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Theoretical Physics
University of Cologne

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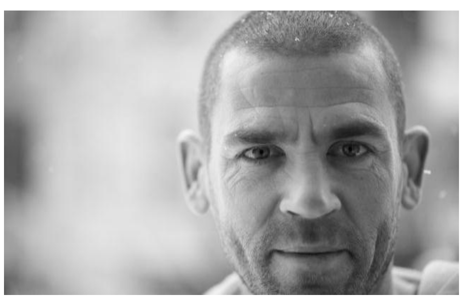
ENTANGLED STATES OF MATTER

Goldstone fluctuations in the SYK model

Dmitry Bagrets



Nucl. Phys. B 911, 191 (2016)

Nucl. Phys. B 921, 727 (2017)

to be published ...

Alexander Altland
Univ. zu Köln

Alex Kamenev
Univ. of Minnesota

“Chaos and Order ...” , KITP, Santa-Barbara, 10th October, 2018

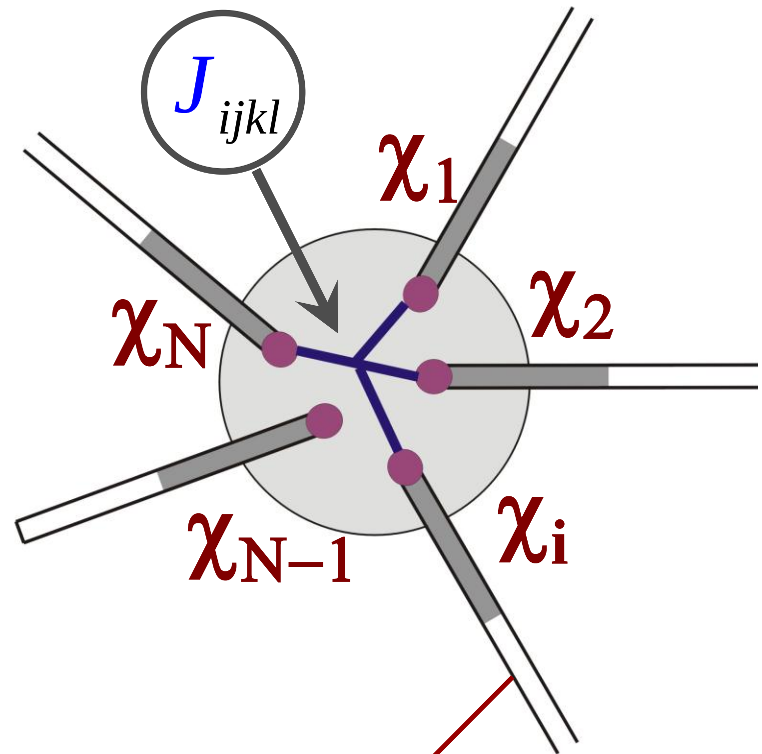
Sachdev-Ye-Kitaev model

S. Sachdev, PRX 5 (2015) 041025
A. Kitaev, talks at KITP, April & May 2015

$$\hat{H} = \frac{1}{4!} \sum_{ijkl}^{N} J_{ijkl} \chi_i \chi_j \chi_k \chi_l$$

Couplings J's are quenched random Gaussian variables,

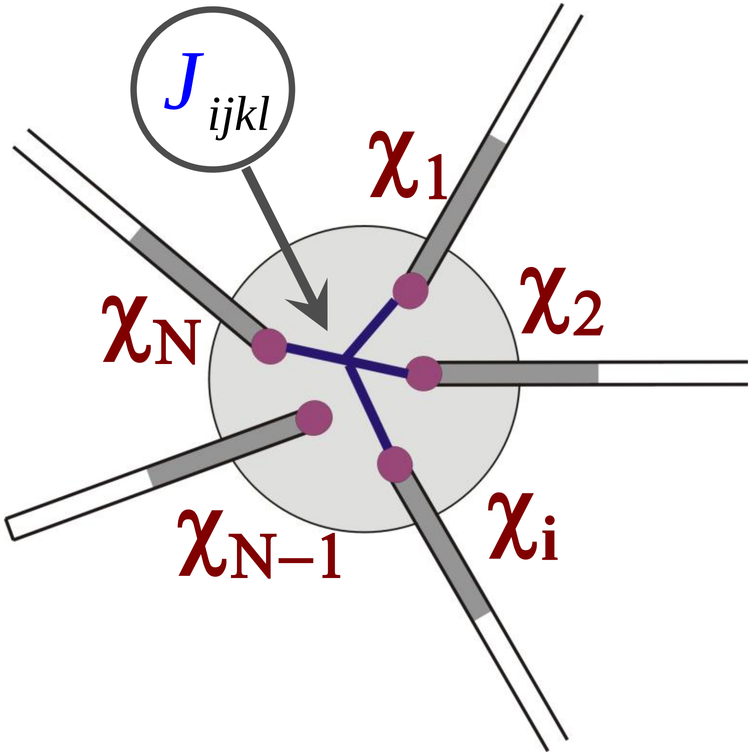
$$\left\langle \left(J_{ijkl} \right)^2 \right\rangle = 3!\, J^2 / N^3$$



Class D Majorana wire

$$\{\chi_i, \chi_j\} = \delta_{ij}$$

SYK model

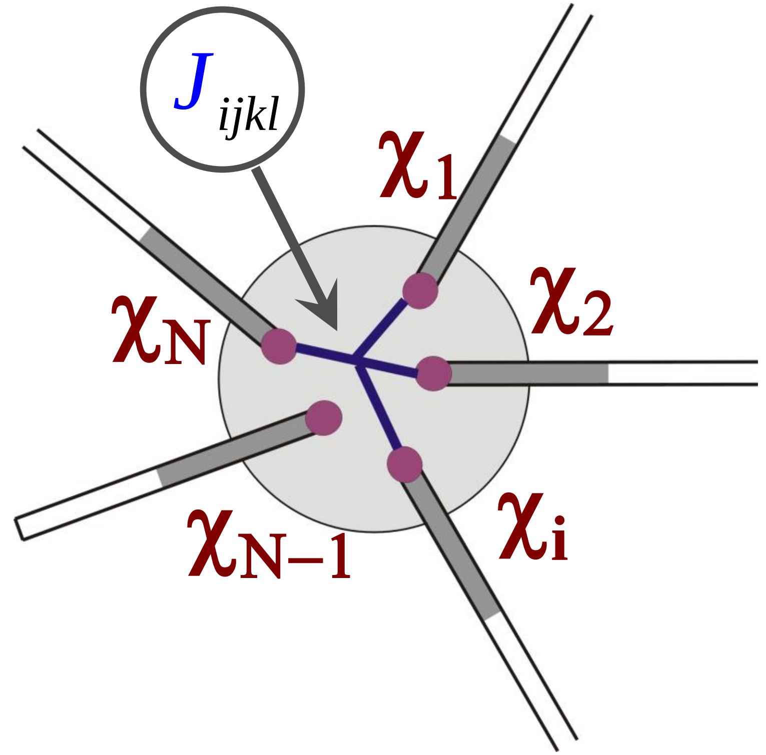


- **AdS_2/CFT_1 holography**

  black-hole physics, dilaton gravity, quantum information paradox, gravitational shock-wave scattering, etc.

