\frac{p^2}{2m} + \frac{m\omega^2 r^2}{2}\right)} - 1\right]^{-1}$

The effect of (weak) interactions on BEC

Treat the interactions at the mean field level: $V_{\text{eff}}(r) = \frac{m\omega^2 r^2}{2} + 2g n_{\text{mf}}(r)$

where the mean field density is obtained from the self-consistent equation

$$n_{\text{mf}}(r) = \int \rho_{\text{mf}}(r,p) \frac{d^2p}{h^2} \qquad \rho_{\text{mf}}(r,p) = \left[e^{\beta(\frac{p^2}{2m} + V_{\text{eff}}(r))} - 1 \right]^{-1}$$

Two remarkable results

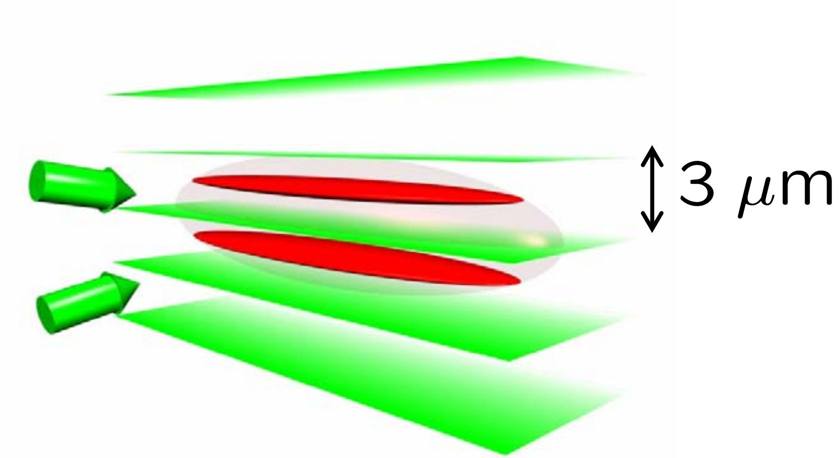
- One can accommodate an arbitrarily large atom number. Badhuri et al
- The effective frequency deduced from $V_{\text{eff}}(r) \simeq m\omega_{\text{eff}}^2 r^2/2$ tends to zero when $\mu \to 2g n_{\text{mf}}(0)$ Holzmann et al.

Similar to a 2D gas in a flat potential…

…BEC suppressed, expect BKT (?)

How to make an ultracold 2D Bose gas

3D BEC + 1D optical lattice



2 independent 2D clouds
(no tunnelling)

10^5 atoms/plane
plane thickness: 0.2 μm, separation: 3 μm

(other 2D experiments at MIT, Innsbruck, Oxford, Florence, NIST, Heidelberg etc.)

Why 2 planes?

Crucial info in the *phase* of Ψ, and $g_1(x,y) = \langle \psi^*(x,y)\,\psi(0) \rangle$

→ accessible in an interference experiment

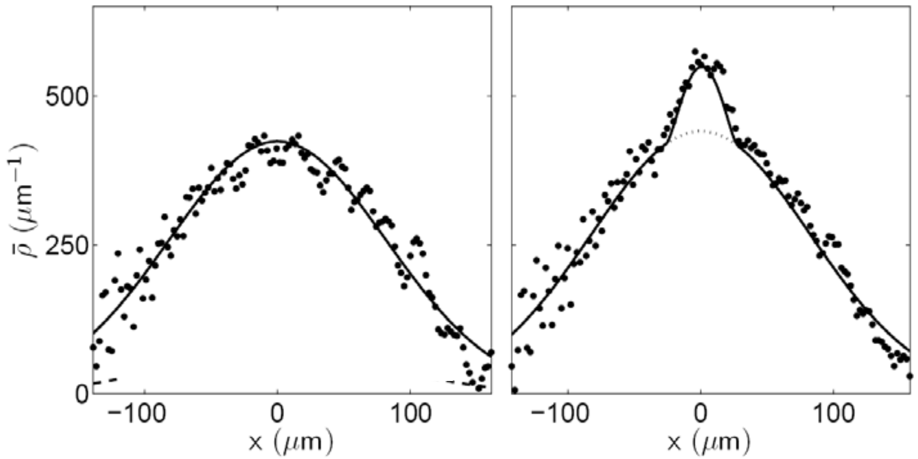
2.
Critical point of an interacting 2D Bose gas

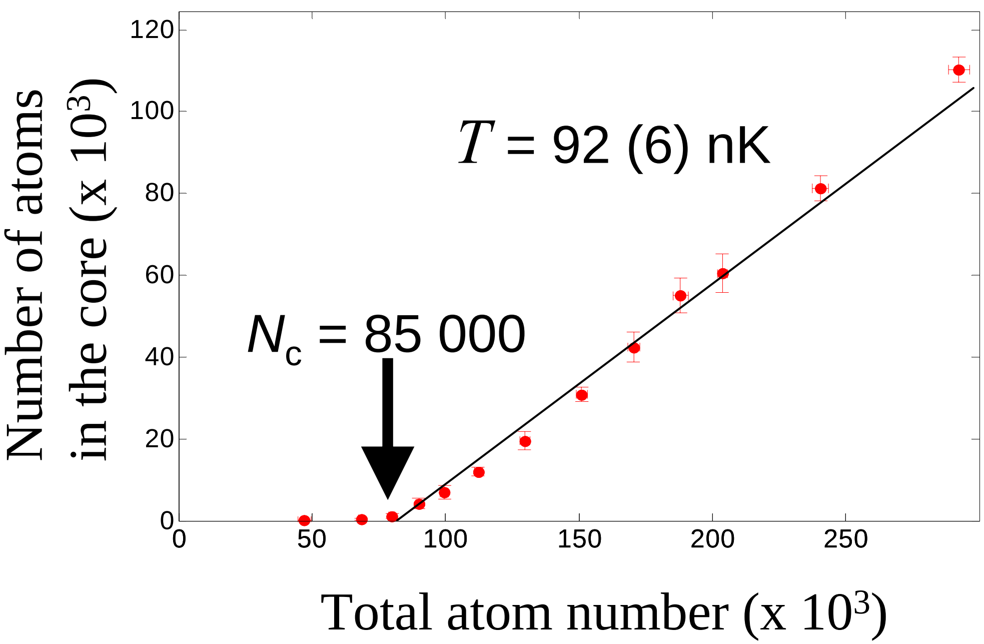
P. Krüger, Z. H. and J. Dalibard, cond-mat/0703200

