

# Dark Matter Caustics

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...

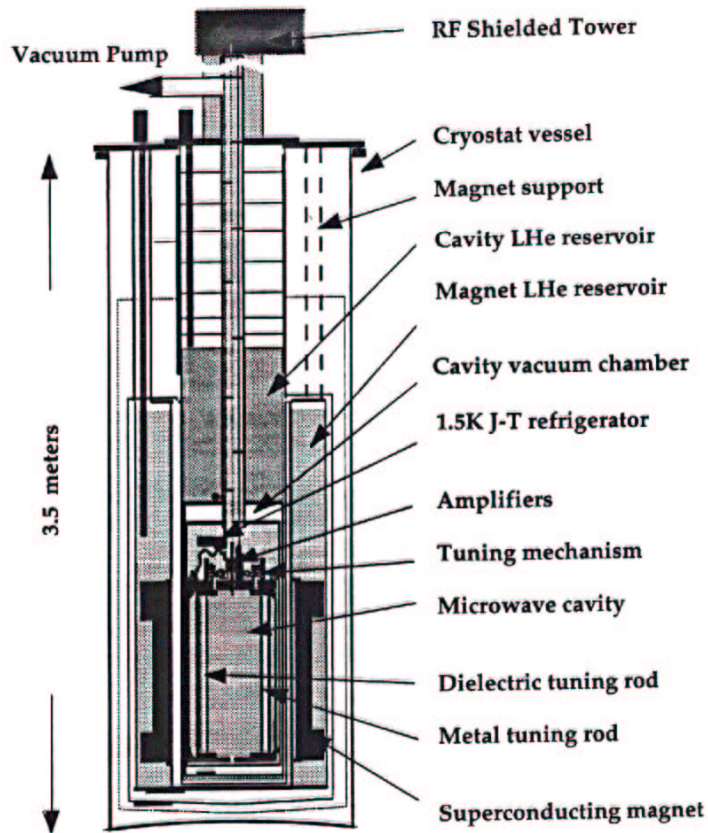
see also: S. Tremaine, C. Hogan, ... →

## Motivation

Elucidating the structure  
of galactic halos is  
centrally important for

- understanding galactic  
dynamics
- predicting signals for  
dark matter searches

The US large scale axion dark matter search.



ADMX

cfr. D. Kinion  
M. Mueck

Galactic halo models

Model

Complaint

1. Isothermal model

Infalling particles do not thermalize

2. N body simulations (NFW, Moore, ...)

With  $10^7$  particles the resolution is inadequate

3. Caustic ring model

## Outline

- Topology  $\rightarrow$  dark matter caustics

outer caustic spheres

inner caustic rings

- Self-similar infall

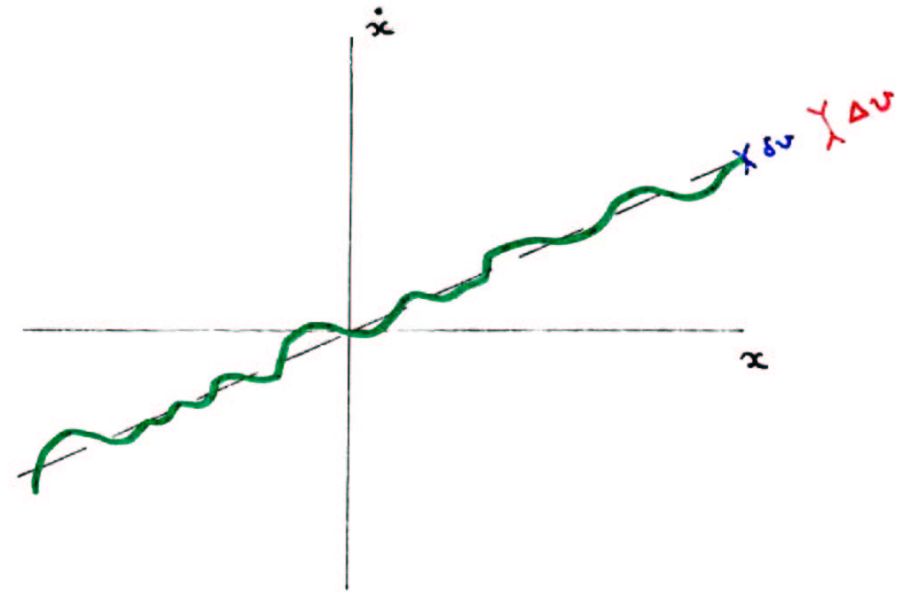
$\rightarrow$  caustic ring radii

$$a_n \approx \frac{1}{n} 40 \text{ kpc} \cdot \frac{N_{\text{rot}}}{220 \text{ km/s}} \cdot \frac{j_{\text{max}}}{0.27}$$

- Observational evidence

The initial phase space distribution of CDM at time  $t_i$ :

