# N=(0, 2) SYK Chaos and Higher-Spins

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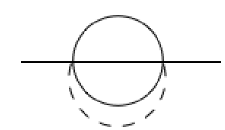
# The Sachdev-Ye-Kitaev (SYK) model

(Sachdev, Ye, 1993; Parcollet, Georges, 1998; Kitaev 2015,2017,...)

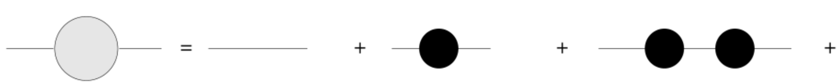
- Strongly coupled Quantum Mechanics model

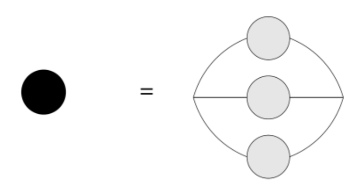
$$H = (i)^{\frac{q}{2}} \sum_{1\le i_1<i_2<\cdots<i_q\le N} j_{i_1 i_2\cdots i_q}\psi_{i_1}\psi_{i_2}\cdots\psi_{i_q}$$

$$\langle j^2_{i_1\cdots i_q}\rangle = \frac{J^2(q-1)!}{N^{q-1}} = \frac{2^{q-1}}{q}\frac{\mathcal{J}^2(q-1)!}{N^{q-1}}$$



- Perturbatively solvable in the large $N$ limit





$$\int d\tau' G(\tau,\tau')\Sigma(\tau',\tau'') = -\delta(\tau-\tau'') \ , \qquad \Sigma(\tau,\tau') = J^2\left[G(\tau,\tau')\right]^{q-1}$$

# *Use of the SYK-like models*

- New insights into old problems
  - Perturbatively solvable
  - Holographic
  - Chaotic
  - Tower of operators

- Question: emergence of higher spin symmetry

  - Large number of symmetries
  - Another corner of AdS/CFT correspondence
  - Tensionless/High energy limit of string theory, a very symmetric phase

# *Higher dimensions*

- Why higher dimensions ?
  - Sensible notion of spins
  - Have better studied higher-spin/string models
  - Simplest example is 1+1D

# *Higher dimensional models*

- Lattice models (Gu, Qi, Stanford, 2016, many generalizations)
- Random Thirring Model (Berkooz, Narayan, Rozali, Simon, 2017)
- Topological model (Turiaci, Verlinde, 2017)
- Tensor models (Giombi, Klebanov, Tarnopolsky, 2017
  Giombi, Klebanov, Popov, Prakash, Tarnopolsky, 2018)
- Supersymmetric models
  - $\mathcal{N} = (1,1)$ model (Murugan,Stanford, Witten, 2017)
  - $\mathcal{N} = (2,2)$ model (Murugan,Stanford, Witten, 2017; Bulycheva, 2018)

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# *Supersymmetric models*

- SUSY is important to reach the SYK-like fix point
- Reduce the number of supersymmetries
- Look for connections with other established models

# The $\mathcal{N}$=(0,2) model

(CP, 2018)

- $$S = \int d^2z d\theta d\bar{\theta} \left( -\bar{\Phi}^a \partial_{\bar{z}} \Phi^a + \frac{1}{2} \bar{\Lambda}^i \Lambda^i \right) + \int d^2 z d\theta \frac{J_{ia_1 \dots a_q}}{q!} \Lambda^i \Phi^{a_1} \dots \Phi^{a_q}$$

  Chiral: $\Phi^a = \phi^a + \sqrt{2}\theta\psi^a + 2\theta\bar{\theta}\partial_z \phi^a \,,\qquad a = 1 \dots N$

  Fermi: $\Lambda^i = \lambda^i - \sqrt{2}\theta G^i + 2\theta\bar{\theta}\partial_z \lambda^i \,,\qquad i = 1 \dots M$

- $N, M \gg 1$ , with $\mu = \frac{M}{N}$ free

- IR solution $G_c^I(z_1, z_2) = \dfrac{n_I}{(z_1 - z_2)^{2h_I} (\bar{z}_1 - \bar{z}_2)^{2\tilde{h}_I}} \,,\qquad I = \phi, \psi, \lambda, G$

$$h_\phi = \frac{\mu q - 1}{2\mu q^2 - 2}, \quad h_\psi = \frac{\mu q^2 + \mu q - 2}{2\mu q^2 - 2}, \quad h_\lambda = \frac{q - 1}{2\mu q^2 - 2}, \quad h_G = \frac{\mu q^2 + q - 2}{2\mu q^2 - 2}$$

$$\tilde{h}_\phi = \frac{\mu q - 1}{2\mu q^2 - 2}, \quad \tilde{h}_\psi = \frac{\mu q - 1}{2\mu q^2 - 2}, \quad \tilde{h}_\lambda = \frac{\mu q^2 + q - 2}{2\mu q^2 - 2}, \quad \tilde{h}_G = \frac{\mu q^2 + q - 2}{2\mu q^2 - 2} \,.$$

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  $q$=2 is not free

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# Range of $\mu$

- Convergence of the FT

$$\int r dr d\theta \, r^{2\frac{\mu q-1}{\mu q^2-1}-3} e^{i\theta} e^{ir\cos\theta}$$