Phase transition in a 2D atomic gas

Fix the temperature T
Vary the atom number N

Bimodal distribution
for $N > N_C$

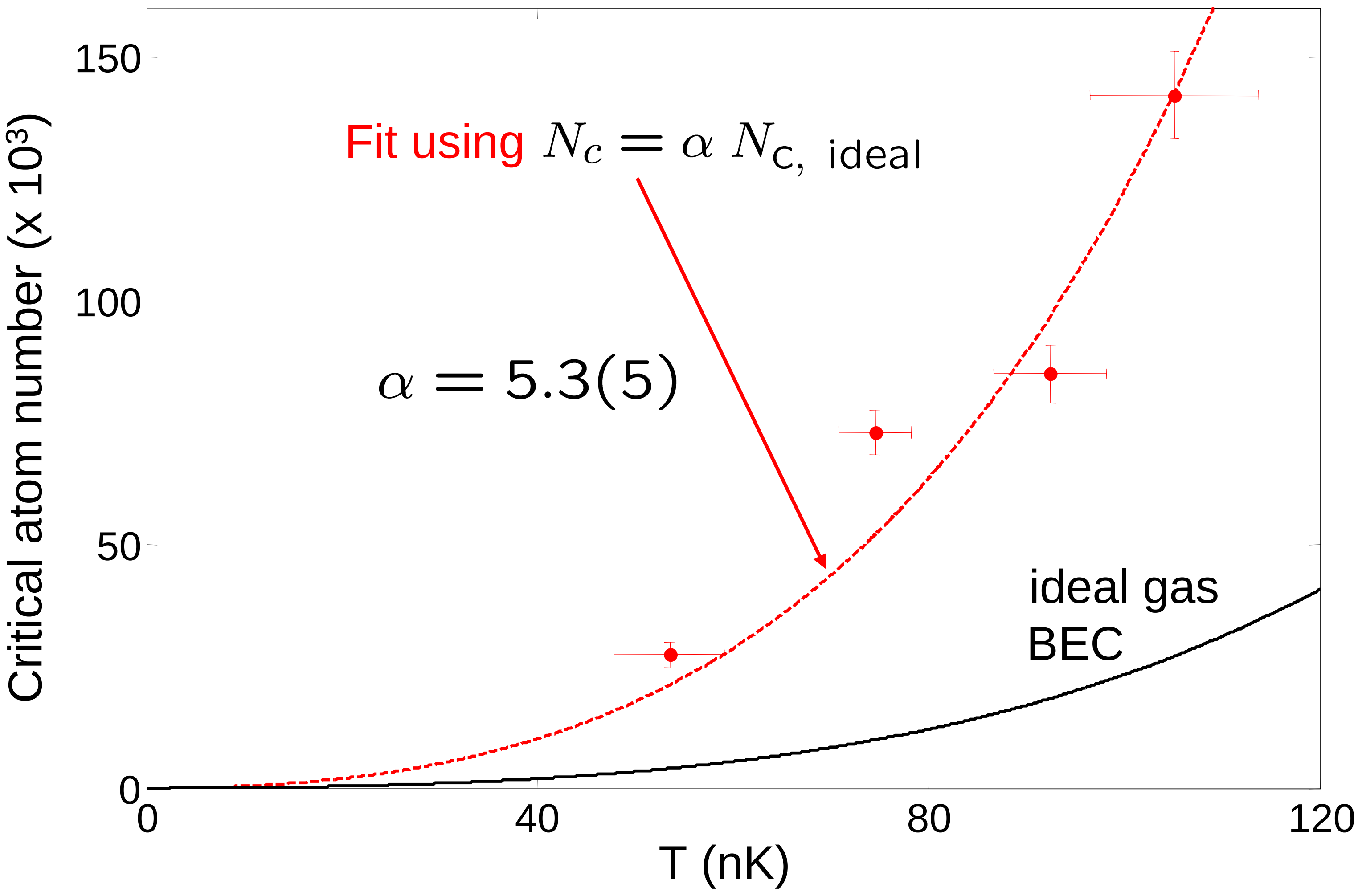




Similar signature to 3D BEC

Dense core follows the
Thomas-Fermi law
in time-of-flight expansion,
characteristic of
superfluid hydrodynamics

Critical atom number vs. *T*



5.3 times larger than the ideal gas BEC prediction!

Can it be the Kosterlitz-Thouless critical point?

$n_{\text{superfluid}}\lambda^2 = 4$ is universal and elegant, but not the whole story

Total critical density depends on microscopics (long standing problem!)

