Dark Matter Caustics

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Motivation

Elucidating the structure of galactic halos is centrally important for

- understanding galactic dynamics
- predicting signals for dark matter searches

see also: S. Tremaine, C. Hogan, ...
The US large scale axion dark matter search.

Galactic halo models

Model

1. Isothermal model

Complaint

Infalling particles do not thermalize

2. N body simulations (NFW, Moore, ...)

3. Caustic ring model

With $10^7$ particles the resolution is inadequate

ADMX

cfr. D. Kinion
M. Mueck
Outline

- Topology → dark matter caustics
  - outer caustic spheres
  - inner caustic rings
- Self-similar infall
  → caustic ring radii
  \[ a_n \approx \frac{1}{m} \frac{40 \text{ kpc}}{220 \text{ km/s}} \cdot \frac{N_{\text{core}}}{0.27} \]
- Observational evidence

The initial phase space distribution of CDM at time \( t_i \):

\[ t_{eq} < t_i \ll t_0 \]

\[ \delta \nu \approx 10^{-17} \left( \frac{t_0}{E} \right)^{2/3} \quad \text{for axions} \]

\[ 10^{-12} \left( \frac{t_0}{E} \right)^{2/3} \quad \text{for WIMPs} \]

\[ \Delta N \approx \frac{\delta p}{p} \approx t^{2/3} \]
The outer caustics are
topological spheres \((A_2\) catastrophe\).

The inner caustics are
closed tubes \((rings)\) whose
cross-section is a \(D_4\) catastrophe.

galactic center

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**Figure 1.** Infall of all particles on a given initial turnaround sphere. The sphere crosses itself between frames b) and c). The cusps in frames d) and e) are at the intersection of the ring caustic with the plane of the figure.
Caustic rings of dark matter form around isolated galaxies if

1) Collisionless dark matter exists (axions, WIMPs, neutrinos)

2) The velocity dispersion of infalling dark matter is less than 30 km/s

3) The angular momentum distribution of infalling dark matter is dominated by a smooth component which carries net angular momentum
Galactic halo formation by CDM infall is self-similar, in the zero angular momentum case, if

$$\frac{\delta M_i}{M_i} = \left( \frac{M_0}{M_i} \right)^\varepsilon$$

Fillmore + Goldreich
Bertscheing

$$0 \leq \varepsilon \leq 1$$

It produces flat rotation curves if

$$0 \leq \varepsilon \leq \frac{2}{3}$$

In standard \((n=1)\) CDM models of structure formation

$$\varepsilon \approx 0.2 - 0.35$$
\[ e = 0.2 \quad \dot{\varphi} = 0 \quad h = 0.7 \]
Galactic halo formation by CDM infall, with angular momentum, is self-similar if in addition

\[ I(\mathcal{F}, t) = j(\mathcal{F}) \frac{R^2(t)}{t} \]

P.S. Tkachen Wang

Angular momentum has the effect of producing an effective core radius \( b \)

For the Milky Way galaxy

\[ b \approx 12 \text{kpc} \Rightarrow j \approx 0.2 \]

\[ R \approx 2 \text{ Mpc} \]
In the self-similar infall model

Fillmore + Goldreich
Bertschinger

with angular momentum

PS, Thacker + Wang

the caustic ring radii are predicted:

\[ a_n \approx 40 \text{ kpc} \cdot \frac{1}{n} \left( \frac{\mathcal{N}_{\text{rot}}}{220 \text{ km/s}} \right). \]

\[ n = 1, 2, 3, ... \]

\[ \left( \frac{J_{\text{max}}}{0.27} \right) \cdot \left( \frac{0.7}{h} \right) \]

\[ l_n = J_{\text{max}} \mathcal{N}_{\text{rot}} R_n, \]
With Will Kinney (astro-ph/9906049), we looked at 32 well-measured extended rotation curves selected by R.H. Sanders and K.G. Begeman et al.

\[ a_m = (41, 20, 13.3, 10, \ldots) \text{ kpc} \times \left( \frac{\sigma_{\text{rot}}}{220 \text{ km/s}} \right) \left( \frac{\ddot{d}_{\text{max}}}{0.7} \right) \left( \frac{0.7}{h} \right) \]

For each rotation curve:

1) Rescale
   \[ \Gamma \rightarrow \tilde{\Gamma} = \Gamma \left( \frac{220 \text{ km/s}}{\sigma_{\text{rot}}} \right) \]

2) Cut-off
   \[ \tilde{\Gamma} > 10 \text{ kpc} \]

From Begeman, Broeils and Sanders

* MNRAS 249 (1991) 523
3) fit to line or quadratic polynomials

\[ \delta v_i \]

residuals

\[ (\delta v_i)^2 \]

\[ \delta \tilde{v}_i = \frac{\delta v_i}{< \delta v_i^2 >^{1/2}} \]

Average over the sample of 32 galaxies

\[ b_i = \frac{1}{N_i} \sum_{j=1}^{N_i} \delta \tilde{v}_j \]

(Where \( N_i \) is the \( i^{th} \) data points in \( i^{th} \) bin)

With assigned error

\[ \Delta b_i = \frac{1}{\sqrt{N_i}} \]
North Galactic Rotation Curve
D. P. Clemente (1985)
derived from the Massachusetts - Stony Brook Galactic plane CO survey

Rises in galactic rotation curves have been interpreted in the past as due to the presence of spiral arms.