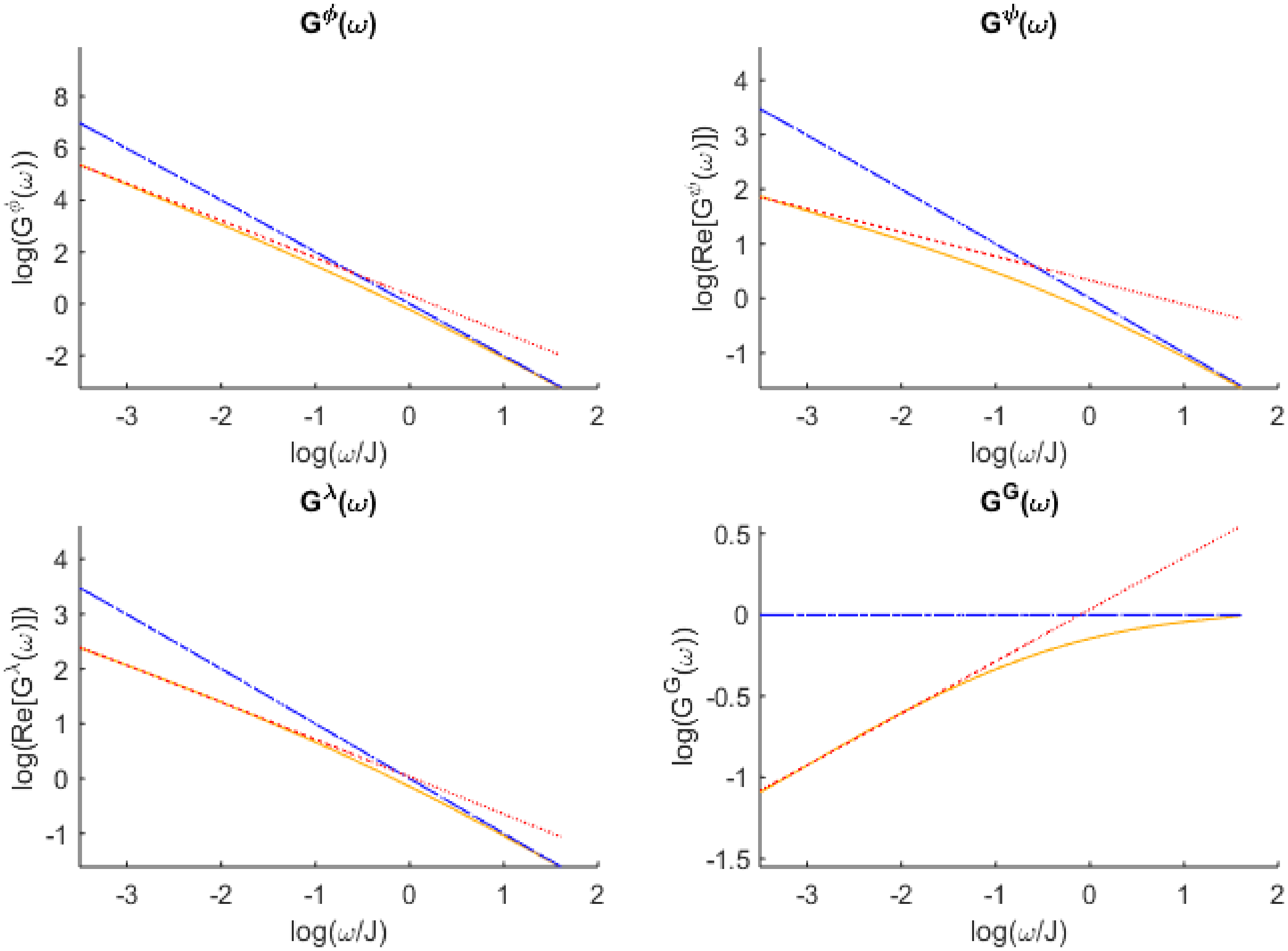
$$\Rightarrow \int dr d\theta r^{2\frac{\mu q-1}{\mu q^2-1}-2}(1+ir\cos\theta+\ldots)e^{i\theta}$$

$$\Rightarrow \int dr d\theta \, i r^{2\frac{\mu q-1}{\mu q^2-1}-1}\cos\theta e^{i\theta}$$

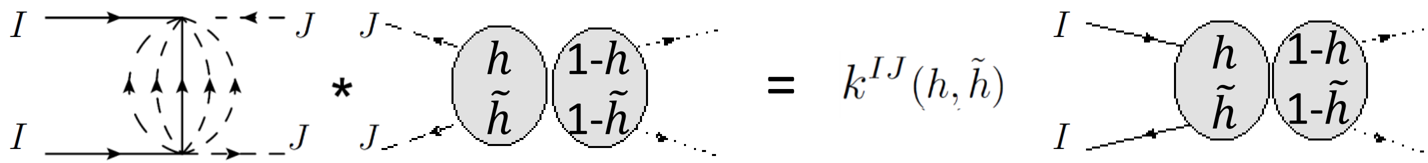
$$\boxed{\mu > \frac{1}{q}}$$

- In this range the model flows to the SYK-like fixed point

# Numerical Confirmation



# *4-point function*

- $\langle \bar{\phi}^i \phi^i \bar{\phi}^j \phi^j \rangle$, $\langle \bar{\phi}^i \phi^i \bar{\psi}^j \psi^j \rangle$ , $\langle \bar{\phi}^i \phi^i \bar{\lambda}^j \lambda^j \rangle$, …
- 


$$\Phi^I(z_1, z_2) = (z_{12})^{h-2h_I} (\bar{z}_{12})^{\tilde{h}-2\tilde{h}_I} \,, \qquad I = \phi, \psi, \lambda, G$$

$$k^{\psi\phi} = -\frac{2\mu(q-1)^2 q(\mu q-1)^2 \Gamma\left(\frac{(q-1)q\mu}{q^2\mu-1}\right)^2 \Gamma\left(\frac{-h\mu q^2+\mu q^2+\mu q+h-2}{q^2\mu-1}\right) \Gamma\left(\tilde{h}-\frac{(q-1)q\mu}{q^2\mu-1}\right)}{(\mu q^2-1)^3 \Gamma\left(\frac{\mu q^2+\mu q-2}{q^2\mu-1}\right)^2 \Gamma\left(\frac{-h\mu q^2+(q-1)\mu q+h}{q^2\mu-1}\right) \Gamma\left(\tilde{h}+\frac{(q-1)q\mu}{q^2\mu-1}\right)}$$

