Thermal Evolution of Fluid Permeability in Sea Ice

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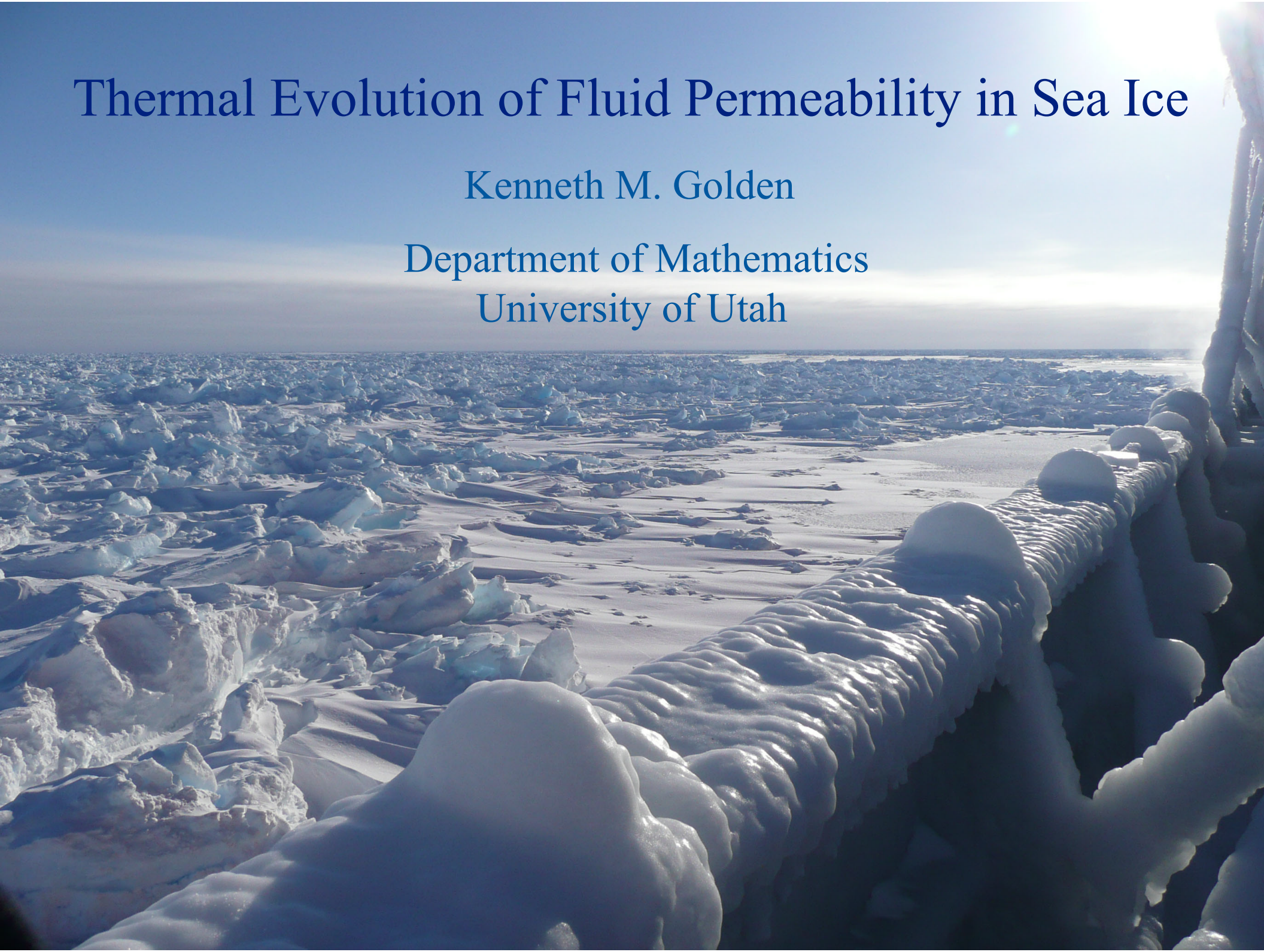

Collaborators

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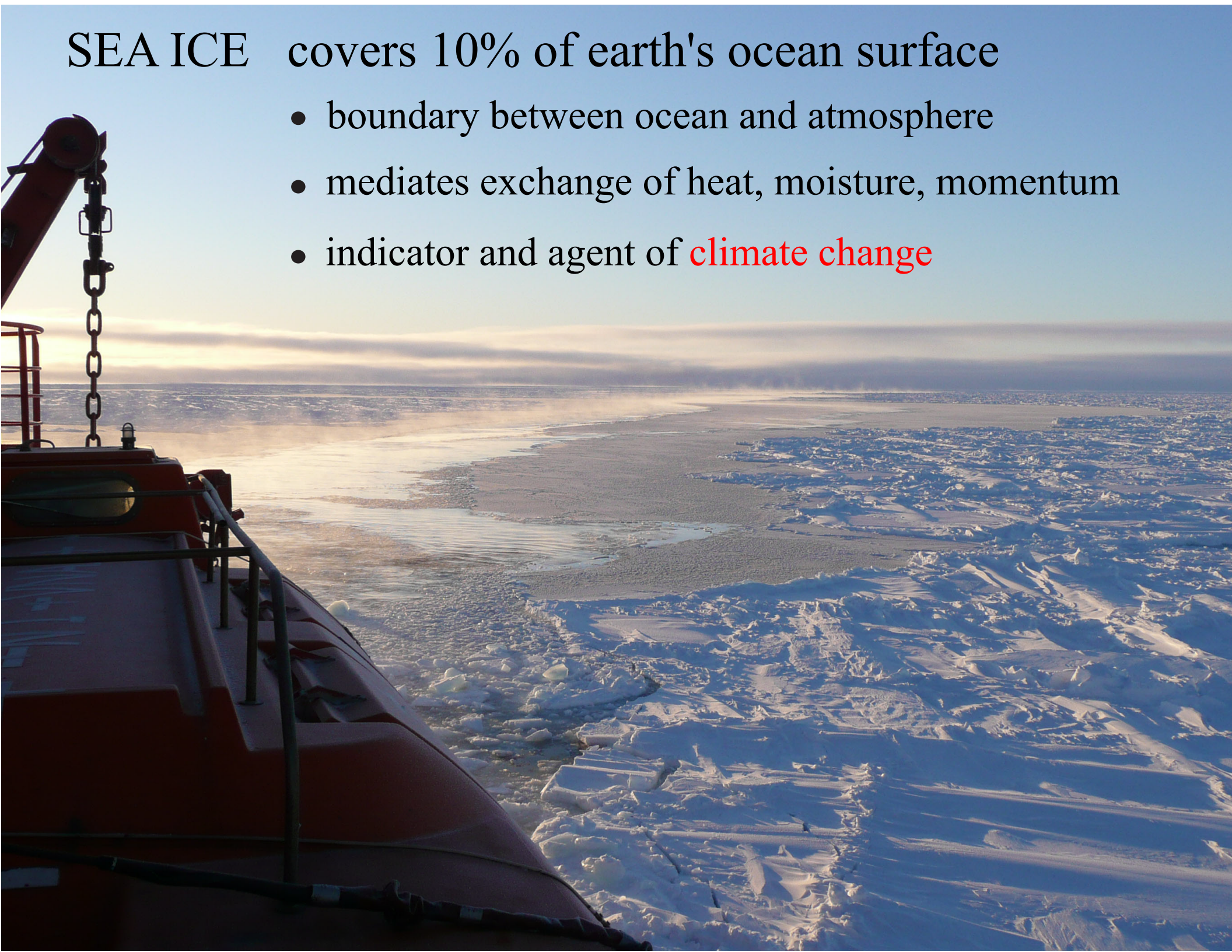
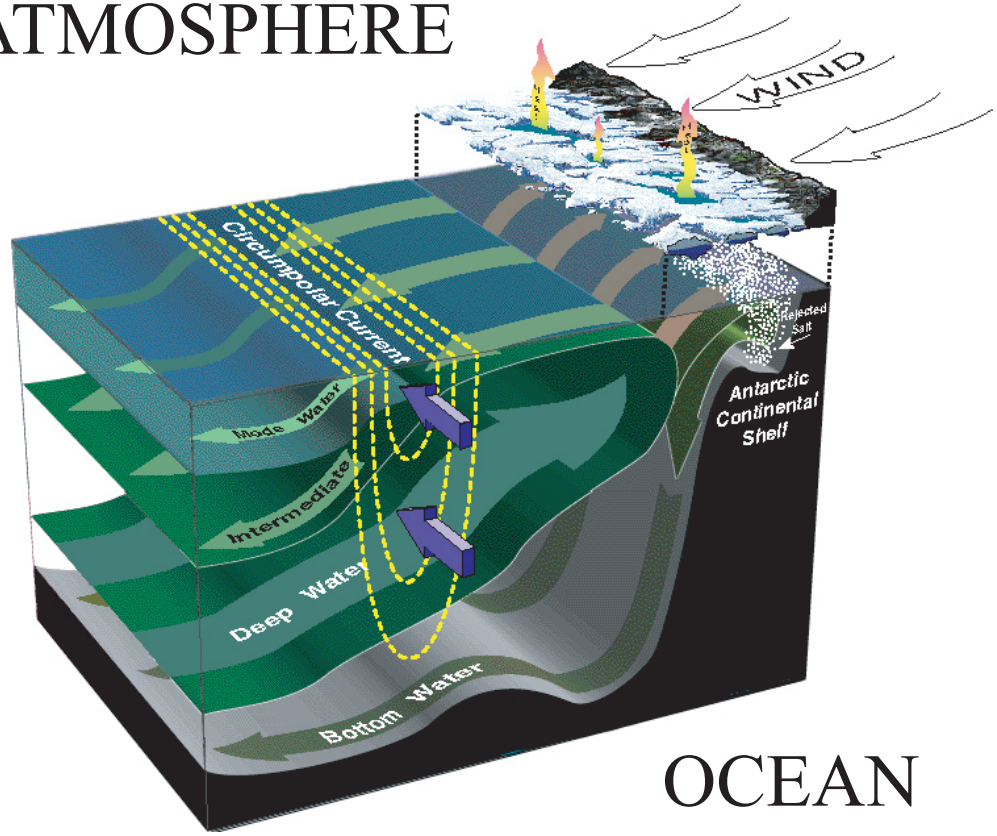

SEA ICE covers 10% of earth's ocean surface
• boundary between ocean and atmosphere
• mediates exchange of heat, moisture, momentum
• indicator and agent of climate change

Southern Ocean off Wilkes Land
Antarctica, September 2007

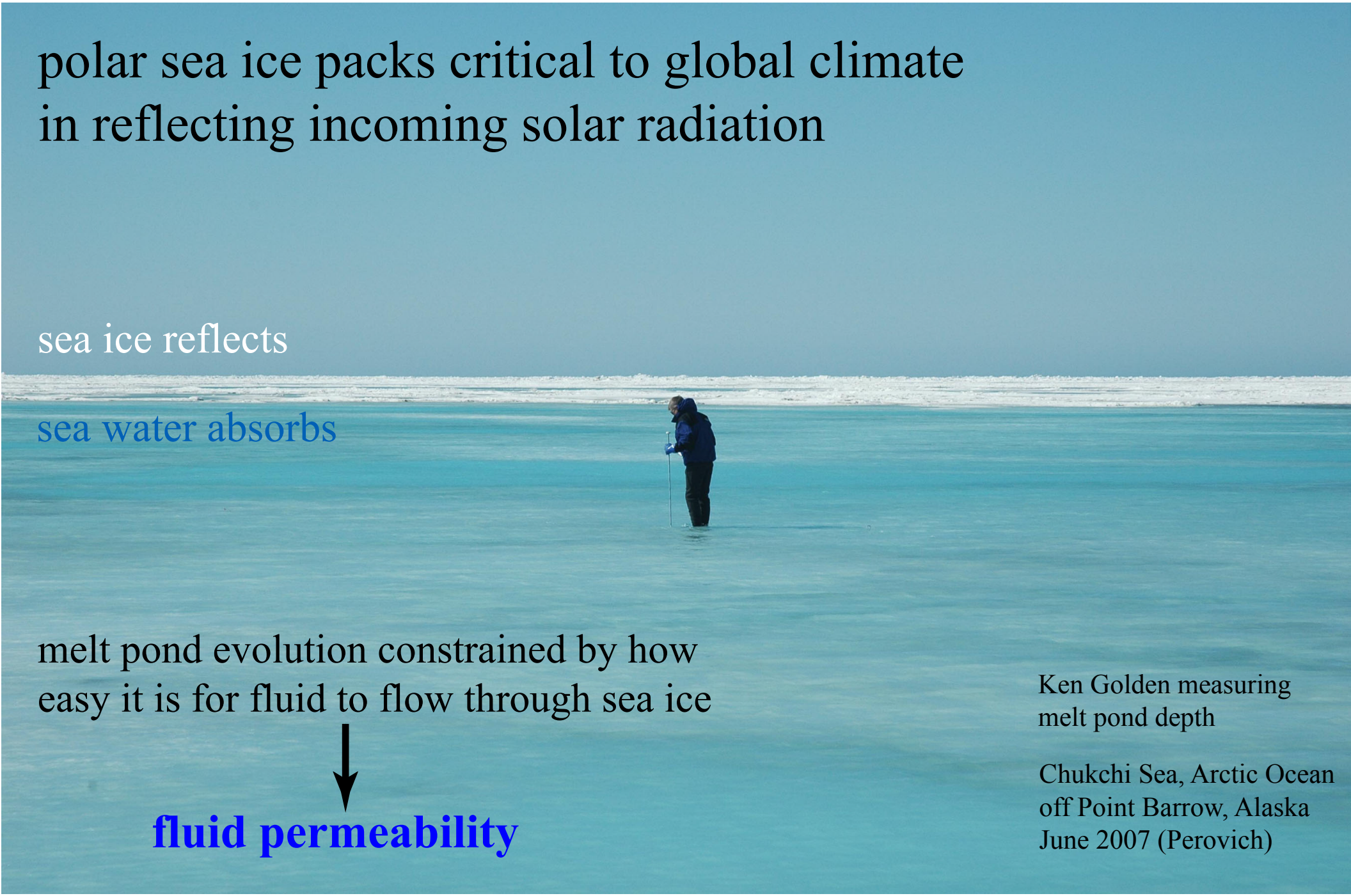
ATMOSPHERE



OCEAN

OCEAN CIRCULATION

- **freezing sea ice rejects dense brine**
- **melting sea ice drains fresh water**



polar sea ice packs critical to global climate
in reflecting incoming solar radiation

sea ice reflects

sea water absorbs

melt pond evolution constrained by how
easy it is for fluid to flow through sea ice

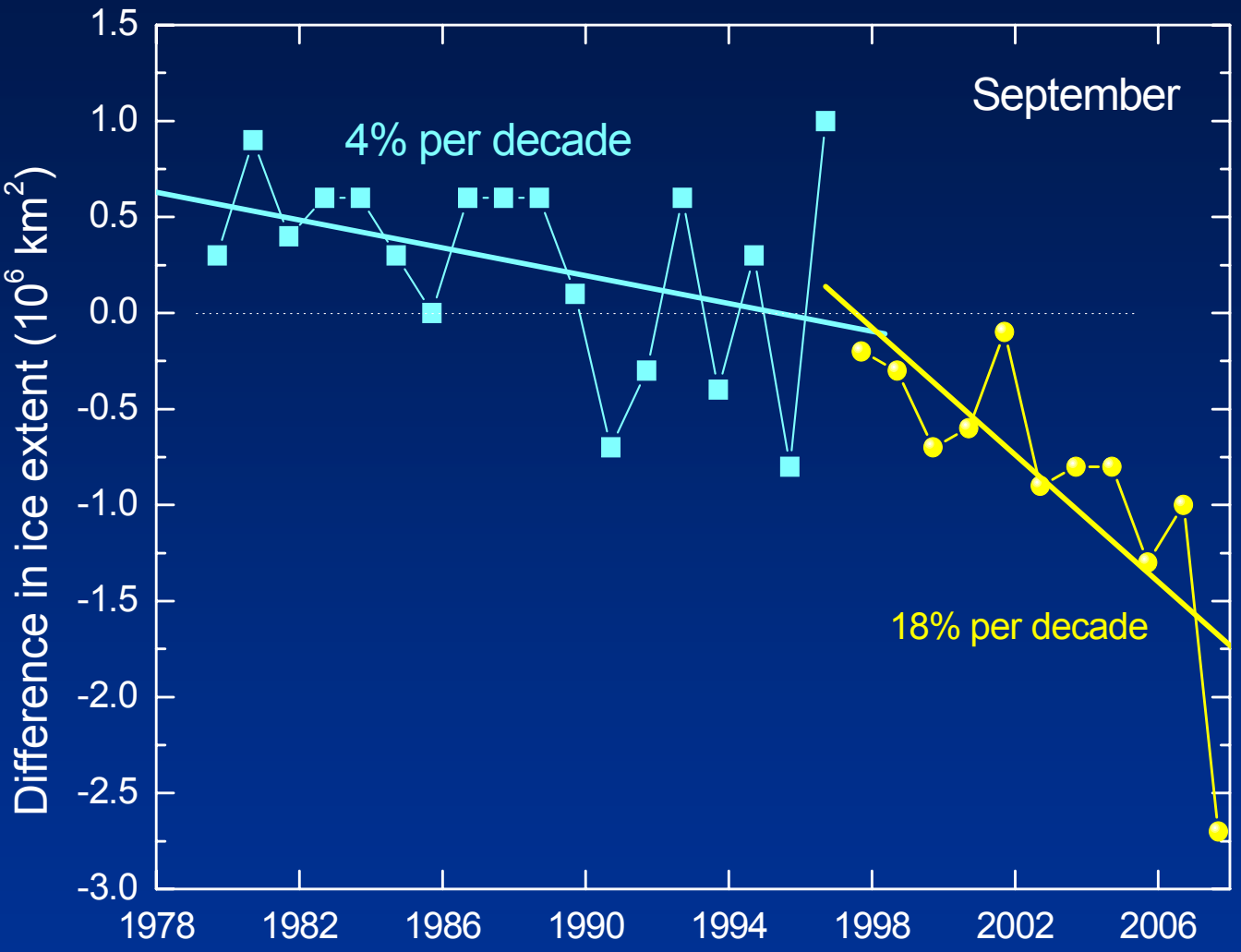
↓

fluid permeability

Ken Golden measuring
melt pond depth

Chukchi Sea, Arctic Ocean
off Point Barrow, Alaska
June 2007 (Perovich)

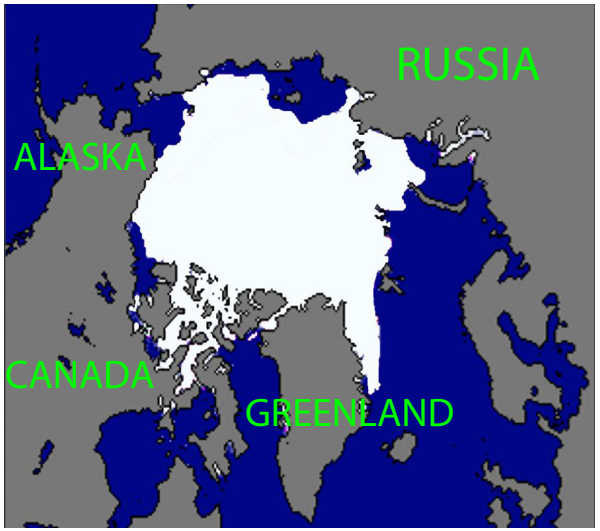
Arctic sea ice extent

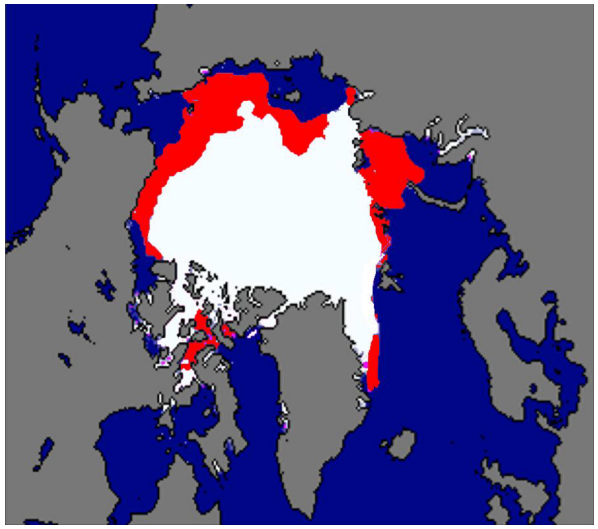


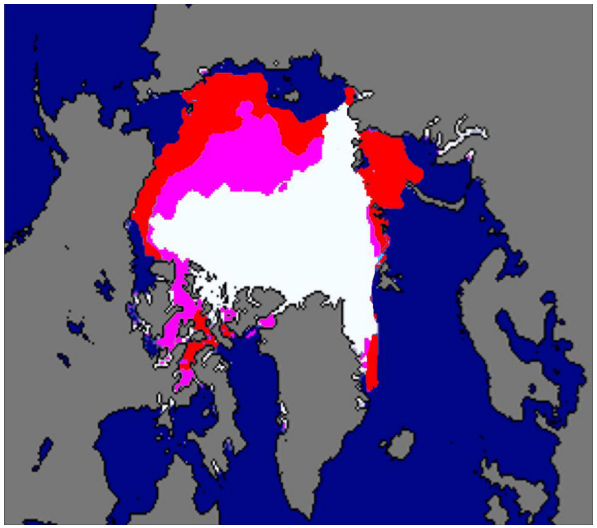
Record decrease

Record minimum

Rapid change .... tipping point? (D. Perovich, Nov. 07)