- **Strong-correlation physics**

  Many-body (de)-localization, RMT-like level statistics, quantum chaos, strange metals, …



This talk

- Reparametrization ('conformal') symmetry in SYK
- Schwarzian action & Liouville quantum mechanics
- Complex SYK model: emergent Coulomb blockade
- Quantum chaos & OTO correlators

Infra-red 'conformal' symmetry

A. Kitaev' 2015

- averaging over disorder with the replica trick

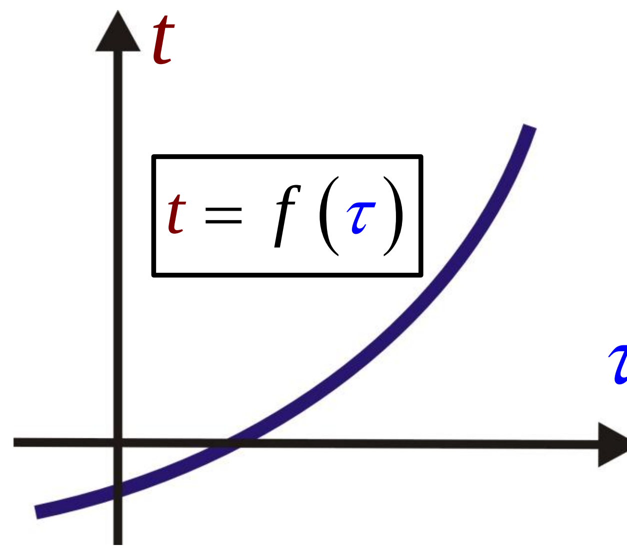
$$\left\langle Z^R = \exp\left(-\sum_{a=1}^{R} \int \left(\dot{\chi}^a \chi^a - H^a \right) dt \right) \right\rangle \qquad \hat{H}^a = \frac{1}{4!} \sum_{ijkl}^{N} J_{ijkl} \chi_i^a \chi_j^a \chi_k^a \chi_l^a$$

Replicated partition sum

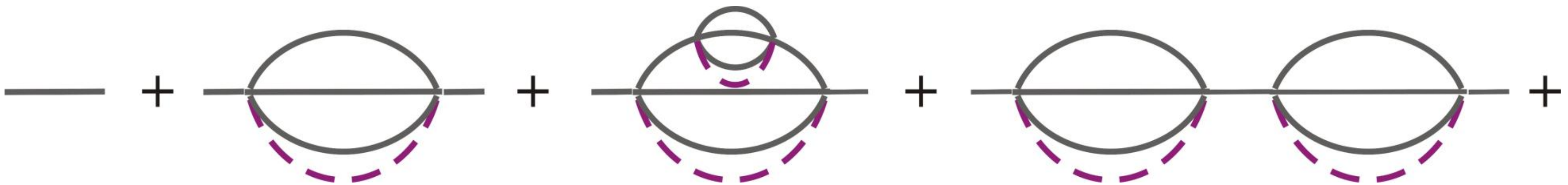
- reparametrizations of the time

$$\tilde{\chi}^a(\tau) = \left[f'(\tau) \right]^{1/4} \chi^a(t)$$

fermion's scaling dimension

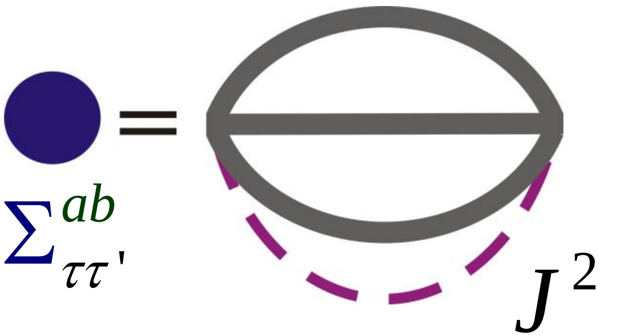


Mean field solution



- **Self-consistent Dyson equation** **(S. Sachdev, J. Ye '1993)**

$$-(\partial_\tau + \Sigma)\bullet G = 1, \quad \Sigma^{ab}_{\tau\tau'} = J^2\left[G^{ab}_{\tau\tau'}\right]^3$$



- **Mean-field solution (T=0)**

$$\bar{G}^{ab}_{t-t'} \propto -\frac{\delta^{ab}}{\sqrt{J}}\frac{1}{|t-t'|^{1/2}}$$

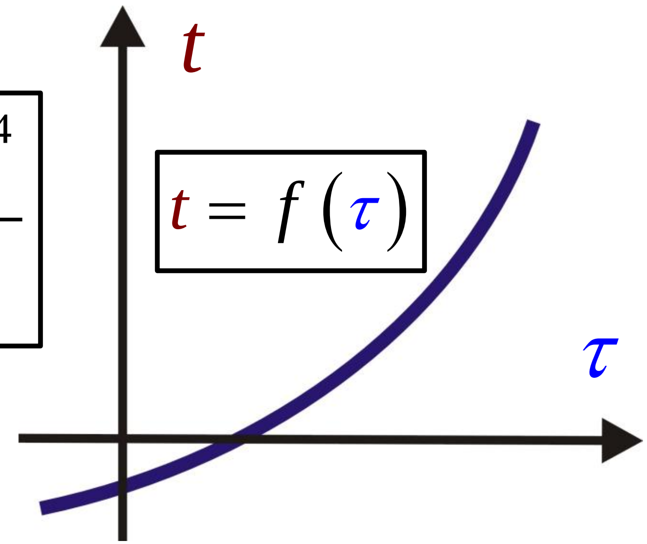
- is of conformal form with scaling dimension $\Delta = 1/4$

Goldstone mode manifold

Kitaev' 2015

- **Mean-field solution is not unique!**
  - in the conformal limit one has infinite set of solutions

$$G(\tau_1,\tau_2) \propto \mp \frac{[f'(\tau_1)]^{1/4}[f'(\tau_2)]^{1/4}}{|f(\tau_1)-f(\tau_2)|^{1/2}}$$