…

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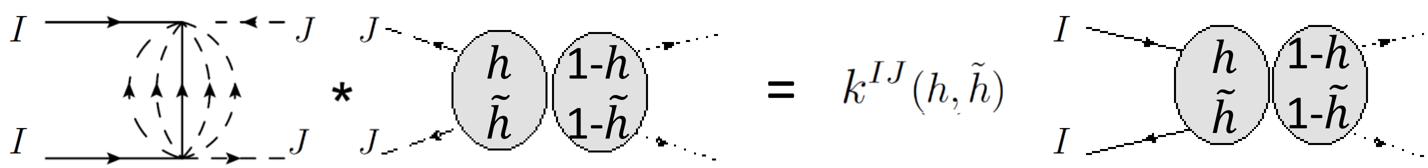
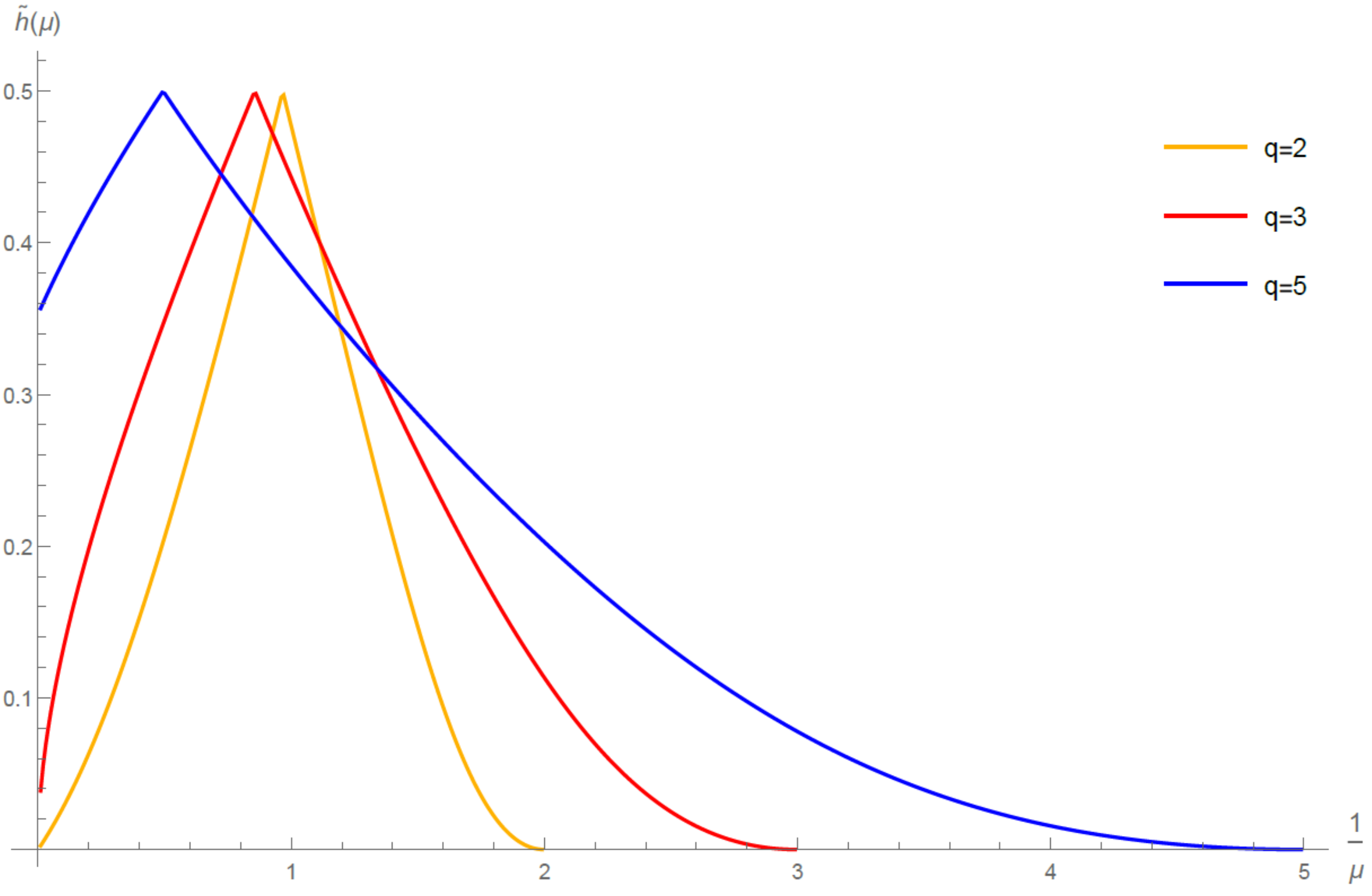
- 

- $\begin{pmatrix} k^{\phi\phi} & k^{\phi\psi} & k^{\phi\lambda} & k^{\phi G} \\ k^{\psi\phi} & 0 & k^{\psi\lambda} & 0 \\ k^{\lambda\phi} & k^{\lambda\psi} & 0 & 0 \\ k^{G\phi} & 0 & 0 & 0 \end{pmatrix}$ whose eigenvalue x satisfies

$$E_c(x,h,\tilde h,\mu,q) = x^4 - k^{\phi\phi}x^3 - \left(k^{\phi G}k^{G\phi} + k^{\phi\psi}k^{\psi\phi} + k^{\phi\lambda}k^{\lambda\phi} + k^{\psi\lambda}k^{\lambda\psi}\right)x^2$$
$$+ \left(k^{\phi\phi}k^{\psi\lambda}k^{\lambda\psi} - k^{\phi\psi}k^{\psi\lambda}k^{\lambda\phi} - k^{\phi\lambda}k^{\psi\phi}k^{\lambda\psi}\right)x + k^{\phi G}k^{\psi\lambda}k^{\lambda\psi}k^{G\phi} = 0$$

- Solve x=1 to get the spectrum of $O^{\tilde h, h}$, spin s = $|h - \tilde h|$.

# *Lightest scalar operators*



# The Lyapunov exponent

- $K_R^{(ij)} * \Psi_R^j = k_R^{ij}\Psi_R^i$ (Kitaev 2015, Maldacena Stanford, 2016)

$$\Psi_R^I(1,2) = \frac{e^{-\frac{1}{2}(h+\tilde{h})(t_1+t_2)-\frac{1}{2}(h-\tilde{h})(x_1+x_2)}}{(2\cosh\frac{x_{12}-t_{12}}{2})^{h_1+h_2-h}(2\cosh\frac{x_{12}+t_{12}}{2})^{\tilde{h}_1+\tilde{h}_2-\tilde{h}}}$$

