Instability of the Conformal Phase in Some Tensor and SYK Models

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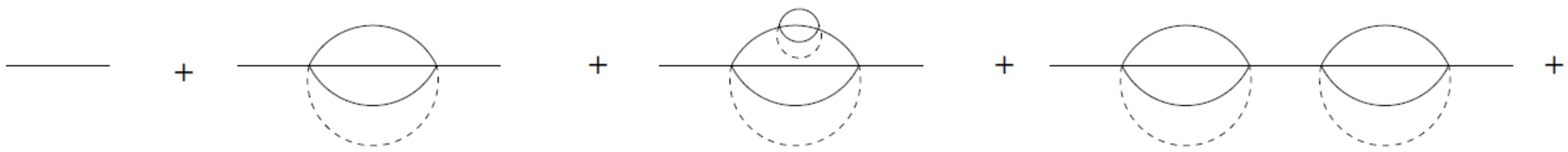
The Sachdev-Ye-Kitaev Model

- Quantum mechanics of a large number N_{SYK} of anti-commuting variables with action

$$I = \int \mathrm{d}t \left(\frac{\mathrm{i}}{2} \sum_i \psi_i \frac{\mathrm{d}}{\mathrm{d}t} \psi_i - \mathrm{i}^{q/2} j_{i_1 i_2 \dots i_q} \psi_{i_1} \psi_{i_2} \dots \psi_{i_q} \right)$$

- Random couplings j have a Gaussian distribution with zero mean.
- The model flows to strong coupling and becomes nearly conformal. Sachdev, Ye; Georges, Parcollet, Sachdev; Kitaev; Polchinski, Rosenhaus; Maldacena, Stanford; Jevicki, Suzuki, Yoon; Kitaev, Suh

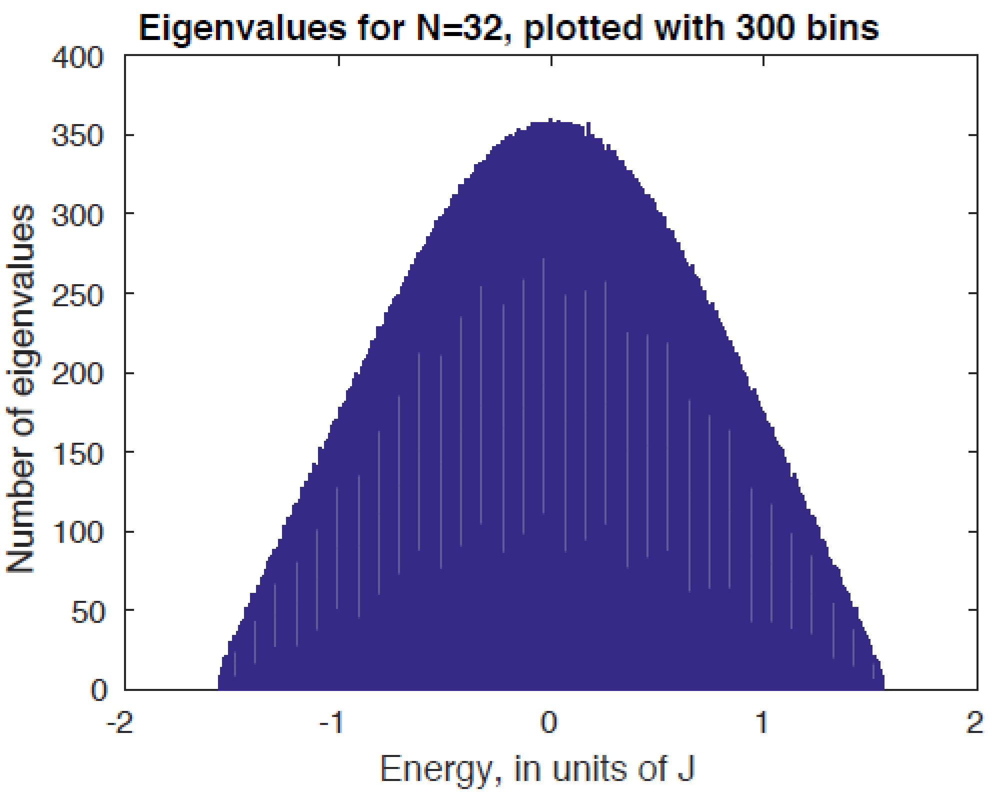
- The simplest dynamical case is q=4.
- Exactly solvable in the large N_{SYK} limit because only the melonic Feynman diagrams contribute



- Solid lines are fermion propagators, while dashed lines mean disorder average.
- The exact solution shows resemblance with physics of certain two-dimensional black holes.

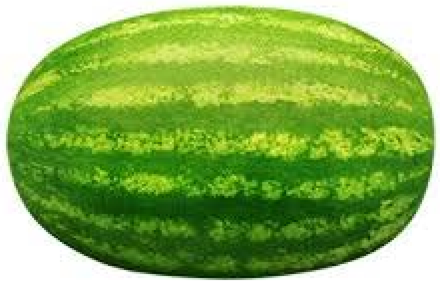
Kitaev; Almheiri, Polchinski; Sachdev; Maldacena, Stanford, Yang; Engelsoy, Mertens, Verlinde; Jensen; Kitaev, Suh; …

- Spectrum for a single realization of N_{SYK}=32 model with q=4. Maldacena, Stanford
- No exact degeneracies, but the gaps are exponentially small. Large low T entropy.



SYK-Like Tensor Quantum Mechanics

- E. Witten, “An SYK-Like Model Without Disorder,” arXiv: 1610.09758.
- Appeared on the evening of Halloween: October 31, 2016.


- It is sometimes tempting to change the term “melonic diagrams” to “pumpkinlike diagrams.”

$O(N)^3$ Tensor Model

- Interactions of N^3 Majorana fermions without randomness IK, Tarnopolsky

$$\{\psi^{abc}, \psi^{a'b'c'}\} = \delta^{aa'}\delta^{bb'}\delta^{cc'}$$

$$H = \frac{g}{4}\psi^{abc}\psi^{ab'c'}\psi^{a'bc'}\psi^{a'b'c} - \frac{g}{16}N^4$$

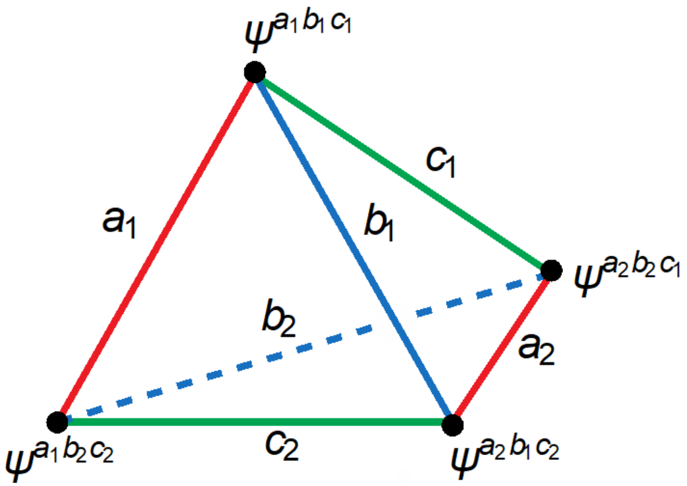
- Has $O(N)_a$x$O(N)_b$x$O(N)_c$ symmetry under

$$\psi^{abc} \to M_1^{aa'} M_2^{bb'} M_3^{cc'} \psi^{a'b'c'}, \qquad M_1, M_2, M_3 \in O(N)$$

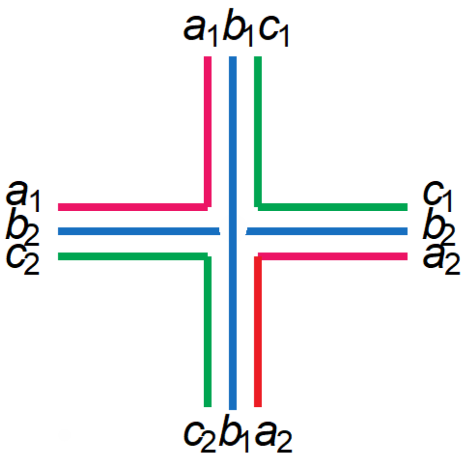
- The SO(N) symmetry charges are

$$Q_1^{aa'} = \frac{i}{2}[\psi^{abc}, \psi^{a'bc}] , \qquad Q_2^{bb'} = \frac{i}{2}[\psi^{abc}, \psi^{ab'c}] , \qquad Q_3^{cc'} = \frac{i}{2}[\psi^{abc}, \psi^{abc'}]$$

- The 3-tensors may be associated with indistinguishable vertices of a tetrahedron.

