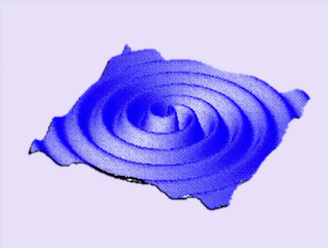

Exploring the Possibility of Floating Orbits for Extreme Mass Ratio Binary Black Holes

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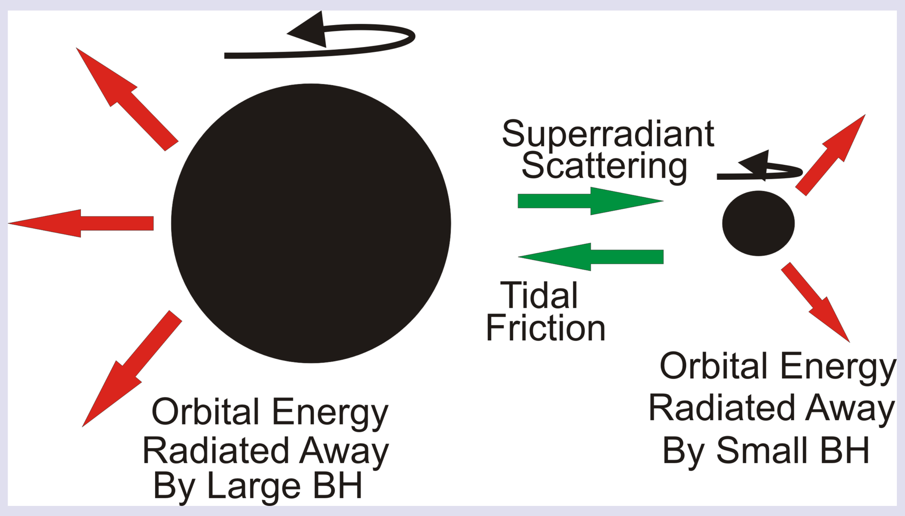
Motivation

- Constant frequency gravitational waves (GWs) emitted by floating (non-decaying) orbits would appear as delta function-like peaks in frequency space, standing out from the background noise.
- Floating orbits, **if they exist**, would be excellent sources of detectable GWs for LISA in the case of extreme mass ratio (EMR) binary black holes (BHs), and LIGO in the case of equal mass binary BHs.

Outline



- EMR binary BHs lose orbital energy via GWs.
- **Flux gained from BH spins = Flux lost via GWs ⇒ FLOATING/NON-DECAYING ORBIT.**
- Using the Teukolsky formalism, we compute, numerically, the flux ($|L_S|$) gained from the spin of the larger BH via *superradiant scattering.*
- Using the results of Poisson et al, we compute, analytically, the flux ($\dot{E}_{orbital}$) gained from the spin of the smaller BH via *tidal friction.*
- We compare the sum of the orbital fluxes gained with the flux (L_∞) lost via gravitational radiation reaction.

Superradiant Scattering

- Superradiant scattering is similar to the *Penrose Process*; GWs emitted by the smaller BH possess negative energy when moving through the ergoregion of the larger BH.
- The larger BH thus loses spin energy to orbital energy.
- Numerical results for circular equatorial orbits and BH masses $M = 10^9 M_\odot$ and $m = 10^3 M_\odot$:

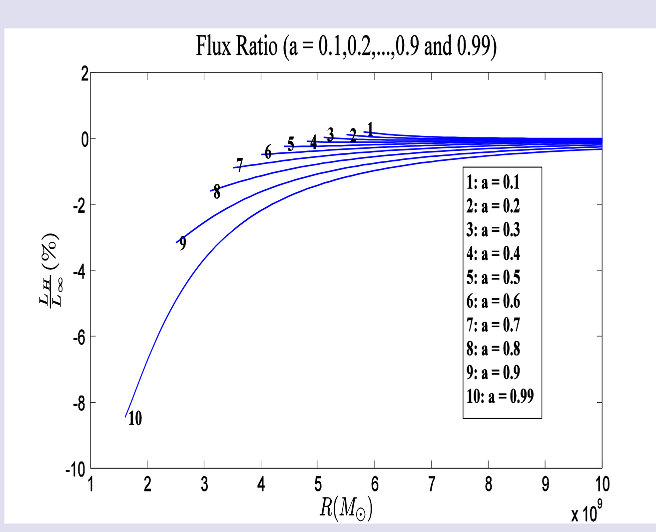


Figure 1: Ratio of the energy flux from the smaller non-spinning BH towards the larger spinning BH (L_H), and infinity (L_∞). The radii R span from the ISCO radius to $R = 10M$. $a = J/M^2$ (J = ang mom) is varied from 0.1 to 0.9 in increments of 0.1, and $a = 0.99$ is also included. $L_H < 0$ signifies superradiant scattering .