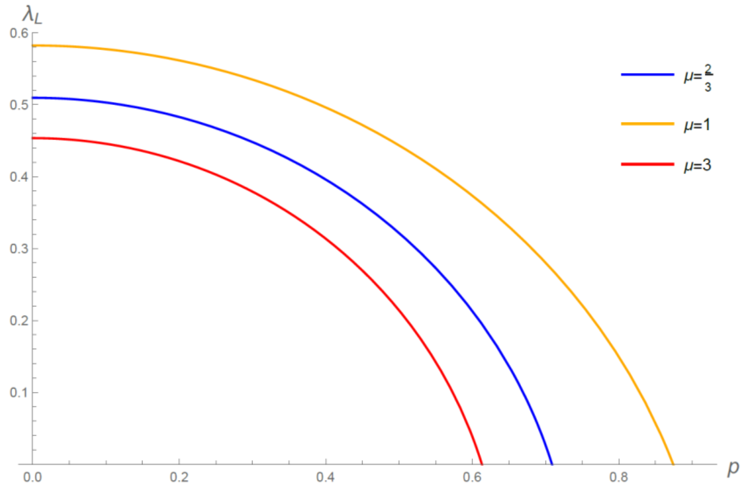
$$h = -\frac{\lambda_L}{2} + i\frac{p}{2} \qquad \tilde{h} = -\frac{\lambda_L}{2} - i\frac{p}{2}$$

(Murugan,Stanford, Witten, 2017)

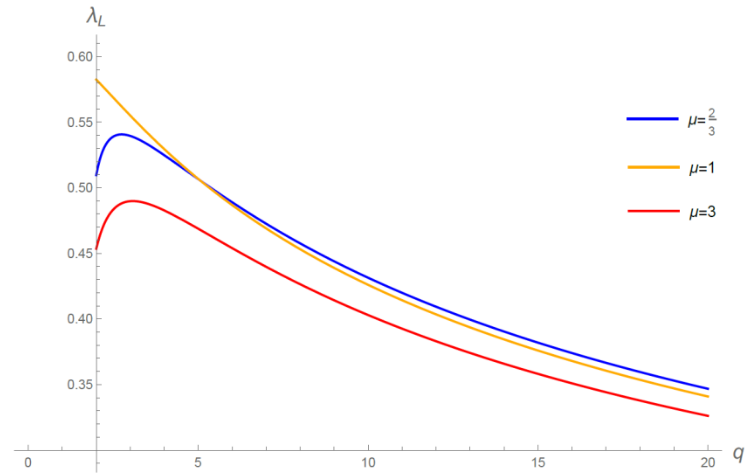
- $$E_R(x,h,\tilde{h},\mu,q) = x^4 - k_R^{\phi\phi}x^3 - \left(k_R^{\phi G}k_R^{G\phi} + k_R^{\phi\psi}k_R^{\psi\phi} + k_R^{\phi\lambda}k_R^{\lambda\phi} + k_R^{\psi\lambda}k_R^{\lambda\psi}\right)x^2 + \left(k_R^{\phi\phi}k_R^{\psi\lambda}k_R^{\lambda\psi} - k_R^{\phi\psi}k_R^{\psi\lambda}k_R^{\lambda\phi} - k_R^{\phi\lambda}k_R^{\psi\phi}k_R^{\lambda\psi}\right)x + k_R^{\phi G}k_R^{\psi\lambda}k_R^{\lambda\psi}k_R^{G\phi} = 0$$

- Find $\lambda_L$ by solving x=1

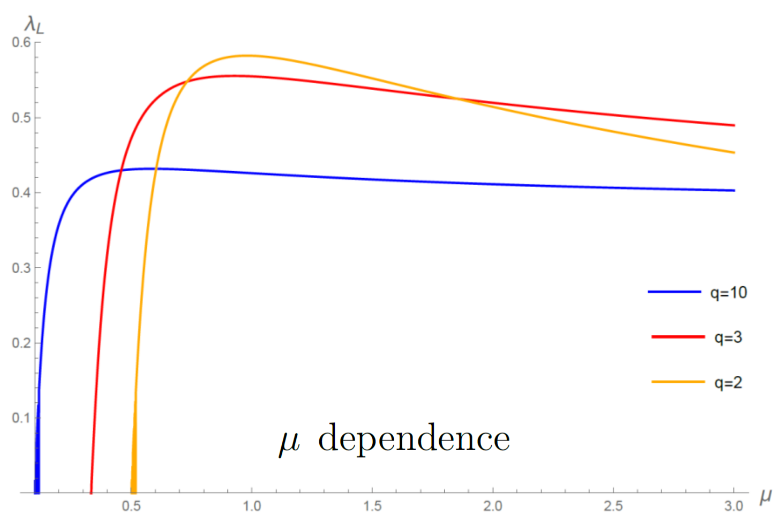
# *The Lyapunov exponent*




(a) $p$ dependence.