- **Mean-field solution**
  - Invariant under conformal transformation SL(2,R)

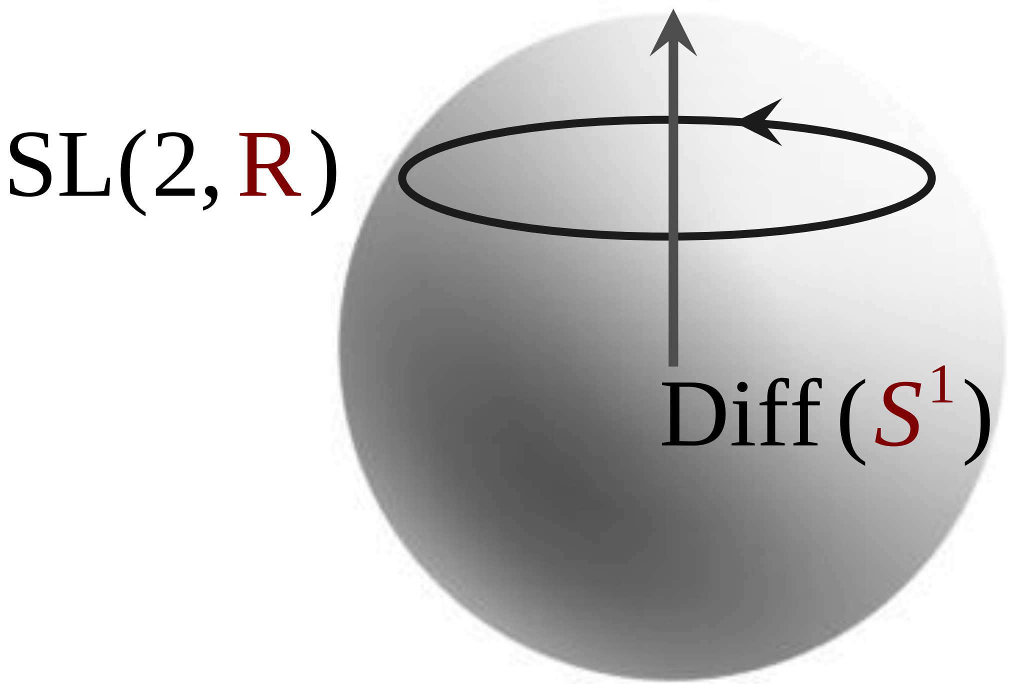
$$\bar{G}^{ab}_{t-t'} \propto -\frac{\delta^{ab}}{\sqrt{J}}\frac{1}{|t-t'|^{1/2}}$$

$$f(\tau) = \frac{a\tau+b}{c\tau+d} \in \mathrm{SL}(2,\mathrm{R})$$

Goldstone mode manifold

- emergence of infinite dimensional soft-mode manifold

$$\mathrm{Diff}(S^1)/\mathrm{SL}(2,\mathbb{R})$$



- **Heisenberg magnet:** $SU(2)/U(1) \sim S^2$

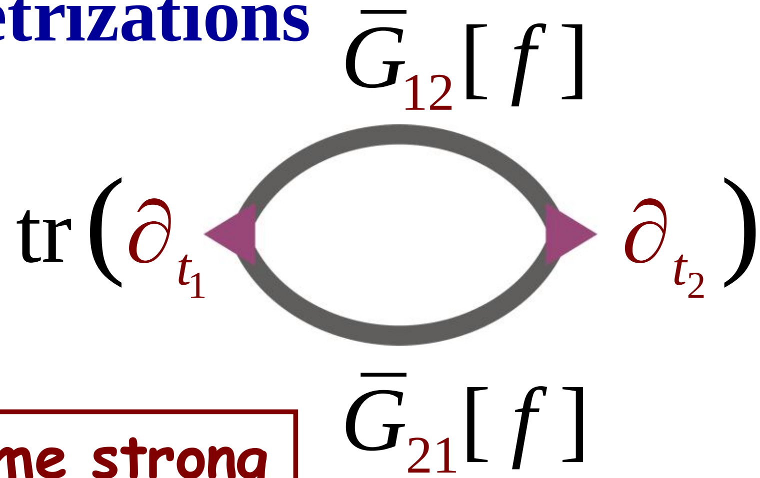
Schwarzian action &
Liouville quantum mechanics

Goldstone action

Kitaev' 2015; J. Maldacena & D. Stanford '2015

- **Schwarzian action of reparametrizations**

$$S_0[f] = -M \int_{-\infty}^{+\infty} \{f,\tau\}\, d\tau$$



at t>M fluctuations become strong

- the Schwarzian derivative is defined by

$$\{f,\tau\} \equiv \frac{f'''}{f'} - \frac{3}{2}\left(\frac{f''}{f'}\right)^2, \qquad M \propto N \ln N / J$$

- and respects the coset structure versus H=SL(2,R)

$$\{h \circ f, \tau\} = \{f,\tau\} \quad \text{if} \;\; h(t) = \frac{at+b}{ct+d} \in \mathrm{SL(2,R)}$$

Green's function

Q: What is the IR limit of Green's function?

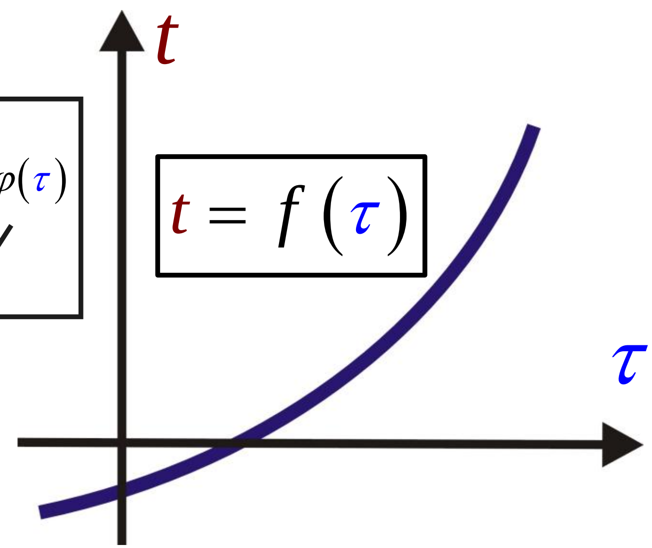
$$G(\tau_1-\tau_2)\propto \mp \int_{G/H} Df(\tau)\times \frac{\left[f'(\tau_1)\right]^{1/4}\left[f'(\tau_2)\right]^{1/4}}{\left|f(\tau_1)-f(\tau_2)\right|^{1/2}}\times e^{-S_0[f]}$$