If so,
- why are there eight rises between 3 and 7 kpc?
- why are the rises so sharp?
\[ \epsilon = 0.3 \]

\[ j_{\text{max}} = 0.263 \]

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<td>14</td>
<td>( 3.16 )</td>
<td>( 3.15 )</td>
<td>( -0.3 % )</td>
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\[ \text{rmsd} = 3.1 \% \]

Caustic in the Sky?

Look in the direction tangent to the ring to search for its imprint on the distribution of gas in the galactic disk.
IRAS100.20.gif

from http://skyview.gsfc.nasa.gov/

IRAS12.mod.gif
5\textsuperscript{th} Caustic parameters

(assuming \( R_0 = 8.5 \text{ kpc} \))

\[ a_5 = 8.31 \pm 0.01 \text{ kpc} \]
\[ a_5 + p_5 = 8.45 \pm 0.01 \text{ kpc} \]
\[ p_5 = 134 \pm 10 \text{ pc} \]
\[ q_5 = 199 \pm 14 \text{ pc} \]

Local densities and velocities of the \( n=5 \) flows

\[ d^+ = 1.7 \times 10^{-24} \text{ g cm}^{-3} \]

\[ d^- = 0.15 \times 10^{-24} \text{ g cm}^{-3} \]

\[ \Sigma^\pm = (470 \phi \pm 100 \chi) \text{ km s}^{-1} \]

\( (\phi, \chi, r) \) are galactocentric cylindrical coordinates

3X previous estimates of the total local DM density!
The caustics are smoothed by the velocity dispersion \( \delta v_{DM} \) of infalling dark matter over distances of order

\[ \delta a = R \frac{\delta v_{DM}}{v} \]

where \( R \) is the turnaround radius and \( \nu \) is the flow velocity.

From the sharpness of the edges of the triangular feature of the IRAS map:

\[ \delta a \approx 20 \text{ pc} \]

\[ \delta v_{DM} \approx 50 \text{ m/s} \]

Signal to noise ratio in axion dark matter search:

\[ \frac{1}{m} = \frac{P_{signal}}{T_{noise}} \sqrt{\frac{t}{B}} \]

where \( t \) is the measurement integration time and \( B \) is the band width.

- Full signal: \( B \approx 10^{-6} m_a \)
- Narrow peak: \( B \approx 10^{-10} m_a \)
\[
\frac{\text{Signal}}{\sqrt{B}} \text{ is increased by the factor}
\]
\[
\frac{3}{\sqrt{10^{-4}}} = 300
\]

\[
\therefore \frac{A}{n} \text{ increased by a factor } \approx 100
\]

Also, noise is decreased by a factor \(\frac{1}{100}\) by using SQUIDs instead of HEMTs.

The annual modulation in WIMP searches is the reverse of what is usually thought, i.e., the flux is lowest in June and highest in December.

J. Vergados
A. Green
G. Gelmini + P. Gondolo
M. Kamionkowski + P. Ullio
FIG. 1. Two trajectories past a spherically symmetric mass distribution. The incident flow is assumed to be uniform and to have vanishing velocity dispersion. If the scatterer is a point mass, the particles with small impact parameter are scattered by an angle close to 180°, and hence the number of flows $n = 2$ everywhere. If the scatterer is distributed over a finite radius, there is a maximum scattering angle $\Theta_{\text{max}} < 180°$. In that case, $n = 1$ upstream of the scatterer, and $n = 3$ downstream.
Scattering Angle

Initial velocities: 200 km/sec and 500 km/sec
dashed line: 1/r potential

Density Wake vs $\theta$

$v_\infty = 255$ km/sec
inset: skirt caustic at $\theta = 115^\circ$
Summary of Evidence for Dark Matter Caustics

- Why does the 32 galaxy composite rotation curve fluctuate upwards at 40 kpc (3.1σ) and 20 kpc (2.6σ)?
  [probability ~ 3 \times 10^{-4}]

- Why is there a triangular feature in the IRAS map of the galactic plane near 80° longitude?

- Why is the triangle equilateral, with axis parallel to the galactic plane, and pointing away from the galactic center?

- Why does the triangle (8.31 to 8.44 kpc) coincide with a rise in the rotation curve (8.28 to 8.43 kpc)?
  [probability ~ 10^{-3}]

- Why are the rises sharp?
The Big Flow:

\[ d = 1.7 \times 10^{-24} \text{ g} \text{ cm}^{-3} \]

3X previous estimates of the total local DM density

\[ \vec{n} = (470 \hat{\phi} \pm 100 \hat{\phi}) \frac{\text{km}}{\text{sec} \cdot \text{Hz}} \]

\[ \vec{n}_\odot = 220 \text{ km} \text{ sec}^{-1} \hat{\phi} \]

\[ \hat{\phi} \text{ in the direction of galactic rotation} \]