Supersymmetry and superstring: model building for dark energy

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The problems to solve

Vacuum energy or "cosmological constant" problem

Set $\lambda = 0$ but assume that there is a nonzero vacuum (*i.e.* groundstate) energy:

$$< T_{\mu\nu} >= -< \rho > g_{\mu\nu}$$

then

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G_N T_{\mu\nu} + 8\pi G_N < \rho > g_{\mu\nu}$$

effective cosmological constant:

$$\lambda_{eff} = 8\pi G_N < \rho > \equiv \Lambda^4 / m_P^2$$

$$\frac{\Lambda^4}{m_P^2} = |\lambda_{eff}| \leq H_0^2 = 10^{-120} m_P^2$$

$$\Lambda \leq 10^{-30}\, m_P \sim 10^{-3} \text{ eV}$$

Cosmic coincidence problem

Why is $\rho_\Lambda \equiv \Lambda^4 \sim \rho_H$ now?

$$\dot\rho = -3H(p+\rho) = -3\frac{\dot a}{a}(p+\rho)$$

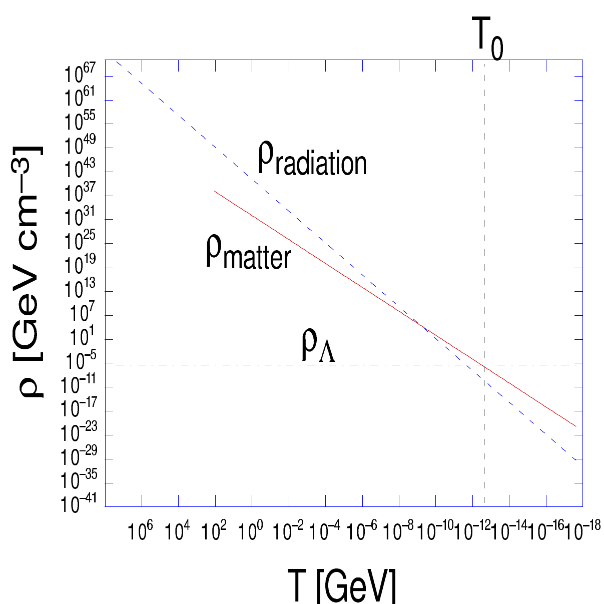
$\rightarrow$ Cosmological constant $\rho_\lambda = \text{cst}$

Matter ($p \sim 0$) $\rho_m \propto a^{-3}$

Or, to avoid any reference to us:

Why does the vacuum energy starts to dominate at a time t_Λ ($z_\Lambda \sim 1$) which almost coincides with the epoch t_G ($z_G \sim 1$ to 3) of galaxy formation?

COSMIC COINCIDENCE PROBLEM



Arkani-Hamed, Hall, Kolda, Murayama, astro-ph/0005111

Supersymmetry, a very special symmetry

Quantities conserved in interactions:

- Q electric charge
- $P_\mu = (E, \mathbf{p})$ energy-momentum $\leftrightarrow$ invariance by

  vector quantity translation
- $M_{\mu\nu}$ angular momentum $\leftrightarrow$ invariance by

  tensor quantity rotation

$\updownarrow$

spacetime transformations

Coleman-Mandula: these are the only conserved quantities which transform non-trivially under Lorentz transformations.

Exception: Q_r spinor charge (spin 1/2)

SUPERSYMMETRY

Supersymmetry is a spacetime transformation:

BOSON $\bullet$ x^μ $\overset{\text{SUSY}}{\rightarrow}$ FERMION $\overset{\text{SUSY}}{\rightarrow}$ BOSON $\bullet$ $x^\mu + a^\mu$

$$Q_r Q_s + Q_s Q_r = (\gamma^\mu)_{rs} P_\mu$$

$$\updownarrow$$

spacetime translations

Conséquence: $H = P^0 \sim \sum_r Q_r^2$

- $< 0|H|0 >= 0 \Leftrightarrow \; Q_r|0 >= 0$

Vacuum is invariant under supersymmetry if and only if vacuum energy (i.e. cosmo. constant) vanishes.

- $[H, Q_r] = 0$

Bosons and fermions associated by supersymmetry are degenerate in mass. Non observed in nature

$\rightarrow$ supersymmetry is spontaneously broken.