Fisher & Hohenberg + Prokof'ev et al.:

$$n_{\text{total}}\lambda^2 = \ln\left(\frac{C}{\bar{g}}\right) \qquad C = 380 \pm 3$$

$$\bar{g} = \frac{mg}{\hbar^2}$$ dimensionless interaction strength

For our setup: $\bar{g} = 0.13$ ➡ $n_{\text{total}}\lambda^2 \simeq 8.0$

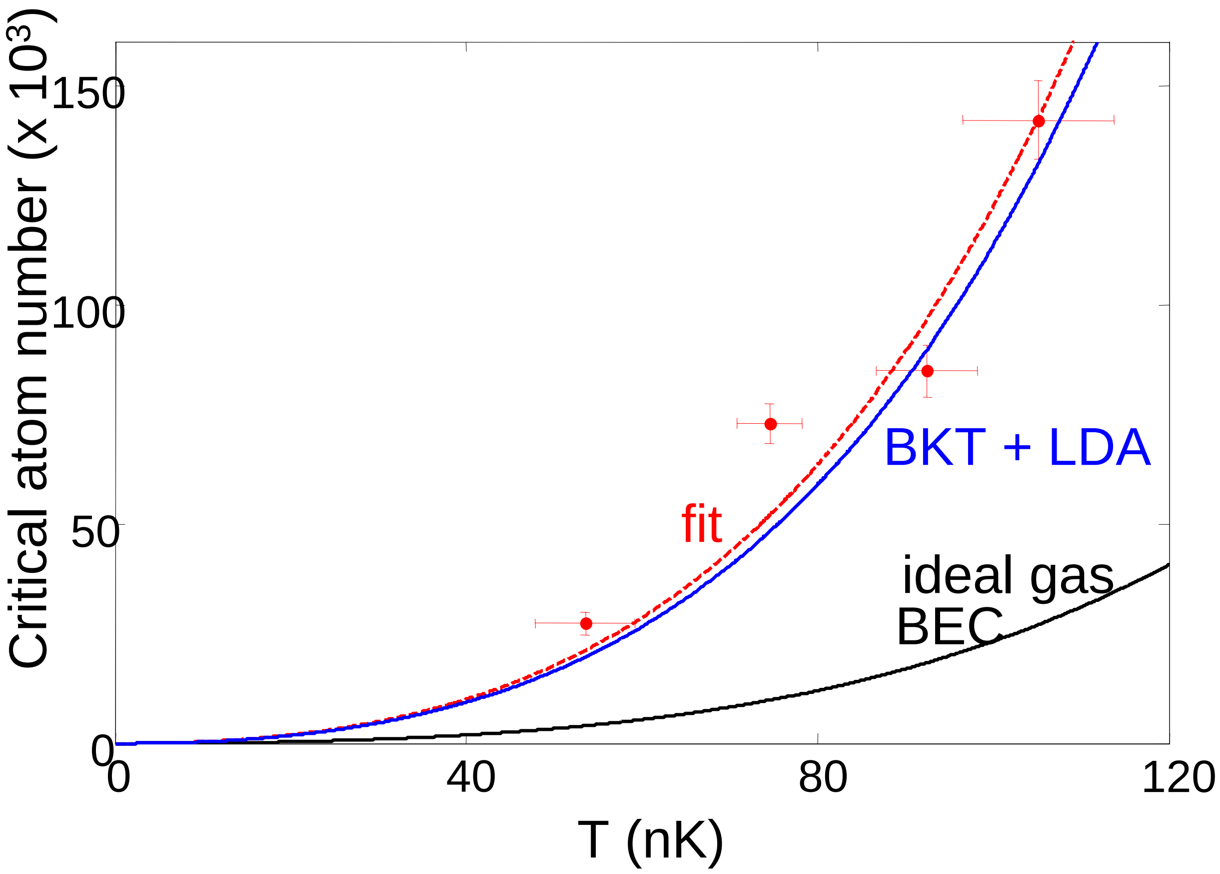
Extract from the experiment: $n_c\lambda^2 = 8.6 \pm 0.8$

(in the center of the cloud)

Critical atom number vs. T

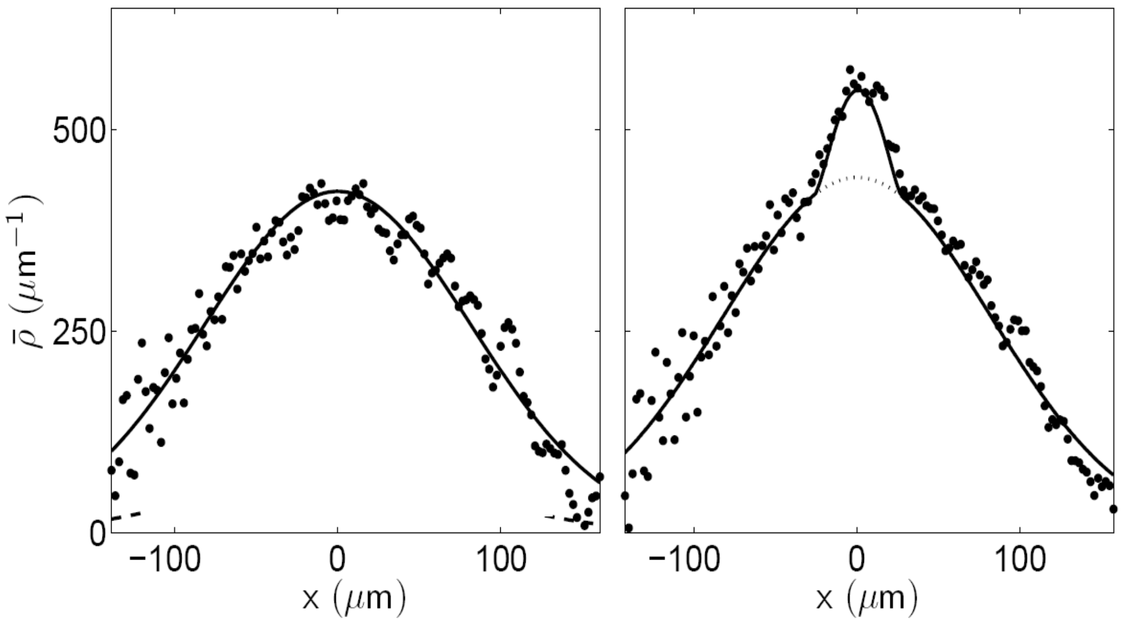
BKT + LDA + experimentally observed Gaussian profiles:

$$N_{\mathrm{c,\ KT}} = 4.9\ N_{\mathrm{c,\ ideal}}$$



Not bad…

Equation of state?



Bimodal distribution fitted well by Gaussian + Thomas-Fermi

…but why?

3.
Coherence of an interacting 2D Bose gas

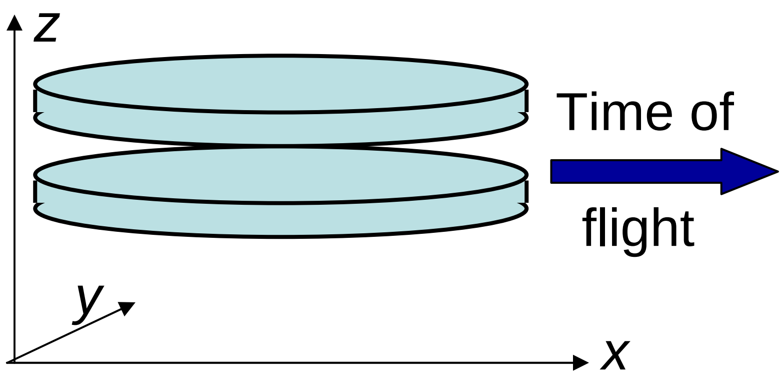
Z. H., P. Krüger, M. Cheneau, B. Battelier, S. Stock, and J. Dalibard

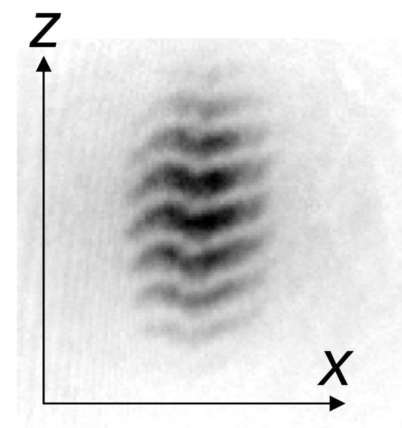
Phys. Rev. Lett. **95**, 190403 (2005)
Nature **441**, 1118 (2006)
cond-mat/0703200

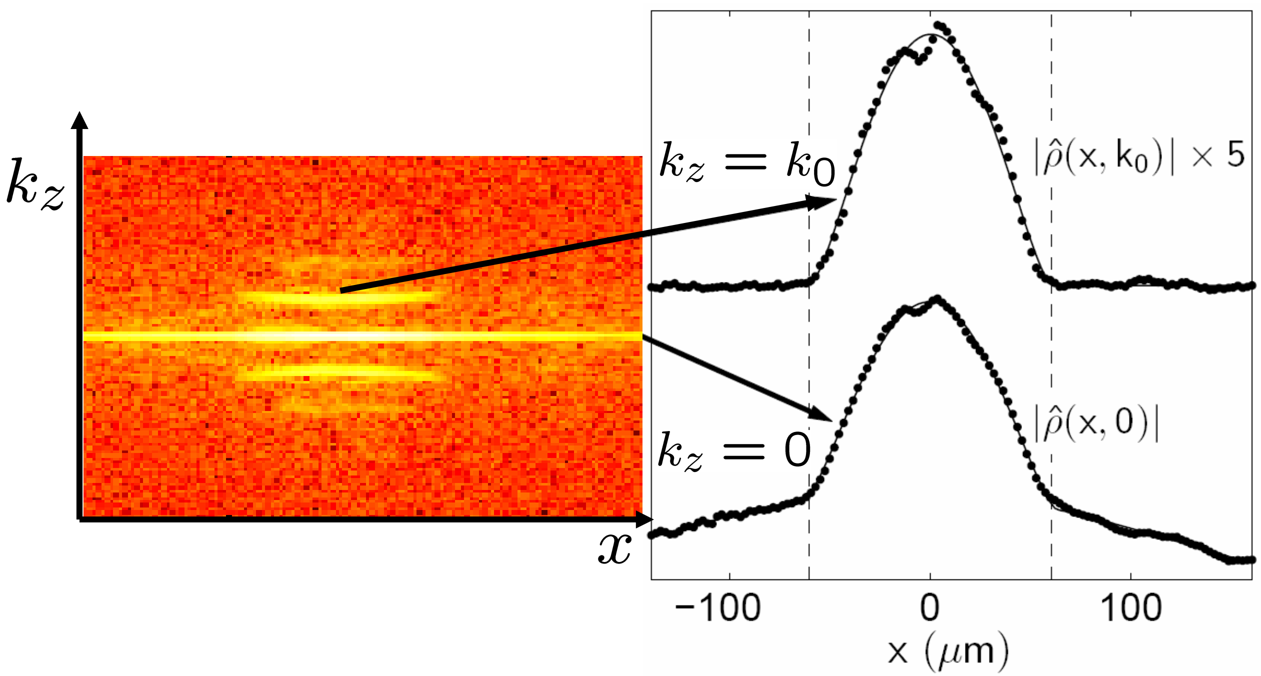
+ Schweikhard, Tung and Cornell, cond-mat/0704.0289