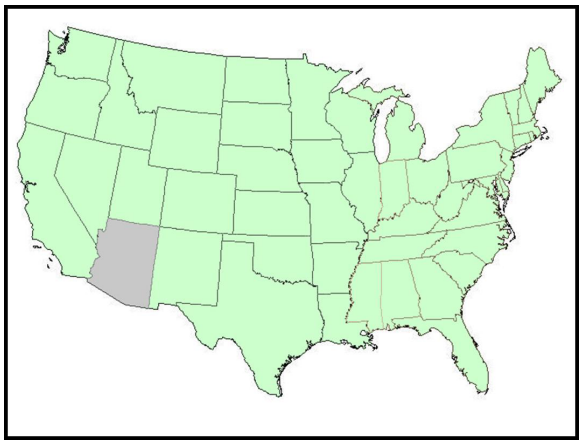


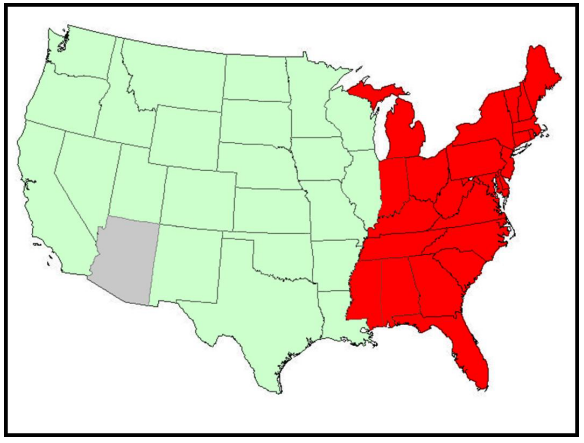


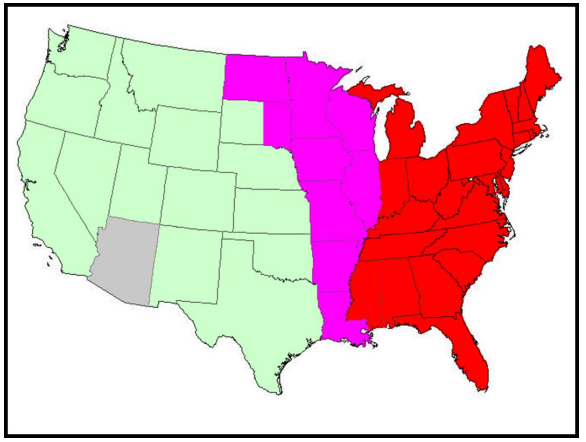
1980

2005

2007







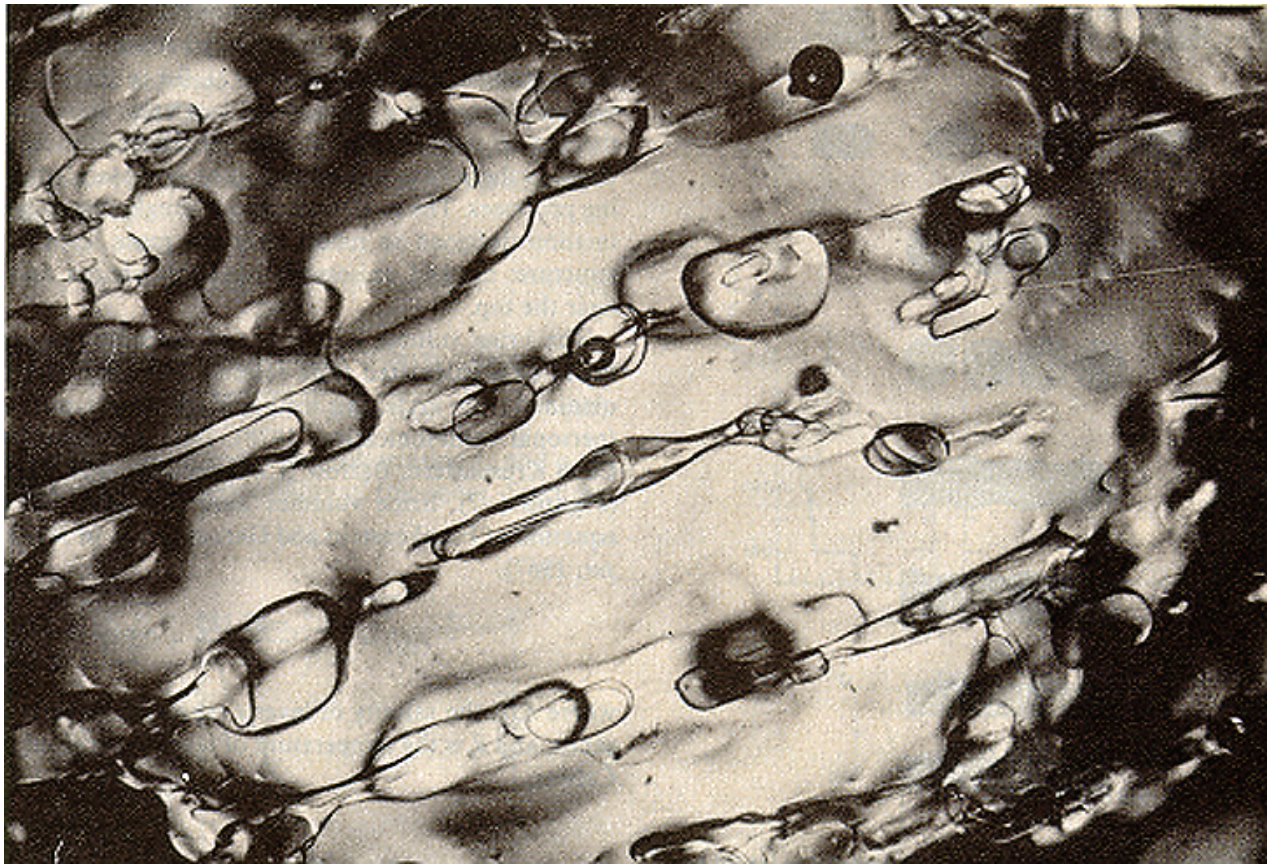
What this talk is about:

- salt water flowing through sea ice is important to geophysics and biology of the polar regions

- our work on permeability of sea ice can be used in
  Global Climate Models
  Biogeochemical Models

  to refine predictions of climate change & ecosytem response

sea ice is a *composite material*



pure ice with **brine**, air, and salt inclusions

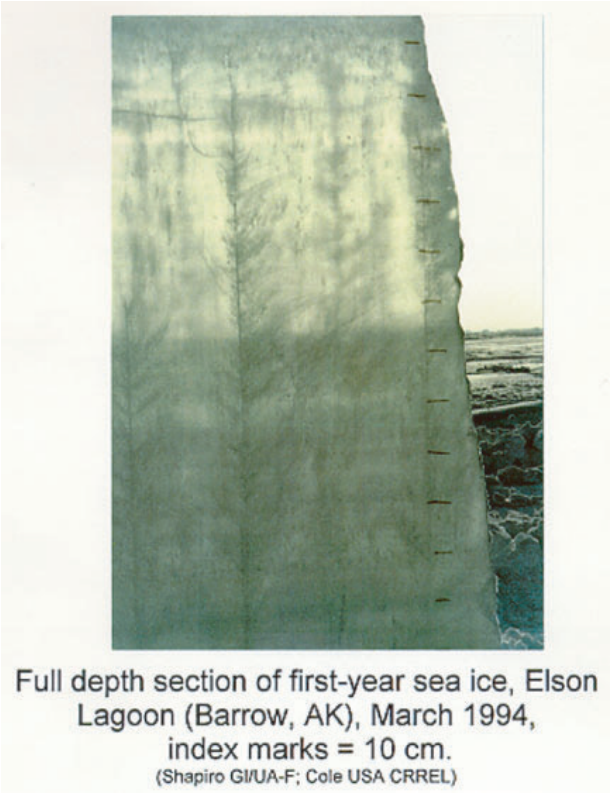
... with multiscale structure

millimeter scale

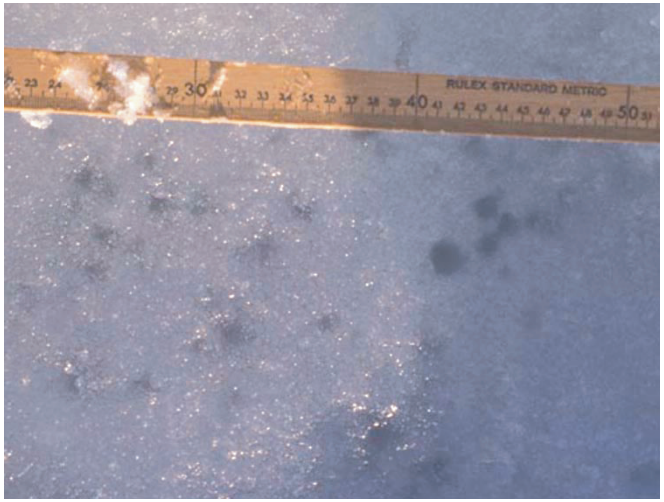
brine inclusions

centimeter scale

polycrystalline

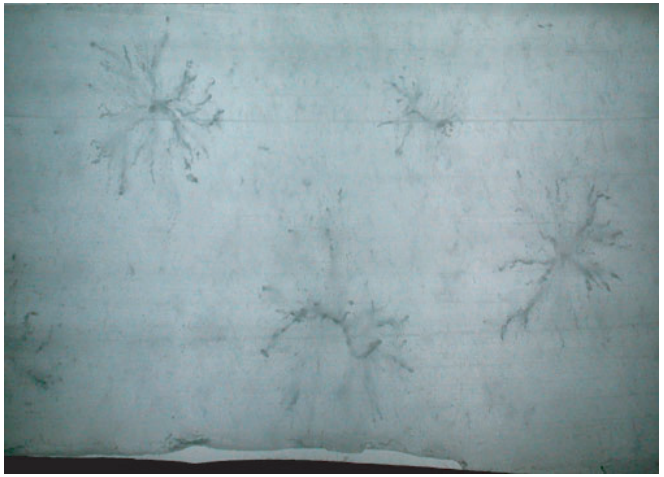


Full depth section of first-year sea ice, Elson Lagoon (Barrow, AK), March 1994, index marks = 10 cm.
(Shapiro GI/UA-F; Cole USA CRREL)



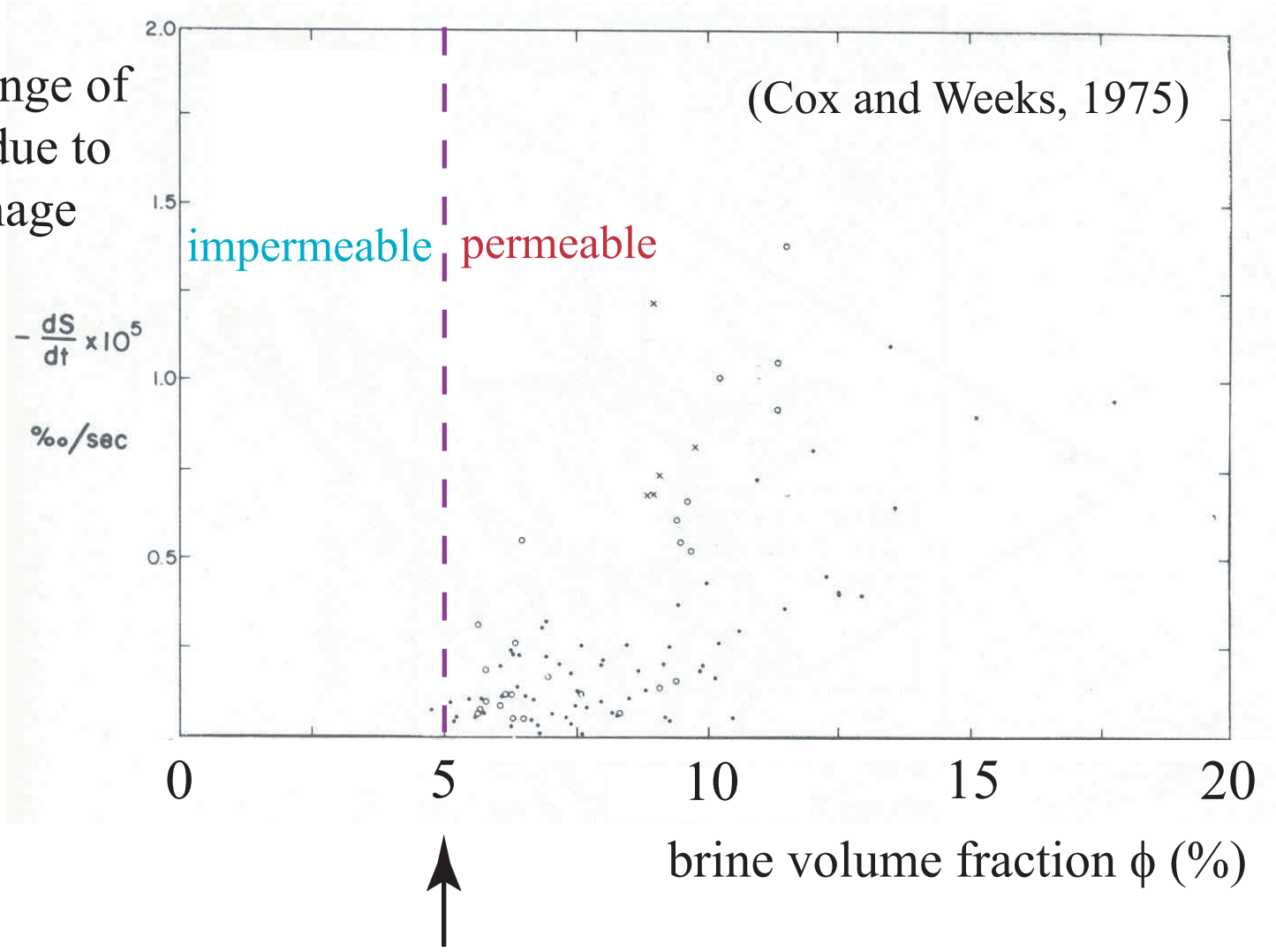
meter scale

brine channels



Critical behavior of fluid transport in sea ice

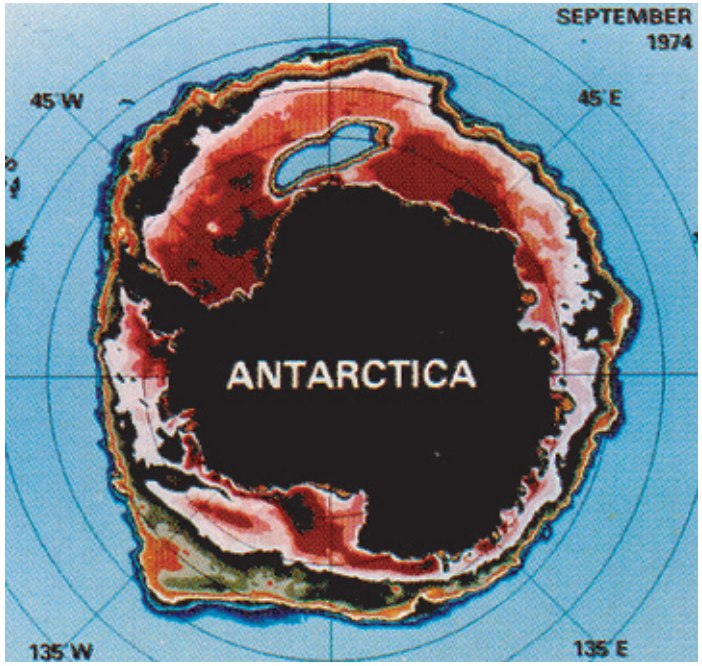
rate of change of salinity S due to brine drainage



critical brine volume fraction $\phi_c \approx 5\%$ ⟷ $T_c \approx -5^\circ$C, $S \approx 5$ ppt

RULE OF FIVES

sea ice permeability constrains:



Antarctic Zone Flux Experiment (ANZFLUX)

Eastern Weddell Gyre
July - August, 1994

- **snow-ice production -** freezing of flooded ice surfaces

  Ackley, Lytle, Golden, Darling, Kuehn, 1995
  Maksym and Jeffries, 2001, ...

- enhancing **thermal fluxes**

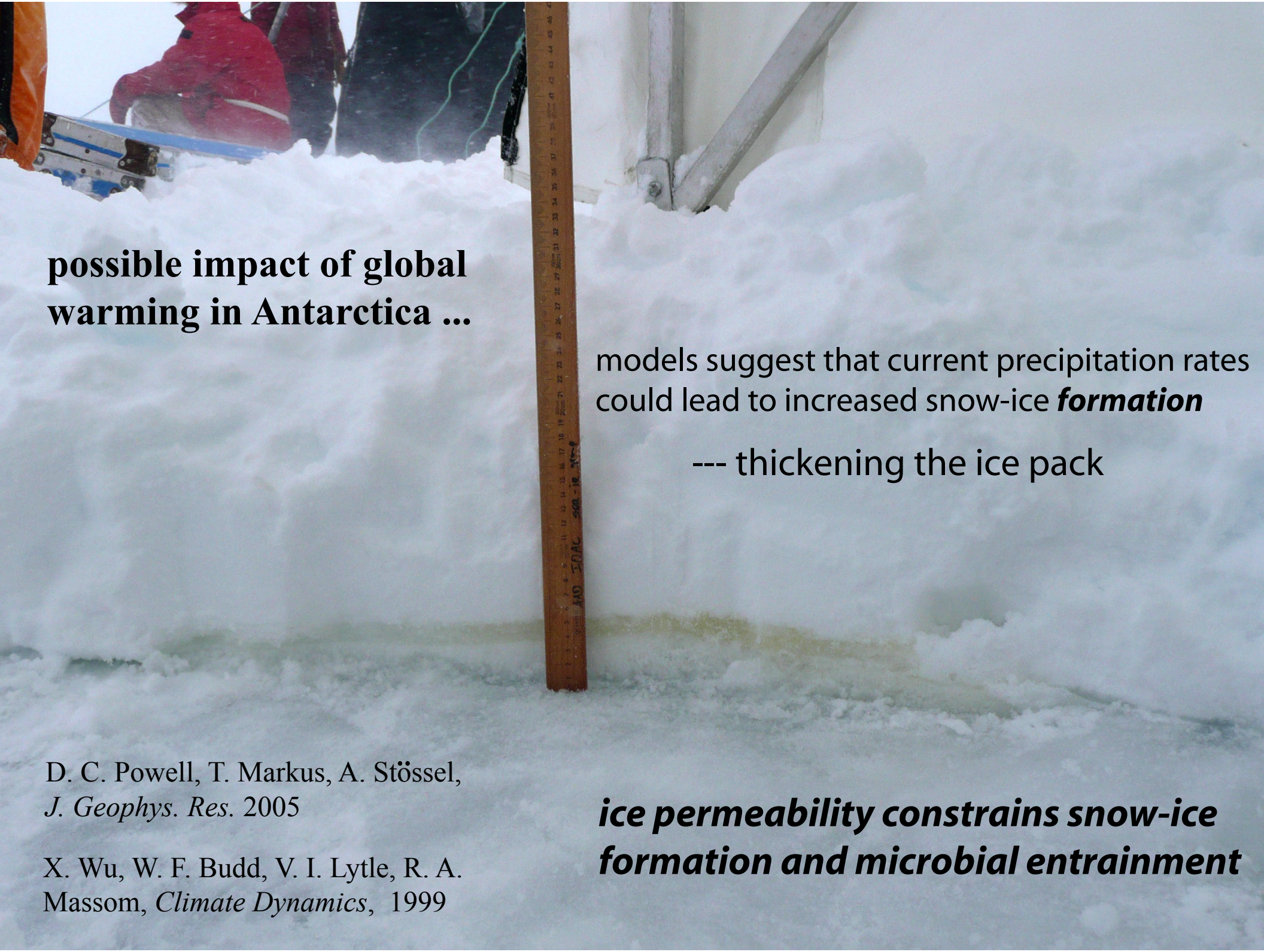
  Lytle and Ackley, 1996
  Trodahl, *et. al.,* 2000, 2001

- microwave signatures in **remote sensing**

  Lytle and Golden, 1995
  Hosseinmostafa, *et. al.,* 1995, ...