- average the mean-field result over Goldstone modes

- **Phase representation (measure is flat!)**

$$S_0[\varphi]=\frac{M}{2}\int_{-\infty}^{+\infty}\left[\varphi'(\tau)\right]^2 d\tau,\qquad f'(\tau)=e^{\varphi(\tau)}$$

non-compact phase



Green's function

Q: What is the IR limit of Green's function?

$$G(\tau_1 - \tau_2) \propto \int_0^{+\infty} \frac{d\alpha}{\sqrt{\alpha}} \left\langle e^{\frac{1}{4}\varphi(\tau_1)} e^{\frac{1}{4}\varphi(\tau_2)} \exp\left[-\alpha \int_{\tau_1}^{\tau_2} e^{\varphi(\tau)} d\tau\right] \right\rangle_{\varphi}$$

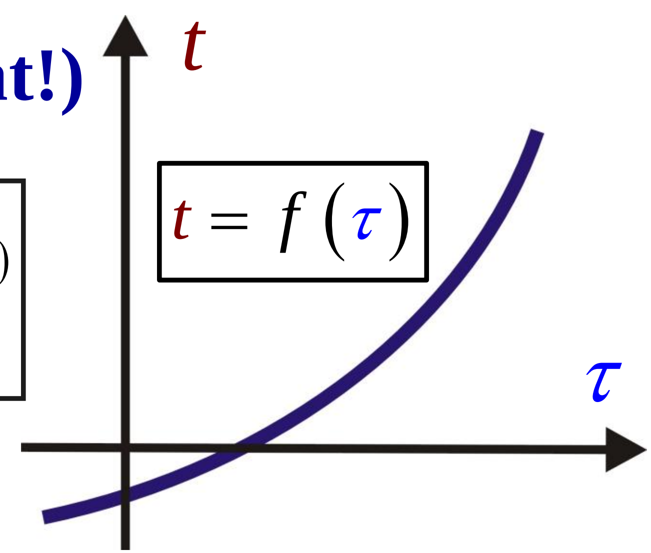
Vertex operators

Liouville potential

- **Phase representation (measure is flat!)**

$$S_0[\varphi] = \frac{M}{2} \int_{-\infty}^{+\infty} \left[\varphi'(\tau)\right]^2 d\tau, \quad f'(\tau) = e^{\varphi(\tau)}$$

non-compact phase



Liouville QM

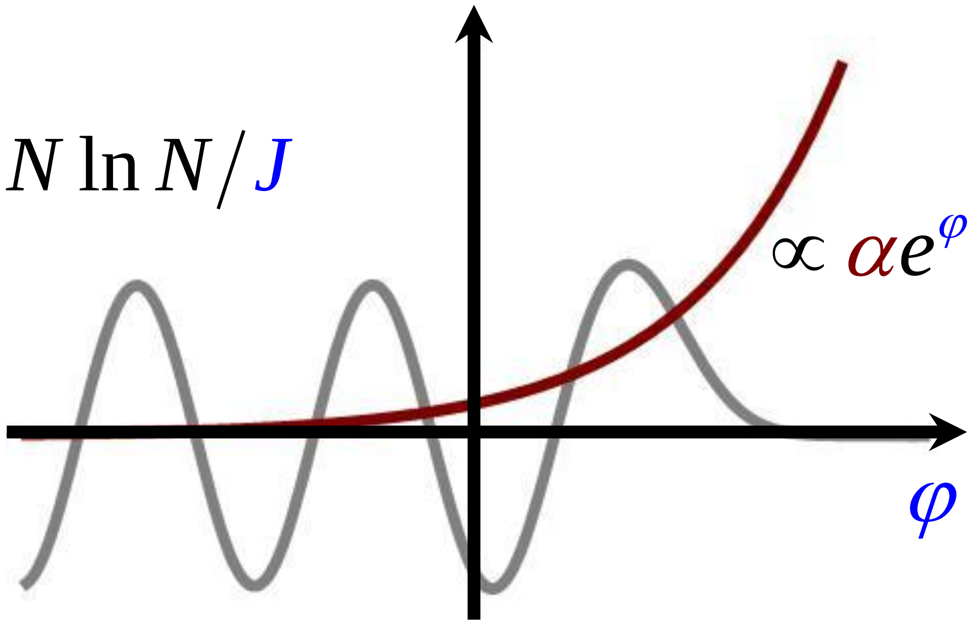
$$\langle \varphi | k \rangle \propto K_{2ik}\left(\sqrt{8M\alpha}\, e^{\varphi/2}\right)$$

- **Effective Hamiltonian**

$$\hat{H} = -\frac{\partial_\varphi^2}{2M} + \alpha e^{\varphi}, \quad M \sim N \ln N / J$$

"effective mass"