Local supersymmetry/Supergravity

$$\begin{array}{ccc} g_{\mu\nu} & \overset{SUSY}{\Longrightarrow} & \psi_\mu \\ \text{graviton} & & \text{gravitino} \end{array}$$

Possible to cancel the vacuum energy by the supersymmetric combination: ("super-cosmological constant term"):

$$\mathcal{S} = \int d^4x \sqrt{g} \left[3m_P^2 {m_{3/2}}^2 - m_{3/2} \bar{\psi}_\mu \sigma^{\mu\nu} \psi_\nu \right]$$

$$\begin{array}{ccc} \text{Global SUSY br.} & & \text{Local SUSY br.} \\ \Lambda^4 \neq 0 & \longrightarrow & m_{3/2} \neq 0 \\ & & \Lambda = \text{ or } \neq 0 \end{array}$$

It remains true that, in many models, a supersymmetric background is associated with a vanishing vacuum energy.

Flat directions of the scalar potential: directions in field space along which the scalar potential vanishes.

Quantum theory study of strong interactions led to the study of long flux tubes

or open strings

closed string

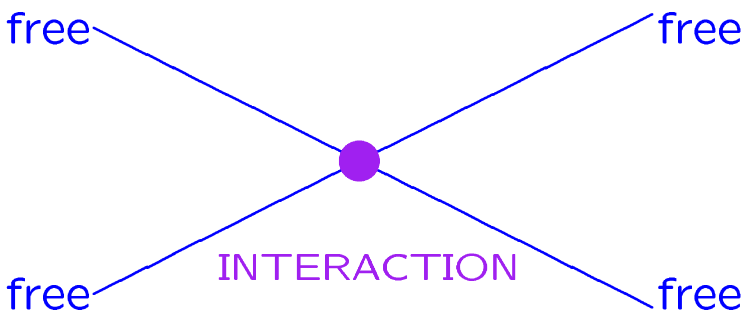
$M_S \sim 1$ GeV

oscillation mode $\ni$ massless resonance of spin 2 (graviton)

$\rightarrow M_S \sim 10^{19}$ GeV

General relativity
Quantum theory $\Big\} \rightarrow$ String theory $\leftarrow$ extra dimensions

String theory is based, as field theory, on the concept of S-matrix



or in string theory

Easy to implement in an asymptotically flat or AdS spacetime.

Notoriously difficult in de Sitter spacetime

Susskind...

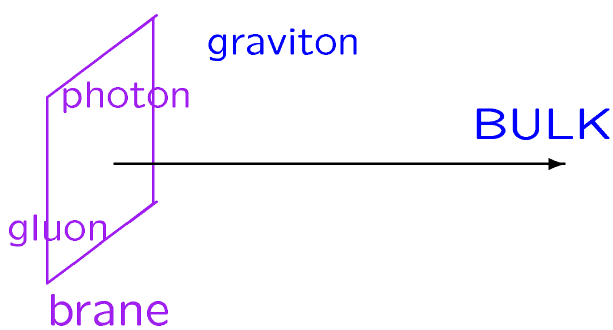
String theory is a laboratory for new concepts:

- many scalar fields, often associated with flat directions

dilaton ϕ $\Leftrightarrow$ string coupling	$\leftrightarrow$	scalar – tensor theory of gravity
moduli $\leftrightarrow$ radii, $\cdots$		

- brane = locus of end points of open strings

Braneworld set up



Allows to use supersymmetry to cancel the bulk cosmological constant in the case of asymptotically flat spacetime: supersymmetry breaking is only effective on the brane (non-BPS)

- presence of tensor fields coupled to the brane world volume

Solutions to the problems?