- evolution of **albedo (reflectance)** of pack

  Eicken, Grenfell, Perovich,
  Richter-Menge, Frey, 2004



possible impact of global warming in Antarctica ...

models suggest that current precipitation rates could lead to increased snow-ice ***formation***

--- thickening the ice pack

D. C. Powell, T. Markus, A. Stössel, *J. Geophys. Res.* 2005

X. Wu, W. F. Budd, V. I. Lytle, R. A. Massom, *Climate Dynamics*, 1999

ice permeability constrains snow-ice formation and microbial entrainment

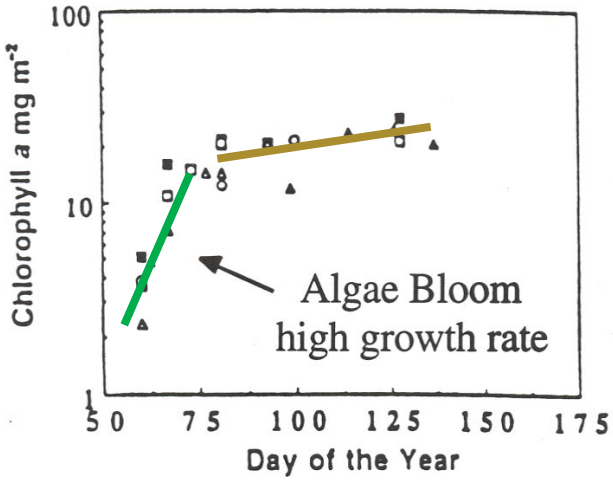
sea ice biology

robust algal and bacterial communities support rich ***food webs*** in polar oceans

nutrient replenishment controlled by ice permeability

Convection-fueled algae bloom
Ice Station Weddell

(Fritsen, Lytle, Ackley, Sullivan, *Science* 1994)



algae

krill

penguins

leopard seal

Darcy's Law for slow viscous flow in a porous medium

averaged fluid velocity

pressure gradient

$$\mathbf{v} = -\frac{k}{\eta}\nabla p$$

viscosity

k = fluid permeability of porous medium

example of *homogenization*

mathematics for analyzing effective behavior of heterogeneous systems

Unified approach to understanding fluid permeability in sea ice:

Thermal evolution of permeability and microstructure in sea ice, K. M. Golden, H. Eicken, A. L. Heaton, J. Miner, D. Pringle, and J. Zhu, *Geophysical Research Letters,* vol. 34, 2007 (+ cover).

1. Homogenization and Darcy's law
2. Rigorous bounds
3. Percolation theory
4. Hierarchical models
5. Network model

+

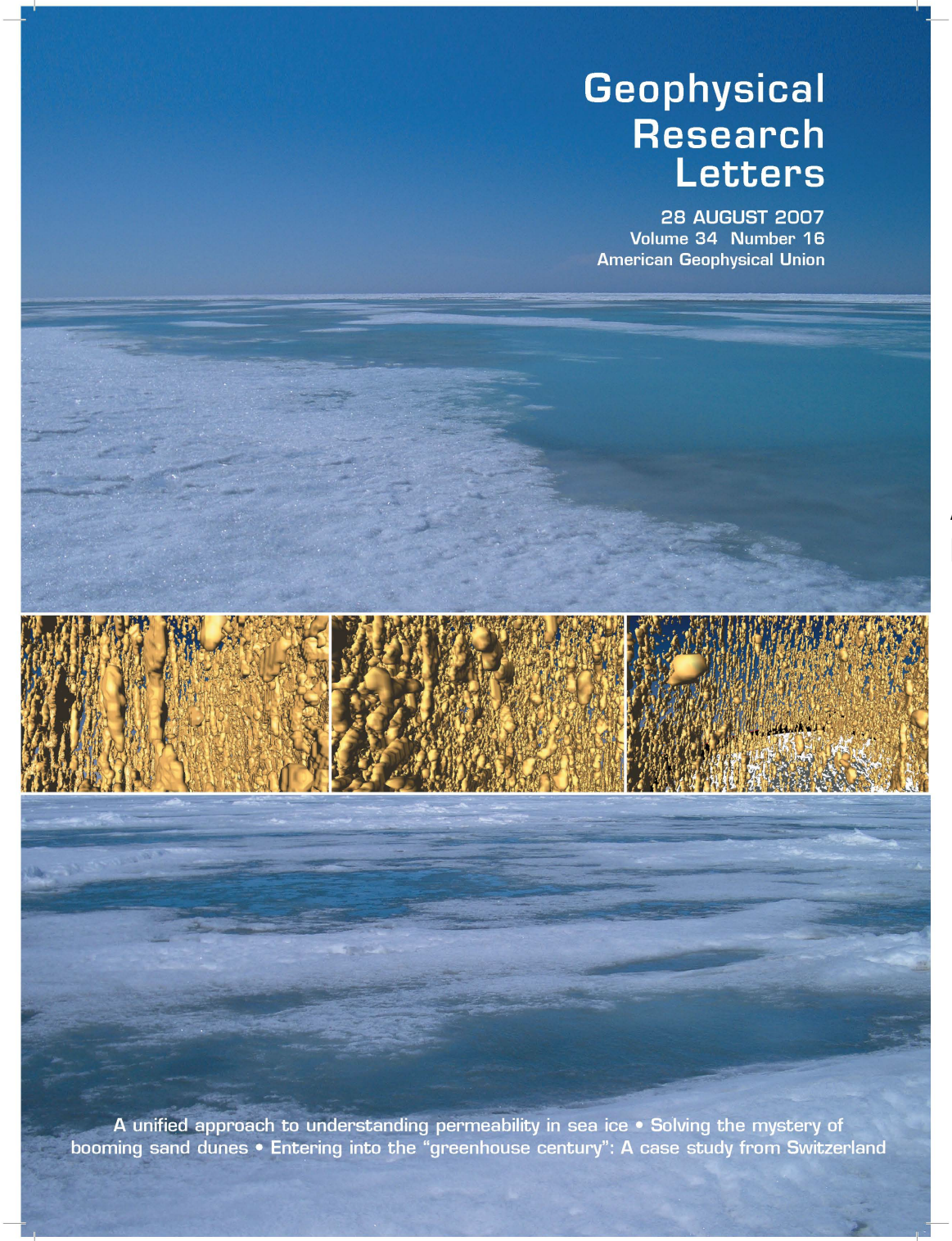
X-ray CT imaging and pore analysis provide unprecedented look at temperature evolution of brine phase and its connectivity

Validated with lab and Arctic field data.

micro-scale

controls

macro-scale processes in global climate



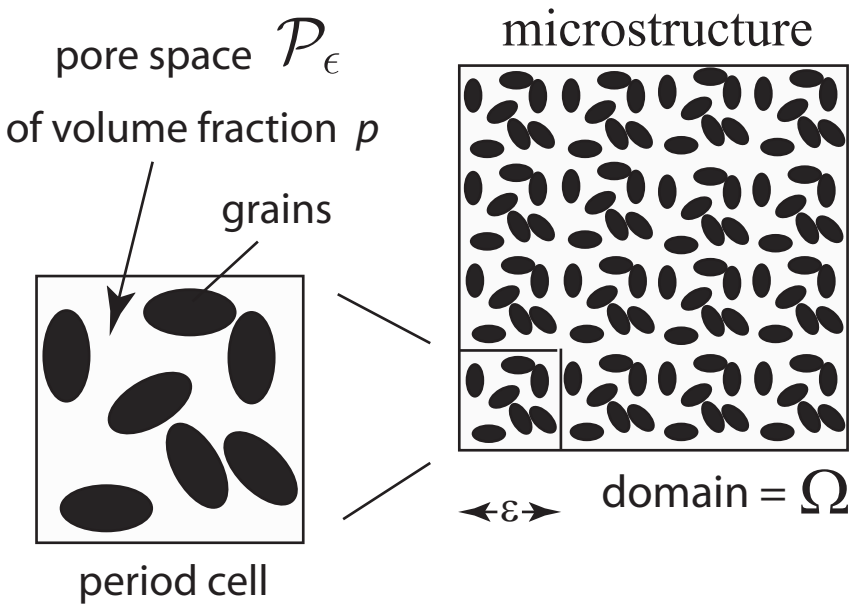
Arctic Ocean near Point Barrow, June 2007

brine inclusion tomography

Arctic Ocean near Point Barrow, June 2004

HOMOGENIZE as $\epsilon \to 0$

Stokes equations for fluid velocity $\mathbf{v}^\epsilon$, pressure p^ϵ, force $\mathbf{f}$:



$$\nabla p^\epsilon - \epsilon^2 \eta \Delta \mathbf{v}^\epsilon = \mathbf{f}, \quad x \in \mathcal{P}_\epsilon$$

$$\nabla \cdot \mathbf{v}^\epsilon = 0, \quad x \in \mathcal{P}_\epsilon$$

$$\mathbf{v}^\epsilon = 0, \quad x \in \partial \mathcal{P}_\epsilon$$

η = fluid viscosity

via two-scale expansion

MACROSCOPIC EQUATIONS $\mathbf{v}^\epsilon \to \mathbf{v}$, $p^\epsilon \to p$ as $\epsilon \to 0$

Darcy's law (f = 0)

$$\mathbf{v} = -\frac{1}{\eta}\, \mathbf{k}\, \nabla p, \quad x \in \Omega$$

$$\nabla \cdot \mathbf{v} = 0, \quad x \in \Omega$$

$\mathbf{k}(x)$ = effective fluid permeability tensor

[Keller '80, Tartar '80, Sanchez-Palencia '80, Lions '81, Allaire '89, '91,'97]

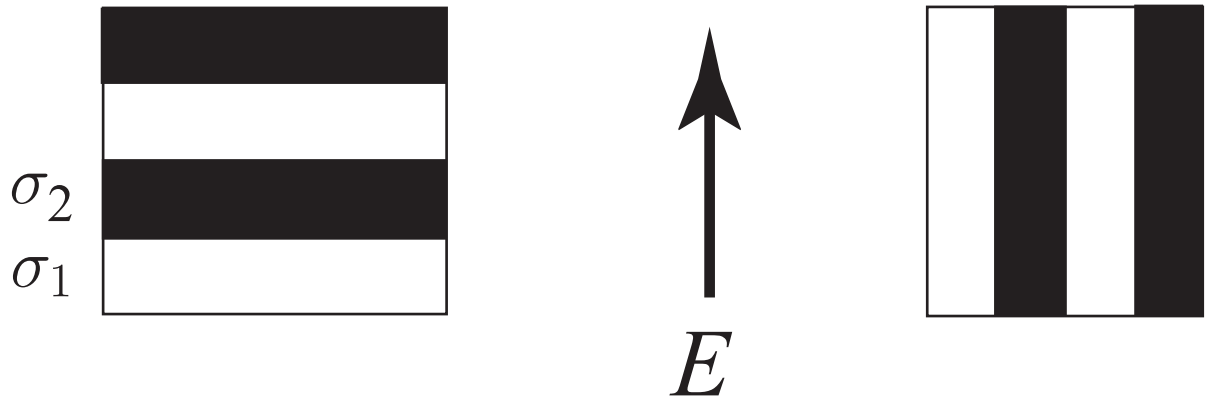
Bounds on the fluid permeability of sea ice

arithmetic and harmonic mean bounds for electrical conductivity σ (dielectric constant ε)

Effective electrical conductivity σ^* for two phase composite of σ_1 and σ_2

optimal bounds on σ^* for known volume fractions p_1 and p_2 :

$$\frac{1}{\frac{p_1}{\sigma_1}+\frac{p_2}{\sigma_2}} \leq \sigma^* \leq p_1\sigma_1 + p_2\sigma_2$$



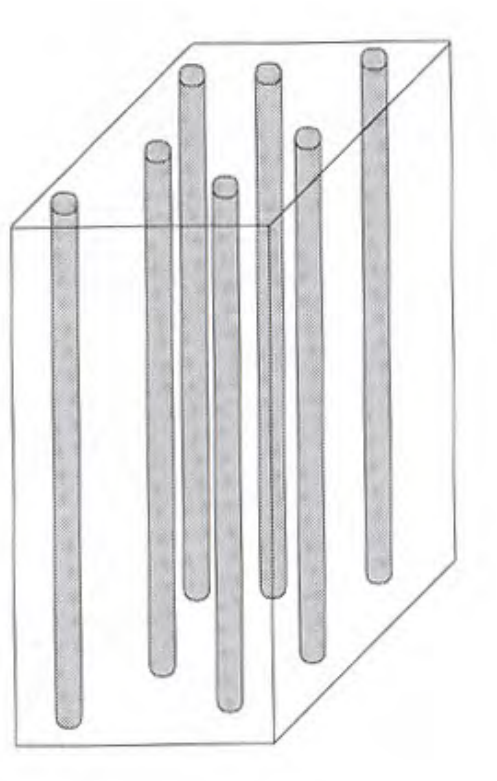
Wiener, 1912

PIPE BOUNDS on vertical fluid permeability k :

vertical pipes

with appropriate radii

maximize k



special case of optimal VOID BOUNDS

Torquato and Pham, *Phys. Rev. Lett.* 2004

analyze via **trapping constant** for diffusion process in pore space

diffusing tracer with concentration $c(x,t)$

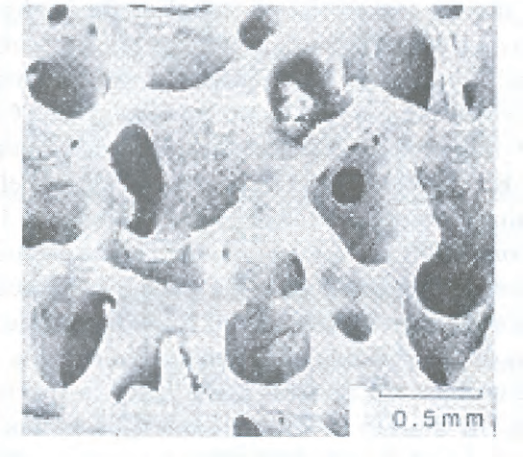
reacts with traps on pore boundaries

$$\frac{\partial c}{\partial t} = D\Delta c + G, \qquad x \in \Omega_b(\omega)$$

on the boundary:

$$D\frac{\partial c}{\partial n} + \kappa c = 0, \qquad x \in \partial\Omega_b(\omega)$$

bone



fluid filled
porous medium

trapping constant: $\gamma^{-1} = \langle u \rangle$

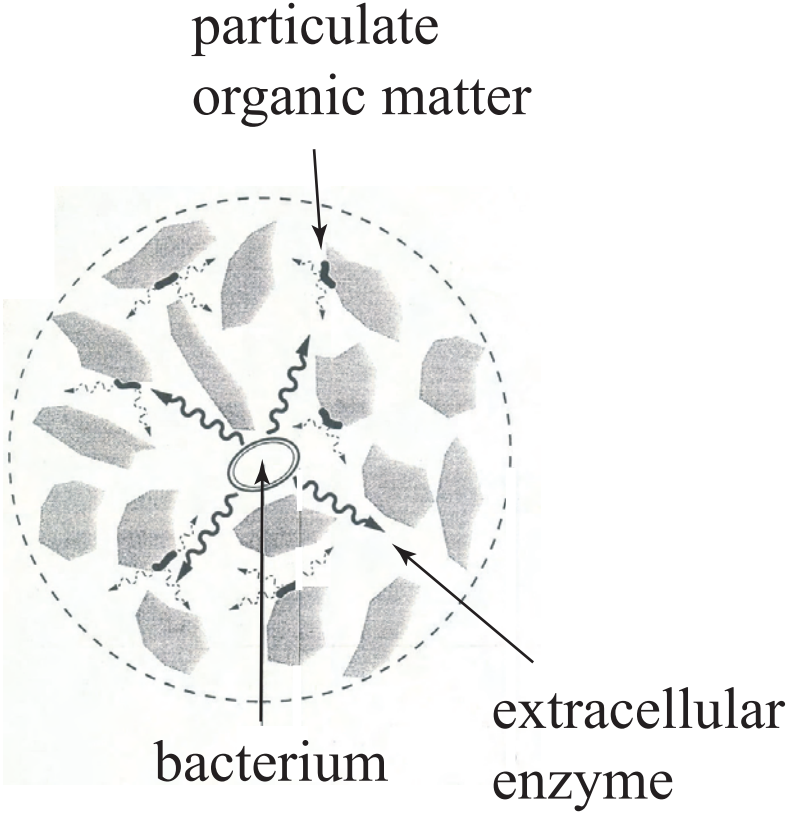
$u(x)$ is scaled concentration field in steady state with absorbing b.c. (reaction-controlled limit)

mean survival time: $\tau = \dfrac{1}{\gamma\phi D} = \dfrac{\langle u \rangle}{\phi D}$

Diffusive trapping and survival

- nutrient uptake by bacteria in fluid-filled porous medium

Vetter, *et al.*,
Microbial Ecol. 1998



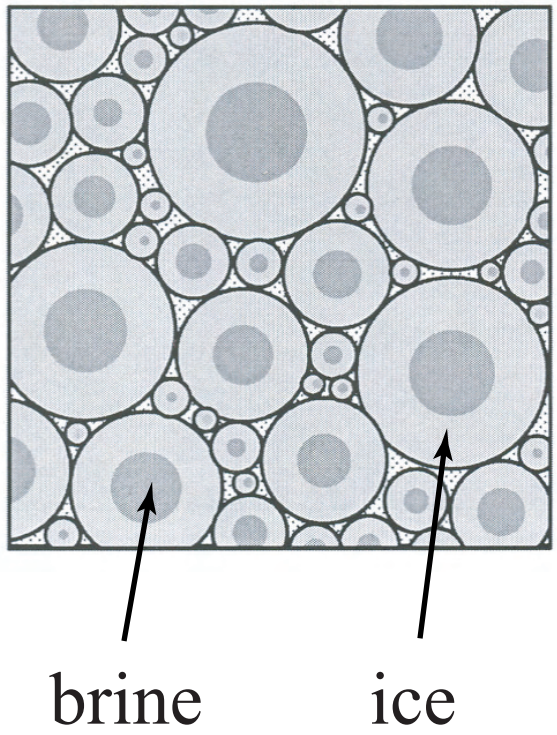
- NMR imaging of brine microstructure

magnetized nuclei diffuse in
porous microstructure

variational analysis yields a general lower VOID BOUND

EVALUATE void bound for Hashin-Shtrikman coated cylinders (d=2) (Torquato and Pham, 2004)

optimal coated cylinder geometry



$$\gamma \geq \frac{8\langle R_I^2 \rangle}{\phi \langle R_I^4 \rangle}, \quad d = 2$$

$$\mathbf{k} \leq \gamma^{-1}\mathbf{I}$$

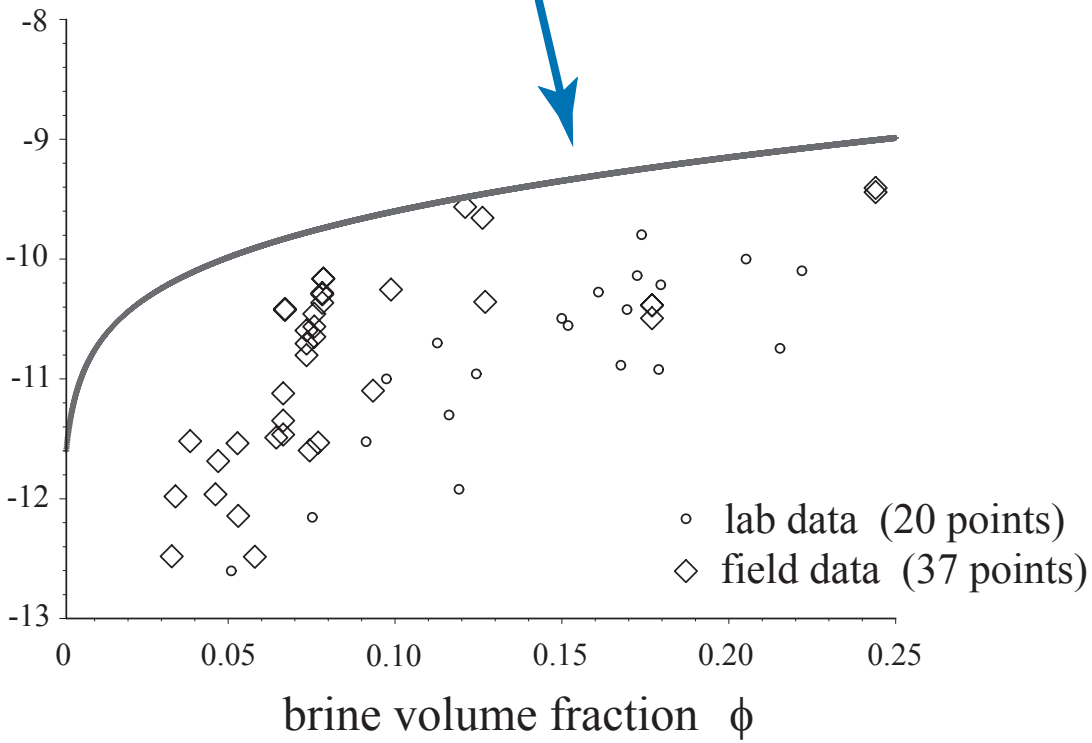
for any ergodic porous medium (Torquato, 2002)

lognormal pipe bound: inclusion cross sectional areas *A* lognormally distributed

ln(*A*) is normally distributed with mean μ and variance σ²

$$k \leq \frac{\phi\langle R^4\rangle}{8\langle R^2\rangle} = \frac{\phi}{8}\langle R^2\rangle e^{\sigma^2}$$