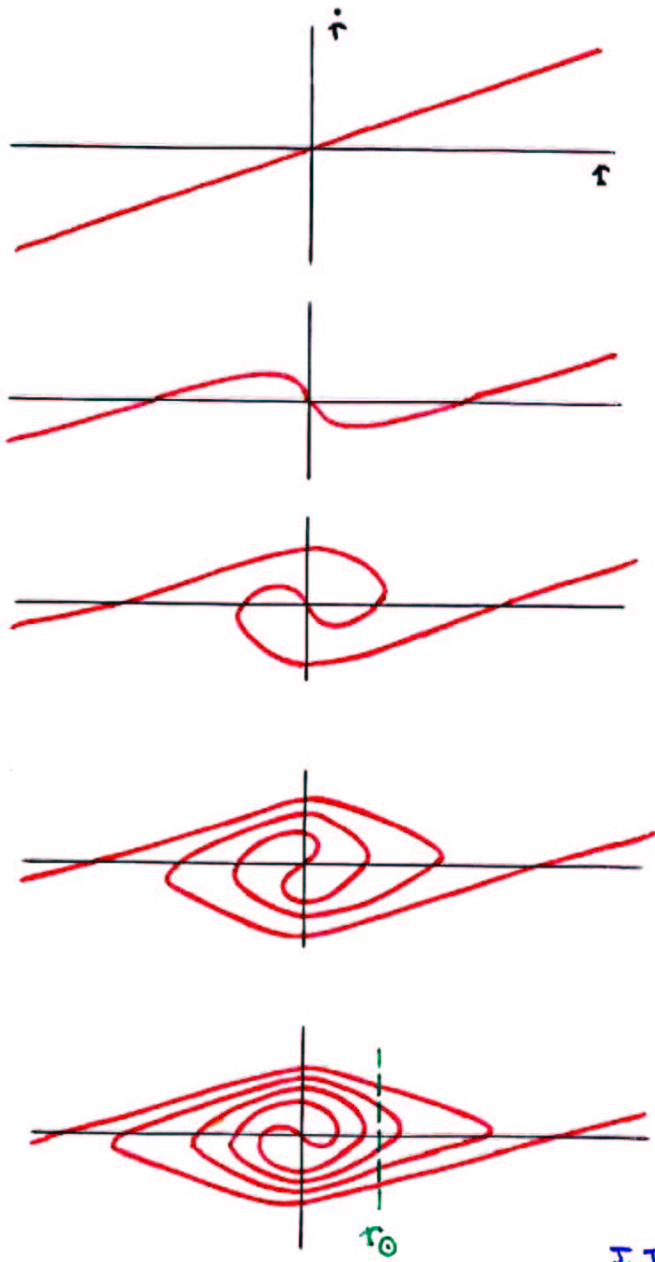
$$t_{\text{eq}} < t_i \ll t_0$$



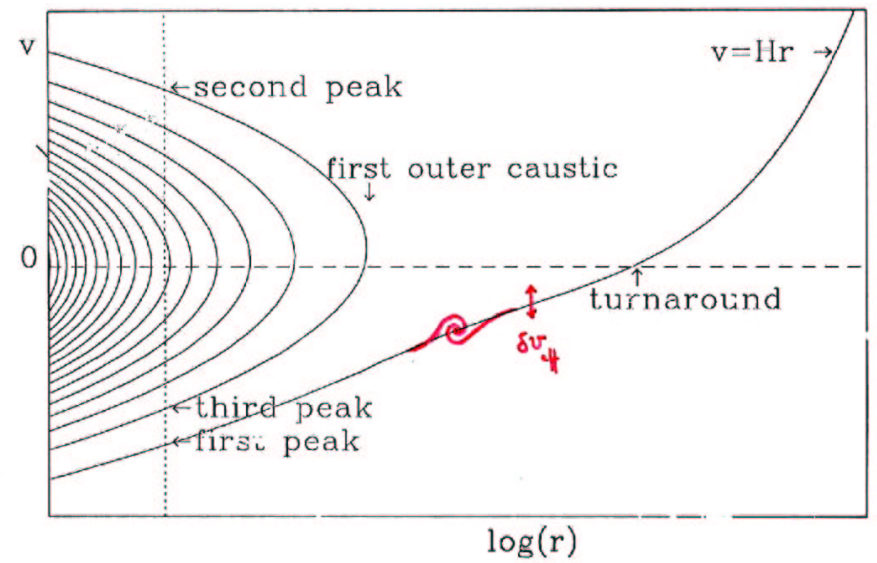
$$\delta v \approx 10^{-17} \left( \frac{t_0}{t} \right)^{2/3} \quad \text{for axions}$$

$$\approx 10^{-12} \left( \frac{t_0}{t} \right)^{2/3} \quad \text{for WIMPs}$$

$$\Delta N \sim \frac{\delta P}{P} \sim t^{2/3}$$



J. Ipser + P.S.



The outer caustics are  
topological spheres ( $A_2$  catastrophe)

The inner caustics are  
closed tubes (rings) whose  
cross-section is a  $D_4$  catastrophe)

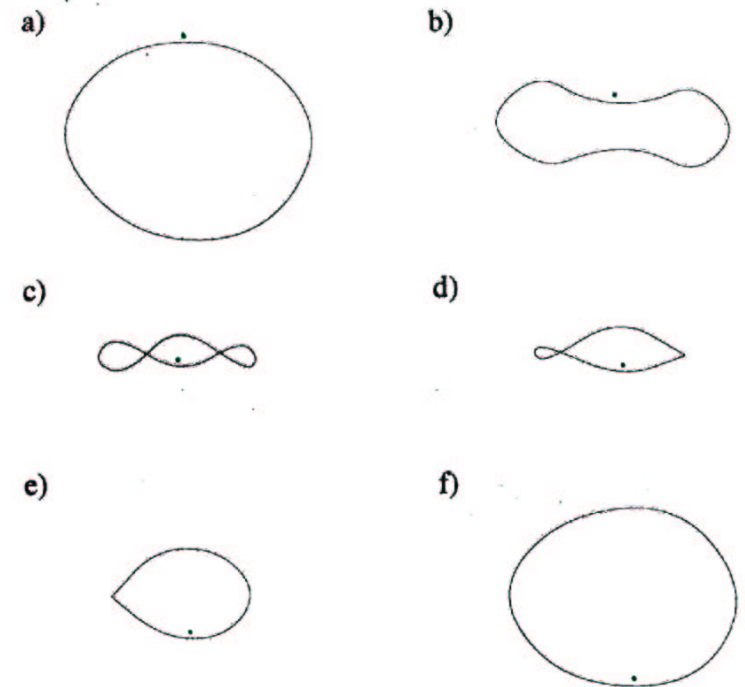
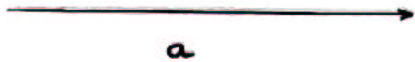
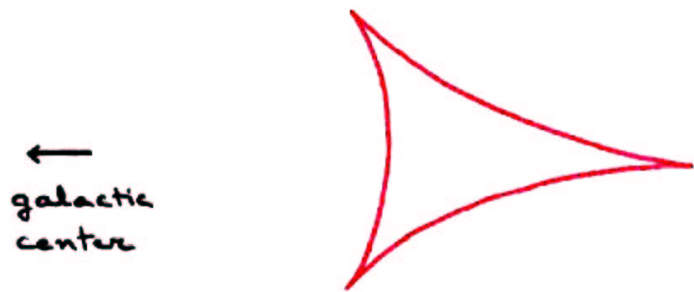
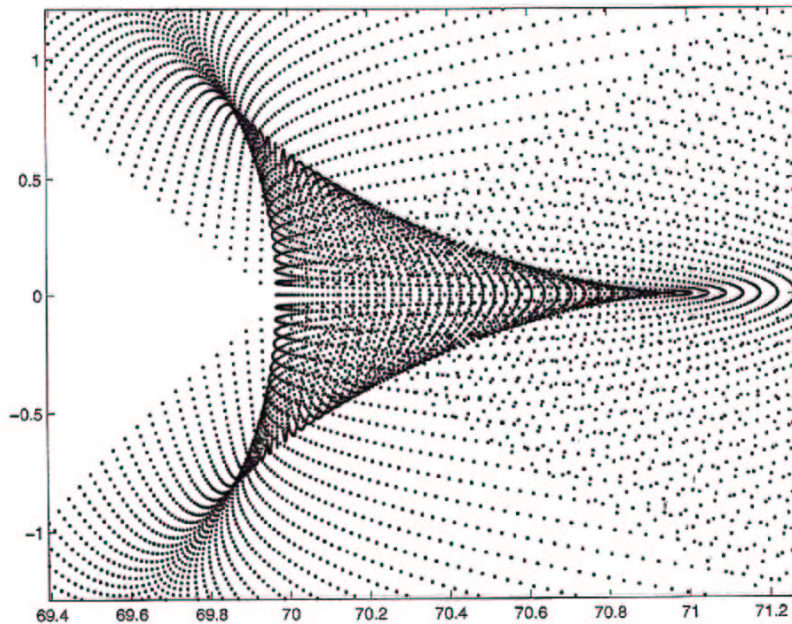


FIG. 1. Infall of all particles on a given initial turnaround sphere. The sphere crosses itself between frames b) and c). The cusps in frames d) and e) are at the intersection of the ring caustic with the plane of the figure.



Charmousis, Ormali,  
Qiu + P.S.

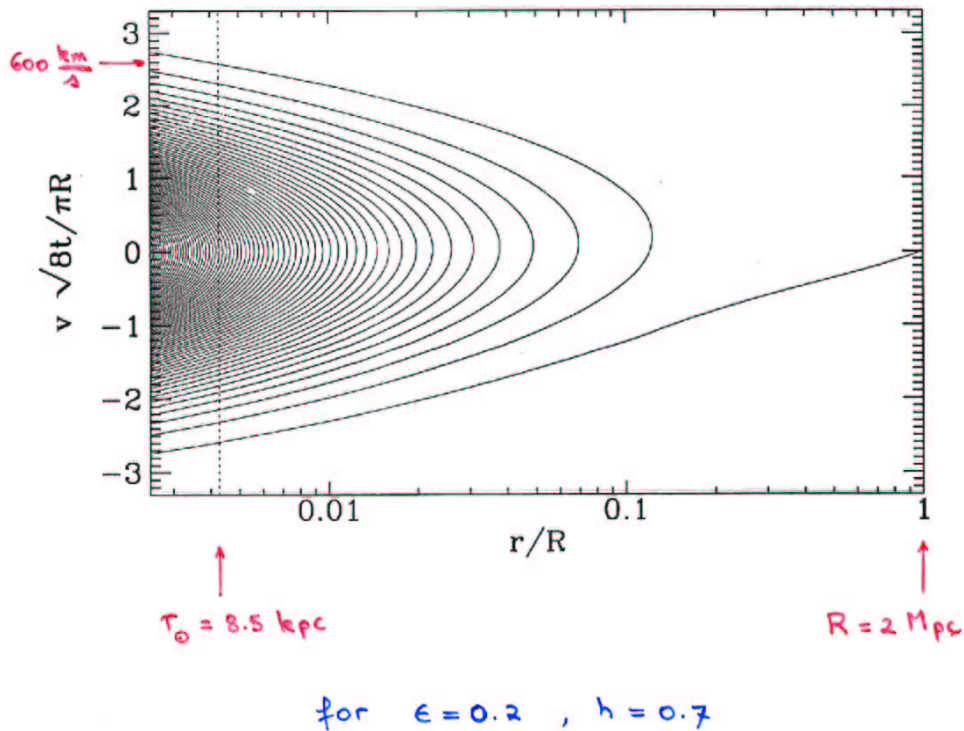
Caustic rings of dark matter  
form around isolated galaxies