• Vacuum energy: Models of relaxation of the cosmological constant

$\rightarrow$ violations of Lorentz invariance

• Cosmic coincidence: Models of quintessence

$\rightarrow$ time variation of constants

$\rightarrow$ violations of equivalence principle

A no-go theorem

Weinberg

Not possible to have a vanishing λ as a consequence of the equation of motion of some fields

Consider constant fields ϕ_n: eq. of motion $\delta\mathcal{L}/\delta\phi_n = 0$

Remember that $\delta\left(\sqrt{g}\lambda\right) = \sqrt{g}g^{\mu\nu}\delta g_{\mu\nu}\lambda/2$

Hence would need

$$g_{\mu\nu}\frac{\delta\mathcal{L}}{\delta g_{\mu\nu}} = \sum_n \frac{\delta\mathcal{L}}{\delta\phi_n} f_n(\phi)$$

Amounts to a symmetry cond: invariance of $\mathcal{L}$ under

$$\delta g_{\mu\nu} = 2\epsilon g_{\mu\nu},\;\; \delta\phi_n = -\epsilon f_n(\phi)$$

Possible to redefine the fields $\phi_n \to \phi, \sigma_a$ such that

$$\delta g_{\mu\nu} = 2\epsilon g_{\mu\nu},\;\; \delta\phi = -\epsilon,\;\; \delta\sigma_a = 0$$

Then $\quad \mathcal{L} = e^{4\phi}\sqrt{g}\mathcal{L}(\sigma).$

Key assumption: Lorentz invariance, finite number of fields

- Dolgov compensation mechanism

involves free massless vector or tensor (non-gauge) fields e.g. $\mathcal{L} \sim \eta D_\mu A_\nu D^\mu A^\nu$

⊖ leads to strong breaking of Lorentz invariance

Rubakov, Tinyakov

- Brown-Teitelboim mechanism

generalizes particle creation by electric field in 1+1 dimensions to higher dimensions (and a 3-form field coupled to a membrane in 3+1 dimensions)

⊕ generalizable to string set up

⊖ requires very small membrane charges:

non-perturbative effects

Feng, March-Russell, Sethi, Wilczek

multiple 4-forms

Bousso, Polchinski

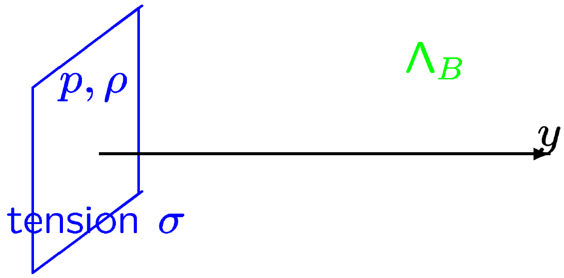
- Rubakov relaxation of cosmological cst at inflation

⊕ couples the problem to inflation

Brane universes and self-tuning

Arkani-Hamed, Dimopoulos, Kaloper, Sundrum; Kachru, Schultz, Silverstein

Braneworld set up



Cosmological constant on the brane:

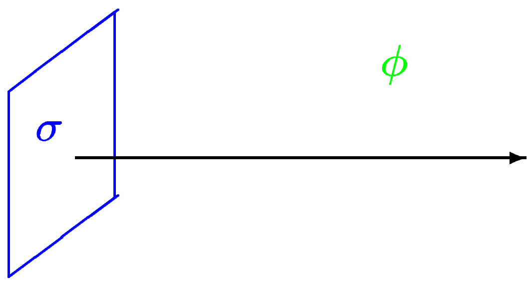
$$\lambda = \frac{1}{2M_5^3}\,\Lambda_B + \frac{1}{12M_5^6}\,\sigma^2$$

M_5 5-dimensional Planck scale

⊖ requires fine-tuning between string tension and bulk vacuum energy (Randall-Sundrum condition)

⊕ decouples the cosmological constant from the brane vacuum energy i.e. the tension σ

SELF TUNING: introduce a bulk scalar field ϕ which couples conformally to matter on the brane



$$\mathcal{S} \;=\; \mathcal{S}_{bk} + \mathcal{S}_{br} = M_5^3 \int d^5x \sqrt{|g_5|} \left[\frac{1}{2} R^{(5)} - \frac{3}{2} \partial^m \phi \, \partial_m \phi - 3\mathcal{V}(\phi) \right] + \int_{\text{brane}} d^4x \sqrt{|g_4|} \, f^2(\phi) \, (-\sigma),$$

One finds static spatially flat solutions to the classical equations of motion for any value of the brane tension

e.g. $\quad \mathcal{V}(\Phi) = 0 \quad , \quad f^2(\phi) = Ce^{\mp\phi}$

Warped background $ds^2 = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$

$$A(y) \;=\; \frac{1}{2} \ln \left(1 - \frac{|y|}{y_c} \right)$$

$$\phi(y) \;=\; \phi_0 \pm \ln \left(1 - \frac{|y|}{y_c} \right)$$

Naked singularity at $|y| = y_c > 0$

• Fine tuning associated with the presence of the singularity: one may cure the singularity by adding a second brane but the content of the second brane is then fine-tuned.