Theory: Shlyapnikov-Gangardt-Petrov, Holtzman *et al.,* Kagan *et al.*, Stoof *et al.*, Mullin *et al.*, Simula-Blackie, Hutchinson *et al.* Polkovnikov-Altman-Demler

Interference of two 2D gases

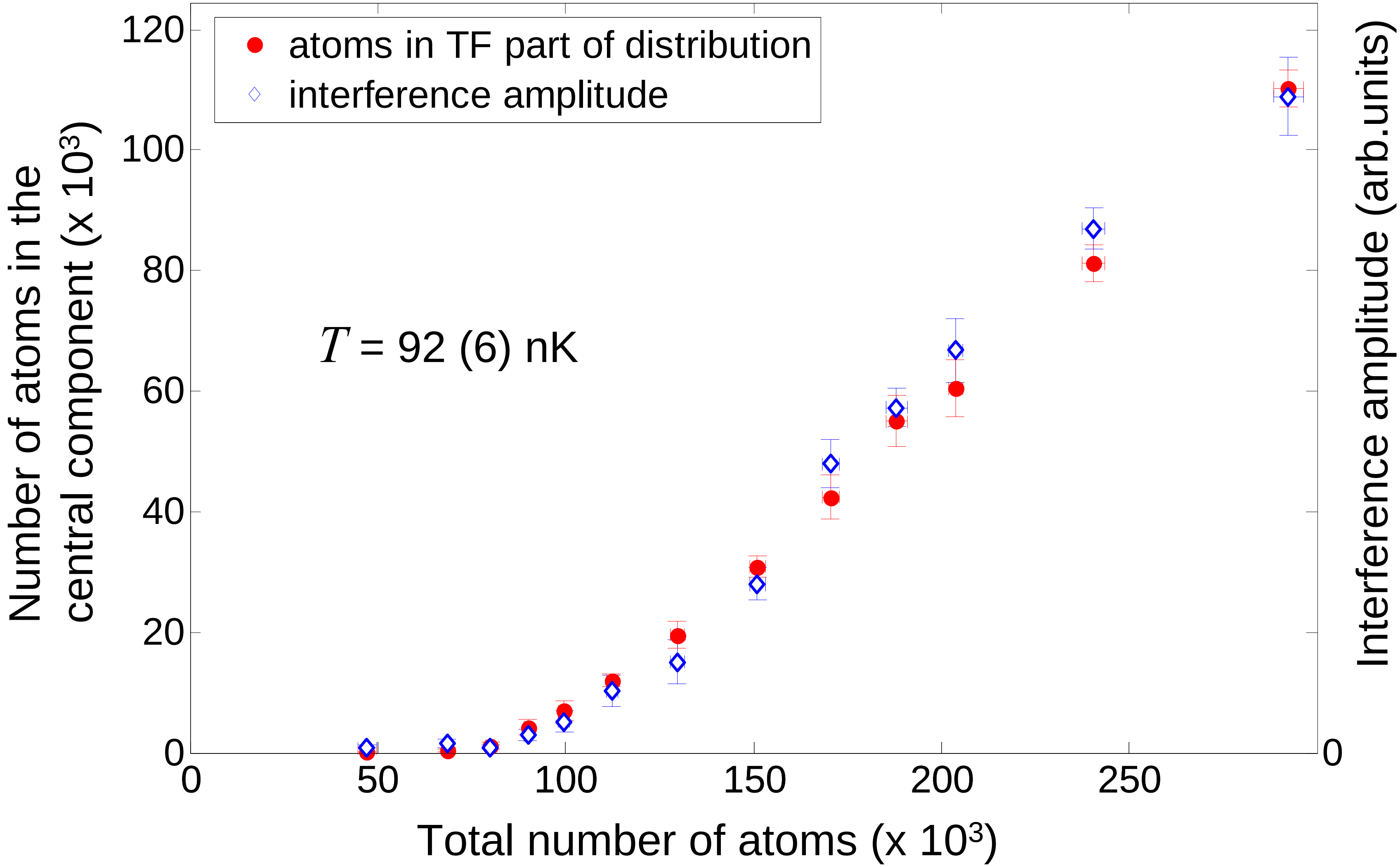






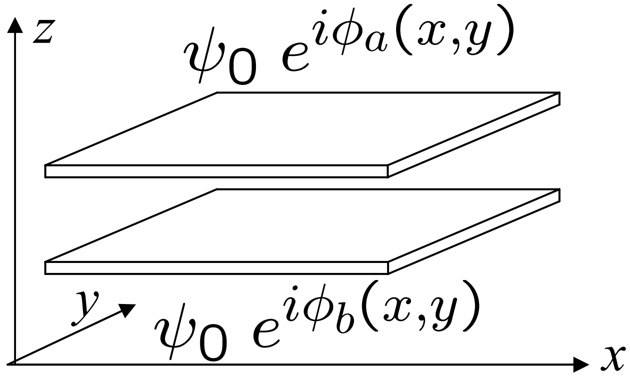
The interfering part coincides with the central part of the bimodal distribution