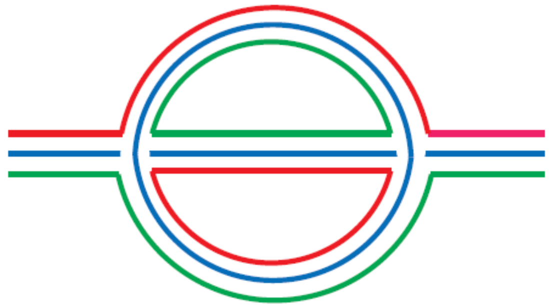


- This is equivalent to

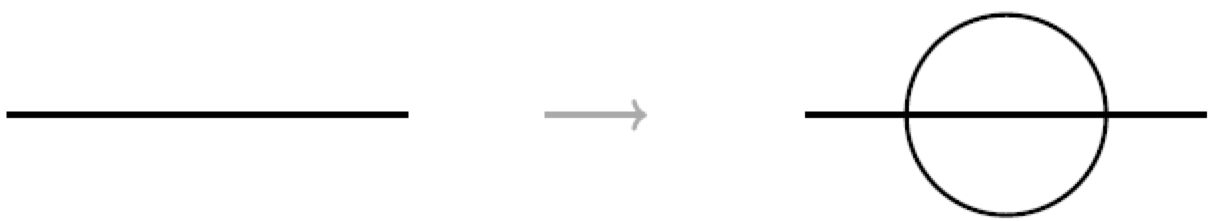


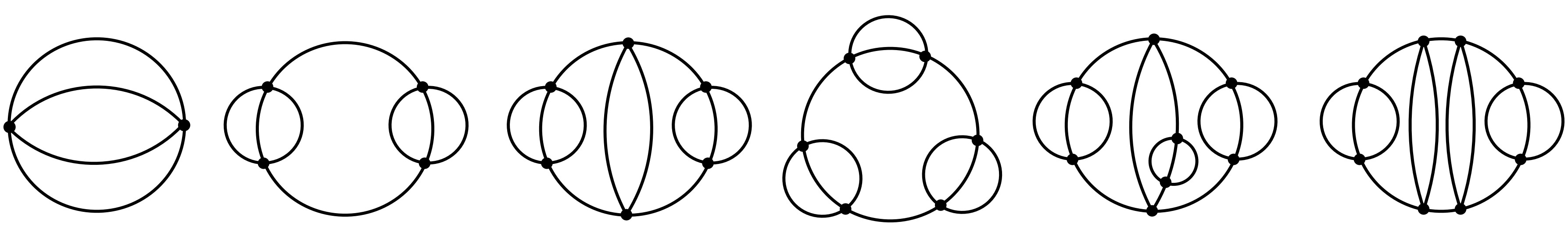
- The triple-line Feynman graphs are produced using the propagator

- Leading correction has 3 index loops

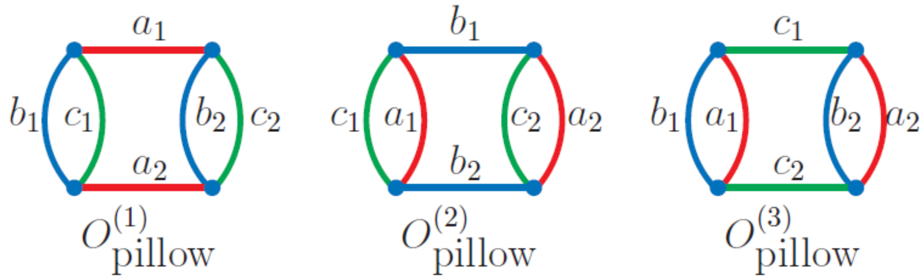


- This "melon" insertion is of order 1 if $\lambda = gN^{3/2}$ is held fixed in the large N limit.
- "Melonic graphs" obtained by Bonzom, Gurau, Rivasseau





- The tetrahedral term is the unique dynamical quartic interaction with $O(N)^3$ symmetry.
- The other possible terms are quadratic Casimirs of the three SO(N) groups.



$$O^{(1)}_{\text{pillow}} = \sum_{a_1<a_2} Q_1^{a_1 a_2} Q_1^{a_1 a_2} , \qquad O^{(2)}_{\text{pillow}} = \sum_{b_1<b_2} Q_2^{b_1 b_2} Q_2^{b_1 b_2} , \qquad O^{(3)}_{\text{pillow}} = \sum_{c_1<c_2} Q_3^{c_1 c_2} Q_3^{c_1 c_2}$$

- In the model where $SO(N)^3$ is gauged, they vanish.

$O(N)^3$ vs. SYK Model

- Using composite indices $I_k = (a_k b_k c_k)$

$$H = \frac{1}{4!} J_{I_1 I_2 I_3 I_4} \psi^{I_1} \psi^{I_2} \psi^{I_3} \psi^{I_4}$$

The couplings take values $0, \pm 1$

$$J_{I_1 I_2 I_3 I_4} = \delta_{a_1 a_2} \delta_{a_3 a_4} \delta_{b_1 b_3} \delta_{b_2 b_4} \delta_{c_1 c_4} \delta_{c_2 c_3} - \delta_{a_1 a_2} \delta_{a_3 a_4} \delta_{b_2 b_3} \delta_{b_1 b_4} \delta_{c_2 c_4} \delta_{c_1 c_3} + 22 \text{ terms}$$

- The number of distinct terms is

$$\frac{1}{4!} \sum_{\{I_k\}} J^2_{I_1 I_2 I_3 I_4} = \frac{1}{4} N^3 (N-1)^2 (N+2)$$

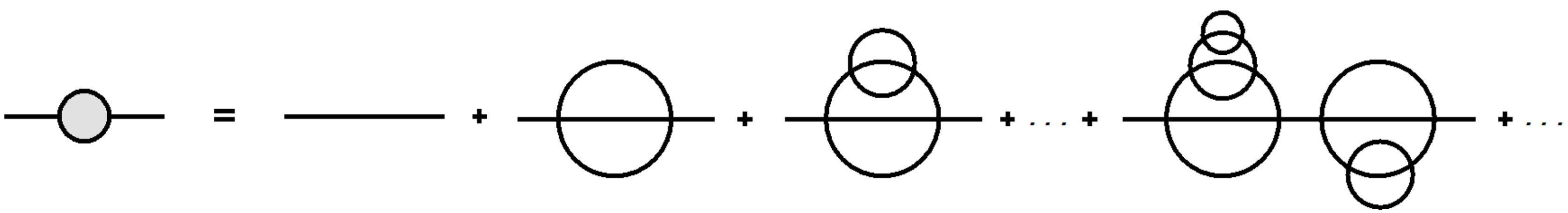
- Much smaller than in SYK model with $N_{\mathrm{SYK}} = N^3$

$$\frac{1}{24} N^3 (N^3 - 1)(N^3 - 2)(N^3 - 3)$$

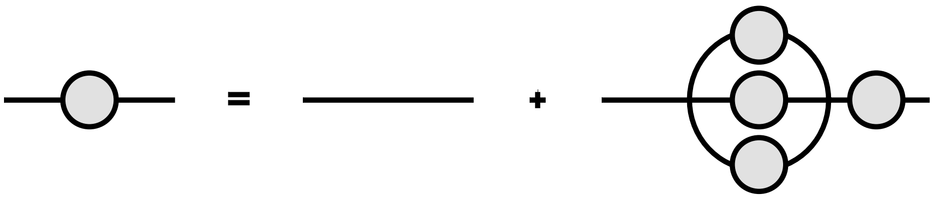
- The tensor Hamiltonian is much sparser.
- In the SYK model interactions are all-to-all.
- In the corresponding tensor model they are not (recent discussions with L. Susskind and E. Witten).
- The number of fermion species coupled to a given one by the Hamiltonian grows only as $N_{\mathrm{SYK}}^{2/3}$
- Nevertheless, the tensor quantum mechanics is also maximally chaotic in the large N limit.