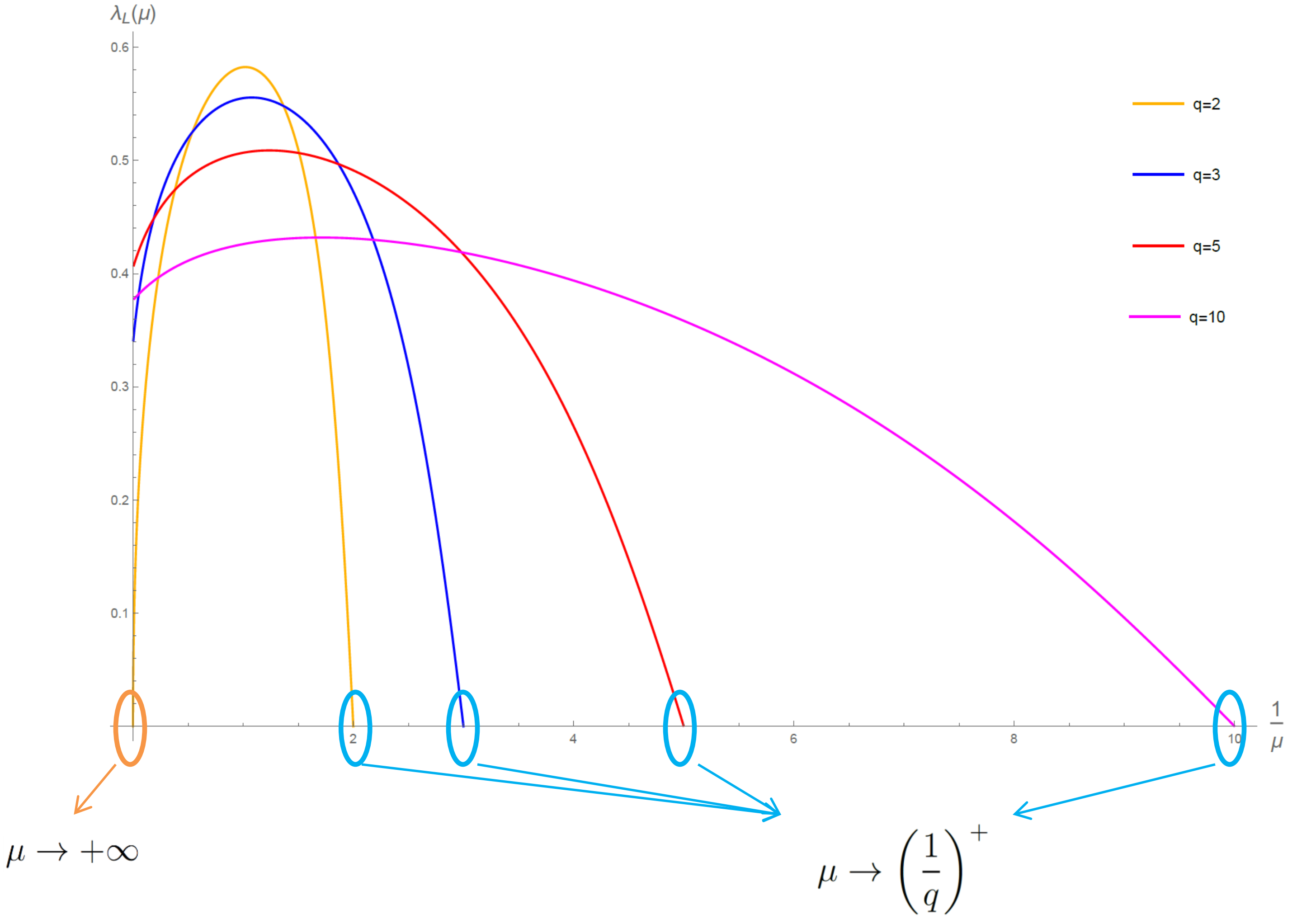

(b) $q$ dependence.




$\mu$ dependence

Fix $q$, check the $\mu$ dependence of the model

# *Two interesting limits*



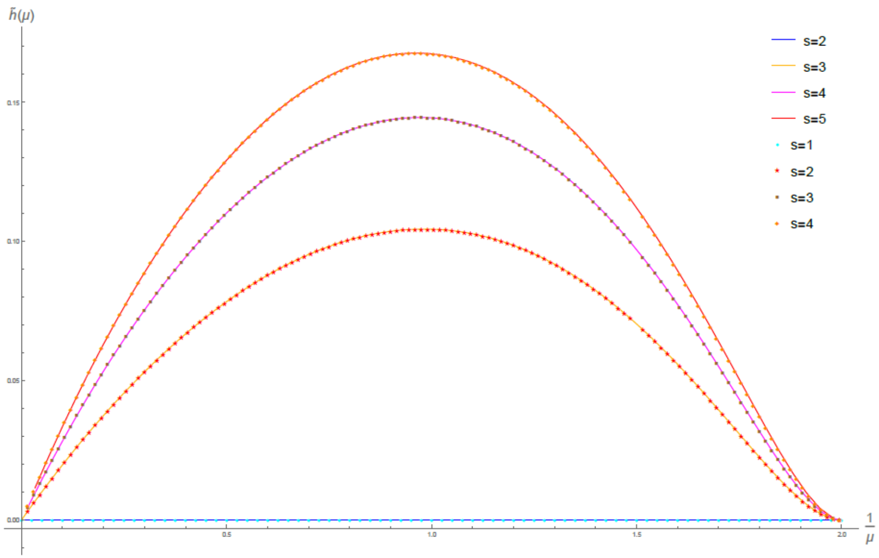
# *Two interesting limits*

- Lyapunov exponent drops to zero
- “Integrablity” takes over ?
- Large symmetries ?

$$(\tilde{h}, h) = (\gamma, \gamma + s) \text{ or } (\gamma + s, \gamma), \qquad s \in \mathbb{Z}/2$$

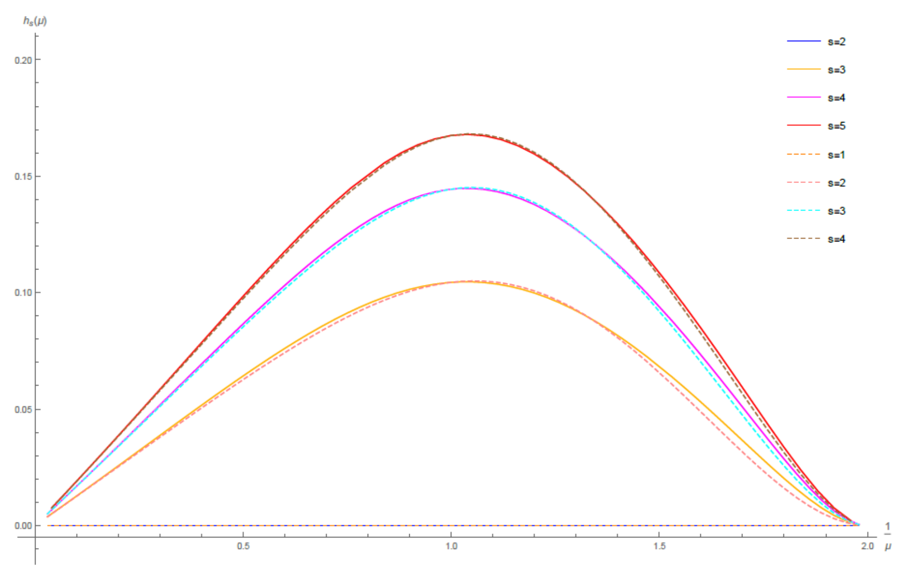
looking for the smallest $\gamma$ for each $\mu$