if

- 1) collisionless dark matter exists  
(axions, WIMPs, neutrinos)
- 2) the velocity dispersion of  
infalling dark matter is  
less than 30 km/s
- 3) the angular momentum distribution  
of infalling dark matter is  
dominated by a smooth component  
which carries net angular momentum

The minimum number of flows on Earth is of order 100.

Of order  $(100)^6 = 10^{12}$  particles are needed to resolve this structure



Galactic halo formation by CDM infall is self-similar, in the zero angular momentum case, if

$$\frac{\delta M_i}{M_i} = \left( \frac{M_0}{M_i} \right)^\epsilon$$

Fillmore  
+ Goldreich  
Bertschinger

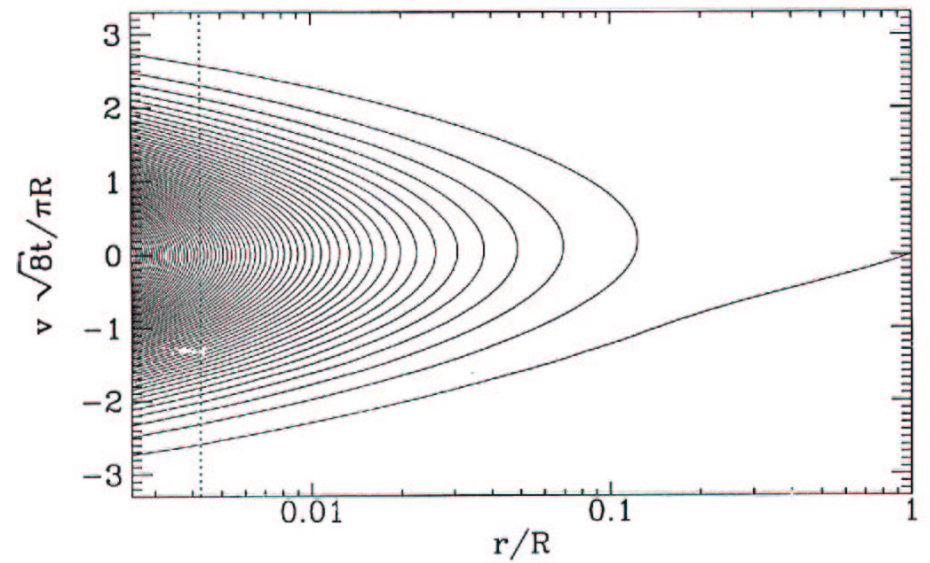
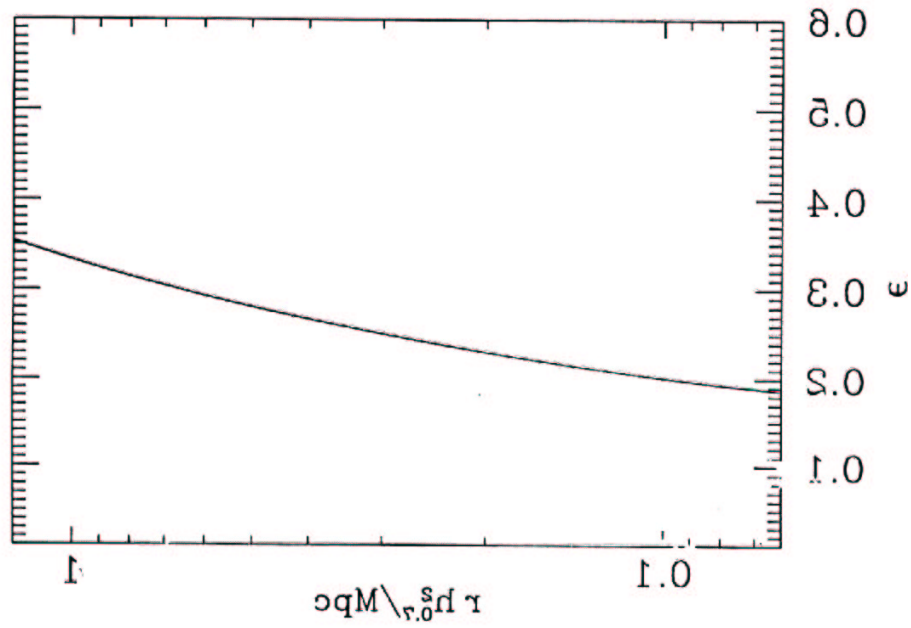
$$(0 \leq \epsilon \leq 1)$$

It produces flat rotation curves if

$$0 \leq \epsilon \leq \frac{2}{3}$$

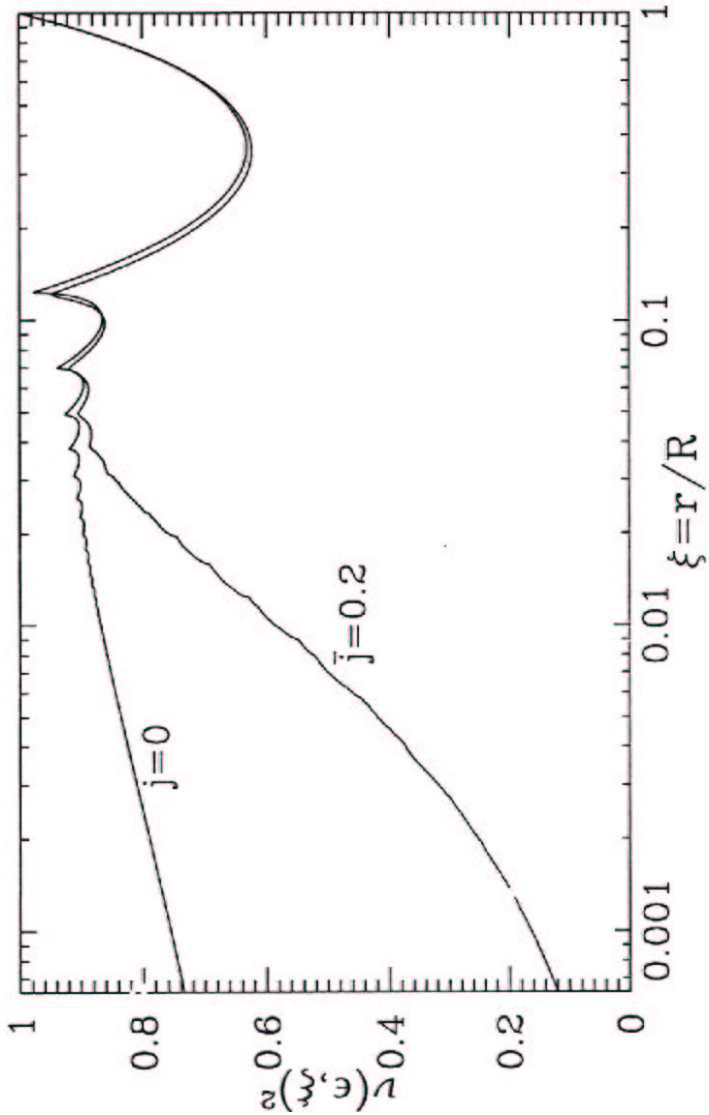
In standard ( $n=1$ ) CDM models of structure formation