Schwinger-Dyson Equations

- Some are the same as in the SYK model Kitaev; Polchinski, Rosenhaus; Maldacena, Stanford; Jevicki, Suzuki, Yoon; Kitaev, Suh



$$G(t_1 - t_2) = G_0(t_1 - t_2) + g^2 N^3 \int dt dt' G_0(t_1 - t) G(t - t')^3 G(t' - t_2)$$

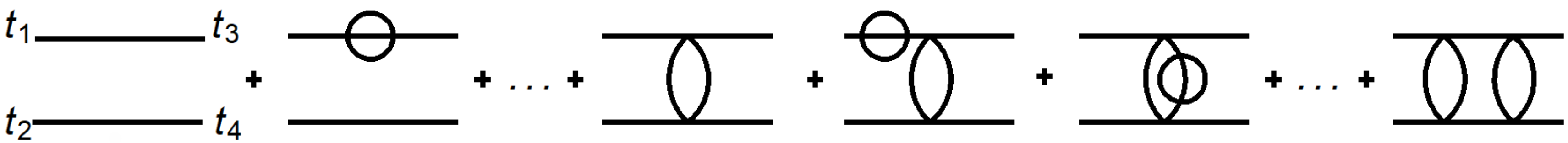


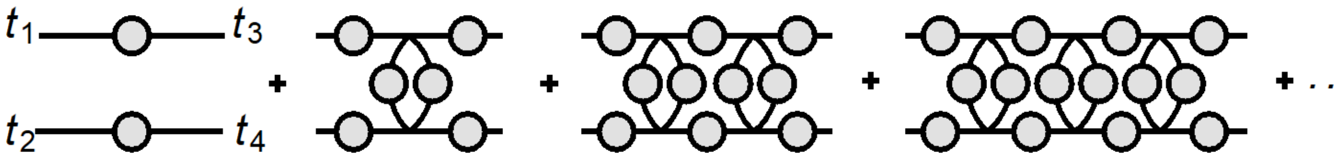
- Neglecting the left-hand side in IR we find

$$G(t_1 - t_2) = -\left(\frac{1}{4\pi g^2 N^3}\right)^{1/4} \frac{\mathrm{sgn}(t_1 - t_2)}{|t_1 - t_2|^{1/2}}$$

- Four point function

$$\langle\psi^{a_1b_1c_1}(t_1)\psi^{a_1b_1c_1}(t_2)\psi^{a_2b_2c_2}(t_3)\psi^{a_2b_2c_2}(t_4)\rangle = N^6 G(t_{12})G(t_{34}) + \Gamma(t_1,\ldots,t_4)$$





- If we denote by Γ_n the ladder with n rungs

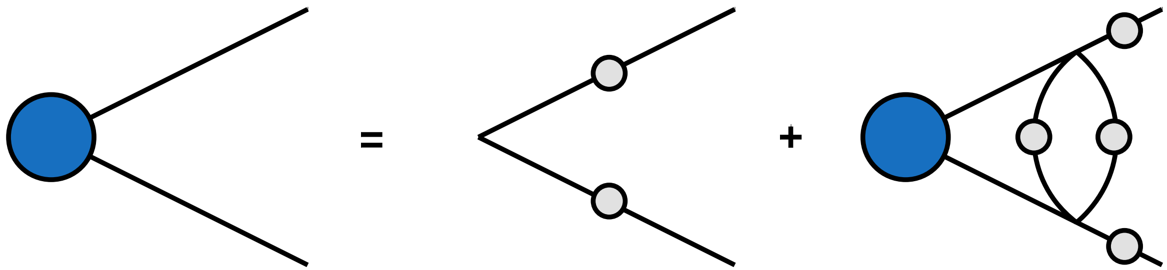
$$\Gamma = \sum_n \Gamma_n$$

$$\Gamma_{n+1}(t_1,\ldots,t_4) = \int dt dt' K(t_1,t_2;t,t')\Gamma_n(t,t',t_3,t_4)$$

$$K(t_1,t_2;t_3,t_4) = -3g^2N^3G(t_{13})G(t_{24})G(t_{34})^2$$

Spectrum of two-particle operators

- S-D equation for the three-point function Gross, Rosenhaus



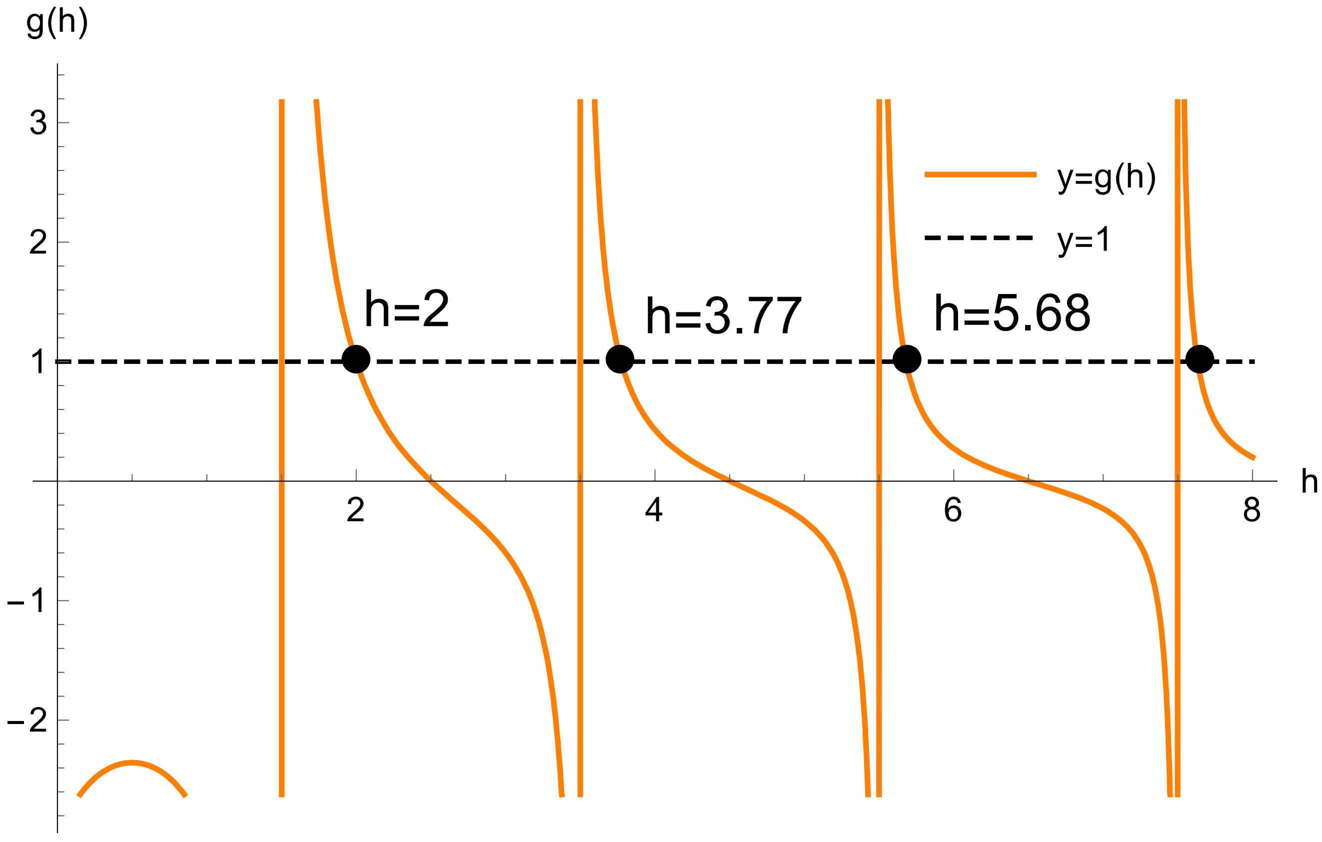
$$v(t_0,t_1,t_2) = g(h)\int dt_3 dt_4 K(t_1,t_2;t_3,t_4)v(t_0,t_3,t_4)$$

$$v(t_0,t_1,t_2) = \langle O_2^n(t_0)\psi^{abc}(t_1)\psi^{abc}(t_2)\rangle = \frac{\operatorname{sgn}(t_1-t_2)}{|t_0-t_1|^h|t_0-t_2|^h|t_1-t_2|^{1/2-h}}$$