μ increases with *T* and σ = 1
(Perovich and Gow, '96, ...)



k = vertical fluid permeability of sea ice

similar calculations in math-finance (stock prices tend to be lognormally distributed)

Golden, Eicken, Heaton, Miner, Pringle, Zhu, *GRL* 2007
Golden, Heaton, Eicken, Lytle, *Mech. Materials* 2006

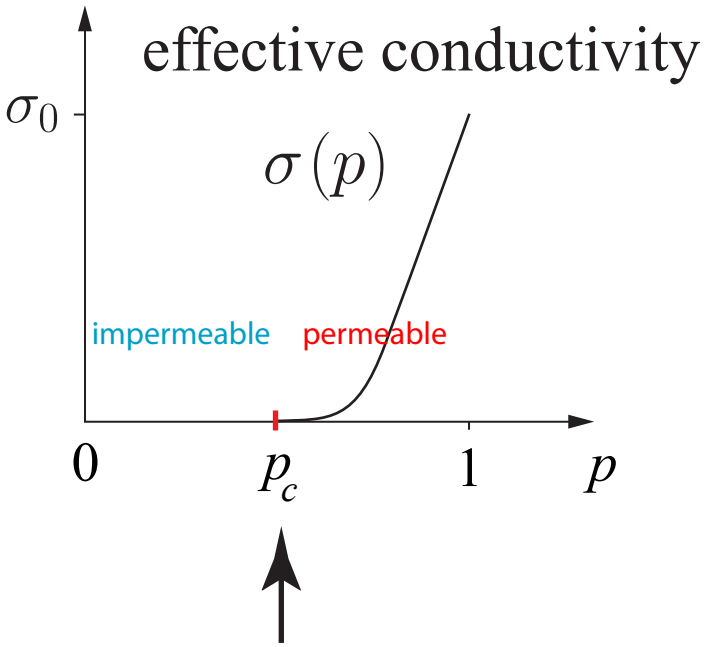
Theoretical predictions of ice permeability using percolation and hierarchical models

Lattice Percolation Model

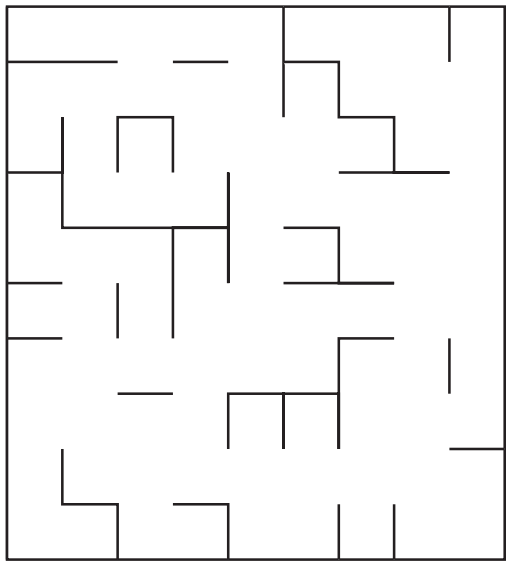
Random Resistor Network from physics of semiconductors

local conductivity **electrical** **or** **fluid**

bond → $\begin{cases} \sigma_0 & \text{probability } p \\ 0 & \text{probability } 1 - p \end{cases}$

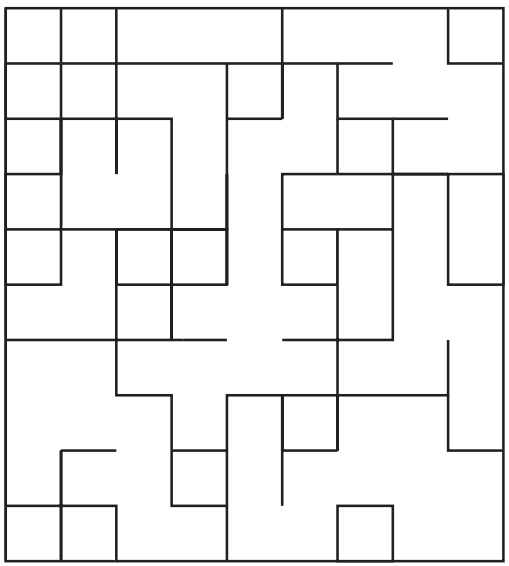


impermeable



$p = 1/3$

permeable



$p = 2/3$

square bond lattice

percolation threshold

$p_c = 1/2$ for $d = 2$

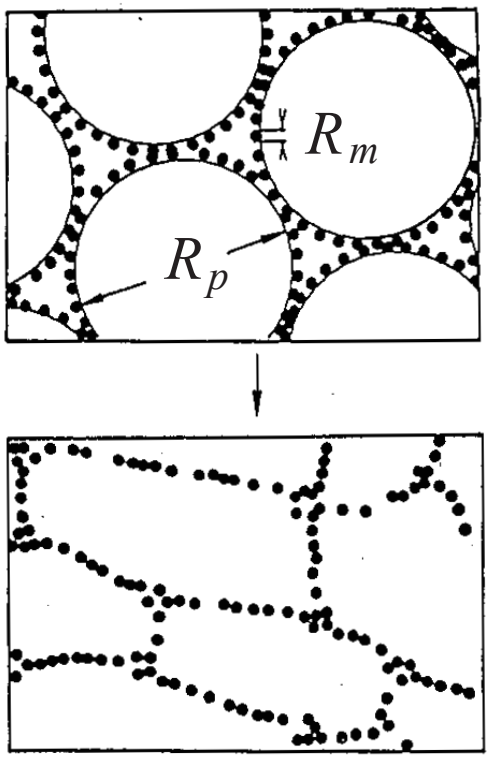
onset of an infinite, connected cluster

tipping point

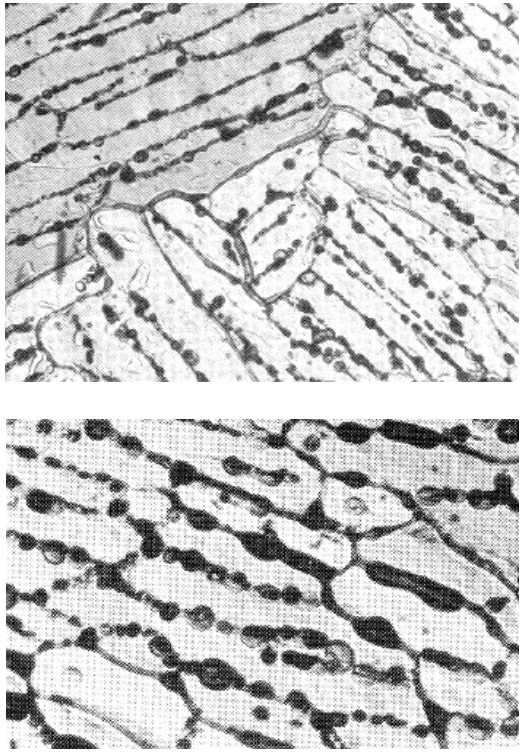
p_c depends on lattice type and dimension

Continuum percolation model for stealthy materials applied to sea ice microstructure explains **Rule of Fives** and Antarctic data on ice production and algal growth

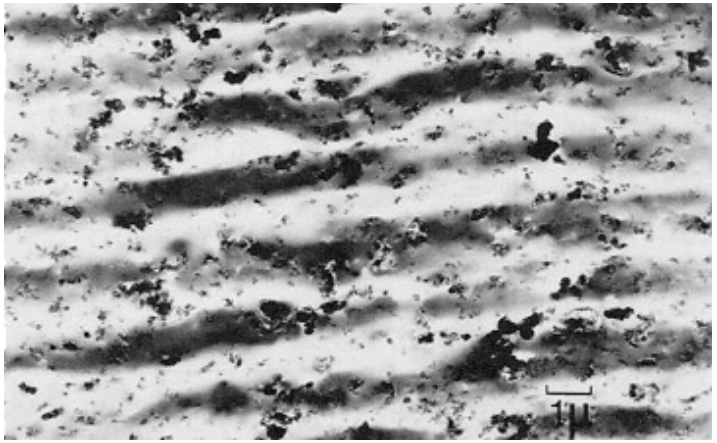
$\phi_c \approx 5\ \%$ Golden, Ackley, Lytle, *Science,* 1998



compressed powder



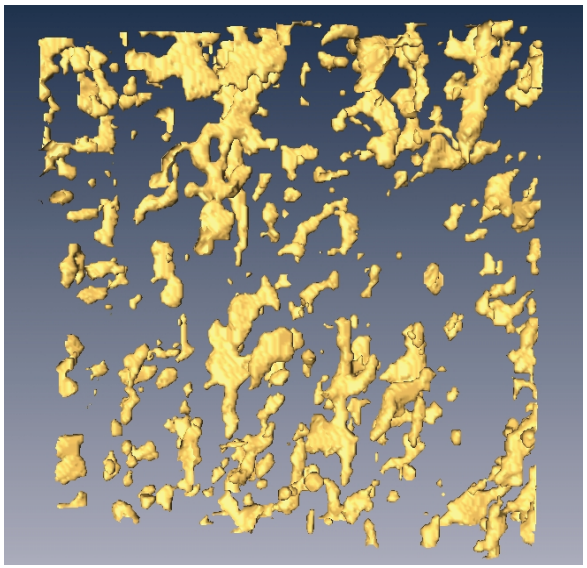
sea ice



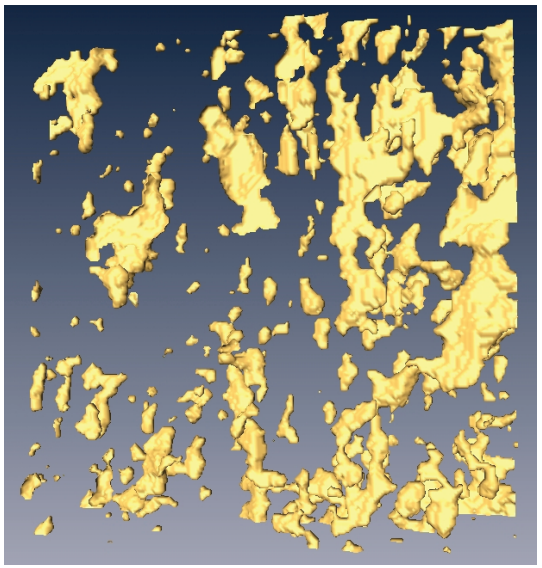
microstructure of radar absorbing composite

brine connectivity (over cm scale)

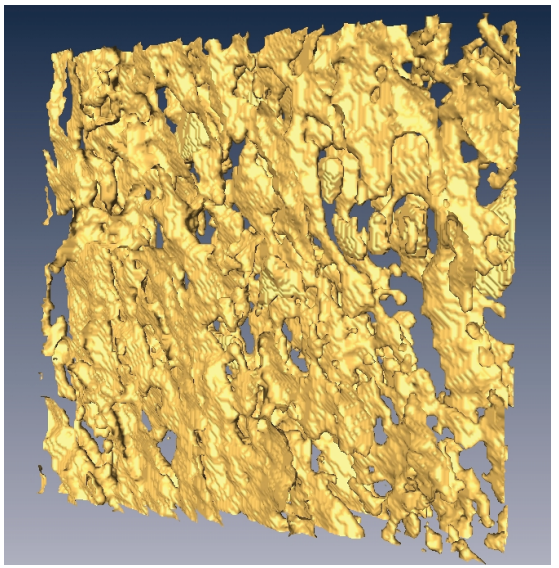
8 x 8 x 2 mm



-15 °C, $\phi = 0.033$

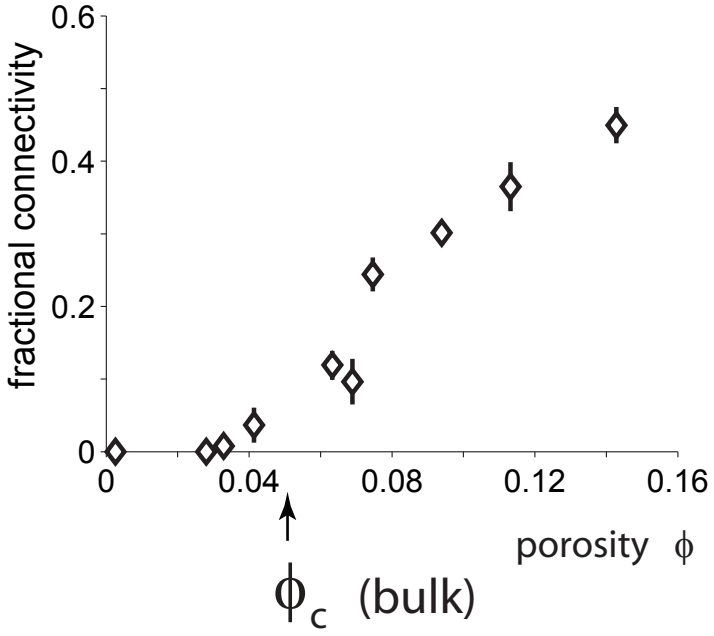


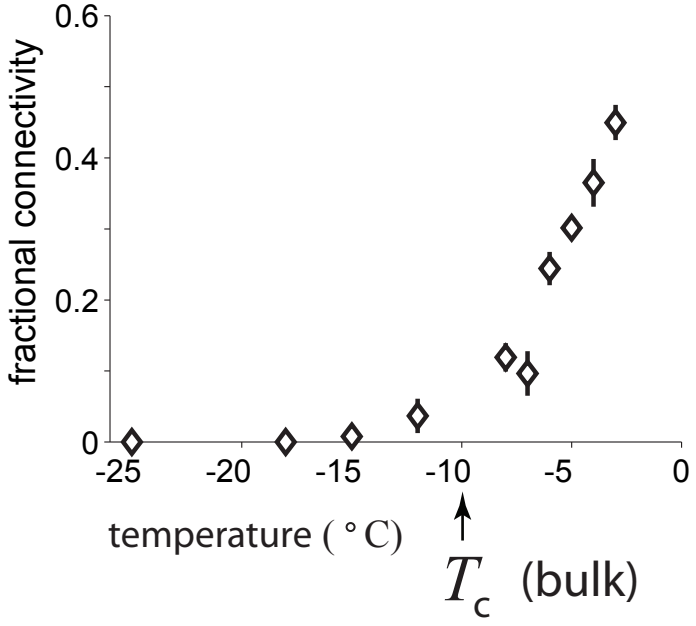
-6 °C, $\phi = 0.075$



-3 °C, $\phi = 0.143$

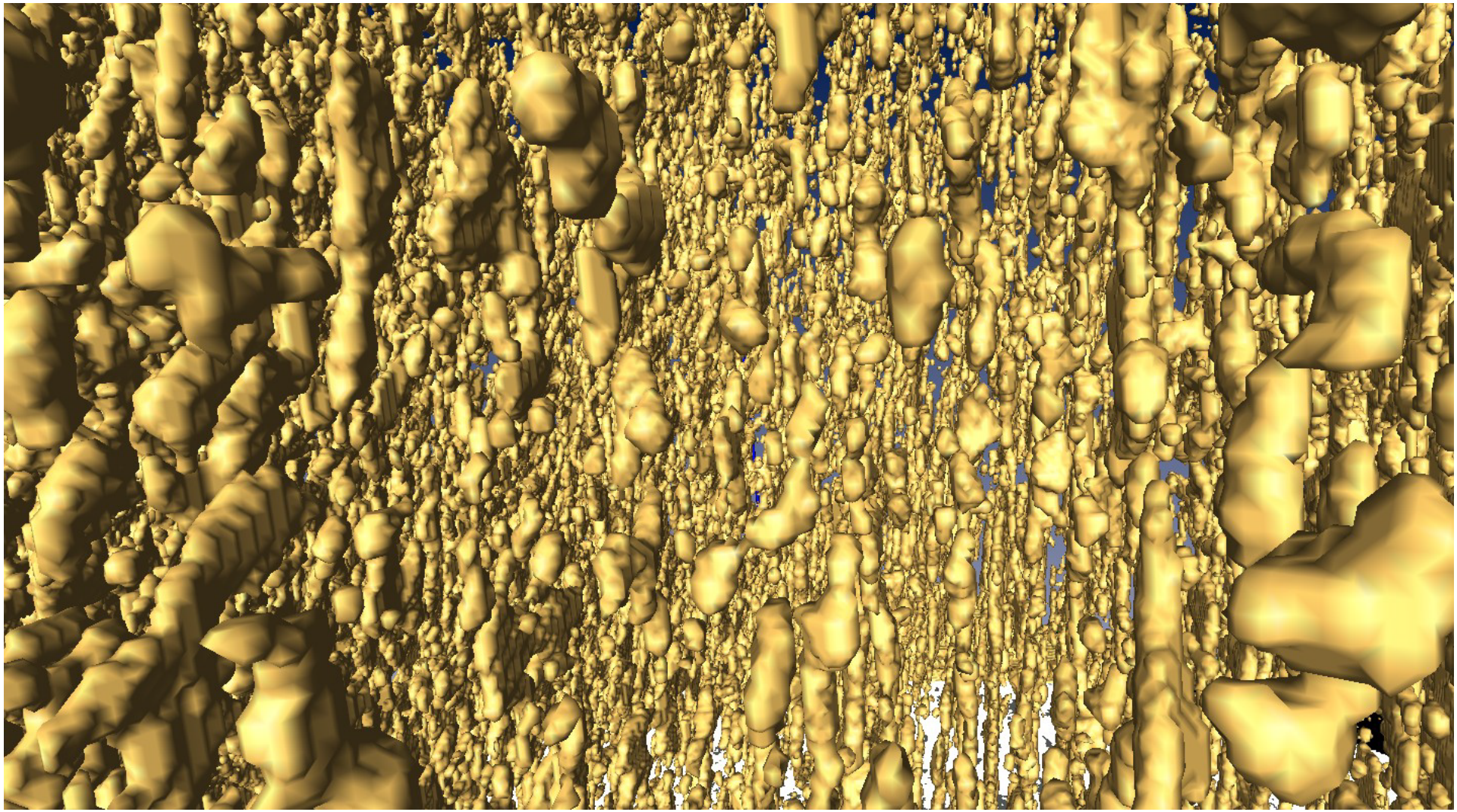
across cylinder
8 mm height
(21 mm diam.)