Forste, Lalak, Lavignac, Nilles

• Include the one-loop corrections to gravity in the bulk

P.B., C. Charmousis, Stephen Davis, Jean-François Dufaux

$$
\begin{aligned}
S_{\text{bulk}} &= \frac{M_5^3}{2}\int d^4x dy \sqrt{-g}\,\{R - \zeta(\nabla\phi)^2 \\
&\quad + \alpha e^{-\zeta\phi}\left[\mathcal{L}_{GB} + c_2(\nabla\phi)^4\right] - 2\Lambda_B e^{\zeta\phi}\} \\
S_{\text{brane}} &= -\int d^4x \sqrt{|g_4|}\,\sigma e^{\chi(\phi)}
\end{aligned}
$$

with the Gauss-Bonnet combination

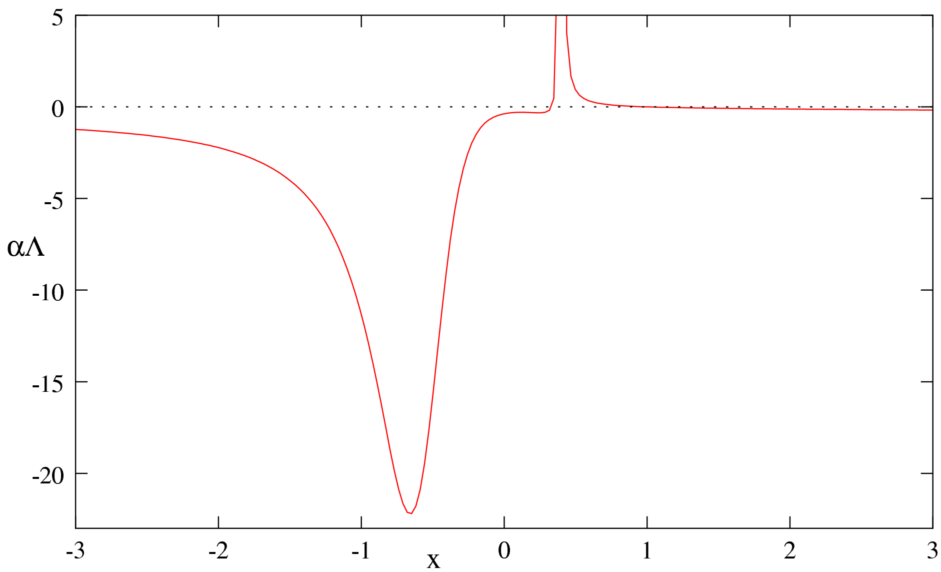
$$\mathcal{L}_{GB} = R^2 - 4R_{ab}R^{ab} + R^{abcd}R_{abcd}$$

There exist solutions with no naked singularity at finite distance from the brane

cf. singularity-free cosmological solutions of Antoniadis, Rizos, Tamvakis

$$
\begin{aligned}
A(y) &= A_0 + x \ln\left(1 + \frac{|y|}{y_c}\right) \\
\phi(y) &= \phi_0 - \frac{2}{\zeta} \ln\left(1 + \frac{|y|}{y_c}\right)
\end{aligned}
$$

$\alpha\Lambda_B$ as a function of x:



$$
\begin{aligned}
M_P^2 &= M_5^3 \int_0^\infty dy e^{2A(y)} \left(1 + 4\alpha e^{-\zeta\phi(y)} (3A'^2(y) + 2A''(y)]\right) \\
&\propto \left[\frac{y_c}{2x+1}\left(1 + \frac{y}{y_c}\right)^{2x+1}\right]_0^\infty
\end{aligned}
$$

Planck mass finite for $x < -1/2$

• Replace the bulk scalar field by a black hole configuration in the bulk

→ Flat branes in black hole backgrounds

Csaki, Erlich, Grojean

Possible to find flat brane solutions without naked singularities, with enough parameters to avoid fine tuning of the theory

In the case of AdS-Reissner-Nordstrom black hole, enough parameters (M, Q) so that they can be tuned to take into account variations of the brane vacuum energy