- Superradiant scattering increases with increasing spin parameter a and decreasing orbital radius R.
- Maximum superradiant flux ratio for each spin parameter value a corresponding to Figure: 1:

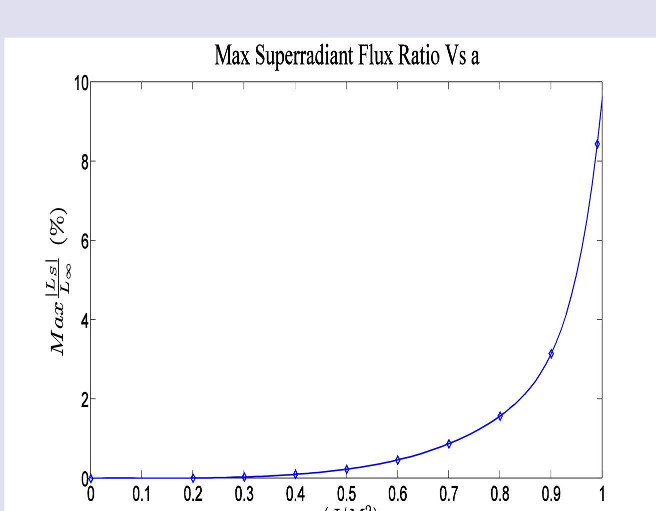


Figure 2: We define the superradiant flux L_S as equal to L_H when $L_H < 0$, and zero otherwise. For each a, we determine the maximum superradiant flux ratio $Max\frac{|L_S|}{L_\infty}$ and plot this as a function of a. We extrapolate the curve to $a = 1$ using a 12th degree polynomial fit.

- $|L_S| < 10\% L_\infty \Rightarrow$ orbital flux gained from spin of larger BH alone cannot sustain a circular equatorial floating orbit.

Tidal Friction

- Superradiant scattering is functionally equivalent to tidal friction, as proposed in [1].

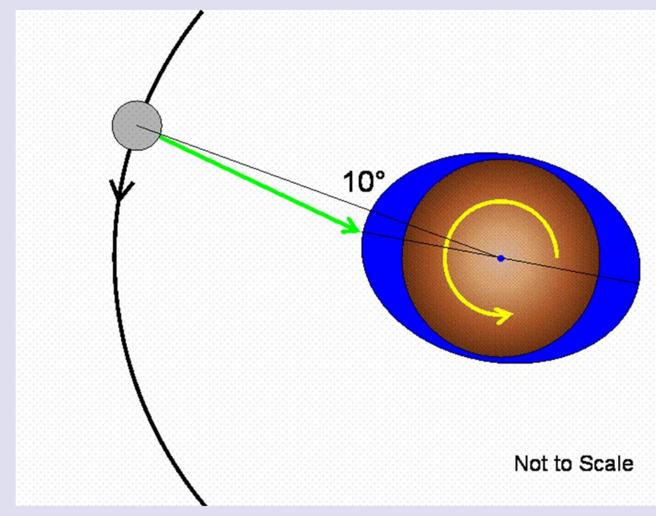


Figure 3: Tidal interaction between the earth and the moon is familiar from Newtonian mechanics. Analogously, in the case of BHs, the interaction between the orbiting BH and the distorted event horizon of the second spinning BH causes a transfer of energy from spin to orbit. (Picture from: astronomy.ohio-state.edu)

- We use tidal friction to calculate orbital flux gained from the spin of the smaller BH.
- From [2], the orbital flux gained from the smaller BH due to tidal friction for circular equatorial orbits is:

$$\dot{E}_{orbital} = \frac{4}{5}\frac{m^5}{M^5}(1+3b^2)V^{12}\Gamma_K \frac{b^2}{1+\sqrt{1-b^2}} \tag{1}$$

where $V = \sqrt{\frac{M}{R}} < V_{ISCO}(a)$, b = spin parameter of smaller BH, and:

$$\Gamma_K(a,b,V) = \frac{1-2V^2+a^2V^4}{(1-3V^2+2aV^3)^2}$$
$$\left(1 - \frac{4+27b^2}{4+12b^2}V^2 - \frac{4-3b^2}{2+6b^2}aV^3 + \frac{8+9b^2}{4+12b^2}a^2V^4\right) \tag{2}$$