# *Lightest operators with spins*




$(\gamma, \gamma + s)$

$(\gamma + s, \gamma)$

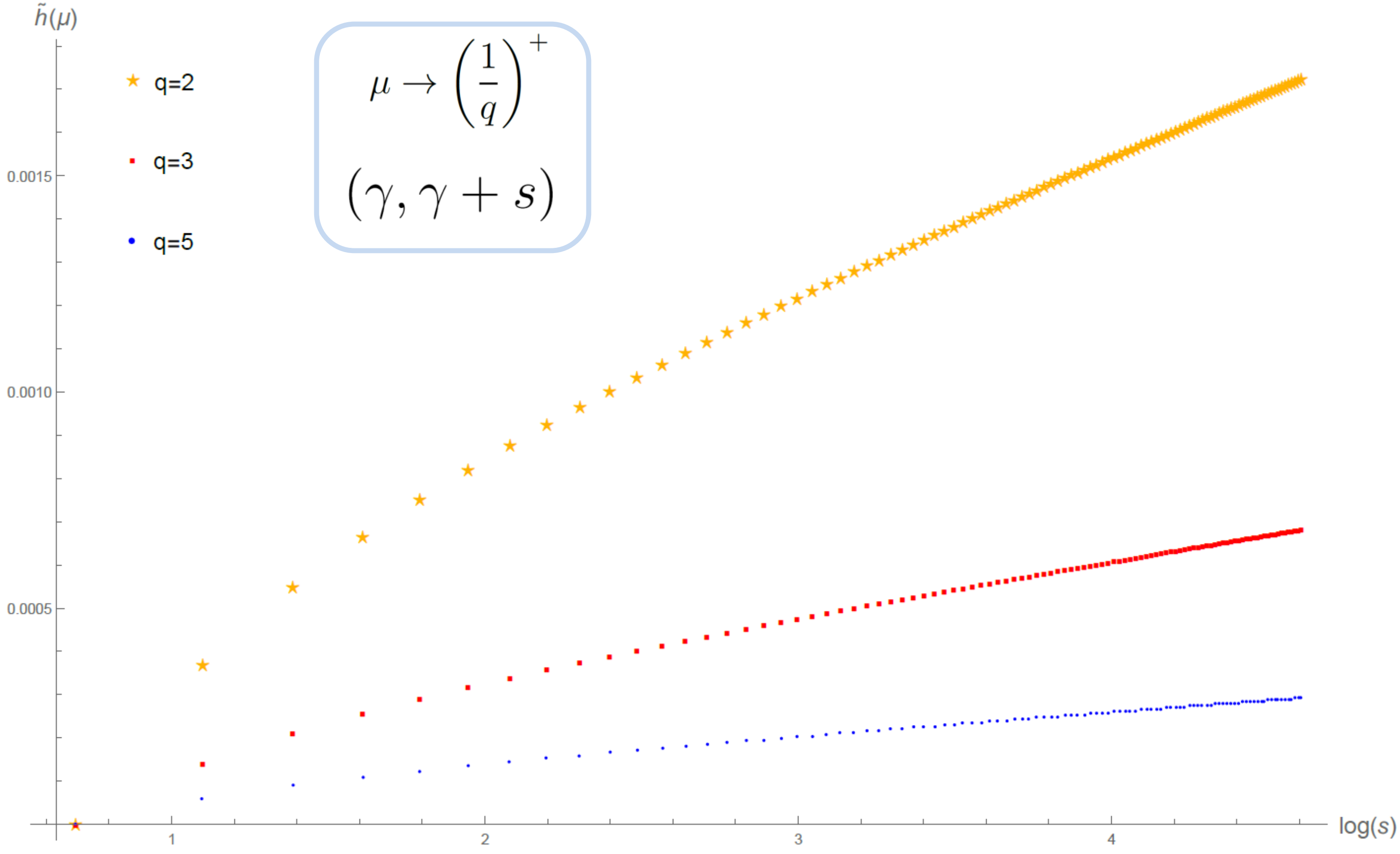



- Emergent higher-spin operators in the two limits!

- Generate large symmetry
  ➡ nonchaotic

# *Dispersion relation*

- How does the anomalous dimension γ depend on spin s ?