$$\epsilon \approx 0.2 - 0.35$$



$\epsilon = 0.2$  ,  $j = 0$  ,  $h = 0.7$





$\bar{j} = 0.2$

Galactic halo formation by CDM infall, with angular momentum, is self-similar if in addition

$$l(\hat{r}, t) = j(\hat{r}) \frac{R^2(t)}{t}$$

P.S. Tkachev Wang

↑  
ang. mom. per unit mass distribution

↑  
dimensionless time-independent distribution

Angular momentum has the effect of producing an effective core radius  $b$

For the Milky Way galaxy

$$b \approx 12 \text{ kpc} \leftrightarrow \bar{j} \approx 0.2$$

$$R \approx 2 \text{ Mpc}$$

In the self-similar infall model

Fillmore + Goldreich  
Bertschinger

with angular momentum

PS, Thacker + Wang

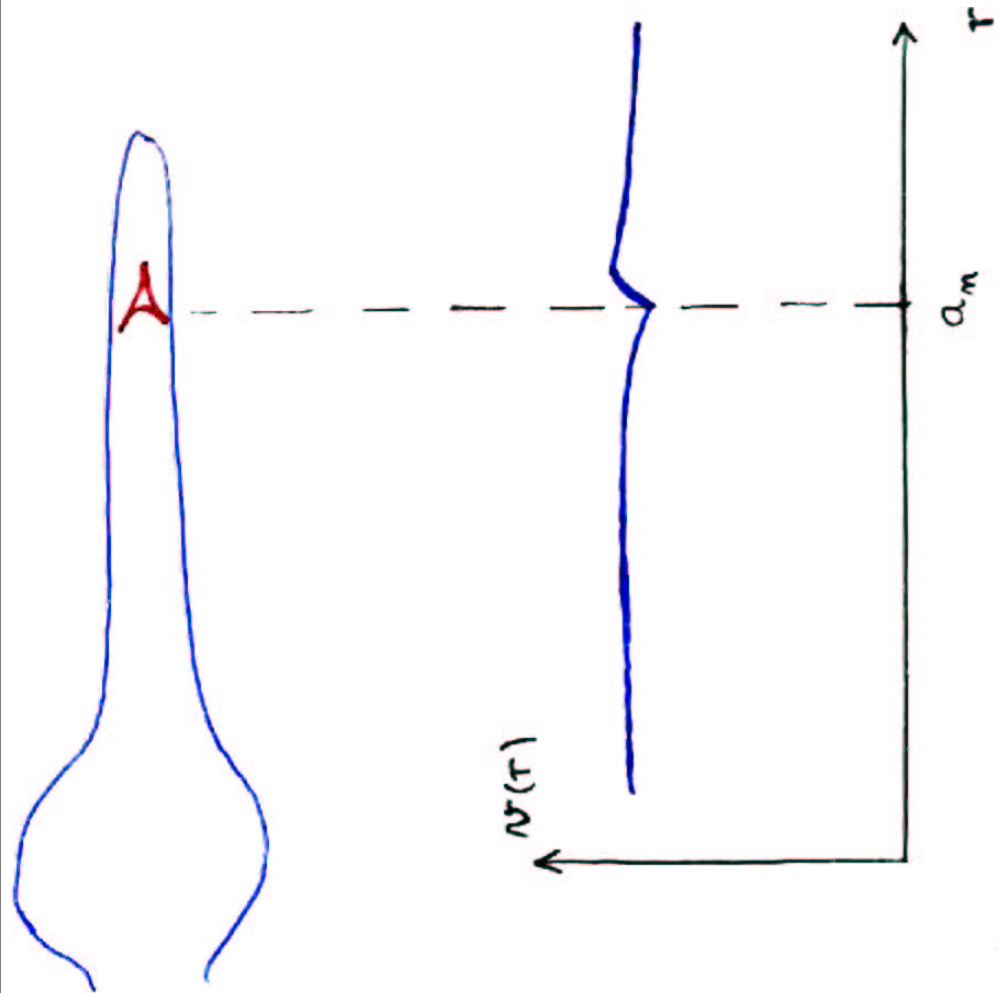
the caustic ring radii are predicted:

$$a_n \approx 40 \text{ kpc} \cdot \frac{1}{n} \cdot \left( \frac{v_{\text{rot}}}{220 \text{ km/s}} \right) \cdot$$

$n=1, 2, 3, \dots$

$$\cdot \left( \frac{j_{\text{max}}}{0.27} \right) \cdot \left( \frac{0.7}{h} \right)$$

$$l_n = j_{\text{max}} v_{\text{rot}} R_n$$



With Will Kinney (astro-ph/9906049),  
 we looked at 32 well-measured  
 extended rotation curves selected  
 by R.H. Sanders and K.G. Begeman et al.

$$a_m = (41, 20, 13.3, 10, \dots) \text{ kpc} \times$$

$$\times \left( \frac{v_{rot}}{220 \text{ km/s}} \right) \left( \frac{j_{max}}{0.27} \right) \left( \frac{0.7}{h} \right)$$