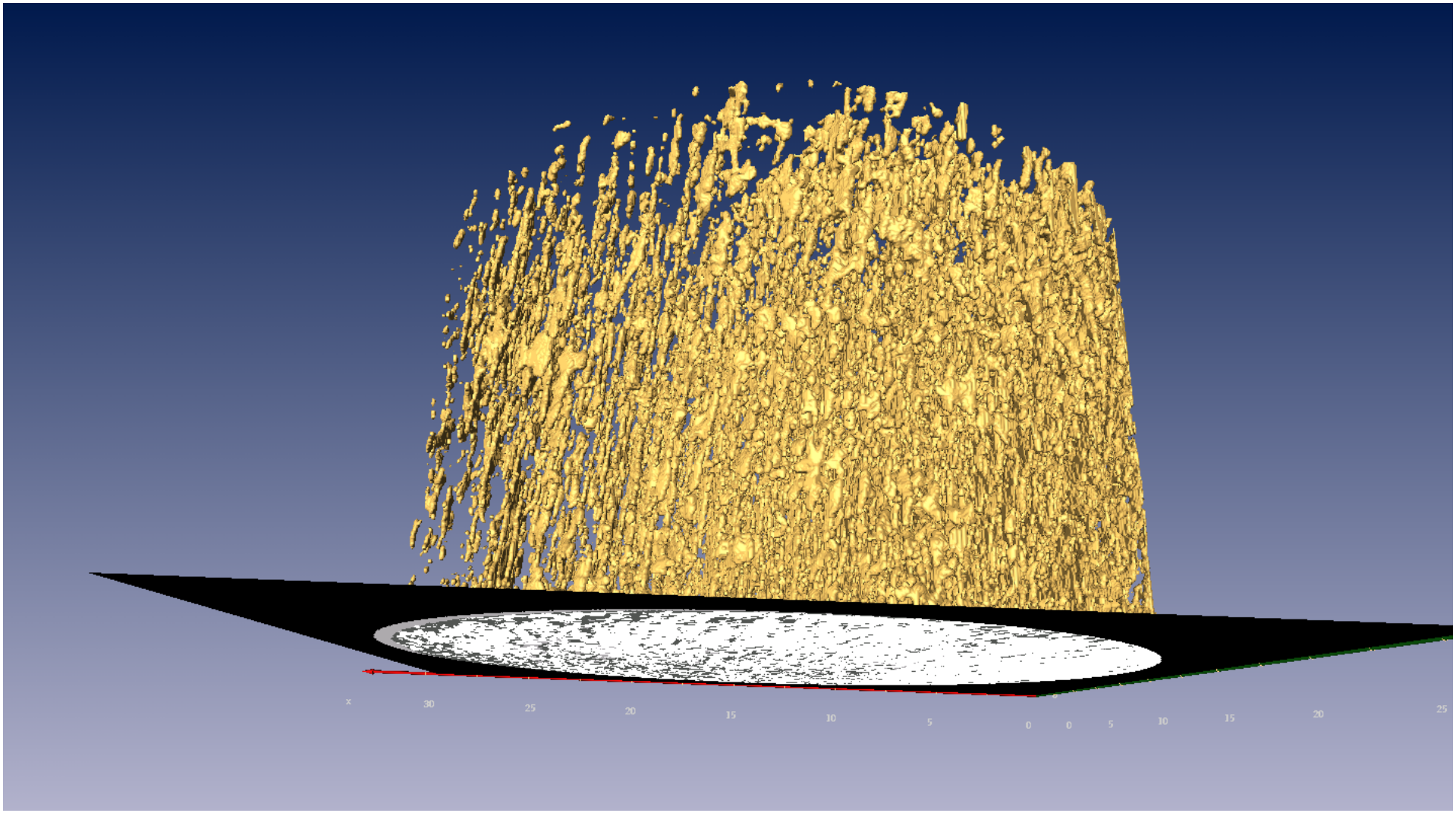


First direct evidence of conjectured transition in connectivity.

X-ray computed tomography (CT) and medial axis analysis (Prodanovic, et al., 2006) of lab-grown sea-ice single crystals doped with CsCl to improve ice/brine resolution.

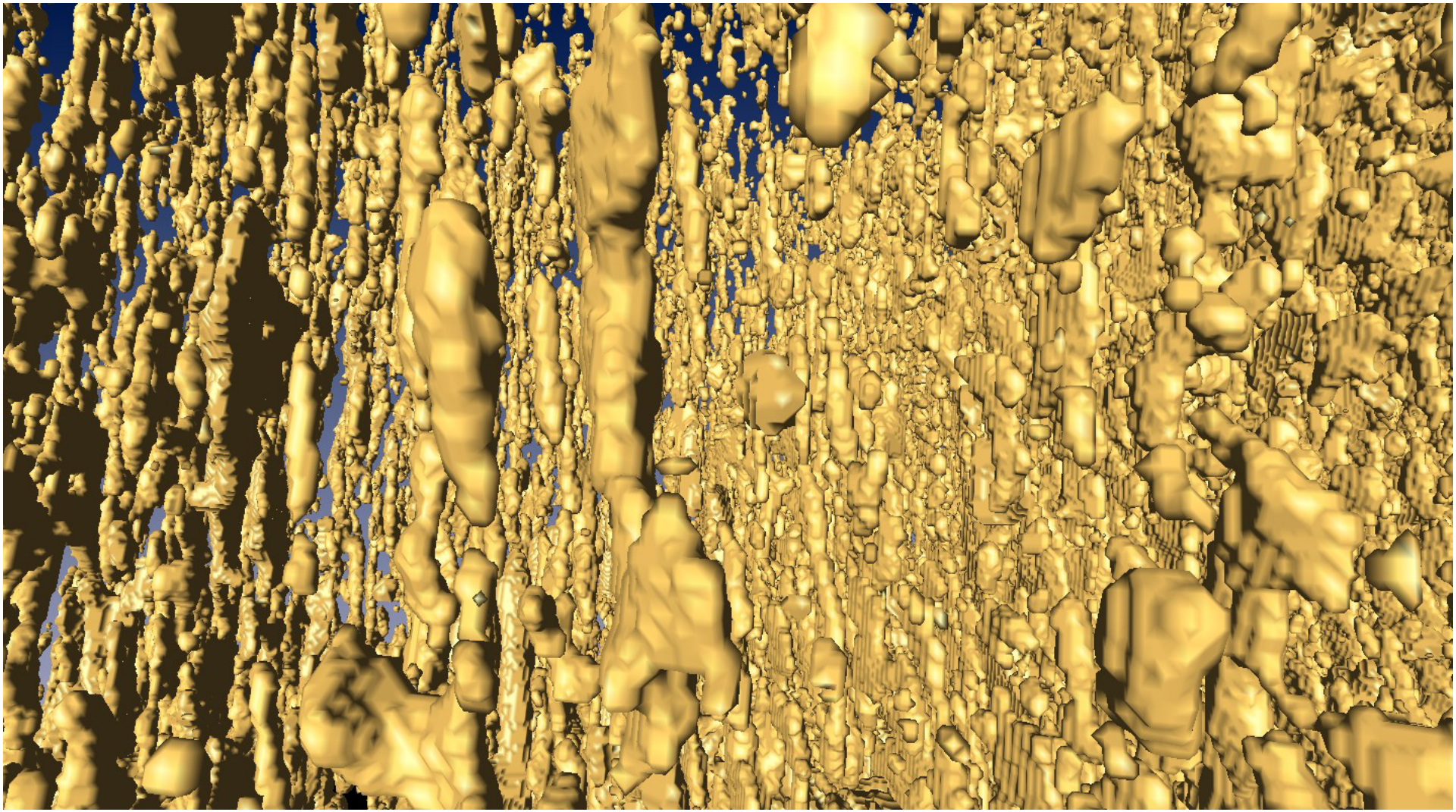


$\phi = 5.7\ \%$ $\quad$ $T = -8$

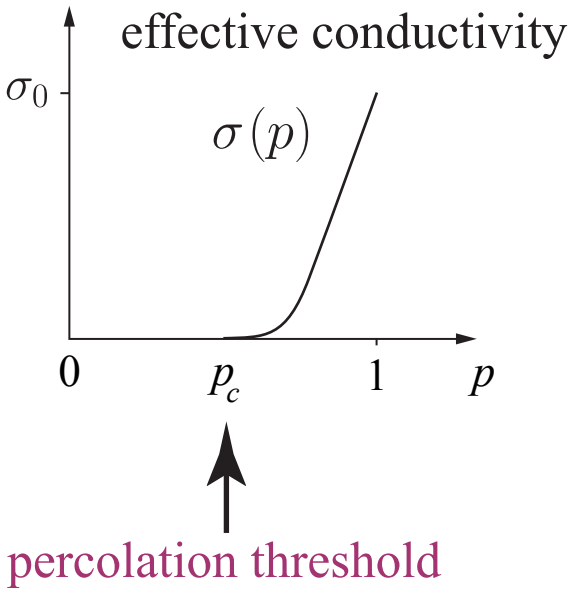


$\phi = 7.8\ \%$ $T = -6$

zoom



$\phi = 11.3\ \%$ $T = -4$



Critical behavior of transport near percolation threshold

conductivity

$$\sigma(p) \sim \sigma_0 (p - p_c)^t, \quad p \to p_c^+$$

permeability

$$\kappa(p) \sim \kappa_0 (p - p_c)^e, \quad p \to p_c^+$$

exponent t

UNIVERSAL for lattices

depends only on dimension d

$e = t$

critical exponent t for conductivity and permeability of lattices

numerical estimates in $d = 3$: $t = 2.0$

Derrida, Stauffer, Herrmann, Vannimenus 1983
Gingold, Lobb 1990
Adler, Meir, Aharony, Harris, Klein 1990
Berkowitz, Balberg 1992
⋮

rigorous inequality for t :

$$1 \leq t \leq 2, \qquad d = 2, 3$$

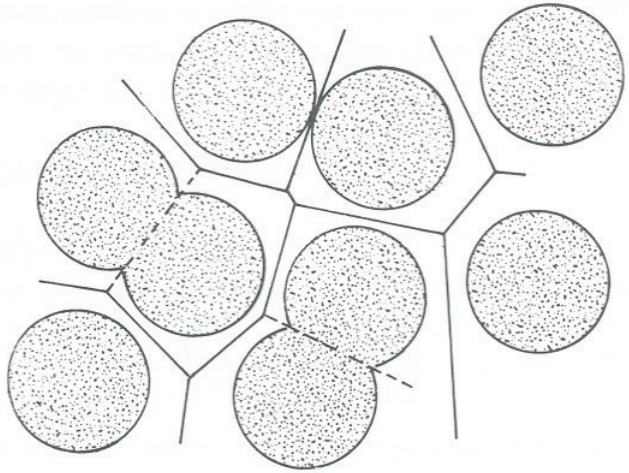
Golden, *Phys. Rev. Lett.* 1990
Comm. Math. Phys. 1992

Non-universal behavior in the continuum:

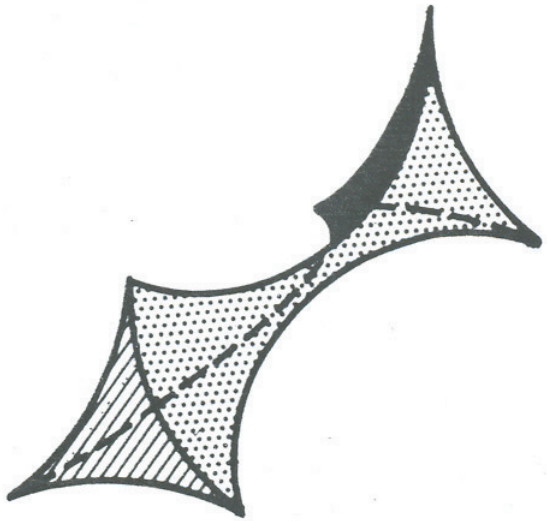
critical exponents for transport in Swiss cheese model take values different than for lattices, e.g. $t > 2$

$e \neq t$

Halperin, Feng, Sen, *Phys. Rev. Lett.* 1985



Swiss cheese model
$d = 2$



conducting neck in $d = 3$
Swiss cheese model

in general, non-universal exponents arise from a singular distribution of local conductances

In sea ice, this distribution is lognormal.

(excluding inclusions below cutoff)

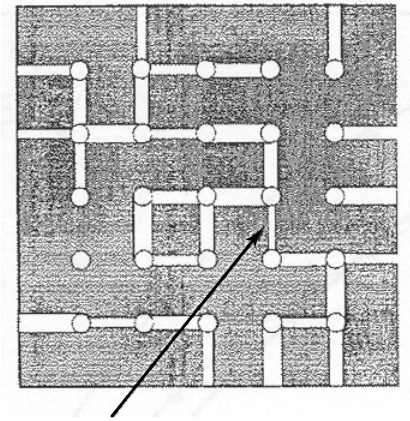
Thus, the permeability exponent for sea ice is **2**, the universal lattice value.

ESTIMATE fluid conductivity scaling factor $k_0 = r^2 / 8$

for media with broad range of conductances

CRITICAL PATH ANALYSIS

bottlenecks control flow



critical pore

Ambegaokar, Halperin, Langer 1971: CPA in electronic hopping conduction
Friedman, Seaton 1998: CPA in fluid and electrical networks
Golden, Kozlov 1999: rigorous CPA on long-range checkerboard model

$k_0 \approx r_c^2 / 8$ critical fluid conductivity

Microstructural analyses yield $r_c \approx$ 0.5 mm

Thus, for vertical fluid permeability of sea ice near the percolation threshold

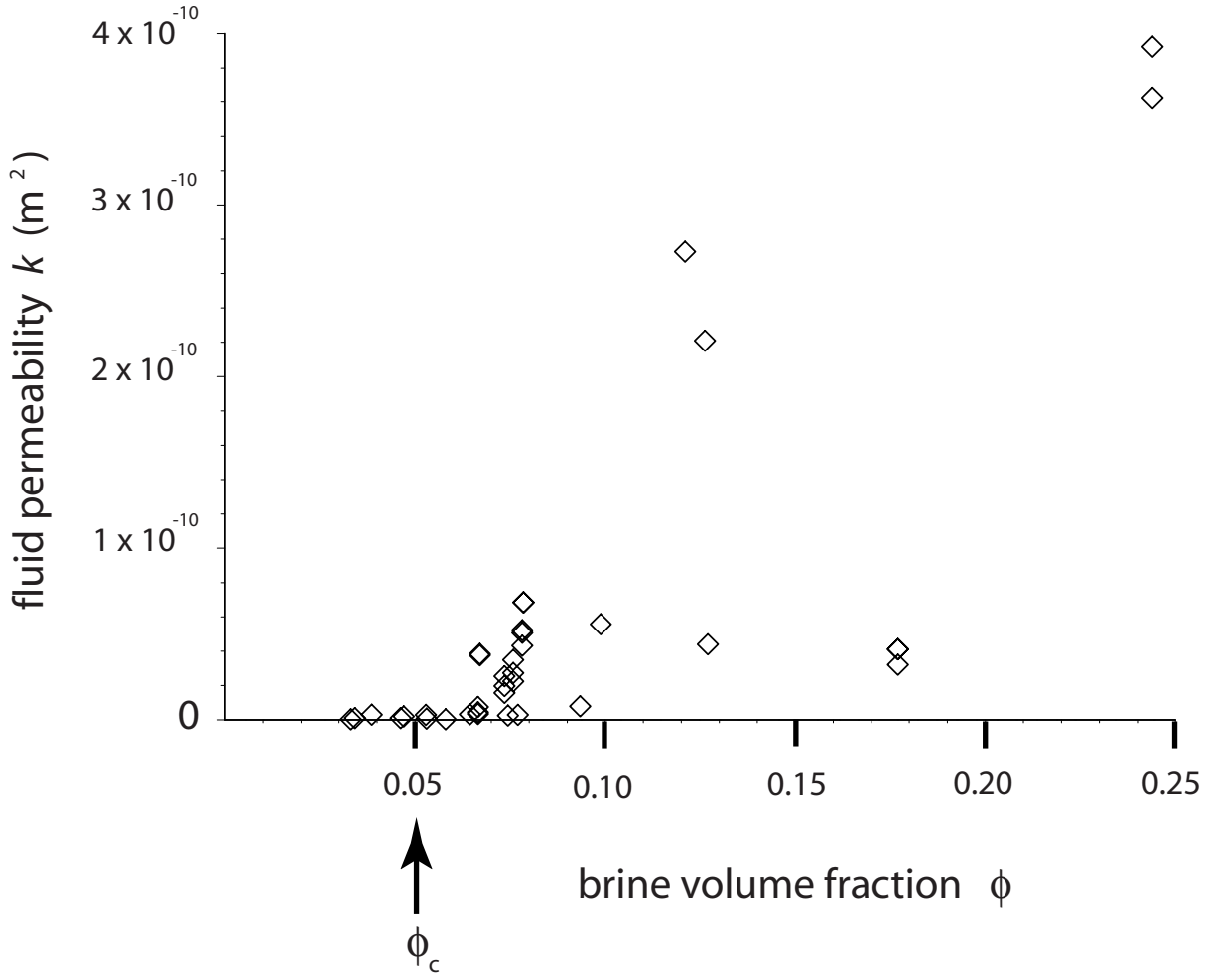
$$k(\phi) = k_0 (\phi - \phi_c)^2$$

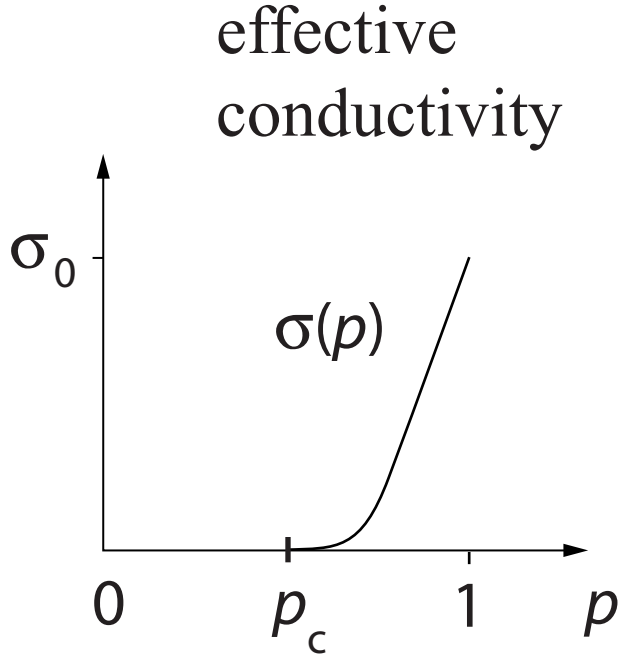
critical exponent t

$$k_0 = 3 \times 10^{-8} \text{ m}^2$$

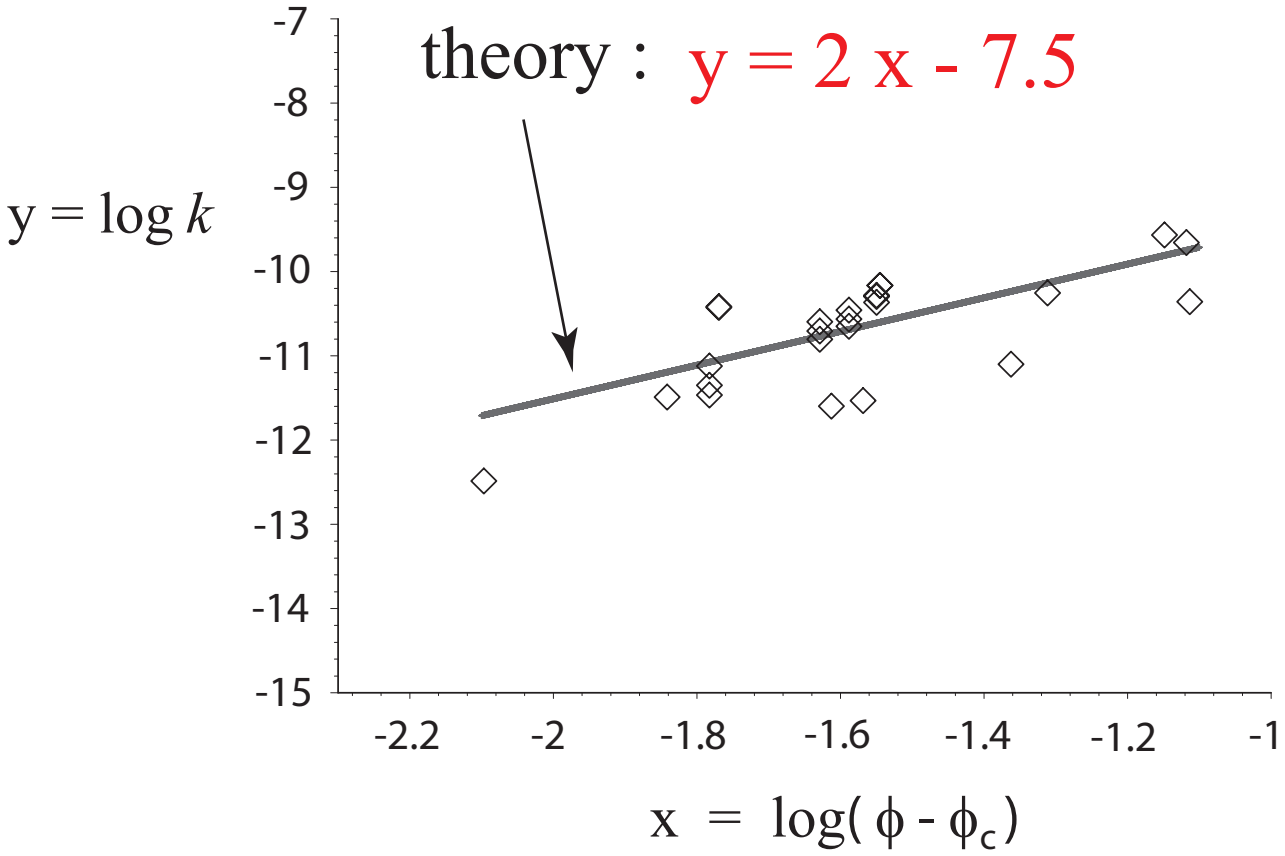
- universal lattice value $t \approx 2.0$ due to general stucture of brine inclusion PDF - true for all saline ice, e.g. Europa?
- sedimentary rocks like sandstones in same universality class
- no free parameters -- microstructural input only