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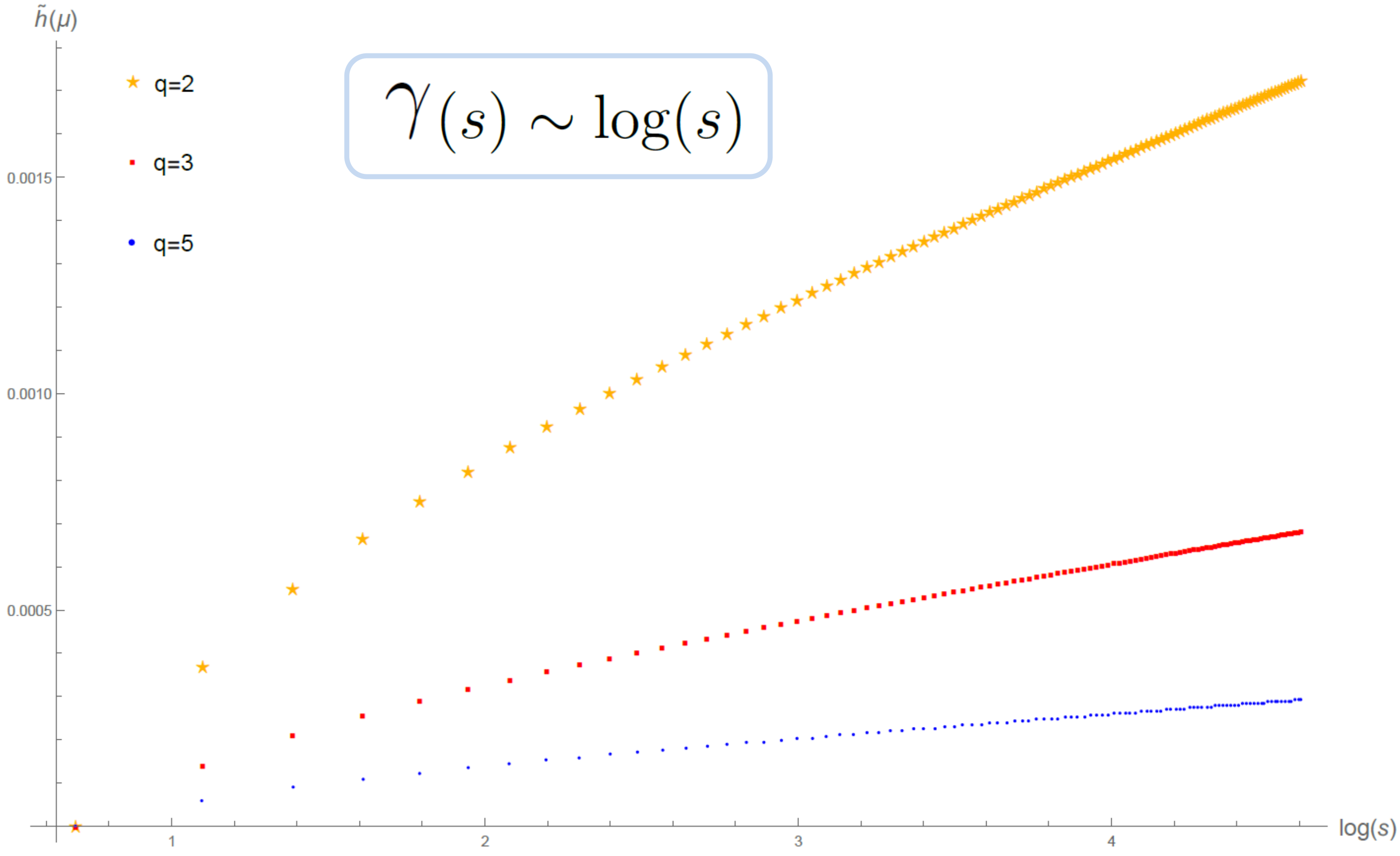
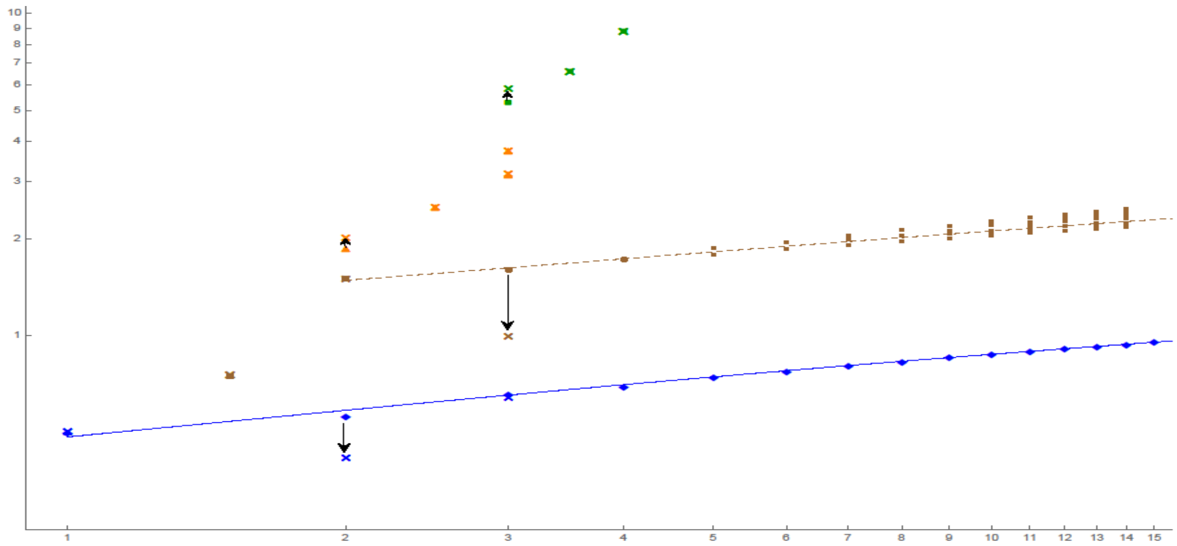


# *Relations with Higher-spin theory*

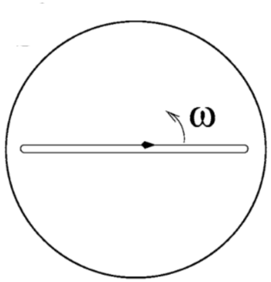
- Higher-spin perturbation computation (Gaberdiel, CP, Zadeh, 2015)



- Rotating folded closed long string in AdS (Gubser, Klebanov, Polyakov, 2002)

$$E - S = \frac{\sqrt{\lambda}}{\pi} \ln\left(S/\sqrt{\lambda}\right) + \cdots . \qquad \lambda = g_{\rm YM}^2 N$$

logarithmic due to the AdS geometry



# *Relations with Higher-spin theory*

- Logic reversed
- Regarded as deformation away from higher spin point
- Consistent with previous results
- A toy model that mimics the process of turning off the string tension where the tuning is explicit

# *Comments on the higher-spin limits*

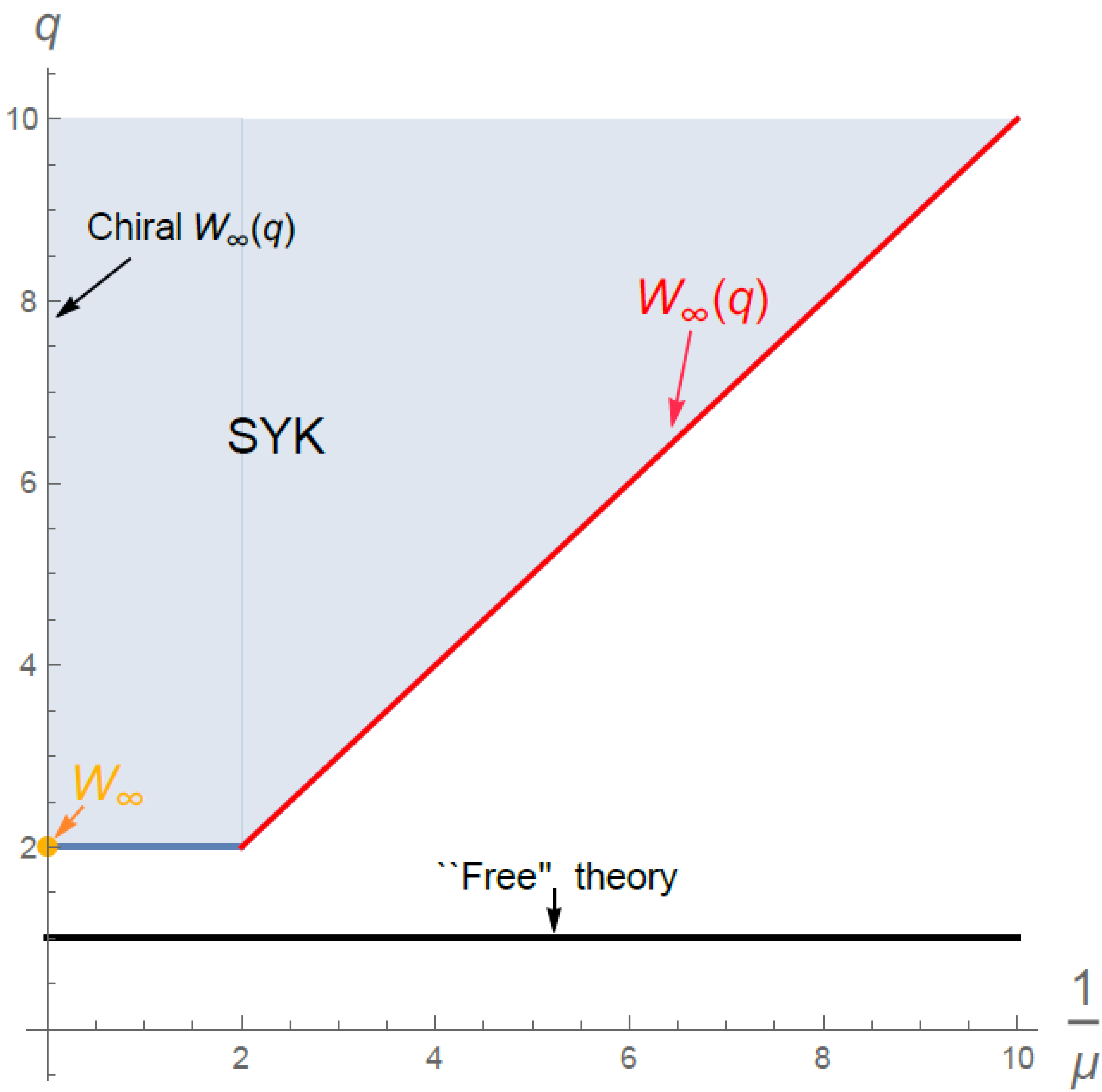
- A tower of higher-spin operators, generate a higher-spin algebra $W_\infty(q)$ for each q, similar to the $W_\infty[\lambda]$ algebra in higher-spin holography