For each rotation curve

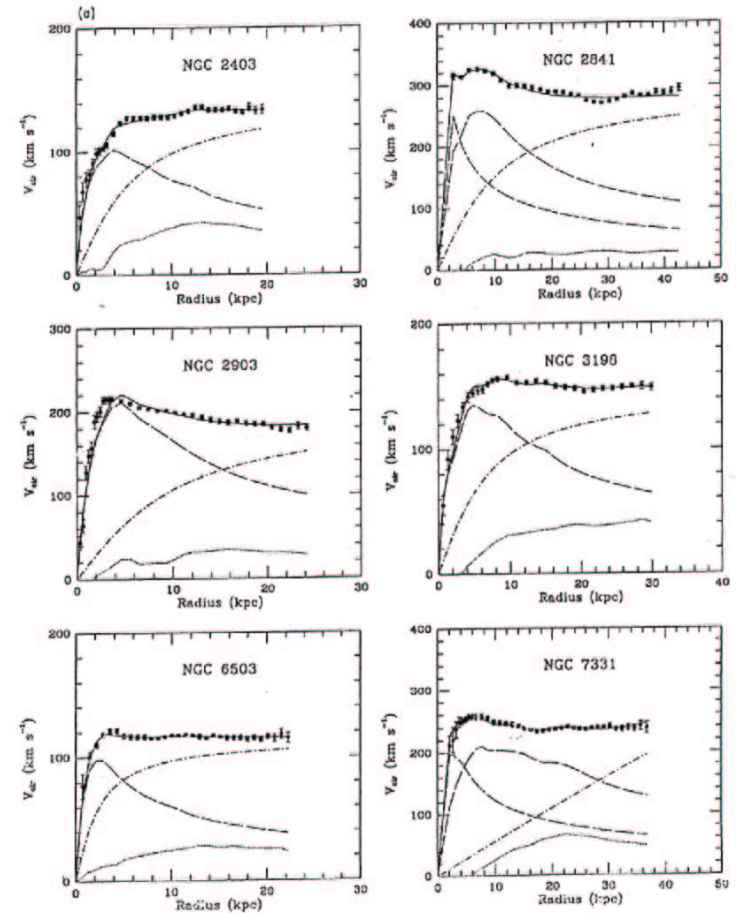
1) rescale

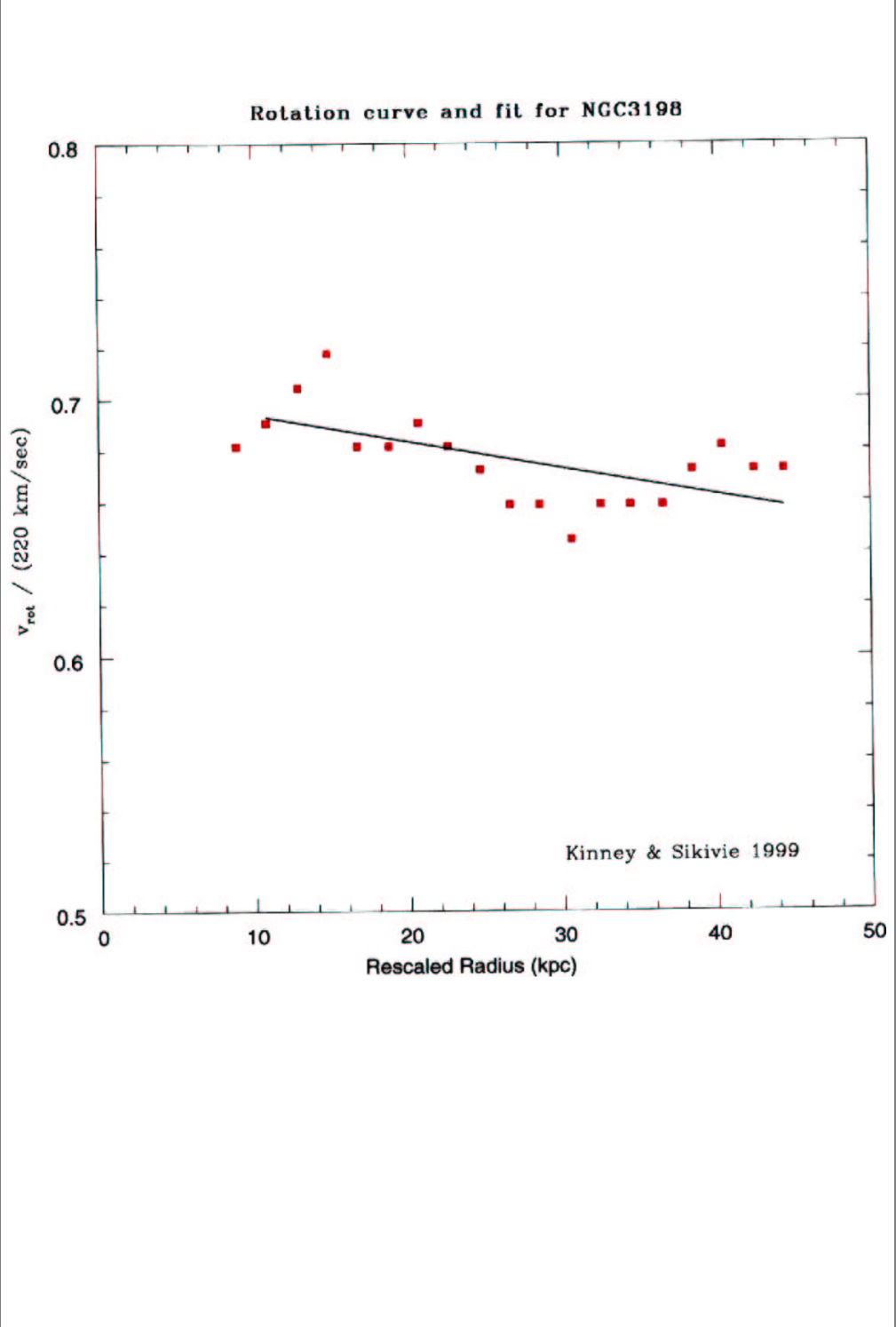
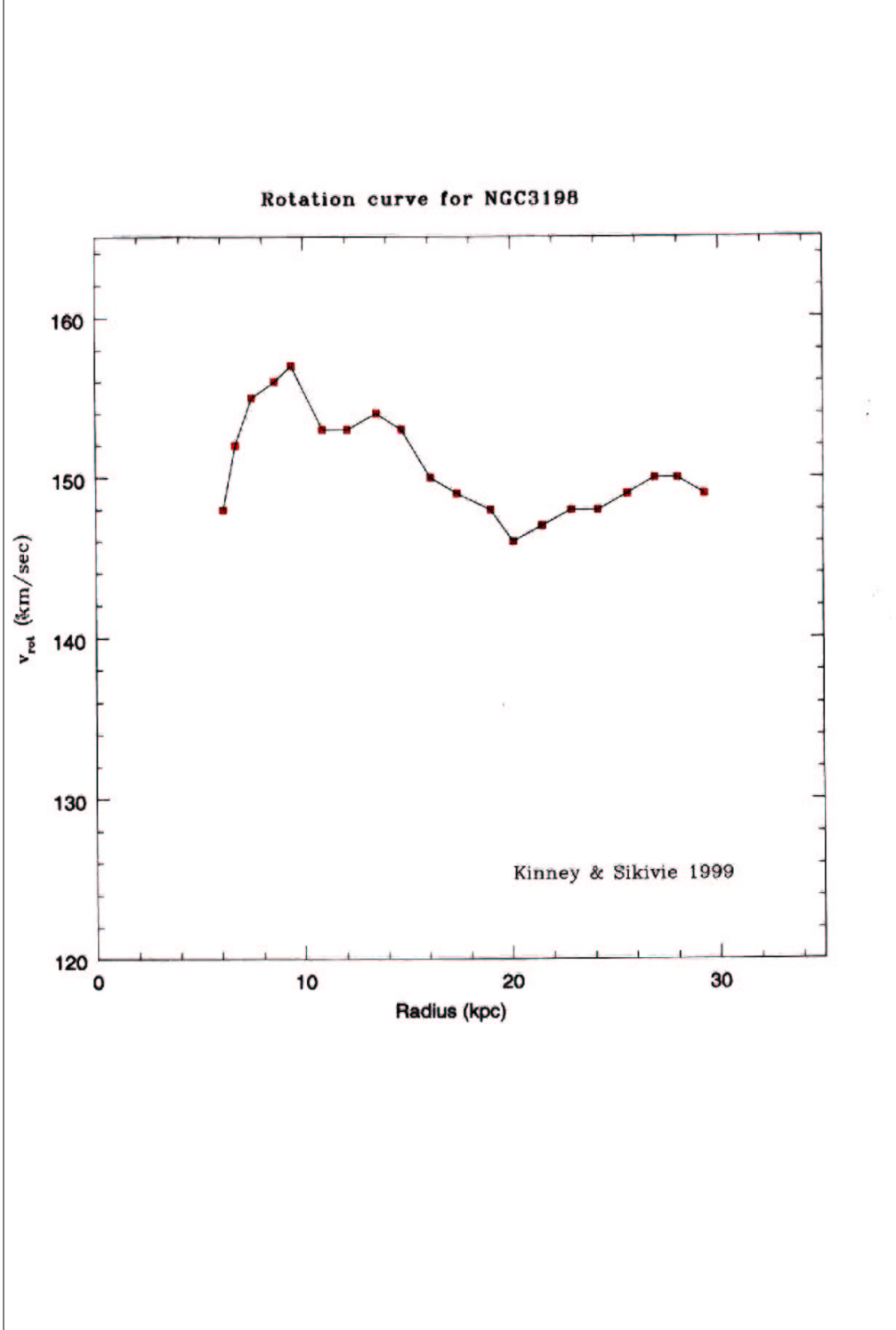
$$r \rightarrow \tilde{r} = r \left( \frac{220 \text{ km/s}}{v_{rot}} \right)$$

2) cut-off

$$\tilde{r} > 10 \text{ kpc}$$

From Begeman, Broeils and Sanders  
 MNRAS 249 (1991) 523





3) fit to line or quadratic polynomials

$\therefore \delta v_j$  residuals

$\therefore \langle \delta v^2 \rangle^{1/2}$

$$\therefore \delta \tilde{v}_j = \frac{\delta v_j}{\langle \delta v^2 \rangle^{1/2}}$$