- Scaling dimensions of operators $O_2^n = \psi^{abc}(D_t^n\psi)^{abc}$

$$g(h) = -\frac{3}{2}\frac{\tan(\frac{\pi}{2}(h-\frac{1}{2}))}{h-1/2} = 1$$

- The first solution is h=2; dual to JT gravity.



- The higher scaling dimensions are

$h \approx 3.77,\ 5.68,\ 7.63,\ 9.60$ approaching $h_n \to n + \frac{1}{2}$

Gauged Model

- To eliminate large degeneracies, focus on the states invariant under $SO(N)^3$.
- Their number can be found by gauging the free theory IK, Milekhin, Popov, Tarnopolsky

$$L = \psi^I \partial_t \psi^I + \psi^I A_{IJ} \psi^J$$

$$A = A^1 \otimes \mathbb{1} \otimes \mathbb{1} + \mathbb{1} \otimes A^2 \otimes \mathbb{1} + \mathbb{1} \otimes \mathbb{1} \otimes A^3$$

$$\#\text{singlet states} = \int d\lambda_G^N \prod_{a=1}^{M/2} 2\cos(\lambda_a/2)$$

$$d\lambda_{SO(2n)} = \prod_{i<j}^{n} \sin\left(\frac{x_i - x_j}{2}\right)^2 \sin\left(\frac{x_i + x_j}{2}\right)^2 dx_1 \dots dx_n$$

- There are no singlets for odd N due to a QM anomaly for odd numbers of flavors.
- The number grows very rapidly for even N

N	# singlet states
2	2
4	36
6	595354780

Table 1: Number of singlet states in the $O(N)^3$ model

$$\#\text{singlet states} \sim \exp\left(\frac{N^3}{2}\log 2 - \frac{3N^2}{2}\log N + O(N^2)\right)$$

- The large low-temperature entropy suggests tiny gaps for singlet excitations ~ c^{-N^3}

Discrete Symmetries

- Act within the $SO(N)^3$ invariant sector and can lead to small degeneracies.
- Z_2 parity transformation within each group like

$$\psi^{1bc} \to -\psi^{1bc}$$

- Interchanges of the groups flip the energy

$$P_{23}\psi^{abc}P_{23} = \psi^{acb} \ , \qquad P_{12}\psi^{abc}P_{12} = \psi^{bac}$$

$$P_{23}HP_{23} = -H \ , \qquad P_{12}HP_{12} = -H$$

- Z_3 symmetry generated by $P = P_{12}P_{23}$, $\qquad P^3 = 1$

$$P\psi^{abc}P^\dagger = \psi^{cab} \ , \qquad PHP^\dagger = H$$

- At non-zero energy the gauge singlet states transform under the discrete group $A_4 \times Z_2$.
- Spectrum for N=4. Pakrouski, IK, Popov, Tarnopolsky

$$\pm\sqrt{32(447 \pm \sqrt{125601})}$$

$$\pm\sqrt{32(187 \pm \sqrt{11481})}$$

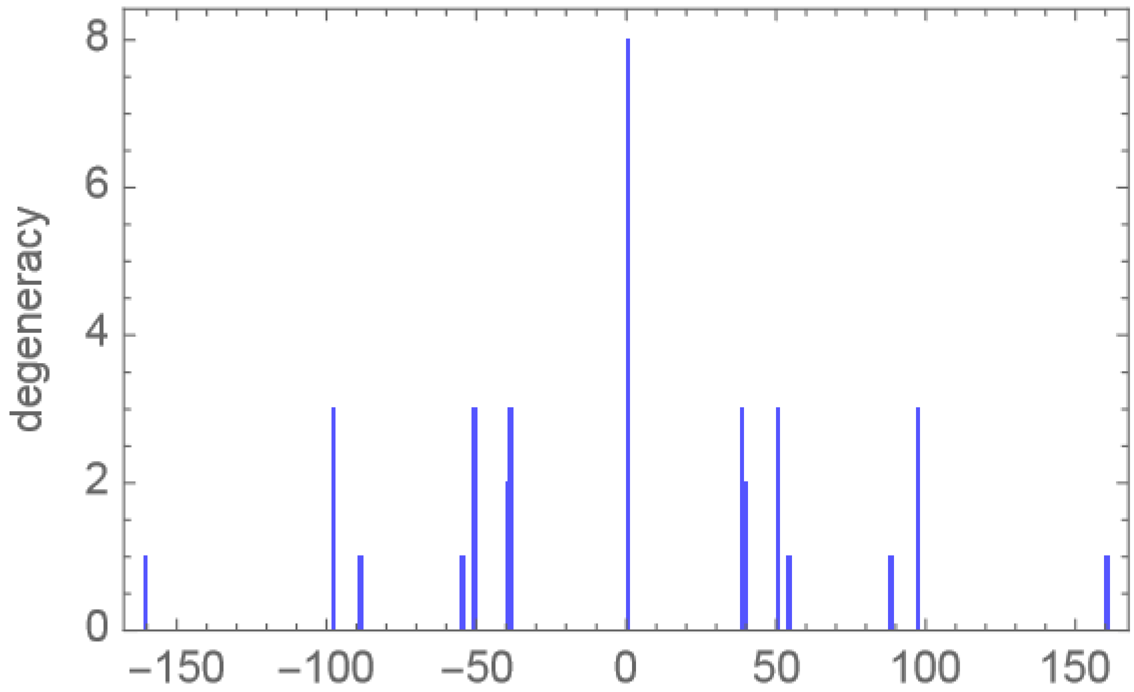
$$8\sqrt{24} =$$

$$8\sqrt{23} =$$

E	P_1	P_2	P_3	E	P_1	P_2	P_3
−160.140170	1	1	1	160.140170	1	1	1
−97.019491	1	1	−1	97.019491	1	1	−1
−97.019491	−1	1	1	97.019491	−1	1	1
−97.019491	1	−1	1	97.019491	1	−1	1
−88.724292	−1	−1	−1	88.724292	−1	−1	−1
−54.434603	1	1	1	54.434603	1	1	1
−50.549167	1	1	−1	50.549167	1	1	−1
−50.549167	−1	1	1	50.549167	−1	1	1
−50.549167	1	−1	1	50.549167	1	−1	1
−39.191836	1	1	1	39.191836	1	1	1
−39.191836	1	1	1	39.191836	1	1	1
−38.366652	1	−1	−1	38.366652	1	−1	−1
−38.366652	−1	1	−1	38.366652	−1	1	−1
−38.366652	−1	−1	1	38.366652	−1	−1	1
0.000000	1	1	1	0.000000	−1	−1	−1
0.000000	−1	1	1	0.000000	1	−1	−1
0.000000	1	−1	1	0.000000	−1	1	−1
0.000000	1	1	−1	0.000000	−1	−1	1

arXiv:1808.07455

Energy Distribution for N=4



- For N=6 there will be over 595 million states packed into energy interval <1932. So, the gaps should be tiny.