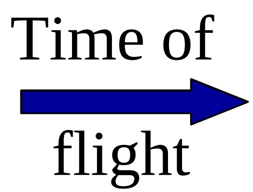
Bimodality and interferences

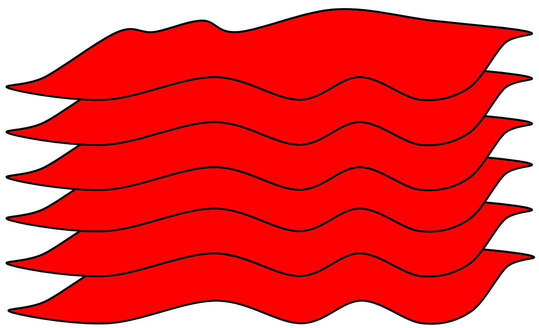


Within our accuracy, onsets of bimodality and interference coincide

Local vs. long-range coherence







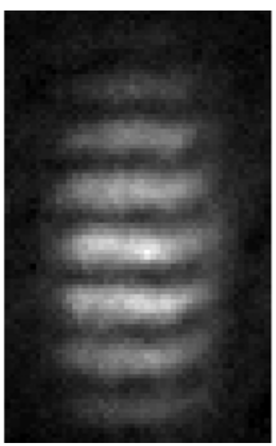
Phonons ("spin waves")