- The quadrupole formula is a Newtonian order approximation to L_∞:

$$L_Q = \frac{32}{5}\left(\frac{m}{M}\right)^2 V^{10} \tag{3}$$

- $\dot{E}_{orbital} \sim (m/M)^5 = 10^{-30}$ and $L_Q \sim (m/M)^2 = 10^{-12}$, except for orbital radii $R = R_F$ when Γ_K blows up:

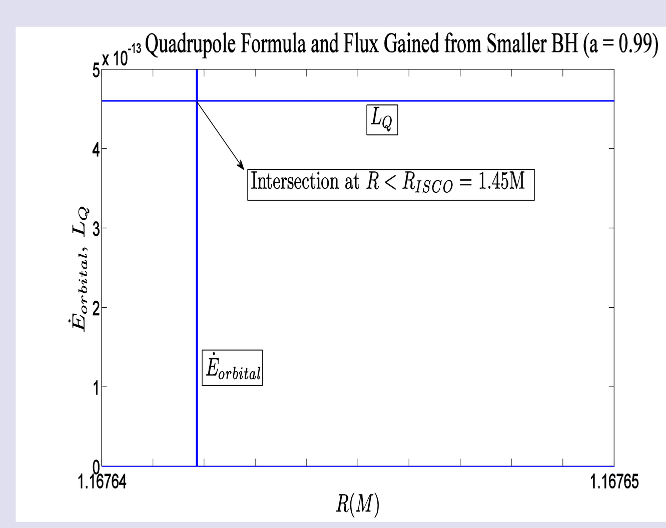


Figure 4: Fluxes $\dot{E}_{orbital}$ and L_Q as a function of R for $a = 0.99$. $\dot{E}_{orbital}$ blows up to equal L_Q, but only within the ISCO where orbits are unstable.

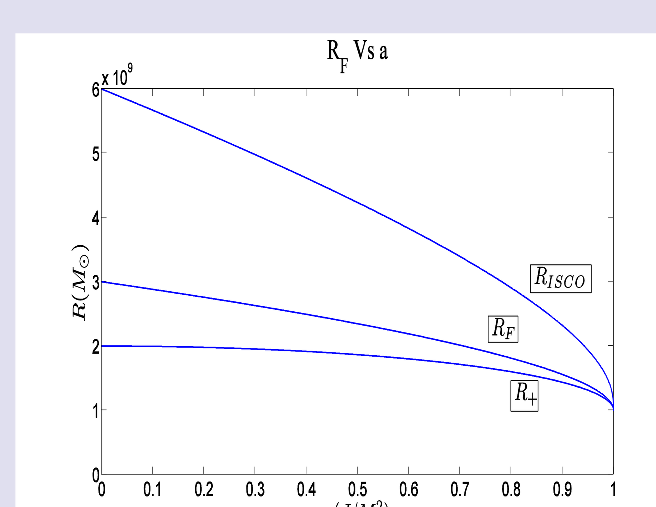


Figure 5: Orbital radii R_F at which $\dot{E}_{orbital}$ blows up, as a funtion of the spin parameter a of the larger spinning BH. R_F always lies between the event horizon radius R_+ and the ISCO radius R_{ISCO}.

- $\dot{E}_{orbital} << L_Q \sim L_\infty \Rightarrow$ orbital flux gained from spin of smaller BH alone cannot sustain a circular equatorial floating orbit.

Total Orbital Flux Gained

- The total orbital flux gained is $L_T = |L_S| + \dot{E}_{orbital}$.
- $|L_S| \sim 0.1 L_\infty$ and $\dot{E}_{orbital} \sim 10^{-18} L_\infty \Rightarrow \dot{E}_{orbital}/|L_S| \sim 10^{-17}$ $\Rightarrow L_T \simeq |L_S| < 10\% L_\infty$.
- The total flux gained remains insufficient to sustain a circular equatorial floating orbit.