Tetrahedral Bosonic Tensor Model

- Action with a potential that is not positive definite IK, Tarnopolsky; Giombi, IK, Tarnopolsky

$$S = \int d^d x \Big(\frac{1}{2} \partial_\mu \phi^{abc} \partial^\mu \phi^{abc} + \frac{1}{4} g \phi^{a_1 b_1 c_1} \phi^{a_1 b_2 c_2} \phi^{a_2 b_1 c_2} \phi^{a_2 b_2 c_1} \Big)$$

- Schwinger-Dyson equation for 2pt function
Patashinsky, Pokrovsky

$$G^{-1}(p) = -\lambda^2 \int \frac{d^d k d^d q}{(2\pi)^{2d}} G(q) G(k) G(p+q+k)$$

- Has solution

$$G(p) = \lambda^{-1/2} \Big(\frac{(4\pi)^d d\Gamma(\frac{3d}{4})}{4\Gamma(1-\frac{d}{4})} \Big)^{1/4} \frac{1}{(p^2)^{\frac{d}{4}}}$$

Spectrum of two-particle spin zero operators

- Schwinger-Dyson equation

$$\int d^d x_3 d^d x_4 K(x_1, x_2; x_3, x_4) v_h(x_3, x_4) = g(h) v_h(x_1, x_2)$$

$$K(x_1, x_2; x_3, x_4) = 3\lambda^2 G(x_{13}) G(x_{24}) G(x_{34})^2$$

$$v_h(x_1, x_2) = \frac{1}{[(x_1 - x_2)^2]^{\frac{1}{2}(\frac{d}{2} - h)}}$$

$$g_{\text{bos}}(h) = -\frac{3\Gamma\left(\frac{3d}{4}\right)\Gamma\left(\frac{d}{4} - \frac{h}{2}\right)\Gamma\left(\frac{h}{2} - \frac{d}{4}\right)}{\Gamma\left(-\frac{d}{4}\right)\Gamma\left(\frac{3d}{4} - \frac{h}{2}\right)\Gamma\left(\frac{d}{4} + \frac{h}{2}\right)}$$

- In d<4 the first solution is complex $\frac{d}{2} + i\alpha(d)$

Complex Fixed Point in 4-ε Dimensions

- The tetrahedron

$$O_t(x) = \phi^{a_1b_1c_1}\phi^{a_1b_2c_2}\phi^{a_2b_1c_2}\phi^{a_2b_2c_1}$$

mixes at finite N with the pillow and double-sum operators

$$O_p(x) = \frac{1}{3}\left(\phi^{a_1b_1c_1}\phi^{a_1b_1c_2}\phi^{a_2b_2c_2}\phi^{a_2b_2c_1} + \phi^{a_1b_1c_1}\phi^{a_2b_1c_1}\phi^{a_2b_2c_2}\phi^{a_1b_2c_2} + \phi^{a_1b_1c_1}\phi^{a_1b_2c_1}\phi^{a_2b_1c_2}\phi^{a_2b_2c_2}\right),$$

$$O_{ds}(x) = \phi^{a_1b_1c_1}\phi^{a_1b_1c_1}\phi^{a_2b_2c_2}\phi^{a_2b_2c_2}$$

- The renormalizable action is

$$S = \int d^dx\left(\frac{1}{2}\partial_\mu\phi^{abc}\partial^\mu\phi^{abc} + \frac{1}{4}\left(g_1O_t(x) + g_2O_p(x) + g_3O_{ds}(x)\right)\right)$$

- The large N scaling is

$$g_1 = \frac{(4\pi)^2 \tilde{g}_1}{N^{3/2}}, \quad g_2 = \frac{(4\pi)^2 \tilde{g}_2}{N^2}, \quad g_3 = \frac{(4\pi)^2 \tilde{g}_3}{N^3}$$

- The 2-loop beta functions and fixed points:

$$\tilde{\beta}_t = -\epsilon \tilde{g}_1 + 2\tilde{g}_1^3,$$

$$\tilde{\beta}_p = -\epsilon \tilde{g}_2 + \left(6\tilde{g}_1^2 + \frac{2}{3}\tilde{g}_2^2\right) - 2\tilde{g}_1^2 \tilde{g}_2,$$

$$\tilde{\beta}_{ds} = -\epsilon \tilde{g}_3 + \left(\frac{4}{3}\tilde{g}_2^2 + 4\tilde{g}_2\tilde{g}_3 + 2\tilde{g}_3^2\right) - 2\tilde{g}_1^2(4\tilde{g}_2 + 5\tilde{g}_3)$$

$$\tilde{g}_1^* = (\epsilon/2)^{1/2}, \quad \tilde{g}_2^* = \pm 3i(\epsilon/2)^{1/2}, \quad \tilde{g}_3^* = \mp i(3 \pm \sqrt{3})(\epsilon/2)^{1/2}$$

- The scaling dimension of $\phi^{abc}\phi^{abc}$ is

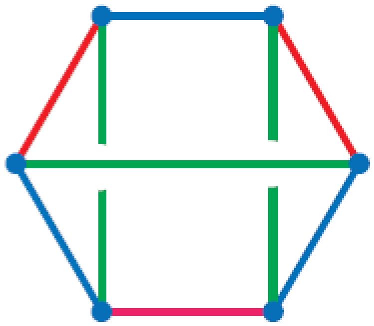
$$\Delta_O = d - 2 + 2(\tilde{g}_2^* + \tilde{g}_3^*) = 2 \pm i\sqrt{6\epsilon} + \mathcal{O}(\epsilon)$$

Prismatic Bosonic Tensor Model

- Large N limit dominated by the positive sextic "prism" interaction Giombi, IK, Popov, Prakash, Tarnopolsky

$$S = \int d^d x \left(\frac{1}{2}(\partial_\mu \phi^{abc})^2 + \frac{g_1}{6!} \phi^{a_1 b_1 c_1} \phi^{a_1 b_2 c_2} \phi^{a_2 b_1 c_2} \phi^{a_3 b_3 c_1} \phi^{a_3 b_2 c_3} \phi^{a_2 b_3 c_3} \right)$$

- To obtain the large N solution it is convenient to rewrite



$$S = \int d^d x \left(\frac{1}{2}(\partial_\mu \phi^{abc})^2 + \frac{\lambda}{3!} \phi^{a_1 b_1 c_1} \phi^{a_1 b_2 c_2} \phi^{a_2 b_1 c_2} \chi^{a_2 b_2 c_1} - \frac{1}{2} \chi^{abc} \chi^{abc} \right)$$

- Tensor counterpart of a bosonic SYK-like model.

  Murugan, Stanford, Witten

- The IR solution in general dimension:

$$3\Delta_\phi + \Delta_\chi = d\ , \qquad d/2 - 1 < \Delta_\phi < d/6$$

$$\frac{\Gamma(\Delta_\phi)\Gamma(d-\Delta_\phi)}{\Gamma(\frac{d}{2}-\Delta_\phi)\Gamma(-\frac{d}{2}+\Delta_\phi)} = 3\frac{\Gamma(3\Delta_\phi)\Gamma(d-3\Delta_\phi)}{\Gamma(\frac{d}{2}-3\Delta_\phi)\Gamma(-\frac{d}{2}+3\Delta_\phi)}$$