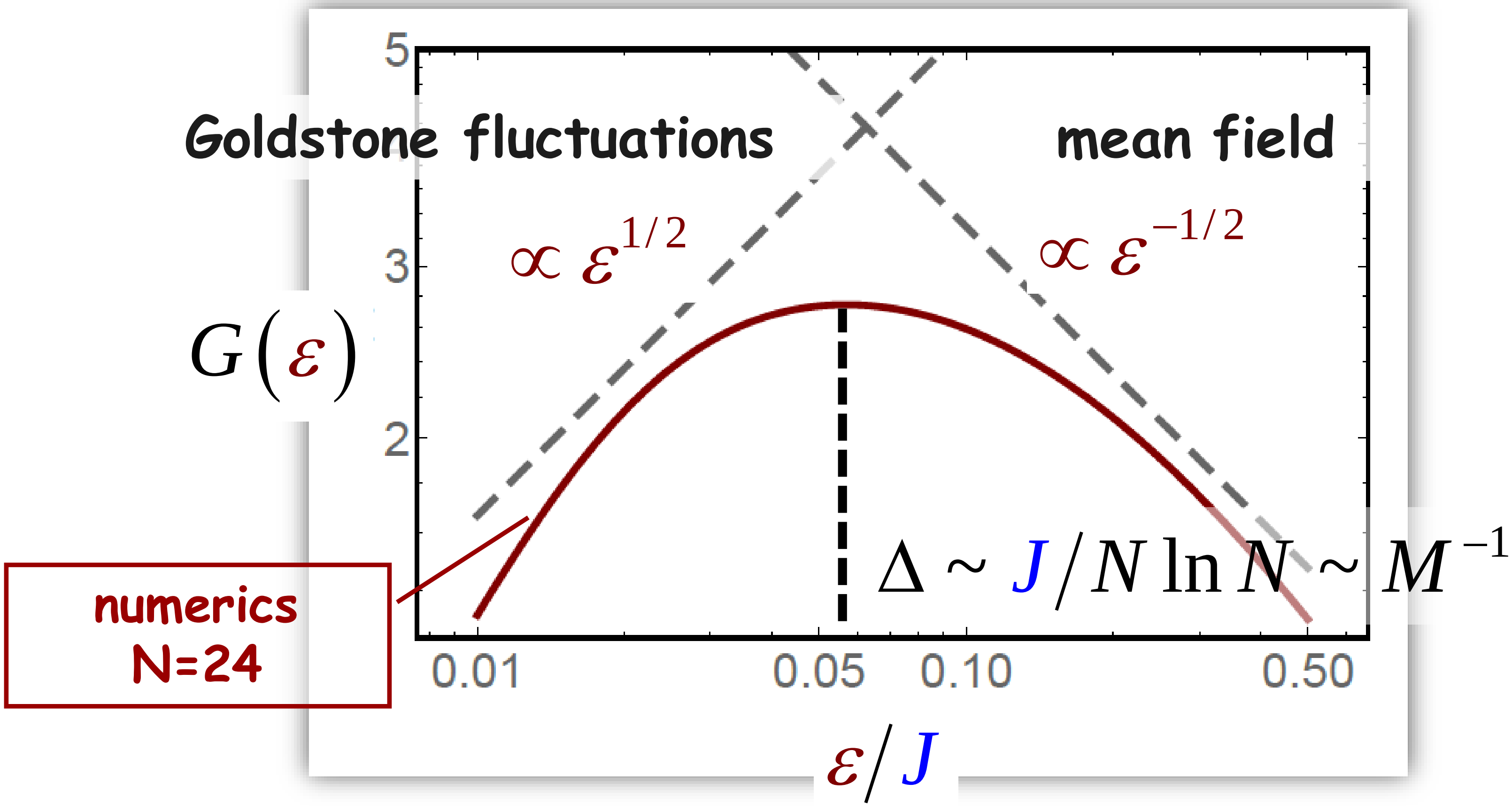
$$k \in \mathbb{R}^+$$



- **Spectral decomposition of the Green's function**

$$G(\tau) \propto \mp \int_0^{+\infty} \frac{d\alpha}{\sqrt{\alpha}} \sum_k \langle 0 | e^{\frac{1}{4}\varphi} | k_\alpha \rangle e^{-|\tau| k^2/2M} \langle k_\alpha | e^{\frac{1}{4}\varphi} | 0 \rangle$$

Green's function



- **Time domain:**

$$G(\tau) \propto \pm \frac{1}{\sqrt{J}} \begin{cases} |\tau|^{-1/2}, & \tau < 1/\Delta \\ \Delta^{-1} |\tau|^{-3/2}, & \tau > 1/\Delta \end{cases}$$

Four-point Green's function

$$G_4(\tau_1,\tau_2,\tau_3,\tau_4)=\frac{1}{N^2}\sum_{i,j}^{N}\langle\chi_i(\tau_1)\chi_i(\tau_2)\chi_j(\tau_3)\chi_j(\tau_4)\rangle$$

- **Time ordering:**

$\tau_1,\tau_4\approx 0^{\pm}$ $\qquad$ $\tau_2,\tau_3\approx\tau^{\pm}$

$$G_4(\tau)\propto\begin{cases}|\tau|^{-1}, & \tau<1/\Delta\\ \Delta^{-1/2}|\tau|^{-3/2}, & \tau>1/\Delta\end{cases}$$

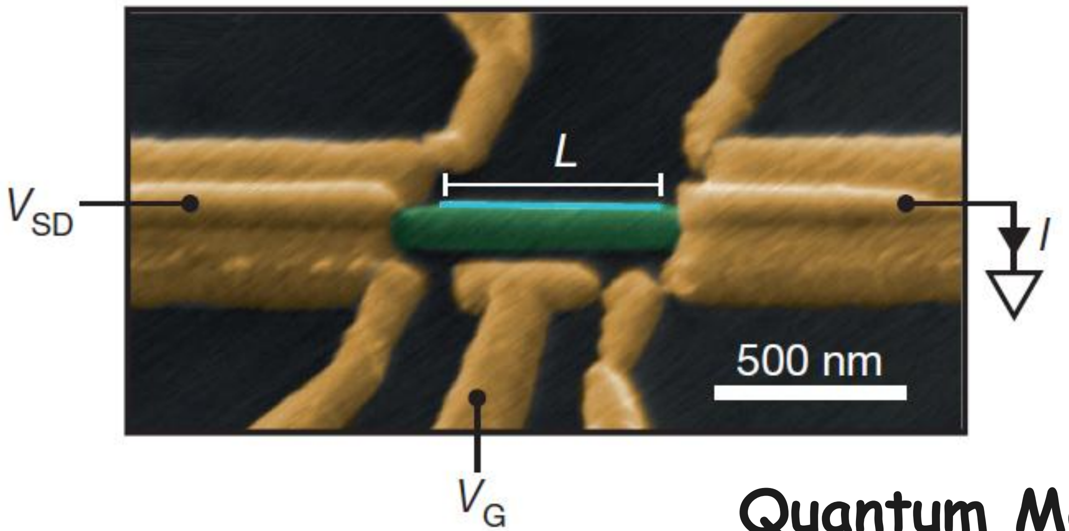
$\Delta\sim J/N\ln N$

single-partilce level spasing

universal long-time decay

Random mass Dirac model

L. Balents & M.Fisher '97, D. Shelton & A. Tsvelik '98



$$\hat{H} = \begin{pmatrix} -iu\partial_x & m(x) \\ m(x) & iu\partial_x \end{pmatrix}$$

$$\langle m(x)m(x')\rangle_{\text{dis}} \propto \delta(x-x')$$

Quantum Majorana wire at criticality

- **Statistics of zero-energy wave functions**

$$\left\langle \left|\psi_0(x)\psi_0(0)\right|^p \right\rangle_{\text{dis}} \sim L^{-1}|x|^{-3/2}$$