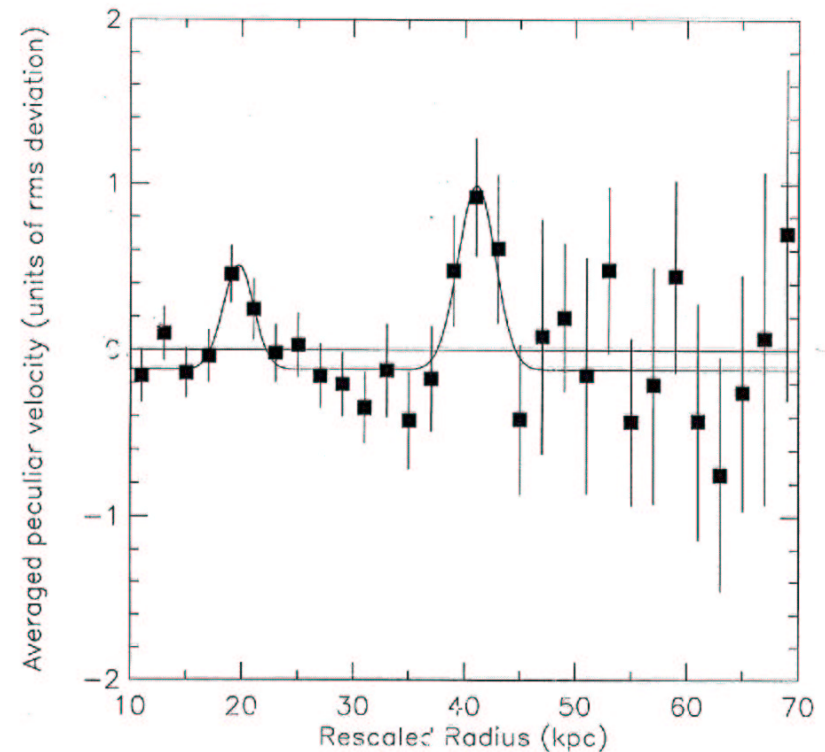
Average over the sample of 32 galaxies

$$b_i \equiv \frac{1}{N_i} \sum_{j=1}^{N_i} \delta \tilde{v}_j$$

(where  $N_i$  is the # data points in  $i^{\text{th}}$  bin)

with assigned error

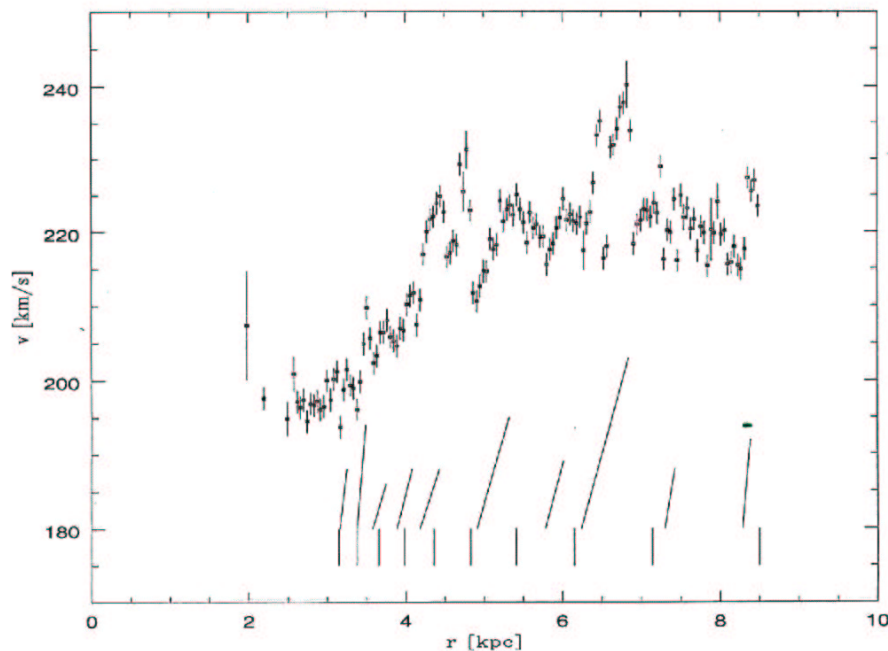
$$\Delta b_i = \frac{1}{\sqrt{N_i}}$$



W. Kinney + PS

## North Galactic Rotation Curve

D. P. Clements (1985)

derived from the Massachusetts -  
Stony Brook Galactic plane CO survey

Rises in galactic rotation curves  
have been interpreted in the past  
as due to the presence of  
**spiral arms**.

If so,

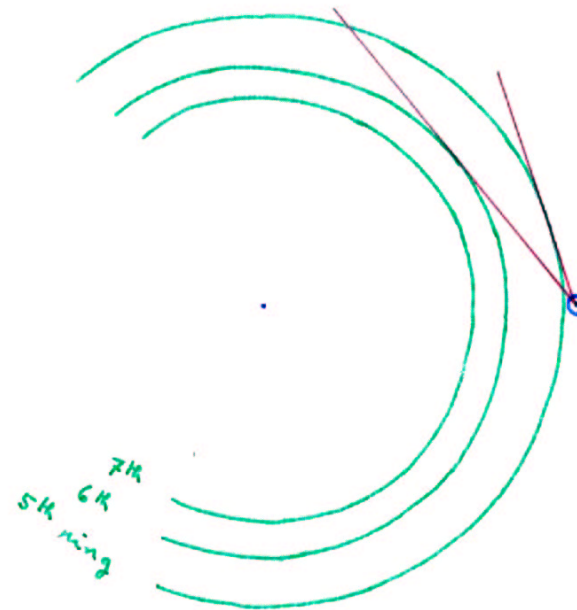
- why are there **eight** rises  
between 3 and 7 kpc?
- why are the rises so **sharp**?

$\epsilon = 0.3$   
 $j_{\max} = 0.263$   
↓

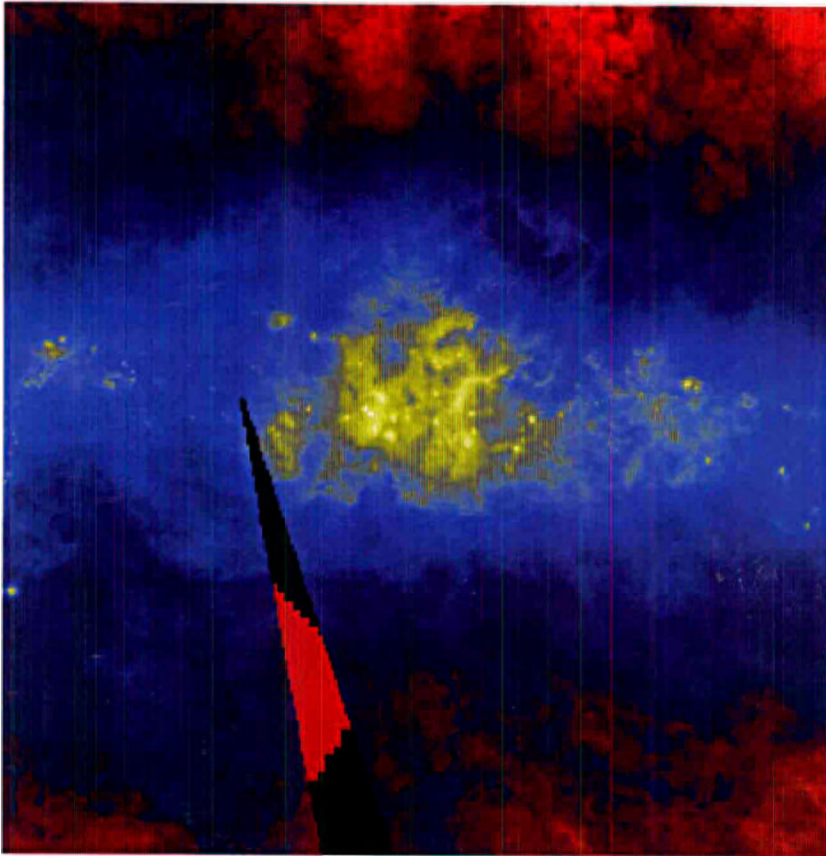
$n$	$a_n^{\text{obs}}$ (kpc)	$a_n^{\text{theor}}$ (kpc)	
1		41.2	
2		20.5	
3		13.9	
4		10.5	
5	8.28	8.50	+ 2.6 %
6	7.30	7.14	- 2.2 %
7	6.24	6.15	- 1.5 %
8	5.78	5.41	- 6.4 %
9	4.91	4.83	- 1.6 %
10	4.18	4.36	+ 4.3 %
11	3.89	3.98	+ 2.3 %
12	3.58	3.66	+ 2.2 %
13	3.38	3.38	0.0 %
14	3.16	3.15	- 0.3 %