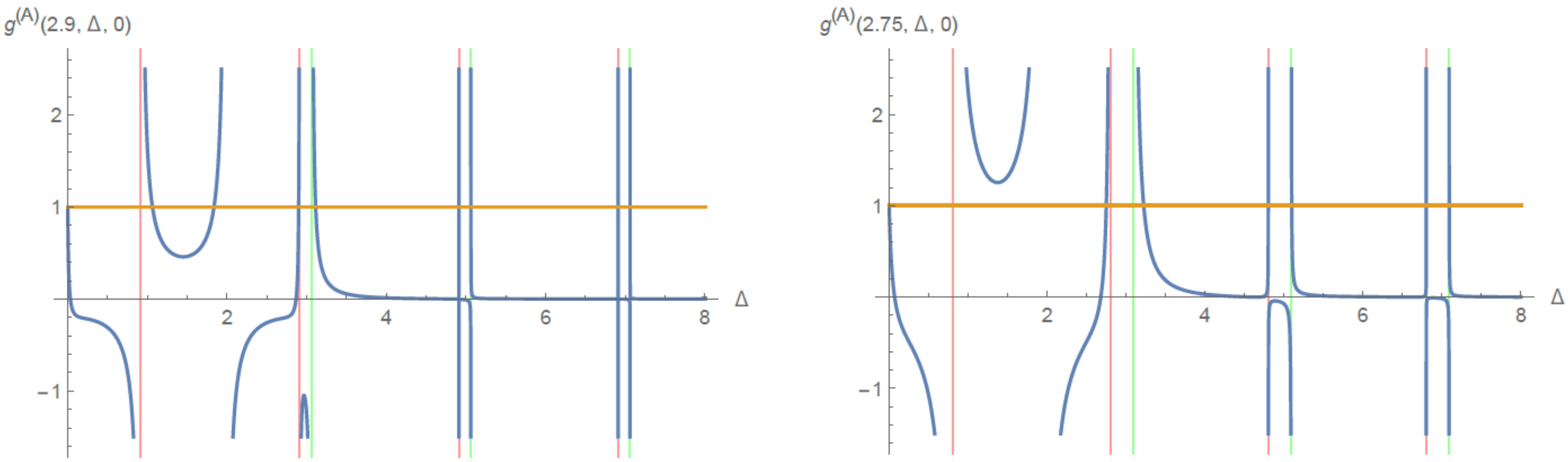
- In $d = 3 - \epsilon$

$$\Delta_\phi = \frac{1}{2} - \frac{\epsilon}{2} + \epsilon^2 - \frac{20\epsilon^3}{3} + \left(\frac{472}{9} + \frac{\pi^2}{3}\right)\epsilon^4 + \left(7\zeta(3) - \frac{12692}{27} - \frac{56\pi^2}{9}\right)\epsilon^5 + O\left(\epsilon^6\right)$$

- For d=2.9 find numerically

$$\Delta_\phi = 0.456264\ , \qquad \Delta_\chi = 1.53121$$

- Dimensions of bilinear operators in d=2.9 and 2.75



- The first root has expansion

$$\Delta_{\phi^2} = 1 - \epsilon + 32\epsilon^2 - \frac{976\epsilon^3}{3} + \left(\frac{30320}{9} + \frac{32\pi^2}{3}\right)\epsilon^4 + O\left(\epsilon^5\right)$$

- For $1.6799 < d < 2.8056$ Δ_{ϕ^2} becomes complex

$$\frac{d}{2} + i\alpha(d)$$

Complex Fixed Points

- May appear after two real fixed points have merged. Dymarsky, IK, Roiban; Rastelli, Pomoni; Kaplan, Lee, Son, Stephanov: Gorbenko, Rychkov, Zan
- Correspond to (weakly) first-order transitions.
- Theories where operator dimensions are formally complex recently dubbed "complex CFTs." Gorbenko, Rychkov, Zan
- For large N an operator of dimension $\frac{d}{2} + i\alpha(d)$ corresponds in dual AdS to a scalar violating the Breitenlohner-Freedman stability bound:

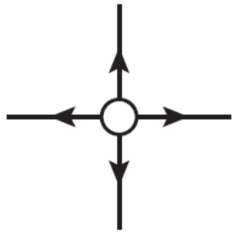
$$m^2 < -d^2/4$$

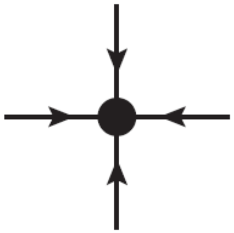
Bipartite Fermionic Model

- A model with a complex tensor and $O(N)^3$ symmetry IK, Tarnopolsky (based on a similar model by Gurau).

$$S = \int dt \left(i\bar{\psi}^{abc}\partial_t \psi^{abc} + \frac{1}{4} g \psi^{a_1b_1c_1}\psi^{a_1b_2c_2}\psi^{a_2b_1c_2}\psi^{a_2b_2c_1} + \frac{1}{4}\bar{g}\bar{\psi}^{a_1b_1c_1}\bar{\psi}^{a_1b_2c_2}\bar{\psi}^{a_2b_1c_2}\bar{\psi}^{a_2b_2c_1} \right)$$

- Two types of vertices

$\bar{\psi}^4 =$ 

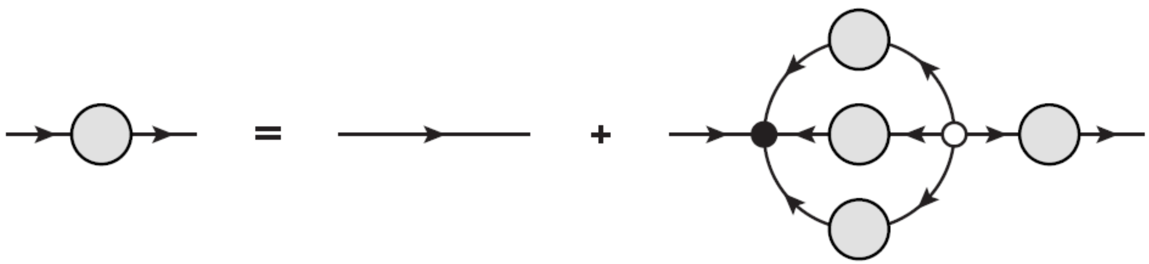
$\psi^4 =$ 

- Propagator connects different vertices

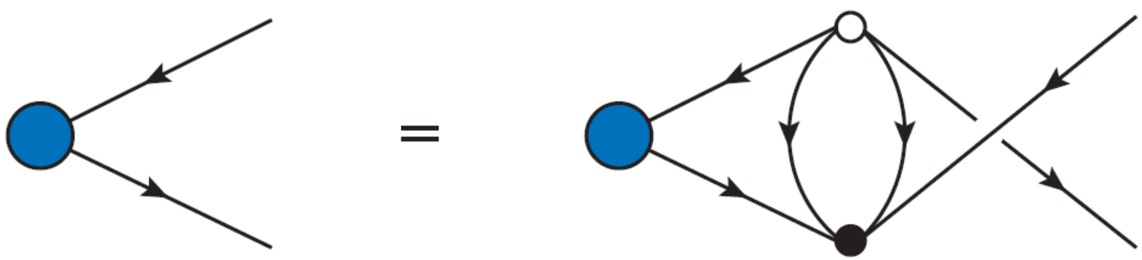
Schwinger-Dyson Equations

- The 2-point function satisfies arxiv:1808.09434

$$G(t_1-t_2)=G_0(t_1-t_2)+g\bar{g}N^3\int dt dt' G_0(t_1-t)G(t-t')^3G(t'-t_2)$$



- Scaling dimensions of $O_n=\bar{\psi}^{abc}\partial_t^n\psi^{abc}$



- For even and odd n, we find, respectively

$$g_S(h) = \frac{3}{2}\frac{\tan(\frac{\pi}{2}(h+\frac{1}{2}))}{h-1/2} \qquad g_A(h) = -\frac{3}{2}\frac{\tan(\frac{\pi}{2}(h-\frac{1}{2}))}{h-1/2}$$

- The n=0 scaling dimension is complex:

$$h \approx \frac{1}{2} + 1.5251i$$

- This signals an instability of the conformal phase of the large N bipartite model.
- The true phase of the theory appears to be gapped, with no ground state entropy.