universal (p-independent) decay

Complex Sachdev-Ye model

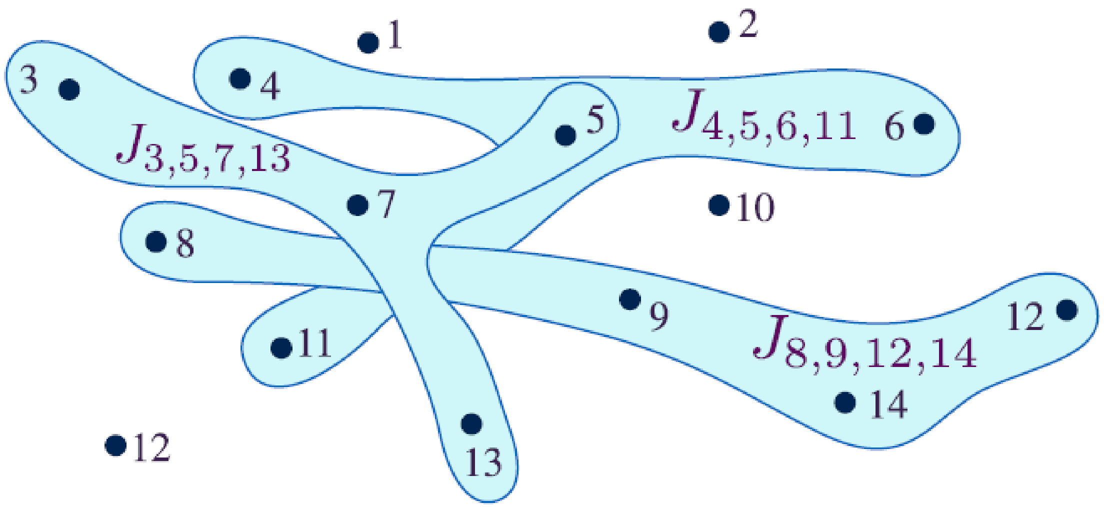
Sachdev-Ye model

S. Sachdev, PRX 5 (2015) 041025

$$\hat{H} = \frac{1}{(2N)^{3/2}} \sum_{ij,kl}^{N} J_{ij;kl} c_i^+ c_j^+ c_k c_l - \mu \sum_i c_i^+ c_i$$

Complex couplings J's are quenched Gaussian distributed variables:

$$\left\langle \left| J_{ijkl} \right|^2 \right\rangle = J^2$$



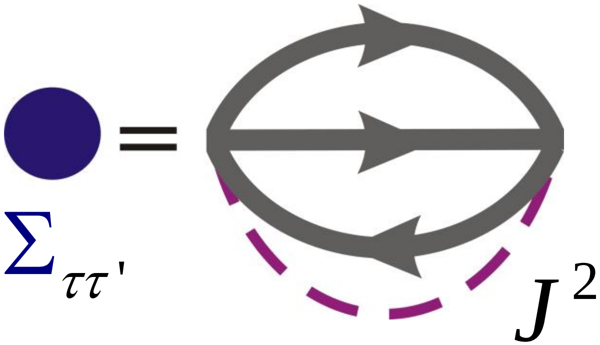
$\{c_i^+, c_j\} = \delta_{ij}$ - true spinless fermions

Mean field solution

- **Self-consistent Dyson equation** **(S. Sachdev, J. Ye '1993)**

$$-\left(\cancel{\partial_\tau}+\Sigma\right)\bullet G=1,\quad \Sigma_{\tau\tau'}=-J^2\left[G_{\tau\tau'}\right]^2 G_{\tau'\tau}$$

- **Mean-field solution (T=0)**



$$\bar{G}_{t-t'}=-\left\langle c_i(t)\,c_i^+(t')\right\rangle\propto\mp\frac{\sin\left(\pi/4\pm\theta\right)}{|t-t'|^{1/2}}$$

- **Charge**

$$Q=\frac{1}{2}-\frac{\theta}{\pi}+\frac{\sin(2\theta)}{4}$$

Analogue of Fermi momentum

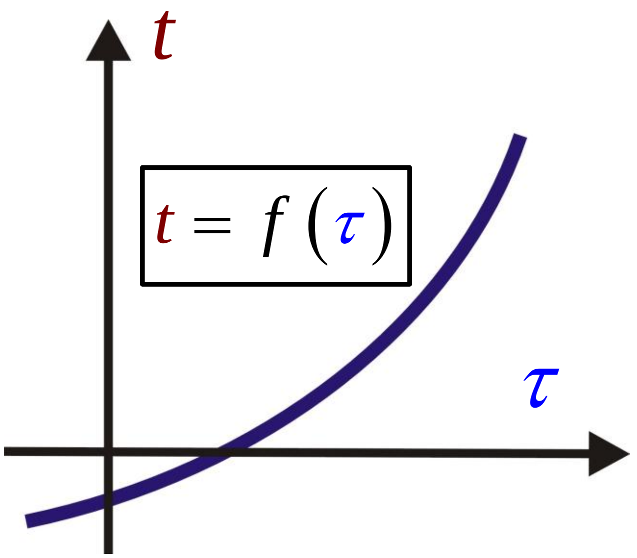
Soft-mode manifold

- **Mean-field solution is not unique!**
  - in the conformal limit one has infinite set of solutions

$$G(\tau_1,\tau_2) \propto \mp \sin\left(\pi/4 \pm \theta\right) \frac{\left[f'(\tau_1) f'(\tau_2)\right]^{1/4}}{\left|f(\tau_1) - f(\tau_2)\right|^{1/2}} e^{i(\sigma(\tau_1)-\sigma(\tau_2))}$$

- **Two Goldstone modes:**
  - f(t) ∈ Diff(R)
  - σ(t) ∈ U(1)

Emergent U(1)-gauge symmetry!



High-T saddle point

$$\bar{G}_{\tau} \propto \mp \frac{\sin\left(\pi/4 \pm \theta\right)}{\sin^{1/2}[\pi|\tau|/\beta]} e^{-2\pi\Xi\tau/\beta}$$