in situ data on vertical fluid permeability of Arctic sea ice





two dimensional
lattice percolation



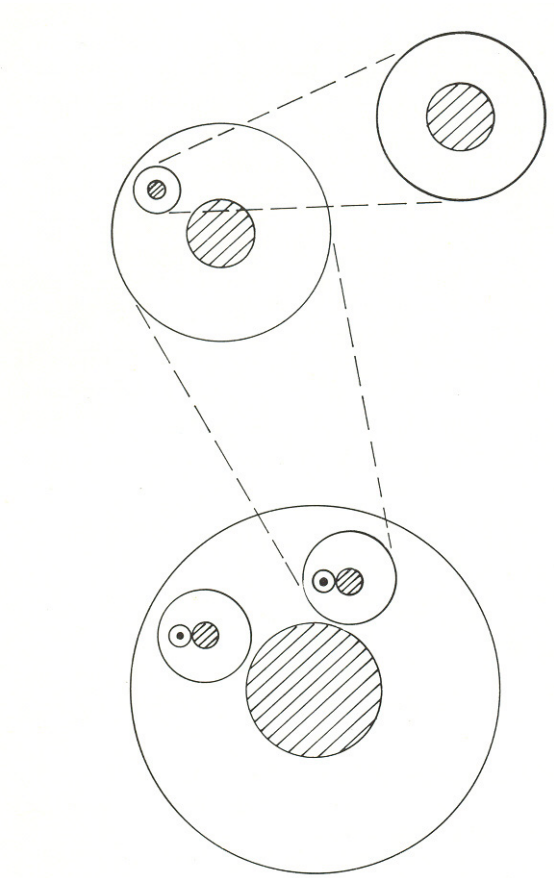
statistical best fit of data:

$$y = 2.07\ x - 7.45$$

Percolation theory applies near threshold, what about full range of ϕ ?

self-similar, hierarchical model for porous rocks

Sen, Scala, Cohen 1981
Sheng 1990
Wong, Koplick, Tomanic 1984



Archie's Law for conductivity

$$\sigma(\phi) = \sigma_w \phi^m, \quad m = 3/2$$

for sea ice fluid permeability

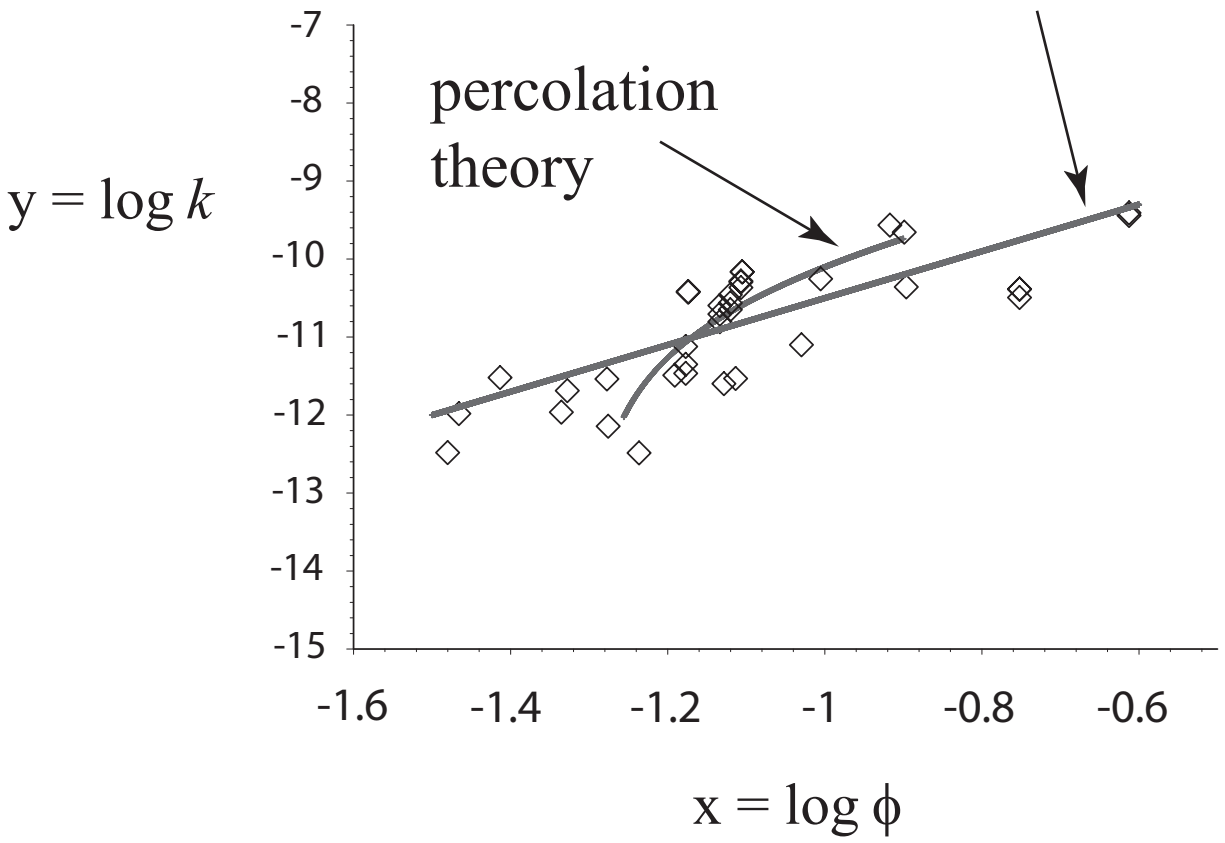
$$k(\phi) = k_0 \phi^{2m}, \quad m = 3/2$$

$$k(\phi) \sim k_0 \phi^3 \ \mathrm{m}^2$$

brine-coated spherical ice grains

and variations

hierarchical model : $y = 3\ x - 7.5$



statistical best fit of data: $y = 3.05\ x - 7.50$

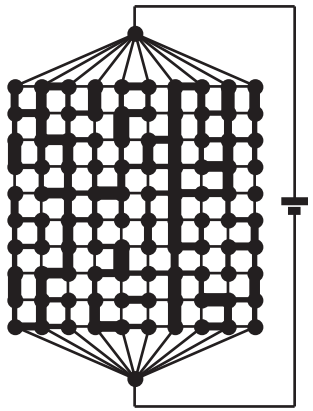
A network model for fluid transport through sea ice

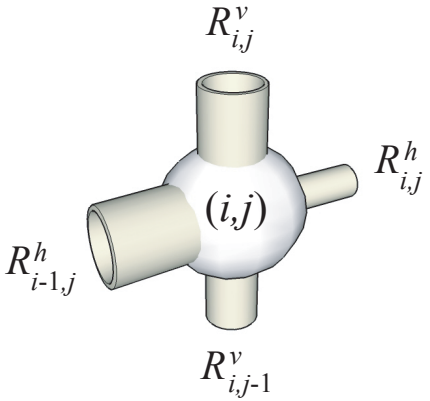
J. Zhu, A. Jabini, K. M. Golden, H. Eicken, M. Morris

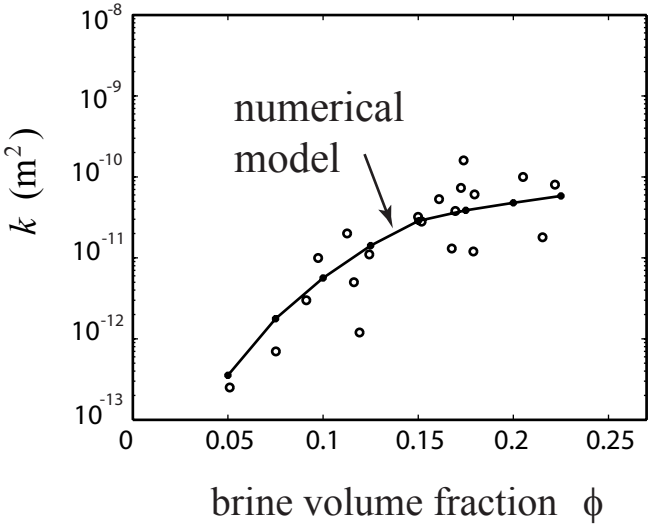
Annals of Glaciology, 2006

- random pipe network with radii chosen from measured inclusion distributions, solved with fast multigrid method

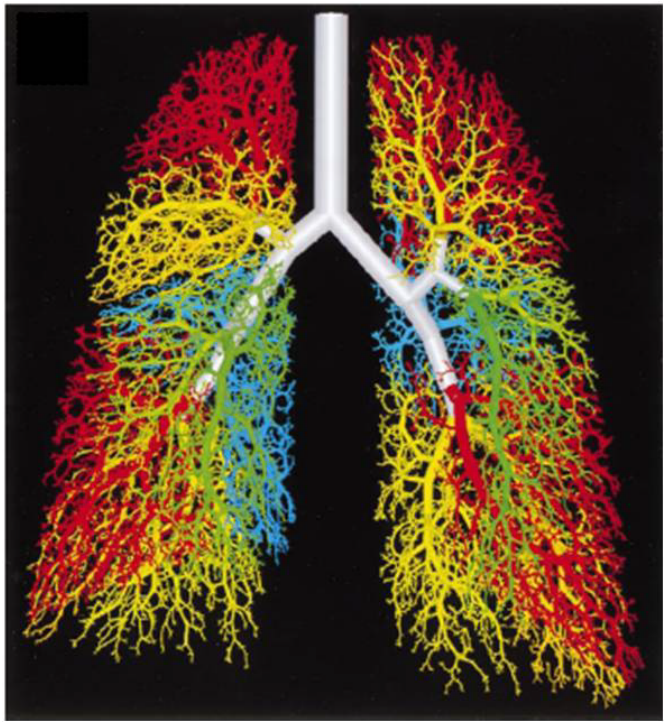
- local velocity field computed, as well as effective permeability





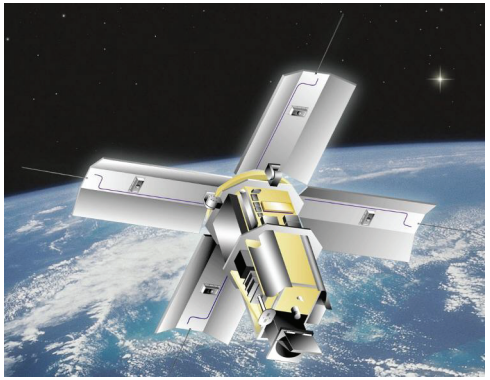


based on connectivity analysis of brine inclusions, introduce *disconnections* into network (Golden et al., GRL 2007)



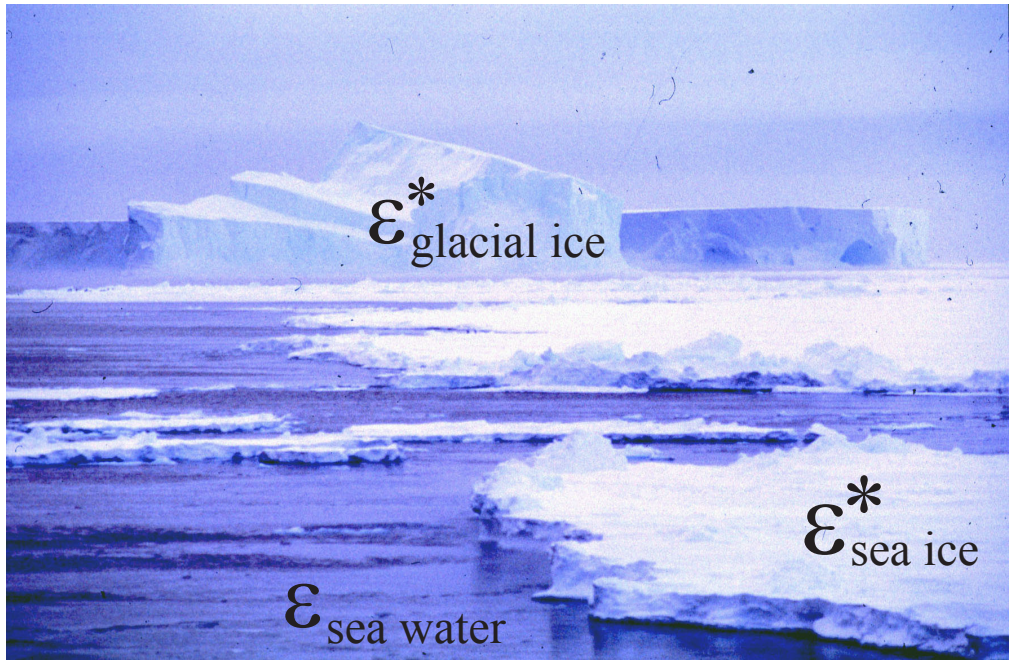
Applying similar model to air flow in lungs (Megan Morris)

Remote sensing of sea ice



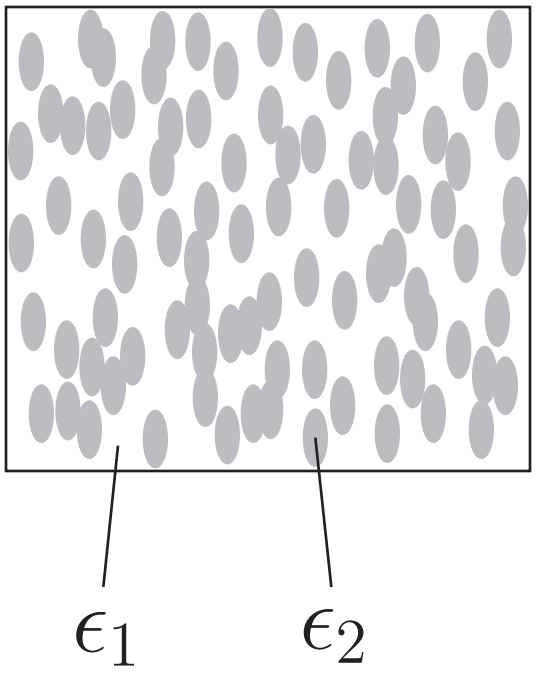
Recover sea ice properties from electromagnetic (EM) data

thickness
roughness
brine porosity ...



INVERSE PROBLEM

Effective complex permittivity of a two phase composite in the quasistatic (long wavelength) limit



ϵ^*

$$D = \epsilon E$$

$$\nabla \cdot D = 0$$

$$\nabla \times E = 0$$

$$\langle D \rangle = \epsilon^* \langle E \rangle$$

p_1 , p_2 = volume fractions of the components

$$\epsilon^* = \epsilon^* \left(\frac{\epsilon_1}{\epsilon_2} \text{ , composite geometry} \right)$$

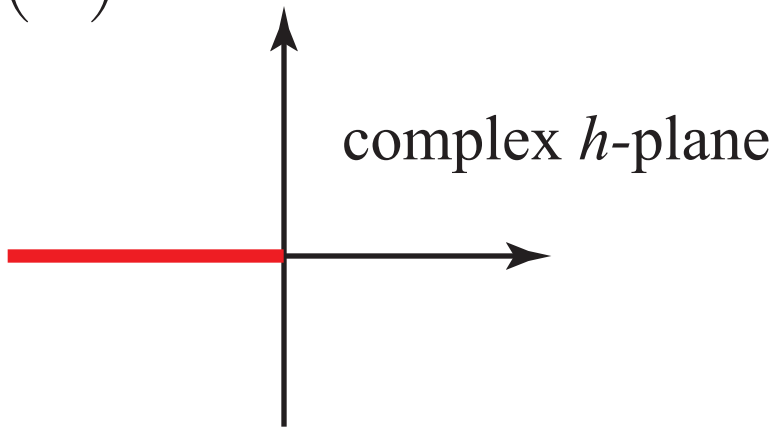
Analytic continuation method for bounding complex ϵ^*

Bergman (1978), Milton (1979), Golden and Papanicolaou (1983)

$$m(h) = \frac{\epsilon^*}{\epsilon_2}\left(\frac{\epsilon_1}{\epsilon_2}\right) \qquad h = \frac{\epsilon_1}{\epsilon_2}$$

Exploit analytic properties of $m(h)$

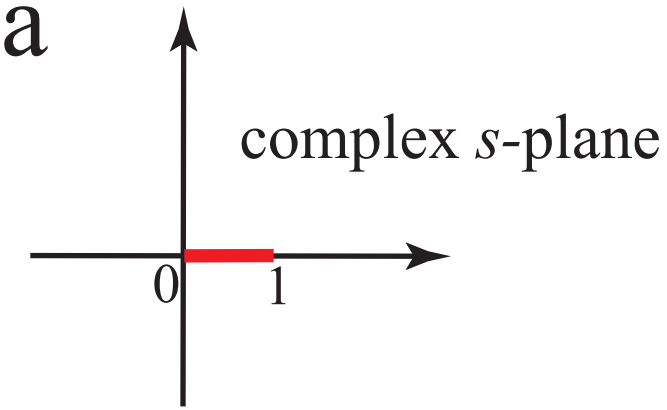
- $m(h)$ analytic off negative real axis



- $m(h)$: UHP $\longrightarrow$ UHP

Integral representation formula

$$F(s) = 1 - m(h), \quad s = \frac{1}{1-h}$$



$$F(s) = \int_0^1 \frac{d\mu(z)}{s - z}$$

μ

- spectral measure of self adjoint operator
- mass = p_1
- higher moments depend on n-point correlations

separates **GEOMETRY** from medium parameters

Obtain bounds from integral representation

Fix s and vary over admissible measures μ

- complex arithmetic and harmonic mean bounds
  complex Hashin-Shtrikman bounds

  (Milton 1980, Bergman 1980)

- extension to multicomponent media

  (Golden and Papanicolaou 1985, Golden 1986, Milton 1987)

- spectral constraints for matrix-particle composites

  real bounds (Bruno 1991)

  complex bounds (Sawicz and Golden 1995, Golden 1997)