Two-Flavor O(N)^3 Model

- Interaction of two rank-3 Majorana tensors with O(N)^3 symmetry, with a parameter α
- Jaewon Kim, Princeton Senior Thesis (2018), arXiv:1811.04330; Kim, IK, Tarnopolsky, paper in preparation.

$$H = \frac{g}{4}\left(\psi_1^{a_1b_1c_1}\psi_1^{a_1b_2c_2}\psi_1^{a_2b_1c_2}\psi_1^{a_2b_2c_1} + \psi_2^{a_1b_1c_1}\psi_2^{a_1b_2c_2}\psi_2^{a_2b_1c_2}\psi_2^{a_2b_2c_1}\right)$$

$$+\frac{g\alpha}{2}\left(\psi_1^{a_1b_1c_1}\psi_1^{a_1b_2c_2}\psi_2^{a_2b_1c_2}\psi_2^{a_2b_2c_1} + \psi_1^{a_1b_1c_1}\psi_2^{a_1b_2c_2}\psi_1^{a_2b_1c_2}\psi_2^{a_2b_2c_1} + \psi_1^{a_1b_1c_1}\psi_2^{a_1b_2c_2}\psi_2^{a_2b_1c_2}\psi_1^{a_2b_2c_1}\right)$$

- Reduces to the bipartite model for $\alpha = -1$
- Melonic S-D equations give 2-pt function

$$G(t_2 - t_1) = -\left(\frac{1}{4\pi(3\alpha^2+1)g^2N^3}\right)^{\frac{1}{4}} \frac{\mathrm{sgn}(t_2 - t_1)}{|t_2 - t_1|^{1/2}}$$

Bilinear Operators

- Even under Z_2 symmetry $\psi_2^{abc} \to -\psi_2^{abc}$

$$O_1^{2n+1} = \psi_1 \partial_t^{2n+1} \psi_1 + \psi_2 \partial_t^{2n+1} \psi_2 \qquad O_2^{2n+1} = \psi_1 \partial_t^{2n+1} \psi_1 - \psi_2 \partial_t^{2n+1} \psi_2$$

- Odd

$$O_3^{2n} = \psi_1 \partial_t^{2n} \psi_2 - \psi_2 \partial_t^{2n} \psi_1 \qquad O_4^{2n+1} = \psi_1 \partial_t^{2n+1} \psi_2 + \psi_2 \partial_t^{2n+1} \psi_1$$

- Scaling dimensions determined from

$$g_1(h) = -\frac{3}{2} \frac{\tan(\frac{\pi}{2}(h-1/2))}{h-1/2} \qquad g_2(h) = -\frac{3}{2} \frac{-\alpha^2+1}{3\alpha^2+1} \frac{\tan(\frac{\pi}{2}(h-1/2))}{h-1/2}$$

$$g_3(h) = \frac{3\alpha^2 - 3\alpha}{3\alpha^2+1} \frac{\tan(\pi h/2 + \pi/4)}{h-\frac{1}{2}} \qquad g_4(h) = -\frac{3\alpha^2 + 3\alpha}{3\alpha^2+1} \frac{\tan(\pi h/2 - \pi/4)}{h-\frac{1}{2}}$$

Duality

- Use transformation $\psi_1 = \frac{1}{\sqrt{2}}(\tilde{\psi}_1 + \tilde{\psi}_2),\ \psi_2 = \frac{1}{\sqrt{2}}(\tilde{\psi}_1 - \tilde{\psi}_2)$
- Find equivalence $(g,\ \alpha) \sim (g',\ \alpha')$

$$g' = \frac{(3\alpha+1)g}{2} \qquad \alpha' = \frac{-\alpha+1}{3\alpha+1}$$

- Apart from overall scaling of energies, can restrict to $-1 \leq \alpha \leq \frac{1}{3}$
- For $\alpha < 0$ operator $\psi_1^{abc}\psi_2^{abc}$ has complex dimension $\frac{1}{2} \pm i f(\alpha)$ where $f\tanh(\pi f/2) = \frac{3\alpha^2 - 3\alpha}{3\alpha^2+1}$
- For small α

$$f(\alpha) = \sqrt{\frac{-6\alpha}{\pi}}(1 + O(\alpha))$$

Coupled SYK Models

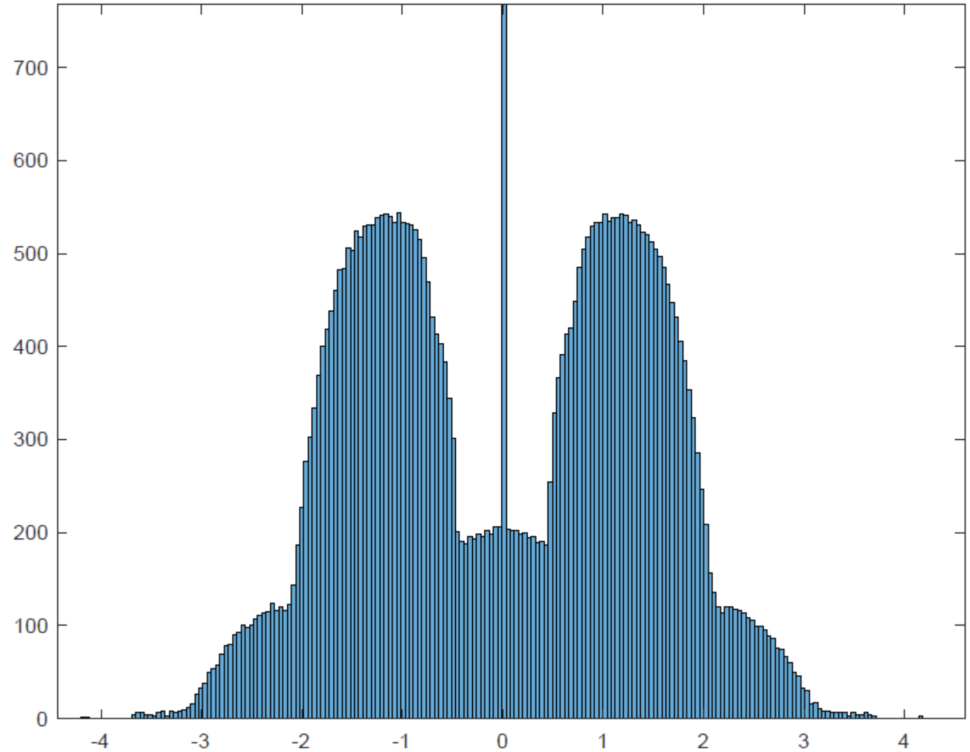
- To study low-energy properties numerically, replace the two-flavor tensor model by its SYK counterpart.
- Double SYK model with a quartic coupling

$$H = \frac{1}{4!} J_{ijkl} \left(\chi_1^i \chi_1^j \chi_1^k \chi_1^l + \chi_2^i \chi_2^j \chi_2^k \chi_2^l + 6\alpha \chi_1^i \chi_1^j \chi_2^k \chi_2^l \right)$$