rmsd = 3.1%

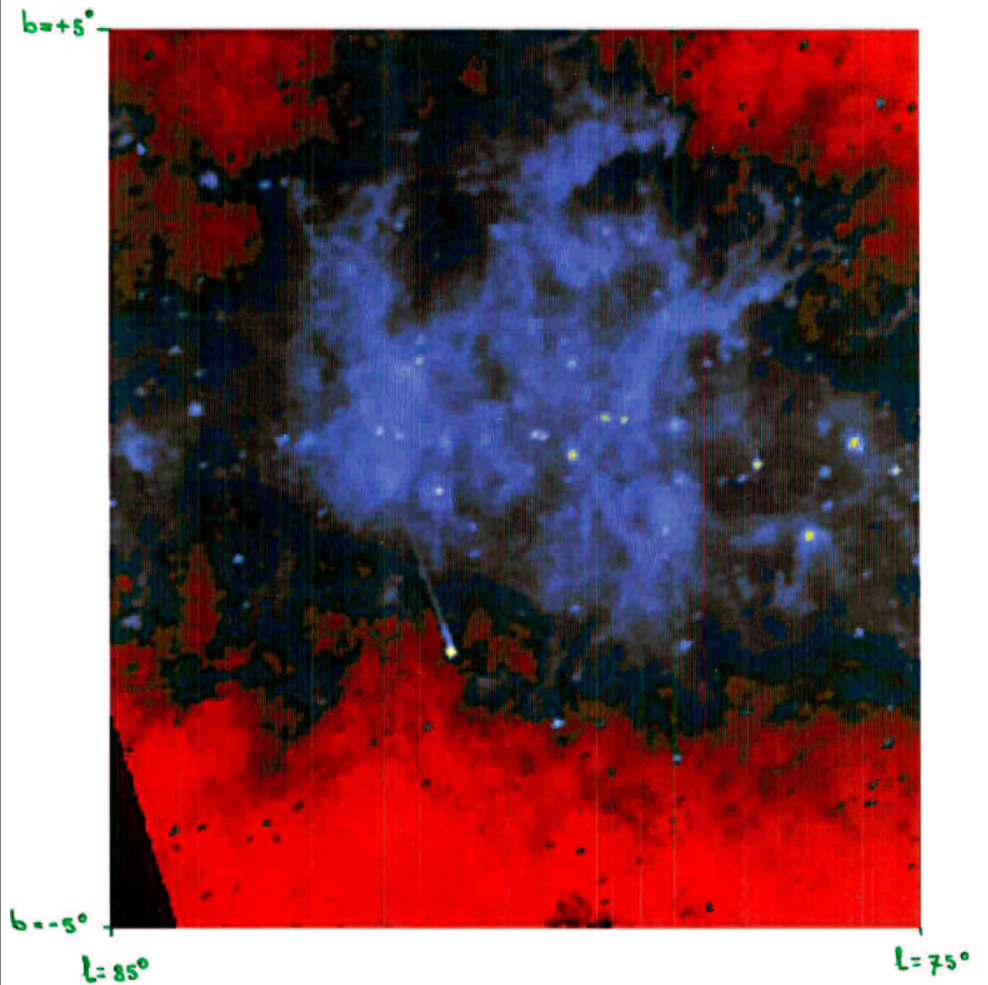
## Caustic in the Sky?



Look in the direction tangent to the ring to search for its imprint on the distribution of gas in the galactic disk



IRAS100.30.gif



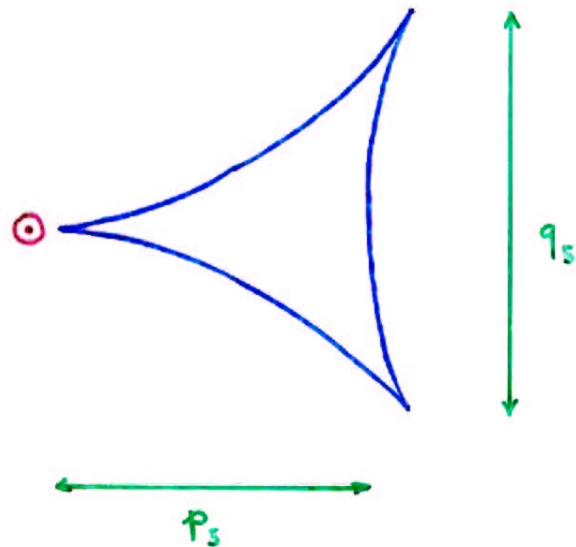
from <http://skyniew.gsfc.nasa.gov/>

IRAS12.mod.gif



## 5<sup>th</sup> Caustic parameters

(assuming  $R_{\odot} = 8.5$  kpc)



$$a_5 = 8.31 \pm 0.01 \text{ kpc}$$

$$a_5 + p_5 = 8.45 \pm 0.01 \text{ kpc}$$

$$p_5 = 134 \pm 10 \text{ pc}$$

$$q_5 = 199 \pm 14 \text{ pc}$$

Local densities and velocities  
of the  $n=5$  flows

$$d^+ = 1.7 \cdot 10^{-24} \text{ g/cm}^3$$

3X previous  
estimates of  
the total local  
DM density!

$$d^- = 0.15 \cdot 10^{-24} \text{ g/cm}^3$$

$$\vec{v}^{\pm} = (470 \hat{\phi} \pm 100 \hat{z}) \text{ km/s}$$

$(z, \phi, r)$  are galactocentric  
cylindrical coordinates

The caustics are smoothed  
by the velocity dispersion  $\delta v_{DM}$   
of infalling dark matter over  
distances of order

$$\delta a \approx R \frac{\delta v_{DM}}{v}$$

$R = \text{turnaround radius}$   
 $v = \text{flow velocity}$

From the sharpness of the edges  
of the triangular feature of the IRAS map:

$$\delta a \lesssim 20 \text{ pc}$$

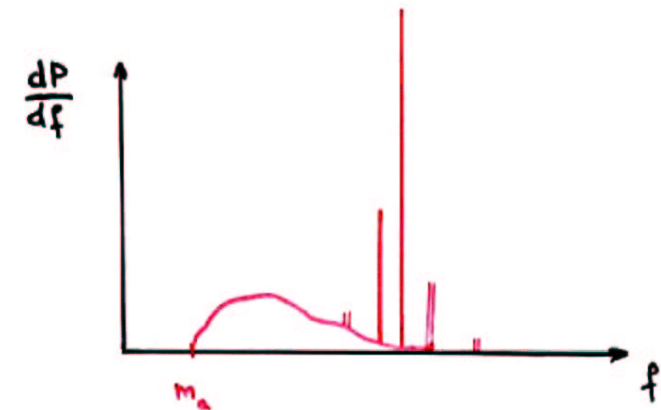
$$\therefore \delta v_{DM} \lesssim 50 \text{ m/s}$$