Capacitance probe measurements of ϵ^*

Lars Backstrom and Hajo Eicken

data taken from { outdoor tank, Fairbanks
Chukchi Sea, Arctic Ocean
McMurdo Sound, Antarctica

ice $\epsilon_1 = 3.06$

brine $\epsilon_2 = 63.3 + i\ 1930$

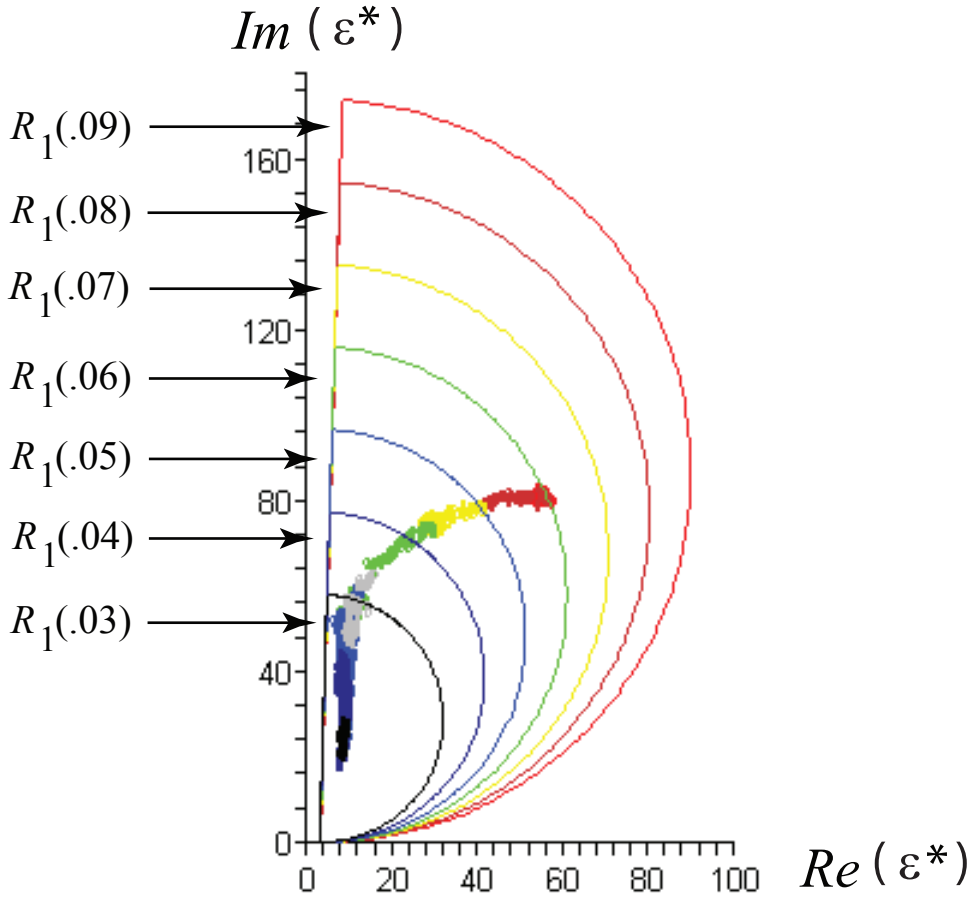
50 Mhz

Hydraprobe

in situ

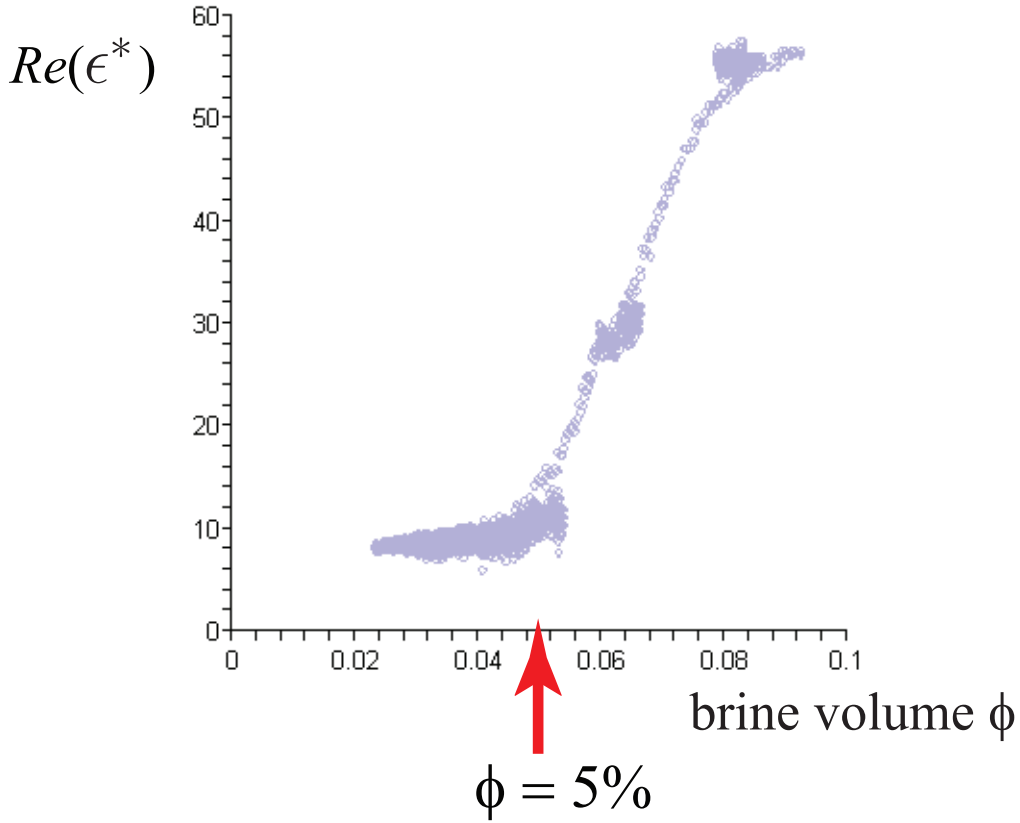
array

Comparison of forward bounds with data



Complex bounds and microstructural recovery from measurements of sea ice permittivity, A. Gully, L. G. E. Backstrom, H. Eicken, and K. M. Golden, *Physica B,* 2007.

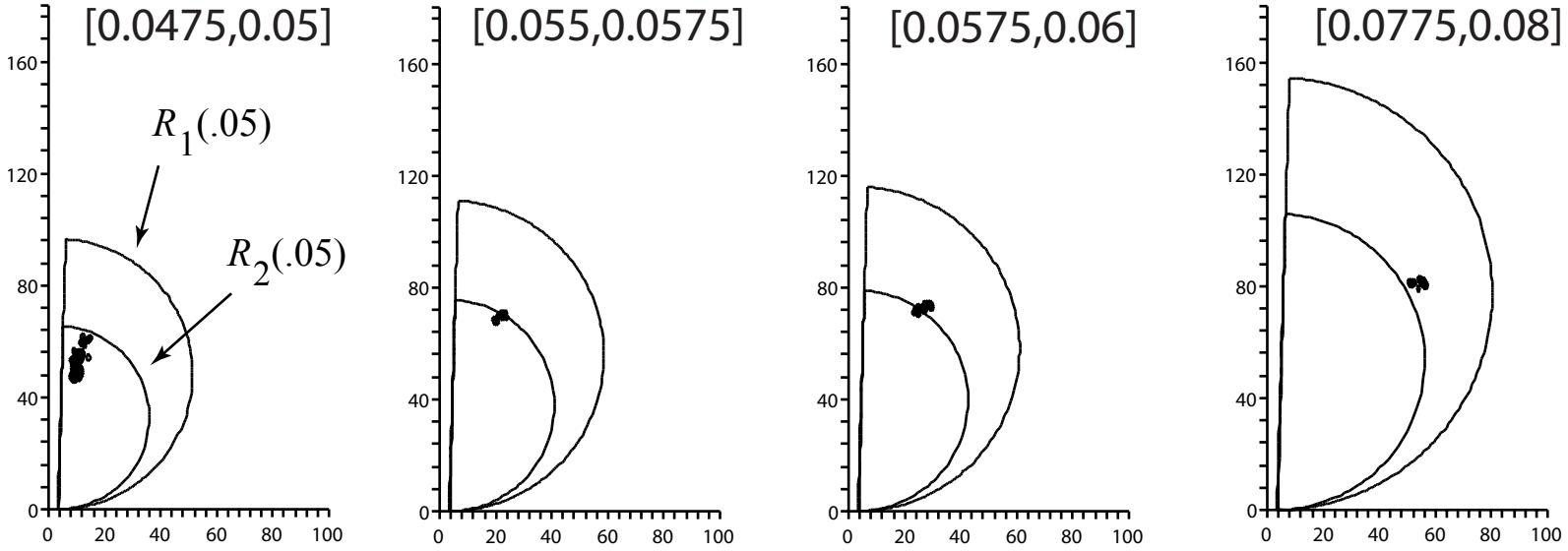
transitional behavior of electrical transport



- implications for obtaining sea ice thickness from EM data

- analysis of dielectric critical behavior and exponents
  Ben Murphy and Adam Gully

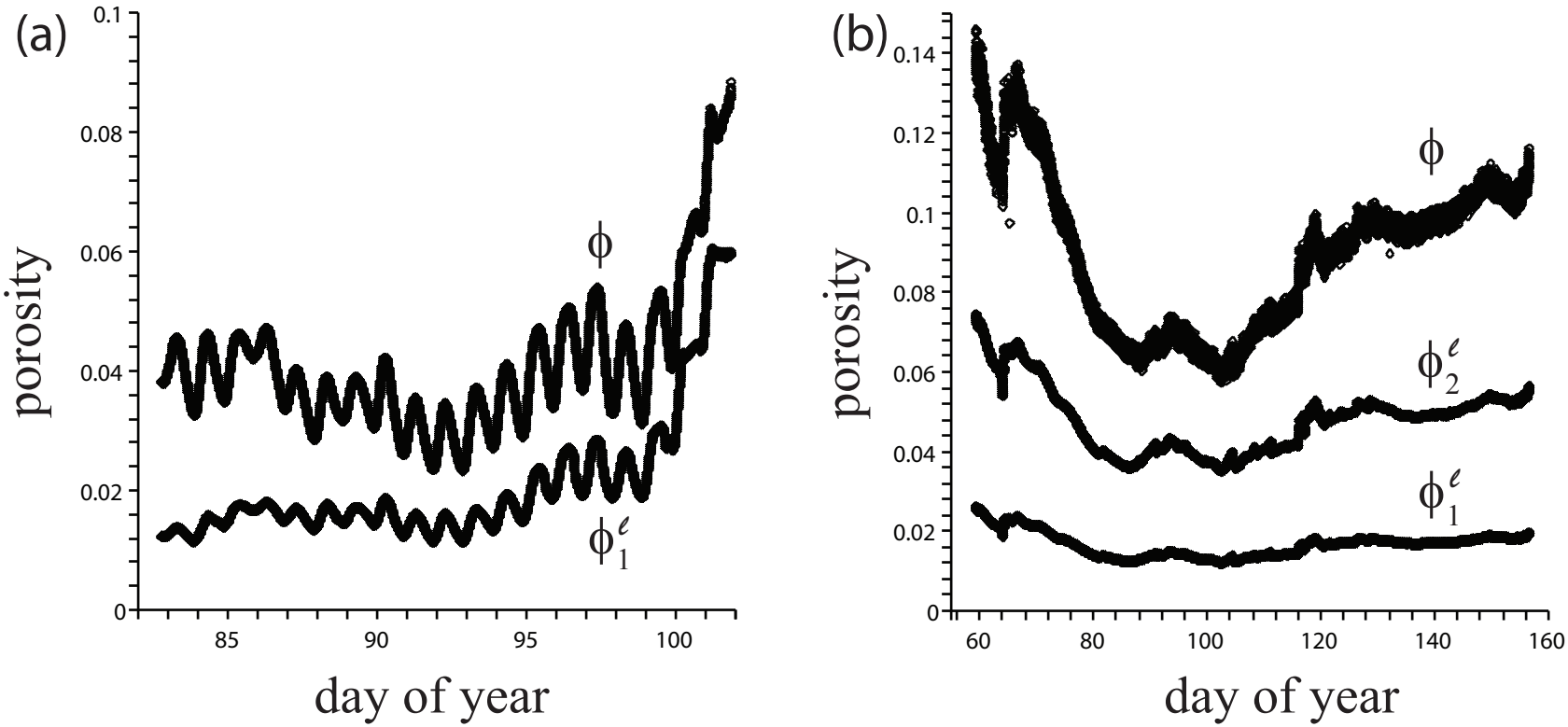
Violation of isotropic bounds near percolation



ice tank data with the elementary and isotropic bounds

isotropic bounds are violated as a vertically oriented, connected brine network develops

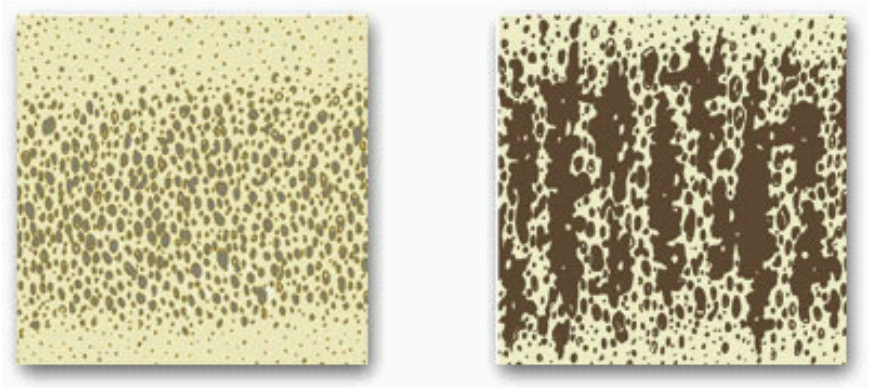
Inverse bounds and microstructural recovery



Inverse Homogenization for Evaluation of Bone Structure

Carlos Bonifasi-Lista, Elena Cherkaev, 2006

bone porosity can be estimated from the effective torsional modulus using reconstruction of spectral measure



normal osteoporotic

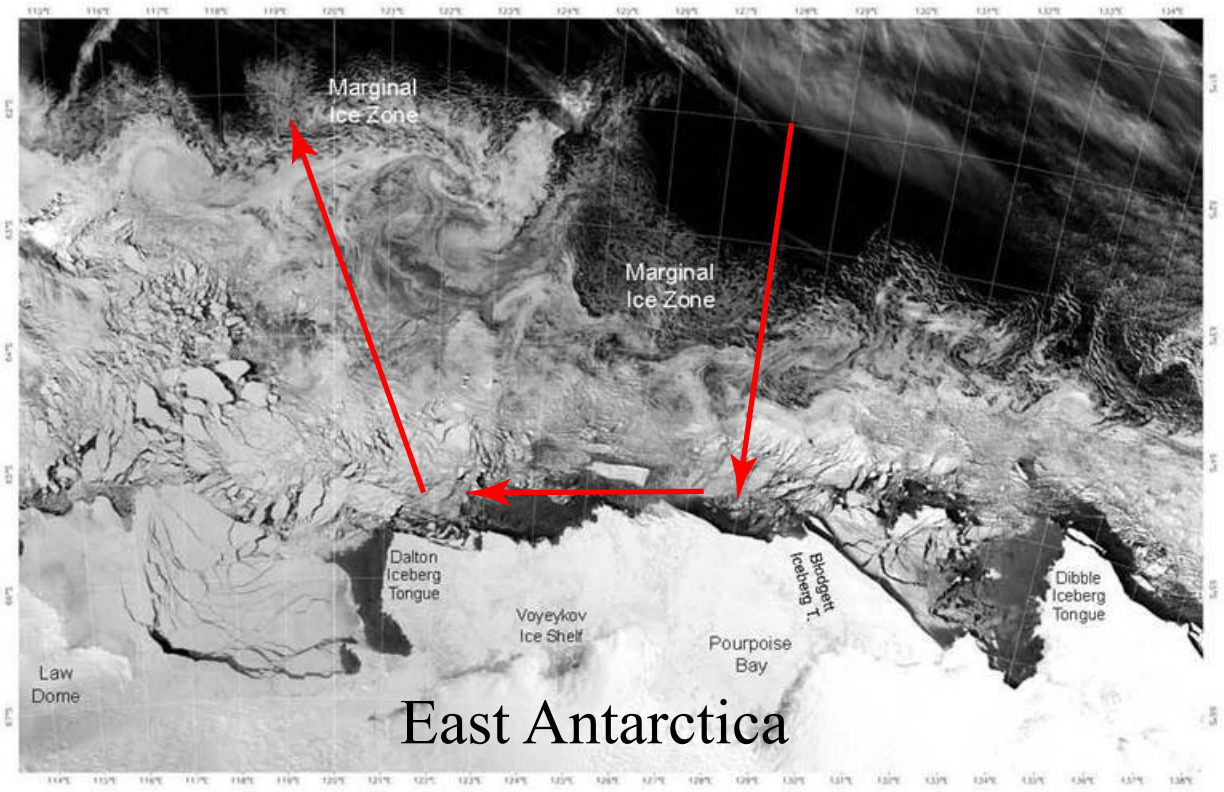
***Application*:**

Monitoring osteoporosis

the math doesn't care if it's sea ice or bone

Sea Ice Physics and Ecosystem eXperiment (SIPEX)

4 September - 17 October 2007



Australian Government
Department of the Environment and Water Resources
Australian Antarctic Division

International Polar Year (IPY) 2007-2008

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Measuring fluid and electrical properties during SIPEX

with Adam Gully

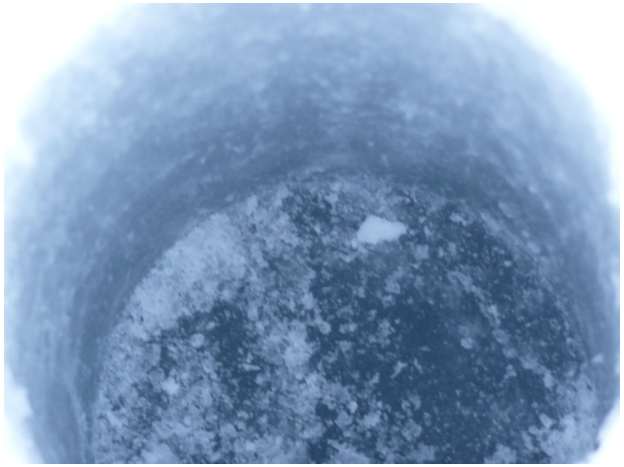
1. Fluid permeability -- first measurements in Antarctic pack ice

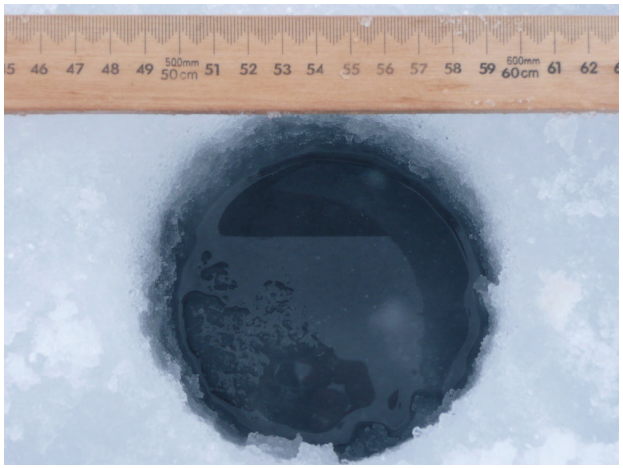
2. Horizontal DC conductivity - Wenner array *surface impedance tomography*: invert resistance data to get profile (w/ T. Worby and J. Reid)