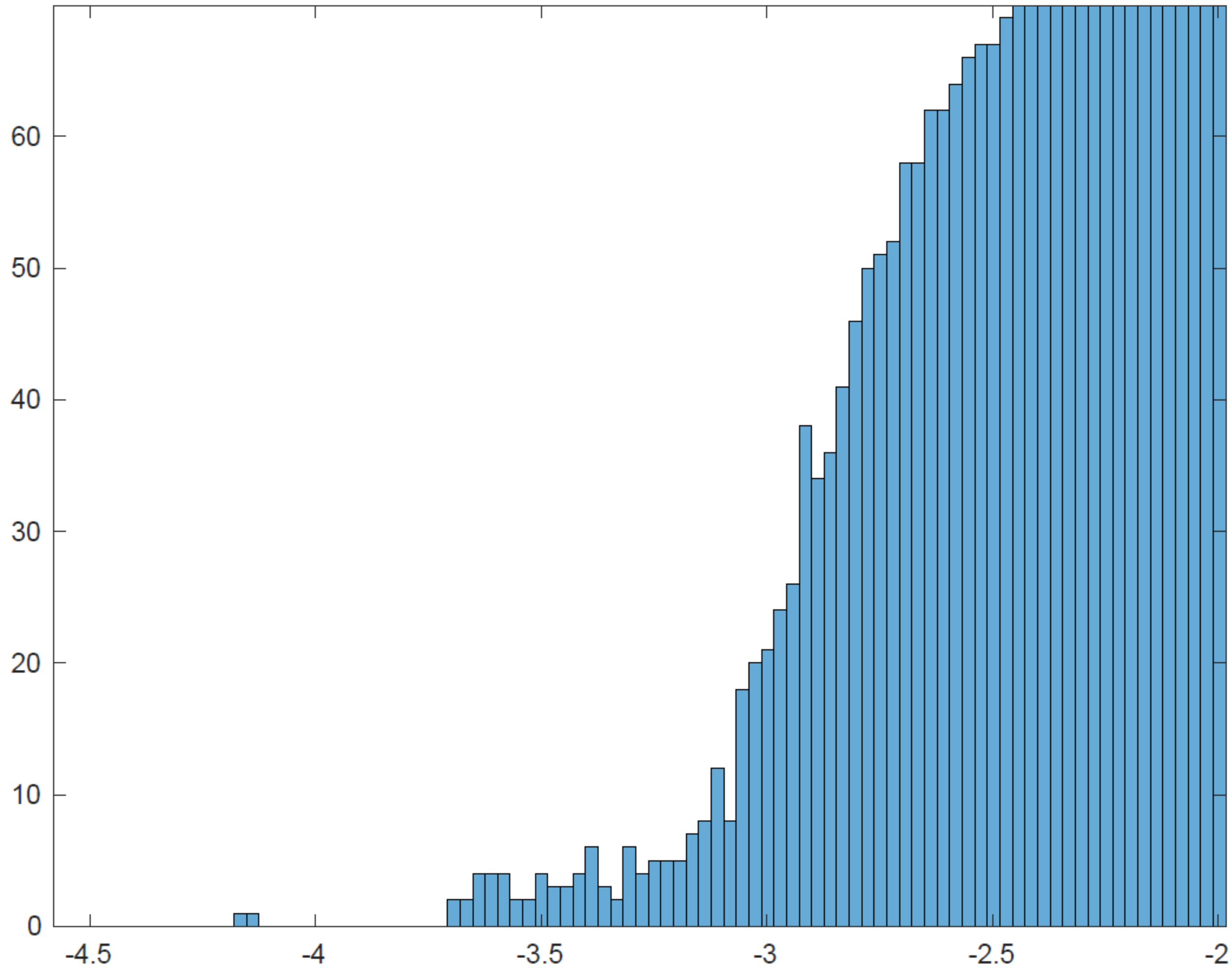
- A generalization of the Gross-Rosenhaus two-flavor model.
- Gives the same large N S-D equations and scaling dimensions as the tensor model.

Gapped Spectrum

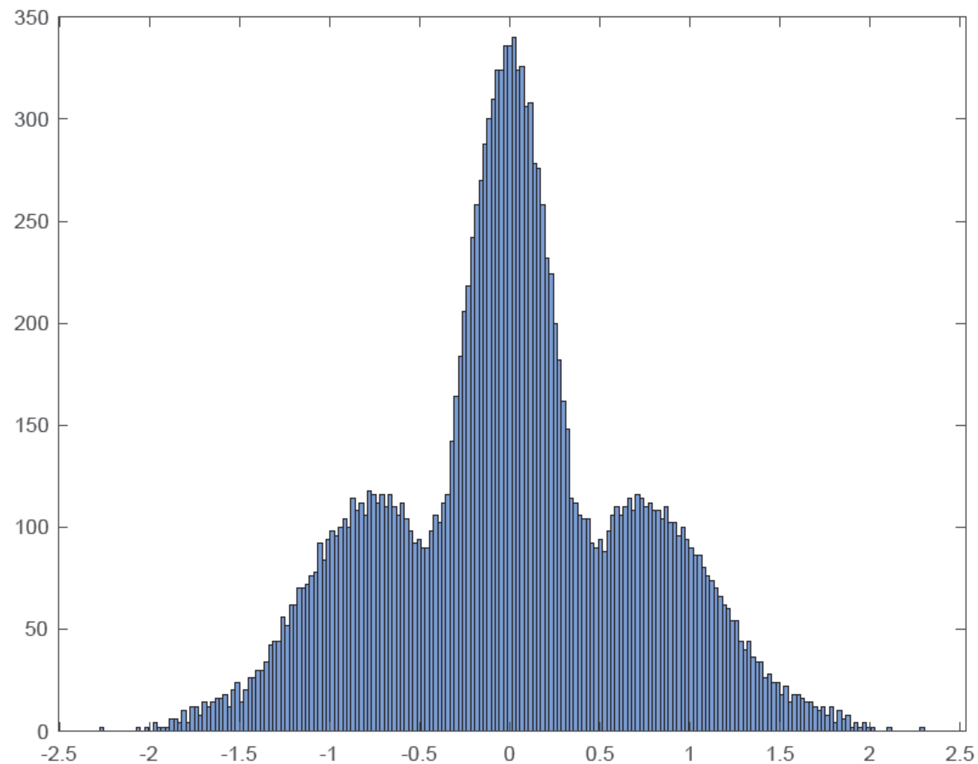
- For a single realization of random couplings and N_{SYK}=16 observe for $\alpha = -1$



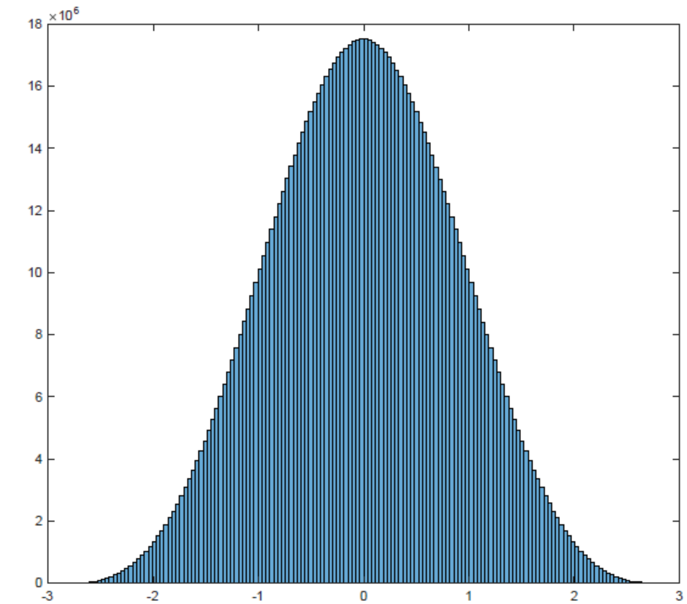
- Zoom in to show that a gap is present near the ground state:



- Spectrum for α=-0.5



- And for α=0



Symmetry Breaking

- These results suggest that, in the large N limit, there is spontaneous breaking of the Z_2 symmetry $\psi_2^{abc} \to -\psi_2^{abc}$ via formation of expectation value of operator $\psi_1^{abc}\psi_2^{abc}$
- This leads to spontaneous mass generation and should connect to the work of Maldacena and Qi where the Z_2 symmetry was broken explicitly.

Dual of a Wormhole?

- Tempting to interpret the gapped phase with small low-T entropy as the dual of a wormhole geometry. Maldacena, Qi
- It appears only for one sign of the coupling:

$$\alpha < 0$$

- Similar to the Gao-Jafferis-Wall model.

Conclusions

- The $O(N)^3$ fermionic tensor quantum mechanics seems to be the closest counterpart of the basic SYK model for Majorana fermions.
- Solution of S-D equations indicates a (nearly) conformal phase with real scaling dimenions.
- Bosonic or fermionic generalizations can lead to complex scaling dimensions with real part d/2, indicating an instability of the conformal phase.

- Studied quantum mechanics of two rank-3 Majorana tensors with $O(N)^3$ symmetry and quartic terms coupling the two, and its SYK counterpart.
- A complex scaling dimension appears only for one sign of the coupling, where numerical calculation also indicates a gapped spectrum.
- Relation to the Gao-Jafferis-Wall wormhole construction?
- Relation to Juan Maldacena’s talk?
- Relation to Cenke Xu’s talk?
- Physical applications?