→ smooth phase variations

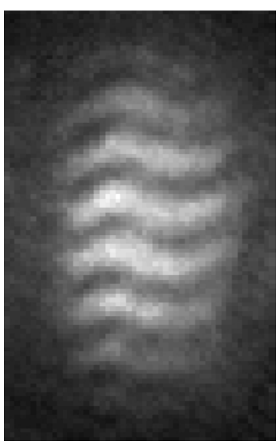
Vortices

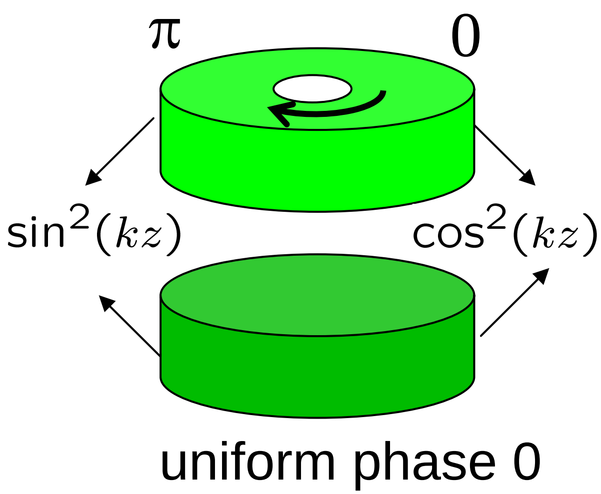
→ sharp dislocations

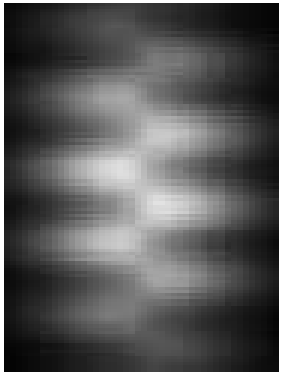
cold



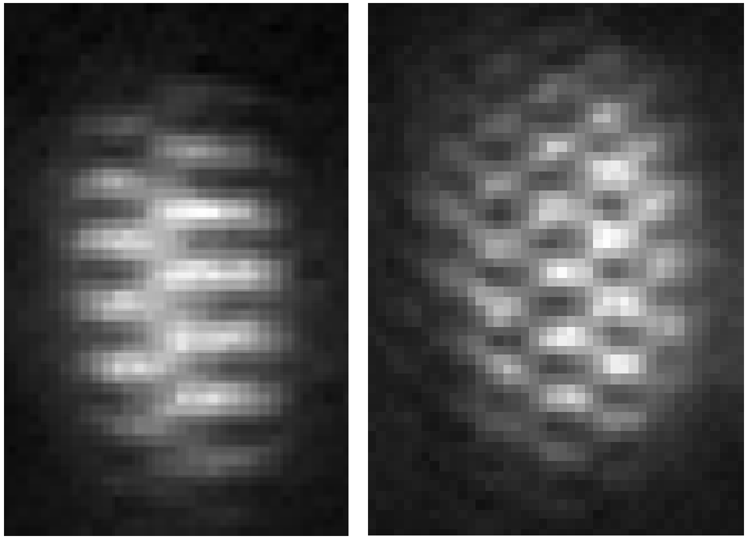
hot





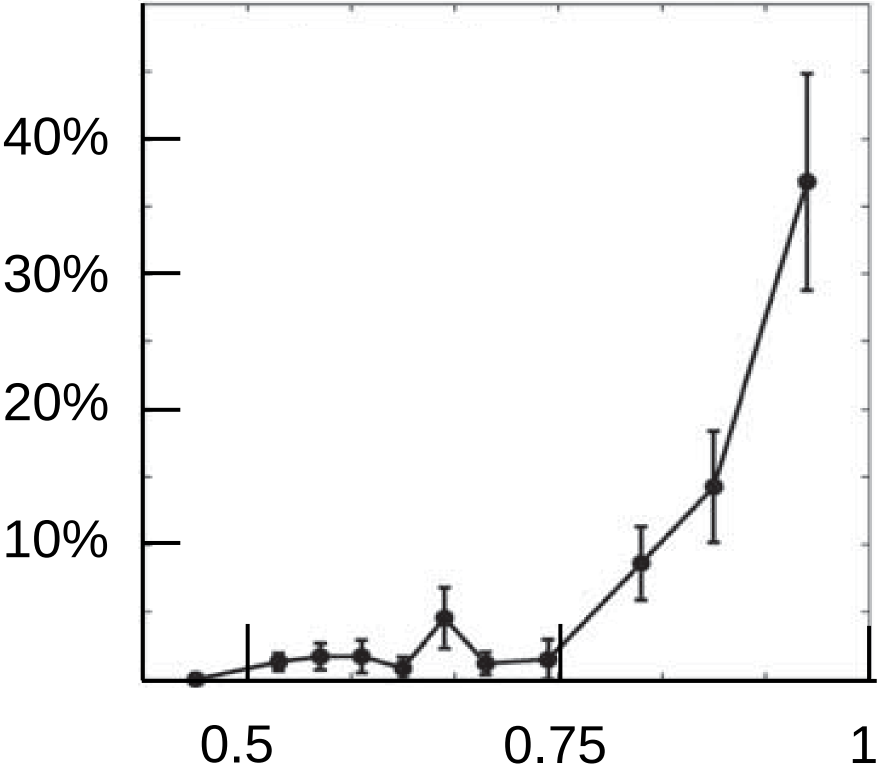


Free vortices in 2D clouds



(Similar results at NIST)

Fraction of images showing at least one dislocation in the central region:



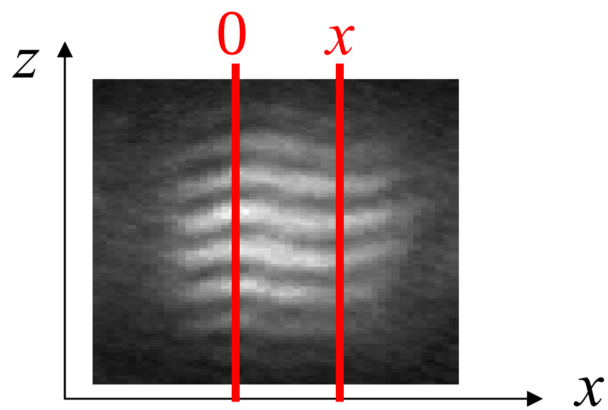
Long-range coherence

Embedded in: $g_1(\vec{r}) = \langle \psi^*(\vec{r})\, \psi(0) \rangle$

The interference signal between $\psi_a(x,y)$ and $\psi_b(x,y)$ gives:

$$|\psi_a|^2 + |\psi_b|^2 + \psi_a^* \psi_b\, e^{ikz} + \psi_a \psi_b^*\, e^{-ikz}$$

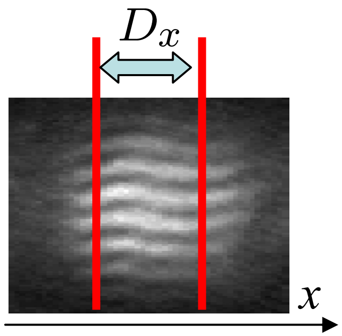
$\psi_a^* \psi_b \rightarrow \kappa(x,y)$



$$\langle \kappa(x,y)\kappa(0) \rangle = \langle \psi_a^*(x,y)\psi_b(x,y)\psi_a(0)\psi_b^*(0) \rangle$$

$$= \langle \psi_a^*(x,y)\psi_a(0) \rangle \, \langle \psi_b(x,y)\psi_b^*(0) \rangle = |g_1(x,y)|^2$$

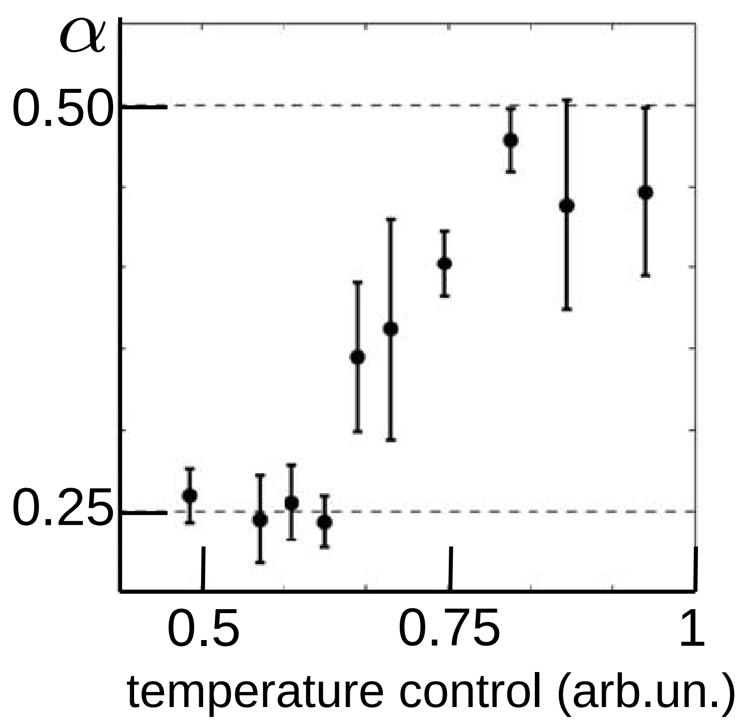
Long-range coherence



Polkovnikov, Altman, Demler:

Integrated contrast: $C(D_x) = \frac{1}{D_x} \int_0^{D_x} \kappa(x)\, dx$

$\langle C^2(D_x) \rangle \sim \frac{1}{D_x} \int_0^{D_x} (g_1(x,0))^2\, dx$ scales as: $1/(D_x)^{2\alpha}$

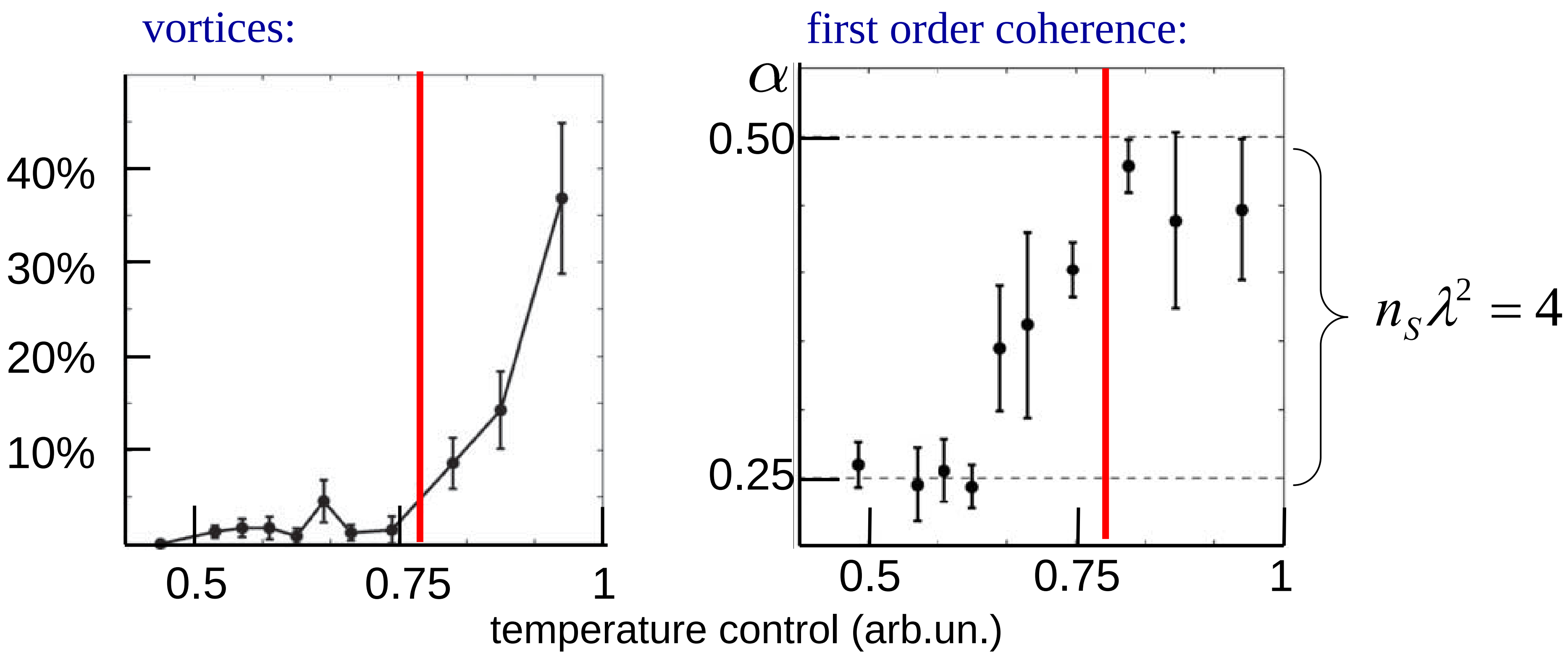


"universal jump in superfluid density"

➡ drop in α from 0.5 to 0.25

(in an infinite uniform system)

Vortices vs. Correlations vs. Temperature

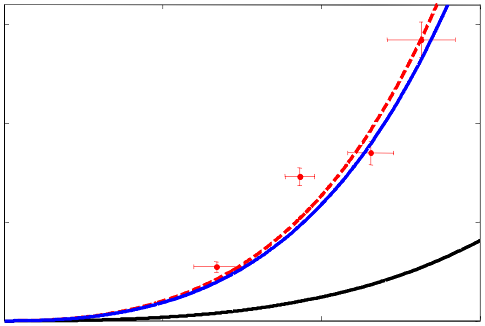


The onset of vortex proliferation coincides with the loss of quasi-LRO

Z. Hadzibabic *et al.,* Nature **441**, 1118 (2006)

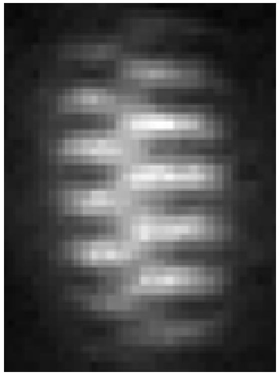
see also Schweikhard, Tung and Cornell, cond-mat/0704.0289 for KT in a lattice

So far in atomic Flatland…



Phase transition with a critical point $\left(N_C, T_C\right)$:

- eliminates conventional BEC
- agrees quantitatively with BKT + LDA



Direct visualization of free vortices:

- coincides with loss of quasi-long-range order
- supports the microscopic basis of the theory

Open questions/future:

Equation of state?

Tune the interactions from g ~ 1 to g ~ 10^{-4}

Superfluidity – transport, dissipation?

Resolve tightly bound vortex pairs in the superfluid state?

THE END