3. Vertical DC conductivity - direct from cores

4. Tracer studies

Measurements of fluid permeability

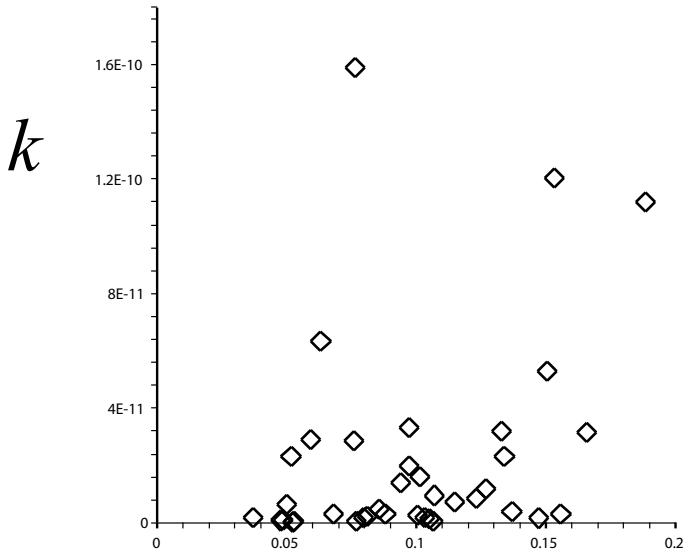


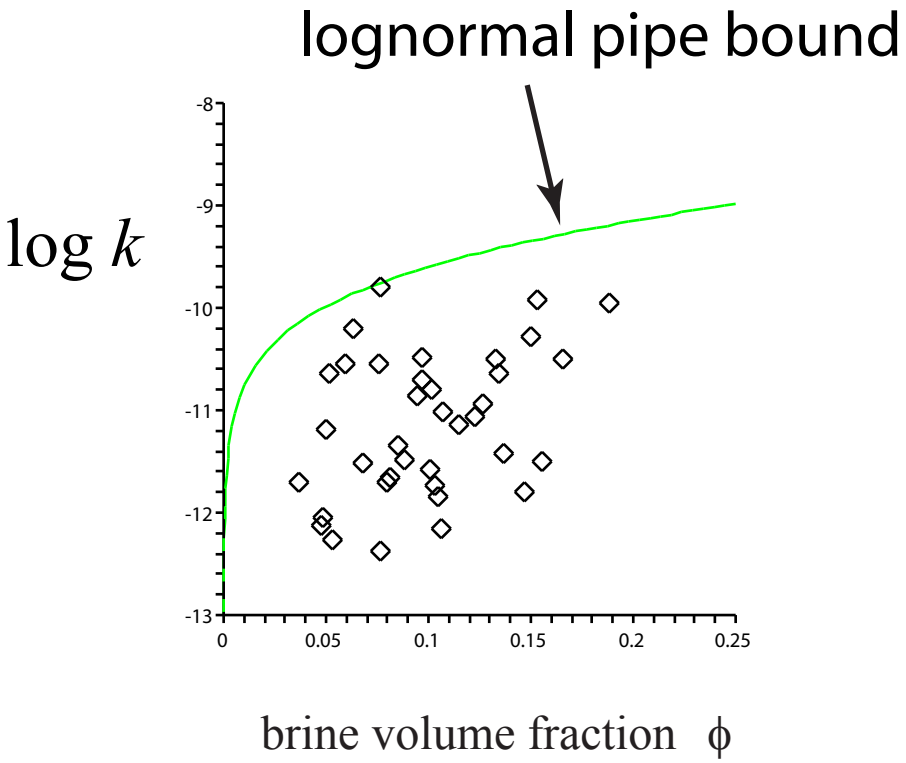


(Photo by Jan Lieser)

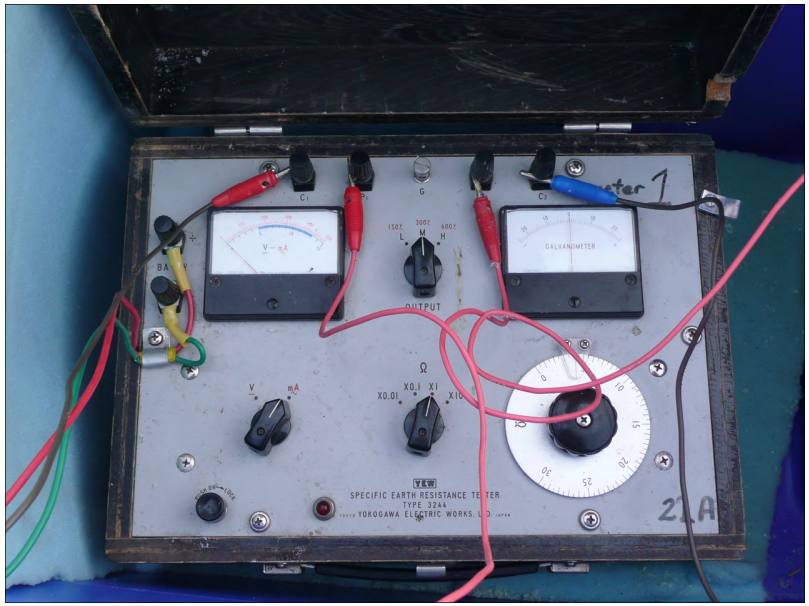
demonstrated Rule of Fives

corrected permeability data -- excludes horizontal flow

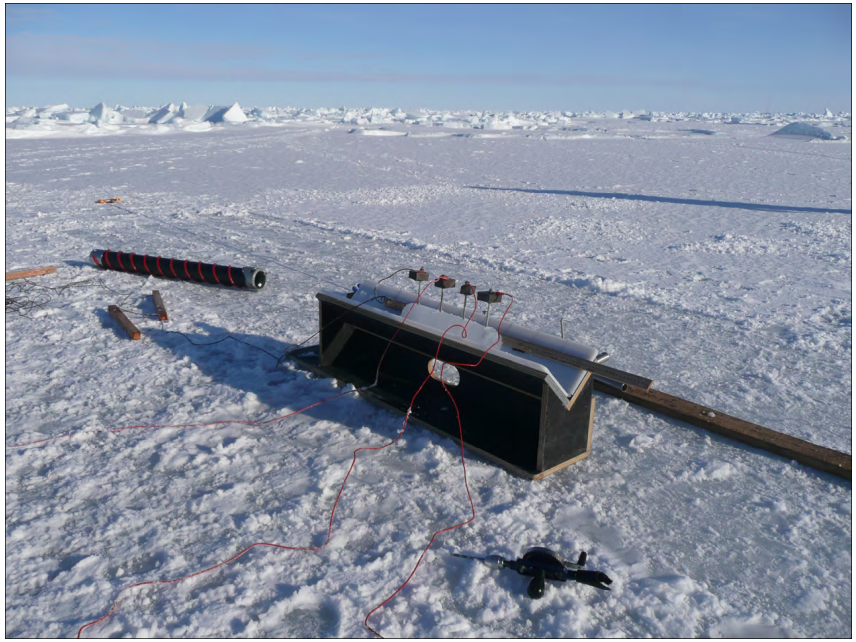


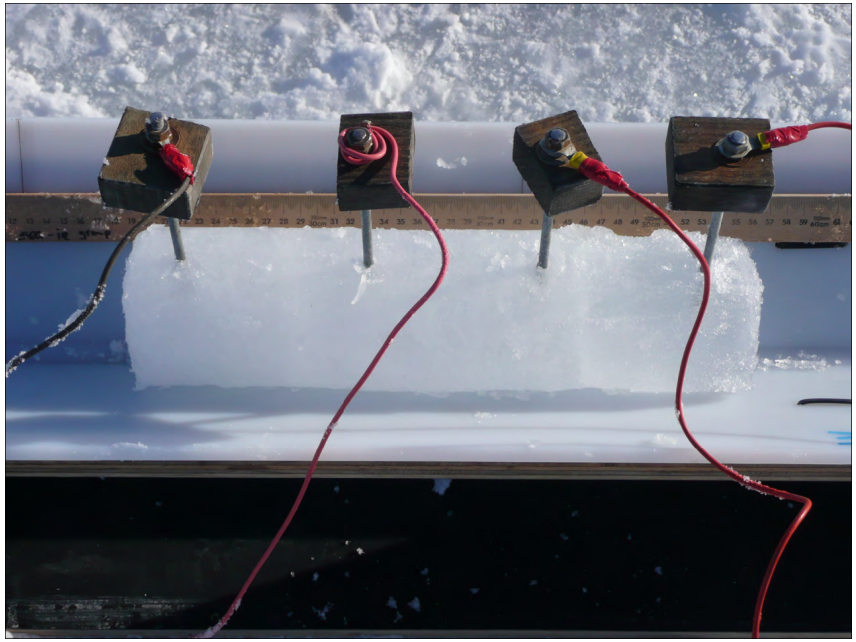


electrical measurements



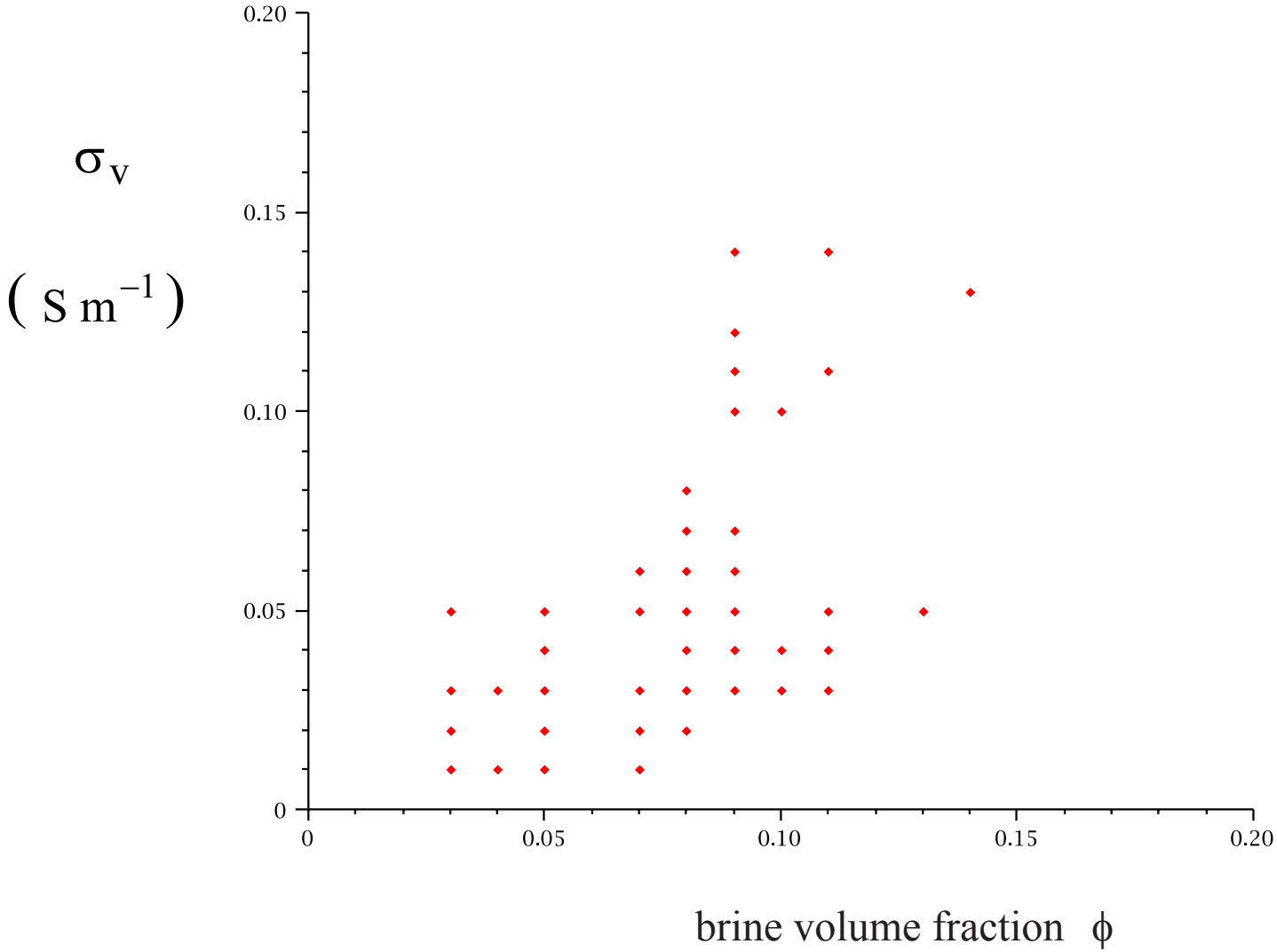
Wenner array





vertical conductivity

vertical conductivity data for first year Antarctic sea ice

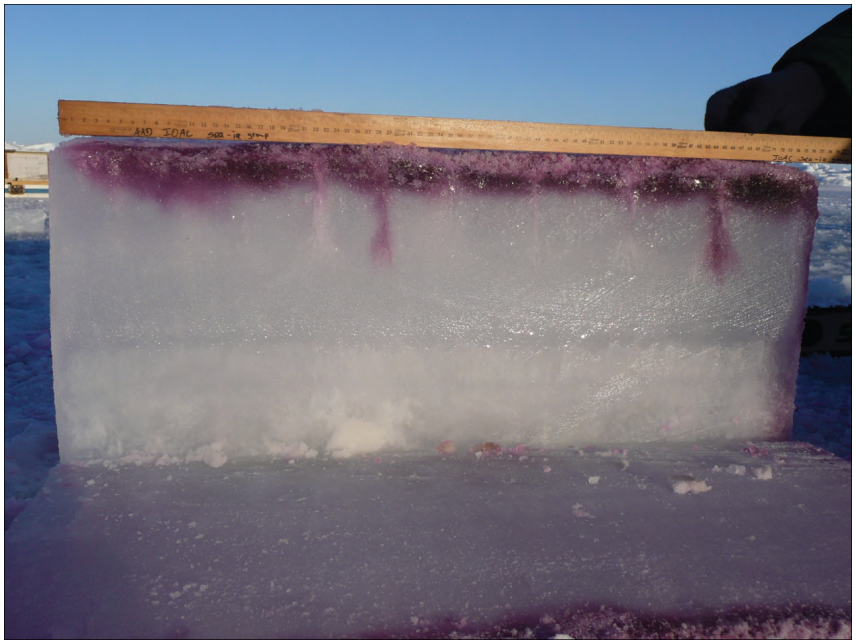


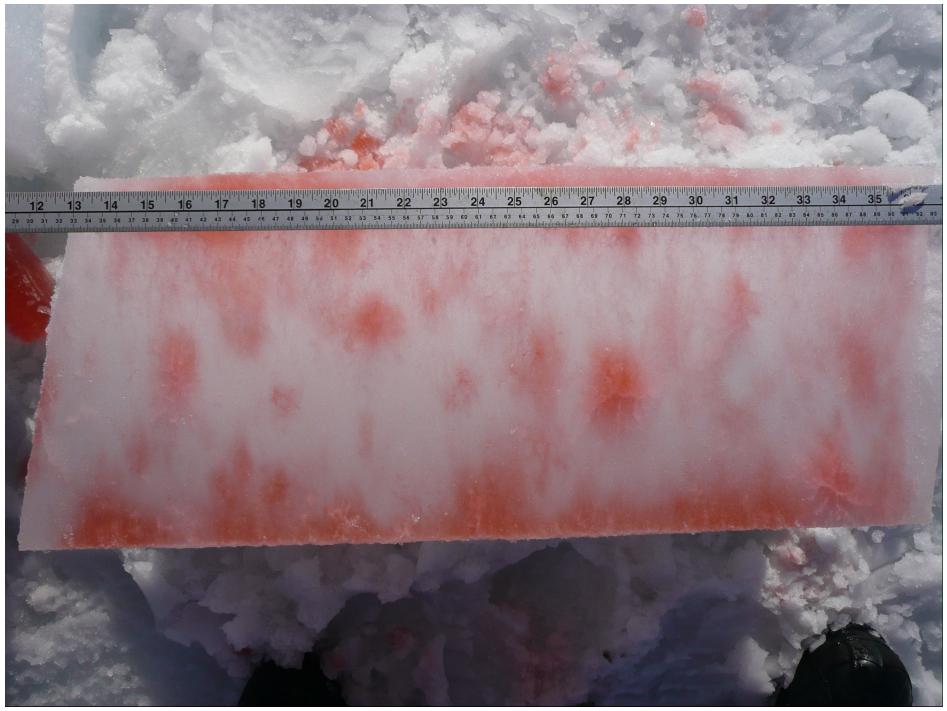
correlate with EM soundings of thickness

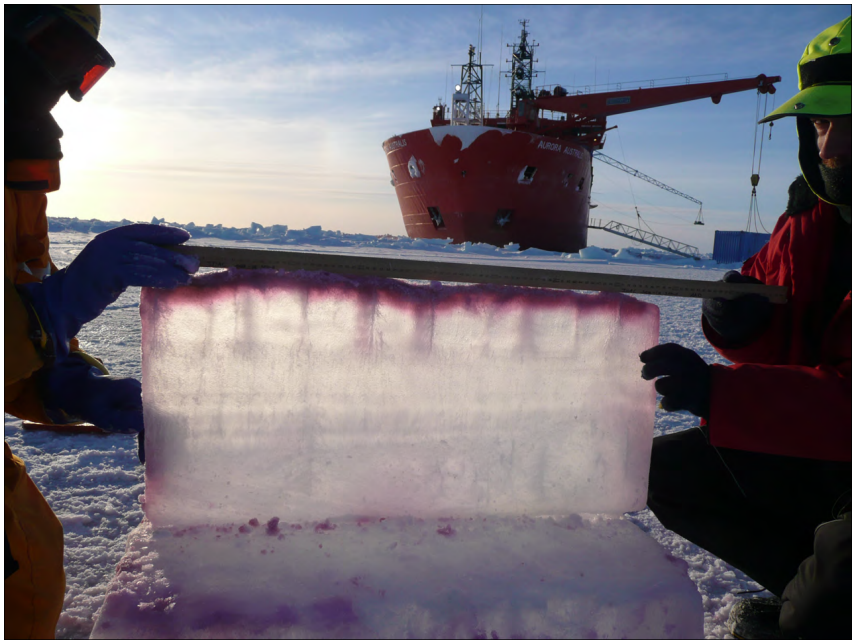
Kazu Tateyama
Adam Gully

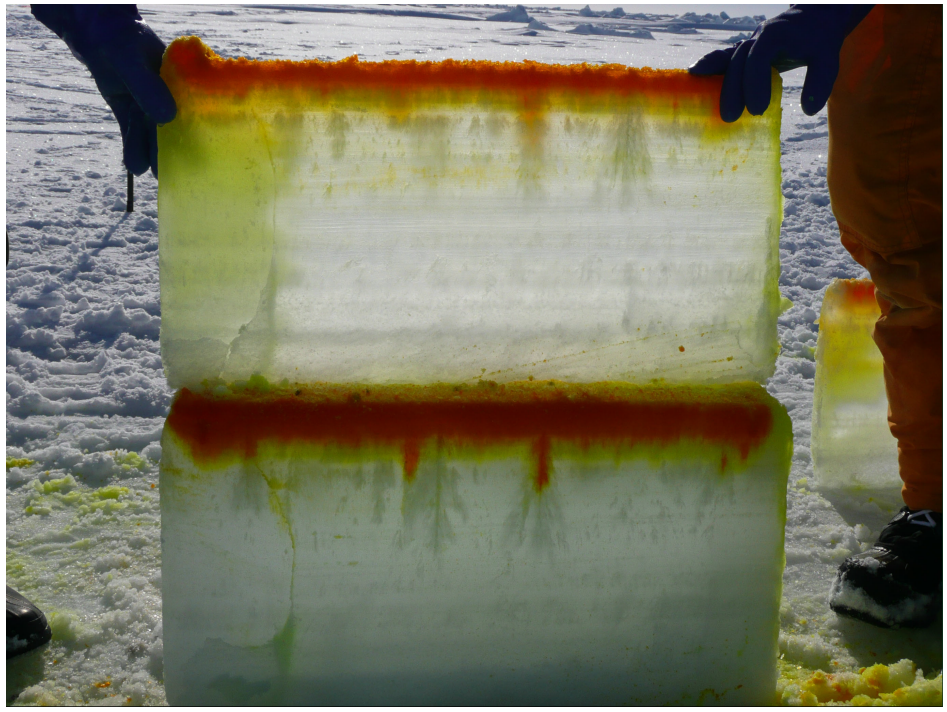
Worbot (T. Worby)

tracers flowing through inverted sea ice blocks









Conclusions

1. Brine flow through sea ice is a key to geophysics and biology of polar regions.

2. Comprehensive theory of fluid permeability, using mathematical techniques from solid state physics.

3. Measured sea ice transport properties in Antarctica.

4. Sea ice processes such as melt pond evolution, snow-ice formation, nutrient flux can be modeled more realistically.

5. Results can help to predict how climate change may affect sea ice packs and polar ecosystems.

THANKS to:

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CMG Program

(Collaboration in Mathematical Geosciences)

VIGRE Program

REU Program

THE UNIVERSITY OF UTAH

UAF UNIVERSITY OF ALASKA FAIRBANKS

Buchanan Bay

Antarctica

July 1999

Mertz Glacier

Polynya Experiment

NSF

Australian Government

Department of the Environment and Water Resources

Australian Antarctic Division

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