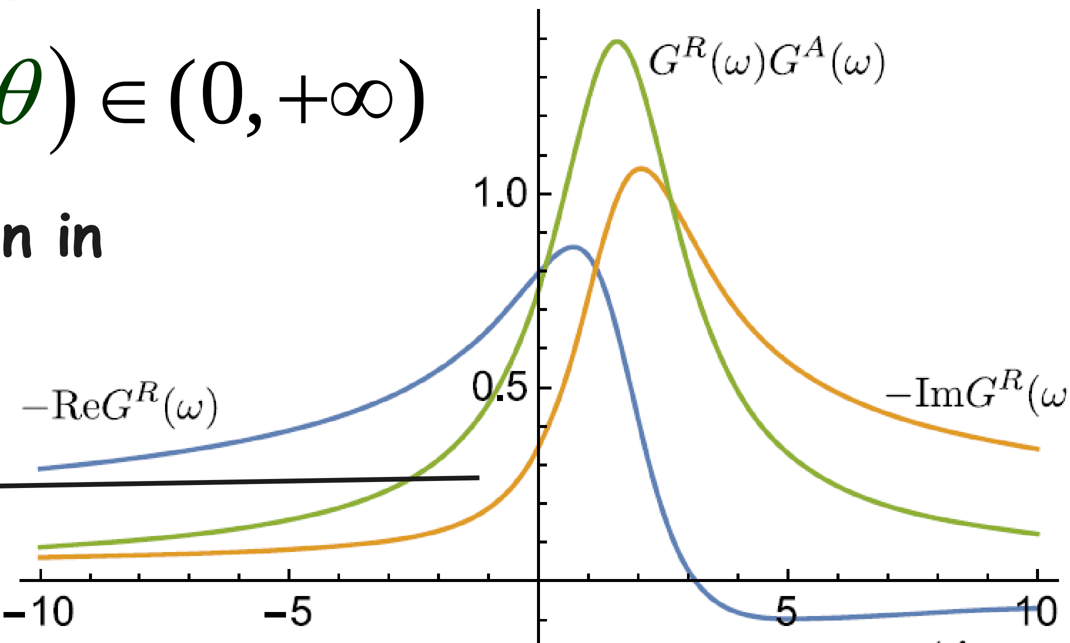
- reparametrization & U(1) phase: $f(\tau) = \tan[\pi\tau/\beta]$, $\sigma(\tau) = 2\pi i\,\Xi\,\tau/\beta$

- **Spectral asymmetry:**

$$e^{2\pi\Xi} = \tan\left(\pi/4 + \theta\right) \in (0, +\infty)$$

p/h-symmetry is broken in complex SYK model

$$T > J/N \ln N$$



Goldstone action

- **Schwarzian action coupled to U(1) gauge field**

$$S_0[\varphi,\sigma] = \frac{M}{2}\int_{-\infty}^{+\infty}\left[\varphi'(\tau)\right]^2 d\tau + \frac{K}{2}\int_{-\infty}^{+\infty}\left[\sigma'(\tau)\right]^2 d\tau + \frac{i\Lambda}{2}\int_{-\infty}^{+\infty}\varphi'(\tau)\sigma'(\tau)d\tau$$

non-compact phase/Schwarzian

U(1) compact phase

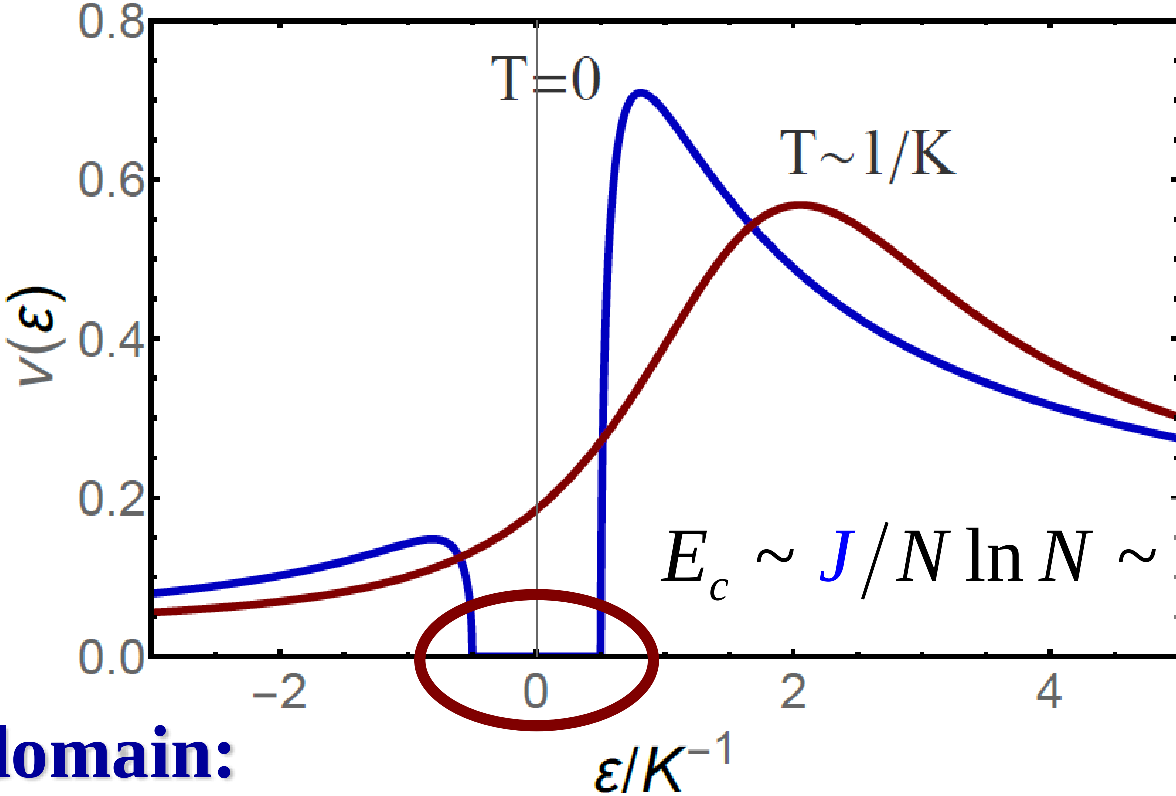
- Coupling constants: $M \sim K \propto N \ln N / J\,, \quad \Lambda \sim K/\ln N << K$

- **Two-point Green's function beyond mean-field:**

$$G(\tau_{12}) \propto \mp \int_{G/H} D[f_\tau, \sigma_\tau] \times \frac{\left[f'(\tau_1) f'(\tau_2)\right]^{1/4}}{\left|f(\tau_1) - f(\tau_2)\right|^{1/2}} e^{i(\sigma(\tau_1)-\sigma(\tau_2))} \times e^{-S_0[f,\sigma]}$$