Eccentric and Inclined Orbits

- Eccentricity provides a pool of energy other than the spin of the BHs.
- Our code can deal with eccentricities up to $e = 0.6$:

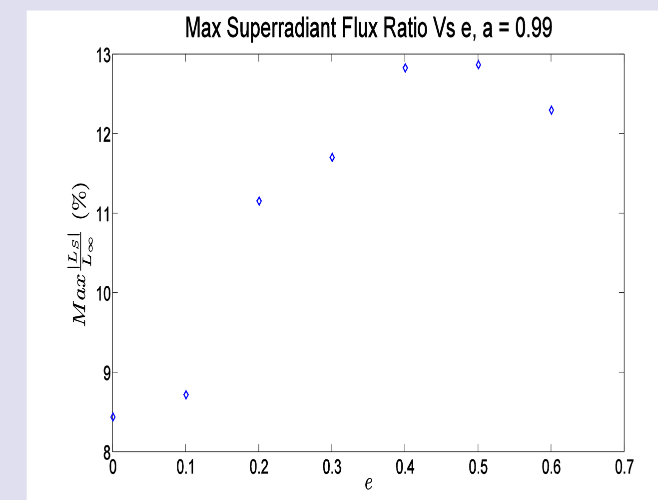


Figure 6: For each e up to $e = 0.6$, we determine the maximum superradiant flux ratio $Max\frac{|L_S|}{L_\infty}$ and plot this as a function of e, for $a = 0.99$. Adding moderate eccentricities does not increase $|L_S|$ significantly enough to equal L_∞.

- Table below suggests that high eccentricities will likely not provide the necessary boost to $|L_S|$ needed to equal L_∞:

e	E
0	0.73597
0.1	0.74678
0.3	0.79123
0.6	0.87330
0.99	0.99657

Table 1: e = eccentricity, E = specific energy of the smaller BH and $a = 0.99$. There is only a 35% increase in specific energy when going from a circular ($e = 0$) to a highly eccentric ($e = 0.99$) orbit. If the total amount of extra energy available in a very eccentric orbit is small, it seems hardly likely that the orbital flux gained will increase significantly.

- Orbital inclination may be a source of orbital energy, but Hughes [3] has shown that the orbital flux gained from this is small.
- Neither eccentricity nor inclination seem to help increase the orbital flux gained significantly enough to equal the flux lost.

Conclusions

- EMR binary BHs will, in all likelihood, be unable to float.
- Results for the energy interchange from the spins of the BHs to the orbit of the binary is large enough to play a significant role in the evolution of EMR inspirals and is therefore critical to data analysis in GW detection for LISA.
- It is clear that the larger BH is the dominant contributor to the orbital flux. This suggests that, in the case of equal large mass binary BHs, (of interest to LIGO), the nature of the inspiral will be crucially determined by the orbital energy gained from the spins of the BHs, perhaps much more so than for EMR binaries.

Other Scenarios

- For what value of a will the superradiant flux gained equal the flux lost?

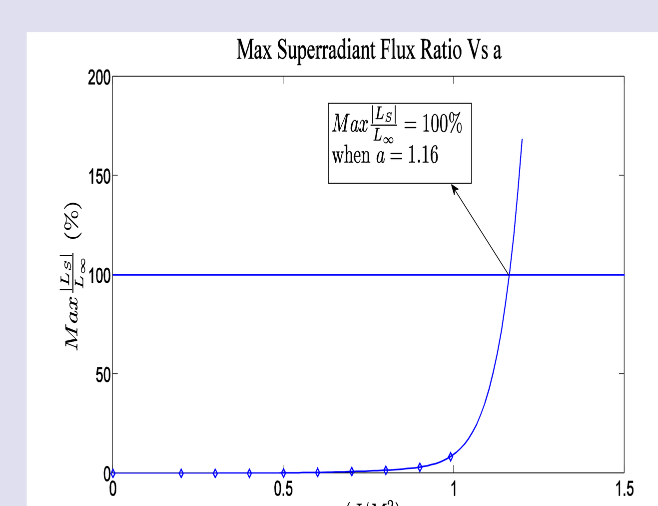


Figure 7: Extrapolating Figure: 2 to a flux ratio of 100%.

- Quark stars may be able to attain $a = 1.16$ [4], perhaps an indication of the possibility of floating orbits around them.

References

[1] K.Thorne; R.Price; D.Macdonald, "Black Holes: The Membrane Paradigm" (Yale University Press, New Haven, CT, 1986)

[2] S. Comeau, E. Poisson, Phys. Rev. D 80, 087501 (2009)

[3] S.Hughes, Phys. Rev. D 64, 064004 (2001)

[4] K.Lo, L.Lin, The Astrophysical Journal 728, 12 (2011)