Signal to noise ratio in axion  
dark matter search

$$\frac{1}{n} = \frac{P_{\text{signal}}}{T_{\text{noise}}} \sqrt{\frac{t}{B}}$$

$t = \text{measurement integration time}$

$B = \text{bandwidth}$



Full signal

$$B \sim 10^{-6} m_a$$

Narrow peak

$$B \sim 10^{-10} m_a$$

$\frac{P_{\text{signal}}}{\sqrt{B}}$  is increased by the factor

$$\frac{3}{\sqrt{10^{-4}}} = 300$$

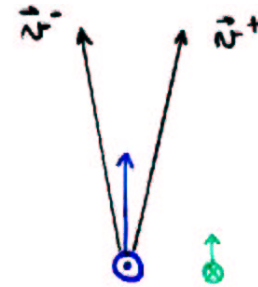
$\therefore \frac{1}{m}$  increased by a factor  $\sim 100$

Also  $T_{\text{noise}}$  is decreased by

a factor  $\frac{1}{100}$  by using

SQUIDS instead of HEMTs

J. Clarke  
et al.



The annual modulation in

WIMP searches is the reverse

of what is usually thought, i.e.

the flux is

lowest in June

highest in December

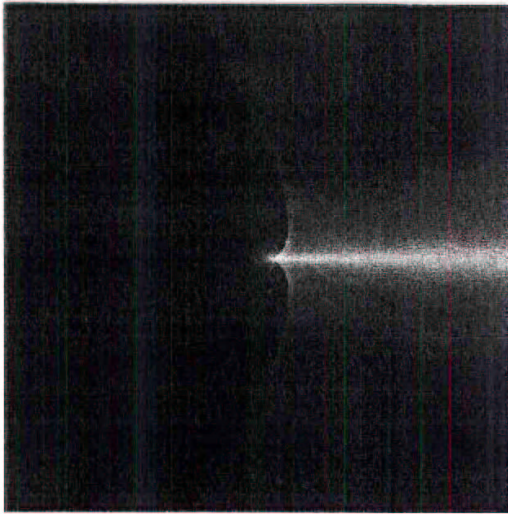
J. Vergados

A. Green

G. Gelmini + P. Gondolo

M. Kamionkowski

+ P. Ullio



picture by Ben Bromley

Solar wakes of dark matter flows

S. Wick + P.S. astro-ph/02...

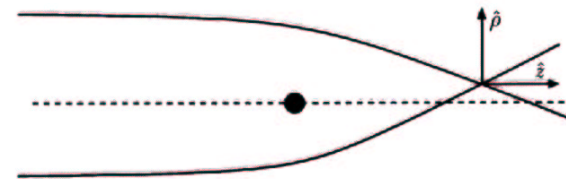
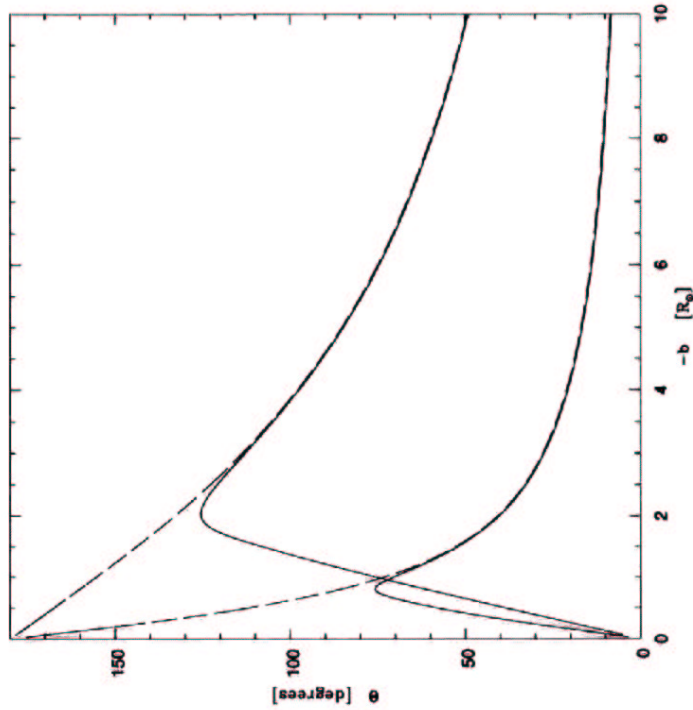


FIG. 1. Two trajectories past a spherically symmetric mass distribution. The incident flow is assumed to be uniform and to have vanishing velocity dispersion. If the scatterer is a point mass, the particles with small impact parameter are scattered by an angle close to  $180^\circ$ , and hence the number of flows  $n = 2$  everywhere. If the scatterer is distributed over a finite radius, there is a maximum scattering angle  $\Theta_{\max} < 180^\circ$ . In that case,  $n = 1$  upstream of the scatterer, and  $n = 3$  downstream.

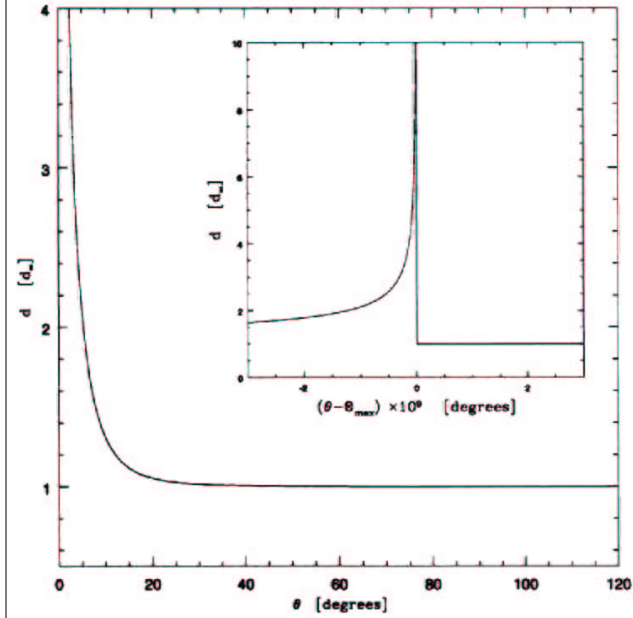
# Scattering Angle

Initial velocities: 200 km/sec and 500 km/sec  
 dashed line: 1/r potential



# Density Wake vs $\theta$

$v_\infty = 255$  km/sec  
 inset: skirt caustic at  $\theta = 115^\circ$



## Summary of Evidence for Dark Matter Caustics

- Why does the 32 galaxy composite rotation curve fluctuate upwards at 40 kpc ( $3.1\sigma$ ) and 20 kpc ( $2.6\sigma$ )?  
[probability  $\sim 3 \cdot 10^{-4}$ ]
- Why eight rises in the Milky Way rotation curve between 3 and 7 kpc?
- Why are the rises sharp?
- Why is there a triangular feature in the IRAS map of the galactic plane near  $80^\circ$  longitude?
- Why is the triangle equilateral, with axis parallel to the galactic plane, and pointing away from the galactic center?
- Why does the triangle (8.31 to 8.44 kpc) coincide with a rise in the rotation curve (8.28 to 8.43 kpc)?  
[probability  $\sim 10^{-3}$ ]

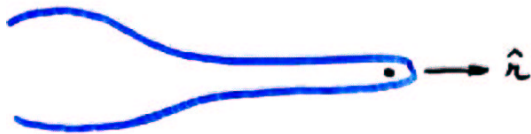


## The Big Flow:

$$d = 1.7 \cdot 10^{-24} \text{ g/cm}^3$$

3X previous estimates of the total local DM density

$$\vec{v} = (470 \hat{\varphi} \pm 100 \hat{r}) \frac{\text{km}}{\text{s}}$$



$\hat{\varphi}$  is the direction of galactic rotation

$$\vec{v}_0 = 220 \frac{\text{km}}{\text{s}} \hat{\varphi}$$