- Model is not free in this limits. Special property is from some delicate scaling/cancelling in the IR dynamics

- Singular: cannot simply plug in $\mu = \frac{1}{q}$ , rather take

$$\mu = \frac{1}{q} + \delta \text{ with } 1 \gg \delta > 0$$

- The other limit $\mu \to +\infty$ is similar, although not identical



q
10
8
6
4
2
Chiral $W_\infty(q)$
SYK
$W_\infty(q)$
$W_\infty$
``Free'' theory
2
4
6
8
10
$\frac{1}{\mu}$

*Thank you!*

# *For a single realization*

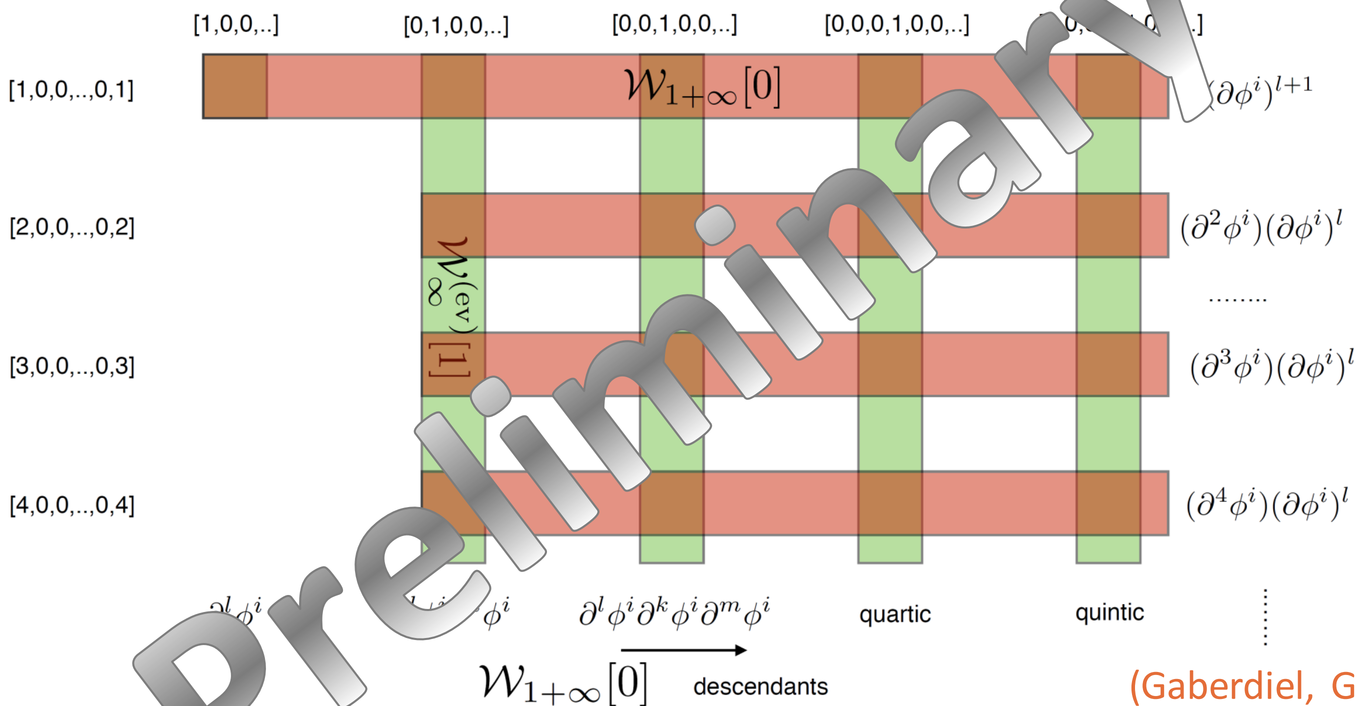
(Ahn, CP, in progress)

- Symmetry that is independent of realization?
- $\bar{Q}$-cohomological algebra ( $\bar{Q}$^2 =0 )
- In UV, does not depend on the value of the random coupling
  Remain so along the RG

# *The cohomological chiral algebra*

(Ahn, CP, in progress)

- Observe a similar structure as stringy “higher spin square”




(Gaberdiel, Gopakumar 2015)

- Another connection to higher-spin/string theories
- Relation with the previous emergent higher-spin symmetries

*Thank you again!*