Tunneling DoS

complex SYK model shows Coulomb blockade at low T

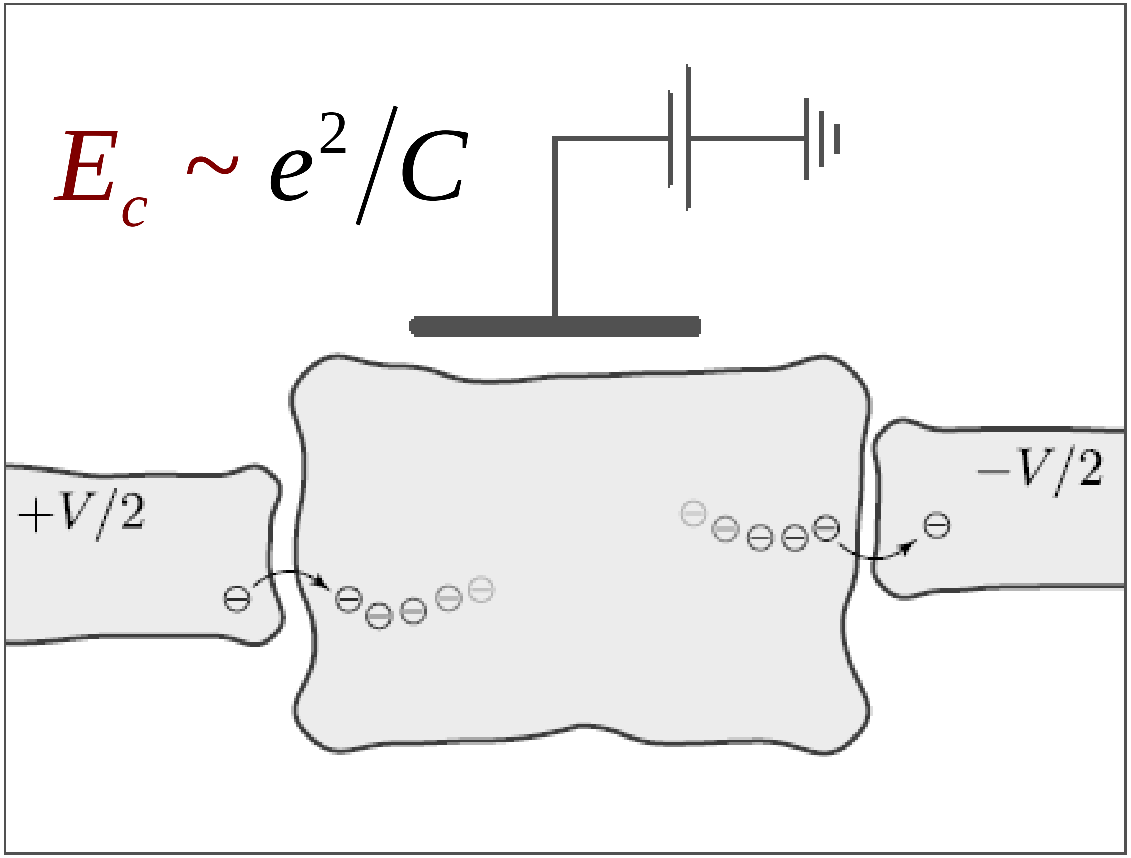


$$E_c \sim J/N \ln N \sim K^{-1}$$

- **Time domain:**

$$G(\tau) \propto \pm \begin{cases} |\tau|^{-1/2}, & \tau < 1/E_c \\ E_c^{-1} |\tau|^{-3/2} e^{-E_c|\tau|/2}, & \tau > 1/E_c \end{cases}$$

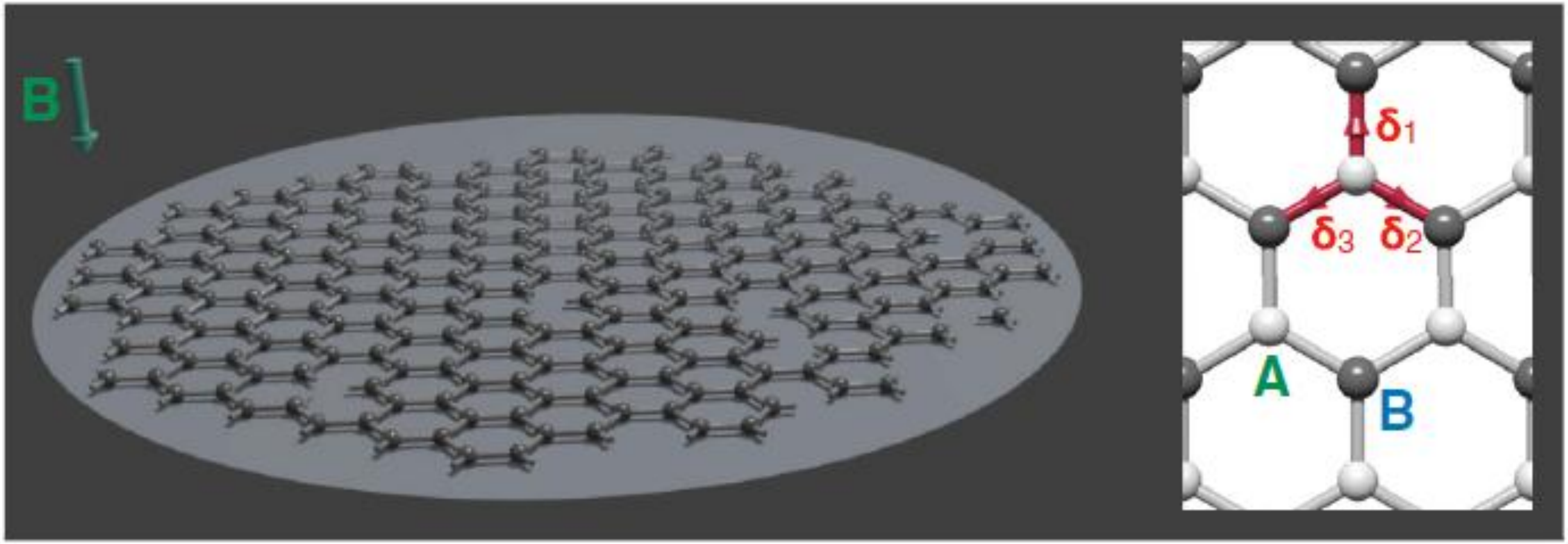
Coulomb blockade



- Mesoscopic quantum dot: transport is blocked at low $eV < E_c$; U(1) phase stems from real e.m. field

- **Quantum Holography in a Graphene Flake with an Irregular Boundary**

- the proposal for an experimental realization of the complex SYK model



A. Chen, R. Ilan, F. de Jaun, D. Pikulin and M. Franz, PRL 121, 036403 (2018)

Summary

- **Conformal symmetry breaking in SYK model leads to large Goldstone mode fluctuations**

- **Fluctuations qualitatively affect physics at large time scales and low energies,**

$$t > N \ln N / J$$

- **… and modify correlation functions**

- **Complex SYK model shows emergent Coulomb blockade at low T**