Leads to violations of Lorentz invariance

Kalbermann,Halevi;Chung,Freese;Ishihara;Chung,Kolb,Riotto;

Csaki,Erlich,Grojean

Bulk metric:

$$ds^2 = -\ h(y)\ dt^2 + y^2\ d\mathbf{x}^2 + \frac{1}{h(y)} dy^2$$

$h(y)$ describes the black hole configuration

Speed of light (confined to the brane)

$\neq$

Speed of gravity waves

Quintessence

Why not replace λ by a dynamical (i.e. time-dependent) component with negative pressure (to have acceleration $\rho + 3p < 0$)

Hence equation of state:

$$p = w\rho \qquad (-1 \leq) w \leq 0$$

Several candidates:

- network of light, nonintercommuting topological defects

Vilenkin:Spergel,Pen

$w = -n/3$ $\qquad$ n dimension of the defect

- very light pseudo-Goldstone bosons

Frieman,Hill,Stebbins,Waga

- quintessence

Wetterich; Ratra, Peebles; Caldwell, Dave, Steinhardt ...

Tracker field

Ratra, Peebles; Caldwell, Dave, Steinhardt

$$V(\phi) = \lambda \frac{\Lambda^{4+\alpha}}{\phi^\alpha}, \quad \alpha > 0$$

motivated by dynamical supersymmetry breaking

P.B.

$$\frac{\rho_\phi}{\rho_B} \sim \frac{a^{-3\alpha(1+w_B)/(2+\alpha)}}{a^{-3(1+w_B)}} \text{ decreases}$$

$$w_\phi = -1 + \frac{\alpha(1+w_B)}{2+\alpha}$$

Example of models

- k-essence

Armendariz-Picon, Mukhanov, Steinhardt

Addresses the question as to why the quintessence component has started to dominate only recently i.e. during radiation domination

Model uses non-linear kinetic terms for the scalar field:

$$\mathcal{L}_{\rm kin} = \phi^{-2}\mathcal{K}\left[(\nabla\phi)^2\right]$$

Radiation-dominated era: $\rho_\phi/\rho_{\rm rad} \sim$ cst

Matter-dominated era: $\rho_\phi < 0$

$\rho_\phi/\rho_{\rm mat} \nearrow$ until ρ_ϕ dominates.

- Run-away dilaton

Gasperini, Piazza, Veneziano

Quintessence field is the dilaton in the strong (string) coupling limit $\phi \to \infty$

Saturation assumed in this limit:

$$\alpha_{GUT} \sim C_A + \mathcal{O}(e^{-\phi}) \quad , \quad (M_P/_S)^2 \sim N + \mathcal{O}(e^{-\phi})$$

⊖ remains to explain why $\lambda = 0$ in this limit

How to test experimentally quintessence?

$\rightarrow$ time i.e. redshift dependence of the equation of state parameter $w_\phi(z)$ over the range $0 < z < 2$

Huterer, Turner...

$\rightarrow$ time variation of couplings

Wetterich...

$\rightarrow$ violations of the equivalence principle

Damour, Piazza, Veneziano

Quintessential problems

Kolda, Lyth

Main problem → the quintessence field must be extremely weakly coupled to ordinary matter:

- difficult to achieve a monotonically decreasing potential because higher order corrections are increasing for large field values

- quintessence field must be very light

$$m_\phi \sim \Lambda \left(\frac{\Lambda}{m_P}\right)^{1+\alpha/2} \sim H_0 \sim 10^{-33}\ \mathrm{eV}$$

favors pseudo-Goldstone boson?

- difficult to find a symmetry which prevents coupling to gauge fields $\beta\phi^n F^{\mu\nu}F_{\mu\nu}$

Conclusion

⊕ Until now supersymmetry provides the only symmetry argument that would “protect” the vacuum energy

Is it a sign that supersymmetry is a good symmetry in most of the Universe?

It does not seem to be in the observable part of our Universe.

⊕ String theory provides a (tight) framework for a large number of new or not so new concepts

Extra dimensions, brane, scalar fields, tensor fields...

⊖ If string theory is the quantum theory, it is a worry that it has not given yet the solution to the cosmological constant problem

Do we miss something or does it mean that the theory needs some major revamping?